

## SE 504 (Formal Methods and Models)

Spring 2017

### HW #1: Predicate Strength/Weakness and Hoare Triple Laws

Due: 10pm, Thursday, Feb 9

For each of Problems 1 through 7, indicate the weakness/strength relationship that exists between the two given predicates,  $P$  and  $Q$ . Recall that there are four possibilities:  $P$  and  $Q$  are equivalent,  $P$  is strictly stronger than  $Q$ ,  $P$  is strictly weaker than  $Q$ , or none of the above. For a more detailed treatment, follow the *On the Strength/Weakness Relationship between Predicates* link on the course web page.)

You must justify your answers, but you need not provide formal justifications for “obvious” theorems of arithmetic, such as  $x > y \Rightarrow x \geq y$  or  $x \geq y + 4 \Rightarrow x \geq y$  or  $x < y \Rightarrow x \neq y$ .

1.  $P : x > -5 \wedge y > x - 3$  and  $Q : x \geq 1 \wedge y > x$
2.  $P : x > 0 \wedge y \geq x - 1$  and  $Q : x \geq 2 \vee y > x$
3.  $P : x \geq 2 \vee y > x$  and  $Q : x > 0$
4.  $P : x \geq 2 \wedge y > x$  and  $Q : x > 0$
5.  $P : x > 8 \wedge y < x$  and  $Q : x = 0$
6.  $P : x \geq 0 \Rightarrow y > 0$  and  $Q : x = 1 \Rightarrow y \geq 0$
7.  $P : f.i = 7$  and  $Q : (\exists i \mid : f.i = 7)$

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For the last two problems, use the **Strengthening the Precondition**, **Weakening the Postcondition**, **Precondition Disjunctivity**, and **Postcondition Conjunctivity** Laws of Hoare Triples (a link to which you can find on the course web page), as well as “obvious” theorems of arithmetic and theorems from Gries and Schneider, to prove the stated implications.

8.  $(Q_0 \Rightarrow Q_1) \implies (\{P\} S \{Q_0\} \wedge \{P\} S \{Q_1\} \equiv \{P\} S \{Q_0\})$

*Hint:* Assume the antecedent and prove the consequent.

9. If  $\{x \geq y \geq 0\} S \{x > 3y \wedge y > z\}$  and  $\{y < 12\} S \{x > 2y \wedge y < z - 2\}$ , then  $\{x > y \geq 0 \vee y \leq 7\} S \{x \geq 2y \wedge y \neq z\}$