CCT class solved assignment 2.

Q1) $6x+1 \equiv 2(x+2) \pmod 7$

$6x+1 \equiv 2x+4 \pmod 7$

$6x+1-1 \equiv 2x+4-1 \pmod 7$

$6x \equiv 2x+3 \pmod 7$

$4x \equiv 3 \pmod 7$

$x = 3 \cdot 4^{-1} \pmod 7$

M.I of 4.

$4 \times ? \equiv 1 \pmod 7$

$4 \times 2 \equiv 1 \pmod 7$

$\therefore$ 2 is the multiplicative inverse of 4.

$\therefore x \equiv 6 \pmod 7$

Q2) $12x \equiv 30 \pmod{38}$

$\Downarrow$

$12x = 30 + 38k$

$6x = 15 + 19k$

$\Downarrow$

$6x \equiv 15 \pmod{19}$

$3 \cdot 2x \equiv 3 \cdot 5 \pmod{19}$

$2x \equiv 5 \pmod{19}$

$x = 5 \times 2^{-1} \pmod{19}$

$2 \times ? \equiv 1 \pmod{19}$

Multiplicative Inverse of 2 is 10

$\therefore x \equiv 5 \times 10 \pmod{19}$

$\equiv 50 \pmod{19}$

$\equiv 12 \pmod{19}$

Q3) $4x \equiv 5 \pmod{14}$

gcd (4, 14) = ?

q	r_1	r_2	r
0	4	14	4
3	14	4	2
2	4	2	0
x	2	0	x

$\therefore$ gcd (4,14) = 2
but gcd (4,14) is not divisibleby 5
$\therefore$ congruence has no solutions.

Q4)

q	r_1	r_2	r	t_1	t_2	t
6	20	3	2	0	1	-6
1	3	2	1	1	-6	7
2	2	1	0	-6	7	-20
x	1	0	x	7	-20	x
2	20	9	2	0	1	-2
4	9	2	1	1	-2	9
2	2	1	0	-2	9	-20
x	1	0	x	9	-20	x
1	20	11	9	0	1	-1
1	11	9	2	1	-1	2
4	9	2	1	-1	2	-9
2	2	1	0	2	-9	8
x	1	0	x	-9 ↳ 20-9=11	8	x
1	20	13	7	0	1	-1
1	13	7	6	1	-1	2
1	7	6	1	-1	2	-3
6	6	1	0	2	-3	+20
x	1	0	x	-3 → 20-3=17	20	x

1	20	19	1	0	1	-1
19	19	1	0	1	-1	20
x	1	0	x	-1	20	x

$\hookrightarrow 20-1 = 19$

The M.I. pairs in modulus 20 are
(1,1), (3,7), (9,9), (11,11), (13,17), (19,19)

Q5) a) $180x + 38y = 1$

$180 = 38 \times 4 + 28$

$38 = 28 \times 1 + 10$

$28 = 10 \times 2 + 8$

$10 = 8 \times 1 + 2$

$8 = 2 \times 4 + 0$

$2 = 10 - 8 \times 1$
$= 10 - 1 \times (28 - 10 \times 2)$
$= 10 - 1 \times 28 + 10 \times 2$
$= 3 \times 10 - 1 \times 28$
$= 3(38 - 28 \times 1) - 1 \times 28$
$= 3 \times 38 - 3 \times 28 - 1 \times 28$
$= 3 \times 38 - 4 \times 28$
$= 3 \times 38 - 4(180 - 38 \times 4)$
$= 3 \times 38 - 4 \times 18 + 16 \times 38$
$= 19 \times 38 - 4 \times 180$

M.I. of 38 = 19 //

b) $180x + 7y = 1$

$180 = 7 \times 25 + 5$

$7 = 5 \times 1 + 2$

$5 = 2 \times 2 + 1$

$2 = 1 \times 2 + 0$

$1 = 5 - 2 \times 2$
$= 5 - 2 \times (7 - 5 \times 1)$
$= 5 - 2 \times 7 + 5 \times 2$
$= 3 \times 5 - 2 \times 7$
$= 3 \times (180 - 7 \times 25) - 2 \times 7$
$= 3 \times 180 - 75 \times 7 - 2 \times 7$
$= \cancel{23 \times 7 - 1 \times 180}$
$= 3 \times 180 - 77 \times 7$

M.I. of 7 = -77
or
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c) $180x + 132y = 1$

$180 = 132 \times 1 + 48$

$132 = 48 \times 2 + 36$

$48 = 36 \times 1 + 12$

$36 = 12 \times 3 + 0$

M.I. of $132 = -4$ or 128

$12 = 48 - 36 \times 1$

$= 48 - 1 \times (132 - 48 \times 2)$

$= 48 - 1 \times 132 + 2 \times 48$

$= 3 \times 48 - 1 \times 132$

$= 3 \times (180 - 132) - 132$

$= 3 \times 180 - 4 \times 132$

(d) $180x + 24y = 1$

$180 = 24 \times 7 + 12$

$24 = 12 \times 2 + 0$

$12 = 180 - 27 \times 7$

M.I. of $24 = -7$ or 17

Q6) a) $25x + 10y = 15$

$a = 25$, $b = 10$, $c = 15$

$\gcd(25, 10) = 5 = d$

5 divides 15 $\therefore$ there are infinite solutions.

we can divide both the sides by 5

$5x + 2y = 3 \Rightarrow \cancel{5s + 2t = 3}\ 1$

find s and t such that

$5s + 2t = \cancel{3}\ 1$

$5 = 2 \times 2 + 1$

$2 = 1 \times 2 + 0$

$1 = 5 - 2 \times 2$

$s = 1$ $t = -2$

Particular soln: $x_0 = (c/d)\, s = (15/5)\, 1 = 3$

$y_0 = (c/d)\, t = (15/5)(-2) = -6$.

General soln: $x = x_0 + k(b/d) \Rightarrow x = 3 + (10/5)\, k$

$x = 3 + 2k$

$y = y_0 \cancel{+} - k(a/d) \Rightarrow y = -6 - k(25/5) = -6 - 5k$

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(b) $19x + 13y = 20$

$a = 19 \quad b = 13 \quad c = 20$

$\gcd(19, 13) = 1 = d$

1 divides 20

$\therefore d = 1$ and there are infinite solutions.

Find s and t such that

$19s + 13t = 1$

$19 = 13 \times 1 + 6$

$13 = 6 \times 2 + 1$

$6 = 1 \times 6 + 0$

$1 = 13 - 6 \times 2$

$= 13 - 2 \times (19 - 13 \times 1)$

$= 13 - 2 \times 19 + 2 \times 13$

$= 3 \times 13 - 2 \times 19$.

$s = -2 \quad t = 3$

Particular soln : $x_0 = (20/1)(-2) = -40$

$y_0 = (20/1)(3) = 60$

G.S. $x = x_0 + 13k \Rightarrow x = -40 + 13k$

$y = y_0 - 19k \Rightarrow y = 60 - 19k$

(c) $14x + 21y = 77$

$\rightarrow a = 14 \quad b = 21 \quad c = 77$

$\gcd(14, 21) = 7$

$\therefore d = 7$ and ~~it has~~ there are infinite solns.

Find s and t such that

$14s + 21t = 1$

$21 = 14 \times 1 + 7$

$14 = 7 \times 2 + 0$

$7 = 21 - 14 \times 1$

$s = -1 \quad t = 1$

P. Soln : $x_0 = (77/7) \cdot (-1) = -11 \quad y_0 = (77/7)(1) = 11$

G·Soln : $x = -11 + 3k$ $y = 11 - 2k$.

(d) $40x + 16y = 88$

→ $a = 40$ $b = 16$ $c = 88$

$\gcd(40, 16) = 8$

$d = 8$ there are infinite soln. $(5x + 2y = 11)$

find s and t such that

$5s + 2t = 1$

$5 = 2 \times 2 + 1$ $1 = 5 - 2 \times 2$

$2 = 1 \times 2 + 0$ $s = 1$ $t = -2$.

P. Soln: $x_0 = 11 \times 1 = 11$ $y_0 = -22$

G-Soln: $x = 11 + 2k$ $y = -22 - 5k$.

Q7) $3^{31} \bmod 7$

→ We know that by fermat's little theorem

$3^6 \equiv 1 \pmod 7$

$\therefore (3^6)^5 \cdot 3^1 \bmod 7 \equiv (1)^5 \cdot 3^1 \bmod 7$

$\cancel{\equiv 3^6 \bmod 7}$ $\equiv 3 \bmod 7$.

$\therefore 3^{31} \bmod 7 \equiv 3 \bmod 7$.

Q8) $128^{129} \bmod 17$

→ $128^{16} \equiv 1 \bmod 17$ (by thm).

$\cancel{(128^{16})^8 \cdot 128 \equiv 1 \bmod 17}$.

$(128)^{129} \bmod 17 \equiv (128^{16})^8 \cdot 128 \bmod 17$.

$\equiv 128 \bmod 17$

$\equiv 9 \bmod 17$.

Q9) $2^{20} + 3^{30} + 4^{40} + 5^{50} + 6^{60} \bmod 7$

$\rightarrow$ $2^{20} \bmod 7 + 3^{30} \bmod 7 + 4^{40} \bmod 7 + 5^{50} \bmod 7$
$+ 6^{60} \bmod 7$

we know by fermat's little thm.

~~$2^6 \equiv 1 \bmod 7$, $3^6 \equiv 1 \bmod 7$~~

$2^6 \equiv 3^6 \equiv 4^6 \equiv 5^6 \equiv 6^6 \equiv 1 \bmod 7$.

$(2^6)^3 \cdot 2^2 + (3^6)^5 + (4^6)^6 \cdot 4^4 + (5^6)^8 \cdot 5^2 + (6^6)^{10} \bmod 7$

$\equiv 4 + 1 + 256 + 25 + 1 \bmod 7$.

$\equiv 287 \bmod 7$

$\equiv 0 \bmod 7 = 0$.