

# CSE 3318

Week of 07/31/2023

Instructor : Donna French

```
1 Pichu|1.00|4.4|B|Tiny Mouse|Static|1|172
2 Pikachu|1.04|13.2|B|Mouse|Static|2|025
3 Raichu|2.07|66.1|B|Mouse|Static|3|026
4 Charmander|2.00|18.7|B|Lizard|Blaze|1|004
5 Charmeleon|3.7|41.9|B|Flame|Blaze|2|005
6 Charizard|5.7|199.5|B|Flame|Blaze|3|006
7 Squirtle|1.08|19.8|B|Tiny Turtle|Torrent|1|007
8 Wartortle|3.03|49.6|B|Turtle|Torrent|2|008
9 Blastoise|5.03|188.5|B|Shellfish|Torrent|3|009
10 Iggybuff|1.00|2.2|B|Balloon|Cute Charm, Competitive|1|174
11 Jigglypuff|1.08|12.1|B|Balloon|Cute Charm, Competitive|2|039
12 Wigglytuff|3.03|26.5|B|Balloon|Cute Charm, Competitive|3|040
13 Ponyta|3.03|66.1|B|Fire Horse|Run Away, Flash Fire|1|077
14 Rapidash|5.07|209.4|B|Fire Horse|Run Away, Flash Fire|2|078
15 Psyduck|2.07|43.2|B|Duck|Damp, Cloud Nine|1|054
16 Golduck|5.07|168.9|B|Duck|Damp, Cloud Nine|2|055
17 Vaporeon|3.03|63.9|B|Bubble Jet|Water Absorb|2|134
18 Eevee|1.00|14.3|B|Evolution|Run Away, Adaptability|1|133
19 Jolteon|2.07|54.0|B|Lightning|Volt Absorb|3|135
20 Meowth|1.04|9.2|B|Scratch Cat|Pickup, Technician|1|052
21 Persian|3.03|70.5|B|Classy Cat|Technician, Limber|2|053
22 Bulbasaur|2.04|15.2|B|Seed|Overgrow|1|001
23 Ivysaur|3.03|28.7|B|Seed|Overgrow|2|002
24 Venusaur|6.07|220.5|B|Seed|Overgrow|3|003
25 Smoochum|1.04|13.2|F|Kiss|Oblivious, Forewarn|1|238
26 Jynx|4.07|89.5|F|Human Shape|Oblivious, Forewarn|2|124
27 Growlithe|2.04|41.9|B|Puppy|Intimidate, Flash Fire|1|058
28 Arcanine|6.03|341.7|B|Legendary|Intimidate, Flash Fire|2|059
29 Finneon|1.04|15.4|B|Wing Fish|Swift Swim, Storm Drain|1|456
30 Lumineon|3.11|52.9|B|Neon|Swift Swim, Storm Drain|2|457
31 Ditto|1.00|8.8|U|Transform|Limber|0|132
32 Nidoran|1.04|15.4|F|Poison Pin|Poison Point, Rivalry|1|029
33 Nidorina|2.07|11.1|F|Poison|Poison Point, Rivalry|2|030
```

```
11 typedef struct Pokemon
12 {
13     char *name;
14     float height;
15     float weight;
16     char gender;
17     char *category;
18     char *abilities;
19     int evolution;
20     int national_pokedex_number;
21     struct Pokemon *next_ptr;
22 }
23 POKEMON;
```

# Coding Assignment 6

CSE 3318

Coding Assignment 6 is over creating a hash table and using it to do the 3 basic dictionary actions - insert, delete and search.

Your hash table will be created using an integer array with separate chaining to resolve conflicts.

## Step 1

For this assignment, you need to choose a subject for your dictionary. For my version of the assignment, I chose Pokémon – you may not also use Pokémon – you need to pick your own subject. Pick a subject that allows you to gather enough information to fulfill the minimum requirements of the assignment.

## Structure

You will need to create a `struct` to hold each entry of the dictionary. My Pokémon example is

```
typedef struct Pokemon
{
    char *name;
    float height;
    float weight;
    char gender;
    char *category;
    char *abilities;
    int evolution;
    int national_pokedex_number;
    struct Pokemon *next_ptr;
}
POKEMON;
```

In this `struct`, the `char` pointers are used instead of `char` arrays since it is not known how much space those entries will need. Memory to hold those fields will be dynamically allocated using `malloc()` and the pointer returned by `malloc()` will be stored in the `struct`.

For your assignment, you will need a `struct` with a minimum of 5 fields – the parts shown in **bold** are required.

```
typedef struct YourStructName
```

```
{
```

```
    char *chararrayfield;
```

```
    float floatfield;
```

```
    char charfield;
```

```
    int intfield;
```

```
    struct YourStructName *next_ptr;
```

```
}
```

```
YOURSTRUCTNAME;
```

Your `struct` must be `typedefed` and you will be expect to use the `typedef` throughout the program. You may add more fields if you want your dictionary to hold more information. Your member names must adequately describe the data being stored in the variable. Put your `typedef` in a text file and name it `Code6_xxxxxxxxxx_struct.txt` where `xxxxxxxxxx` is your student id.

## Input File

Your input file must contain a minimum of 30 pipe delimited records that represent your dictionary entries. Here's the first 10 lines of my Pokédex file.

```
Pichu|1.00|4.4|B|Tiny Mouse|Static|1|172
Pikachu|1.04|13.2|B|Mouse|Static|2|025
Raichu|2.07|66.1|B|Mouse|Static|3|026
Charmander|2.00|18.7|B|Lizard|Blaze|1|004
Charmeleon|3.7|41.9|B|Flame|Blaze|2|005
Charizard|5.7|199.5|B|Flame|Blaze|3|006
Squirtle|1.08|19.8|B|Tiny Turtle|Torrent|1|007
Wartortle|3.03|49.6|B|Turtle|Torrent|2|008
Blastoise|5.03|188.5|B|Shellfish|Torrent|3|009
Igglybuff|1.00|2.2|B|Balloon|Cute Charm, Competitive|1|174
```

The fields in each entry must be pipe delimited and must be listed in the same order as your struct. Save this as a text file named `Code6_xxxxxxxxxx_InputFile.txt` where `xxxxxxxxxx` is your student id.

## Hashing Function

Pick one of the methods discussed in class for creating your hash function (numeric keys, alphanumeric keys, folding). Pick which field(s) from your struct will be used for the hash function. Remember that whatever method you choose should result in a value that you can MOD with the hash table define to get an index that will always be within your array.

Write a small C program that prompts for input and calls your hash function and prints the result of the hash. You are proving that your hash function will work. Here's an example – your version may be slightly different depending on your input. THIS CODE IS ONLY FOR SHOWING THAT YOUR HASH FUNCTION WORKS. It is ONLY an example – modify it as needed to show that your hash function will work.

```
#include <stdio.h>
#define HASHTABLESIZE 10
int MyHashFunction(pass in value being used in hash)
{
    perform hash method
    return value % HASHTABLESIZE;
}

int main(void)
{
    char HashValue[20];
    printf("Enter value ");
    scanf("%s", HashValue);
    printf("The hash value for %s is %d\n", HashValue, MyHashFunction(HashValue));
    return 0;
}
```

Compile your version and run and confirm that your function works. Save this code as  
Code6\_xxxxxxxxxx\_HashFunction.c where xxxxxxxxx is your student id.

## **Step 2**

Submit your input file, struct and hashing function for approval. Submit these to me by email. If you want to zip the files, you can, but you can just send them as attachments. No pictures or screenshots will be accepted.

Three files must be submitted via email

Code6\_xxxxxxxxxx\_struct.txt

Code6\_xxxxxxxxxx\_InputFile.txt

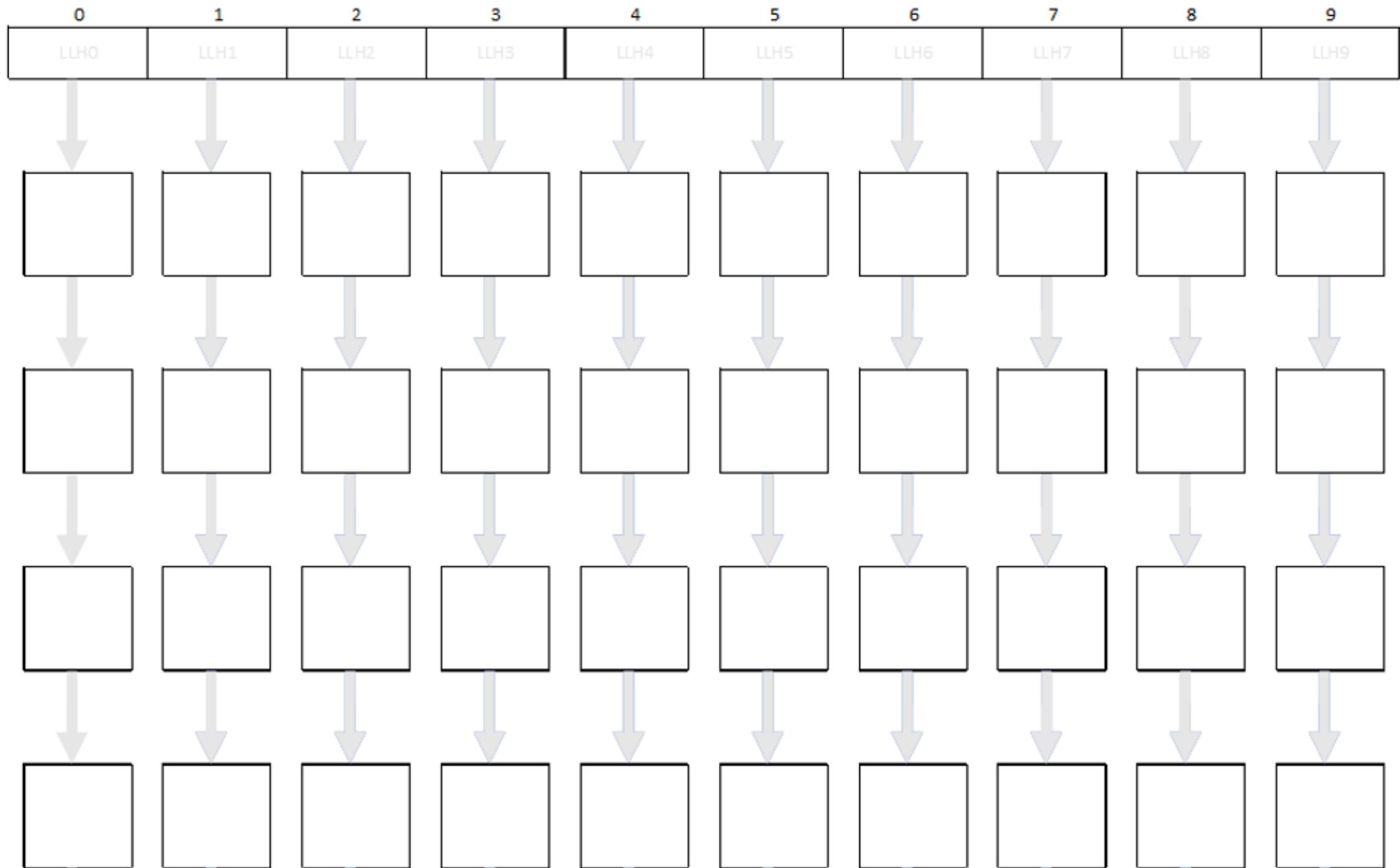
Code6\_xxxxxxxxxx\_HashFunction.c

Once you get approval, you will receive the rest of the instructions for the assignment. If you do not get approval, any code you turn in for Coding Assignment 6 will be assigned a grade of 0 regardless.

Part A – Complete the hash version using the same values/Part A data

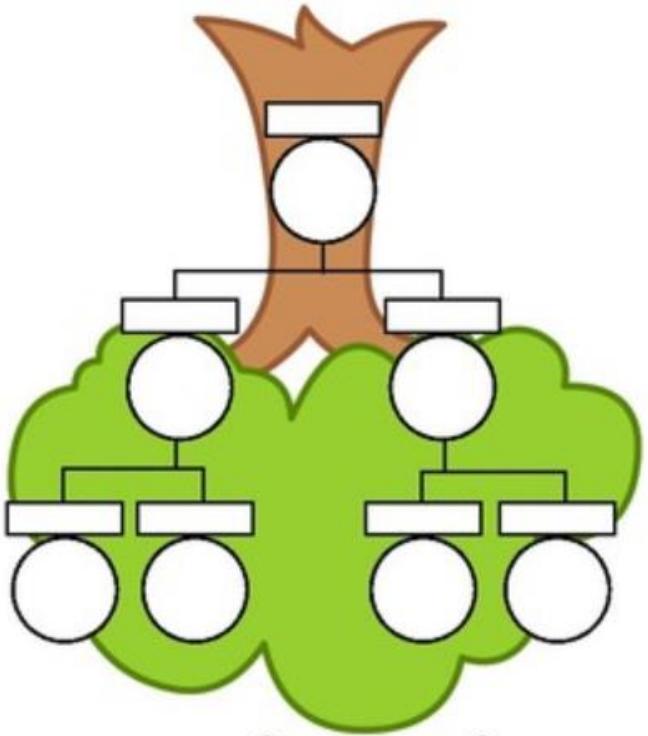
Part B - Create the hash table using the values from the Part B data. Use linear probing for resolving conflicts.

|   |   |   |   |   |   |   |   |   |   |
|---|---|---|---|---|---|---|---|---|---|
| 0 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 |
|---|---|---|---|---|---|---|---|---|---|

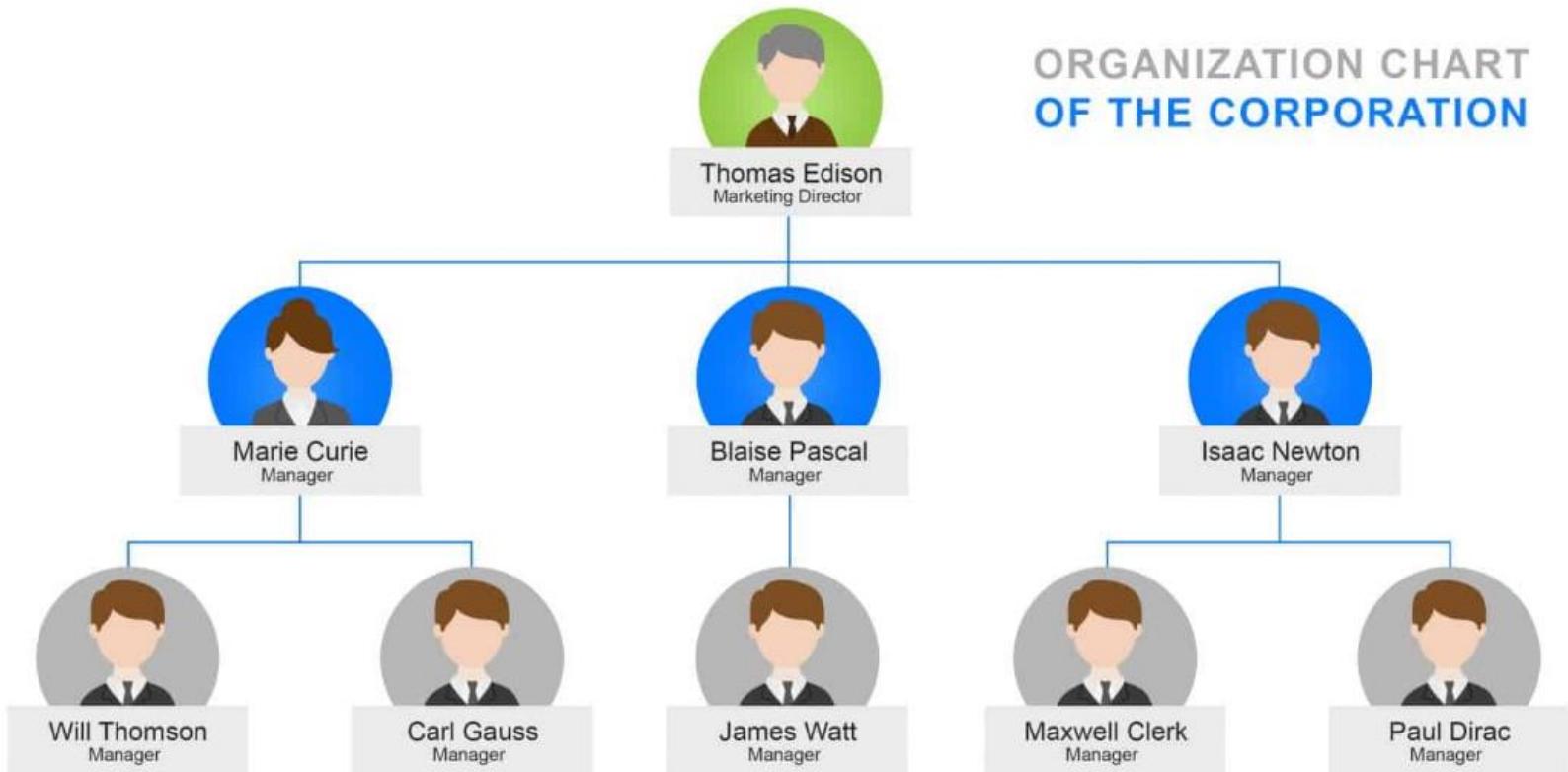


# Tree Data Structure

- Linked lists, stacks, and queues are all **linear** structures
  - one node follows another
  - each node contains a pointer to the next node
- Trees are **non-linear** structures
  - more than one node can follow another
  - each node contains pointers to an arbitrary number of nodes
  - the number of nodes that follow can vary from one node to another.
- Trees organize data hierarchically instead of linearly.



My Family Tree



# Binary Tree

What is a binary tree?

A binary tree is a non-linear tree-like data structure consisting of nodes where each node has up to two child nodes, creating the branches of the tree.

The two children are usually called the left and right nodes.

Parent nodes are nodes with children. Parent nodes can be child nodes themselves.

Binary trees are used to implement binary search trees and binary heaps. They are also often used for sorting data as in a heap sort.



# Introduction to Tree Data Structure

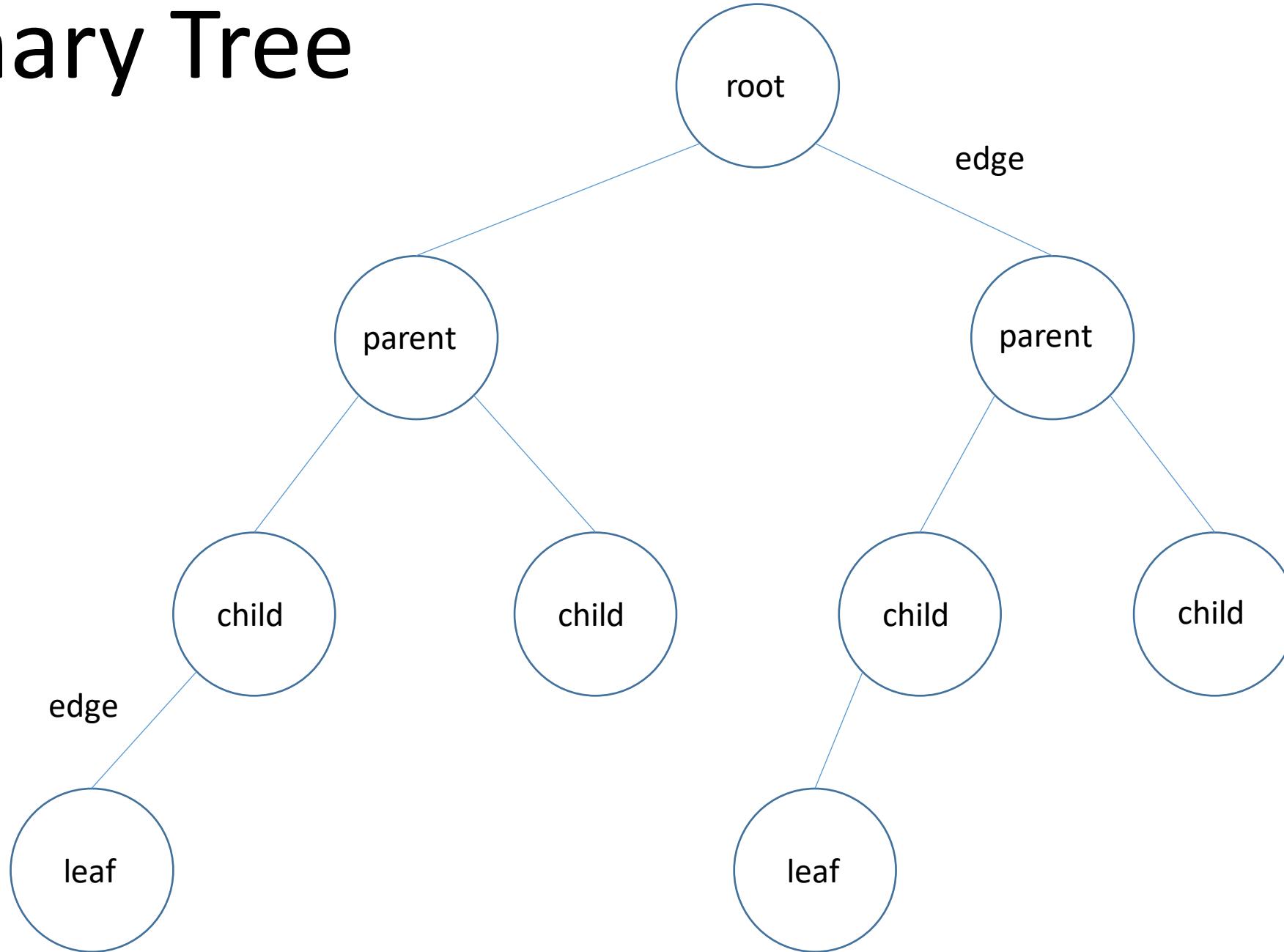
Codearchery



# Binary Tree and Binary Search Tree in Data Structure

Codearchery

# Binary Tree



# Binary Tree

## Tree Vocabulary

topmost node

node directly under another node

node directly above another node

node with no children

link between two nodes

length of the path from the root

length of the path from the node to  
the deepest leaf reachable from it

root of the tree

child

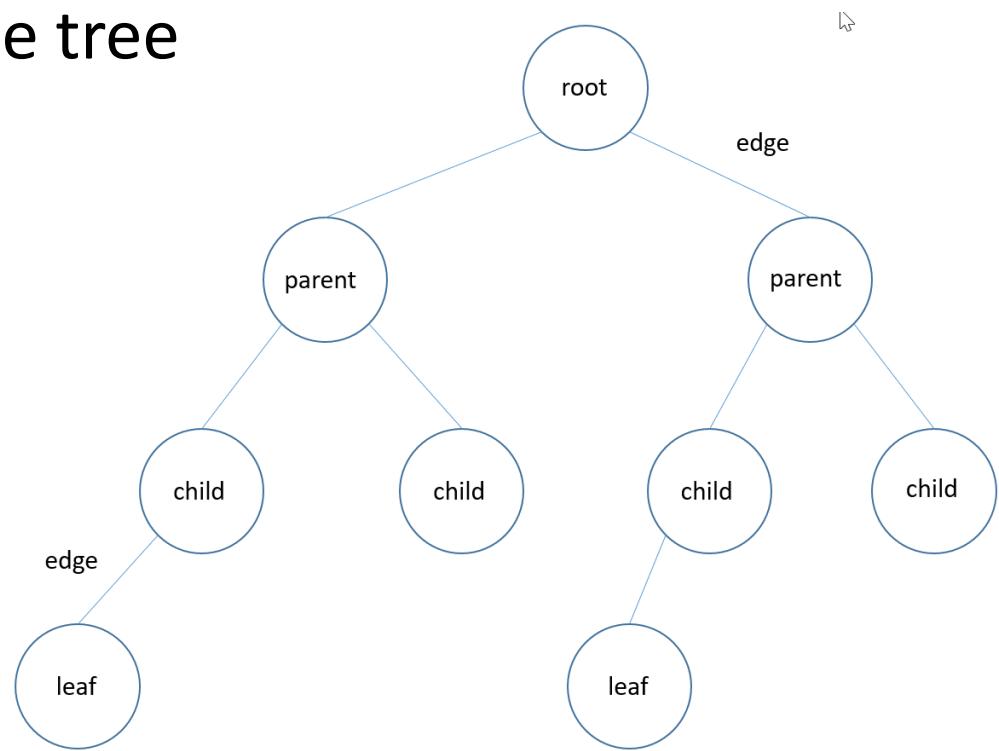
parent

leaf

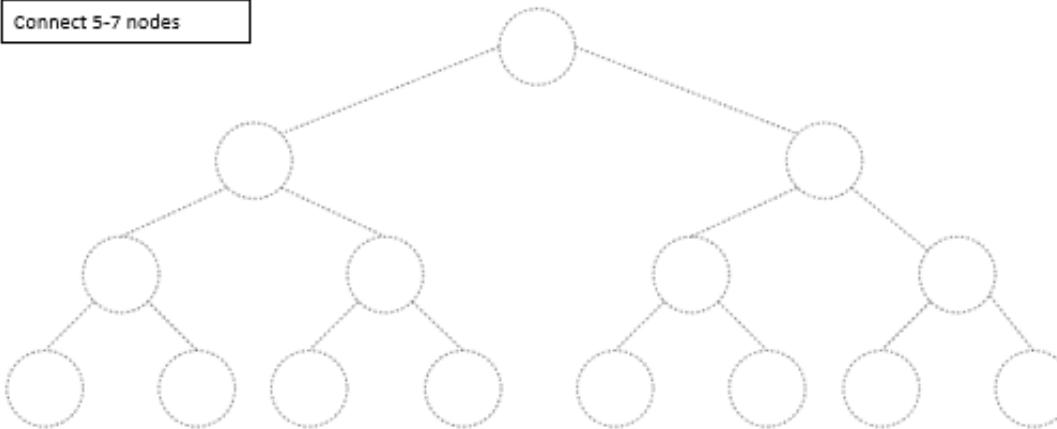
edge

depth

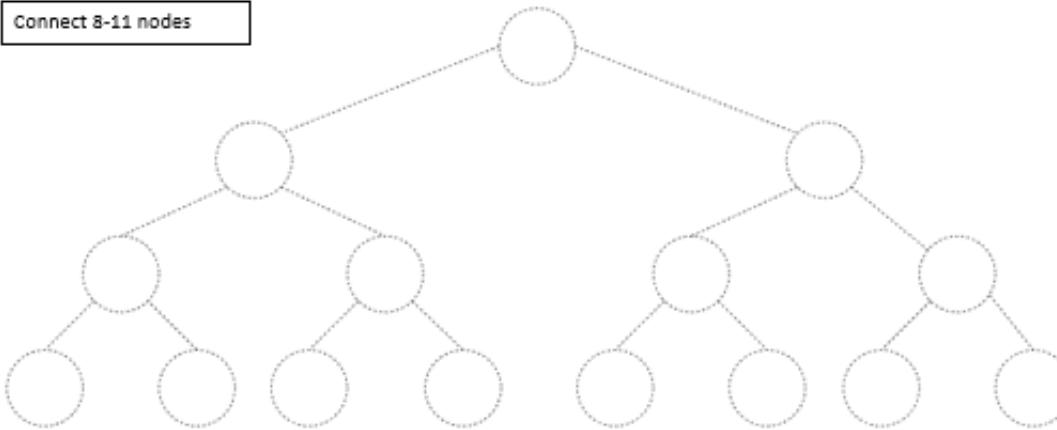
height



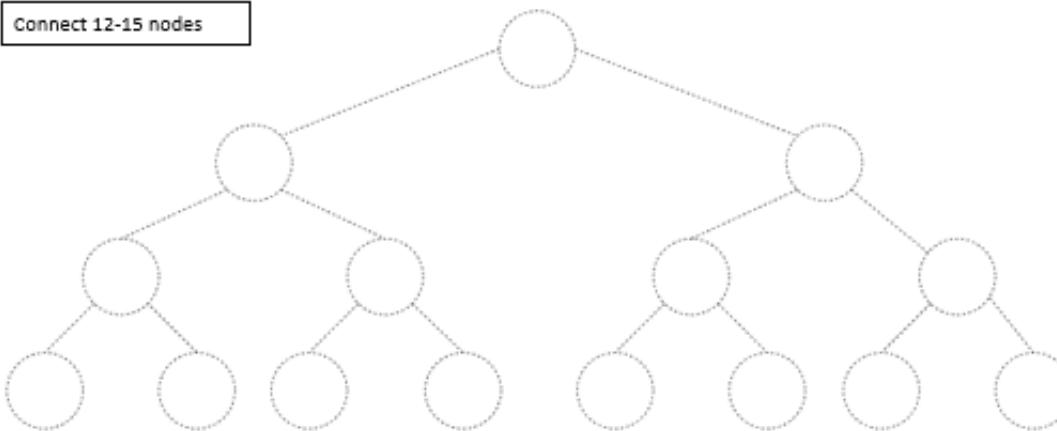
Connect 5-7 nodes



Connect 8-11 nodes



Connect 12-15 nodes



# Binary Trees

Before we talk about heaps, let's talk about some versions of binary trees.

Perfect Binary Tree

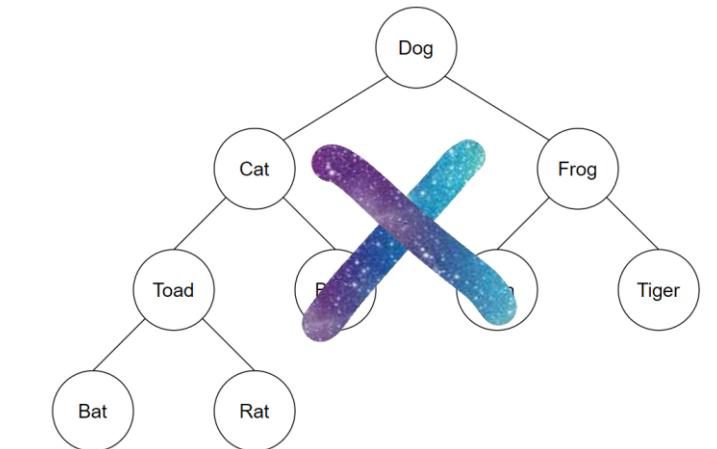
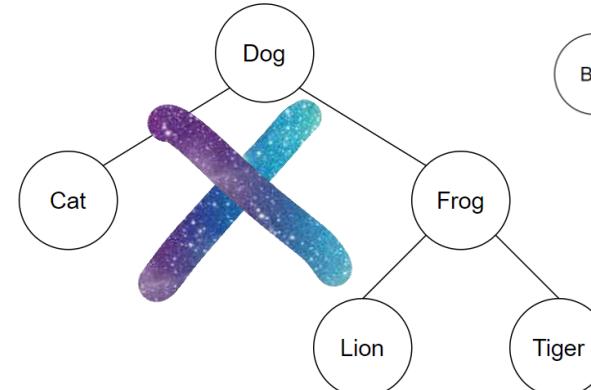
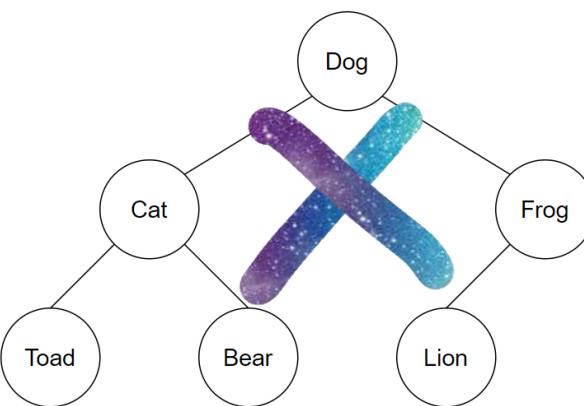
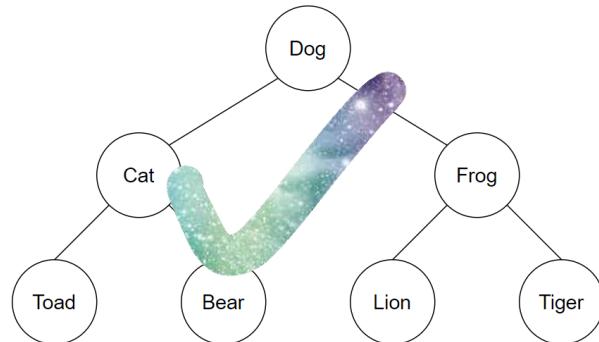
Full Binary Tree

Complete Binary Tree

# Binary Trees

## Perfect Binary Tree

A Binary tree is a **Perfect Binary Tree** if all the internal nodes have two children and all leaf nodes are at the same level.

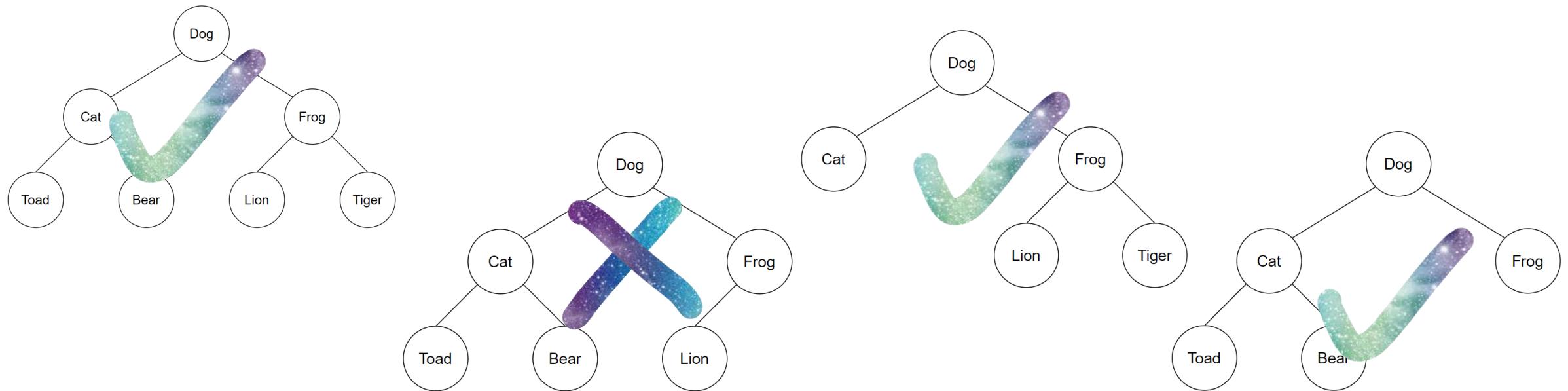


# Binary Trees

## Full Binary Tree

A Binary Tree is a **full binary tree** if every node has 0 or 2 children.

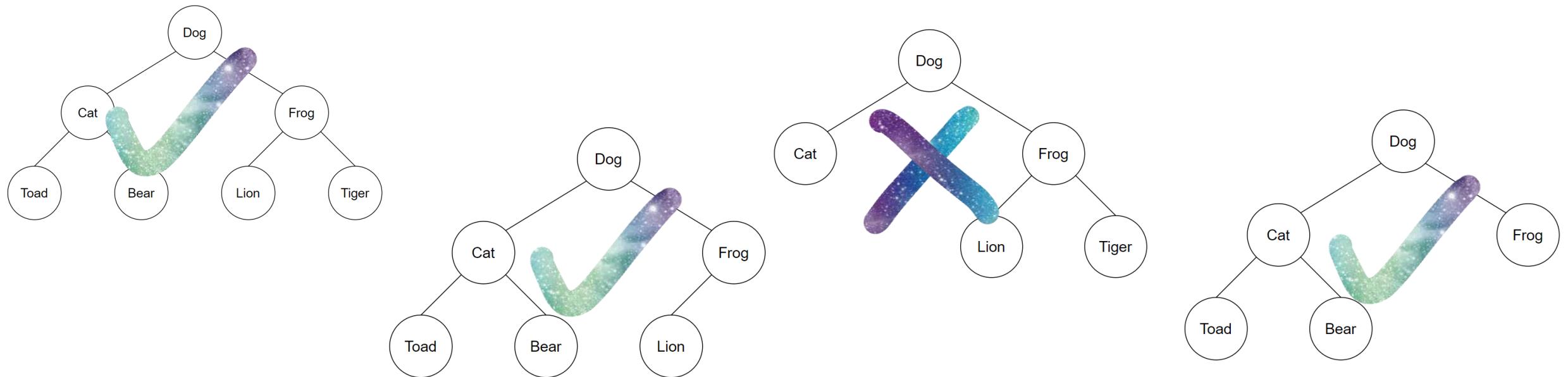
We can also say a full binary tree is a binary tree in which all nodes except leaf nodes have two children.



# Binary Trees

## Complete Binary Tree

A Binary Tree is a **Complete Binary Tree** if all the levels are completely filled except possibly the last level and the last level has all keys as left as possible

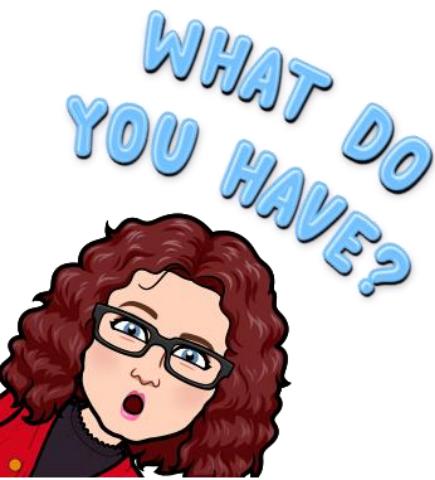


# Binary Trees

A Binary Tree is a **complete binary tree** if all the levels are completely filled except possibly the last level and the last level has all keys as left as possible

A Binary Tree is a **full binary tree** if every node has 0 or 2 children.

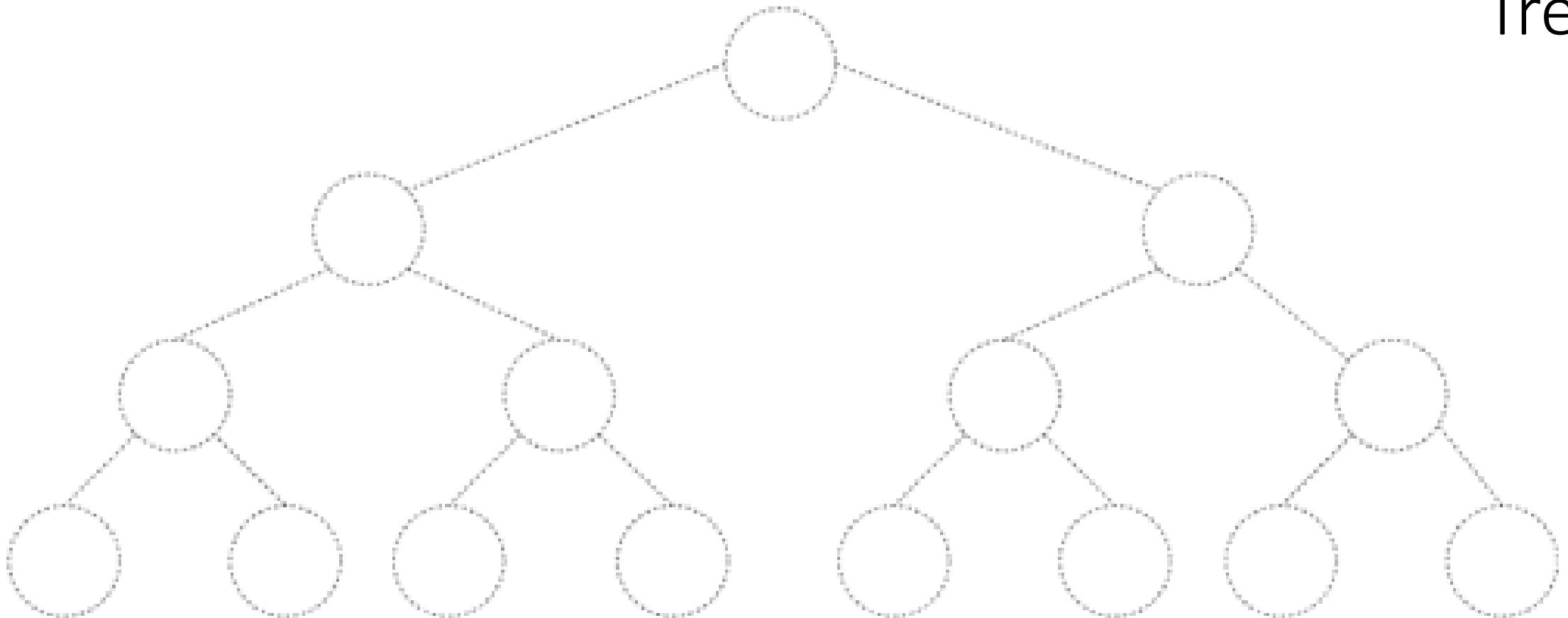
A Binary tree is a **Perfect Binary Tree** if all the internal nodes have two children and all leaf nodes are at the same level.



# Complete Binary Tree

n = 2-3  
n = 4-7  
n = 8-15

Levels = 2  
Levels = 3  
Levels = 4



# Binary Heap

A binary **heap** is a complete binary tree that satisfies the heap property.

**Complete Binary Tree** - all the levels are completely filled except possibly the last level and the last level has all keys as left as possible

Heap is one of the most efficient implementations of an abstract data type called a priority queue.

A priority queue is an abstract data type similar to regular queue data structure in which each element additionally has a "priority" associated with it.

In a priority queue, an element with high priority is served before an element with low priority.

# Priority Queue

Imagine you are an Air Traffic Controller (ATC) working in the control tower of an airport.

Aircraft X is ready to land and the runway is free, so you give them permission to land.

Aircraft Y radios in that they will be arriving at the airport within the next 5 minutes.

You also know that the runway will be occupied/unavailable for any other plane for at least 15 minutes while a plane is landing.

You tell Aircraft Y to go into a holding pattern when they arrive because the runway will be occupied for at least another 10 minutes after their arrival.

You have Aircraft X and Y in a queue.

# Priority Queue

If another aircraft arrives, they will be put in the queue behind Aircraft Y and told to maintain a holding pattern while waiting their turn.

Five minutes later, Aircraft Z radios in that they are 7 minutes away from the airport but critically low on fuel due to going around a big storm system.

They can't land immediately because the runway will still be occupied by Aircraft X.

So you put them in the queue because they have to wait on Aircraft X (which is already landing) but they have priority over Aircraft Y even though Aircraft Y arrived first.

# Binary Heap

A binary heap is a binary tree with the following properties...

1. Complete tree
2. Heap property which means the binary heap is either a
  - Min Heap
  - or
  - Max Heap

# Binary Heap

## Min Heap

The value at root of the tree must be smallest value present in the Binary Heap.

This property must be recursively true for all nodes in the binary tree.

Any parent node should be smaller than its child nodes.

# Binary Heap

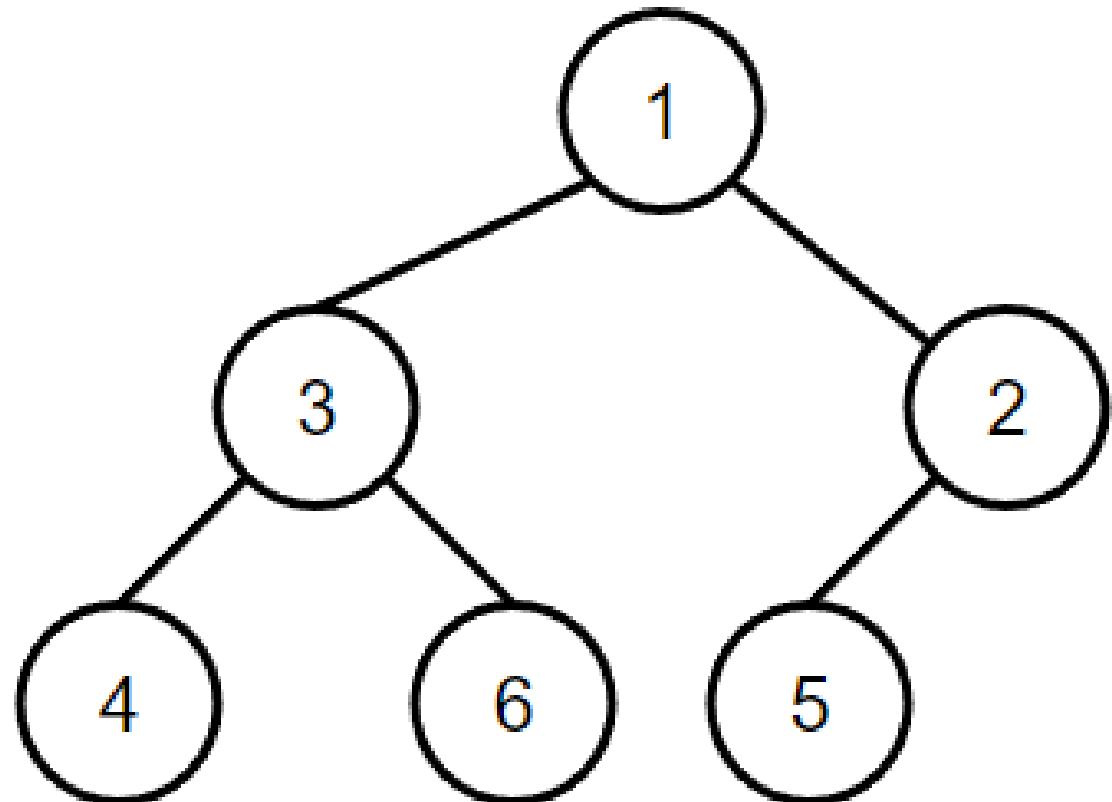
Is this a Min Heap?

1. Is it a complete tree?

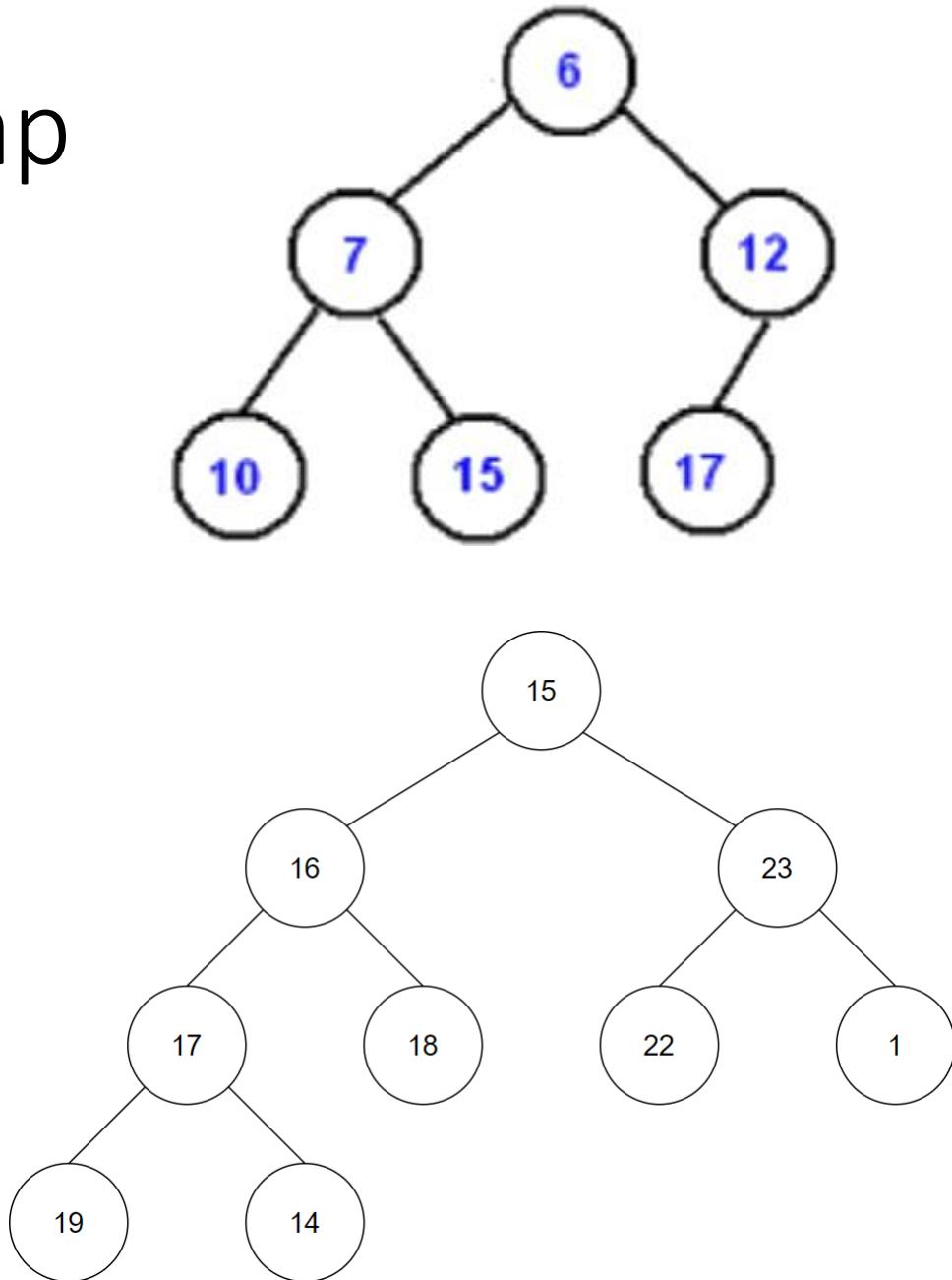
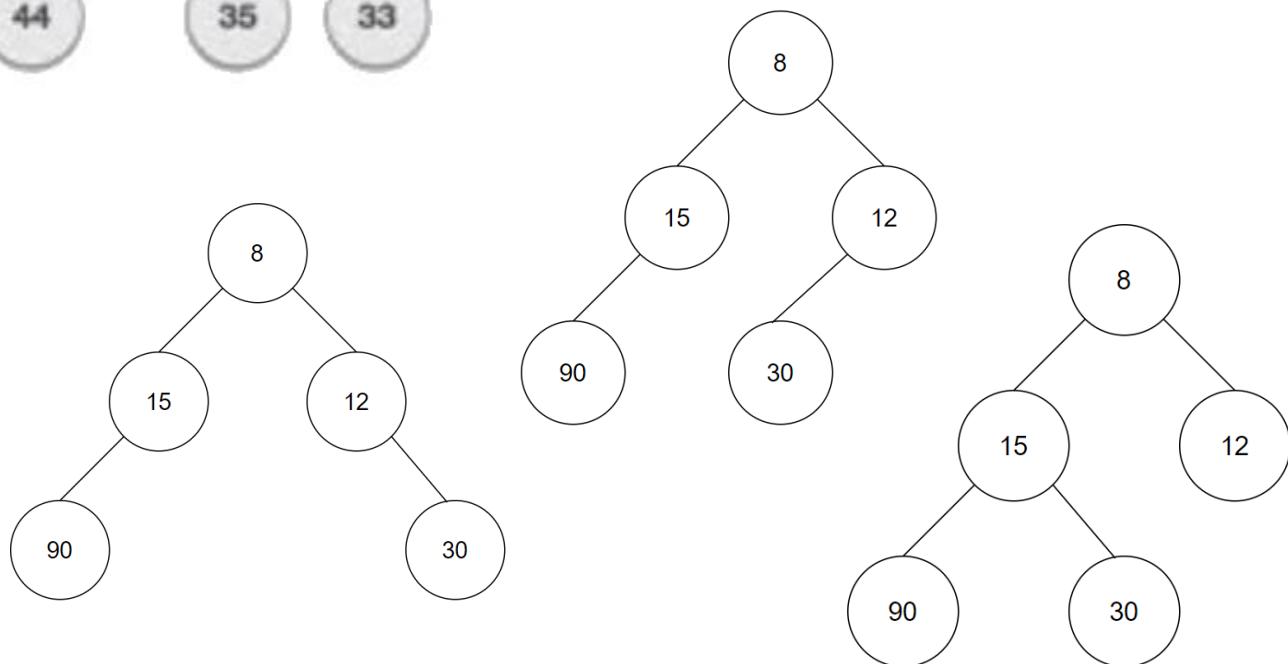
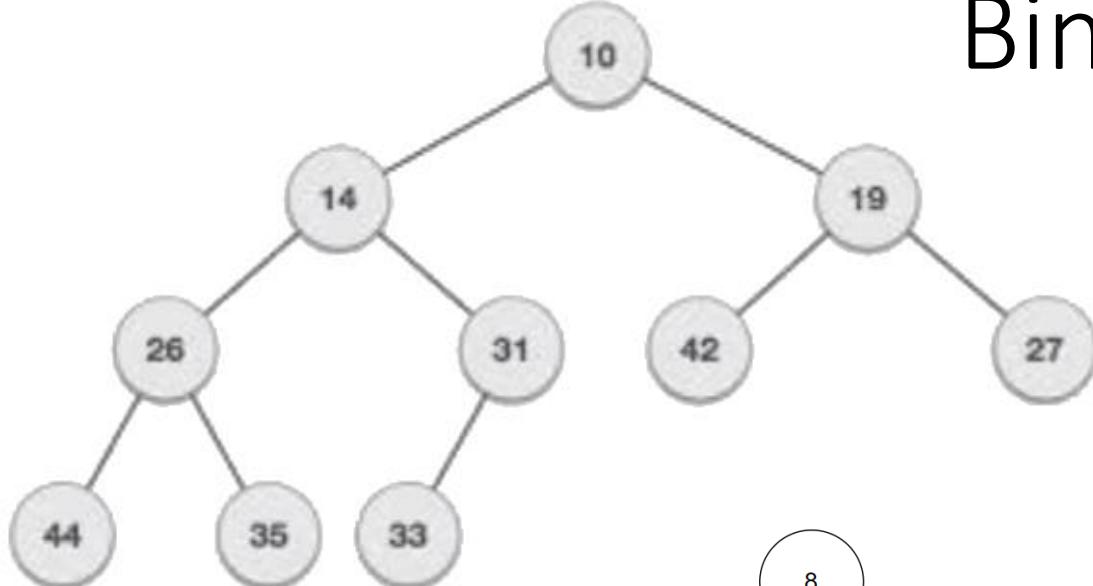
all the levels are completely filled except possibly the last level and the last level has all keys as left as possible

2. Is the value at the root of the tree the smallest value in the heap?

3. Are all parent nodes smaller than their children?



# Binary Heap



# Binary Heap

## Max Heap

The value at root of the tree must be largest value present in the Binary Heap.

This property must be recursively true for all nodes in the binary tree.

Any parent node should be larger than its child nodes.

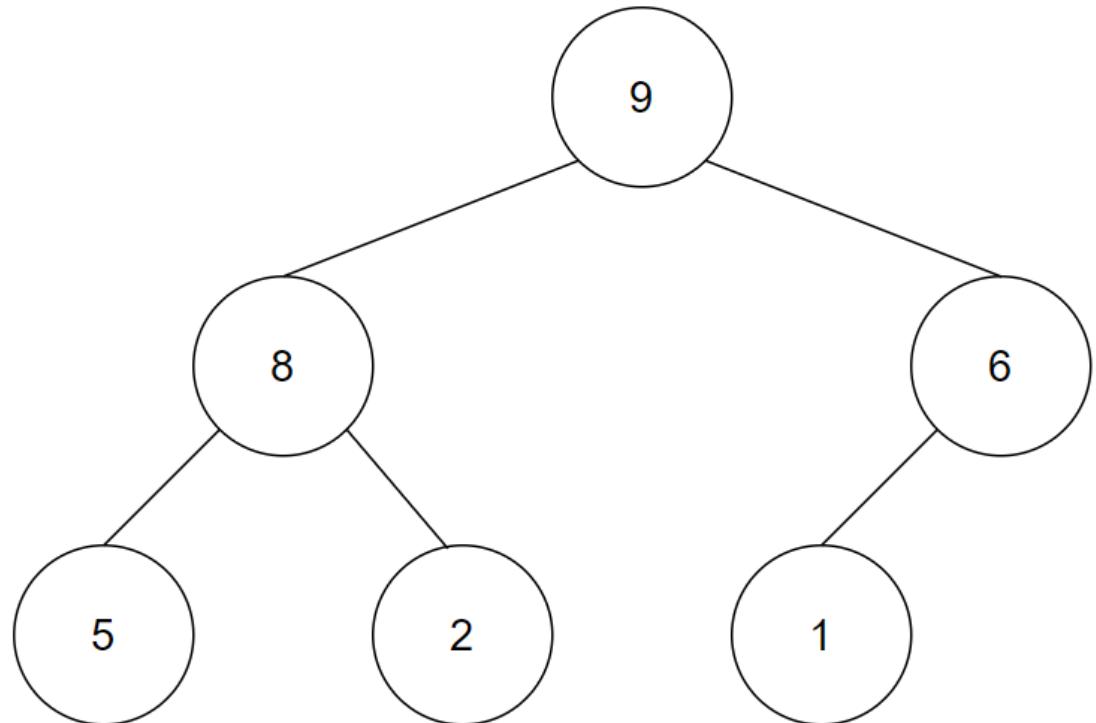
# Binary Heap

Is this a Max Heap?

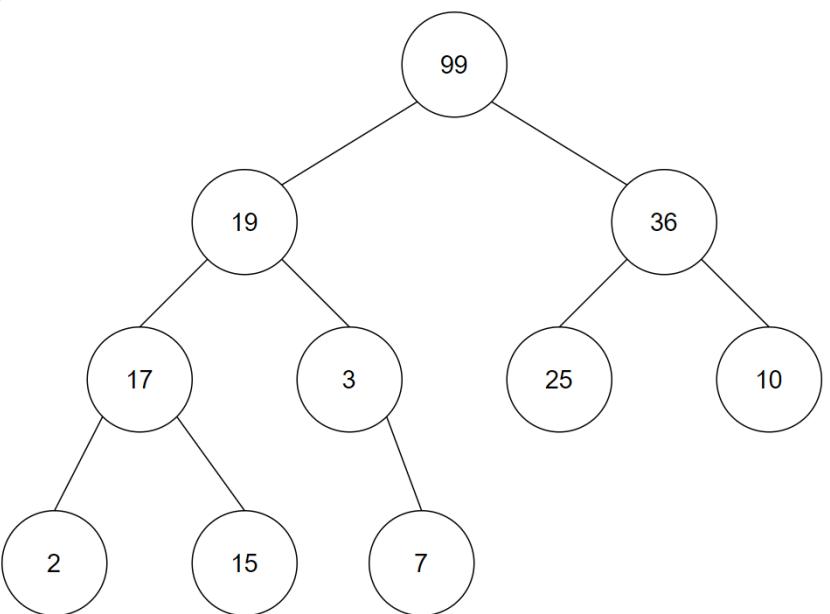
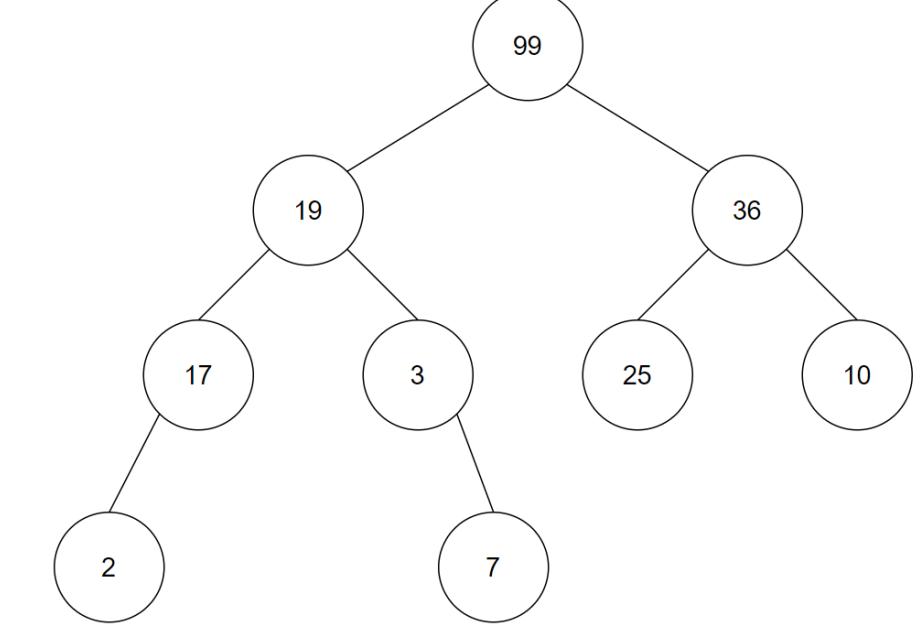
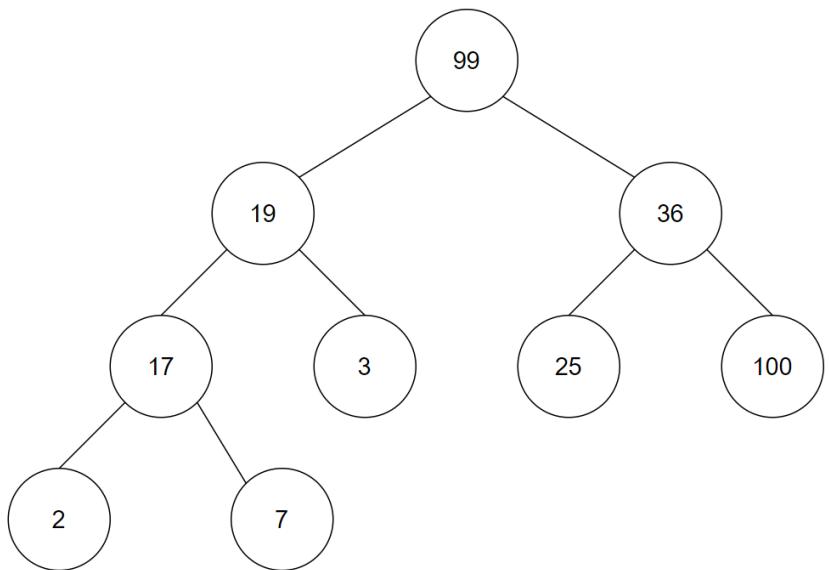
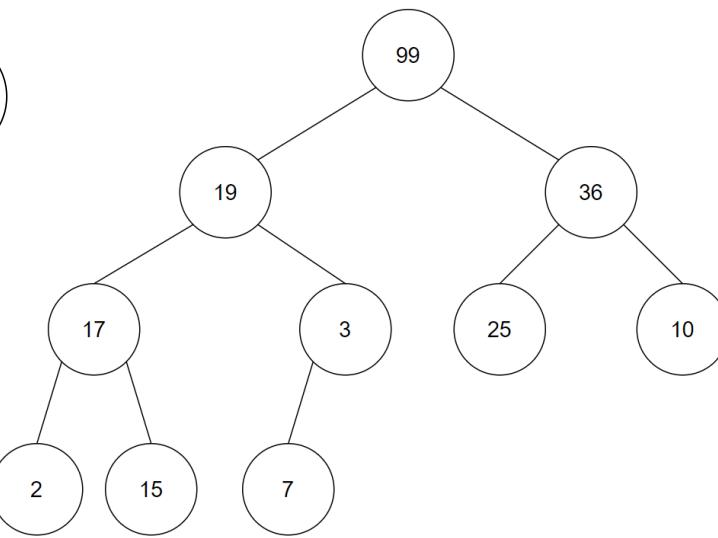
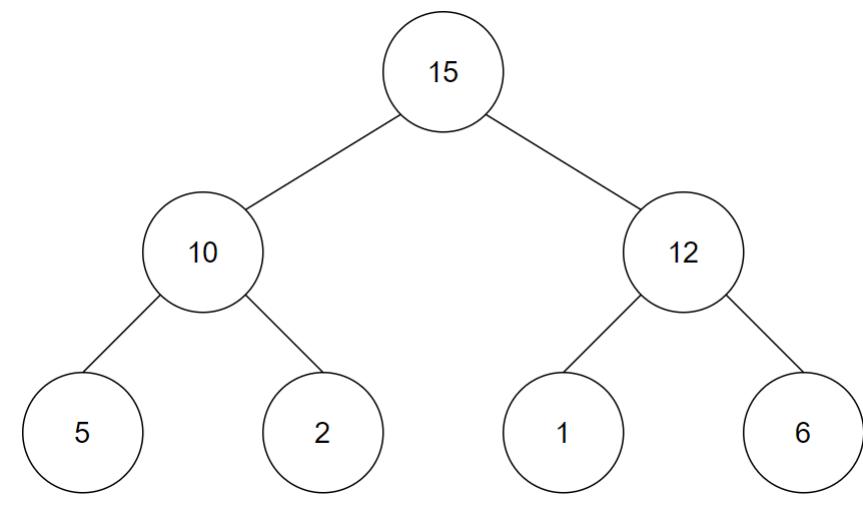
1. Is it a complete tree?

all the levels are completely filled except possibly the last level and the last level has all keys as left as possible

2. Is the value at the root of the tree the largest value in the heap?
3. Are all parent nodes larger than their children?



# Binary Heap



# Binary Heap

Now, here's the good news.

A binary heap is typically represented by an array.

So how do we turn a tree into an array?

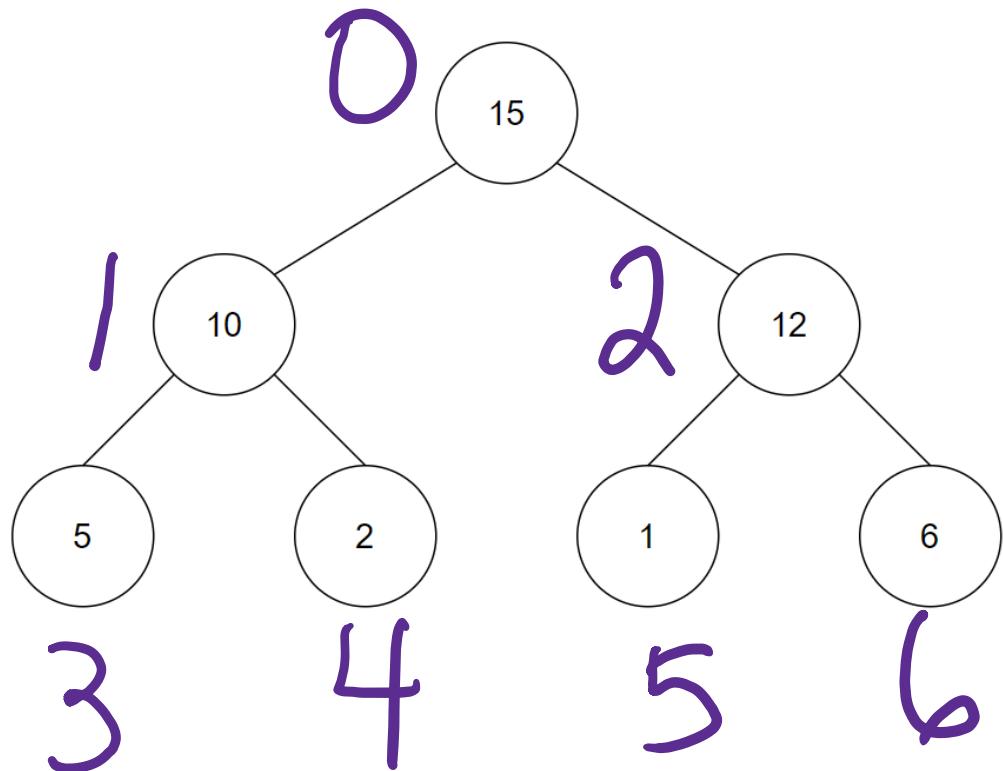


**THERE'S A  
BETTER WAY.**



# Binary Heap

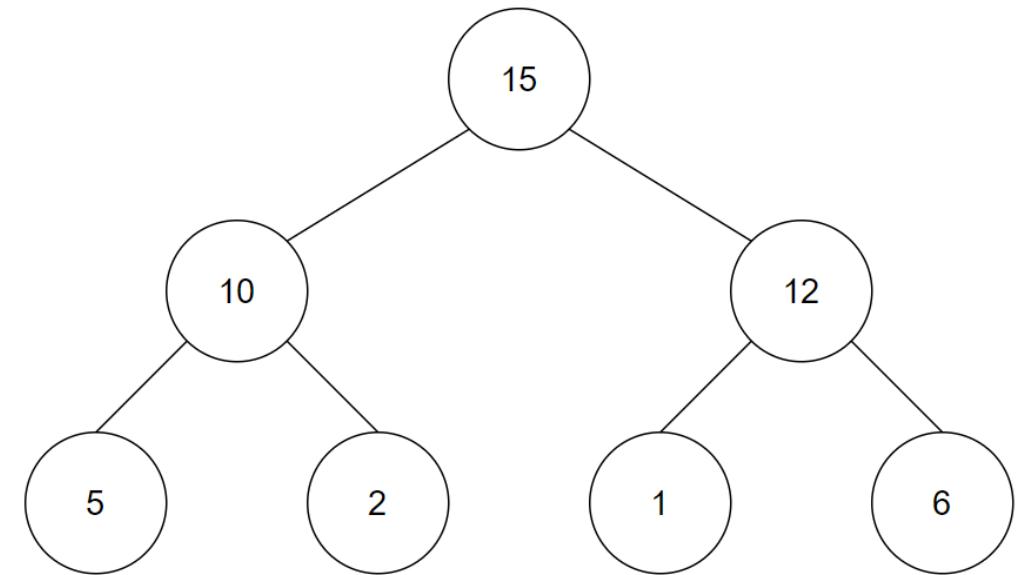
Let's look at it graphically and then we'll formalize the process.



# Binary Heap

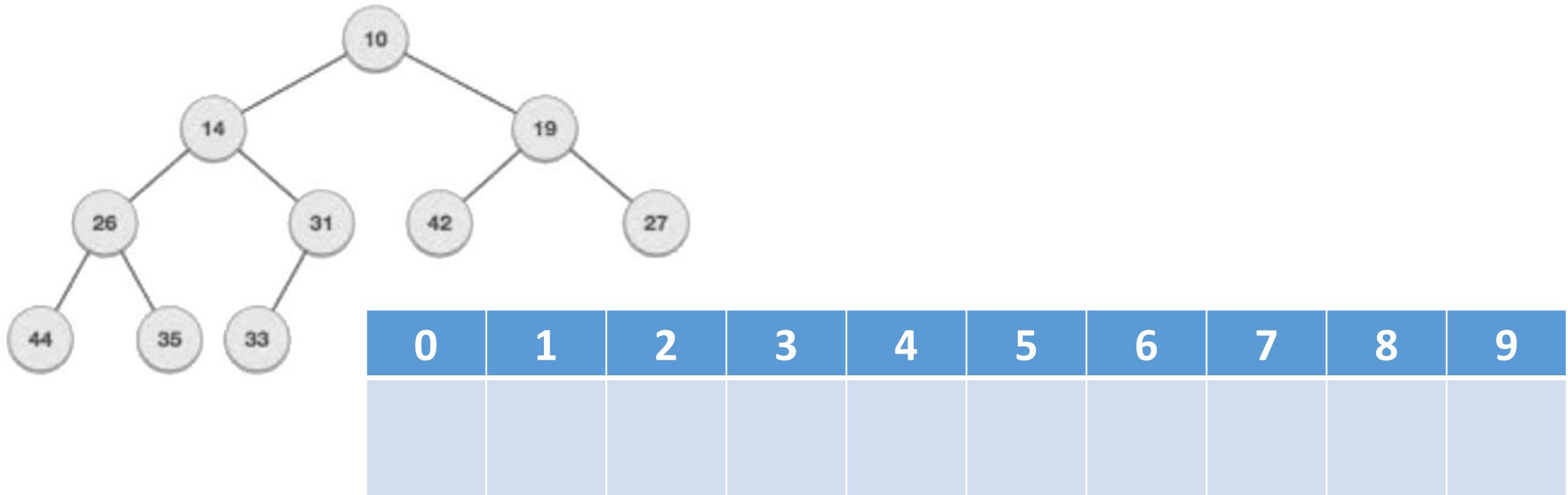
Let's start with the array and see if we correctly recreate the binary heap.

| 0  | 1  | 2  | 3 | 4 | 5 | 6 |
|----|----|----|---|---|---|---|
| 15 | 10 | 12 | 5 | 2 | 1 | 6 |

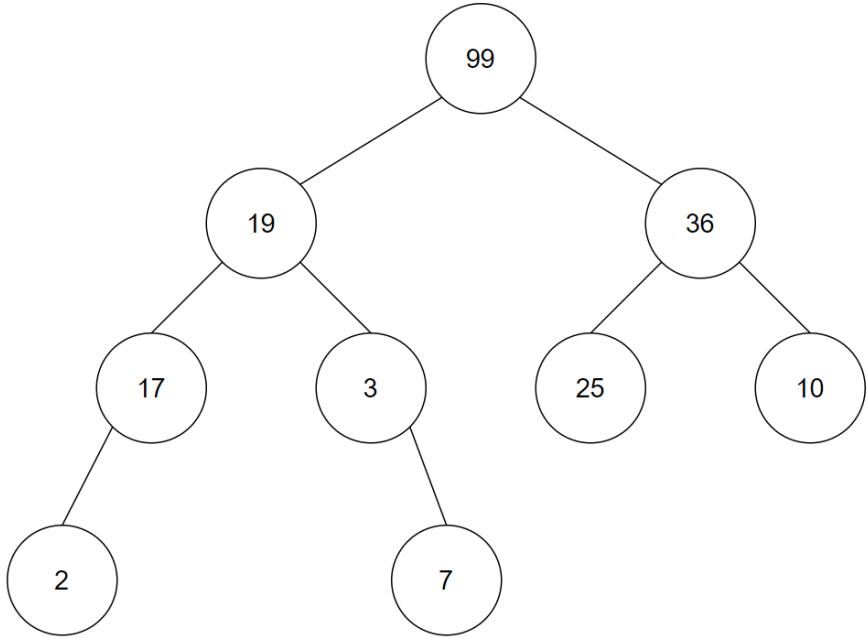


# Binary Heap

That was for a max heap – let's try a min heap...



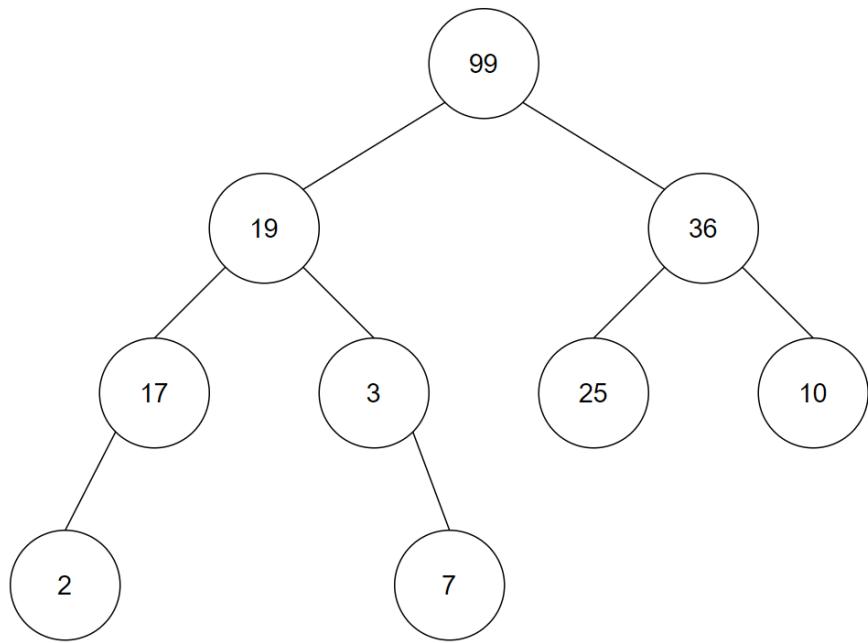
# Binary Heap



What happens if we follow this process with a tree that has the values in the right locations (max parents/root) but was not a complete tree?

all the levels are completely filled except possibly the last level and the last level has all keys as left as possible

# Binary Heap



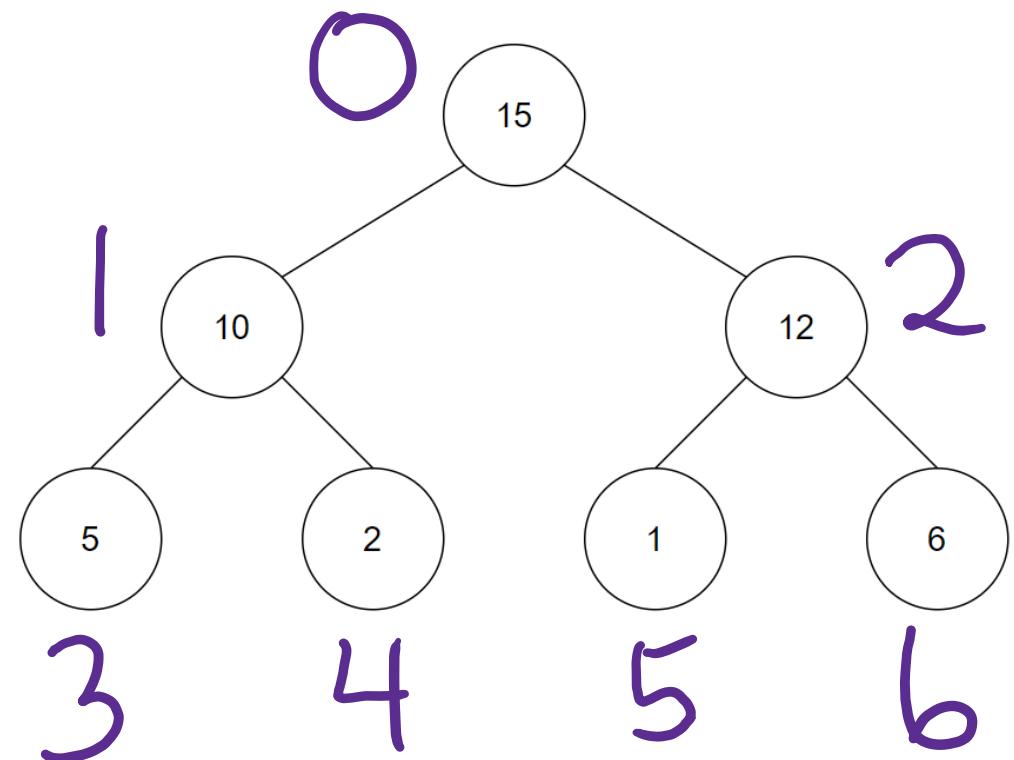
# What if we just left blanks?

# Binary Heap

This traversal method is called Level Order. We go through each level in order.

|    |    |    |   |   |   |   |
|----|----|----|---|---|---|---|
| 0  | 1  | 2  | 3 | 4 | 5 | 6 |
| 15 | 10 | 12 | 5 | 2 | 1 | 6 |

That order corresponds to the array indexes.



# Binary Heap

So root always goes to ARRAY[0].

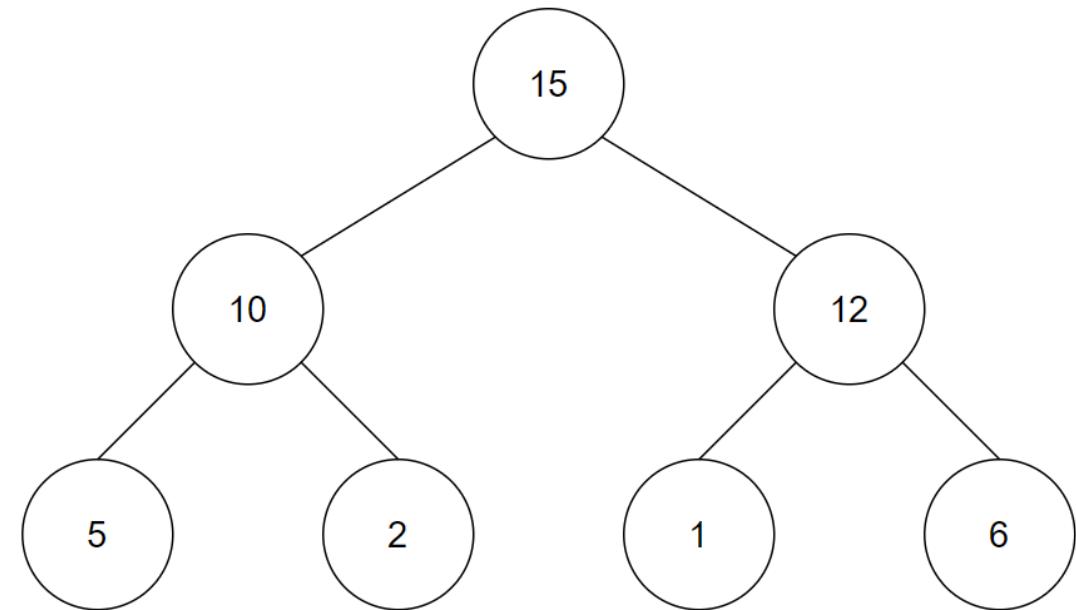
When using zero indexed arrays, we can state that for a index of i

left child =  $2i + 1$

right child =  $2i + 2$

Left child of root ( $i=0$ ) =  $2i + 1 = 1$

Right child of root ( $i=0$ ) =  $2i + 2 = 2$



| 0  | 1  | 2  | 3 | 4 | 5 | 6 |
|----|----|----|---|---|---|---|
| 15 | 10 | 12 | 5 | 2 | 1 | 6 |

# Binary Heap

left child =  $2i + 1$

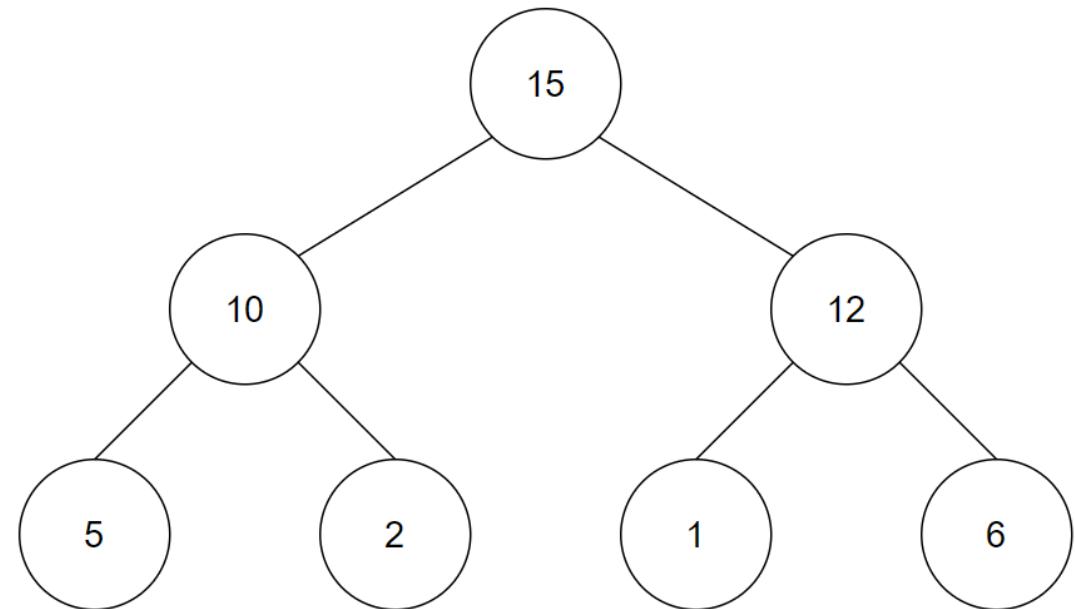
right child =  $2i + 2$

Left child of (10) ( $i=1$ ) =  $2i + 1 = 3$

Right child of (10) ( $i=1$ ) =  $2i + 2 = 4$

Left child of (12) ( $i=2$ ) =  $2i + 1 = 5$

Right child of (12) ( $i=2$ ) =  $2i + 2 = 6$



| 0  | 1  | 2  | 3 | 4 | 5 | 6 |
|----|----|----|---|---|---|---|
| 15 | 10 | 12 | 5 | 2 | 1 | 6 |

# Adding to a Max Heap

How do we add a value?

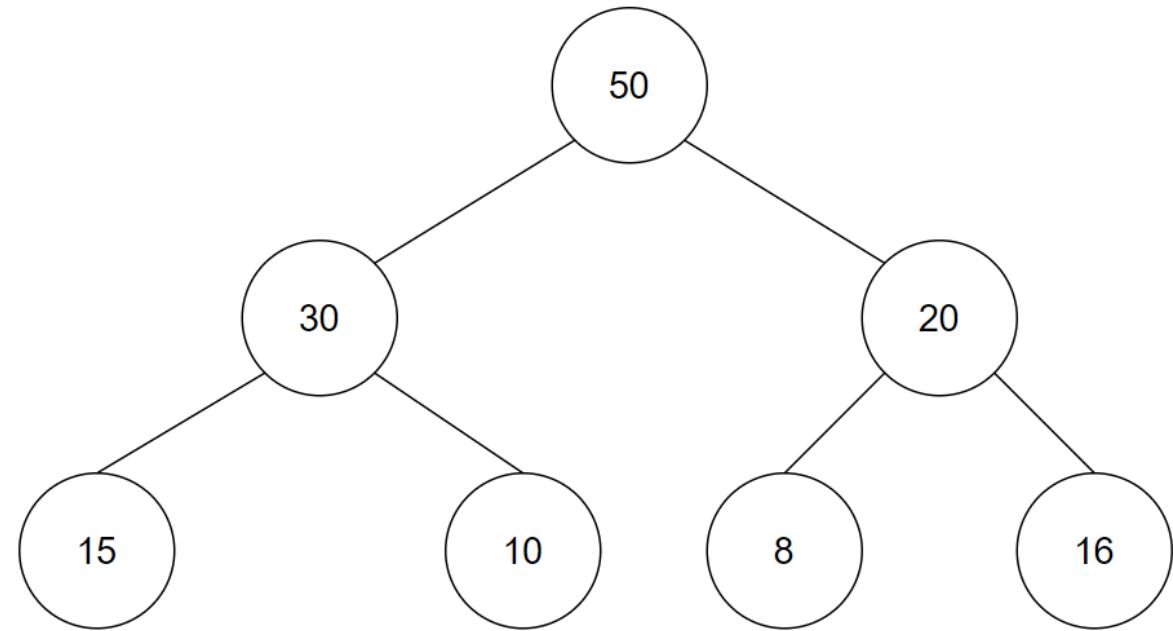
Let's say we want to add 60?

60 is greater than all of the other values in the tree so it should become the root.

But how do we make it the root?

Where does 50 go?

How do we shift around the parents and children?



| 0  | 1  | 2  | 3  | 4  | 5 | 6  |
|----|----|----|----|----|---|----|
| 50 | 30 | 20 | 15 | 10 | 8 | 16 |

# Adding to a Max Heap

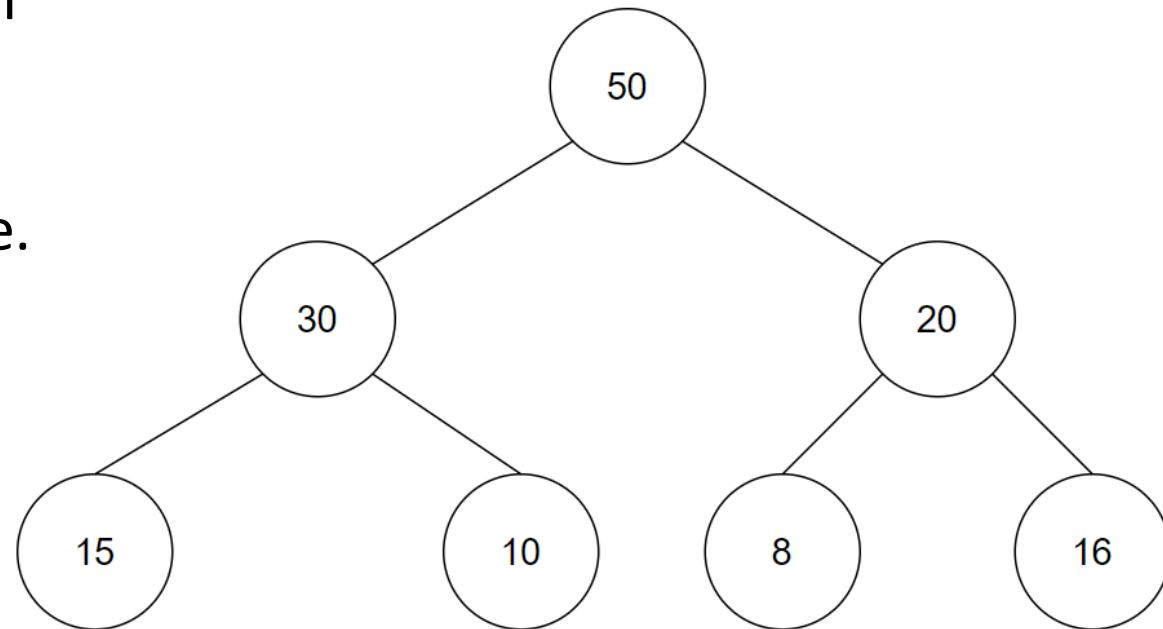
Adding a value to heap is called inserting or pushing.

So let's look at the array instead of the tree.

Where would it make sense to add an element to the array?

At the end would be the easiest

should be room  
won't have to move anyone else



| 0  | 1  | 2  | 3  | 4  | 5 | 6  |
|----|----|----|----|----|---|----|
| 50 | 30 | 20 | 15 | 10 | 8 | 16 |

# Adding to a Max Heap

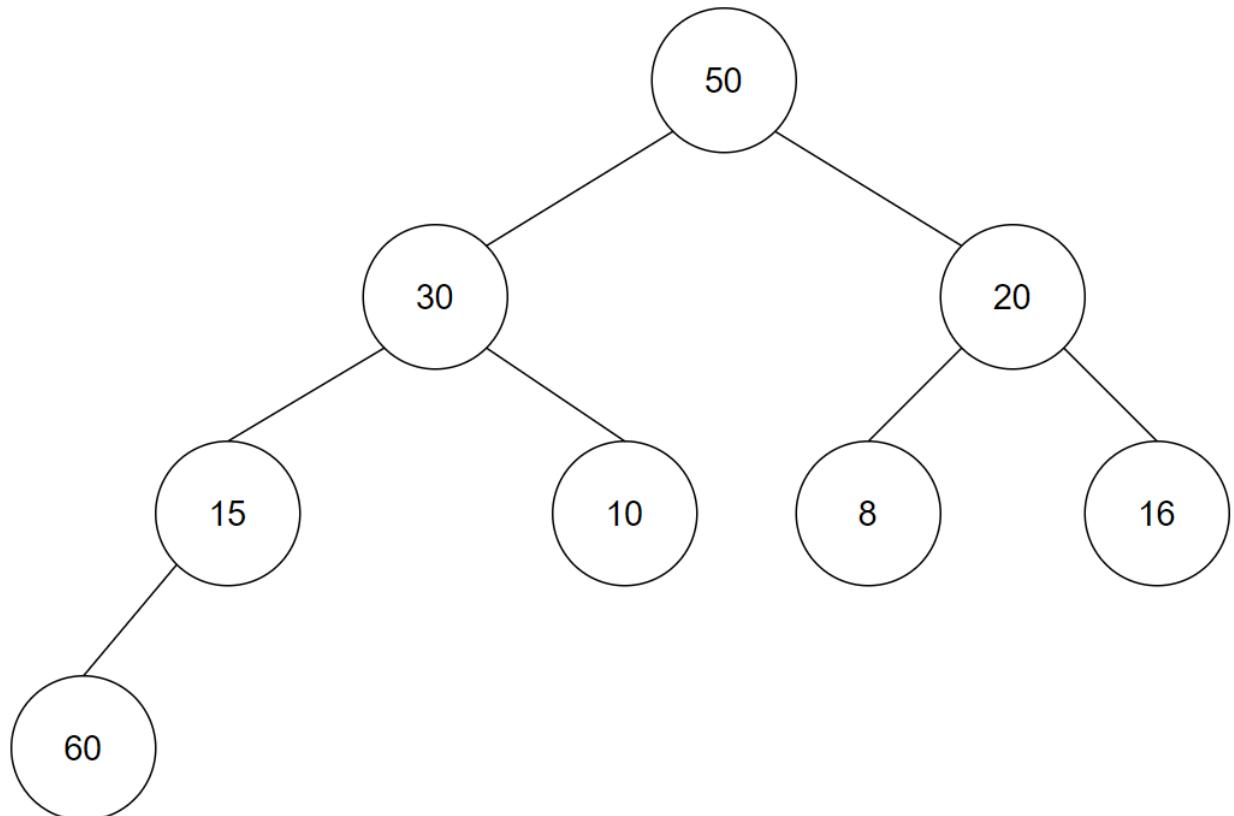
What does that do to our tree?

Where would the 60 go according to the array?

Once we put 60 there, we have violated the heap property

60 is largest value in the tree but it is not the root.

So we need to move it...



| 0  | 1  | 2  | 3  | 4  | 5 | 6  | 7  |
|----|----|----|----|----|---|----|----|
| 50 | 30 | 20 | 15 | 10 | 8 | 16 | 60 |

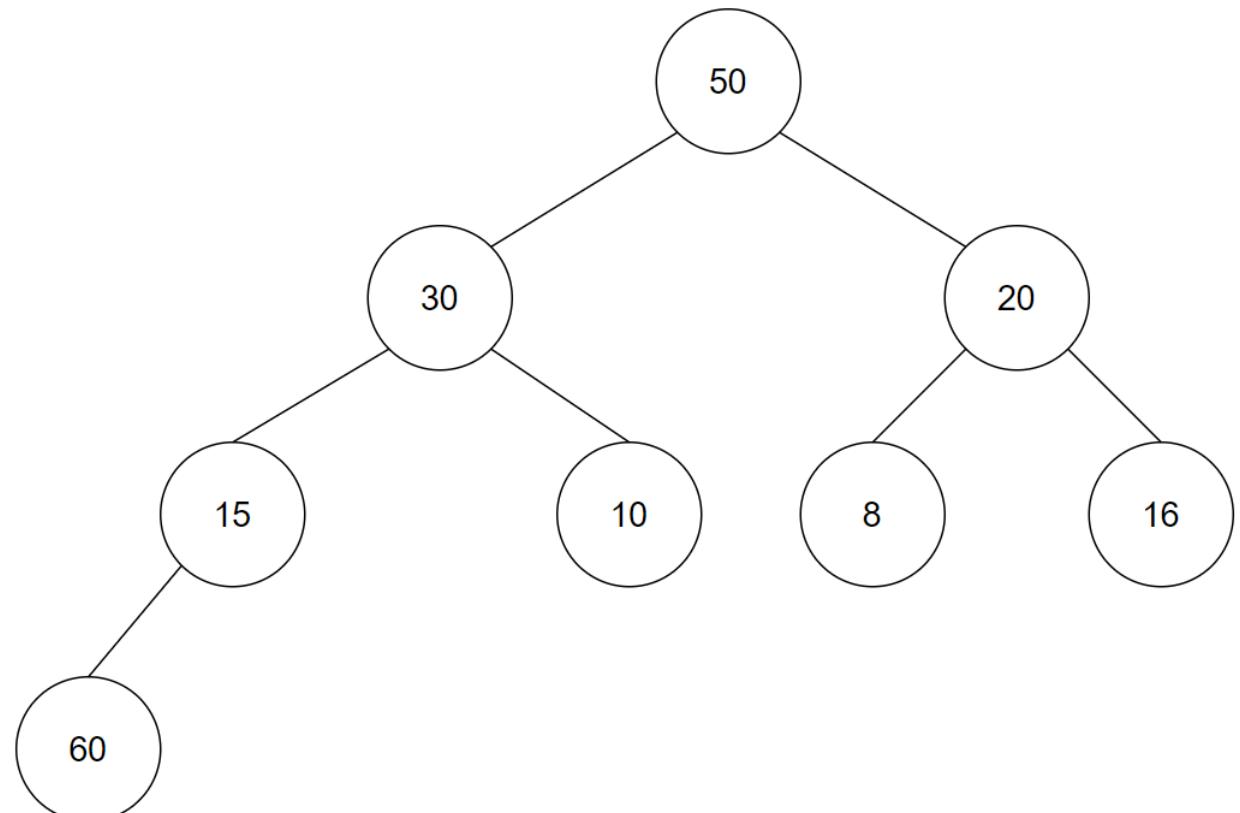
# Adding to a Max Heap

We could swap 15 and 60.

And then swap 30 and 60.

And then swap 50 and 60.

Have we restored the heap property?



| 0  | 1  | 2  | 3  | 4  | 5 | 6  | 7  |
|----|----|----|----|----|---|----|----|
| 50 | 30 | 20 | 15 | 10 | 8 | 16 | 60 |

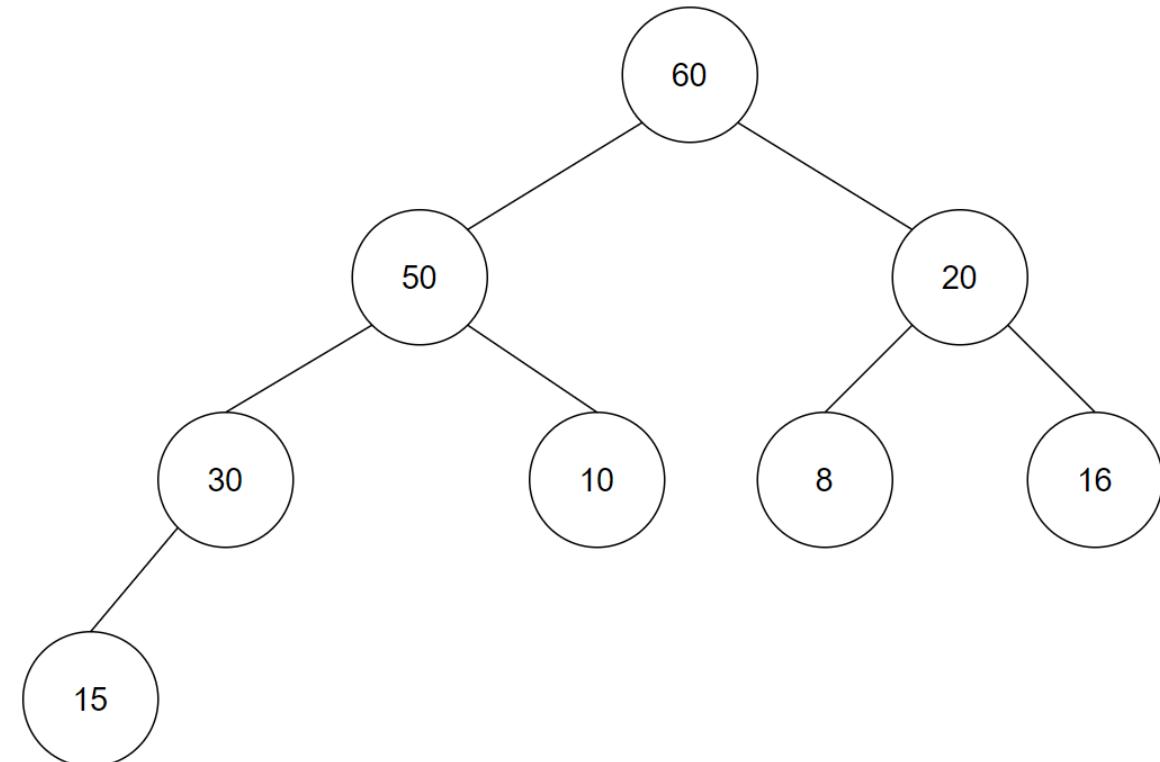
# Adding to a Max Heap

Yes!

We have a complete tree and the root is the greatest value and every child is less than its parent.



HEAPIFY



| 0  | 1  | 2  | 3  | 4  | 5 | 6  | 7  |
|----|----|----|----|----|---|----|----|
| 60 | 50 | 20 | 30 | 10 | 8 | 16 | 15 |

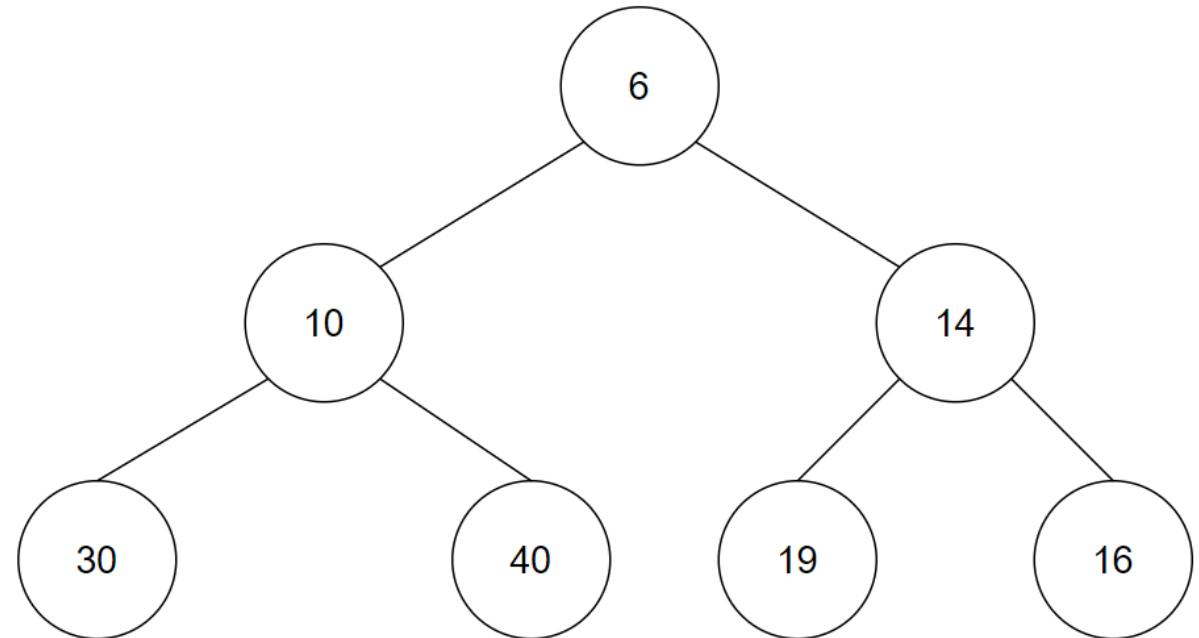
# Adding to a Min Heap

How do we add a value?

Let's say we want to add 2?

2 is smaller than all of the other values in the tree so it should become the root.

Do we follow the same process as we did with max heap?



| 0 | 1  | 2  | 3  | 4  | 5  | 6  |
|---|----|----|----|----|----|----|
| 6 | 10 | 14 | 30 | 40 | 19 | 16 |

# Adding to a Min Heap

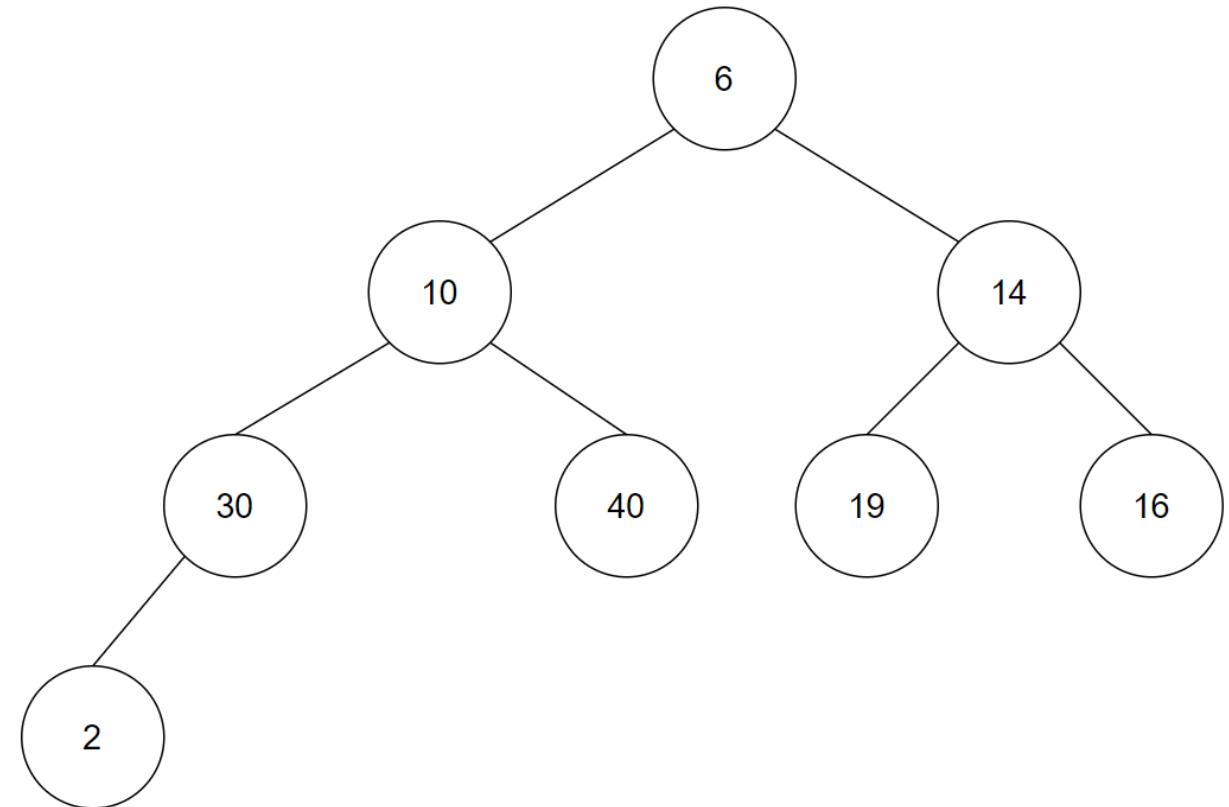
Let's add the 2 just like we did 60.

Now we need to swap 2 and 30.

Now swap 2 and 10.

Now swap 6 and 2.

Have we restored the heap property?



| 0 | 1  | 2  | 3  | 4  | 5  | 6  | 7 |
|---|----|----|----|----|----|----|---|
| 6 | 10 | 14 | 30 | 40 | 19 | 16 | 2 |

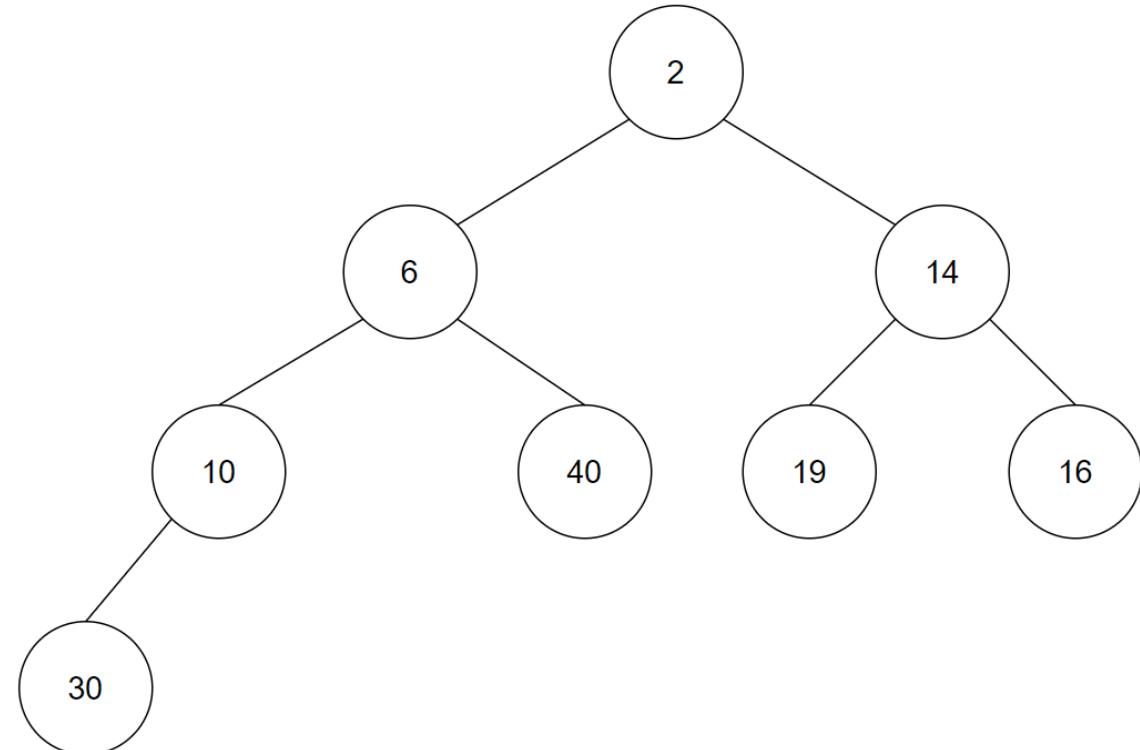
# Adding to a Min Heap

Yes!

We have a complete tree and the root is the smallest value and every child is greater than its parent.



HEAPIFY



| 0 | 1 | 2  | 3  | 4  | 5  | 6  | 7  |
|---|---|----|----|----|----|----|----|
| 2 | 6 | 14 | 10 | 40 | 19 | 16 | 30 |

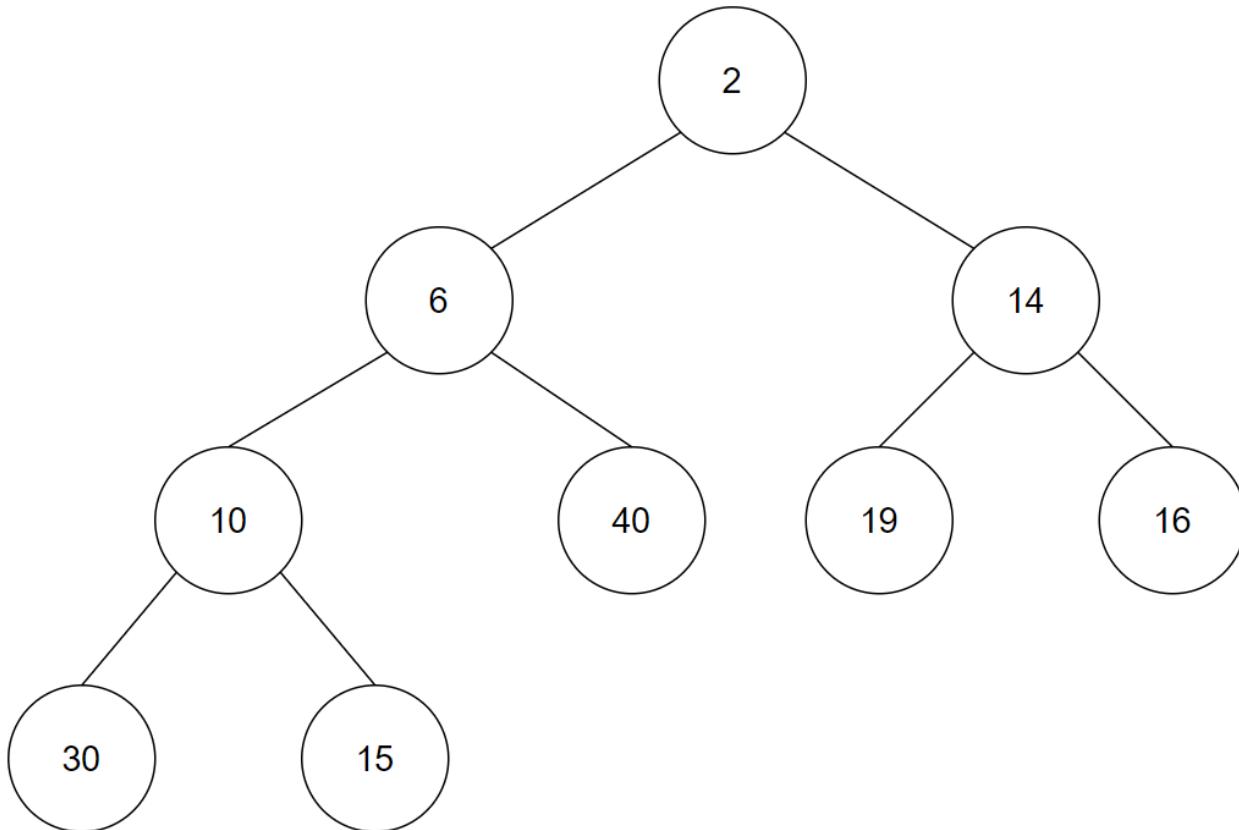
# Adding/Inserting with Heap

What if my heap looks like this and I want to add 1.

Where do I add it?

Follow the same process as before...

Add the new value at the end of the array.



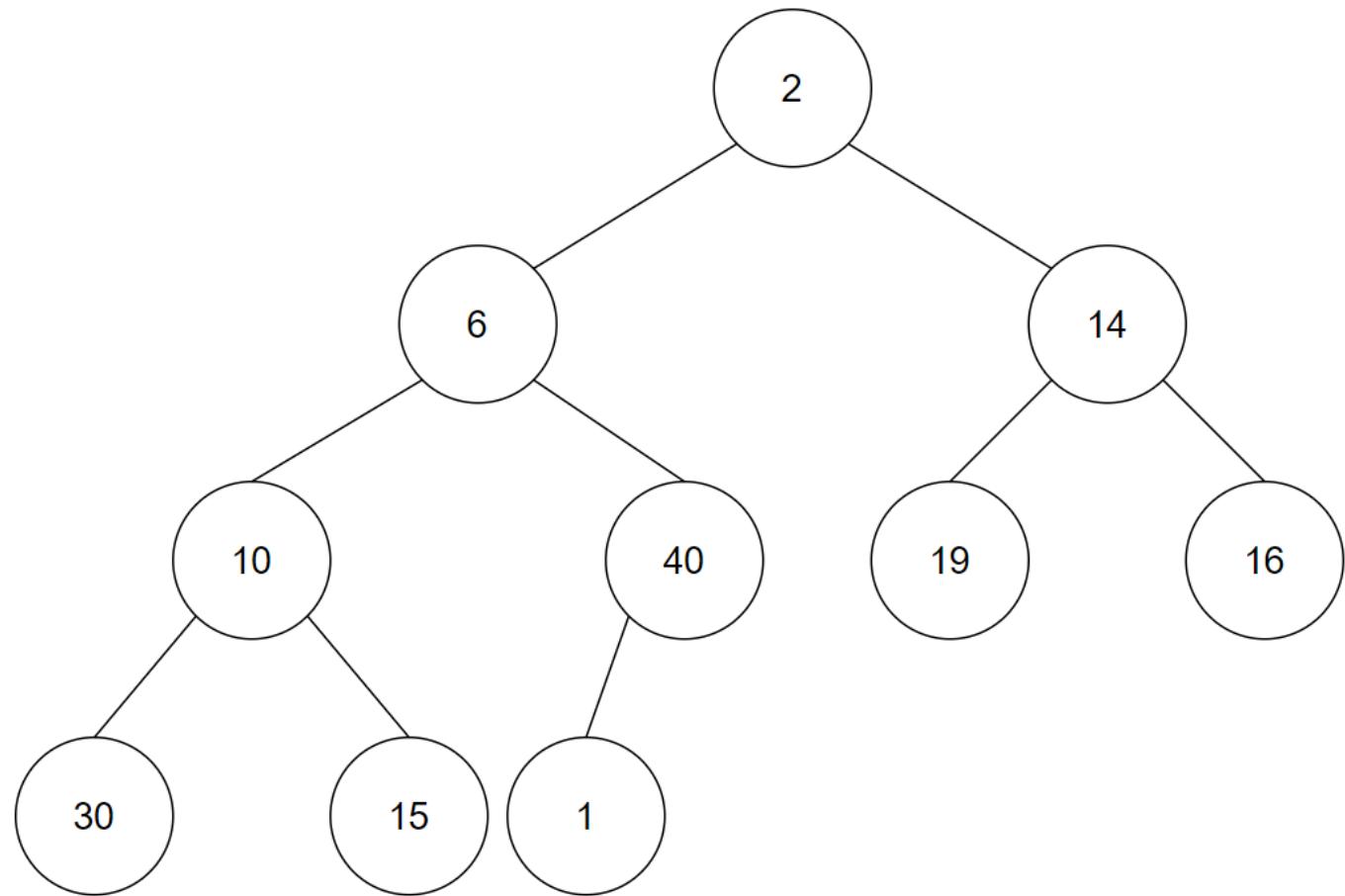
| 0 | 1 | 2  | 3  | 4  | 5  | 6  | 7  | 8  | 9 |
|---|---|----|----|----|----|----|----|----|---|
| 2 | 6 | 14 | 10 | 40 | 19 | 16 | 30 | 15 |   |

# Adding/Inserting with Heap

Now swap 40 and 1

Swap 1 and 6

Swap 1 and 2



| 0 | 1 | 2  | 3  | 4  | 5  | 6  | 7  | 8  | 9 |
|---|---|----|----|----|----|----|----|----|---|
| 2 | 6 | 14 | 10 | 40 | 19 | 16 | 30 | 15 | 1 |

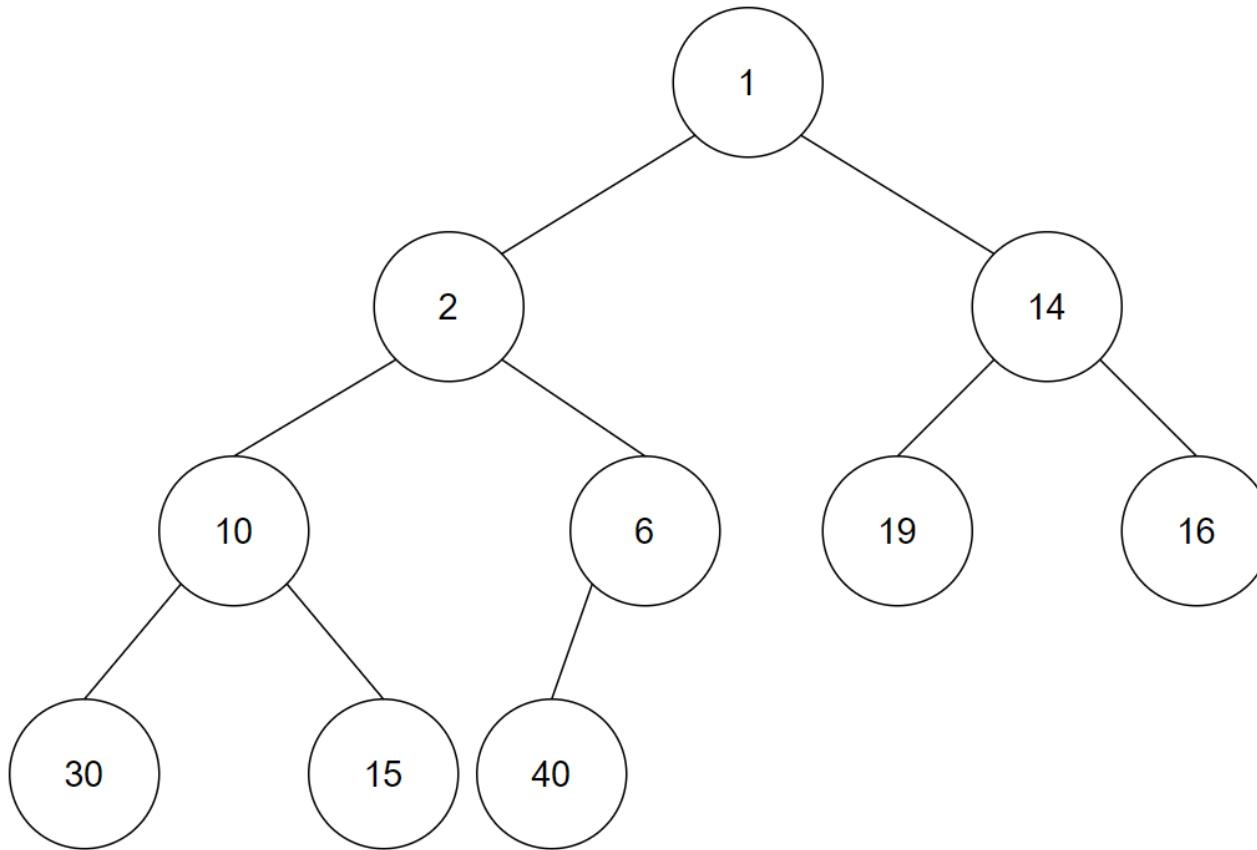
# Adding/Inserting with Heap

Do we still have a min heap?

Complete tree and every parent is less than its children.

Root is the smallest.

Yes!



| 0 | 1 | 2  | 3  | 4 | 5  | 6  | 7  | 8  | 9  |
|---|---|----|----|---|----|----|----|----|----|
| 1 | 2 | 14 | 10 | 6 | 19 | 16 | 30 | 15 | 40 |

# Deleting from a Heap

If you had a stack of cans like this one, how would you remove a can?

Without making a mess or knocking them over?

This is not a game of Jenga

so we would take the top can.



# Deleting from a Heap

We delete from a heap the same way – we take the top element.

For a max heap, the top element/root is the biggest element in the heap.

For a min heap, the top element/root is the smallest element in the heap.

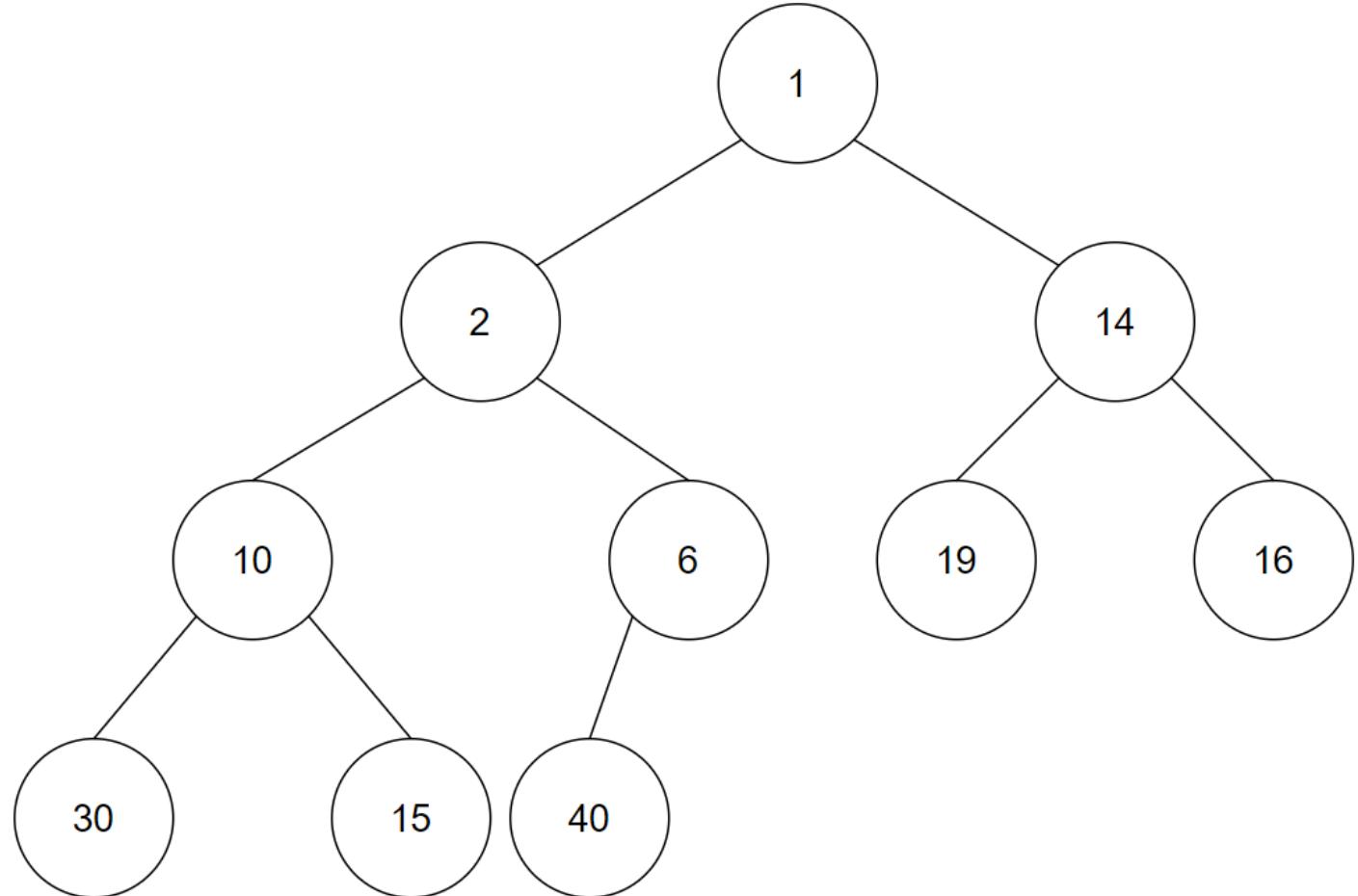
# Deleting from a Min Heap

So what happens when we remove the root?

We need a new root?

Do we pick one of the root's children?

No.



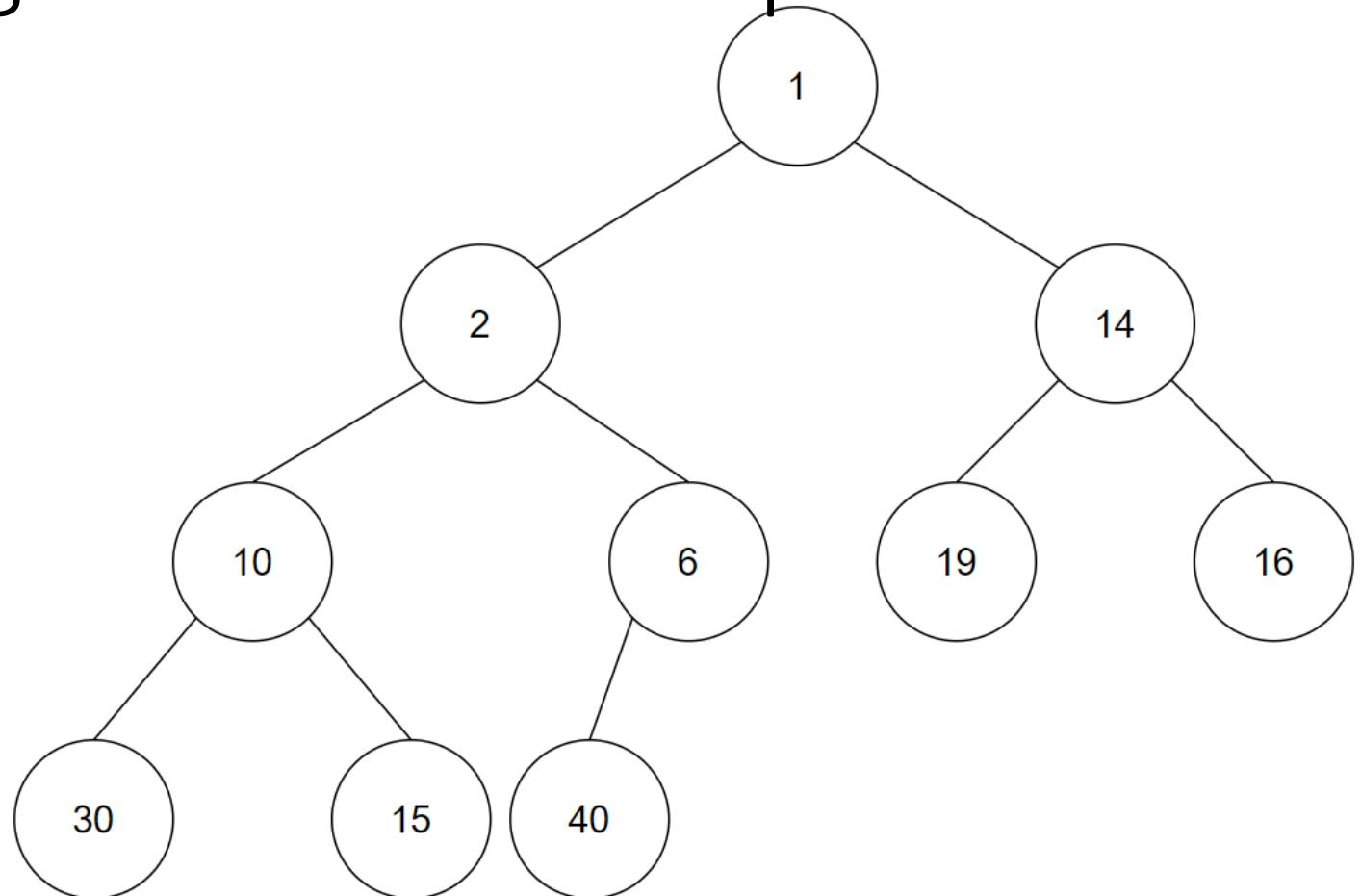
# Deleting from a Min Heap

Let's look at the array again.

If we take the value out of the 0<sup>th</sup> element, what is the easiest replacement value from the array?

If we take element 1, then we'll just have another gap.

We've already said that we don't like gaps in our arrays.



| 0 | 1 | 2  | 3  | 4 | 5  | 6  | 7  | 8  | 9  |
|---|---|----|----|---|----|----|----|----|----|
|   | 2 | 14 | 10 | 6 | 19 | 16 | 30 | 15 | 40 |

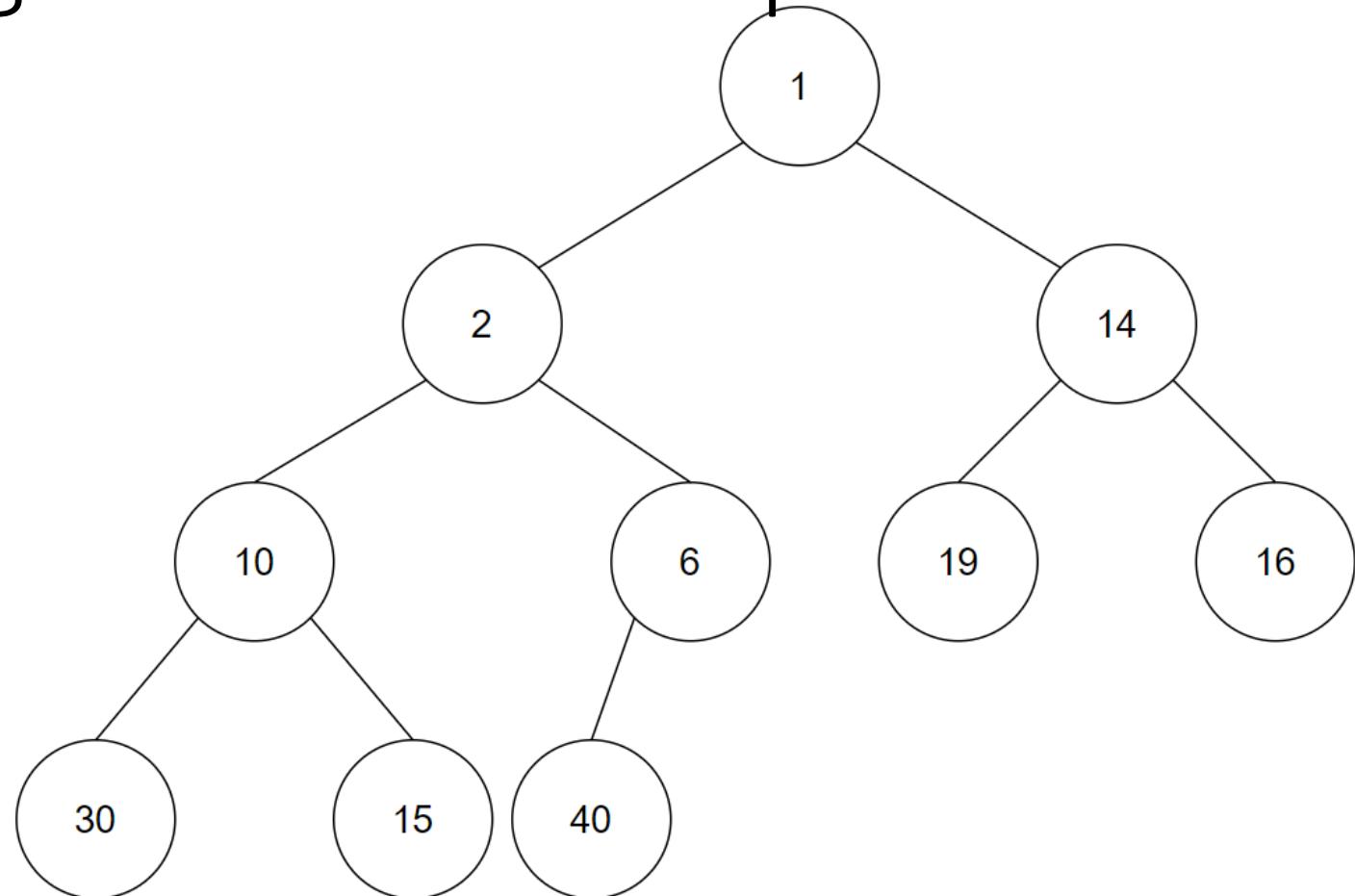
# Deleting from a Min Heap

What if we move the last element to the 0<sup>th</sup> spot?

It's OK to have spaces at the END of the array.

What will the heap look like?

Do we have a problem?



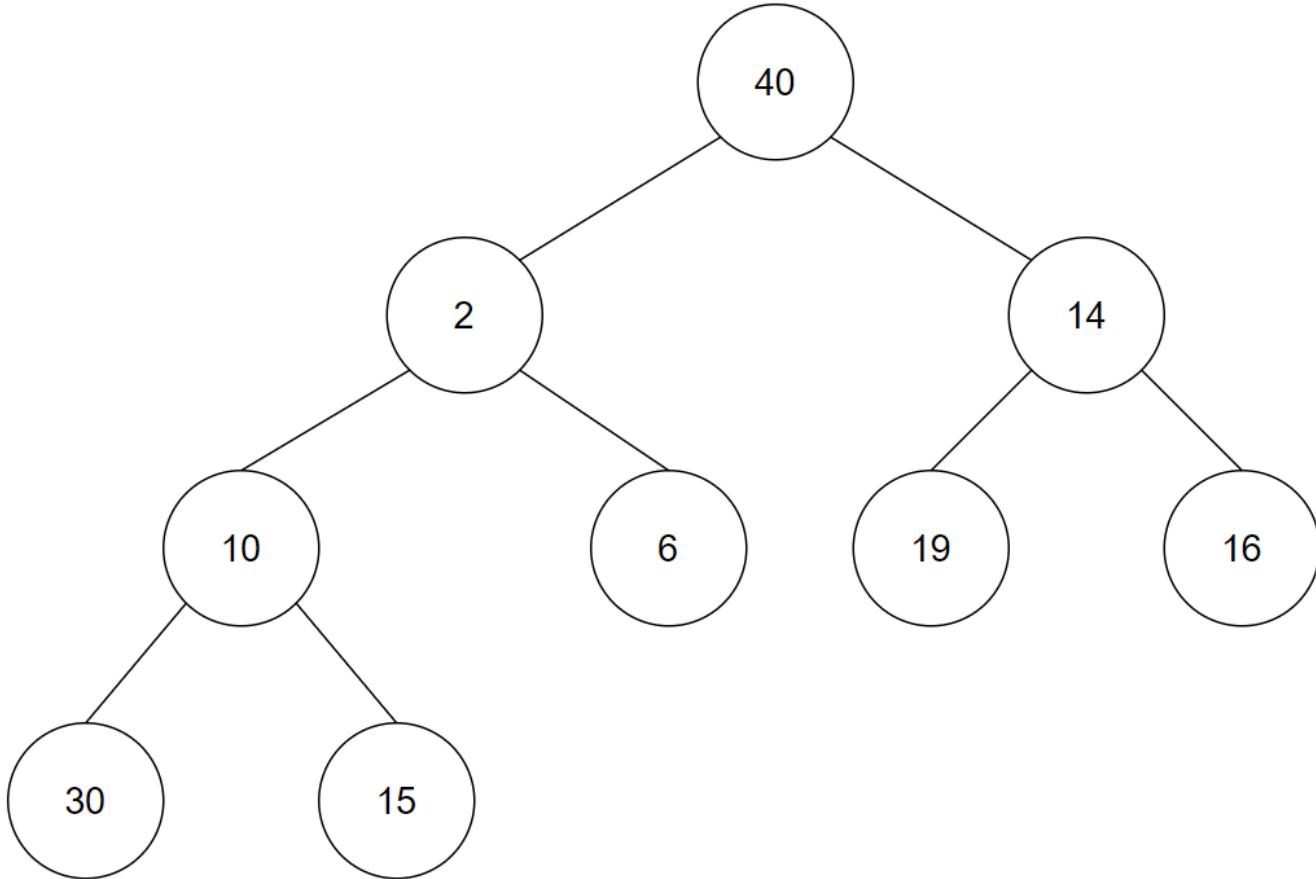
| 0  | 1 | 2  | 3  | 4 | 5  | 6  | 7  | 8  | 9 |
|----|---|----|----|---|----|----|----|----|---|
| 40 | 2 | 14 | 10 | 6 | 19 | 16 | 30 | 15 |   |

# Deleting from a Min Heap

Do we have a problem?

We have lost our heap property again.

The tree is still complete but 40 is too big to be the root of this min heap.



|    |   |    |    |   |    |    |    |    |   |
|----|---|----|----|---|----|----|----|----|---|
| 0  | 1 | 2  | 3  | 4 | 5  | 6  | 7  | 8  | 9 |
| 40 | 2 | 14 | 10 | 6 | 19 | 16 | 30 | 15 |   |

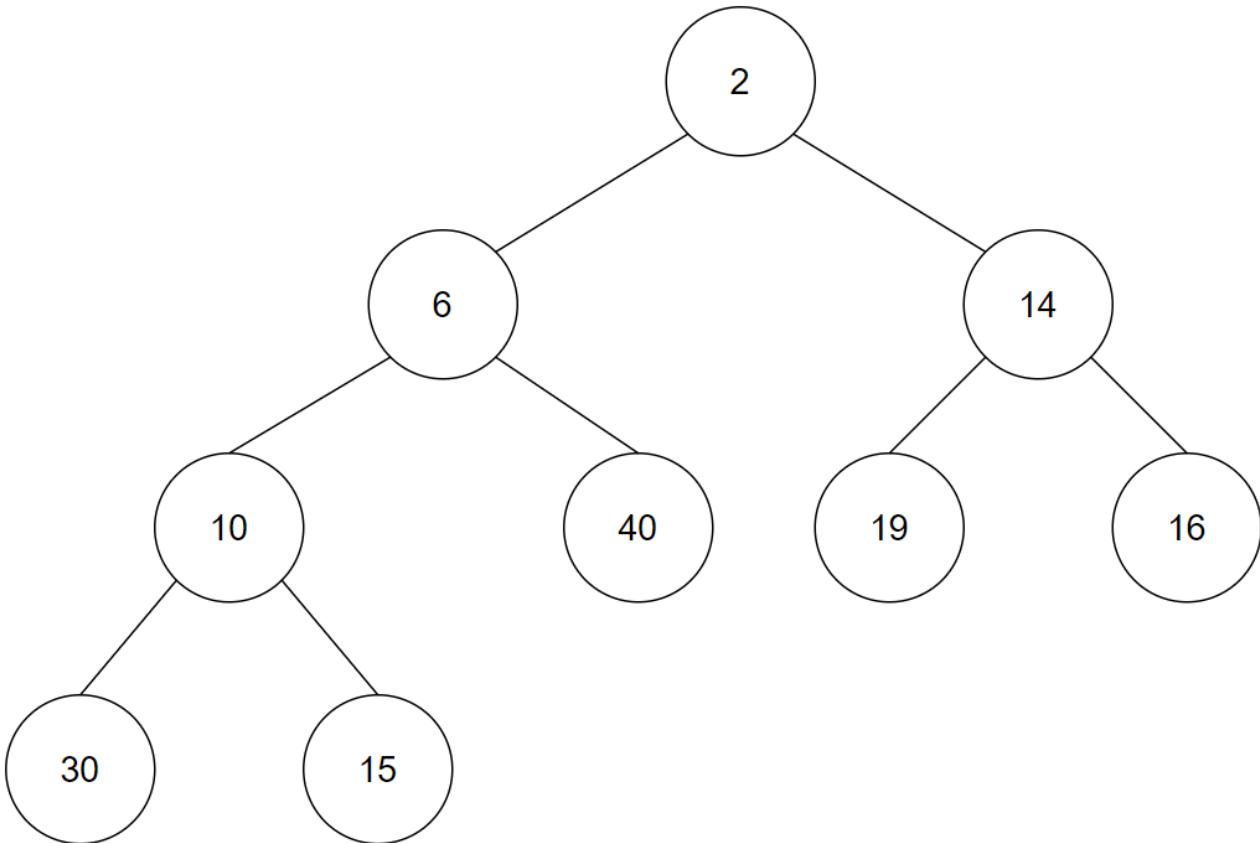
# Deleting from a Min Heap

We are back to a min heap

Complete tree

Parents smaller than children

Root is smallest value.



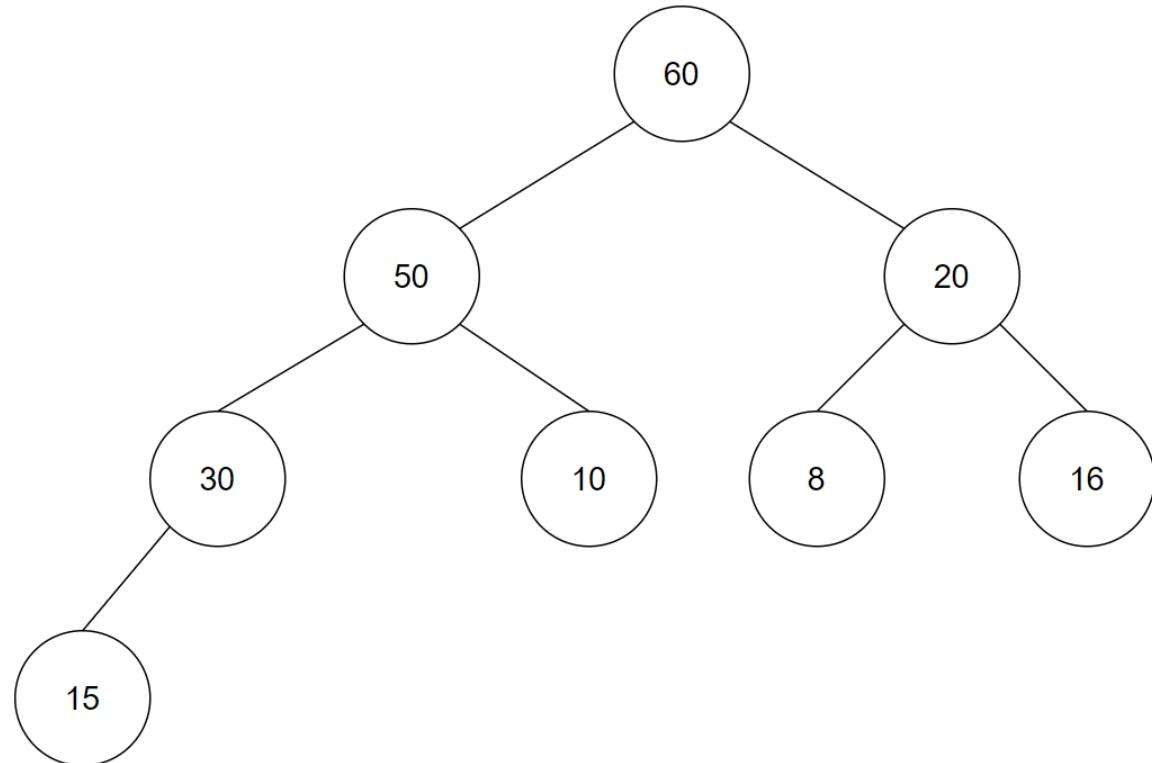
| 0 | 1 | 2  | 3  | 4  | 5  | 6  | 7  | 8  | 9 |
|---|---|----|----|----|----|----|----|----|---|
| 2 | 6 | 14 | 10 | 40 | 19 | 16 | 30 | 15 |   |

# Deleting from a Max Heap

Let's try the same technique with a max heap.

We want remove 60.

We take 60 out and swap it with the last value of 15.



| 0  | 1  | 2  | 3  | 4  | 5 | 6  | 7  |
|----|----|----|----|----|---|----|----|
| 60 | 50 | 20 | 30 | 10 | 8 | 16 | 15 |

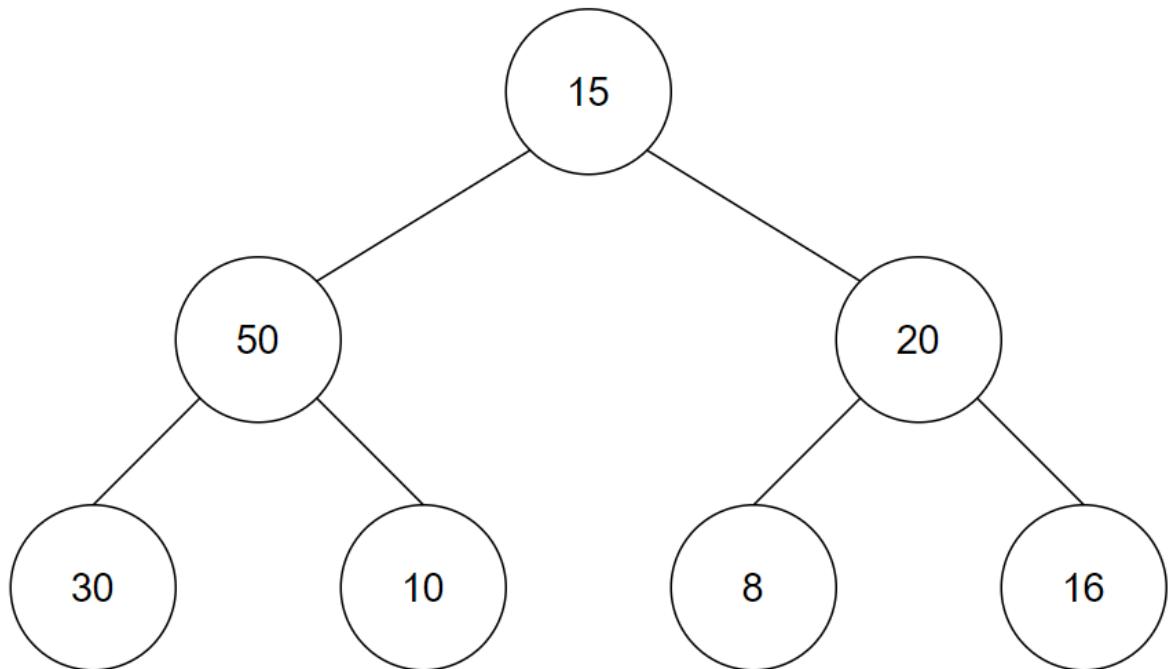
# Deleting from a Max Heap

Now let's fix our heap.

Swap 15 and 50

Swap 15 and 30

Are we OK now?

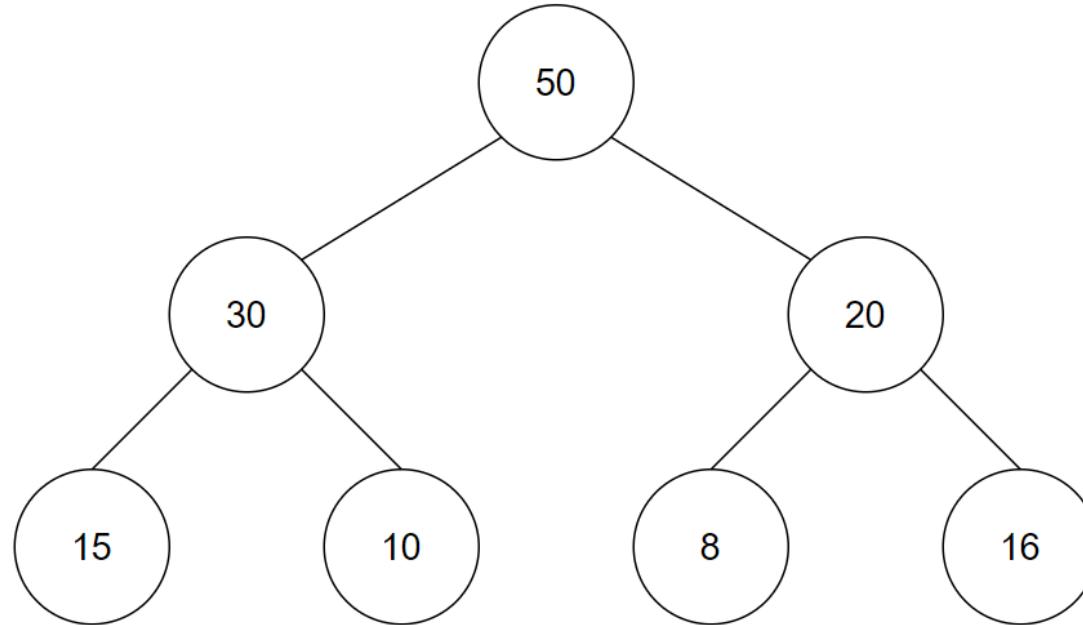


| 0  | 1  | 2  | 3  | 4  | 5 | 6  | 7 |
|----|----|----|----|----|---|----|---|
| 15 | 50 | 20 | 30 | 10 | 8 | 16 |   |

# Deleting from a Max Heap

Yes!

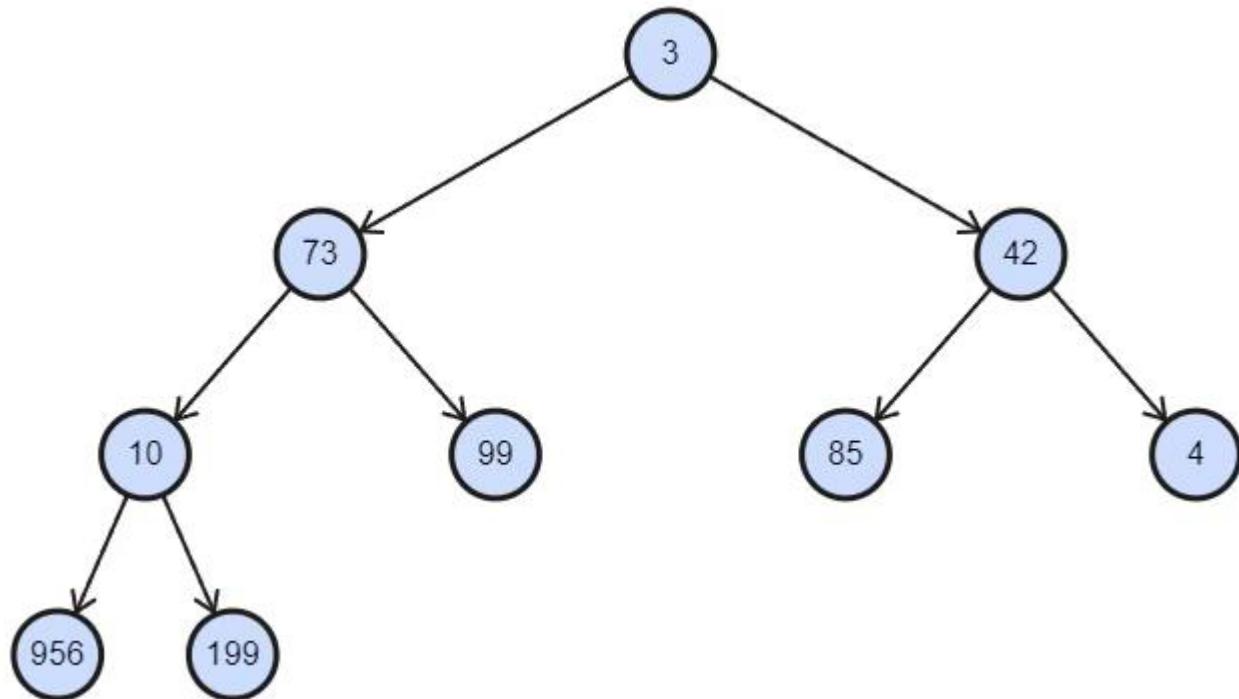
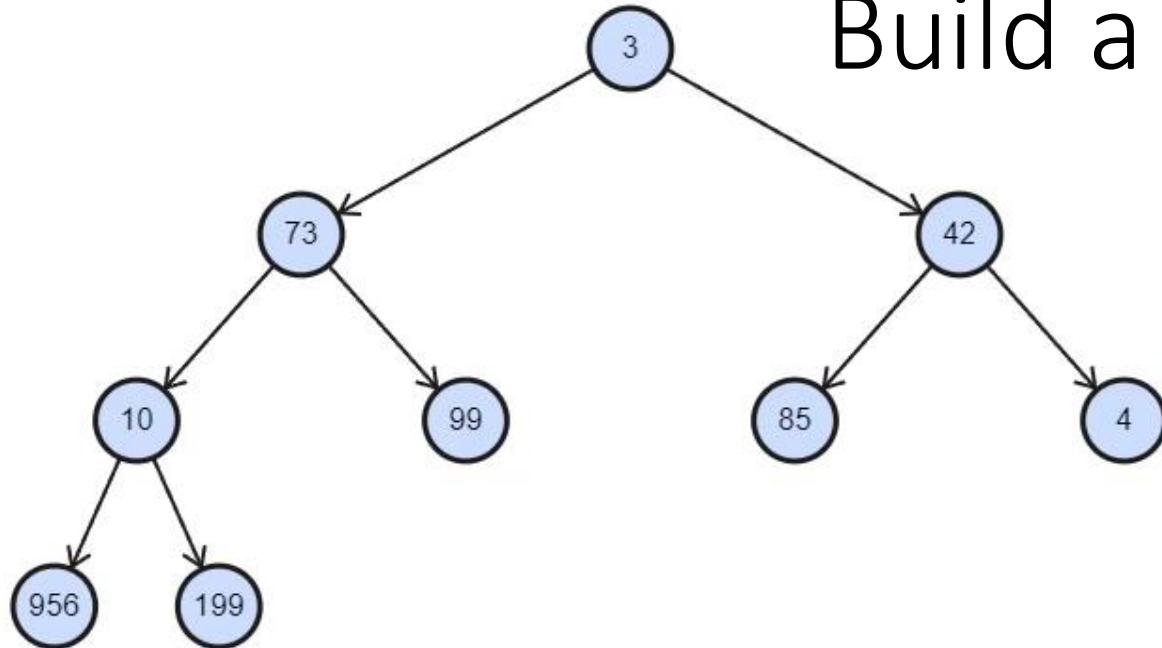
Our tree is complete and we have maintained our heap property.



| 0  | 1  | 2  | 3  | 4  | 5 | 6  | 7 |
|----|----|----|----|----|---|----|---|
| 50 | 30 | 20 | 15 | 10 | 8 | 16 |   |

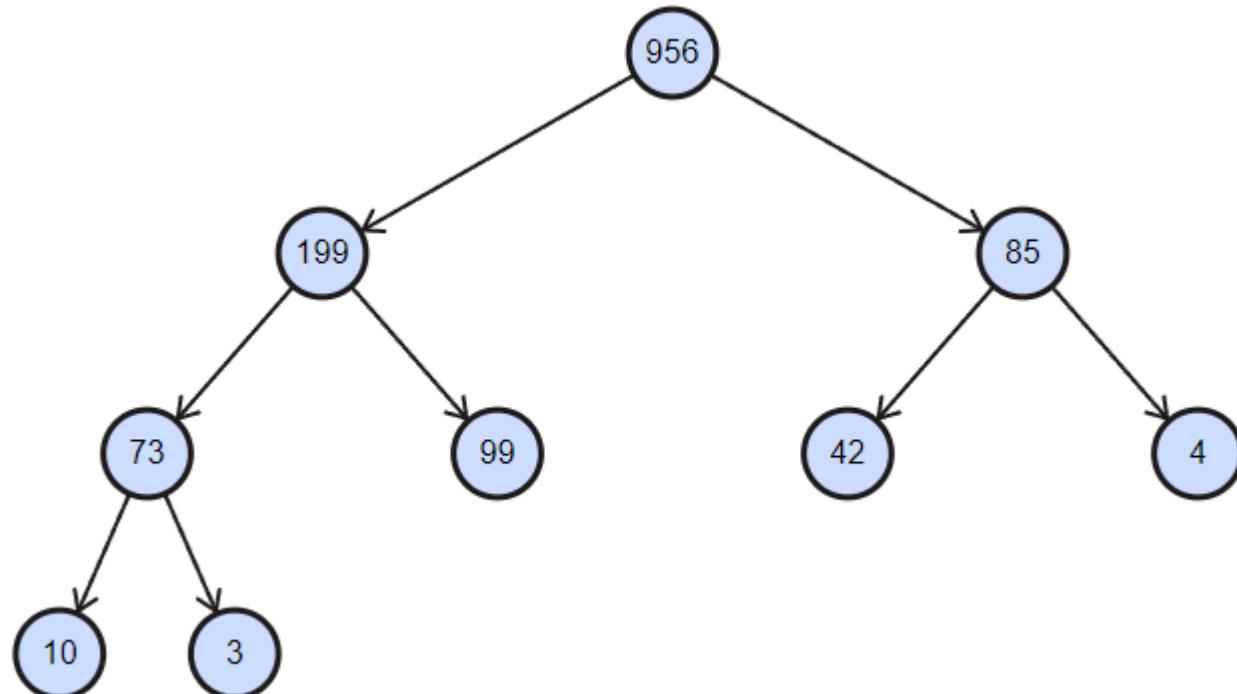
|   |    |    |    |    |    |   |     |     |
|---|----|----|----|----|----|---|-----|-----|
| 3 | 73 | 42 | 10 | 99 | 85 | 4 | 956 | 199 |
|---|----|----|----|----|----|---|-----|-----|

## Build a Max Heap

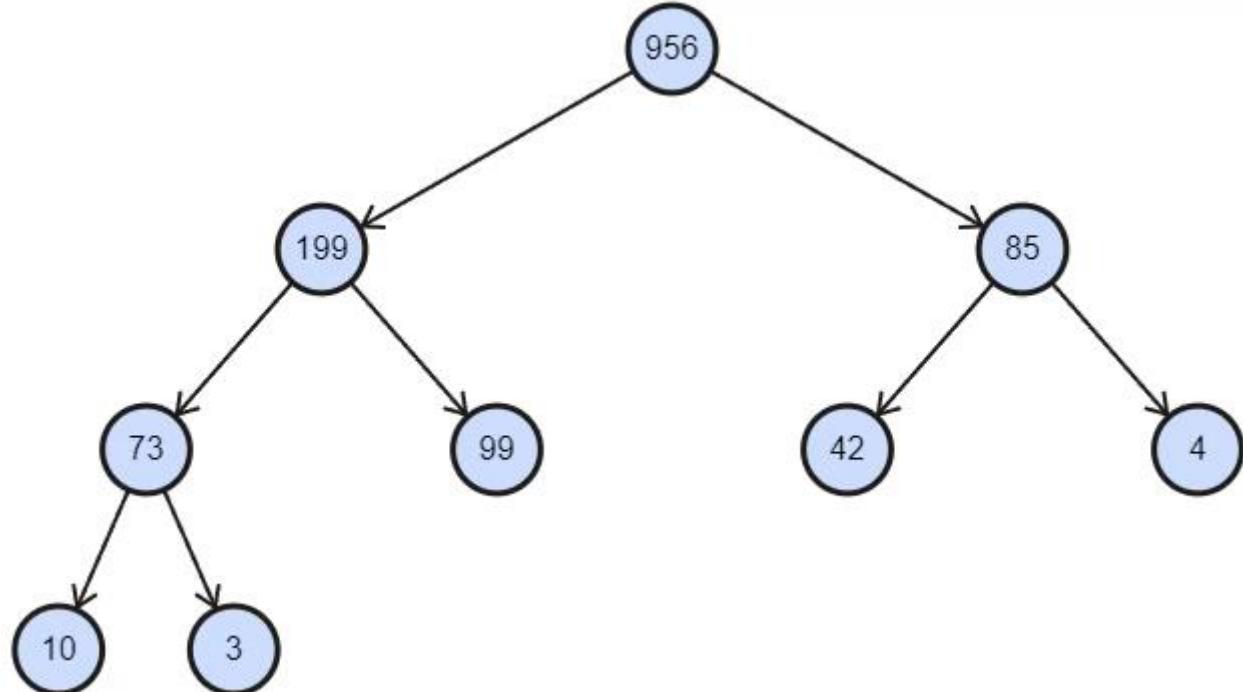


|     |     |    |    |    |    |   |    |   |
|-----|-----|----|----|----|----|---|----|---|
| 956 | 199 | 85 | 73 | 99 | 42 | 4 | 10 | 3 |
|-----|-----|----|----|----|----|---|----|---|

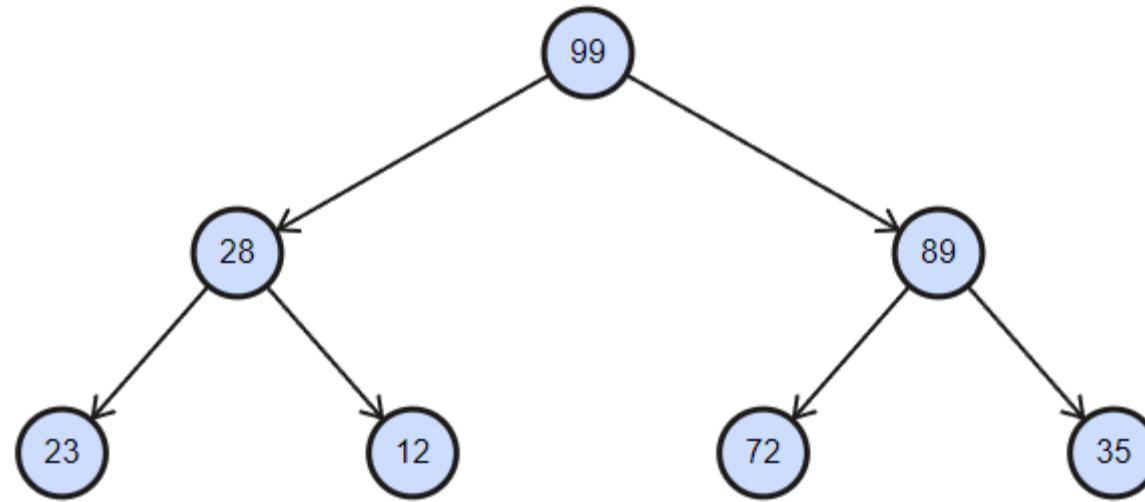
## Insert a New Value in a Max Heap



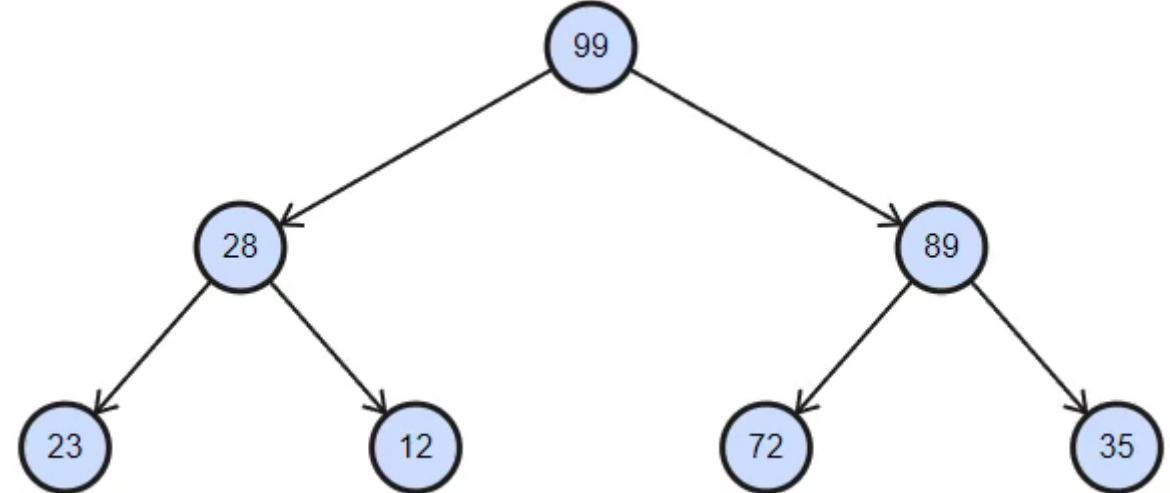
Insert 200



# Delete from Max Heap

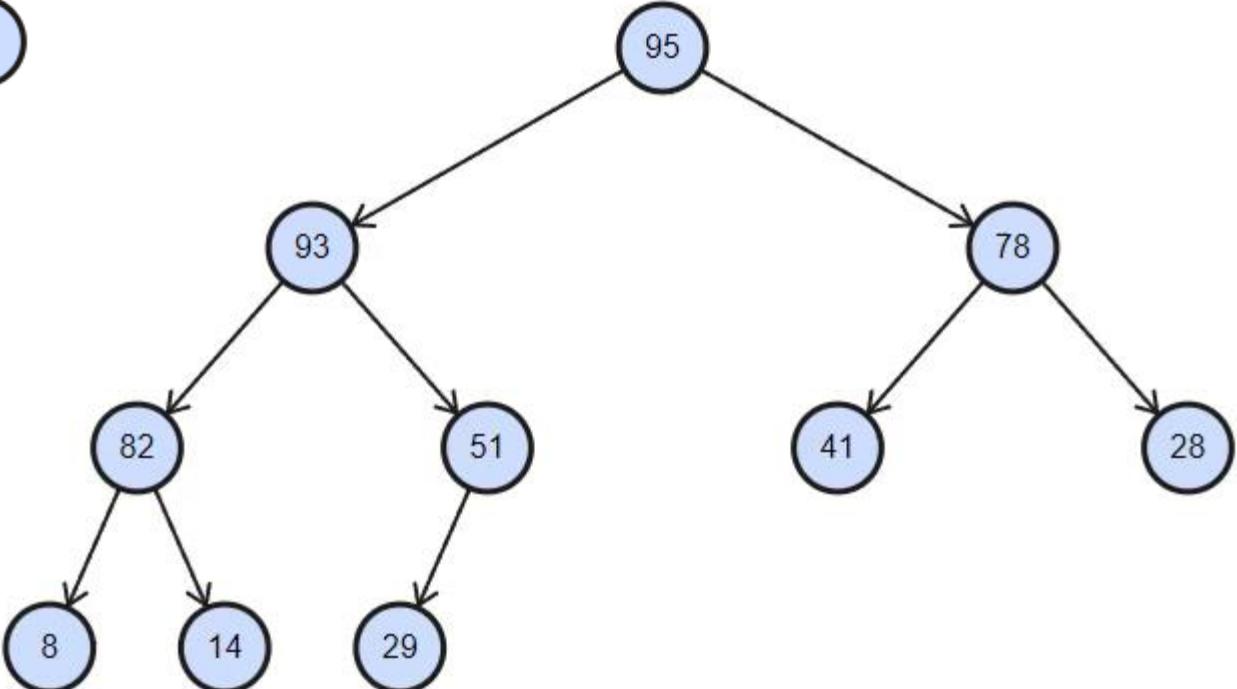
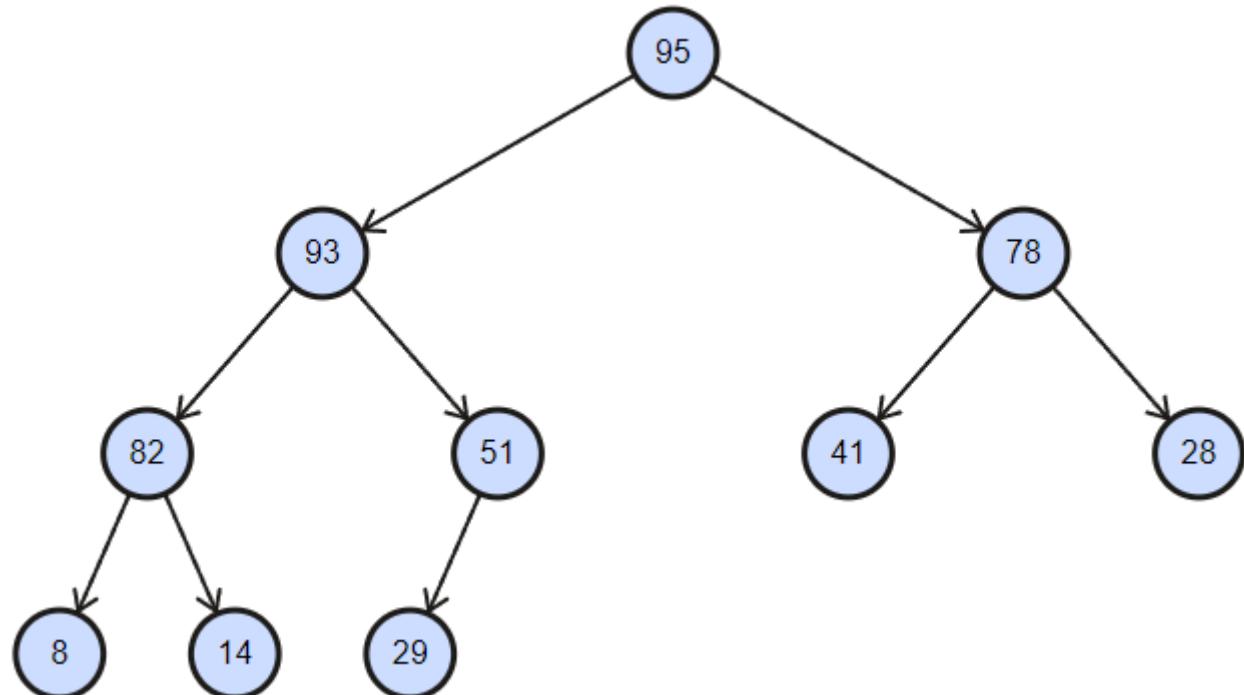


Delete root



|    |    |    |    |    |    |    |   |    |    |
|----|----|----|----|----|----|----|---|----|----|
| 95 | 93 | 78 | 82 | 51 | 41 | 28 | 8 | 14 | 29 |
|----|----|----|----|----|----|----|---|----|----|

## Delete from Max Heap

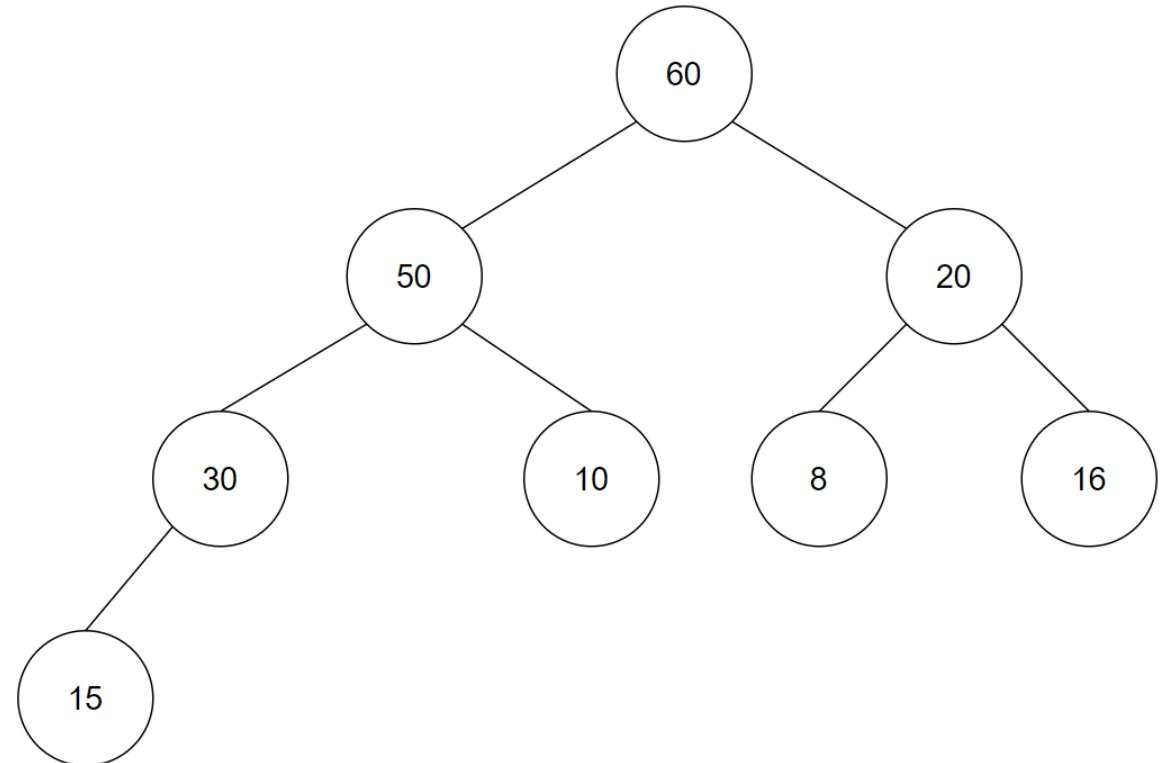


# Sorting Using a Max Heap

What happens if we keep deleting the root from the heap?

When we delete the root, we are just taking it out of index 0 – we are not removing array element 0 – doing so just allows us to put the last element of the array in index 0.

What if we stash that 60 in that spot we just emptied?



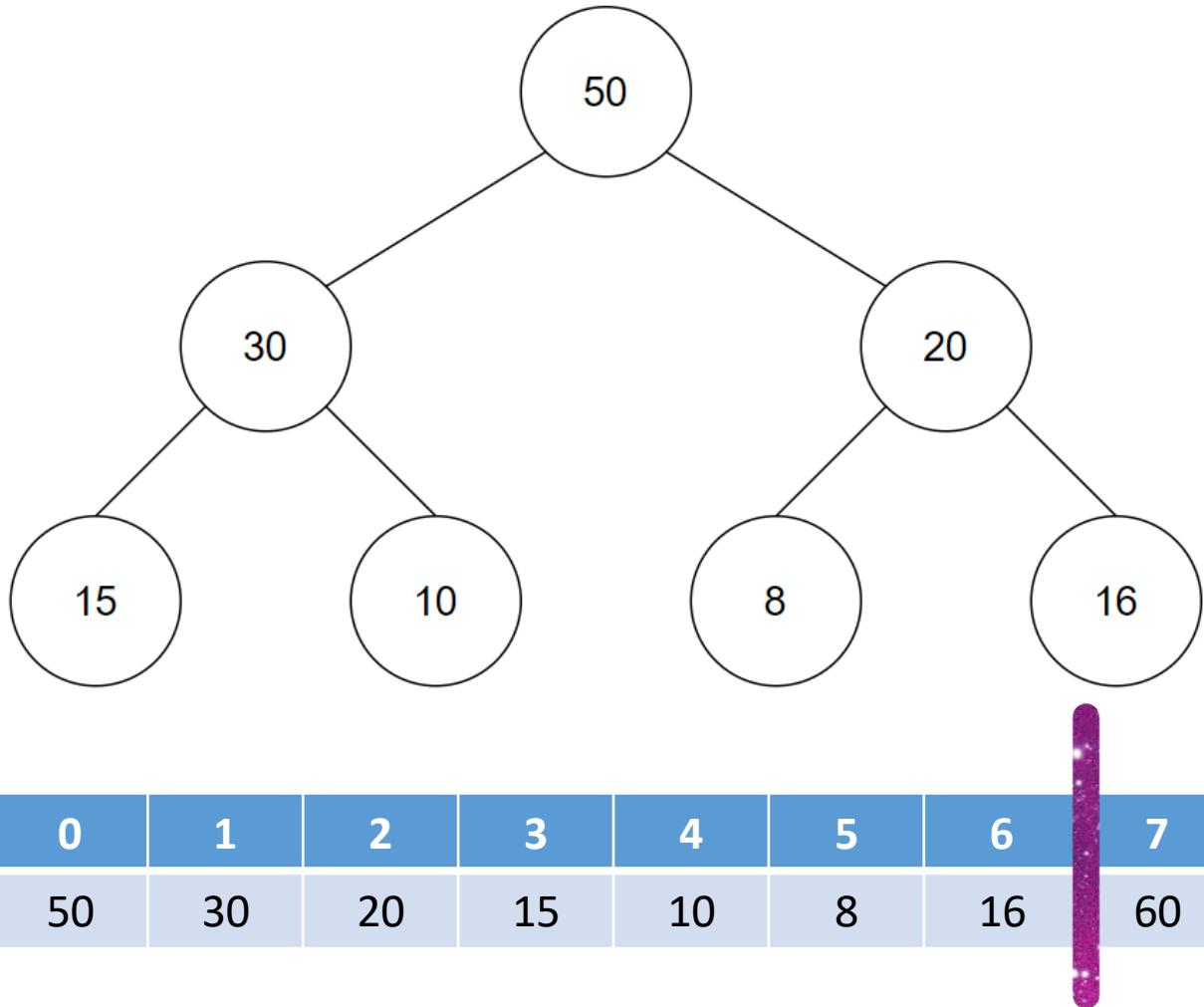
| 0  | 1  | 2  | 3  | 4  | 5 | 6  | 7  |
|----|----|----|----|----|---|----|----|
| 60 | 50 | 20 | 30 | 10 | 8 | 16 | 15 |

# Sorting Using a Max Heap

So after heapifying, our heap is back to a good state and we make note that our array ends at element 6.

We are just using the space in element 7 to store that 60.

So what happens if we repeat this process?



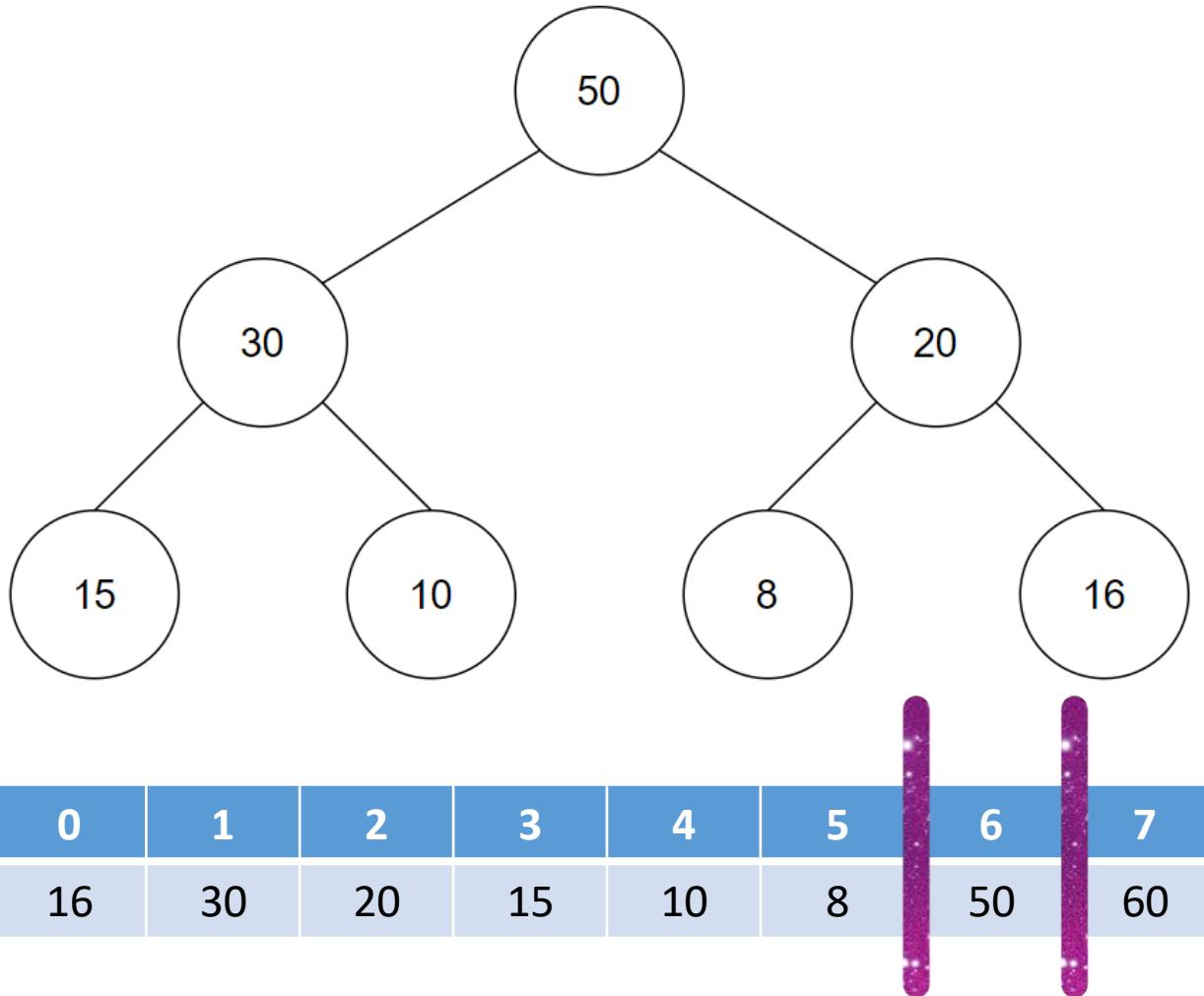
# Sorting Using a Max Heap

We swap the 50 and the 16.

Heapify time!!

We make note that the end of our array is now element 5.

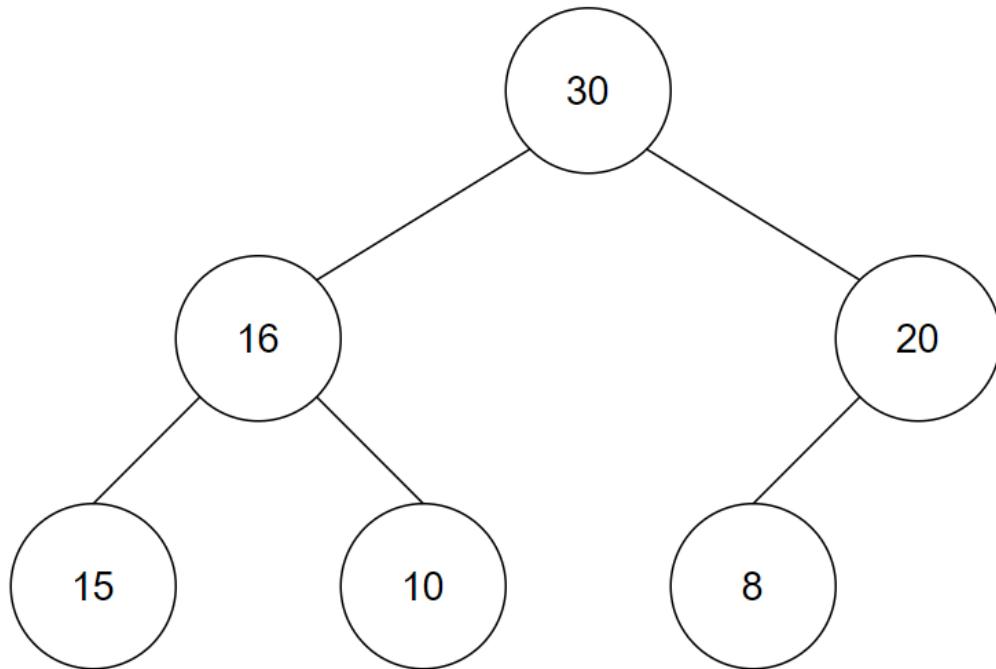
This also means that we only have 6 elements in our heap.



# Sorting Using a Max Heap

Repeat.

Swap 30 and 8 and heapify.



|    |    |    |    |    |   |    |    |
|----|----|----|----|----|---|----|----|
| 0  | 1  | 2  | 3  | 4  | 5 | 6  | 7  |
| 30 | 16 | 20 | 15 | 10 | 8 | 50 | 60 |

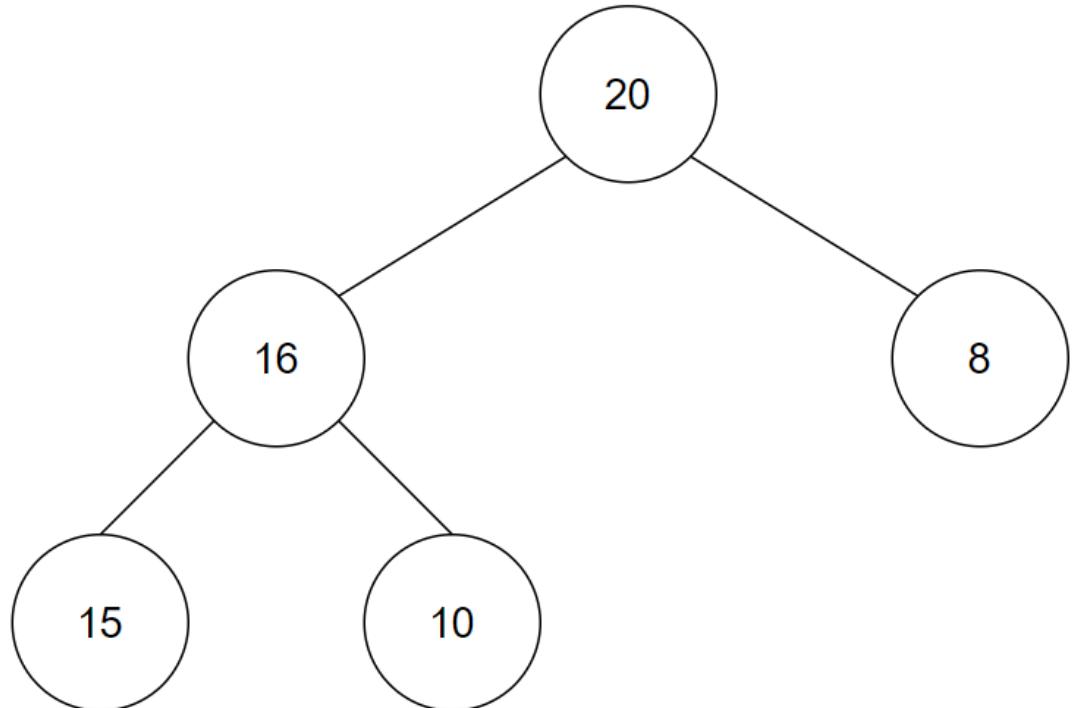
# Sorting Using a Max Heap

Repeat.

Swap 20 and 10

Why not swap 20 and 8?

Heapify.



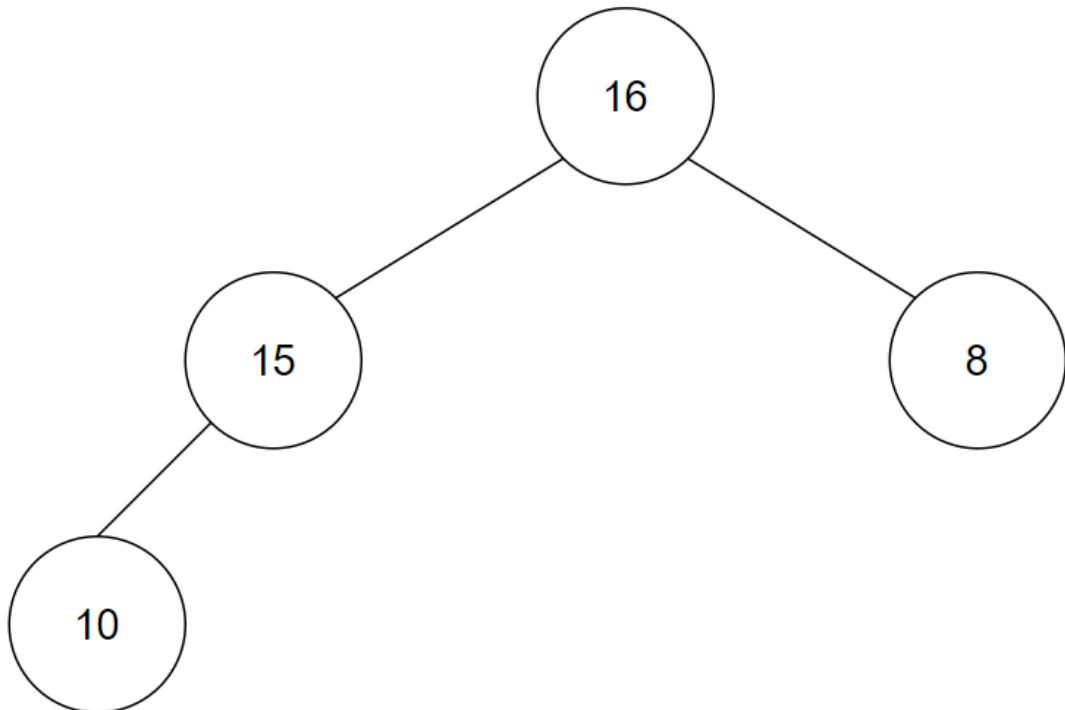
|    |    |   |    |    |    |    |    |
|----|----|---|----|----|----|----|----|
| 0  | 1  | 2 | 3  | 4  | 5  | 6  | 7  |
| 20 | 16 | 8 | 15 | 10 | 30 | 50 | 60 |

# Sorting Using a Max Heap

Repeat.

Swap 16 and 10.

Heapify.



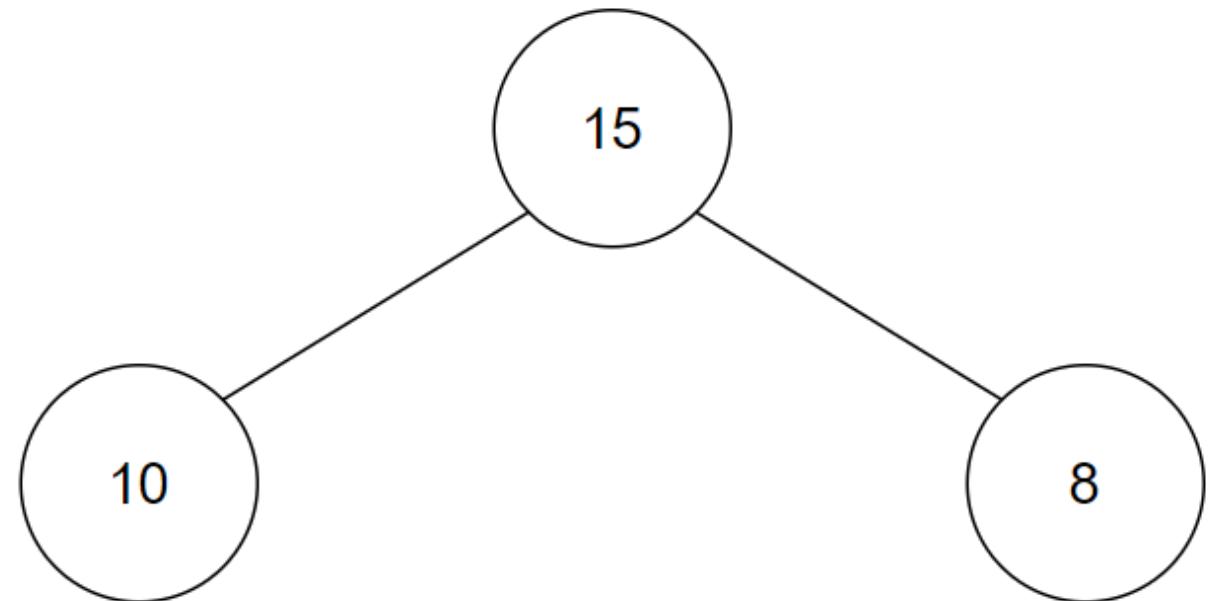
|    |    |   |    |    |    |    |    |
|----|----|---|----|----|----|----|----|
| 0  | 1  | 2 | 3  | 4  | 5  | 6  | 7  |
| 16 | 15 | 8 | 10 | 20 | 30 | 50 | 60 |

# Sorting Using a Max Heap

Repeat.

Swap 8 and 15.

Heapify



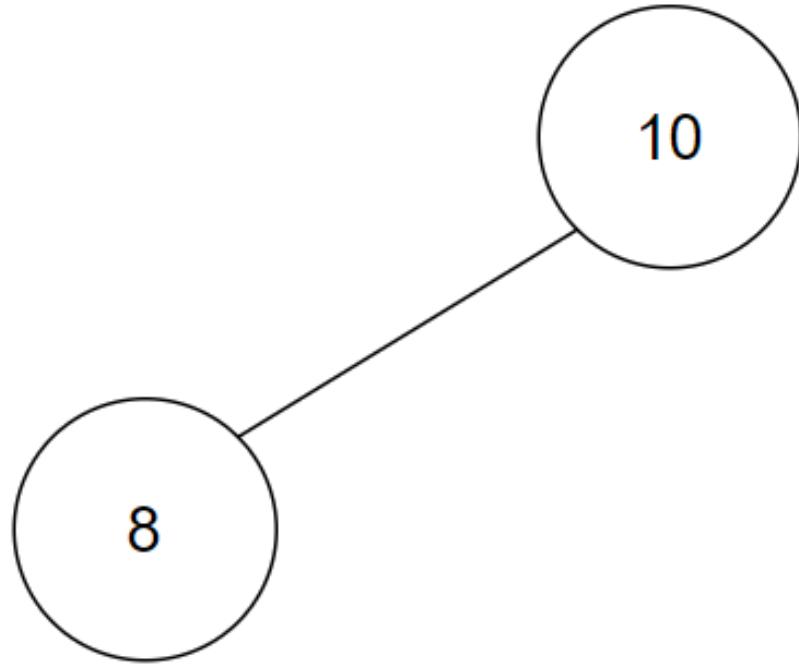
| 0  | 1  | 2 | 3  | 4  | 5  | 6  | 7  |
|----|----|---|----|----|----|----|----|
| 15 | 10 | 8 | 16 | 20 | 30 | 50 | 60 |

# Sorting Using a Max Heap

Repeat.

Swap 10 and 8.

Do we need to heapify?



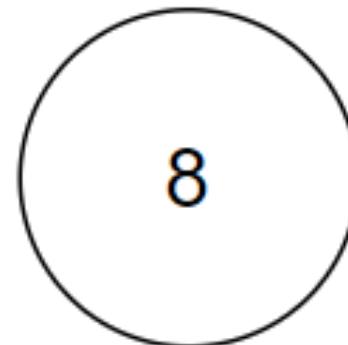
| 0  | 1 | 2  | 3  | 4  | 5  | 6  | 7  |
|----|---|----|----|----|----|----|----|
| 10 | 8 | 15 | 16 | 20 | 30 | 50 | 60 |

A horizontal array of 8 cells, indexed from 0 to 7. The values are: 10, 8, 15, 16, 20, 30, 50, 60. A vertical magenta bar is positioned over the cell at index 1, highlighting the value 8.

# Sorting Using a Max Heap

No.

Final node maintains the heap property.

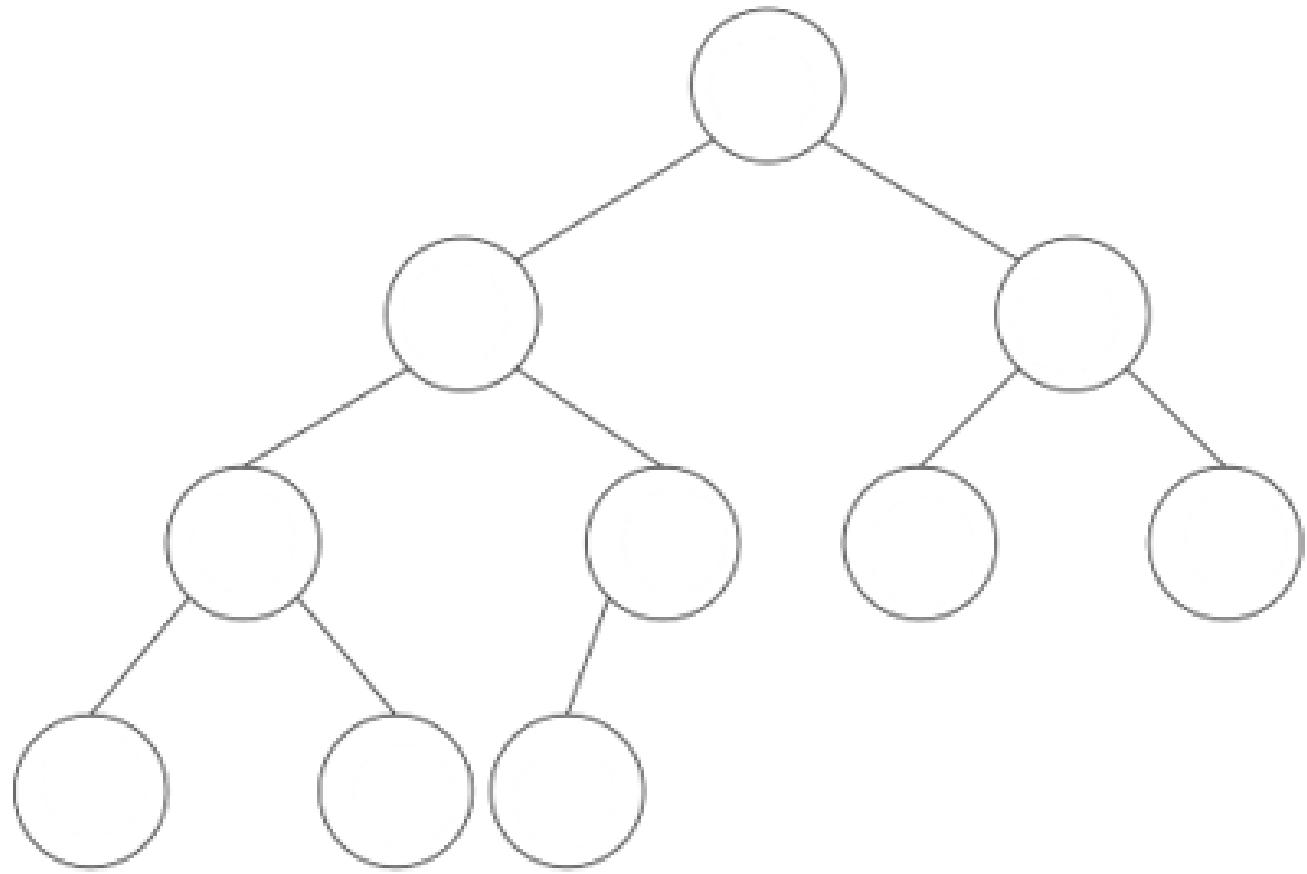


What is in our array now?

It is sorted.

| 0 | 1  | 2  | 3  | 4  | 5  | 6  | 7  |
|---|----|----|----|----|----|----|----|
| 8 | 10 | 15 | 16 | 20 | 30 | 50 | 60 |

- 1.Create the tree given the array.
- 2.Sort the array using a min heap delete



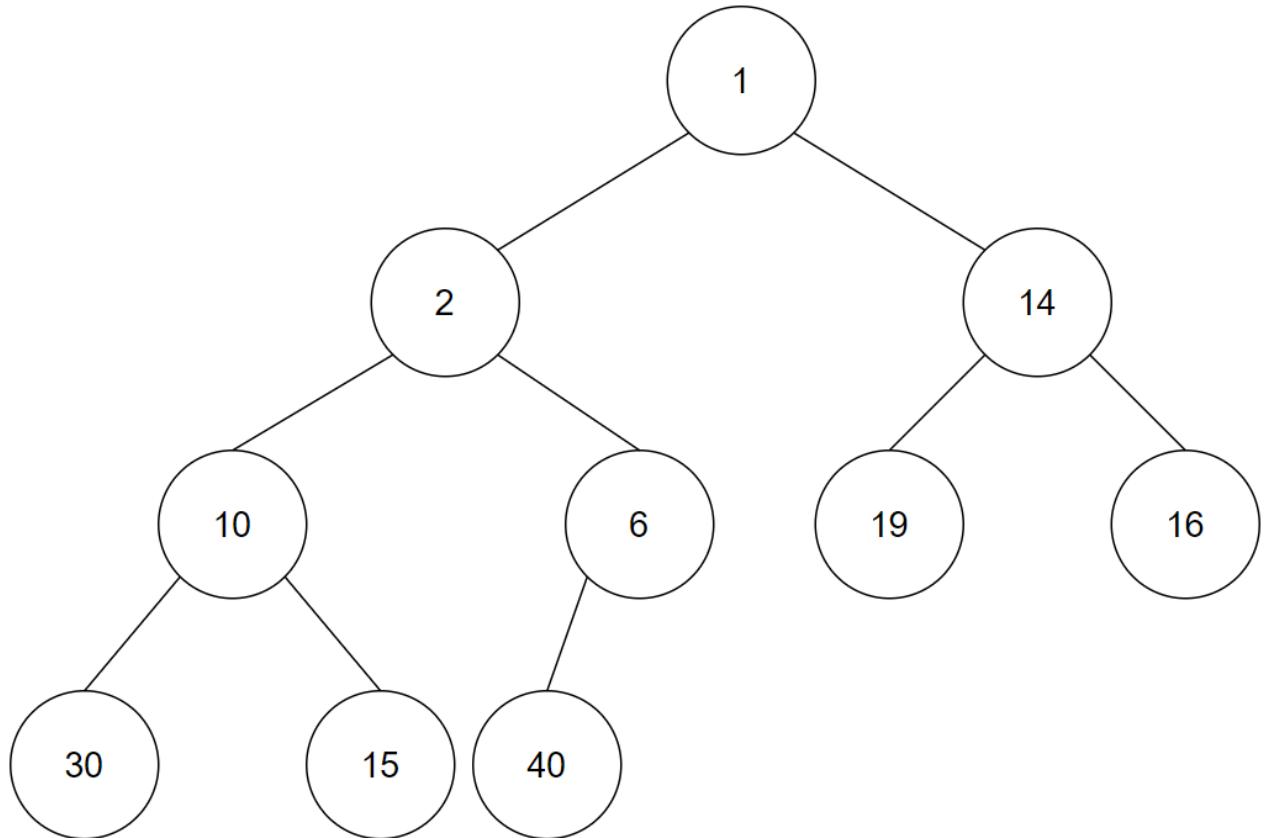
| 0 | 1 | 2  | 3  | 4 | 5  | 6  | 7  | 8  | 9  |
|---|---|----|----|---|----|----|----|----|----|
| 1 | 2 | 14 | 10 | 6 | 19 | 16 | 30 | 15 | 40 |

# Sorting Using a Min Heap

Does the process work differently  
for a Min Heap?

Will the result be different?

Let's try it.

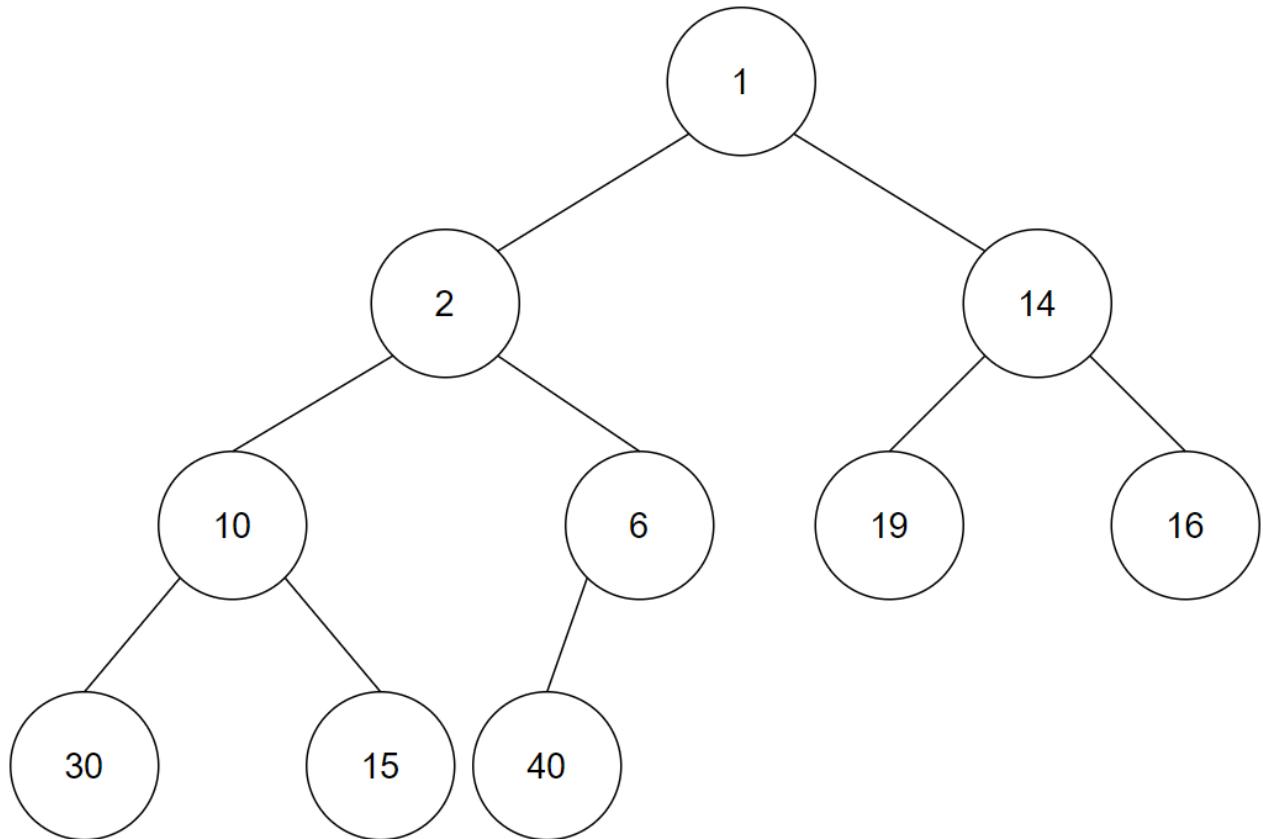


|   |   |    |    |   |    |    |    |    |    |
|---|---|----|----|---|----|----|----|----|----|
| 0 | 1 | 2  | 3  | 4 | 5  | 6  | 7  | 8  | 9  |
| 1 | 2 | 14 | 10 | 6 | 19 | 16 | 30 | 15 | 40 |

# Sorting Using a Min Heap

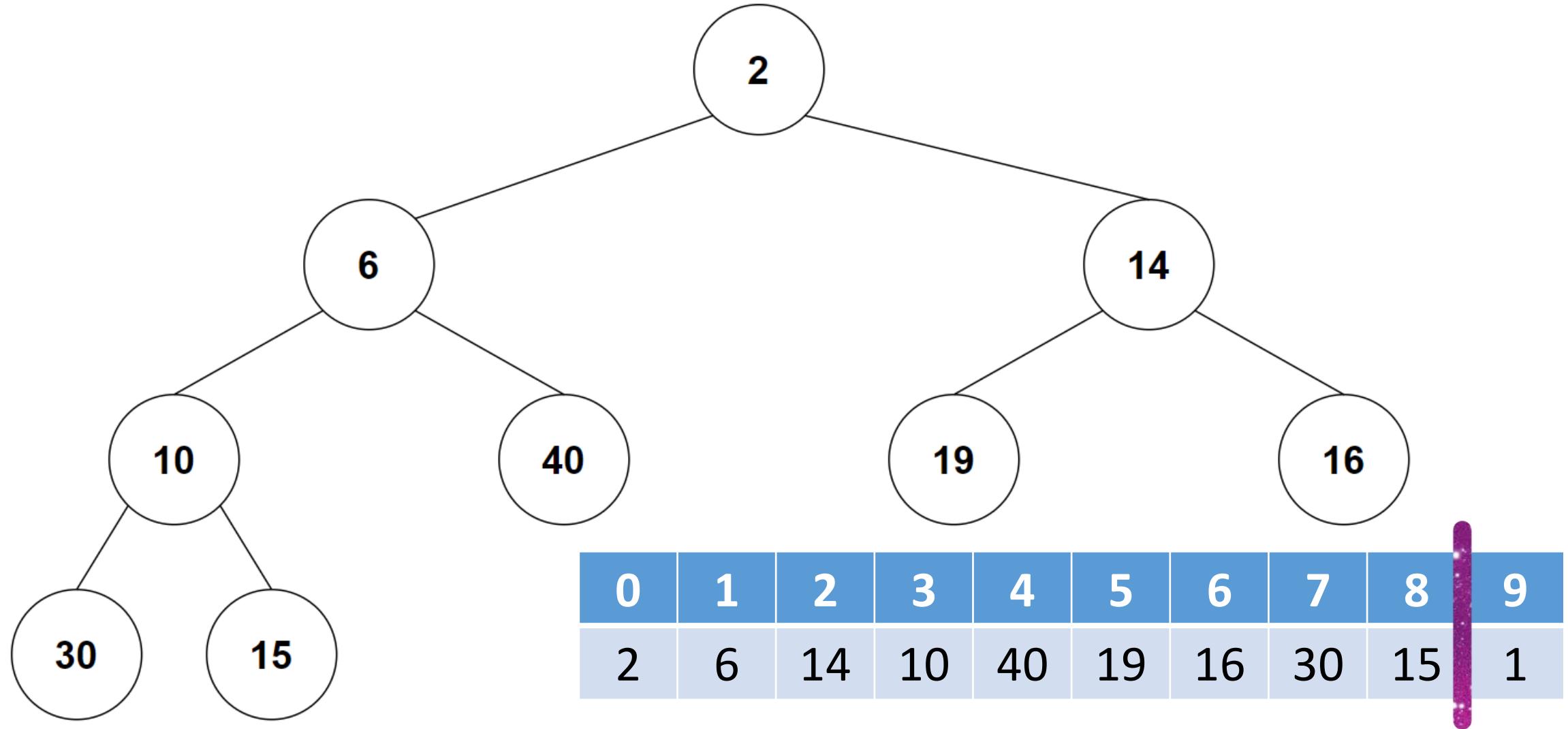
Swap 1 and 40

Heapify!

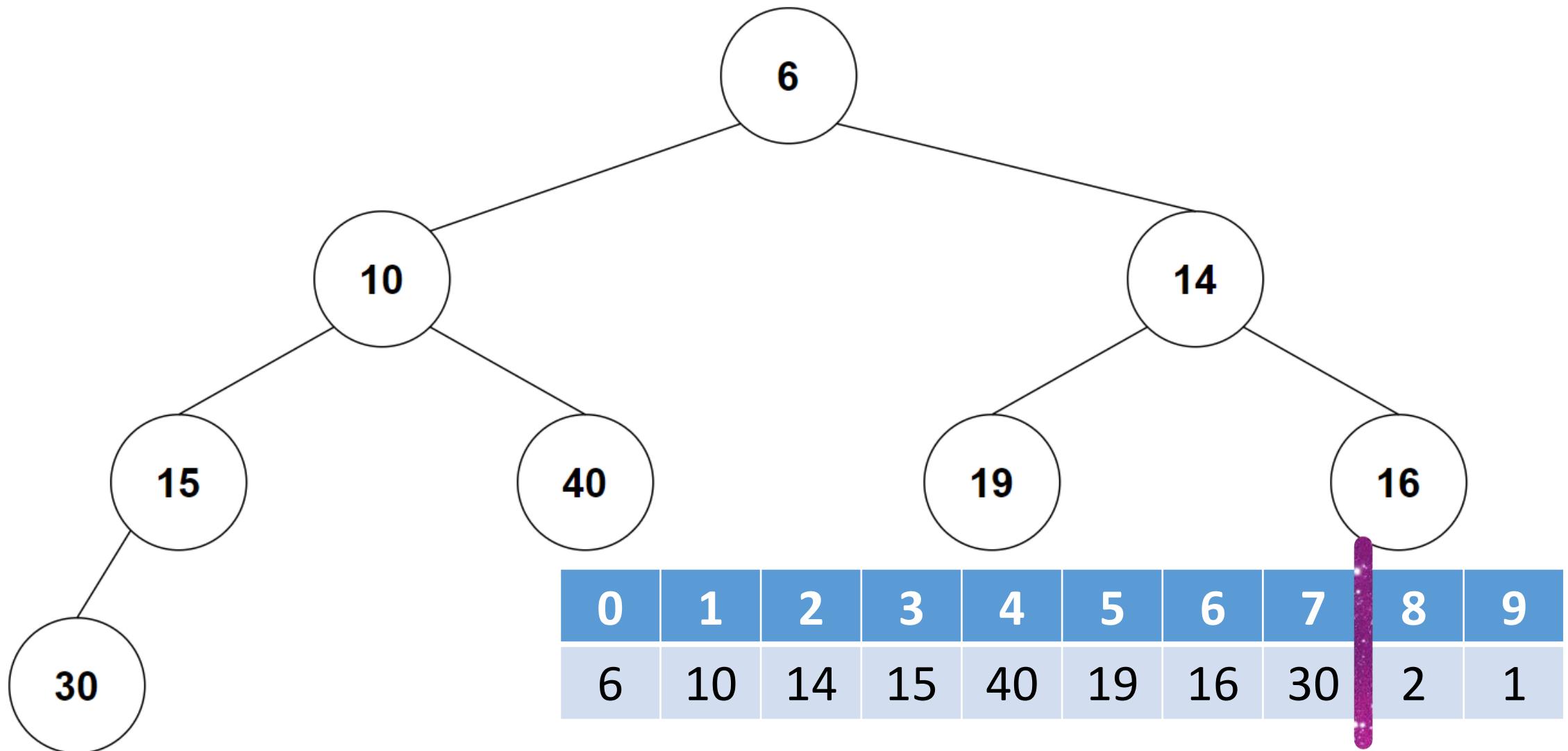


| 0  | 1 | 2  | 3  | 4 | 5  | 6  | 7  | 8  | 9 |
|----|---|----|----|---|----|----|----|----|---|
| 40 | 2 | 14 | 10 | 6 | 19 | 16 | 30 | 15 | 1 |

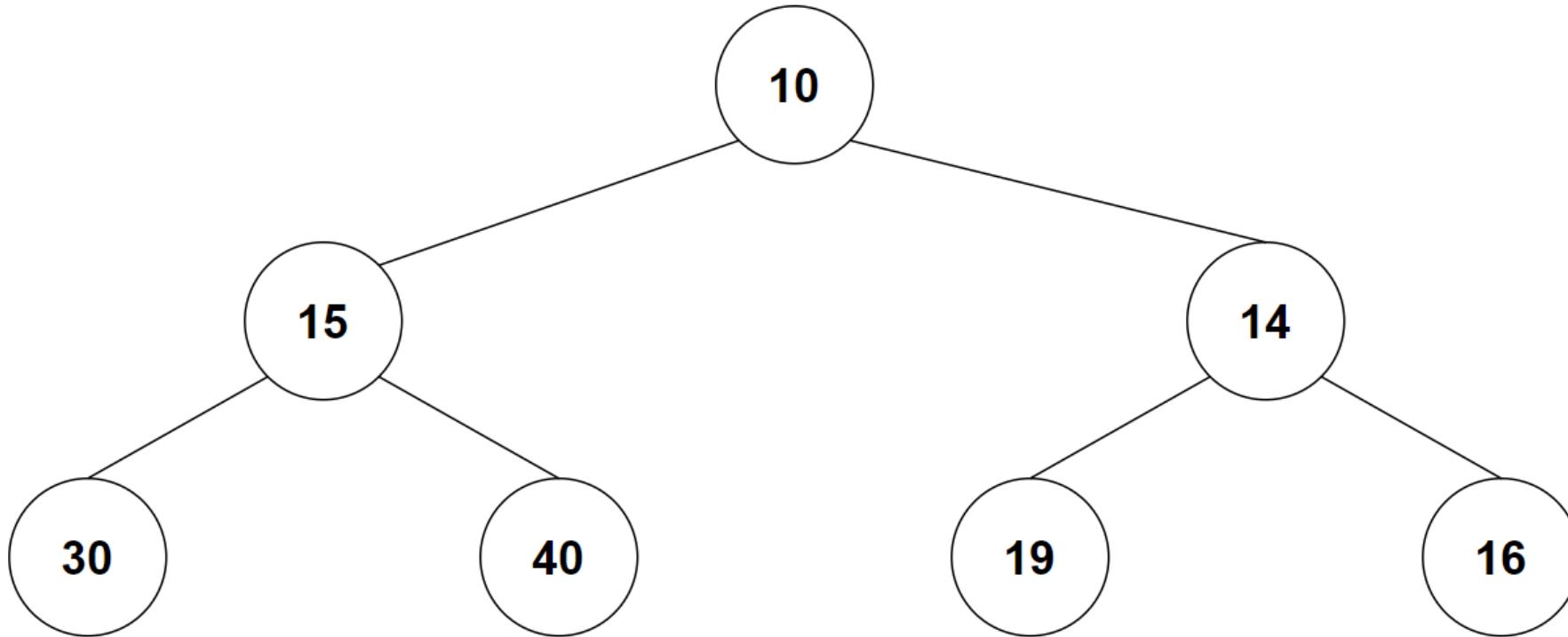
# Sorting Using a Min Heap



# Sorting Using a Min Heap



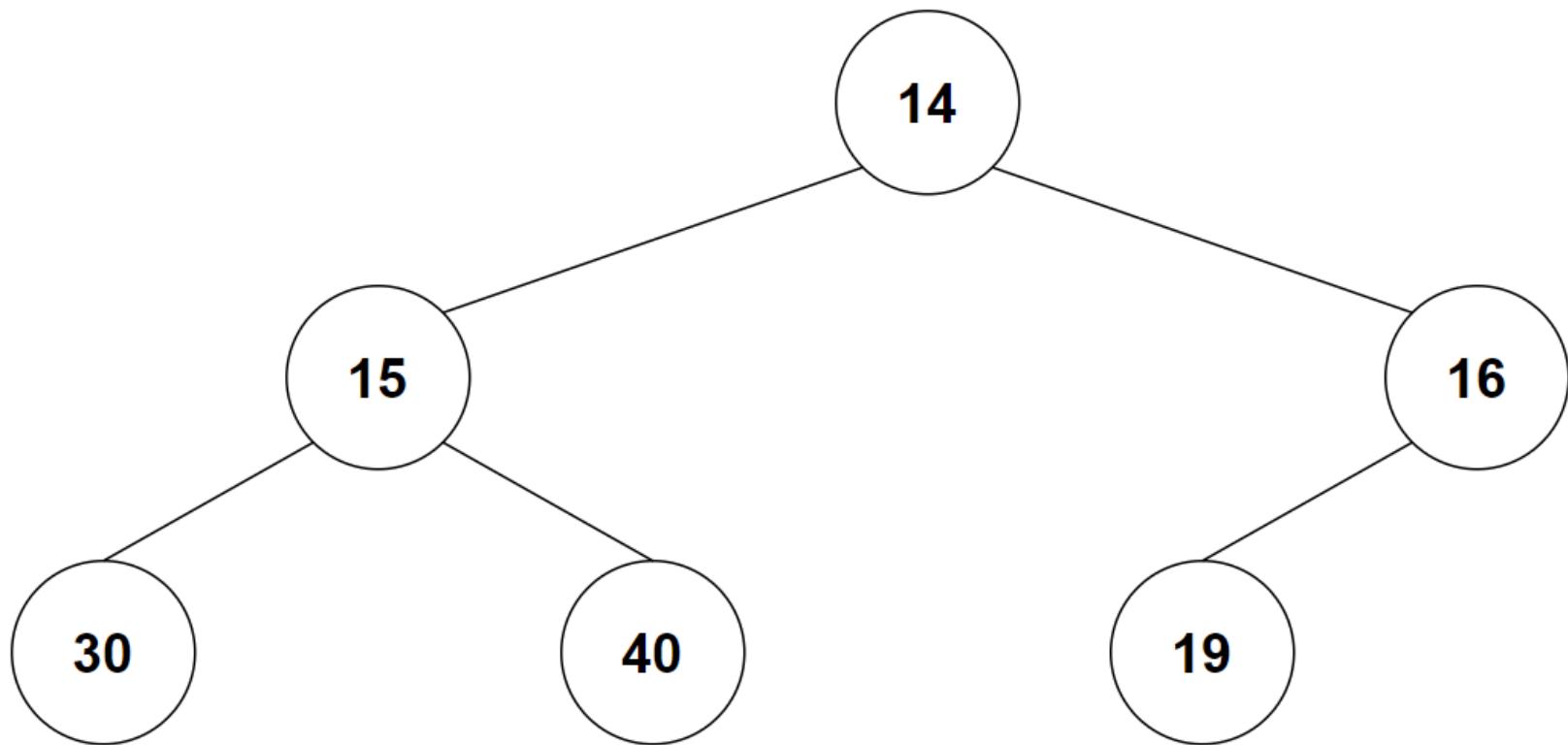
# Sorting Using a Min Heap



|    |    |    |    |    |    |    |   |   |   |
|----|----|----|----|----|----|----|---|---|---|
| 0  | 1  | 2  | 3  | 4  | 5  | 6  | 7 | 8 | 9 |
| 10 | 15 | 14 | 30 | 40 | 19 | 16 | 6 | 2 | 1 |

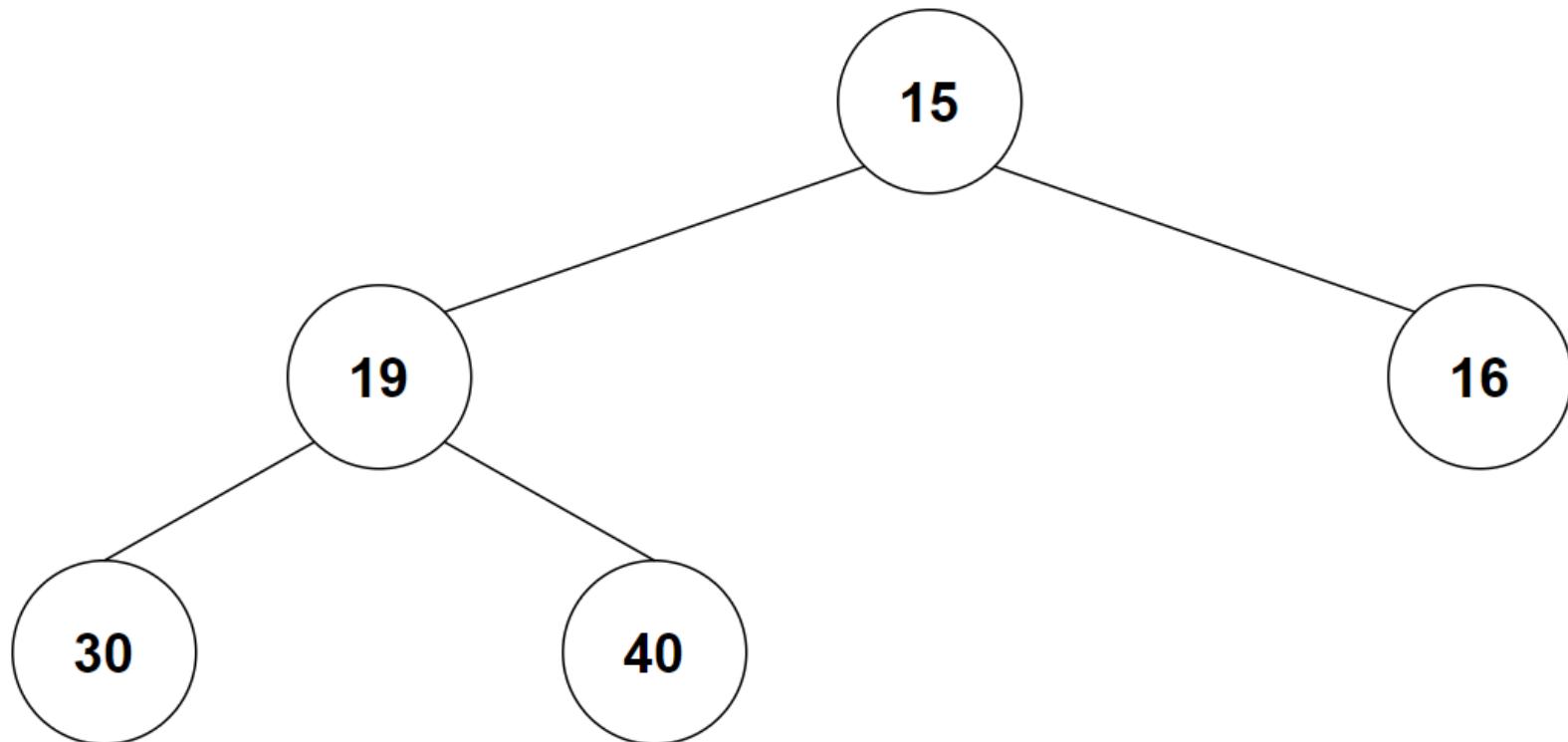
A horizontal array of 10 cells, indexed 0 to 9 from left to right. Cells 0 through 6 contain the values 10, 15, 14, 30, 40, and 19 respectively. Cells 7, 8, and 9 contain the values 6, 2, and 1 respectively. A vertical magenta bar is positioned over cell 6.

# Sorting Using a Min Heap



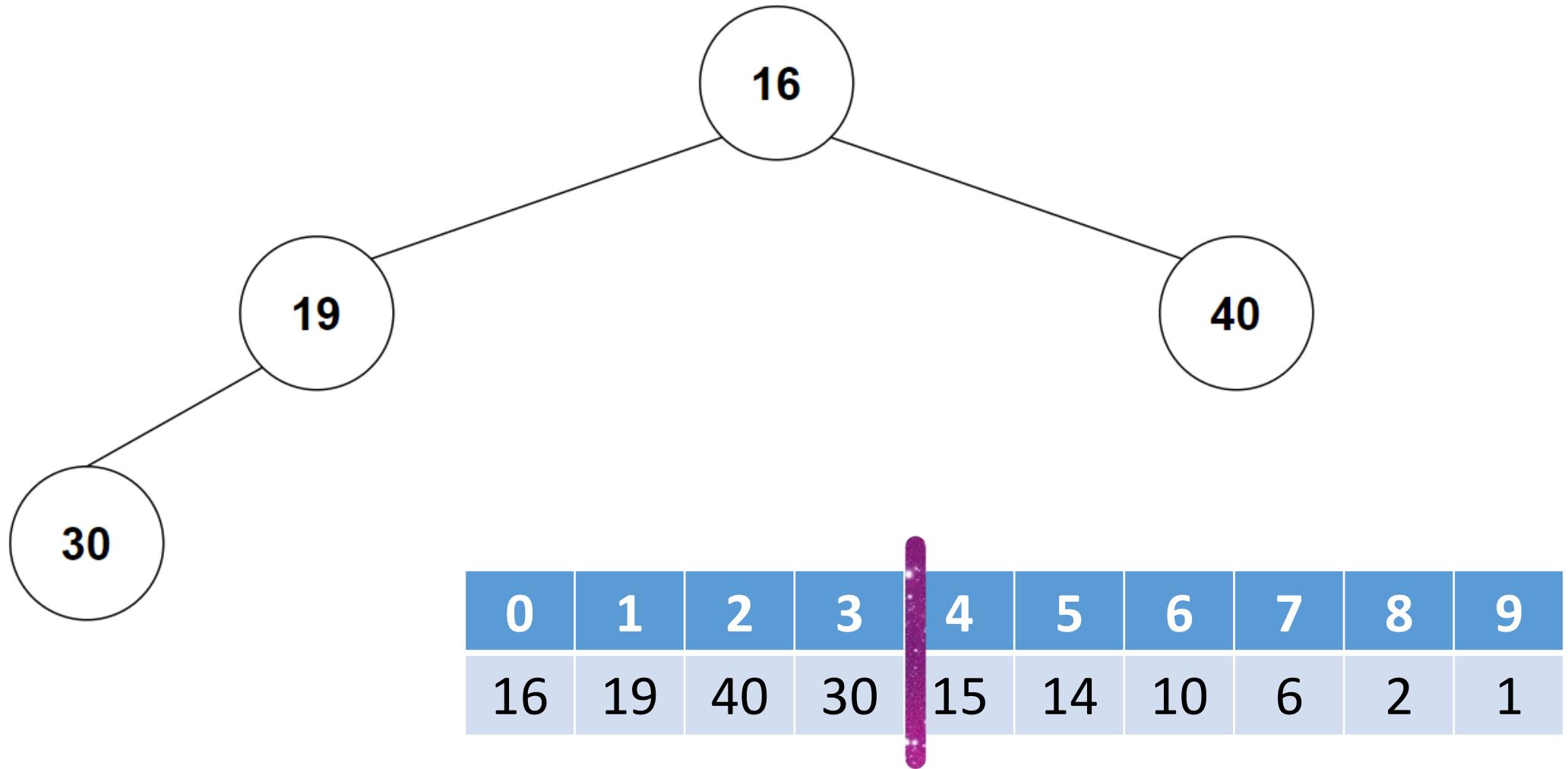
|    |    |    |    |    |    |    |   |   |   |
|----|----|----|----|----|----|----|---|---|---|
| 0  | 1  | 2  | 3  | 4  | 5  | 6  | 7 | 8 | 9 |
| 14 | 15 | 16 | 30 | 40 | 19 | 10 | 6 | 2 | 1 |

# Sorting Using a Min Heap

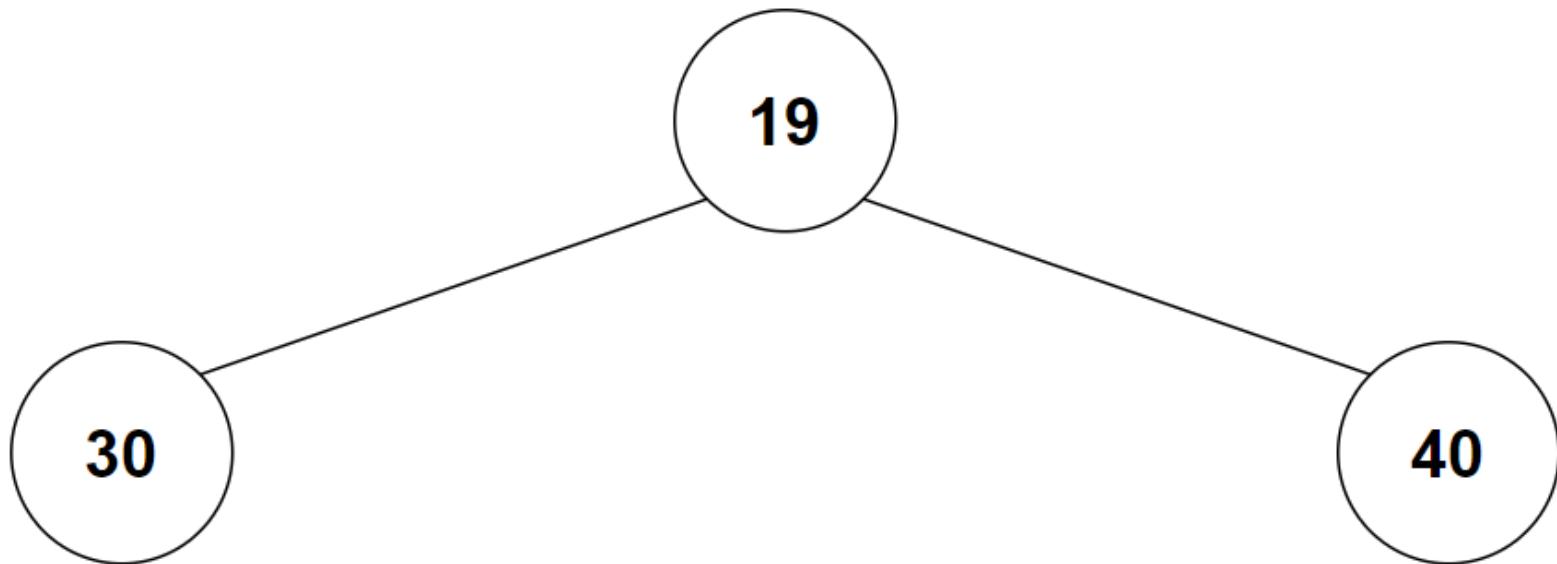


|    |    |    |    |    |    |    |   |   |   |
|----|----|----|----|----|----|----|---|---|---|
| 0  | 1  | 2  | 3  | 4  | 5  | 6  | 7 | 8 | 9 |
| 15 | 19 | 16 | 30 | 40 | 14 | 10 | 6 | 2 | 1 |

# Sorting Using a Min Heap



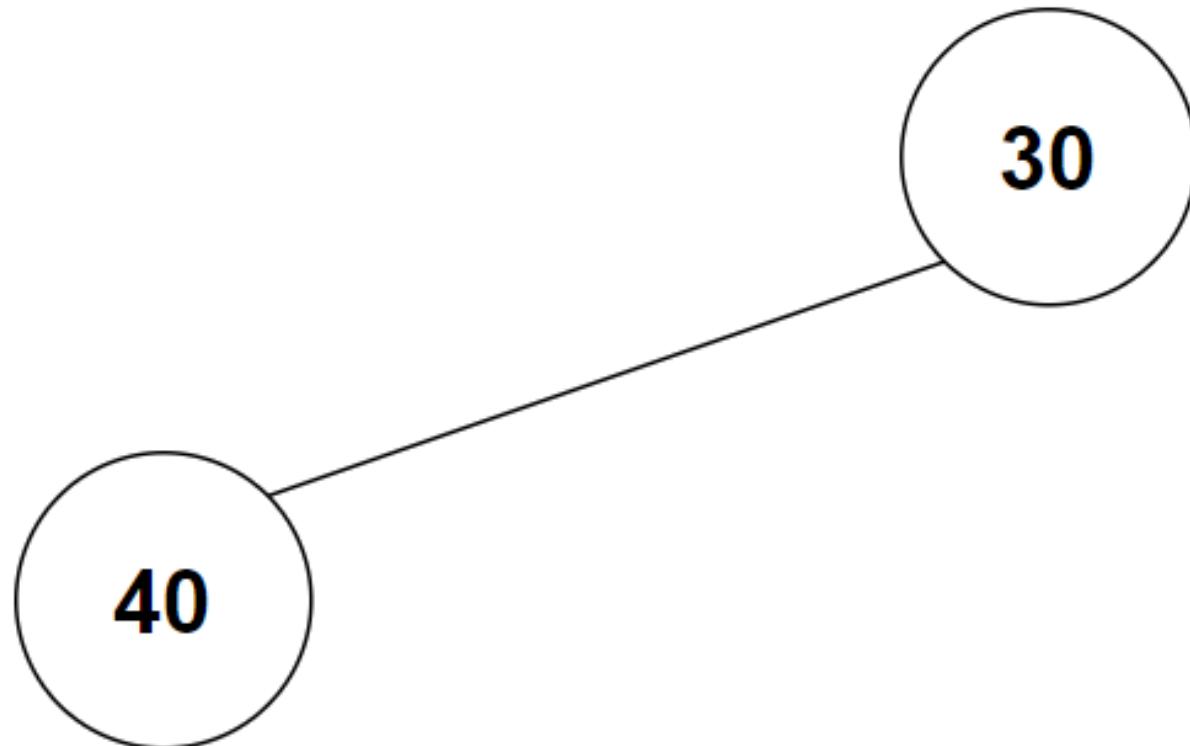
# Sorting Using a Min Heap



|    |    |    |    |    |    |    |   |   |   |
|----|----|----|----|----|----|----|---|---|---|
| 0  | 1  | 2  | 3  | 4  | 5  | 6  | 7 | 8 | 9 |
| 19 | 30 | 40 | 16 | 15 | 14 | 10 | 6 | 2 | 1 |

A horizontal array of 10 cells, indexed 0 to 9 above. Cells 0, 1, 2, 4, 5, 6, 7, 8, 9 contain the values 19, 30, 40, 16, 15, 14, 10, 6, 2, 1 respectively. Cell 3 is highlighted with a thick magenta vertical bar.

# Sorting Using a Min Heap

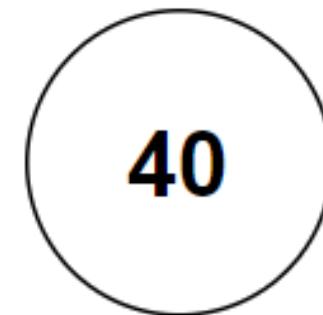


| 0  | 1  | 2  | 3  | 4  | 5  | 6  | 7 | 8 | 9 |
|----|----|----|----|----|----|----|---|---|---|
| 30 | 40 | 19 | 16 | 15 | 14 | 10 | 6 | 2 | 1 |

# Sorting Using a Min Heap

## Summary

Binary Max Heap  
sorted the list  
ascending order.



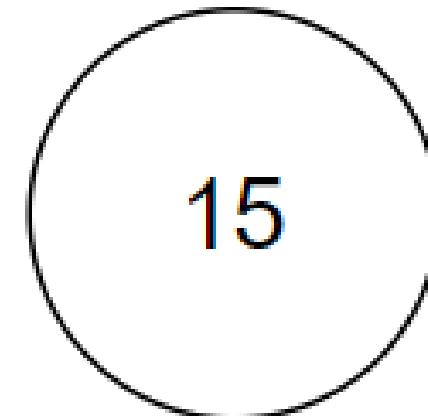
Binary Min Heap  
sorted the list in  
descending order.

| 0  | 1  | 2  | 3  | 4  | 5  | 6  | 7 | 8 | 9 |
|----|----|----|----|----|----|----|---|---|---|
| 40 | 30 | 19 | 16 | 15 | 14 | 10 | 6 | 2 | 1 |

# Creating a Max Heap

Given this array, create a max heap

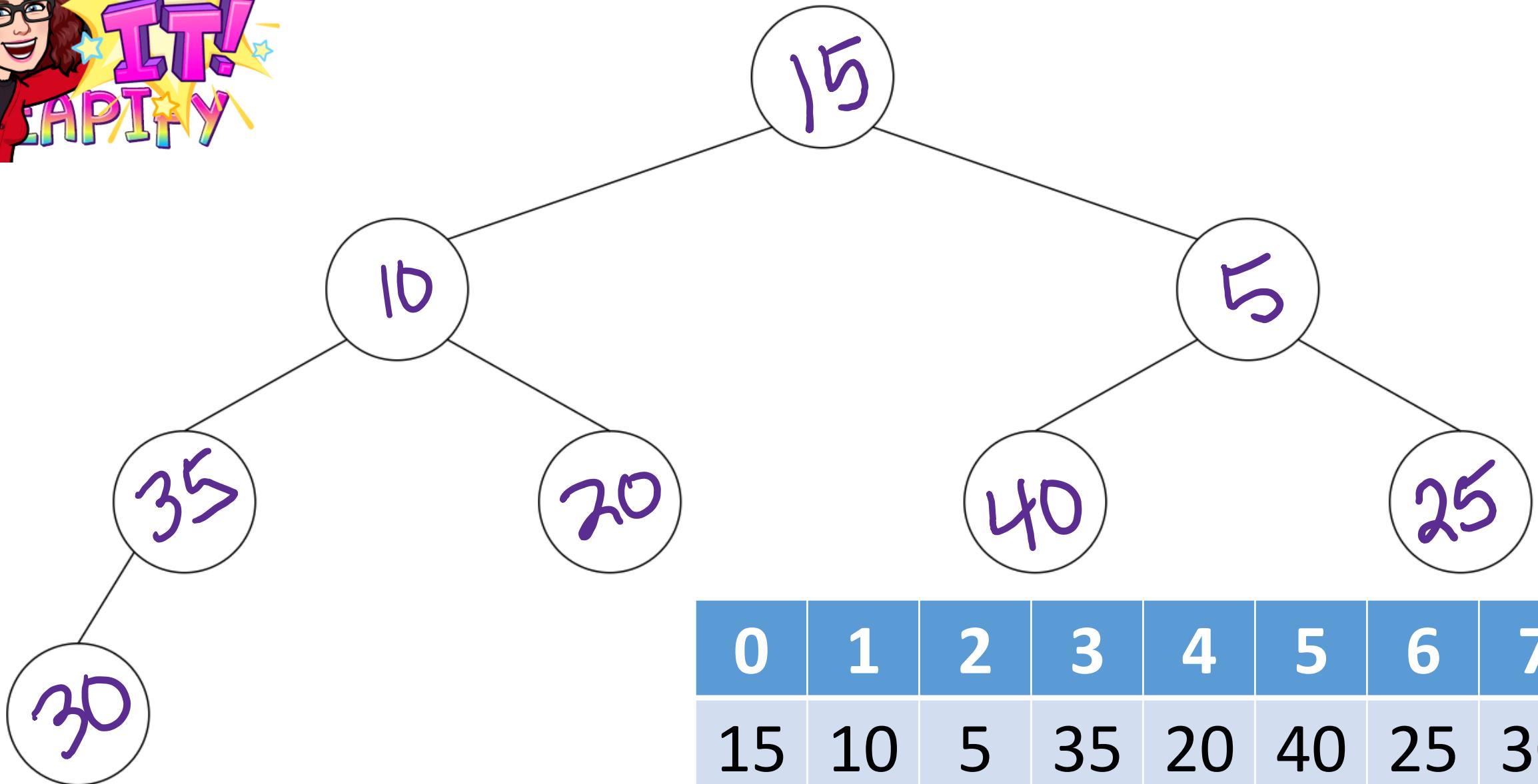
| 0  | 1  | 2 | 3  | 4  | 5  | 6  | 7  |
|----|----|---|----|----|----|----|----|
| 15 | 10 | 5 | 35 | 20 | 40 | 25 | 30 |



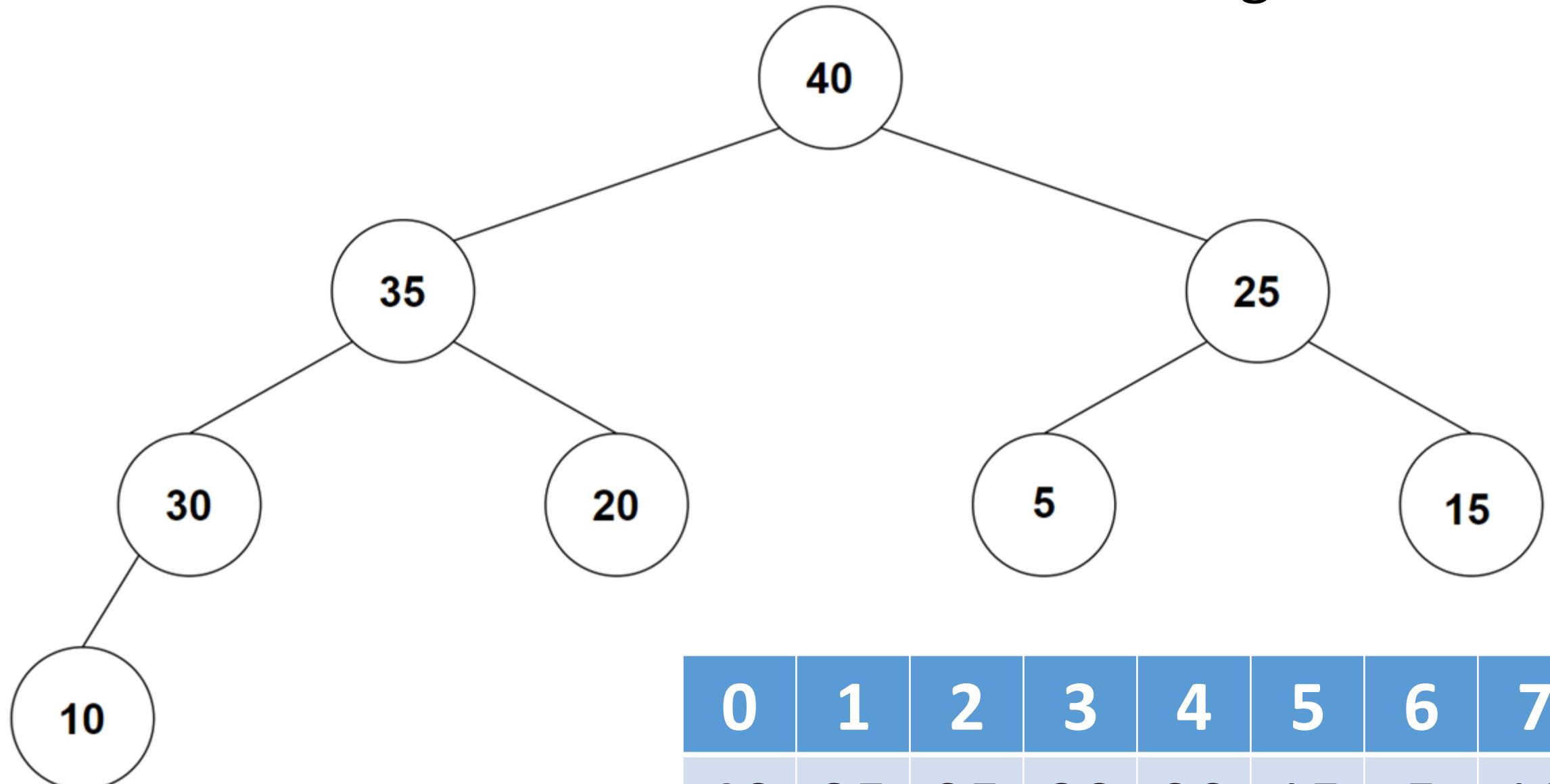
Take the 0<sup>th</sup> element and make it the root.



# Creating a Max Heap



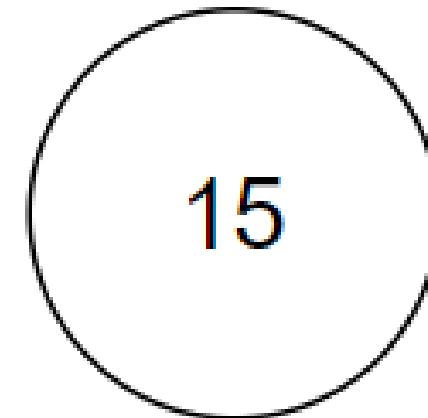
# Creating a Max Heap



# Creating a Min Heap

Given this array, create a min heap

| 0  | 1  | 2 | 3  | 4  | 5  | 6  | 7  |
|----|----|---|----|----|----|----|----|
| 15 | 10 | 5 | 35 | 20 | 40 | 25 | 30 |

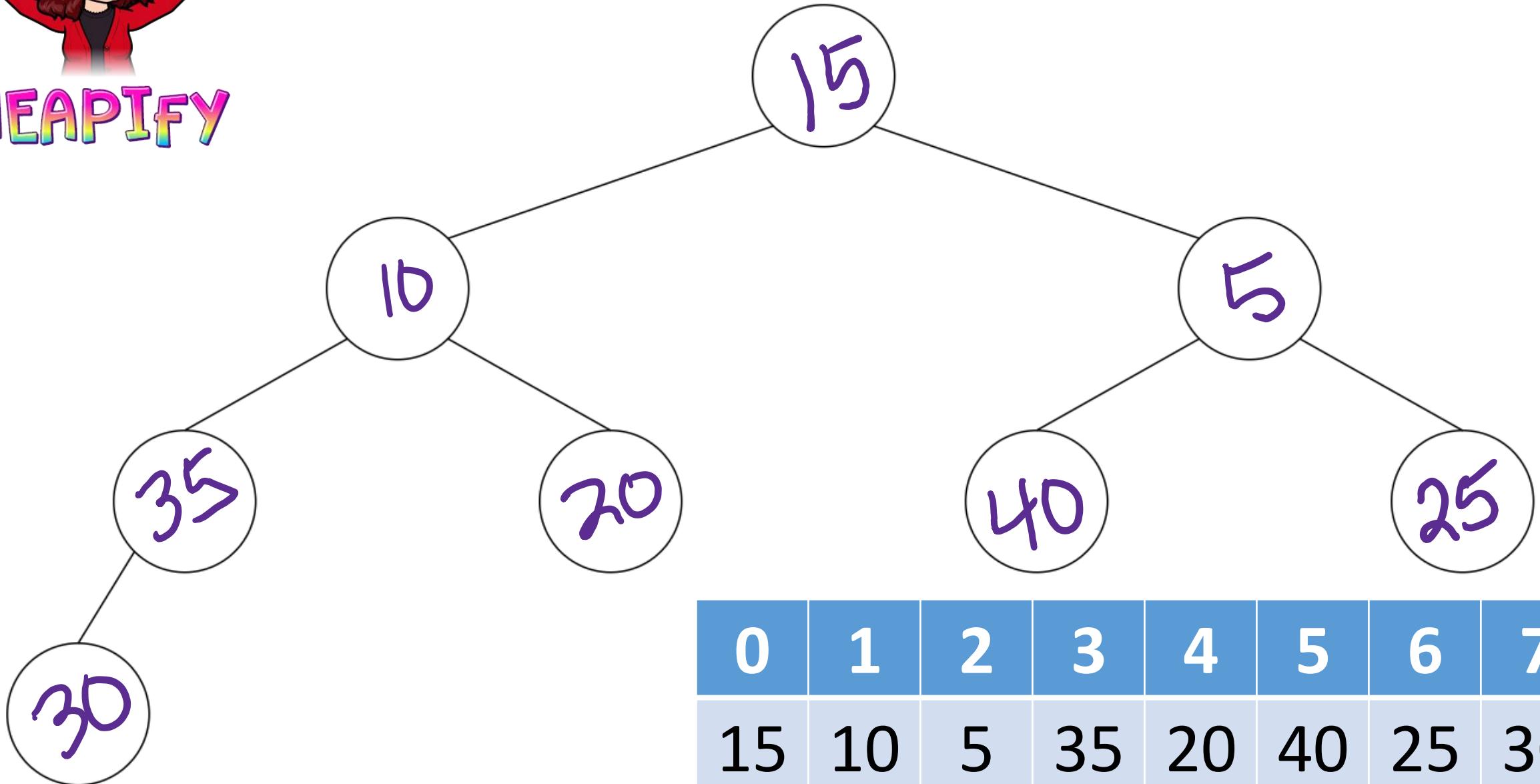


Take the 0<sup>th</sup> element and make it the root.

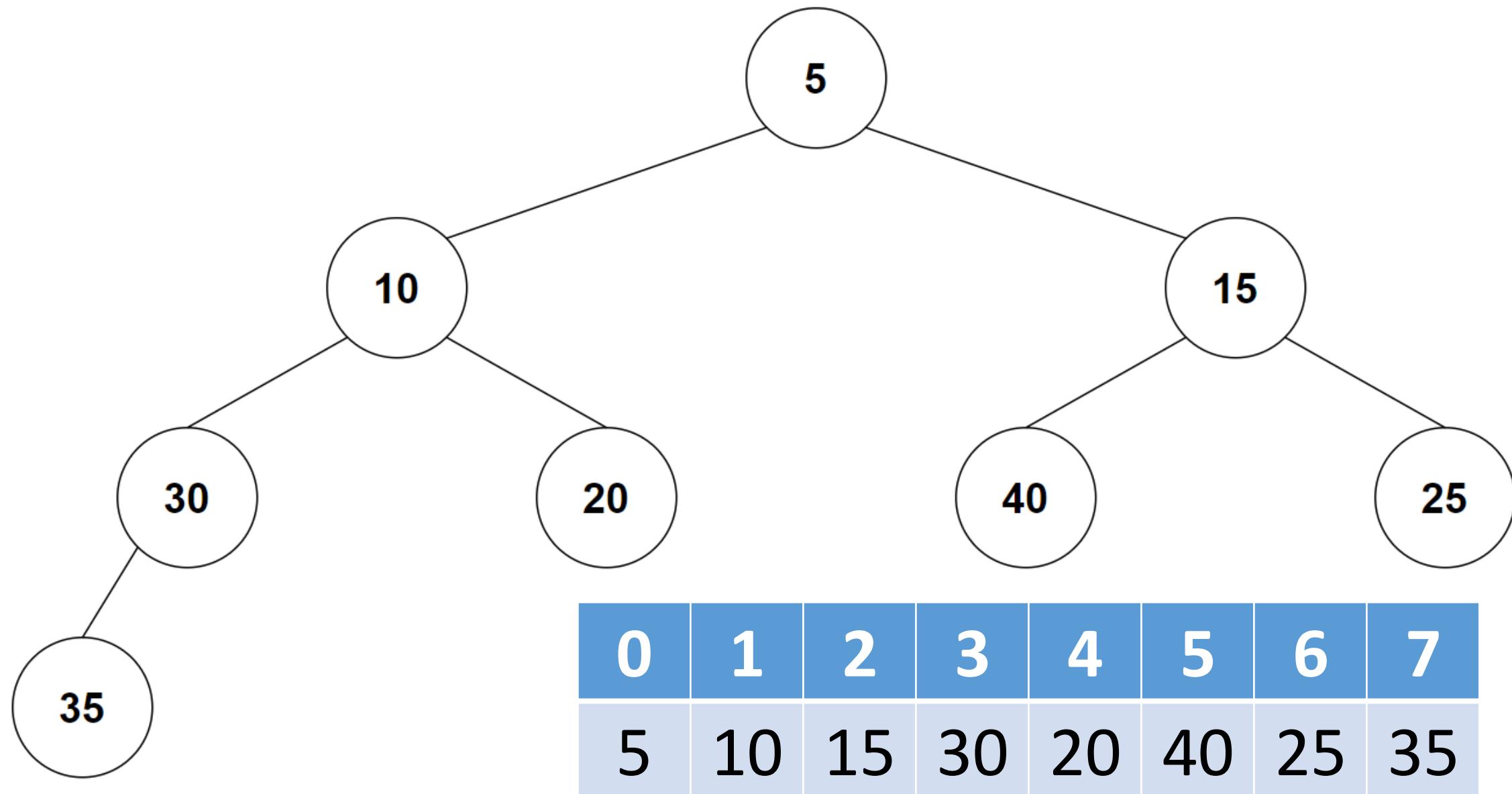
# Creating a Min Heap



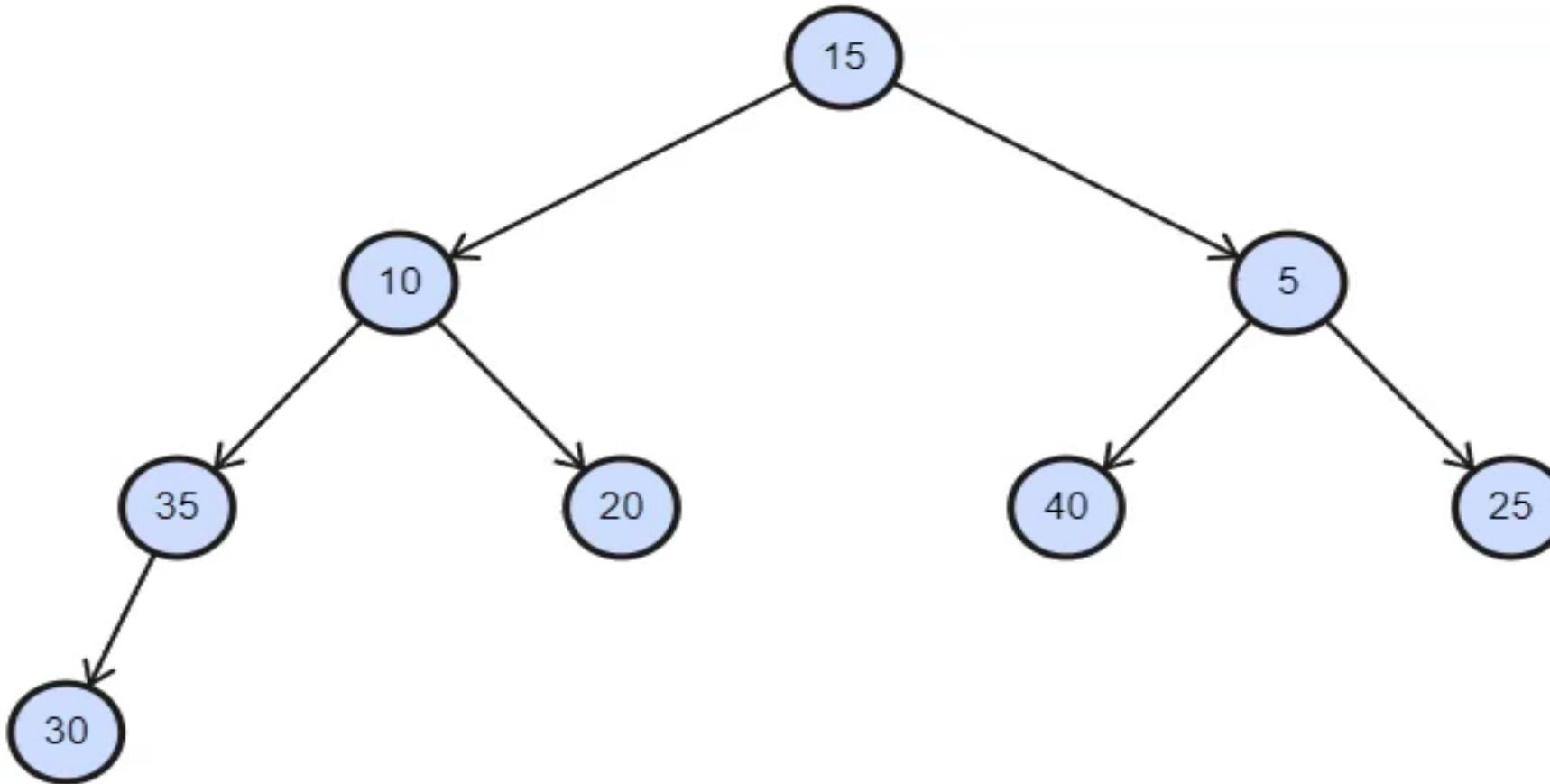
HEAPIFY



# Creating a Min Heap



# Sorting Using a Max Heap



What is the time complexity of building a binary heap?

## Time Complexity

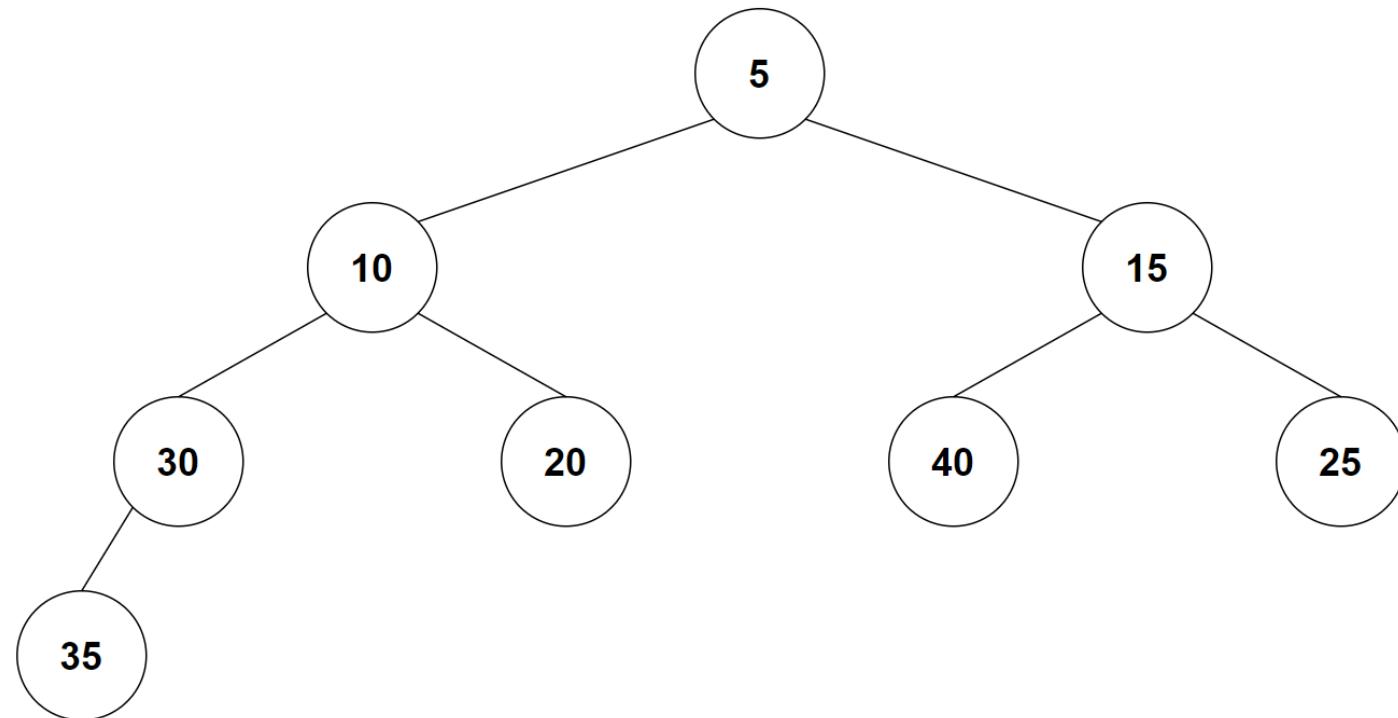
How many elements did we insert?

$n$

The amount of time needed to insert an element into a heap depends on the height of the complete binary tree.

$\log_2 n$

So for worst case, we assume that every element is moved up/down the full height of the tree.



|   |    |    |    |    |    |    |    |
|---|----|----|----|----|----|----|----|
| 0 | 1  | 2  | 3  | 4  | 5  | 6  | 7  |
| 5 | 10 | 15 | 30 | 20 | 40 | 25 | 35 |

$n \log_2 n$

What is the time complexity of deleting a binary heap?

## Time Complexity

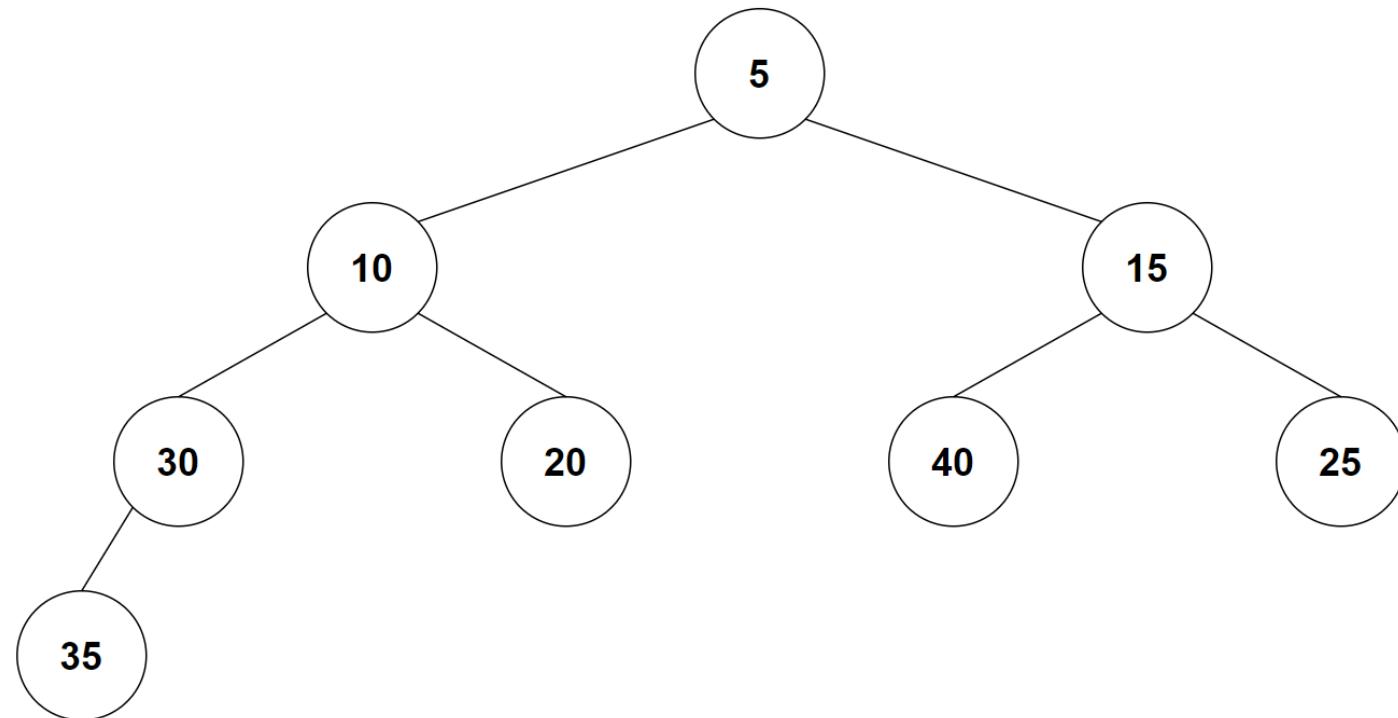
How many elements did we delete?

$n$

The amount of time needed to delete an element into a heap depends on the height of the complete binary tree.

$\log_2 n$

So for worst case, we assume that every element is moved up/down the full height of the tree.



$n \log_2 n$

|   |    |    |    |    |    |    |    |
|---|----|----|----|----|----|----|----|
| 0 | 1  | 2  | 3  | 4  | 5  | 6  | 7  |
| 5 | 10 | 15 | 30 | 20 | 40 | 25 | 35 |

# Time Complexity

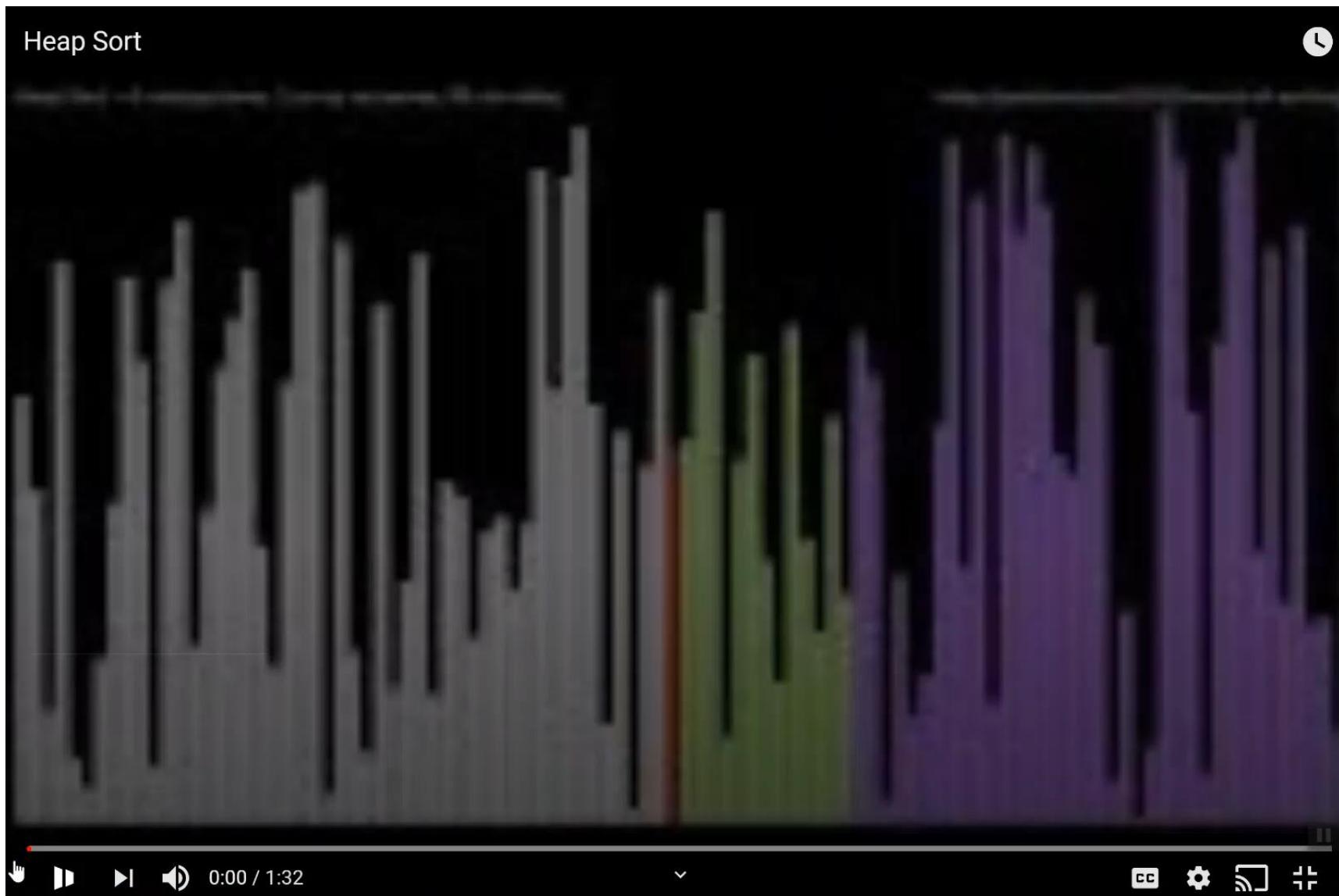
What is the time complexity of sorting using a binary heap?

We sorted by building a binary heap with the data and then deleting it to get the sort.

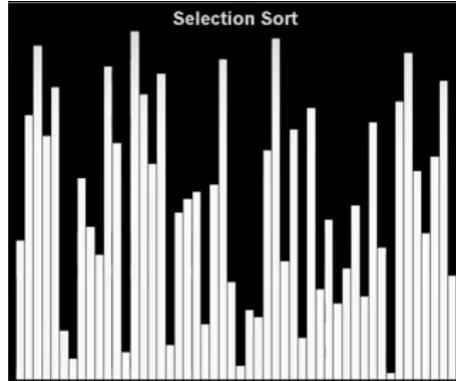
$$\begin{array}{lcl} \text{Building} & + & \text{Deleting} \\ n\log_2 n & + & n\log_2 n \end{array} = \begin{array}{l} \text{Sorting} \\ 2 * n\log_2 n \end{array}$$

$2n\log_2 n = O(n\log_2 n)$  for Heap Sort

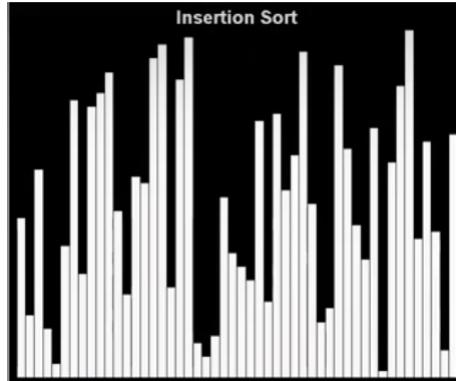
# Visual Heap Sort



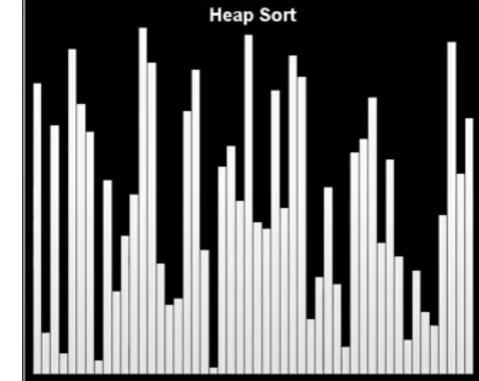
Selection Sort

 $O(n^2)$ 

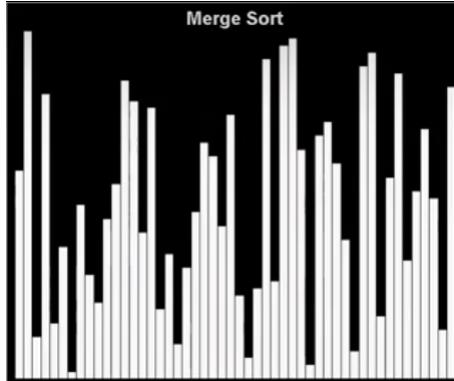
Insertion Sort

 $O(n^2)$ 

Heap Sort

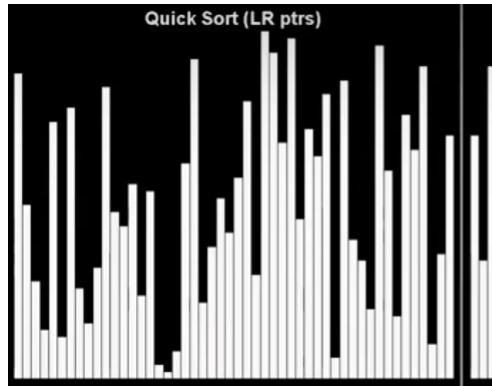
 $O(n \log_2 n)$ 

Merge Sort

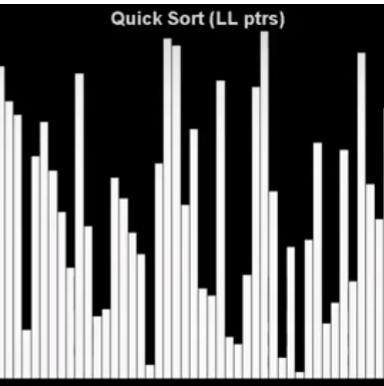
 $O(n \log_2 n)$ 

# Sorts

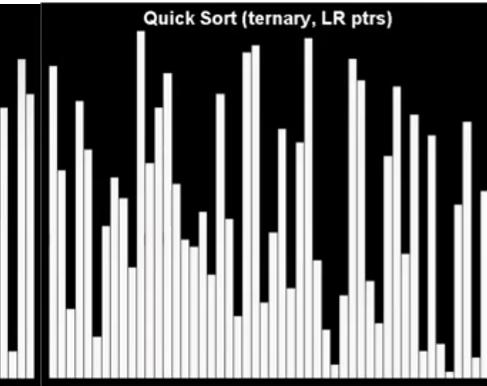
Quick Sort (LR ptrs)

 $O(n \log_2 n)$   
 $O(n^2)$ 

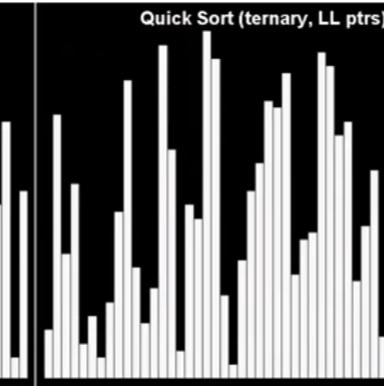
Quick Sort (LL ptrs)



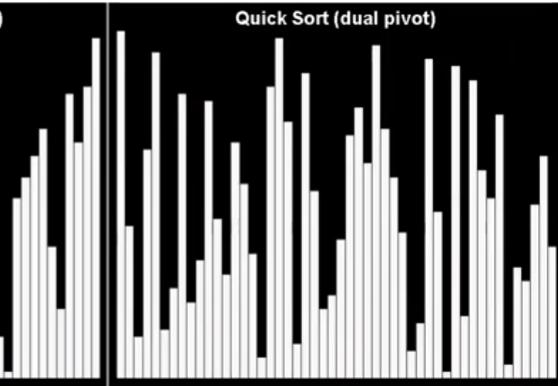
Quick Sort (ternary, LR ptrs)



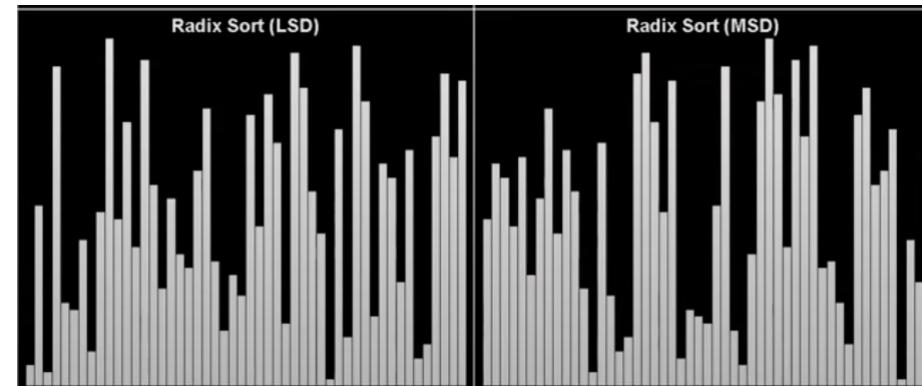
Quick Sort (ternary, LL ptrs)



Quick Sort (dual pivot)



Radix Sort (LSD)



Radix Sort (MSD)



# Radix Sort

Let's start with a list of numbers to sort

645, 86, 182, 35, 601, 8

Step 1

All numbers to be sorted need to be the same length.

# Radix Sort

645, 86, 182, 35, 601, 8

How do we make 8 and 35 and 86 the same length as 648, 182 and 608?

645, 086, 182, 035, 601, 008

We add leading zeros/front pad our numbers with zeroes.

No difference in value between 001 and 1.

Big difference between 1 and 100 – add leading zeros to not change value.

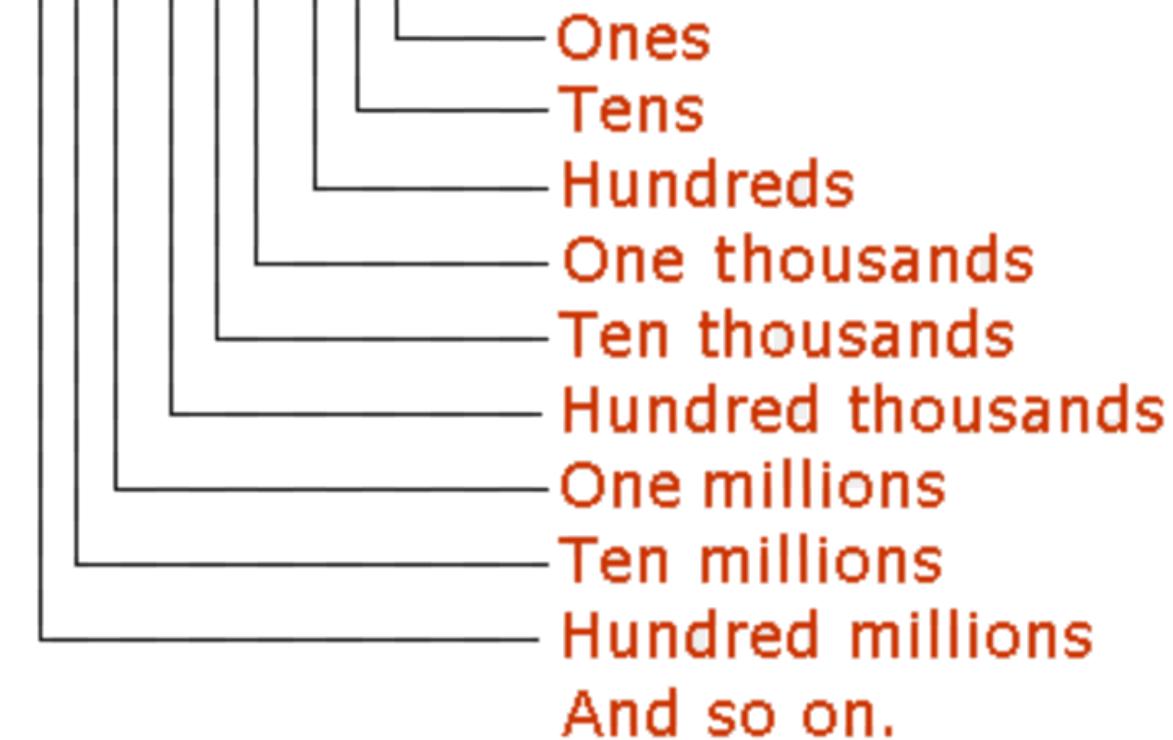
# Radix Sort

64<sup>5</sup>, 08<sup>6</sup>, 18<sup>2</sup>, 03<sup>5</sup>, 60<sup>1</sup>, 00<sup>8</sup>

Now we arrange our list by place value starting with ones.

601, 182, 645, 035, 086, 008

195,782,364



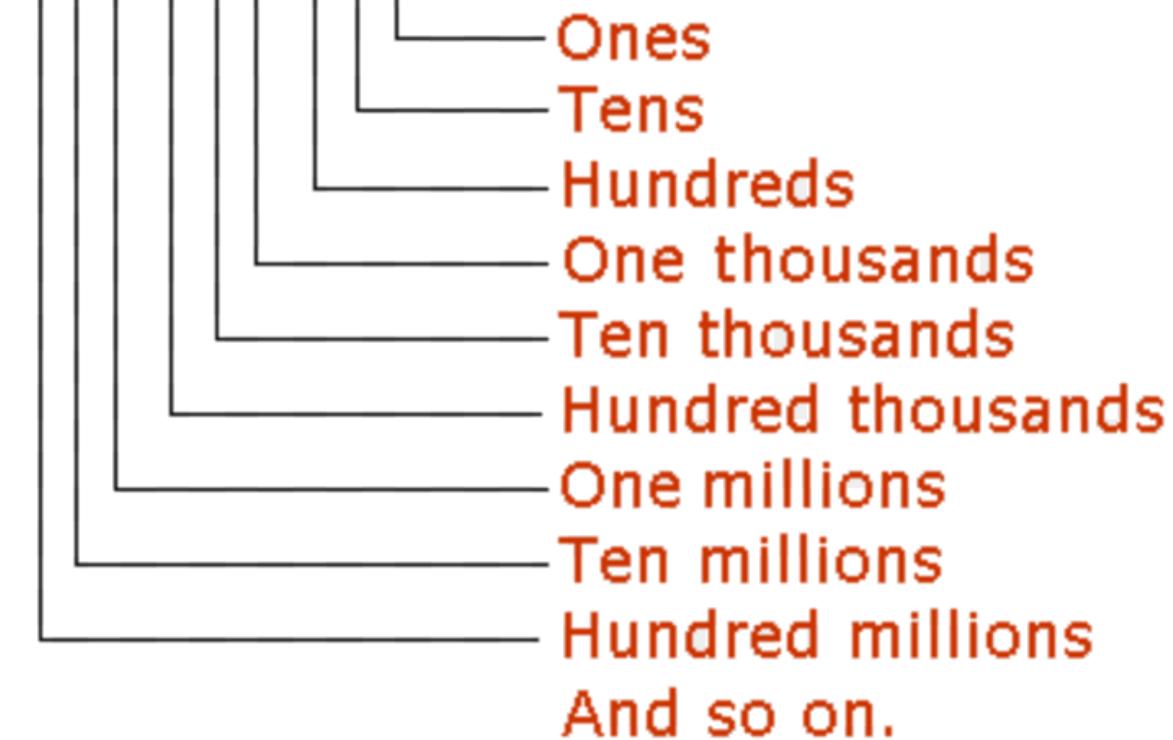
# Radix Sort

601, 182, 645, 035, 086, 008

Now we arrange our list by place value tens.

601, 008, 035, 645, 182, 086

195,782,364



# Radix Sort

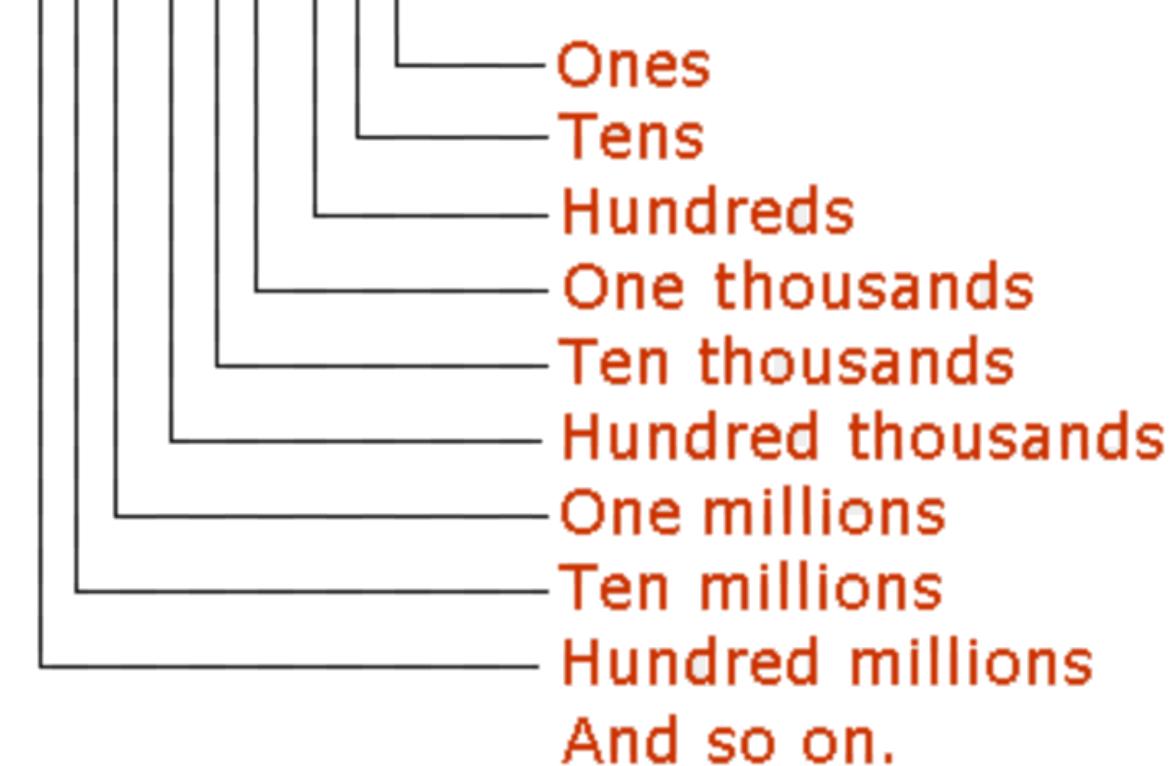
601, 008, 035, 645, 182, 086

Now we arrange our list by place value hundreds.

008, 035, 086, 182, 601, 645



195,782,364



# Radix Sort

645, 086, 182, 035, 601, 008

601, 182, 645, 035, 086, 008

601, 008, 035, 645, 182, 086

008, 035, 086, 182, 601, 645

# Radix Sort

## Radix Definition

The **base** of a system of numeration

So "Radix Sort" means a sort that uses the **base** of a system of numeration.

So can we use Radix Sort on numbers in bases other than 10?

LET'S TRY IT!



# Radix Sort

Let's start with a list of binary numbers

110, 11, 101, 1, 100, 111 , 10

Add leading zeros

110, 011, 101, 001, 100, 111 , 010

110, 011, 101, 001, 100, 111 , 010

Using ones, tens and hundreds place value is more applicable to the decimal system.

So let's talk about Most Significant Bit (MSB) and Least Significant Bit (LSB).

# Radix Sort

MSB

the bit in a binary number which is of the greatest numerical value  
the leftmost/first bit of a number

LSB

the bit in a binary number which is of the lowest numerical value.  
the rightmost/last bit of a number

# Radix Sort

110, 011, 101, 001, 100, 111, 010

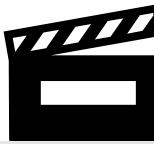
Let's arrange them starting with LSB and work towards the MSB.

110, 100, 010, 011, 101, 001, 111

100, 101, 001, 110, 010, 011, 111

001, 010, 011, 100, 101, 110, 111

# Radix Sort



A screenshot of a blank Microsoft Excel spreadsheet. The interface includes a top menu bar with tabs like 'File', 'Home', 'Insert', etc., and a ribbon. The main area shows a grid of columns labeled A through S and rows labeled 1 through 30. Cell A1 is selected, indicated by a green border and a small plus sign icon in the center. The status bar at the bottom shows 'Sheet1' and 'Sheet2'.

# Radix Sort Time Complexity

So what is the time complexity of Radix Sort?

Let's establish a few terms

d is the number of digits in the numbers in the list – use the number of digits in the biggest number in the list.

n is the number of numbers in the list

b is the base/radix we are using

# Radix Sort Time Complexity

For every number in our list,  $n$ , we checked each digit.

$$d * n$$

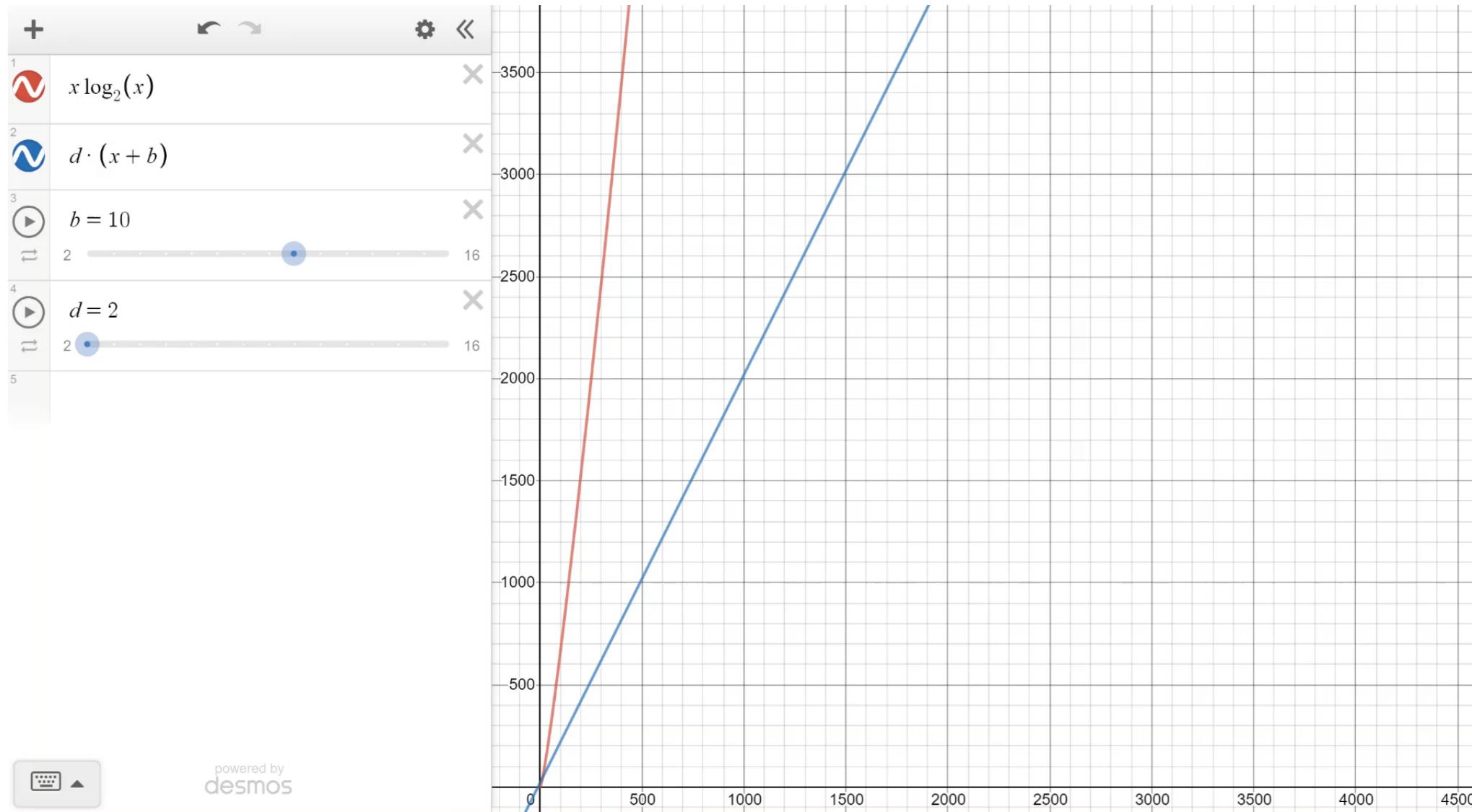
When we had a base 10 number, we had to check 10 digits -> 0-9.

When we had a base 2 number, we had to check 2 digits -> 0 and 1.

So the base influenced how much time we spent on each number.

$$O(d * (n+b))$$

# Radix Sort Time Complexity



# Radix Sort Time Complexity

$$O(d * (n+b))$$

As we saw in the graph, changing the base did not influence the run time very much.

You will also see the time complexity of radix sort written as

$$O(nk)$$

where k is the number of digits

# Radix Sort

## Interesting Thoughts on Radix Sort

Radix Sort does not use any comparisons or swaps.

The time complexity of Radix Sort maintains its coefficient (number of digits) because of the significant impact of that value on the time complexity.

$O(nk)$

Even asymptotic notation does not wave away the impact of the number of digits.

# Radix Sort

So if Radix Sorts has a better run time than most other sorts, why isn't it used more?

The biggest limitation of Radix Sort is that it is limited to data that only consists of digits or letters.

Radix Sort also does not actually move any elements around in the storage container (like an array) – it makes a new version of the storage container; therefore, has a higher space complexity than some other "slower" sorts.  $O(n+k)$ .