CMPS340 Fall 2016
HW #7: Unique Decipherability and Sardinas-Patterson Graphs
Due: 1pm, Wednesday, November 9

Consider the set of codewords $C = \{aaa, ab, abaa, bb, bba\}$.

(a) Construct (and draw a picture of) $SP(C)$, the Sardinas-Patterson graph for C. From observing the graph you can conclude that C is uniquely decipherable. **Explain.** You could have come to the same conclusion, somewhat more easily, by observing that C possesses a certain property pertaining to the suffixes of its members. **Explain.**

(b) Give an example to demonstrate that C has unbounded deciphering delay. (Use a cycle in the graph to help you construct two arbitrarily long strings that are members of C^* (i.e., of the form $x_1 x_2 \cdots x_n$, where $x_i \in C$ for each i), have different first factors, and cannot be distinguished from each other until you reach their right ends.

(c) Let $C' = C \cup \{babb\}$. Augment the graph you constructed in (a) to obtain $SP(C')$. (One new node and three new edges should result.) From observing the graph you can conclude that C' is **not** uniquely decipherable. **Explain.**

(d) Show a disagreeing pair of C'-factorizations for a shortest string for which this is possible.

(e) Show a *prime* disagreeing pair of factorizations, where those factorizations are formed using a path in $SP(C')$ covering as many distinct edges as possible. (The same edge may appear more than once in the path.[1])

[1] Technically, if any edge appears more than once, it is called a *walk* rather than a path. But we digress!