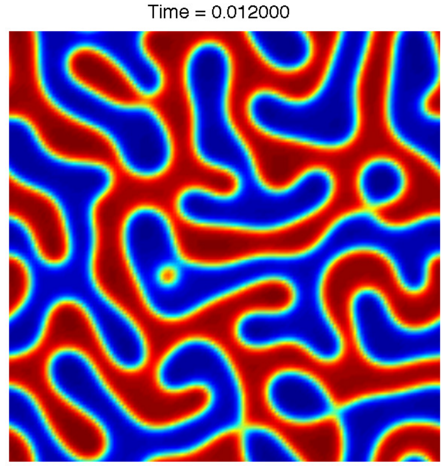
Project 2021 :
Cahn-Hilliard integrator.



The Cahn-Hilliard equation is a mathematical model of the physics which describes the process of separation of microscopic phases of a binary mixture.

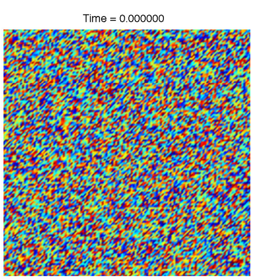
$$\frac{\partial c}{\partial t} = \boldsymbol{\nabla}^2 \left(c^3 - c - a^2 \boldsymbol{\nabla}^2 c\right) = 0$$

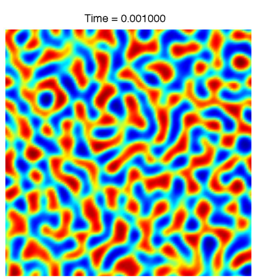
The concentration $c(t, x, y)$ has a value ranging between $[-1, 1]$. The value -1 of 1 corresponds to the pure presence of one or the other component while a zero value corresponds to a perfectly balanced mixture. This equation is a very elegant and effective way to model the segregation of a mixture under the effect of the attraction of the similar particles and the repulsion of the different particles.

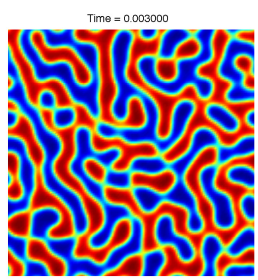
This is to solve this problem with periodic conditions on a square $[0, 1] \times [0, 1]$. We want to know the temporal evolution for the times $0 < t < 0.012$, starting from an initial random situation. A simple way to do this is to use second-order centered differences with an explicit Euler scheme using the following iteration:

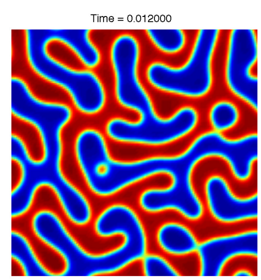
$$\begin{aligned}
F_{i,j} &= \left(C^n_{i,j}\right)^3 - C^n_{i,j} - \frac{a^2}{(\Delta x)^2}\left(C^n_{i-1,j} + C^n_{i+1,j} + C^n_{i,j-1} + C^n_{i,j+1} - 4C^n_{i,j}\right) \\
C^{n+1}_{i,j} &= C^n_{i,j} + \frac{\Delta t}{(\Delta x)^2}\left(F_{i-1,j} + F_{i+1,j} + F_{i,j-1} + F_{i,j+1} - 4F_{i,j}\right)
\end{aligned}$$

Let us consider a timestep $\Delta t = 10^{-6}$ and 12000 iterations, to compute 128^2 nodal values with $a = 0.01$. We mpose boundary conditions Periodic data by imposing that the 0 and 129 indices correspond to 128 and 1 in the equations above.









For the project, we ask you to show that it is quite most efficient to use a discrete cosine transform to invert the linear update matrix by performing a spectral decomposition of the matrix. The time scheme may also be improved, as the selection of the timestep.

1. Write a first `python` program that solves the the Cahn-Hilliard equations with second-order second-order centered differences with an explicit Euler scheme. You must be able to obtain similar pictures as shown above, if you start with a random initial data. You cannot use the MATLAB language ! You are strongly advised to do it as soon as possible : by example, for the next week !

2. Translate 3 codes (by student) of Nicolas Trefethen in the `python` language.
   You must try to preserve the beauty and the simplicity of the codes of Trefethen that can be considered as really nice and well designed. In other words, do not try to improve or to modify the codes : just try to translate them with the idea to respect the original work. It may be a good idea to avoid `import numpy as np` and to use `from numpy import *` in order to keep the concisenss of the original work. Each student will be able to select three codes in three subsets (easy, medium, more difficult). Each code can selected only by one or two students. The selection is based on the rule *first aksed, first served* :-) You are strongly advised to do this part of the work during the first weeks devoted to the lectures about the Trefethen book.

3. Write a second and quite more challenging `python` program that integrates the Cahn-Hilliard equations with a spectral method and a more efficient time integrator. You cannot use the MATLAB language ! Your program program must be as accurate as possible and as fast as possible :-) Using spectral methods is obviously mandatory. The output must be a movie for n^2 nodal values. Investigating large values of n would be nice!

4. As a bonus, you can implement the spectral method in the C or C++ langages. For compiling your C program, detailed explanation and a `cMake` or `Makefile` are expected.

5. Using the BOV library to generate the animation in a C-version of the project is considered as a bonus, but it is not mandatory.

6. Implementing a parallel version is considered as a bonus, but it is not mandatory.

7. Prepare a short movie (podcast) of maximum 10 minutes explaining your projet.
   The best podcast will received a special award from the teacher's team.
   Upload your podcast on `youTube` and provide the URL in your report.

8. Write a rapport of maximum 5 pages. You can explain the numerical method used, describing the efficiency of your program et demonstrating the accuracy of your simulation. Explaining how the physical mathematical model is derived may be a part of the report, but it is not mandatory. Compare results and accuracy obtained with both programs. Analyse the convergence of the spectral results.

9. The deadline to submit the project is Tuesday June 1th 2021.

10. All the project sources and the rapport will be uploaded in the dedicated Teams channel in a single zip file. The name of the will be the names of both students in the so-called camel style. For example, the project of Vincent Legat and Philippe Chatelain will be stored in the following file `LegatChatelain.zip`. No other exotic format as `rar` will be accepted. If you do not strictly follow the naming convention, a small malus will be subtracted from your final grade.

Good luck !