## CS 61A Fall 2023

# Structure and Interpretation of Computer Programs

FINAL SOLUTIONS

#### INSTRUCTIONS

This is your exam. Complete it either at exam.cs61a.org or, if that doesn't work, by emailing course staff with your solutions before the exam deadline.

This exam is intended for the student with email address <EMAILADDRESS>. If this is not your email address, notify course staff immediately, as each exam is different. Do not distribute this exam PDF even after the exam ends, as some students may be taking the exam in a different time zone.

For questions with **circular bubbles**, you should select exactly *one* choice.

Or you must choose either this option
Or this one, but not both!

For questions with **square checkboxes**, you may select *multiple* choices.

You could select this choice.

You could select this one too!

You may start your exam now. Your exam is due at <DEADLINE> Pacific Time. Go to the next page to begin.

### Preliminaries

(a)	What is your full name?
(b)	What is your student ID number?

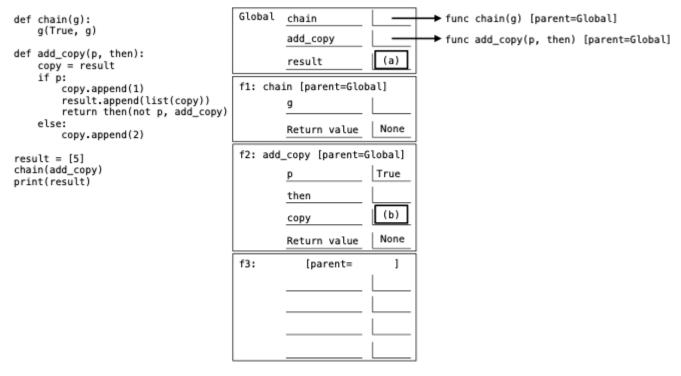
You can complete and submit these questions before the exam starts.

(c)	What is your @berkeley.edu email address?

` '	Sign (or type) your name to confirm that all work on this exam will be your own. The penalty misconduct on an exam is an F in the course.	for academic

#### 1. (5.0 points) Copying Copies

Draw the environment diagram that results from running all of the code below until it is fully executed, an error occurs, or you run out of frames. Then, answer the questions that follow. Blanks and frames with no labels have no questions associated with them and are not scored. You may not need all the spaces or frames.



- (a) (1.0 pt) Which of the following is true about the blanks labeled (a) and (b)?
  - They contain arrows to the same list
  - They contain arrows to different lists with the same contents
  - They contain arrows to different lists with different contents
- (b) (1.0 pt) What error occurs during execution, if any?
  - NameError because a name is referenced before assignment
  - O TypeError because a function was called with the wrong number of arguments
  - O TypeError because a built-in function is called on the wrong argument type(s)
  - RecursionError because of too many recursive calls
  - No error occurs
- (c) (3.0 pt) What would be displayed by evaluating print(result) in the global frame? If an error occurred or you ran out of frames, still evaluate print(result) according to the environment diagram you drew.

```
[5, 1, [5, 1], 2]
```

#### 2. (6.0 points) Path Math

Implement bounds, which takes a Tree instance t and numbers low and high. It returns the number of paths through t from the root to a leaf for which the sum of the labels along the path is at least low and at most high.

```
def bounds(t, low, high):
    """Return the number of root-to-leaf paths in t whose sum is between low and high (inclusive).
    >>> t = Tree(3, [Tree(4), Tree(5, [Tree(1), Tree(2)]), Tree(7)])
    >>> bounds(t, 7, 10) # 3+4=7, 3+5+1=9, 3+5+2=10, 3+7=10
    >>> bounds(t, 9, 10)
    >>> bounds(t, 9, 9)
    11 11 11
    count = 0
    if ____:
         (a)
        count = 1
    return count + ____([____ for b in t.branches])
(a) (2.0 pt) Fill in blank (a).
    O low <= t and t <= high
    t and low <= t and t <= high</pre>
    t.branches and low <= t and t <= high</pre>
     t.is_leaf() and low <= t and t <= high</pre>
    O low <= t.label and t.label <= high
     t and low <= t.label and t.label <= high</pre>
    t.branches and low <= t.label and t.label <= high</pre>
    t.is_leaf() and low <= t.label and t.label <= high</pre>
(b) (1.0 pt) Fill in blank (b).
    Obounds
    \bigcirc max
       sum
    O lambda t:
(c) (3.0 pt) Fill in blank (c).
```

bounds(b, low - t.label, high - t.label)

#### 3. (10.0 points) Talk Like a Pirate Day

Pirate expressions are reversed and substitute some words (such as "aye" for "yes") according to a pirate dialect.

- An Expression instance is constructed from a list of strings and has a dictionary attribute dialect and a Word instance attribute first that represents the first word of the reversed sequence.
- A Word instance is constructed from its attributes: a string w, an Expression instance exp, and a Word instance then representing the next word in the reversed sequence. If there is no next word, then is None. A Word instance's say method returns either w or its substitute if w is a key of the expression's dialect.

Printing a Word prints how a pirate would say that word and the following words in the reversed sequence. Printing an Expression prints all of the words in the reversed sequence using the Expression's dialect.

Reminder: The get method of a dictionary takes two arguments: key and default. If the key is in the dictionary, its value is returned. If not, default is returned. E.g., {1:2}.get(1, 3) evaluates to 2, but {1:2}.get(5, 3) is 3.

```
class Expression:
   """A pirate expression is reversed and substitutes some words using a dialect.
   >>> str(Expression(['I', 'said', 'hi']))
    'ahoy says I'
   >>> e = Expression(['there', 'you', 'are'])
   >>> print(e)
   arrrr you there
   >>> e.dialect['you'] = 'ye' # After adding to the dialect...
   >>> print(e)
                               # ... the result of printing changes
   arrrr ye there
   11 11 11
   def __init__(self, original):
        assert len(original) > 0
        self.dialect = {'yes': 'aye', 'hi': 'ahoy', 'said': 'says', 'are': 'arrrr'}
        previous = None
        for w in original:
            current = _____
                       (a)
             (b)
       self.first = _____(c)
   def __str__(self):
       return _____
                (d)
class Word:
   def __init__(self, w, exp, then):
       self.w = w
        self.exp = exp
        self.then = then
   def say(self):
        return _____
                 (e)
   def __str__(self):
       first = _____
                 (f)
        if self.then:
            return first + ' ' + str(self.then)
```

else:

```
return first
(a) (3.0 pt) Fill in blank (a).
      Word(w, self, previous)
(b) (1.0 pt) Fill in blank (b).
      previous = current
(c) (1.0 pt) Fill in blank (c).
    current
       self.current
    O current.then
      self.current.then
(d) (1.0 pt) Fill in blank (d).
   O self
    O self.first
     str(self)
    str(self.first)
    O print(self)
     print(self.first)
(e) (3.0 pt) Fill in blank (e).
      self.exp.dialect.get(self.w, self.w)
(f) (1.0 pt) Fill in blank (f).
   ○ self.w
       self.exp[w]
    O self.say
    self.say()
       self.say(self)
        self.say(self.w)
```

#### 4. (29.0 points) Six Pages of Pairings (So Please Read the Definition Carefully)

**Definition:** A pairing of a sequence **s** is a list of two-element tuples (called pairs) that contain adjacent elements of **s**. There must be at least one element of **s between** any two pairs in the pairing. The pairs in the pairing must be in the same order as they are in **s**.

The sequence [4, 5, 6, 1, 2, 3, 7, 8] has pairings [(6, 1), (3, 7)] and [(4, 5), (1, 2), (7, 8)] and [] and many others, but the following are not pairings of that sequence:

- [(5, 1), (3, 7)] contains a pair (5, 1), but 5 and 1 are not adjacent in the sequence.
- [(5, 6), (1, 2)] contains two pairs with no element between them, since 6 is adjacent to 1 in the sequence.
- [(1, 2), (4, 5)] contains valid pairs, but those pairs are not in the same order as the sequence.

#### (a) (6.0 points)

return result

Implement longest\_pairing, which takes a list s with 3\*n-1 elements for some positive integer n. It returns the longest pairing of s.

```
def longest_pairing(s):
    """Return the longest pairing for list s that has length 3*n-1 for some positive integer n.
   >>> longest_pairing([1, 2, 3, 4, 5, 6, 7, 8])
    [(1, 2), (4, 5), (7, 8)]
    assert len(s) > 0 and _____, 's must have length 3*n-1 for a positive integer n'
   result, pair, skip = [], [], False
   for x in s:
       if ____:
(b)
            pair.append(x)
        else:
              (c)
        if ____:
             (d)
            results.____
                      (e)
            pair, skip = [], True
```

- i. (1.0 pt) Fill in blank (a)
  - $\bigcirc$  len(s) == 3 \* n 1
  - $\bigcirc$  (len(s) 1) / 3
  - $\bigcirc$  (len(s) 1) % 3 == 0
  - len(s) % 3 == 2
- ii. (1.0 pt) Fill in blank (b)
  - O skip
  - onot skip
  - $\bigcirc$  result
  - O not result
  - O pair
  - $\bigcirc$  not pair
  - len(pair) < 2</pre>
  - len(pair) == 2
- iii. (1.0 pt) Fill in blank (c)
  - skip = False
  - O skip = True
  - pair = []
  - pair.remove(x)
  - O result.append(x)
  - result.append(pair)
- iv. (1.0 pt) Fill in blank (d)
  - O skip
  - $\bigcirc$  not skip
  - O result
  - O not result
  - O pair
  - O not pair
  - $\bigcirc$  len(pair) < 2
  - len(pair) == 2

$\mathbf{v}$ .	$(2.0~\mathrm{pt})$ Fill in blank (e). Select all that apply.
	☐ append(pair)
	☐ extend(pair)
	<pre>append((pair[0], pair[1]))</pre>
	☐ extend((pair[0], pair[1]))
	<pre>append(tuple(pair))</pre>
	☐ extend(tuple(pair))
	☐ append(tuple(pair[0], pair[1]))
	<pre> extend(tuple(pair[0], pair[1]))</pre>

#### (b) (4.0 points)

```
Implement is_pair_sequence, which takes a list s and returns whether it contains only two-element tuples.
```

```
def is_pair_sequence(s):
    """Return whether list s contains only pairs (which are tuples with two elements).
    >>> is_pair_sequence([(1, 2), (3, 4)])
    True
    >>> is_pair_sequence([(1, 2), (3, 4, 5)])
    False
    >>> is_pair_sequence([(1, 2), "not a tuple"])
    False
    >>> is_pair_sequence([(1, 2), (3, (4, 5, 6))]) # (3, (4, 5, 6)) is a two-element tuple
    True
    >>> is_pair_sequence([])
    True
    11 11 11
    return all([_____ for x in s]) and all(map(_____, s))
 i. (2.0 pt) Fill in blank (f). Select all that apply. Assume tuple has no subclasses.
   ☐ tuple(x)
   \square x == tuple
   ☐ x is tuple
   type(x) == tuple
   \square x == type(tuple)
   isinstance(x, tuple)
   ☐ isinstance(x, type(tuple))
   ☐ isinstance(type(x), tuple)
ii. (2.0 pt) Fill in blank (g).
   \bigcirc len(s) == 2
   \bigcirc len(x) == 2
   \bigcirc lambda x: len(s) == 2
    \bigcirc lambda x: len(x) == 2
   lambda s: lambda x: len(s) == 2
   \bigcirc lambda s: lambda x: len(x) == 2
   \bigcirc lambda x: len(s[i]) == 2
   \bigcirc lambda x: len(x[i]) == 2
   lambda i: lambda x: len(s[i]) == 2
```

 $\bigcirc$  lambda i: lambda x: len(x[i]) == 2

#### (c) (6.0 points)

```
Implement is_pairing, which takes a list s and a list of pairs. It returns whether pairs is a pairing of
def is_pairing(s, pairs):
    """Return whether the list of pairs is a pairing for the list s.
    >>> pairs = [(3, 4), (5, 6), (7, 7)]
    >>> is_pairing([3, 3, 4, 5, 4, 5, 6, 0, 7, 7, 7], pairs)
    True
    >>> is_pairing([3, 3, 4, 5, 6, 0, 7, 7, 7], pairs) # Need an element between pairs
    False
    >>> is_pairing([3, 2, 4, 0, 5, 6, 0, 7, 7], pairs) # Elements of a pair must be adjacent
    False
    >>> is_pairing([7, 7, 3, 3, 4, 5, 4, 5, 6], pairs) # Pairing isn't in the same order as s
    False
    11 11 11
    assert is_pair_sequence(pairs)
    if not pairs:
        return True
    if ____:
         (h)
        return False
    if ____ == tuple(s[:2]):
         (i)
        return is_pairing(s[3:], _____) # Note: [0, 1][3:] evaluates to []
    return _____
             (k)
 i. (1.0 pt) Fill in blank (h).
   pairs not in s
   O pairs[0] not in s
   \bigcirc len(s) < 2
    not is_pairing(s, pairs)
ii. (1.0 pt) Fill in blank (i).
      pairs[0]
iii. (1.0 pt) Fill in blank (j).
   O pairs
   ○ pairs[1]
    pairs[1:]
   pairs[:1]
```

iv. (3.0 pt) Fill in blank (k).

```
is_pairing(s[1:], pairs)
```

#### (d) (7.0 points)

Implement unequal\_pairs, a generator function that yields all non-empty pairings of a list s in which no pair contains two equal elements.

```
no pair contains two equal elements.
def unequal_pairs(s):
    """Yield all non-empty pairings for a list s in which each pair's values are unequal.
    >>> sorted(unequal_pairs([4, 2, 2, 4, 4, 1, 1])) # Four different pairings!
    [[(2, 4)], [(4, 1)], [(4, 2)], [(4, 2), (4, 1)]]
    >>> max(unequal_pairs([4, 2, 2, 4, 5, 4, 4, 1, 5, 5, 6]), key=len) # The longest pairing
    [(4, 2), (4, 5), (4, 1), (5, 6)]
    if len(s) >= 2:
        yield from _____
        if ____:
             (m)
            pair = (s[0], s[1])
              (n)
            for rest in unequal_pairs(s[3:]): # Note: [0, 1][3:] evaluates to []
                yield _____
                         (o)
 i. (2.0 pt) Fill in blank (1).
      unequal_pairs(s[1:])
ii. (1.0 pt) Fill in blank (m).
   \bigcirc s[0] == s[1]
   s[0] != s[1]
   pair[0] == pair[1]
   pair[0] != pair[1]
iii. (2.0 pt) Fill in blank (n).
      yield [pair]
```

iv. (2.0 pt) Fill in blank (o).

```
[pair] + rest
```

s.rest.rest.rest

#### (e) (6.0 points)

Implement max\_pair\_sum, which takes a linked list s (either a Link instance or Link.empty). It returns the largest possible sum of the values in a pairing of s. The Link class appears on the Midterm 2 Study Guide (p. 2).

```
def max_pair_sum(s):
    """Return the largest sum of values in a pairing for a linked list of positive numbers s.
    >>> L = Link
                                                                       # Abbreviate Link
    >>> max_pair_sum(L(3, L(4, L(5, L(3, L(4, L(5, L(6))))))))
                                                                       # 4+5 + 5+6
    >>> max_pair_sum(L(3, L(4, L(5, L(3, L(4, L(5, L(6, L(3)))))))) # 3+4 + 3+4 + 6+3
    23
    11 11 11
    if ____:
         (p)
        return 0
    n = ____
          (q)
    if s.rest.rest is Link.empty:
        return n
    else:
        return max(n + max_pair_sum(_____), max_pair_sum(_____))
 i. (2.0 pt) Fill in blank (p). Select all that apply.
   ☐ s is Link.empty
   ☐ s.rest is Link.empty
   s is Link.empty or s.rest is Link.empty
   ☐ s.rest is Link.empty or s is Link.empty
ii. (2.0 pt) Fill in blank (q).
      s.first + s.rest.first
iii. (1.0 pt) Fill in blank (r).
   \bigcirc s
   O s.rest
    s.rest.rest
    s.rest.rest.rest
iv. (1.0 pt) Fill in blank (s).
   () s
     s.rest
    s.rest.rest
```

#### 5. (6.0 points) What Would Scheme Do?

```
Assume the following code has been evaluated.
```

(a) (2.0 pt) What does this expression evaluate to? ((shrink 3 nil) '(3 1 4 1 5 9 2 6))

```
(2 9 1 1)
```

- (b) (1.0 pt) What is the order of growth of the run time of ((shrink 1 nil) s) in terms of the length of list s?
  - O constant
  - linear
  - quadratic
  - O exponential
- (c) (1.0 pt) Which of the following evaluates to 4?
  - (cdr twos)
  - ((cdr twos))
  - (car (cdr twos))
  - ((car (cdr twos)))
  - (car ((cdr twos)))
  - ((car ((cdr twos))))

- - (car ((cdr ((cdr twos)))))

#### 6. (5.0 points) How to Get Promoted

Implement promote, which takes a one-argument procedure f and a list s. It returns a list that begins with all of the elements of s for which calling f on the element returns #t, followed by all of the elements of s for which calling f on the element returns #f. Assume that when f is called on any element of s, it returns either #t or #f.

```
;;; Return a list containing all the elements of s, with the elements for which
;;; f returns #t at the front of the list, but otherwise keeping the order the same.
;;;
;;; scm> (promote even? '(1 2 3 4 5 6 7))
;;; (2 4 6 1 3 5 7)
;;; scm> (promote odd? '(1 2 3 4 5 6 7))
;;; (1 3 5 7 2 4 6)
(define (promote f s)
    (_____ s)))
       (a)
           (b)
(a) (1.0 pt) Fill in blank (a).
    append
    \bigcirc cons
    O list
    promote
    \bigcirc map
    ○ filter
(b) (2.0 pt) Fill in blank (b).
      (filter f s)
(c) (2.0 pt) Fill in blank (c).
      (lambda (x) (not (f x)))
```

(d) This is an A+ question. It is not worth any points. It is not the last question on the exam.

Implement bigger-first without writing lambda or define. You may use promote, curry, and curry-call.

```
(define (curry f) (lambda (x) (lambda (y) (f x y))))
(define (curry-call f) (lambda (x g) (lambda (y) (f (x (g y)) y))))

;;; (bigger-first s) returns a list with all of the elements of s larger than (car s)
;;; at the front, followed by all of the elements of s smaller or equal to (car s).
;;;
;;; scm> (bigger-first '(3 1 4 1 5 9 2 6))
;;; (4 5 9 6 3 1 1 2)
(define bigger-first _____)

((curry-call promote) (curry <) car)</pre>
```

#### 7. (8.0 points) Don't Skip This

**Definition.** A *skip-partition* of a positive integer n is a list of positive integers in increasing order that sums to n and does not contain any duplicates or consecutive numbers.

Implement part, which takes positive integers n and m. It returns a list of all *skip-partitions* of n that contain elements greater than or equal to m. The provided cons-me procedure is used in the implementation.

```
(define (cons-me first) (lambda (rest) (cons first rest)))
```

(a) (2.0 pt) Fill in blank (a).

```
(list (list m))
```

- (b) (1.0 pt) Fill in blank (b).
  - O cons
  - O list
  - append
  - map
- (c) (2.0 pt) Fill in blank (c).

```
(part (- n m) (+ m 2))
```

- (d) (3.0 pt) Which expressions are passed to scheme\_eval when evaluating (if (> 1 2) (+ 1 2) 2)? Check all that apply.
  - ☐ if
  - (> 1 2)

  - 1
  - 2
  - ☐ (+ 1 2)
  - +

#### 8. (6.0 points) Cheap Donuts

There are three tables in a database:

- The donuts table has a row for each menu option at a donut shop. There are columns for the kind (string) of dough and flavor (string). For example, there is one row for chocolate cake donuts (although the store may have many such donuts, they only have one menu option for this kind & flavor combination).
- The price of a donut depends only on the dough. The prices table has a row for each kind of dough. The dough (string) column contains the kind; the price (number) column is for one donut made from that kind of dough.
- Your friends only care about the flavor, not the kind of dough. The quantity table contains one row for every flavor your friends want. The choice (string) column is the flavor they want and the k (number) column is the number of donuts of that flavor they want.

Create a table with two columns, flavor (string) and total (number) with one row for each flavor your friends want. The total column contains the least expensive total cost of buying k donuts of that flavor, where k is the number your friends want.

The rows of the result can appear in any order. Here is an example, but complete the query so that it would work even if the contents or number of rows were different.

donuts:

kind	flavor
cake	chocolate
cake	lemon
cake	vanilla
raised	cinnamon
raised	chocolate

prices:

dough	price
cake	2
raised	3

quantity:

choice	k
chocolate	6
cinnamon	3
vanilla	3

result:

:	flavor	total
	chocolate	12
	cinnamon	9
	vanilla	6

SELECT flavor, \_\_\_\_\_ AS total FROM \_\_\_\_\_ WHERE \_\_\_\_ GROUP BY \_\_\_\_\_;

(a) (b) (c) (d)

(a) (2.0 pt) Fill in blank (a).

```
MIN(price) * k
```

- (b) (1.0 pt) Fill in blank (b).
  - Quantity, prices
  - quantity, donuts, prices
  - O donuts, prices AS a, prices AS b
  - O quantity, prices AS a, prices AS b
- (c) (2.0 pt) Fill in blank (c).

```
kind=dough AND choice=flavor
```

(d) (1.0 pt) Fill in blank (d)	
$\bigcirc$ quantity	
Odough	

 $\bigcirc$  kind

flavor

def match(s, pairs):

#### 9. A+ Questions

These are two separate A+ questions. They can only affect your course grade if you have a high A and might receive an A+. Finish the rest of the exam first! There is a third A+ question earlier in the exam on Page 13.

(a) Complete the definition of fib so that (prefix fib 10) is a list of the first 10 Fibonacci numbers: (0 1 1 2 3 5 8 13 21 34). A Fibonacci number is the sum of the previous two. You may not write lambda or let or define.

```
(define-macro (wait expr) `(lambda () ,expr))
(define (prefix s k) (if (zero? k) nil (cons (car s) (prefix ((cdr s)) (- k 1)))))
(define (add s t) (cons (+ (car s) (car t)) (wait (add ((cdr s)) ((cdr t))))))

(define fib (cons 0 (wait (cons 1 _____))))

(wait (add fib ((cdr fib))))
```

(b) This question uses the definitions and functions from the earlier question called Six Pages of Pairings.

Fill in blank (a) of match, a generator function that takes a list s and a pairing pairs. It yields all non-empty pairings of s that contain the pairs in pairs in order, but which may also contain other pairs as well.

Just fill in just blank (a) as your answer. The remaining blanks are not scored.

```
"""Yield all non-empty pairings of s that contain pairs in order.
>>> for p in sorted(match(range(14), [(3, 4), (8, 9)])):
       print(p)
[(0, 1), (3, 4), (8, 9)]
[(0, 1), (3, 4), (8, 9), (11, 12)]
[(0, 1), (3, 4), (8, 9), (12, 13)]
[(3, 4), (8, 9)]
[(3, 4), (8, 9), (11, 12)]
[(3, 4), (8, 9), (12, 13)]
assert is_pair_sequence(pairs)
if len(s) >= 2:
    first = tuple(s[:2])
    if _____: # Just fill in this blank as your answer
         (a)
       yield [first]
    if ____:
       rest = pairs[1:]
    else:
        rest = pairs
    for p in match(____, rest):
       yield _____ + p
    yield from match(____, pairs)
```

```
(len(pairs) == 1 and pairs[0] == first) or not pairs
```

No more questions.