**IE208 Applied Statistics**

**Fall 2021**

**Midterm Exam (4:30 – 5:45 pm)**

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**ALLOWED:**

* Lecture materials provided by the instructor.
* Other lecture materials prepared by you.

**NOT ALLOWED:**

* All other materials, including any online resources.
* Sharing of materials.
* Any kind of communications: no messaging or emails.

**NOTE:**

* Provide your answers on this Word file for 5 questions and submit it to the e-campus.
* Consider just three digits after the decimal point: only up to three decimal places (ex: 0.123).
* Use the chatting box in the private mode for questions – no verbal questions.
* You cannot leave the Zoom meeting by 5:45 pm.

1. **(20 pts)** A dairy manufacturer claims that at the time of purchase by a consumer the average age of its product is no more than 122 days. In an experiment to test this claim, a random sample of 36 items are found to have ages at the time of purchase with a mean of = 123.5 days and a sample standard deviation of = 12.4 days. Conduct a statistical test to see if the manufacturer’s claim is plausible. Assume that : and : with 05
2. Show the T-statistics.
3. Show the P-Value.
4. Provide the test conclusion.
5. What is the smallest number of samples that changes the test conclusion in c) above? Assume all other information is the same.

* x\_bar=123.5
* s = 12.4
* mu\_0 = 122
* alpha = 0.05
* sample size(n) = 36

1. t-statistics = (x\_bar – mu\_0)\*sqrt(n)/s = 0.7258
2. p-value = P(X > t) = 0.2364
3. p-value > alpha(0.05), so null hypothesis should be accepted
4. n >= 4\*(t\_0.05\*s/
5. **(20 pts)** An experimenter conducted experiments, and a paired experimental design is adopted whereby 14 pairs of the samples are split in half, with one half being examined with the first measuring device () and the other half being examined with the second measuring device (). The differences = – have a sample mean = 7.5 and a sample standard deviation = 7.34. Assume = 0.05.
6. Show the confidence interval to see if and has the same mean.
7. Conduct a hypothesis test to see if and has the same mean. Make sure to include two hypotheses, the p-value, and the test conclusion to your answer.

* Z\_bar = 7.5
* Sample size(n) = 14
* Sample st.dev(s) = 7.34
* Alpha = 0.05
* Mu\_X – mu\_Y = 0

1. Confidence interval  
   alpha = 0.05  
   t\_0.025,13 = 2.1604  
   Lower bound = z\_bar – t\_0.025,14\*s/sqrt(n) = 3.2619  
   Upper bound = z\_bar + t\_0.025,14\*s/sqrt(n) = 11.7380  
   confidence interval : (3.2619, 11.7380)
2. Hypothesis test  
   Null hypothesis : mu\_X = mu\_Y  
   Alternative hypothesis : mu\_X != mu\_Y  
     
   t-statistics = (z\_bar – 0)\*sqrt(n)/s = 3.8232  
   p-value = 2\*P(X > |t|) = 0.0021  
   p-value < alpha(0.05), so reject null hypothesis
3. **(20 pts)** In an unpaired two-sample problem, an experimenter observes = 10, = 7.76, = 1.07 from population A and = 9, = 6.88, = 0.62 from population B.n
4. Construct a 99% one-sided confidence interval − (c,) without assuming equal population variances.
5. Does the value of c increase or decrease if a confidence level 95% is used?
6. Conduct a one-sided hypothesis test with : versus : without assuming equal population variances. Assume = 0.01. Provide the p-value, the test statistics, and the test conclusion.

* alpha = 0.01
* sample error(s.e) = sqrt(s\_x^2/n+s\_y^2/m) = 0.3965
* nju = s.e^4/(s\_x^4/n^2/(n-1)+s\_y^4/m^2/(m-1)) = round down(7.0431) = 7
* mu\_hat = x\_bar-y\_bar = 0.88

1. 99% one-sided confidence interval  
   t\_0.01,7 = -2.998 (2.998)  
   Lower bound = mu\_hat – t\_0.01,7\*s.e = -0.3086  
   confidence interval : (-0.3086, ∞ )
2. T-statistics = mu\_hat/s.e = 2.2195  
   p-value = P(X > t) = 0.031  
   p-value > 0.01, so null hypothesis should be accepted
3. **(20 pts)** A group of 250 patients was randomly split into two groups of 125 patients. The first group of 125 patients was given treatment A, and 62 of them improved their condition. The second group of 125 patients was given treatment B, and 46 of them improved their condition. Perform a hypothesis test to investigate whether there is evidence of a difference between the two treatments with the level of significance of 0.05.

* First group(A)  
  n = 125  
  x = 62  
  p\_a\_hat = x/n = 0.496
* Second group(B)  
  m = 125  
  y = 60  
  p\_b\_hat = y/m = 0.48

**Pooled**

**Null hypothesis: p\_a = p\_b**

**Alternative hypothesis :** **p\_a != p\_b**

* Alpha = 0.05
* P\_hat = (x+y)/(n+m) = 0.488
* Z-statistics = (p\_a\_hat – p\_b\_hat)/sqrt(p\_hat\*(1-p\_hat)\*(1/n+1/m)) = 0.253055103
* P-value = 2\*P(X > |z|) = 0.8002
* P-value > 0.05, null hypothesis should be accepted

1. **(20 pts)** The weights (kg) of a random sample of 16 products from a process were measured, and a sample mean = 6.861 and a sample standard deviation = 0.440 were obtained. The engineers presented a confidence interval (6.668, 7.054) for the average weights of products from the process.

a) What is the confidence level of the confidence interval? Assume that is one of the well-known values you learned in classes.

b) Conduct a hypothesis test to see if the population mean is 6.8. Make sure to provide and as well as the p-value, the test statistics, and the test conclusion. Use you found from a).

* Sample size(n) = 16
* Sample mean(x\_bar) = 6.861
* Sample st.dev(s) = 0.440
* Confidence interval: (6.668, 7.054)

1. L = 7.054 – 6.668 = 0.386  
    = 2\*t\_\*s/sqrt(n)  
   t\_ = L\*sqrt(n)/2/s = 1.7545  
   alpha = 0.04987
2. Mu\_0 = 6.8

t-statistics = (x\_bar-mu\_0)\*sqrt(n)/s = 0.5545  
p-value =2\* P(X >t) = 0.587379464  
so, accept null hypothesis