CSE 5311 Hands-On 6.

Quick Sort

Time complexity

Average case =

The array gets divided into 2 parts of sizes K and (N-K)

$$T(N) = T(N-K) + T(K) = \frac{1}{N}\left[\sum_{i=1}^{N-1} T(i) + \sum_{i=1}^{N-1} T(N-i)\right]$$

As both these terms are equally likely functions =

$$T(N) = \frac{2}{N}\sum_{i=1}^{N-1} T(i)$$

$$N(T(N)) = 2\sum_{i=1}^{N-1} T(i) \quad — ①$$

we can also write this as =

$$(N-1)(T(N-1)) = 2\sum_{i=1}^{N-2} T(i) \quad — ②$$

subtracting ② from ①,

$$N(T(N)) - (N-1)(T(N-1)) = 2T(N-1) + N^2c - (N-1)^2c^2$$

where c is a constant

$$NT(N) = T(N-1)(2+N-1) + \cancel{c} + 2NC - \cancel{c}$$
$$= (N+1)T(N-1) + 2NC$$

Divide both sides by N(N+1);

$$\frac{\cancel{N}T(N)}{\cancel{N}(N+1)} = \frac{\cancel{(N+1)}T(N-1)}{N\cancel{(N+1)}} + \frac{2\cancel{N}C}{\cancel{N}(N+1)} \quad — ③$$

If we put N = N-1

$$\frac{T(N-1)}{N} = \frac{T(N-2)}{(N-1)} + \frac{2C}{N}$$

③ can be expressed as:

$$\frac{T(N)}{N+1} = \frac{T(N-2)}{(N-1)} + \frac{2c}{(N+1)} + \frac{2c}{N}$$

we can get the value of $T(N-2)$ by replacing N by $(N-2)$ in ③

$$T(N) = 2c \log_2 N (N+1)$$

Ignoring all constants

$$T(N) = \log_2 N * (N+1)$$

$$T(N) = N\log_2 N + \log_2 N$$

So = $\boxed{T(N) = \Theta(N \log_2 N)}$ ANSWER