



# CAT-ch me if you can

Version 1.0

(An incomprehensible guide to bell the CAT)

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## Dedication

I whole heartedly dedicate this write-up to those who decide to take CAT with all fanfare at the beginning of the year, but quietly and mysteriously drop out at the last moment, thereby decreasing the total number of applicants.

Had it not been for those nameless altruists, I would surely not have had it easy occupying a seat in the best business school in Asia-Pacific with the toughest entrance procedure in the world – IIM Ahmedabad.

And as an obvious corollary, I would not be writing this. I am deeply indebted to those blessed souls who make this possible.

-The Author

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## Foreword

So you have decided? While deciding to take the CAT itself takes enough grit and determination, sailing through arguably the toughest entrance examination in India needs much more. For one, it takes pure passion; undeterred by initial possible setbacks when you don't live up to your expectation in various mock tests. If there is one thing that can see you through these trying times, it is how passionate you are about studying at IIM. If you are not, then probably you shouldn't be reading this.

Yes, perseverance and hard-work are primary to cracking the CAT. But just working hard doesn't pay off. You need to outsmart the CAT and the clock that ticks away while you sweat it out in the examination hall. The effort should always focus on minimizing the time taken while maximizing the output. That is Time Management in the simplest terms.

While no one can lay down the ground rules for optimum time management, understanding your strengths and weaknesses is crucial to managing time well. It pays to know whether you can comprehend those cryptic Reading Comprehension questions after one and half hours of toiling at the Quantitatives and Data Interpretations; or whether you are left with enough energy to solve Logical Reasonings after breaking your head at Reading Comprehension.

If you are not sure what to do, you might want to change your strategy in the initial few mock tests and build up your own strategy after thoroughly analyzing your performance after each test.

Another aspect of outsmarting CAT is about knowing answers beforehand. There are a lot of situations that are used frequently in CAT and though the question is not always direct, knowing part of the answer beforehand pays really well. That is my experience.

This document attempts to do just that. While certain things mentioned in here may seem trivial, the point it tries to drive home is that knowing even trivial things can save you time in CAT. Most of the other formulas are my way of doing things. Hope they help you find-out your own way of doing things better.

The author urges you to send this document to your friends/relatives who you think might benefit from it. Also, in case you have any doubt or find something mentioned here to be incorrect, please let the author know.

Good Luck!

(The author belongs to IIM Ahmedabad 2005-2007 Batch. He got final admission calls from all the six IIMs. For additional Information on CAT, IIM Interviews, everything else and nothing really, you can send him an email at [get2reach@yahoo.com](mailto:get2reach@yahoo.com) or visit his webpage at <http://www.geocities.com/get2reach>)

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## 1. Arithmetic

### 1.1. Average speed over three equal distances

If a person travels equal distances with speeds  $x$ ,  $y$  and  $z$  respectively, then to calculate the average speed for the whole journey you don't need the time spent in traveling.

$$\text{Average speed} = \text{total distance} / \text{total time taken} = 3d / (d/x + d/y + d/z) = 3xyz / (xy + yz + zx)$$

i.e. Harmonic mean of the individual speeds. You can extend the same to traveling in 4 different speeds and so on.

### 1.2. Infinite GP

Given an infinite GP:  $a, ar, ar^2, ar^3, ar^4 \dots$

In any infinite Geometric Progression, if every term is same as all the successive terms added, then the ratio ' $r$ ' (ratio of successive terms) is equal to  $1/2$ .

Take for example the series:  $2, 1, 1/2, 1/4, \dots$

$$2 = 1 + 1/2 + 1/4 + \dots$$

Extending this further, if every term is twice the addition of all successive terms, the ratio ' $r$ ' is  $1/3$ .

Generalizing, if each term is ' $n$ ' times summation of all successive terms in a GP, then the ratio ' $r$ ' is  $1/(n+1)$

### 1.3. Finding out the APs

Sum of any successive ' $n$ ' terms of an A.P. should always contain the term ' $n^2$ '. Now, if the ratio of sum of first ' $n$ ' terms of the two different APs is  $(2n + 3)/(4n + 2)$ , then how do you find out what the APs are?

Multiply both numerator and the denominator by ' $n$ ' to get the term ' $n^2$ '

Now the ratio is  $(2n^2 + 3n)/(4n^2 + 2n)$

As sum of first ' $n$ ' terms of any AP is  $na + d[n(n-1)/2] = n^2 d/2 + n(a - d/2)$ , compare this with the above expression and equate the powers of  $n$  and  $n^2$  respectively. We get:

$$2 = d_1/2 \text{ and } 3 = a_1 - d_1/2 \dots (i)$$

$$4 = d_2/2 \text{ and } 2 = a_2 - d_2/2 \dots (ii)$$

Now you can find both the APs as you know the first term and the common difference.

### 1.4. Last digit

$n^2 + n$  always ends in either 2, 6 or 0.  $(n^2+n)/2$  would always end in 1, 6, 3, 8, 5, or 0.

### 1.5. Arithmetic mean

Arithmetic mean of  $n$  natural numbers is half of the number increased by 1 i.e.  $(n+1)/2$

Example: Arithmetic mean of first 12 natural numbers is  $13/2 = 6.5$

### 1.6. Maximum value of an expression

$a^m b^n c^p \dots$  will be maximum when  $a/m = b/n = c/p \dots$

Example:  $(a+x)^3 \cdot (a-x)^4$  would be maximum when  $(a+x)/3 = (a-x)/4 \Rightarrow x = -a/7$

### 1.7. Inequality

$x^2 + 1/x^2$  is always greater than or equal to 2.

Because,  $x^2 + 1/x^2 = x^2 + 1/x^2 - 2 + 2 = (x - 1/x)^2 + 2$

So even if  $(x - 1/x)$  is zero, the value would be 2. In all other cases, it's more than 2.

**Caution:**  $y + 1/y$  is not always greater than 2

Say for  $y = -2$ ,  $-2 + 1/-2$  is negative and is not greater than 2.

$y + 1/y$  is greater than or equal to 2 only if  $y$  is positive.

### 1.8. Number of distinct divisors

How do you find out the least number with say 8 distinct divisors?

Any natural number can be expressed as a multiplication of various prime numbers raised to a certain power.

Now if a number  $P$  could be expressed as  $a^x \cdot b^y \cdot c^z$ , the number of distinct divisors that  $P$  has is  $(x+1)(y+1)(z+1)$ , i.e. each of these powers multiplied, after increasing them by 1. This is very important and one should definitely remember this.

Back to the question: Which least number has 8 distinct divisors? Surely, the multiplication of the powers of its prime factors increased by 1 has to be 8.

Can the number be  $2^1 3^3$ ,  $2^3 3^1$  or  $2^1 3^1 5^1$ ? As you can see, the number is  $2^3 3^1$ , i.e. 24. The trick is to start with the smallest prime numbers and see that if power has to be raised, it is done on the smaller ones and NOT the larger ones.

By the way, the divisors of 24 are 1, 2, 3, 4, 6, 8, 12, 24.

### 1.9. Roots and coefficients

Suppose you have an equation of  $n^{\text{th}}$  order:

$$x^n + k_1 x^{n-1} + k_2 x^{n-2} + \dots + k_n = 0$$

Remember that:

$k_1 = (-1)$  . Sum of all the roots

$k_2 = (-1)^2 \cdot \sum$  Product of two roots

$k_3 = (-1)^3 \cdot \sum$  Product of three roots

.

$K_n = (-1)^n$  . Product of all the roots

Where  $\sum$  (sigma) denotes summation.  $\sum$  *product of two roots* means summation of product of all possible combination of two roots.

Example: Take an equation of order 3 with roots a, b and c. The equation would be:

$(x-a)(x-b)(x-c)$ ; Now simplifying this we get:

$$x^3 - (a+b+c)x^2 + (ab+bc+ca)x - abc$$

Here  $k_1 = - (a+b+c)$  i.e.  $(-1)$  . Sum of all the roots

$k_2 = (ab+bd+ca)$  i.e.  $(-1)^2$  .  $\sum$  Product of two roots

$k_3 = -abc$  i.e.  $(-1)^3$  . Product of roots

### 1.10. Binomial expansion

The binomial expansion of  $(a+b)^n$  would be:

$${}^nC_0 a^n + {}^nC_1 a^{n-1} b^1 + {}^nC_2 a^{n-2} b^2 + \dots + b^n \text{ (Important)}$$

Putting  $a=1$ ,  $b=-1$ , you can easily derive that

$${}^nC_0 + {}^nC_2 + {}^nC_4 + \dots = {}^nC_1 + {}^nC_3 + {}^nC_5 + \dots$$

i.e. Sum of all even terms = Sum of all odd terms

### 1.11. Running around a circular track

If two persons start from the same point and run in opposite directions along a circular track, and meet for the first time at the starting point after completing  $n_1$  and  $n_2$  rounds respectively, then  $n_1$  and  $n_2$  must be co-prime.

### 1.12. Divisibility of $a^n - b^n$

$a^n - b^n$  is divisible by  $(a+b)(a-b)$  only if  $n$  is even.

### 1.13. Remainder theorem

Reminder of the division  $(a-1)^n/a$  is 1 if  $n$  is even and the reminder is  $(a-1)$  if  $n$  is odd. However, reminder of  $(a+1)^n/a$  is always 1.

Example:

What is the reminder when 16 is divided by 3? Or 64 divided by 5 for that matter?

Now  $16/3 = (3-1)^4/3$ . As 4 is even, reminder is 1. You can obviously verify that by traditional means.

Similarly  $64/5 = (5-1)^3/5$ . As 3 is odd, the reminder is  $5-1 = 4$ .

What is the reminder when  $101^{101}$  is divided by 25? I am sure now you can answer that.



### 1.14. Minimum number of prime factors

In the expression  $P = x(x+3)(x+6)(x+9)$ , the minimum number of prime factors  $P$  has is 2. Why? (Given that  $x$  is an integer)

Simple.  $P$  has to be even because either  $x$  is even or  $x+3$  is even. Now if  $P$  is even, one prime factor has to be 2. The other one could be even, but it would ultimately have a prime factor other than 2.

In the expression above, if you want  $P$  to have minimum number of prime factors, choose  $x=3$ , because 3, 6, 9 in the expression above are all divisible by 3, thereby reducing the overall number of prime factors. Think about it.

If you put  $x=2$ , you would get additional prime factors like 5 i.e.  $x+3$ , 11 i.e.  $x+9$ . Now you get the point.

**Caution:** It's not necessary that every even number has at least two distinct prime factors. What about  $2^7$ ? It has only one prime factor, i.e. 2.

### 1.15. Summation of all possible numbers formed

Summation of all  $n$ -digit numbers formed using  $n$  distinct non-zero digits (each of the digits being used only once) is:

$$(n-1)! \cdot (\text{sum of all digits}) \cdot (11111\dots n \text{ times})$$

If one of these  $n$  digits is a zero, then the summation would be:

$$(n-2)! \cdot (\text{sum of all digits}) \cdot [(n-1)(11111\dots n \text{ times}) - (11111\dots n-1 \text{ times})]$$

Example: What is the sum of all 3 digit numbers formed using 2, 3 and 5 (use each digit only once)?

The numbers are: 235, 253, 325, 352, 532 and 523. The summation is 2220.

From the formula:  $(3-1)! \cdot (2+3+5) \cdot 111 = 2220$

### 1.16. Finding out an AP, given the average

If average of  $2n+1$  consecutive numbers is ' $x$ ', take the first number as  $x-n$  and the last one as  $x+n$ .

If the numbers are not consecutive, with a common difference of  $d$ , then the first number is  $x-nd$  and the last one is  $x+nd$

The middle number is always same as the average.

Example: What is the 6<sup>th</sup> number in an AP, whose average of first 11 numbers is 24?

The 6<sup>th</sup> Number is 24. The AP is easy to find out, assuming that all numbers are consecutive.

AP would be  $(24-5)\dots 24\dots (24+5)$  i.e. 19, 20, 21, 22, 24, 24, 25, 26, 27, 28, 29; 11 terms

If the numbers were not consecutive, with a common difference of 2, the AP would have been 14, 16, 18, 20, 22, 24, 26, 28, 30, 32, 34

Note that the middle number is always 24 irrespective of the common difference.

**1.17. Summation of series**

$$1X2 + 2X3 + 3X4 + 4X5 \dots aX(a+1) = \sum_{n=1,a} (n^2+n) = \sum_{n=1,a} n(n+1)$$

Where  $\sum_{n=1,a}$  means summation of all terms where n varies from 1 to a.

$$\text{Example: } 1X2 + 2X3 + 3X4 + 4X5 = \sum_{n=1,4} (n^2+n) = \sum_{n=1,4} n^2 + \sum_{n=1,4} n \\ = [4(4+1)(2X4+1)]/6 + 4X(4+1)/2 = 40$$

**1.18. Increase in market size**

If market share of a TV brand is x% and is increased by y%, sale of all other TV brands remaining constant, the total increase in the market size would be  $x.y/100$  %.

**1.19. Usual time of travel**

If walking at 'p' fraction ( $p < 1$ ) of the usual speed, one reaches a place 'h' hours late, the usual time taken to travel (in hours) is  $hp/(1-p)$

Similarly, if one reaches 'h' hours early (in which case  $p > 1$ ), the usual time of travel is  $hp/(p-1)$

**1.20. Divisible by all numbers less than the square root**

There are only 7 numbers which are divisible by all natural numbers less than their square root.

They are: 2, 3, 4, 6, 8, 12 and 24

**1.21. Perfect square to any base**

The number 14641 to any base is always a perfect square as long as the base is more than 6, of course.

Because  $14641_x = x^4 + 4x^3 + 6x^2 + 4x + 1 = (x+1)^4$ , which is a perfect square. This case can be extended to all such numbers which form co-efficient of a perfect square, cube etc. expressions.

**1.22. Power of a term in expansion**

The power of y in the expansion of expression  $(y^p + 1/y^q)^n$  is always of the form  $y^{(p+q)x - qn}$ . Similarly, in the expansion of  $(y^p + y^q)^n$  the power of y is of the form  $y^{(p-q)x + qn}$

This formula would help find out whether a particular term occurs at all in the expansion of an expression.

Example: Is there a  $y^5$  in the expansion of  $(y^3 + 1/y^2)^3$ ?

Now the power has to be of the form  $(3+2)x - 2X3 = 5x - 6$ . Obviously for no value of integral x would the value be equal to 5. Hence  $y^5$  is not there in the expansion.

If you expand the expression given above, it would be:

$$y^9 + y^{-6} + 3y^{-1} + 3y^4$$

As you can see, all these powers satisfy the form  $5x-6$  for integral x.

**1.23. [x]**

If  $[x]$  is defined as the greatest integer less than or equal to  $x$  then

$$[x] = [x/2] + [(x+1)/2]$$

**1.24.  $p^x - p$  is divisible by  $x$** 

If  $x$  is prime, then for any natural number ' $p$ ',  $p^x - p$  is divisible by  $x$ .

**1.25. Expressing a number as product of two co-prime factors**

Every number can be expressed as a product of prime numbers.

Say we have a number  $P = a^x b^y c^z$  where  $a, b, c$  are prime numbers. In how many ways can you express  $P$  as a product  $AXB$  where  $A$  and  $B$  are co-prime to each other?

(Two numbers are called co-prime to each other when they have no common factor except 1)

- i. The answer is  ${}^3C_1$  if there are 3 prime factors
- ii.  ${}^4C_1 + {}^4C_2/2$  if there are 4 prime factors
- iii.  ${}^5C_1 + {}^5C_2$  if there are 5 prime factors
- iv.  ${}^6C_1 + {}^6C_2 + {}^6C_3/2$  if there are 6 prime factors

Can you establish a relationship? Give it a try. The catch lies in grouping the prime factors into 2 groups.

Example: How many ways can you express 420 as a product of two co-prime numbers?

$420 = 2^2 3^1 5^1 7^1$  There are four prime factors 2, 3, 5 and 7.

The number of ways it can be described as a product of two co-prime numbers is:

$$420 = 4 \times 105$$

$$420 = 3 \times 140$$

$$420 = 5 \times 84$$

$$420 = 7 \times 60$$

$$420 = 12 \times 35$$

$$420 = 15 \times 28$$

$$420 = 21 \times 20$$

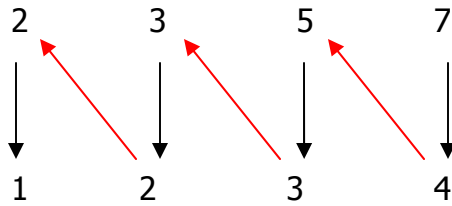
7 ways. That is. Same as  ${}^4C_1 + {}^4C_2/2 = 4+3=7$

**1.26. Recursive division**

If a number divided successively by  $a, b, c, d$  leaves remainder  $w, x, y, z$  respectively then the number must be of the form:

$$((((nd+z)c+y)b+x)a+w)$$

Example: If I ask you what is the least number that leaves remainder 1, 2, 3 and 4 when divided successively by 2, 3, 5 and 7, remembering the formula above might be a little difficult. Check out the following arrangement:



Starting from right, do an addition for the black (down) arrows and multiplication for the red (slant) ones. So your answer would be:

$7+4 = 11$   
 $11 \times 5 = 55$   
 $55+3=58$   
 $58 \times 3=174$   
 $174+2=176$   
 $176 \times 2=352$   
 $352+1=353$

353 is the answer.

### 1.27. *Perfect square*

No number of the form  $3x+2$  can be perfect square

### 1.28. *Division by 12*

If you divide a number having 1 in the beginning followed by all zeros (e.g. 100, 10000 etc) by 12, the remainder is always 4. Of course the number can't be 10.

### 1.29. ${}^nC_x$

${}^nC_x$  is always divisible by  $n$

### 1.30. *First non-zero digit in 10!*

$10!$  obviously would end in zeros. But the first non-zero digit from right is 8. So  $10!$  is of the form xxxxxx800

### 1.31. *Last digit of the remainder*

If you divide a number ' $n$ ' by a number ' $p$ ' where ' $p$ ' ends with 0, last digit of remainder would be same as the last digit of ' $n$ '.

### 1.32. *How many zeros does 100! end with?*

The answer lies in finding out the number of '5's that  $100!$  contains. To find that out, recursively divide 100 with 5 (take just the dividend and ignore the remainder):

$100/5 = 20;$   
 $20/5 = 3$

Add the dividends and that should give you the result. So  $100!$  has 23 zeros in the end. You can extend this to any factorial.

**1.33. Average weight**

If there are 'n' people with average weight 'a' and 'p' people are added with total weight 'w', and the increase in average weight is  $\Delta a$  then:

$$a + \Delta a = (w - n\Delta a)/p \quad \text{i.e. } \Delta a = (w - pa)/(p + n)$$

**1.34. Vehicle breakdown problems**

After a car breakdown, a person travels at 'p' fraction of his original speed. Had the breakdown happened 'd' km earlier, he would have reached his destination 'h' hours later or if it happened 'd' km later, he would have reached 'h' hours earlier.

Then original speed of the person is  $V = d/h \cdot (1/p - 1)$

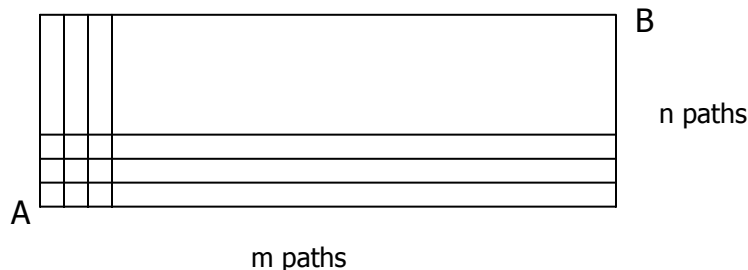
**1.35. Log**

If  $ax = by$  then  $\log_b a = y/x$

**1.36. Theory of de-arrangement:**

If there are 'n' letters and 'n' envelopes with each letter corresponding to just one envelope, the number of ways in which letters can be put into envelopes such that none of the letters go into the correct envelope is:

$$n! - n!/1! + n!/2! - n!/3! + n!/4! + \dots + n!/n! = n! \sum_{k=0, n} (-1)^k / k!$$

**1.37. Way to go**

If there is a grid of paths as shown above with 'm' vertical paths and 'n' horizontal paths, the number of ways one can travel from A to B moving only upward or right-ways and not traversing the same path twice is:  ${}^{m+n}C_n$

**1.38. Duplex of numbers to find out the square**

Duplex of a single digit number x is  $x^2$

Duplex of a two digit number xy is  $2xy$

Duplex of a three digit number xyw is  $2xw + y^2$

Duplex of a four digit number xyzw is  $2xz + 2yw$

Duplex of a five digit number xyzvw is  $2xv + 2yz + w^2$  and so on.

Generalizing, duplex of any number with even no. of digits is twice the sum of product of each pair of digits equidistant from the middle of the number.

Duplex of any number with odd no. of digits is twice the sum of each pair of digits equidistant from the middle digit added to the square of the middle digit.

Now you might wonder why you need to remember duplexes. Well, duplexes can help you find out squares of big numbers in a flash, if you do a little practice. Here is how:

Let's find out the square of 3876

$$\begin{array}{r}
 3 \quad 7 \quad 8 \quad 6 \\
 \hline \dots(1) \\
 \hline \dots(2) \\
 \hline \dots(3) \\
 \hline \dots(4) \\
 \hline \dots(5) \\
 \hline \dots(6) \\
 \hline \dots(7)
 \end{array}$$

Start from the right and proceed towards the left adding one digit at a time while calculating duplex.

1. Duplex of 6=36, write 6 and take 3 as carry
2. Duplex of 86=96;  $96+3=99$ , write 9 and take 9 as carry
3. Duplex of 786=148;  $148+9=157$ , write 7 and take 15 as carry
4. Duplex of 3786=148;  $148+15=163$ ; write 3 and take 16 as carry
5. Duplex of 378=97;  $97+16=113$ ; write 3 and take 11 as carry
6. Duplex of 37=42;  $42+11=53$ ; write 3 and take 5 as carry
7. Duplex of 3=9;  $9+5=14$ , write 14

So the square is 14333796. Calculating this way would be way too faster than doing the multiplication the traditional way.

## 2. Geometry

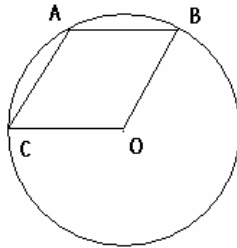
### 2.1. Angle between lines

When calculating the angle between two lines, that is  $\tan \theta = |(m_1 - m_2)/(1 + m_1 m_2)|$  one should consider both the cases before choosing "None of these" as answer:

$$\text{Case 1 : } \tan \theta = (m_1 - m_2)/(1 + m_1 m_2)$$

$$\text{Case 2 : } \tan \theta = (m_2 - m_1)/(1 + m_1 m_2)$$

### 2.2. Rhombus inside a circle

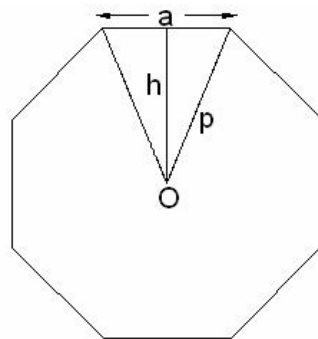


If ABOC is a rhombus with 'O' as the centre of the circle, the triangles ACO and ABO are both equilateral.

### 2.3. Parallelogram and polygons

1. Sum of square of diagonals in any parallelogram is twice the sum of squares of sides.
2. Exterior angle of a regular pentagon is  $360^\circ/5 = 72^\circ$ . Interior angle is  $180^\circ - 72^\circ = 108^\circ$
3. Similarly for a regular heptagon, exterior angle is  $51^\circ$  and interior angle is  $129^\circ$
4. Ratio of the angle the side of a regular polygon subtends at the centre to the exterior angle of the polygon is 1:1.

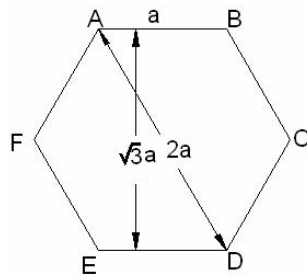
### 2.4. Dimensions of a regular octagon:



$$p = [(\sqrt{2} + 1)/\sqrt{2}] \cdot a$$

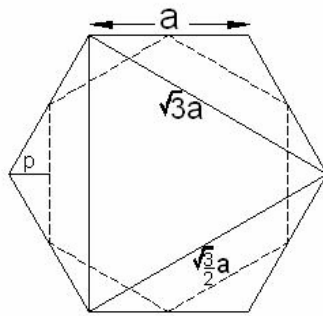
$$h = (\sqrt{2} + 1) \cdot a/2$$

## 2.5. Dimensions of a regular hexagon:



In a regular hexagon with side ' $a$ ', the distance between any two parallel sides is always ' $\sqrt{3}a$ ' and the distance between opposite vertices is ' $2a$ '

If a circle is inscribed inside the hexagon, radius would be ' $\sqrt{3}a/2$ ' and if it's circumscribed, radius would be ' $a$ '.



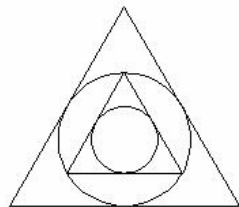
If the alternate vertices of a regular hexagon are joined, the triangle so formed would have ' $\sqrt{3}a$ ' as side. Its area would be half that of the hexagon.

If the midpoints of sides of a regular hexagon are joined, the hexagon so formed would have side equaling ' $\sqrt{3}a/2$ ' i.e.  $\sqrt{3}/2$  times the side of the original hexagon.

Length of perpendicular from a vertex to the corresponding side of the smaller hexagon is  $p = a/4$  i.e. one fourth the side of the original hexagon.

## 2.6. Figures drawn recursively

If similar figures are drawn recursively, the ratio of sides and areas remain constant.

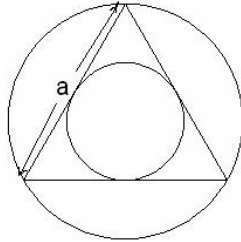




If we draw a circle inside an equilateral triangle and then a circle inside the triangle and so on:

Each  $\Delta$  has  $\frac{1}{4}$  the area of the preceding  $\Delta$  and  
Each circle has  $\frac{1}{4}$  area of the preceding circle.

### 2.7. Incircle and Circumcircle



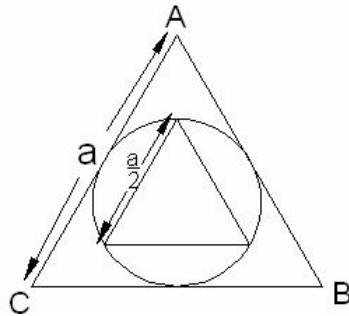
Ratio of radius of incircle to circumcircle of an equilateral triangle is 1:2

If the side of the triangle is 'a' then

Radius of incircle ' $r$ ' =  $a/2\sqrt{3}$

Radius of circumcircle ' $R$ ' =  $a/\sqrt{3}$

### 2.8. Equilateral triangle and circle

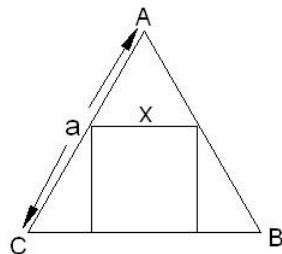


An equilateral triangle inside the incircle of a bigger equilateral triangle would be  $\frac{1}{4}$  in area and  $\frac{1}{2}$  in side length compared to the bigger triangle.

### 2.9. Area to radius

If area of a circle is 'A', radius of the circle would be  $\sqrt{(A/\pi)}$

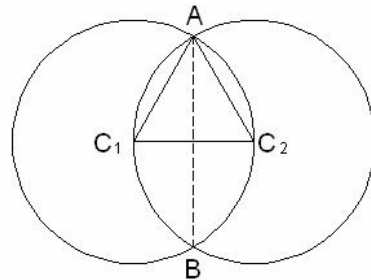
### 2.10. Square inside an equilateral triangle



If there is a square inside an equilateral triangle, the side of the square is

$$x = \frac{\sqrt{3} a}{2 + \sqrt{3}}$$

### 2.11. Overlapping circles



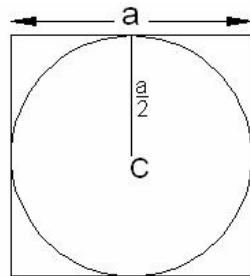
In the figure above, two equal circles intersect each other so that centre of each circle lies on the circumference of the other.

If their radius is 'r' then  $AB = \sqrt{3} r$

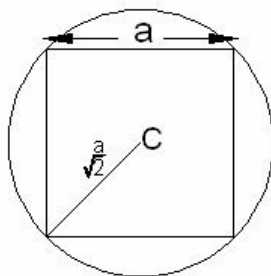
Arc  $AC_2 = 1/6$  of circumference of any circle =  $(\pi/3) r$

$C_1C_2 = AC_1 = AC_2 = r$  i.e. triangle  $AC_1C_2$  is equilateral.

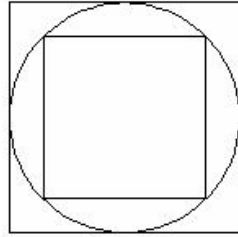
### 2.12. Circle inside a square



If a circle is inscribed inside a square, ratio of area of square to circle =  $4:\pi$

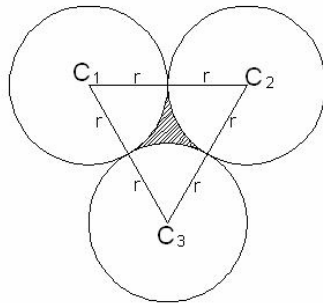


On the other hand, if a square is inscribed inside a circle, ratio of area of square to circle =  $2:\pi$



In the figure above, area of inner square is half that of the outer square.

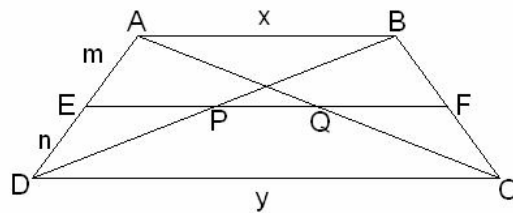
### 2.13. Touching circles



If three equal circles touch each other, area of the enclosed region is:

$$A = \sqrt{3}/4 (2r)^2 - \frac{1}{2}\pi r^2 = (\sqrt{3} - \pi/2) r^2$$

### 2.14. Isosceles trapezium



In the isosceles trapezium given above,  $AB \parallel CD$  and  $AD = BC$ .

- a. If P and Q are midpoints of the diagonals BD and AC respectively, then  $PQ = \frac{1}{2} (\text{Difference between the parallel sides}) = \frac{1}{2} (y-x)$

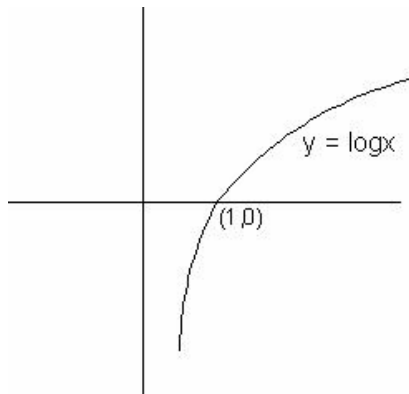
If E and F are the midpoints of AD and BC respectively, then  $EF = \frac{1}{2} (\text{Sum of parallel sides}) = \frac{1}{2} (y+x)$

- b. If E divides AD (or F divides BC) in the ratio  $m:n$  then  $EF = (my+nx)/(m+n)$  and  $PQ = (my-nx)/(m+n)$

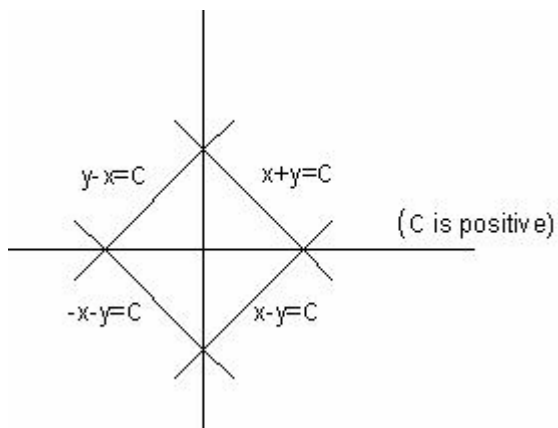
Note that you can arrive at the formula given in (a) if you put  $m=n$

### 3. Common graphs

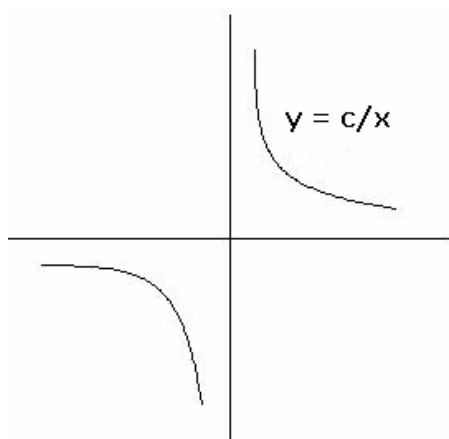
#### 3.1. $y = \log x$

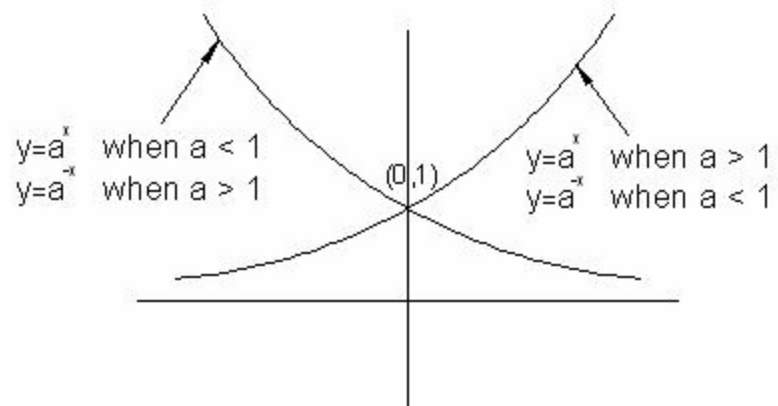
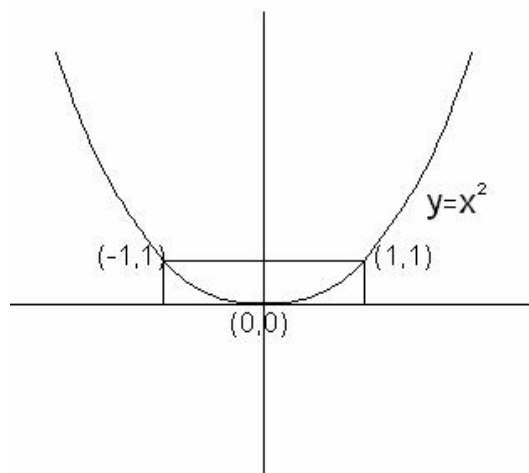
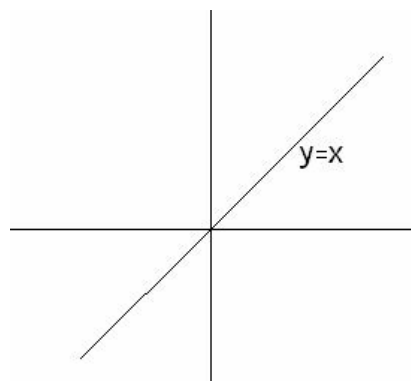


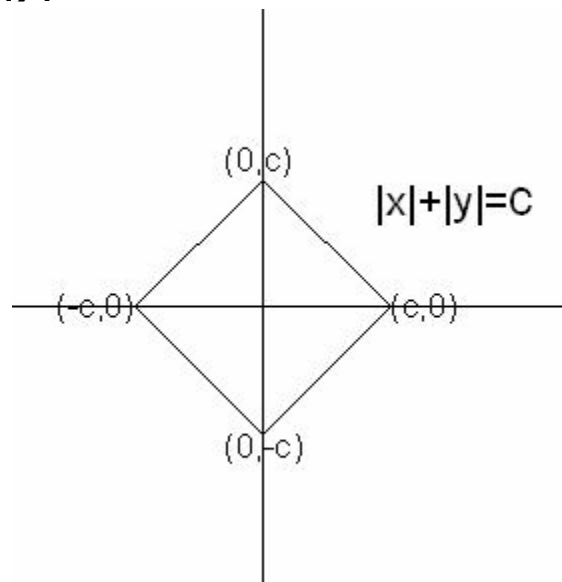
#### 3.2. $x+y=c$



#### 3.3. $y = c/x$



**3.4.  $y=a^x$** **3.5.  $y=x^2$** **3.6.  $y=x$** 

**3.7.  $|x|+|y|=c$** 

The graph above is a rhombus with each side measuring  $\sqrt{2} c$ . Area of the rhombus is  $2c^2$ .

## 4. Appendix

The appendix gives a list of tables that you absolutely need to remember to save those precious seconds in CAT.

### 4.1. Multiplication

1	2	3	4	5	6	7	8	9
13	26	39	52	65	78	91	104	117
14	28	42	56	70	84	98	112	126
15	30	45	60	75	90	105	120	135
16	32	48	64	80	96	112	128	144
17	34	51	68	85	102	119	136	153
18	36	54	72	90	108	126	144	162
19	38	57	76	95	114	133	152	171
20	40	60	80	100	120	140	160	180
21	42	63	84	105	126	147	168	189
22	44	66	88	110	132	154	176	198
23	46	69	92	115	138	161	184	207
24	48	72	96	120	144	168	192	216
25	50	75	100	125	150	175	200	225
26	52	78	104	130	156	182	208	234
27	54	81	108	135	162	189	216	243
28	56	84	112	140	168	196	224	252
29	58	87	116	145	174	203	232	261

## 4.2. Squares

1	1	17	289	33	1089	49	2401
2	4	18	324	34	1156	50	2500
3	9	19	361	35	1225	51	2601
4	16	20	400	36	1296	52	2704
5	25	21	441	37	1369	53	2809
6	36	22	484	38	1444	54	2916
7	49	23	529	39	1521	55	3025
8	64	24	576	40	1600	56	3136
9	81	25	625	41	1681	57	3249
10	100	26	676	42	1764	58	3364
11	121	27	729	43	1849	59	3481
12	144	28	784	44	1936	60	3600
13	169	29	841	45	2025	61	3721
14	196	30	900	46	2116	62	3844
15	225	31	961	47	2209	63	3969
16	256	32	1024	48	2304	64	4096

Remembering squares of number till 65 may be just enough. But, it's always better if you can remember till 100. However, that is a difficult preposition for most mortals!



**4.3. Cubes**

4	64
5	125
6	216
7	343
8	512
9	729
10	1000
11	1331
12	1728
13	2197
14	2744
15	3375
16	4096
17	4913
18	5832
19	6859
20	8000
21	9261

## **5. Disclaimer**

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