Mechanics and Materials for Engineering Design

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A.Y. 2020/2021

Contents

1 Design of a pressure vessel

1.1 Introduction

The Figure 1.1 schematically illustrates a cylindrical pressure vessel with ellipsoidal heads. This design solution assures a more favourable stress state at the junction between the cylindrical part and the heads.

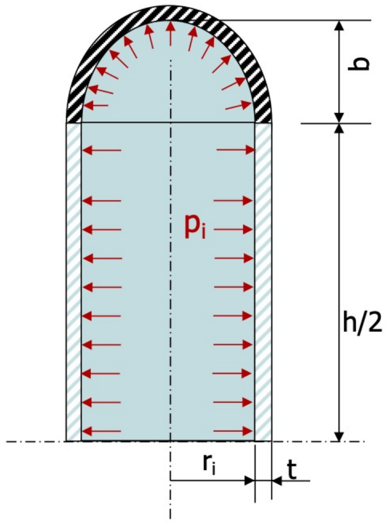


Figure 1.1: Scheme of the vessel.

p_i $[MPa]$	V_0 $[l]$	h_{max} $[m]$	b/r_i
40	1000	3	2

Table 1.1: Data of the homework.

In particular, if the height to radius ratio of the ellipse is 2, the stress concentration factor K_t at the junction is about 1.1. It is requested to design the pressure vessel so as to satisfy the following requirements:

1. Internal volume of fluid equal to V_0.
2. Maximum height of the vessel (including both ellipsoidal and cylindrical parts) not higher than h_{max}.
3. Safety factor against yielding ϕ_y in the most critical location equal to 1.
4. Safety factor against burst ϕ_p (full plasticization, in this case take $K_t = 1$) not lower than 1.2.
5. Vessel made of weldable construction steel.

It is requested to prepare the product design specification and, after completing the design, the technical drawing of the vessel.

1.2 Solution analysis

1.2.1 Material selection and design

For the pressure vessel, the S355JR steel was selected. The yield stress for thicknesses between 40 mm and 63 mm is σ_y = 335 MPa. The satisfaction of the geometrical constraints required was obtained by imposing the following equations:

$$b = 2\, r_i \tag{1.1}$$

$$r_o = r_i + t \tag{1.2}$$

$$h_{max} = h + 2b + 2t \tag{1.3}$$

The last equation ensures that the all the available length is taken advantage of, producing the most slender pressure vessel possible. It is easily verifiable via reiteration of the calculations that this configuration yields the lightest component.
Subsequently, the compliance with the specified internal volume and the yielding condition were imposed:

$$V_0 = \pi\, r_i^2\, h + \frac{4}{3}\pi\, r_i^2\, b \tag{1.4}$$

$$\sigma_{eq} = 2\, p_i \frac{1}{1 - \frac{r_i^2}{r_o^2}} = \frac{\sigma_y}{\phi_y\, K_t} \tag{1.5}$$

A system comprising the equations above was solved to yield the fundamental dimensions of the vessel: r_i, r_o, and t.
The resulting values were rounded for ease of production.

1.3 Loads analysis and verification

1.3.1 Volume and yielding verification

The pressure vessel was than verified with the new dimensions to ensure that the internal volume was bigger than the specified value, and that the safety condition against yielding was still respected:

$$V = \pi\, r_i^2\, h + \frac{4}{3}\pi\, r_i^2\, b \tag{1.6}$$

$$\sigma_{eq} = 2\, p_i \frac{1}{1 - \frac{r_i^2}{r_o^2}} \le \frac{\sigma_y}{\phi_y\, K_t} \tag{1.7}$$

1.3.2 Plastic burst verification

The component was than verified against plastic burst. The internal pressure necessary to cause the burst was calculated, along with the corresponding stress:

$$P_b = \frac{\sigma_y}{2} \ln \frac{r_o^2}{r_i^2} \tag{1.8}$$

$$\sigma_b = 2\,p_i\,\frac{1}{1-\frac{r_i^2}{r_o^2}} \tag{1.9}$$

It was then verified that the ratio between σ_b and the internal pressure σ_y was higher than at least the required safety factor of 1.2:

$$\phi_b \leq \frac{\sigma_b}{\sigma_{eq}} \tag{1.10}$$

1.3.3 Mass calculation

Finally, the mass M of the pressure vessel was calculated:

$$M = \rho\,\left[\pi\,(r_o^2)\,h\,+\,\frac{4}{3}\,\pi\,(r_o^2)\,(b+t) - pi\,(r_i^2)\,h\,\,-\,\frac{4}{3}\,\pi\,(r_i^2)\,b\right] \tag{1.11}$$

1.4 Results

The pressure vessel with the rounded dimensions passed the yielding verification with the requested safety coefficient. The plastic burst verification resulted in a safety coefficient ϕ_b of 1.29, well above the requirement. The main dimensional values obtained are summarized in table 1.2.

$Steel$	$r_i\ [mm]$	$r_o\ [mm]$	$t\ [mm]$	$h_{max}\ [mm]$	$M\ [Kg]$	$V\ [l]$
S355JR	365	426	61	3000	3209.55	1000.89

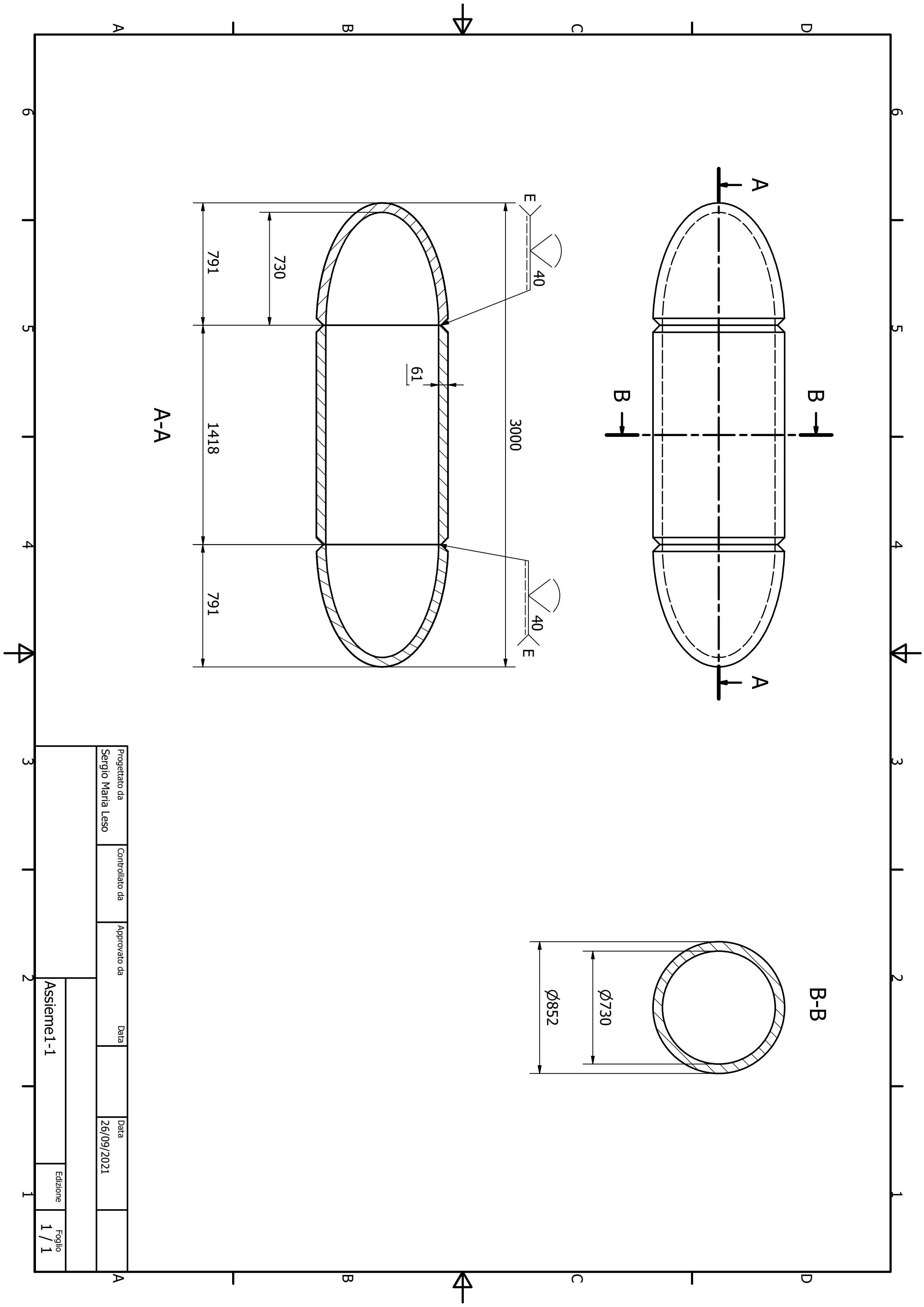
Table 1.2: Resulting specifications of the pressure vessel.

1.5 Conclusions

The pressure vessel was designed respecting the dimensional constraints imposed. The choice of a slender pressure vessel ensures the efficiency of the solution, as it minimizes the amount of material required. The material of choice was the S355JR structural weldable steel. The component passed both the yielding verification and the plastic burst verification with sufficient safety coefficients.

The drawing of the pressure vessel is included in the annexes.

1.6 Annexes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2 Design and verification of the boom of a jib crane

2.1 Introduction

The jib crane is composed of a horizontal beam (boom) over which a hoisting trolley is able to travel. The boom is supported by a vertical column and a tie-rod through brackets, which permit its rotation ($\pm 90°$ with respect to the position shown in the figure) about the vertical axis. The tie-rod is a round bar with threaded ends.

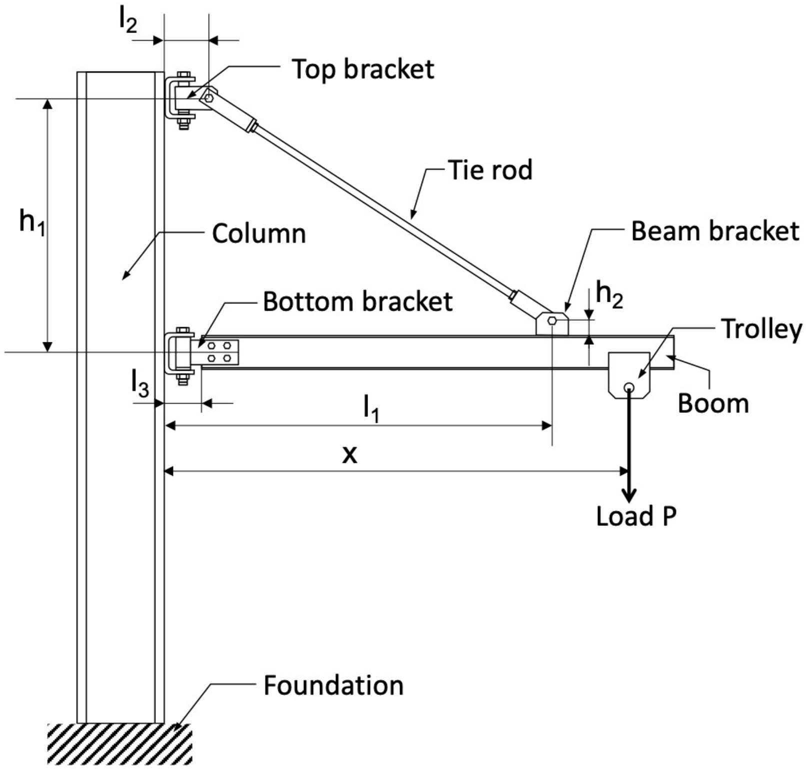


Figure 2.1: Scheme of the jib crane.

l_1 $[mm]$	l_2 $[mm]$	h_1 $[mm]$	h_2 $[mm]$	h_3 $[mm]$
2200	200	1000	120	150

Table 2.1: Data of the homework.

The following requirements must be satisfied:

1. The nominal hoisting capacity is $P = P_{nom} = 1500\ kg$.

2. The crane span is 3200 mm, the load position x can be varied between 600 mm and 3000 mm.

3. The boom is a IPE-beam of standard size. The flange and the web must be large enough to permit the motion of the trolley wheels that can be assumed with 100 *mm* diameter and 20 *mm* thickness.
4. The structure is made of construction steel with good weldability.
5. Low-weight solutions are desirable.

It is requested to select boom size and material able to resist in full elastic regime the nominal load P_{nom} and to assess the maximum overload P_{max} that the boom can resist in its ultimate limit state according to Eurocode 3 prescriptions.

2.2 Solution analysis

2.2.1 Support reactions and internal loads calculations

Since at the beginning of the design, the cross section is not yet selected, the eccentricity of the internal loadings with respect to beams axis is not known a priori. It has been decided to neglect this effect to come to a first-tentative estimation of the cross section. The aforementioned effect is taken into account subsequently. According to EC3 joints category, the brackets can be assumed as hinges with negligible capacity of transmitting bending moments.
Positions of the load (x_2) and position along the beam (x_1) are defined not taking into account the length l_3 and therefore they have the zero value at the beginning of the beam.
To know in which position the load P is more critical for the structure it is possible to write down the solution for the system in terms of the support reactions and the internal actions as function of the position of P (x_2), at least for what concerns the normal action and the shear action and then see which situations are critical. Then it has been decided to select the more critical situations and evaluate the values of the bending moment along the beam.
Once solutions were obtained for different positions of the load P (x_2) it was possible to calculate the stress in critical points along the beam (x_1) and by comparison decide which one was the more critical one.
After this part, the values of the internal actions in the most critical point were considered for the design and verification.

2.2.2 Design and ultimate limit state verification

For this part of the work the material of the beam was selected in order to assure a certain reliability in elastic conditions. After that it was possible to estimate the maximum load sustainable in such conditions and then carry out the ultimate limit state verification according to EC3 to determine the maximum load that the structure can resist in plastic conditions.

2.3 Loads analysis and verification

After some preliminary calculations, in which the eccentricity of the cross-section was neglected, in this section the calculations take into account also this aspect. In particular the selected cross-section is IPE 120.

2.3.1 Support reaction calculation

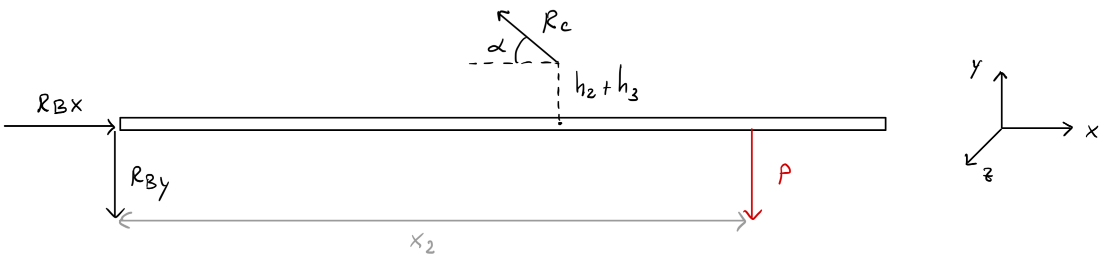


Figure 2.2: Scheme of the free body of the boom.

Resulting equations for the support reactions of the boom are:

$$x: \; R_{Bx} = R_C \, cos(\alpha) \tag{2.1}$$

$$y: \; R_{By} = R_C \, sin(\alpha) \tag{2.2}$$

$$M_z(B): \; -R_C \, cos(\alpha) \; (h_2 + h_3) + P \, x_2 = R_C \, sin(\alpha) \; (l_1 - l_3) \tag{2.3}$$

Solving these equations it is possible to determine the values of the support reactions as function of the position x_2 of the load P.

2.3.2 Internal loads

After some preliminary calculations (see appendix), it was found that the most critical situation is that in which the load is applied in position $x_2 = 2850 \; mm$, therefore internal loads in this section are determined with respect to this specific configuration.
Internal loads are:

$$\begin{cases} N = -R_{Bx}, & 0 \leq x_1 \leq 2050 \\ N = 0, & 2050 < x_1 < \text{end of the beam} \end{cases}$$

$$\begin{cases} V = -R_{By}, & 0 \leq x_1 \leq 2050 \\ V = R_{By} - R_C \, sin(\alpha), & 2050 < x_1 < 2850 \\ V = 0, & 2850 < x_1 < \text{end of the beam} \end{cases}$$

$$\begin{cases} M_z = -V \, x_1, & 0 \leq x_1 \leq 2050 \\ M_z = -R_C \, sin(\alpha) \; (l_1 - l_3) - R_C \, cos(\alpha) \; (h_2 + h_3) - V \, x_1, & 2050 < x_1 < 2850 \\ M_z = 0, & 2850 < x_1 < \text{end of the beam} \end{cases}$$

Corresponding internal loads diagrams are shown in Figures from 2.3 to 2.5.

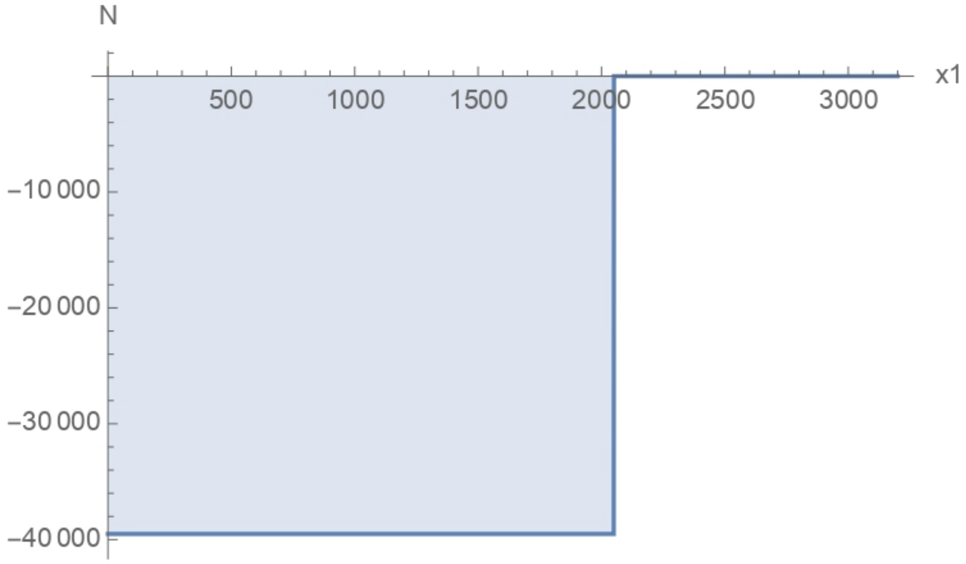


Figure 2.3: Normal action internal loading diagram.

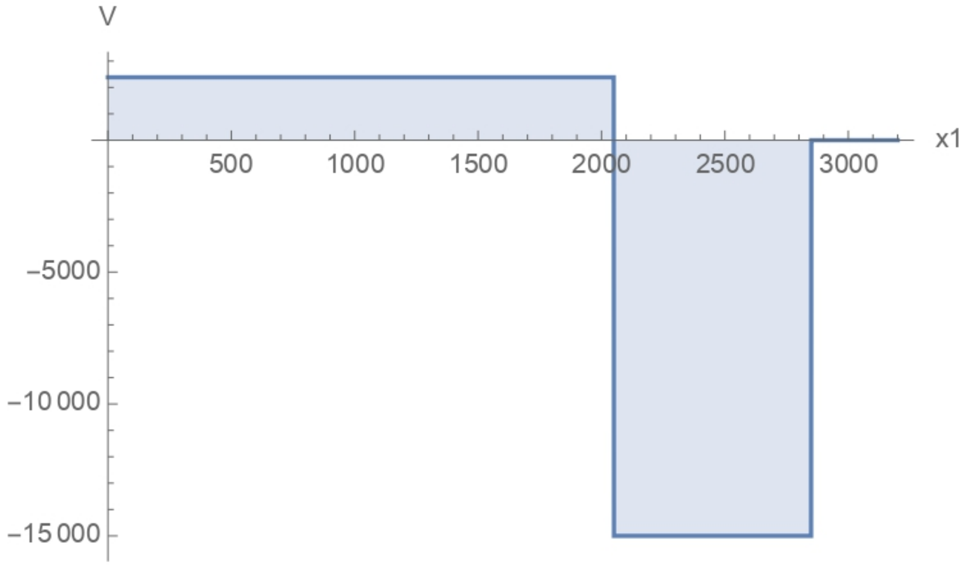


Figure 2.4: Shear action internal loading diagram.

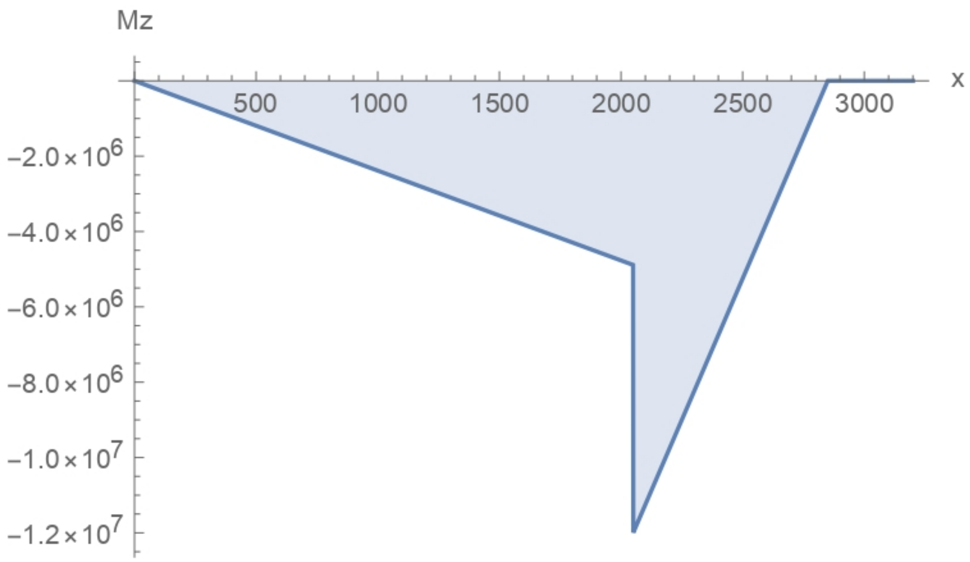


Figure 2.5: Bending moment internal loading diagram.

As it is possible to see from the previous internal loads diagrams, the most critical position in the beam corresponds to $x_1 = 2050\ mm$. The corresponding absolute values of the internal actions are:

$$N = 39510.2\ [N]$$
$$V = 15000\ [N]$$
$$M_z = 1.2\ 10^7\ [N\ mm]$$

2.3.3 Maximum stress

The maximum stress is developed in position $x_1 = 2050$, since in that point all the internal loads reach their maximum value. For the estimation of the maximum stress the shear action has been neglected. The equivalent stress can be expressed as:

$$\sigma_{eq} = \left|\frac{N}{A}\right| + \left|\frac{M_z}{I_z} y\right| \tag{2.4}$$

Where A and I_z are respectively the area and the moment of inertia of the cross-section, and y corresponds to the position $h/2$ with respect to the centre of the cross-section of the beam, that is an IPE 120 (see annexes).
The value of the equivalent stress in $x_1 = 2050$ is:

$$\sigma_{eq} = 256.467\ [MPa] \sim 257\ [MPa]$$

2.3.4 Material selection

For the choice of the material it is necessary to consider the maximum stress developed in the beam, that can be assumed as 257 MPa. With this assumption, a steel S 275 could be a good choice, since the safety coefficient would be > 1, in particular $\phi = 275/257 = 1.07$.
Since the value of the safety factor in this case is very close to 1, it has been decided to select a material that satisfies the condition of $\phi > 1.25$, in order to have a better reliability in the elastic regime. For this reason the selected material is a steel $S355$ and the associated safety factor is $\phi = 355/257 = 1.38$.

2.3.5 Classification of the cross section

The reference material parameter ε is evaluated as:

$$\varepsilon = \sqrt{\frac{235}{f_y}} = \sqrt{\frac{235}{355}} = 0.814 \tag{2.5}$$

The next step is the determination of the parameters α and ψ, that are evaluated according to the stress distribution produced by bending and axial compression. If the cross section belongs to class 1 or 2, the calculation of ψ is not necessary, therefore it has been decided to calculate only α for this part and then ψ if necessary.

$$\alpha = \frac{1}{2}\left(1 + \frac{N_n}{c_w\ t_w\ f_y}\right) = \frac{1}{2}\left(1 + \frac{39510.2}{100\ 4.4\ 355}\right) = 0.626 \tag{2.6}$$

Once α is determined, it is possible to proceed with the classification of the cross section according to the web and the flanges separately.

For what concerns the classification of the cross section according to the web, it is possible to start applying the formula to check if the class is 1, with $\alpha > 1$:

$$\frac{c_w}{t_w} \leq \frac{396\ \varepsilon}{13\ \varepsilon - 1} \tag{2.7}$$
$$22.727 \leq 33.642$$

Since the condition is verified, the cross section, according to the web, belongs to class 1.
Now it is possible to apply the condition to determine the class of the flanges for "I" shaped cross section. The first class to check is the class 1:

$$\frac{c_f}{t_f} \leq 9\ \varepsilon \longrightarrow \frac{25}{6.3} \leq 9\ \varepsilon \tag{2.8}$$
$$3.968 \leq 7.322$$

Since also this condition is verified, and therefore the flanges belong to the first class, it is possible to say that the whole cross section belongs to the first class and therefore it is a section capable of developing a plastic hinge with the rotation capacity required for plastic analysis.

2.3.6 Ultimate limit state verification (application to elastic conditions)

Once the class of the cross-section has been determined and the values of the internal actions are known it is possible to study the conditions for the ultimate limit state verification.
It has been shown that in the most critical point actions are both bending and axial loads, therefore for cross sections of class 1 (like in this case) the ultimate limit state verification is the following:

$$M_{Ed} \leq M_{N,Rd} \tag{2.9}$$

$$M_{N,Rd} = M_{C,Rd}\left[1 - \left(\frac{N_{Ed}}{N_{C,Rd}}\right)^2\right] \tag{2.10}$$

For sections of class 1, $M_{C,Rd}$ is equal to:

$$M_{C,Rd} = \frac{M_{TP}}{\gamma_{M0}} = \frac{f_y\ W_{pl}}{\gamma_{M0}} \tag{2.11}$$

Where γ_{M0} is related to the strength of transversal sections and its value is 1.05.
for what concerns the plastic section modulus W_{pl} or the cross section of the beam IPE 120, it can be determined as follows:

$$W_{pl} = \left[2(b\,t_f)\frac{(h-t_f)}{2} + 2\frac{(h-2t_f)t_w}{2}\ \frac{(h-2t_f)}{4} + 4\left[r^2\left(\frac{h}{2} - t_f - \frac{r}{2}\right) - \frac{\pi\,r^2}{4}\left(\frac{h}{2} - t_f - r + \frac{4\,r}{3\,\pi}\right)\right]\right] \tag{2.12}$$

$$W_{pl} = 104170\ mm^3$$

$$M_{C,Rd} = 3.522x10^7\ Nm$$

For what concerns the axial load that leads to full plasticisation, it can be determined as:

$$N_{C,Rd} = \frac{\Omega\ f_y}{\gamma_{M0}} \tag{2.13}$$

Where Ω is the area of the cross section, and in this case (IPE 120) can be determined as follows:

$$\Omega = h\,b - \left[(b-t_w)(h-2t_f)\right] + 4\left(r^2 - \frac{\pi\,r^2}{4}\right) = 1321mm^2 \tag{2.14}$$

$$N_{C,Rd} = 446624\ N$$

Now it is possible to substitute the axial load and the bending moment in the most critical point in place of N_{Ed} and M_{Ed} to see if the condition is verified:

$$1.200x10^7\ Nm \leq 3.494x10^7\ Nm$$

Of course in this case the condition is verified, since the values of the actions substituted are obtained in the elastic regime.

2.3.7 Critical load for plastic conditions and maximum load sustainable

As said, the previous verification is satisfied since it is carried out considering the elastic regime. Now the aim is to determine for which value of P the structure begins to deform plastically. To do so it is possible to consider the condition of maximum stress and substitute it with the maximum value that the steel can sustain in elastic regime (355 MPa), and, imposing P as a variable, solve the equation:

$$355\ MPa = \left|\frac{N(P)}{A}\right| + \left|\frac{M_z(P)}{I_z}y\right| \tag{2.15}$$

$$P_{crit} \longrightarrow 2076.3\ kg$$

At this point it is possible to use the conditions for the ultimate limit state verification seen before to impose the limit conditions for the resistance and find the value of the maximum load P_{max} sustainable in plastic conditions. It is possible to do this calculation by imposing the equality and by solving the equation as function of P_{max}:

$$M_{Ed}(P_{max}) = M_{C,Rd}\left[1 - \left(\frac{N_{Ed}(P_{max})}{N_{C,Rd}}\right)^2\right] \tag{2.16}$$

$$P_{max} \longrightarrow 4139.98\ kg$$

The value has been rounded but it is possible to show that it verifies all the conditions for the ultimate limit state even if it is almost at the limit. A control has been made with the value of 4200 kg and it was shown that for this value the verification was not satisfied.

2.3.8 Ultimate limit state verification (limit conditions)

Limit conditions (P=4139.98 kg)

$$\varepsilon = 0.814$$
$$\alpha = 0.849$$
$$N_n = 109048\ N$$
$$\text{Class 1 web} \longrightarrow \text{verified}$$
$$\text{Class 1 flanges} \longrightarrow \text{verified}$$
$$M_{Ed} \leq M_{N,Rd} \longrightarrow 3.31198x10^7\ Nm \leq 3.31198x10^7\ Nm$$

Exceeding limit conditions (P=4200 kg)

$$\varepsilon = 0.814$$
$$\alpha = 0.854$$
$$N_n = 109048\ N$$
$$\text{Class 1 web} \longrightarrow \text{verified}$$
$$\text{Class 1 flanges} \longrightarrow \text{verified}$$
$$M_{Ed} \leq M_{N,Rd} \longrightarrow 3.36000x10^7\ Nm \nleq 3.30585x10^7\ Nm$$

2.4 Results

Main results of the design are reported in table 2.2.

Cross section	IPE 120
Material	$S355$
Most critical position of the load $P = 1500\ kg$	$x_2 = 2850\ mm$
Most critical position along the beam	$x_1 = 2050\ mm$
Safety coefficient (elastic conditions), $P = 1500\ kg$	$\phi = 1.38$
Critical load (elastic/plastic limit)	2076.3 kg
Maximum sustainable load (plastic conditions)	4139.98 kg

Table 2.2: Data for the design and verification of the boom of the jib crane.

2.5 Conclusions

For the design of the cross section of the boom of the jib crane, the beam IPE 120 selected (in steel S355) is able to resist with an applied load of $P = 1500\ kg$ in elastic conditions with a safety coefficient of $\phi = 1.38$.

Increasing the load up to $P = 2076.3\ kg$ the structure is still in elastic conditions but overcoming this specific value of loading it enters the plasticity region. In particular, it was estimated a maximum resistance up to a load of $P_{max} = 4139.98\ kg$ in this region, by considering the ultimate limit state verification.

2.6 Annexes

2.6.1 IPE 120 cross section characteristics

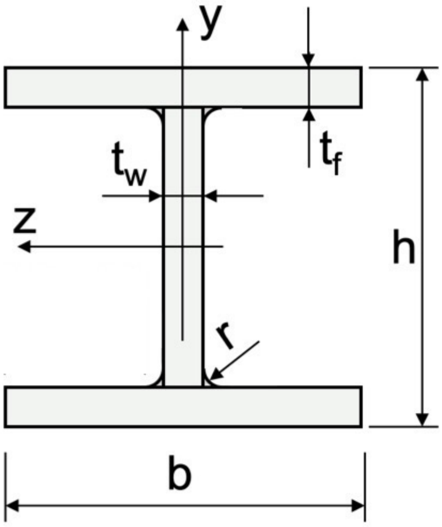


Figure 2.6: Scheme IPE 120.

t_w	t_f	h	b	r
4.4 mm	6.3 mm	120 mm	64 mm	7 mm

Table 2.3: Data for IPE 120 cross section.

2.7 Appendix

2.7.1 Most critical condition determination

Resulting equations for the support reactions are:

$$x:\ R_{Bx} = R_C\ cos(\alpha) \tag{2.17}$$

$$y:\ R_{By} + P = R_C\ sin(\alpha) \tag{2.18}$$

$$M_z(B):\ -R_C\ cos(\alpha)\ (h_2 + h_3) + P\ x_2 = R_C\ sin(\alpha)(l_1 - l_3) \tag{2.19}$$

Solving the previous equations it is possible to determine the values of the support reactions as function of the position x_2 of the load P. Results are graphically shown in Figure 2.7.

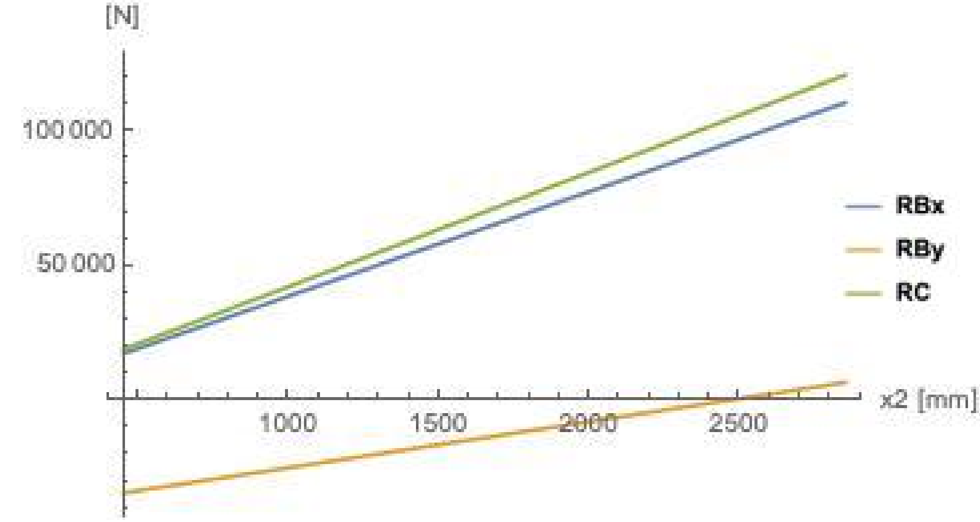


Figure 2.7: Support reactions as function of x_2.

After obtaining the expressions of the support reactions as function of x_2 it is possible to evaluate the internal actions of tension/compression and shear:

Normal actions depending on x2

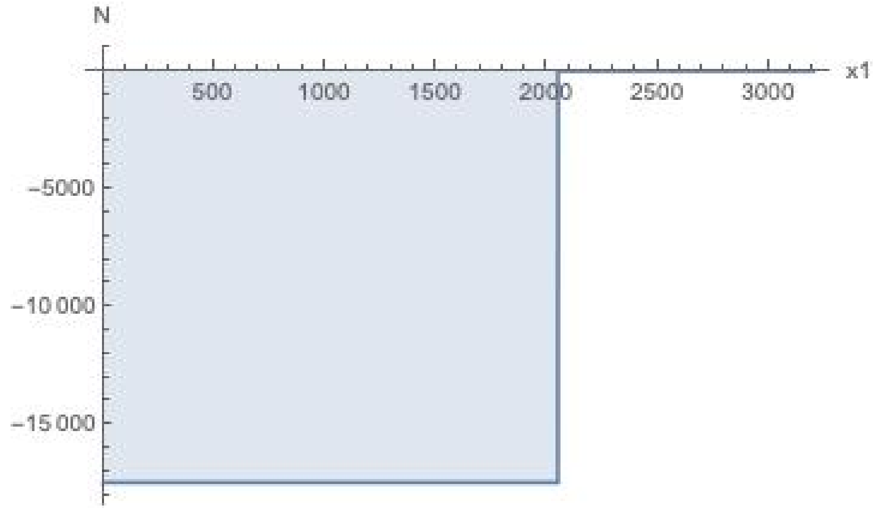


Figure 2.8: Normal action for $x_2 = 450\ mm$.

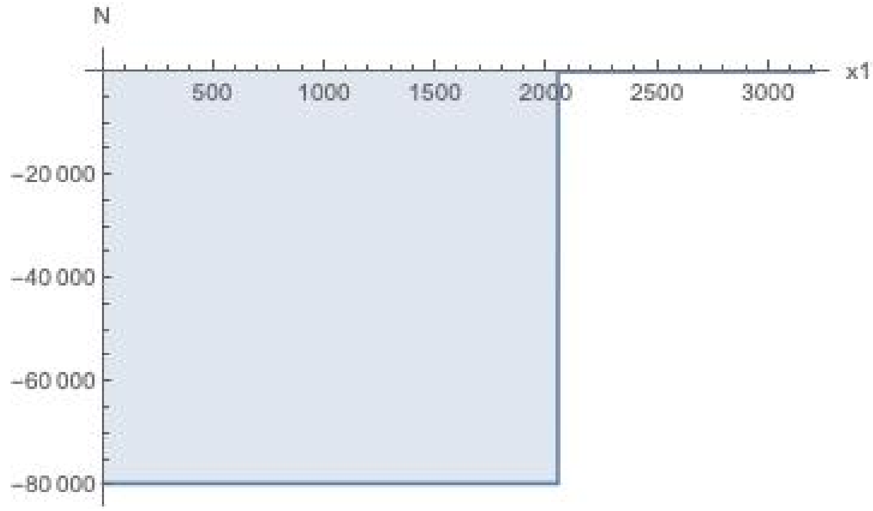


Figure 2.9: Normal action for $x_2 = 2050$ *mm*.

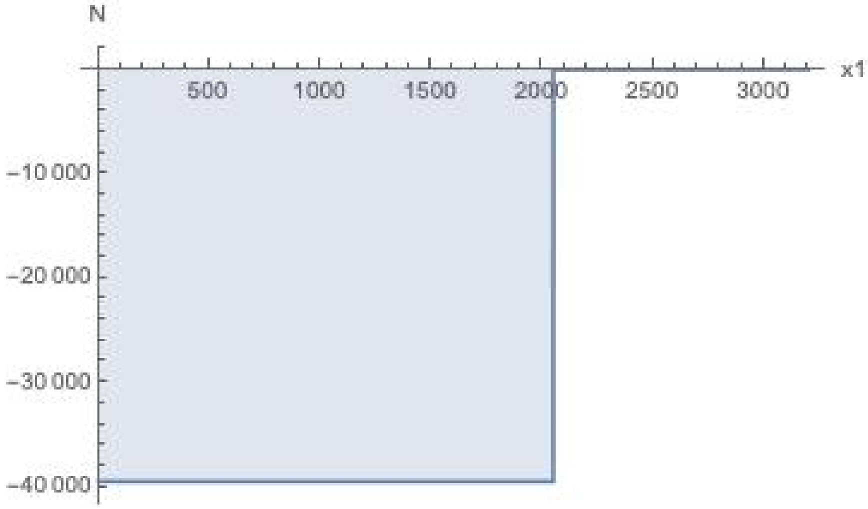


Figure 2.10: Normal action for $x_2 = 2850$ *mm*.

Shear actions depending on x2

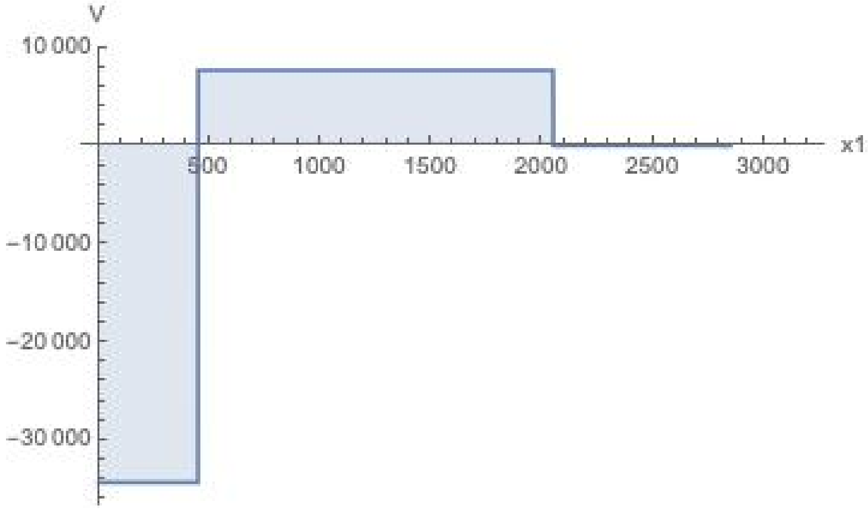


Figure 2.11: Shear action for $x_2 = 450$ *mm*.

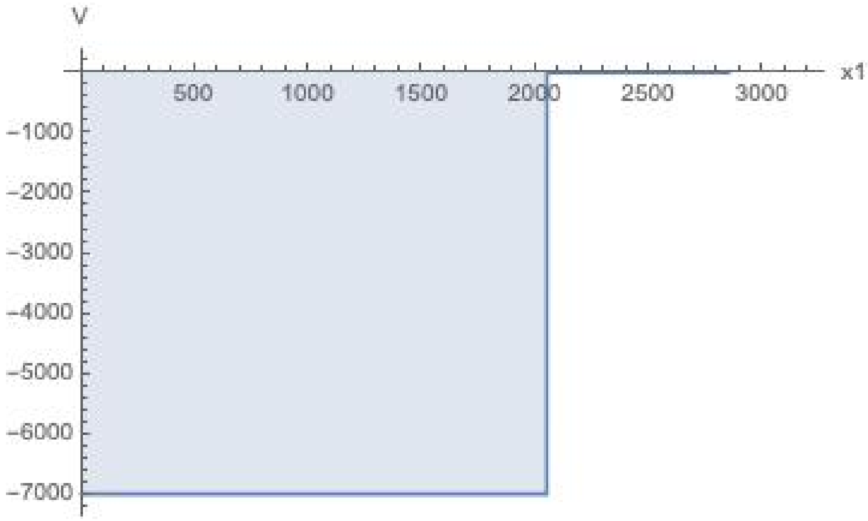


Figure 2.12: Shear action for $x_2 = 2050$ *mm*.

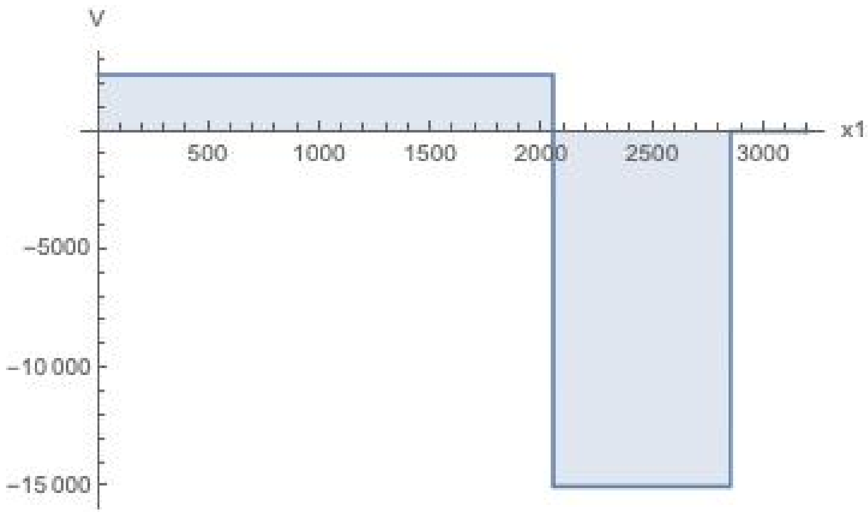


Figure 2.13: Shear action for $x_2 = 2850\ mm$.

Bending moment depending on x2

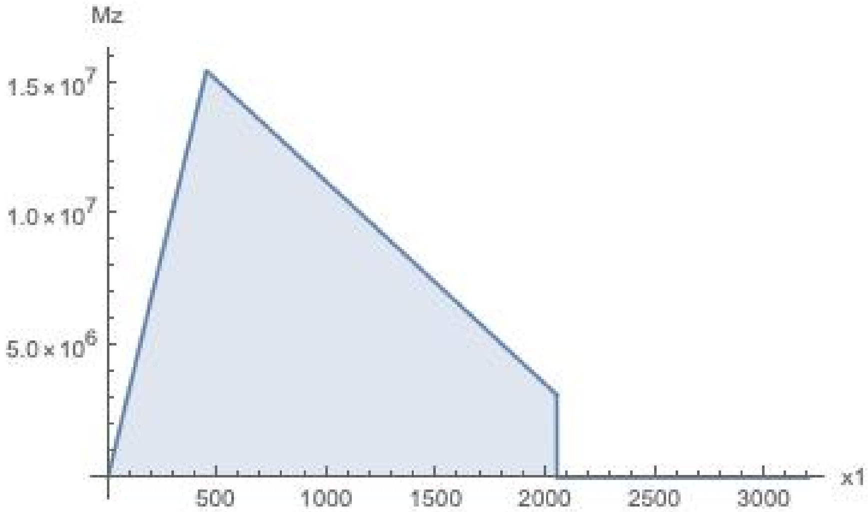


Figure 2.14: Bending moment for $x_2 = 450\ mm$.

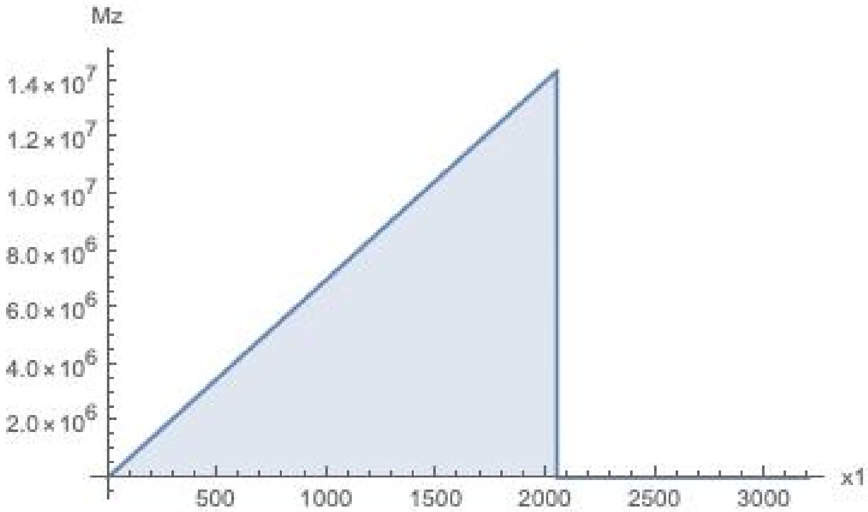


Figure 2.15: Bending moment for $x_2 = 2050\ mm$.

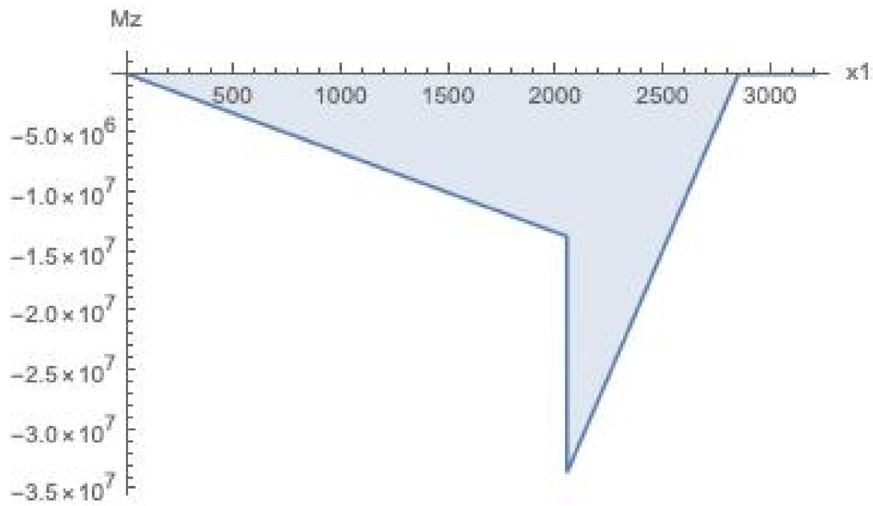


Figure 2.16: Bending moment for $x_2 = 2850\ mm$.

Equivalent stress in key positions

Table 2.4 resumes the calculations of the different equivalent stresses in the various key positions:

x_2 position	x_1 position	Equivalent stress
450	450	108.841 MPa
2050	2050	118.094 MPa
2850	2050	256.467 MPa

Table 2.4: Equivalent stresses in most critical situations.

Of course the selection was made according to the maximum value of the bending moment and normal action along the beam in the different cases, and the applied formula was the following:

$$\sigma_{eq} = \left|\frac{N}{A}\right| + \left|\frac{M_z}{I_z} y\right| \tag{2.20}$$

Looking at the table it is easy to see that the most critical condition is in correspondence of the point $x_1 = 2050\ mm$ when the load is located in position $x_2 = 2850\ mm$.

3 Fatigue design of a pressure vessel

3.1 Introduction

3.1.1 Part 1

The cylindrical vessel designed in Homework 1 is subjected in service to pressurization cycles where the internal pressure fluctuates between 0 and p_1. It is requested to assess the pressure p_1 leading the vessel to failure after 10^5 fatigue cycles. Fatigue verification must be carried out considering both Crossland and Fatemi-Socie multi axial criteria.

Material	$S355JR$
σ_Y ($40mm \leq thickness \leq 63mm$)	335 MPa
σ_{UTS}	470 MPa
r_i	365 mm
r_o	426 mm
Thickness	61 mm
K_t (service factor)	1.1

Table 3.1: Data of pressure vessel under study. Obtained from results of HW1.

3.1.2 Part 2

An inspection made using a NDT revealed a semi-circular crack in the inner surface of the cylindrical part of length $a_0 = 1$ mm. It is requested to assess the residual fatigue life when the vessel is subjected to internal pressure fluctuating between 0 and the lowest value of p_1 estimated in Part 1. The crack is supposed to propagate in mode I.

K_{IC} $[MPa\ m^{0.5}]$	C $[(mm/cycle)/(MPa\ m^{0.5})]$	m
150	$6.89\ 10^{-9}$	3

Table 3.2: Data of the homework.

3.2 Solution analysis

3.2.1 Fatigue verification without crack presence

Methods

The pressure vessel is made by S355JR steel with $\sigma_y = 335\ MPa$ and $\sigma_u = 470\ MPa$ [1]. The analysis was conducted considering σ_{UTS} equal to 470 MPa, in the literature higher value for the

UTS were observed, the lower value was selected in order to perform a conservative analysis. Fatigue behavior can be predicted using the S-N curve (stress to cycle curve) that correlates the stress amplitude applied and the numbers of cycles to failure. Stress-life curves are composed by :

i. low-cycle fatigue where the material undergoes incipient or full cyclic plasticization;

ii. high cycle fatigue with a strong dependence of fatigue lifetime upon the applied stress;

iii. at high numbers of cycle the fatigue limit, asymptotic value of the median fatigue strength as N_f becomes very large.

Not all the materials show fatigue limit. In order to build the S-N curve we cannot relate to σ_{UTS} of the material due to presence of both bending and torsion. Two different analysis were conducted considering the application of only bending and only torsion. The results were used to verify the multi axial fatigue criteria and to find p_i.

Correction Factor

Considering a general component under cyclic load the following correction factor are applied depending on the type of stress present. At low cycle, the stress used to build the S-N curve can be estimated as :

i. $S_{1000} = 0.9S_u$ bending;

ii. $S_{1000} = 0.75S_u$ axial loading;

iii. $S_{1000} = 0.72S_u$ torsion.

Endurance limit depends on different factors :

1. Surface finishing factor : poor surface finish can comport fatigue sensitive locations. Increasing the roughness of the surface will decrease the value of the surface factor reducing in the fatigue limit;

2. Size factor : introduced because the S-N curve that are studied were obtained studying laboratory specimen. Engineering components may have larger sizes . The S-N fatigue resistance is decreased at the component size increases. The value depends on the thickness or the diameter of the component under study.

3. Load factor : endurance limit are obtained from testing with completely reversed bending, when torsional or axial loading is applied decreases the fatigue limit.

For steels fatigue limit of small polished are :
$S_{erb} = 0.5S_u \; for \; S_u < 1400 \, MPa$

$S_{erb} = 700\,Mpa\ for\ S_u > 1400\,Mpa$.
The fatigue limit uses the estimation at high cycle with the influence of factors explained before. The fatigue limit has to be reduced using a service factor (1.1). After the construction of the S-N curve the stress was calculated considering $N_f = 10^5$.

Design decision

The surface finishing of the pressure vessel was considered ground, better surface finishing increase the fatigue limit of the pressure vessel. The main reason of the choice is due to the use of pressure vessel, it's better to have polished surface finish in order to reduce the possibility of localized corrosion or the presence of intensification region. $C_s = 1.58\,S_{UTS}^{-0.085}$.
The size factor $C_d = 1.189\,d^{-0.097}$ where d is the thickness of pressure vessel walls. The thickness is equal to 61 mm. The load factor considered was $C_{load} = 1$ when only bending was considered and $C_{load} = 0.59$ when only torsion was considered.

Multi axial fatigue criteria

Two multi axial fatigue criteria were verified :

1. Crossland : criterion applied only if the stress components vary synchronously and in-phase. The equations to be verified are :

$$\sigma_{a,eq} + \alpha p_{max} = \beta \tag{3.1}$$

$$p_{max} = \frac{\sigma_{max1} + \sigma_{max2} + \sigma_{max3}}{3} \tag{3.2}$$

where α and β are material constants.

$$\alpha = 3\frac{\sqrt{S_{es}} - S_e}{S_e} \tag{3.3}$$

$$\beta = \sqrt{3}S_{es} \tag{3.4}$$

where S_e is the fatigue limit under bending loading, S_{es} is the fatigue limit under torsion loading.

2. Fatemi-Socie : criterion based on the consideration the crack initiates on the plane experiencing the largest shear strain amplitude.

$$\tau_{ns,a,max\theta}(1 + n\frac{\sigma_{n,max}}{S_Y}) = f \tag{3.5}$$

where f= S_{es} and

$$n = 2S_Y\frac{2S_{es} - S_e}{S_e^2} \tag{3.6}$$

3.2.2 Part 2

Crack propagation

The first step was to understand which stress was acting on the crack and causing its propagation. The crack propagates in mode I, which corresponds to the orthogonal direction in respect to the maximum principal stress direction. For that reason, the stress σ_1 used to calculate K_I was taken to be the tangential stress of the pressure vessel (σ_θ).The stress σ_1 was thus calculated as follows:

$$\sigma_1 = \sigma_\theta = \frac{p_i r_i^2}{r_o^2 - r_i^2} + \frac{p_i r_o^2}{r_o^2 - r_i^2} \tag{3.7}$$

With all the data available, the ΔK interval where stable crack growth occurs can be defined. The minimum value of p_i is equal to 0, resulting in a stress ratio R = 0. ΔK was therefore calculated by simply using σ_1 with the maximum internal pressure p_i:

$$\Delta K = Y(a)\,\sigma_1 \sqrt{\pi a} \tag{3.8}$$

Where:

$$Y(a) = 1.12 sec(\frac{\pi a}{2t}) \tag{3.9}$$

To have stable crack growth, the ΔK has to be bigger than the minimum threshold value ΔK_{th}:

$$\Delta K_{th} = 7\,(1 - 0.85R) \tag{3.10}$$

R was equal to 0 in this case, yielding ΔK_{th} = 7. The next step was to verify that the initial crack length a_0 would cause a sufficient stress intensity factor to have propagation:

$$\Delta K = Y(a_0)\,\sigma_1 \sqrt{\pi a_0} \;\geq\; \Delta K_{th} \tag{3.11}$$

Failure mode

The failure of the pressure vessel occurs when the crack reaches a length a_f high enough to either cause full plasticization of the wall, or raise the stress intensity factor to a value equal to the fracture toughness of the material. Since the pressure vessel was built using ductile construction steel, it was initially supposed that fracture would occur by full plasticization. When the crack propagates, the effective thickness of the pressure vessel reduces to a value contained between the outer radius r_o and an equivalent inner radius $r_{i,eq}$, as shown in figure 3.1:

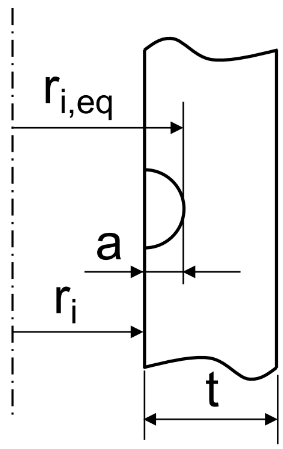


Figure 3.1: Section of the vessel wall during crack growth.

The inner pressure necessary to cause full plasticization P_b decreases as the effective thickness of the wall decreases. When the effective thickness is so low that P_b is equal to the maximum internal pressure acting on the vessel p_i, failure occurs. Therefore the $r_{i,eq}$ for full plasticization was extrapolated:

$$r_{i,eq} = \frac{r_o}{\sqrt{e^{\frac{2p_i}{\sigma_y}}}} \tag{3.12}$$

From this the final crack length was calculated:

$$a_f = r_{i,eq} - r_i \tag{3.13}$$

To verify the initial assumption, the stress intensity factor caused by the final crack length was calculated and verified to be lower than the fracture toughness K_{IC}:

$$K_I(a_f) = Y(a_f)\,\sigma_1\sqrt{\pi a_f} < K_{IC} \tag{3.14}$$

Fatigue life estimation

After having made sure that the assumption was correct, the fatigue life was estimated. The resulting value corresponds to the number of cycles of pressure variation (from 0 MPa to p_i) necessary to propagate the crack from a_0 to a_f:

$$N_{if} = \int_{a_0}^{a_f} \frac{1}{C\Delta K^m} da \tag{3.15}$$

3.3 Results

3.3.1 Part 1

S-N curve construction considering fully reversed bending

The stress limit at low cycle is modified by the factor 0.9, only fully reversed bending is considered. The S_{1000} is modified also by load factor C_{load}.

$$S_{1000} = 0.9S_u \tag{3.16}$$

$$C_{load} = 1 \tag{3.17}$$

$$S_{1000c} = S_{1000}\,C_{load} \tag{3.18}$$

The stress at fatigue limit for commons unnotched, highly polished steel is modified by a factor 0.5 for $S_{UTS} \leq 1400\,MPa$.

$$S_{erb} = 0.5\,S_u \tag{3.19}$$

The endurance limit is also corrected by surface factor, load factor and size factor. Fatigue strength factor was use, $K_s = 1.1$.

$$C_s = 1.58\,S_u^{-0.085} = 0.936545 \tag{3.20}$$

$$C_d = 1.189\,d^{-0.097} = 0.798004 \tag{3.21}$$

$$S_{eb} = \frac{S_{erb}C_{load}C_sC_d}{K_t} \tag{3.22}$$

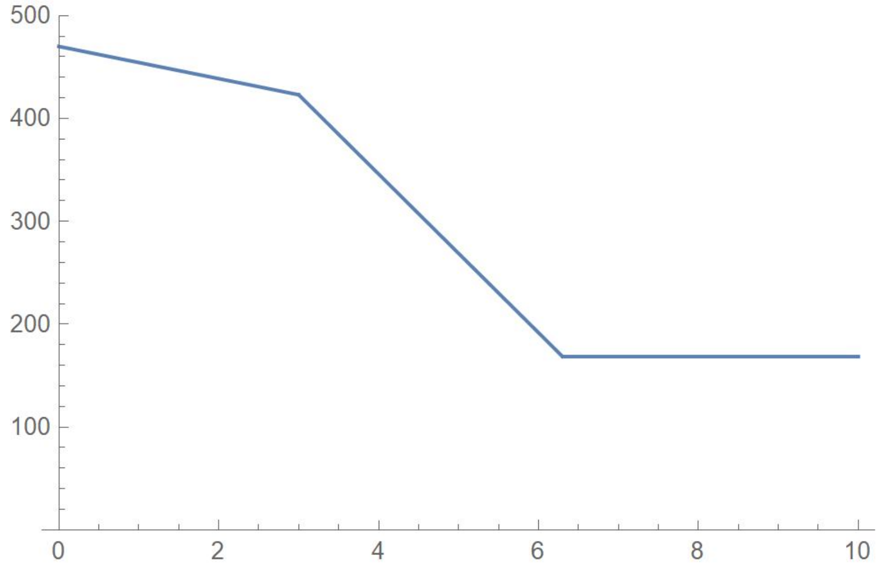


Figure 3.2: SN curve

From the S-N curve obtained the stress at failure after $10^5 N_f$ can be observed.

$$\frac{1}{k} = -\frac{\log \frac{S_{eb}}{S_{1000b}}}{\log \frac{210^6}{1000}} \tag{3.23}$$

$$C = \frac{S_{1000b}}{1000^{-\frac{1}{k}}} \tag{3.24}$$

$$S_a = C(10^5)^{-\frac{1}{k}} \tag{3.25}$$

$$S_{10^5} = 234.451 MPa \tag{3.26}$$

S-N curve considering fully reversed torsion

Load factor is used to correct S_{1000} in case of fully reversed torsion.

$$S_{1000} = 0.72 \ S_u \tag{3.27}$$

$$C_{load} = 0.59 \tag{3.28}$$

$$S_{1000c} = S_{1000} \ C_{load} \tag{3.29}$$

The fatigue limit is corrected with load factor, surface factor and size factor. $K_t = 1.1$ as diminishing factor of the fatigue strength.

$$Sett = \frac{S_{erb} \ Cload \ C_s \ C_d}{K_t} \tag{3.30}$$

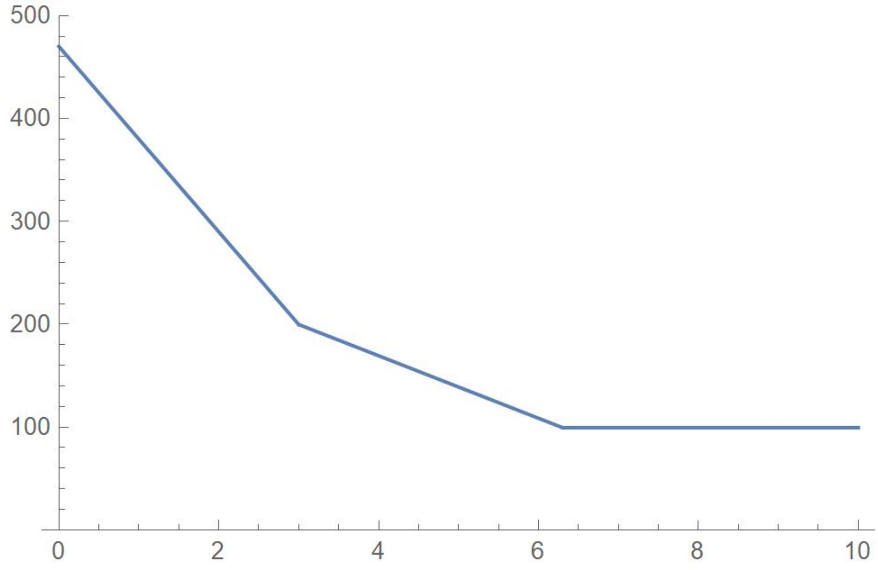


Figure 3.3: SN curve

From the S-N curve obtained the stress at failure after $10^5 N_f$ can be observed.

$$S_{10^5} = 147.76\ MPa \tag{3.31}$$

Crossland criterion verification

Multi axial fatigue criteria are applied to find the p_i at which the pressure vessel fails after $N_f = 10^5$ cycles.

$$p_{max,b} = \frac{S_{eb}}{3} \tag{3.32}$$

$$\sigma_{eq,b} = S_{eb} \tag{3.33}$$

$$p_{max,t} = 0 \tag{3.34}$$

$$\sigma_{eq,t} = \sqrt{3} S_{ett} \tag{3.35}$$

$$\alpha = 3\frac{\sqrt{3}S_{ett} - S_{eb}}{S_{eb}} \tag{3.36}$$

We are not considering the failure at the fatigue limit, the β variable is modified substituting the fatigue limit under torsion with the failure load at $N_f = 10^5$ cycles.

$$\beta = \sqrt{3} S_{10^5} \tag{3.37}$$

$$p_{max} = \frac{S_{10^5,b}}{3} \tag{3.38}$$

$$\sigma_{eq,t} = \beta - \alpha p_{max} \tag{3.39}$$

$$p_i = \frac{2\sigma_{eq,t}}{2}\left(1 - \frac{r_i^2}{r_o^2}\right) \tag{3.40}$$

Obtaining internal pressure that lead to failure equal to $p_i = 33.2\ MPa$.

Fatemi Socie

$$n = 2S_y \frac{2S_{ett} - S_{eb}}{S_{eb}^2} \tag{3.41}$$

$$f = S_{ett} \tag{3.42}$$

$$\sigma_{\frac{r}{\theta}} = \frac{p_i r_i^2}{r_o^2 - r_i^2} \mp \frac{p_i}{r_o^2 - r_i^2} \frac{r_i^2 r_o^2}{r^2} \tag{3.43}$$

$$\tau_{ns,max\theta} = \frac{\sigma_\theta - \sigma_r}{2} = 3.76108 p_i \tag{3.44}$$

$$\sigma_{n,max} = \frac{\sigma_\theta + \sigma_r}{2} = 2.76108 p_i \tag{3.45}$$

$$\tau_{ns,max\theta}(1 + n\frac{\sigma_{n,max}}{S_y}) = f \tag{3.46}$$

$$p_i = 32.56\ MPa \tag{3.47}$$

The internal pressure that will lead to failure the pressure vessel is equal to 32.56 MPa using Fatemi-Socie criterion.

3.3.2 Part 2

The lowest value of pressure obtained in part 1 was obtained with the Fatemi-Socie verification (p_i = 32.56 MPa). Using this value, the initial assumption proved to be correct, that is to say the material fails by full plasticization before reaching the fracture toughness limit. The fatigue life was thus estimated using the final crack length obtained by the full plasticization condition. The resulting length of the final crack and the estimated fatigue life are shown in table 3.4.

a_f $[mm]$	N_{if} $[cycles]$
21.54	96082

Table 3.3: Final crack length and estimated fatigue life.

3.4 Conclusions

3.4.1 Part 1

Crossland and Fatemi Socie multi axial fatigue criteria were used in order to find the internal pressure that will lead the pressure vessel to fail after the application of 10^5 cycles. The internal pressures found are resumed in the following Table 3.4.

Crossland	33.2 MPa
Fatemi Socie	32.56 MPa

Table 3.4: Internal pressure results from multi axial fatigue criteria.

3.4.2 Part 2

The fatigue life of the material was estimated using a the inner pressure value calculated with the Fatemi-Socie criterion. It was initially assumed that the material would fail by full plasticization rather than by reaching the fracture toughness. The assumption was correct, and the final crack length was calculated using the full plasticization condition. The fatigue life was $N_{if} = 96082$ cycles.

4 Design of welded and bolted joints

4.1 Introduction

In Fig 4.1 it is represented the double clamped beam considered in this homework.

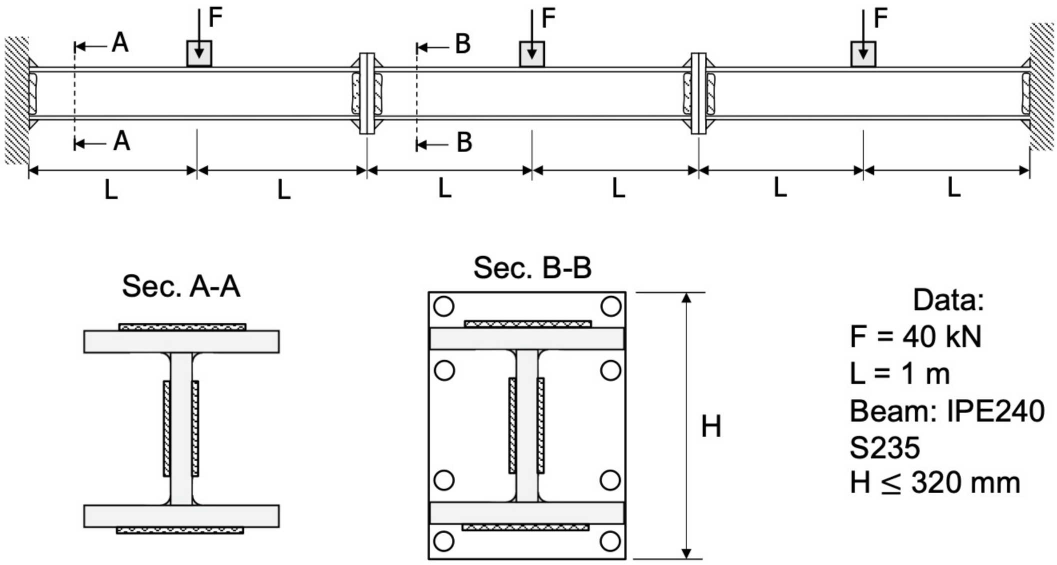


Figure 4.1: Double clamped beam.

The double clamped beam is composed of three segments connected together through bolted joints. The segments terminal sections are fillet welded to the frame or to the bolted members.

4.1.1 Part 1 - Welded joints

It is requested to:

1. Design and verify the fillet welds clamping the two beam extremities. Make sure that the vertical beads are not overlapped on the fillet between web and flange.

2. Design and verify the fillet welds connecting the beam segments to the members.

The dimensions of the fillet welds are design following the EC3 Simplified Method. The joints are subjected bending and shear. After finding the minimum thickness, the weld was verified following the EC3 Dimensional Method.

EC3 Simplified Method

The design resistance of a fillet weld [2] has to follow :

$$f_{vw,d} = \frac{f_u}{\sqrt{3}\beta\gamma} \tag{4.1}$$

where β and γ are material constant. $f_{vw,d}$ is the design shear strenght of the weld.

EC3 Dimensional Method

The design resistance of the fillet weld will be sufficient if the following equations are satisfied :

$$\sqrt{\sigma_{\perp}^2 + 3(\tau_{\perp}^2 + \tau_{\parallel}^2)} \leq \frac{f_u}{\beta\gamma} \tag{4.2}$$

$$\sigma_{\perp} \leq 0.9\frac{f_u}{\gamma} \tag{4.3}$$

4.1.2 Part 2 - Bolted joints

It is requested to design and verify the bolted joints in terms of:

1. Number, location, size, and property class.

2. Member height and thickness.

Consider both a contact (non-preloaded) and a friction (preloaded, friction coefficient f=0.2) connection.

4.2 Solution analysis

4.2.1 Part 0 - Support reactions and internal loads calculation

As first point for the resolution of the homework it is necessary to determine the internal loads, in order to know the values of the actions in the different points of the structure. In particular, it is necessary to determine the value of the loads in correspondence of the welded and bolted joints.
In the analysis it is important to take care that the system is over constrained, but it presents also a symmetrical structure with a symmetrical load distribution, and this aspect will be useful to introduce some simplifications.

4.2.2 Part 1 - Welded joints

The fillet welds design is divided in two :

i. Design of the fillet weld in correspondence of the connection of the beam to walls;

ii. Design of the fillet weld in correspondence of the junction between different beam.

The beams used is IPE240 made of S235, the design was conducted in order to find the minimum thickness of the fillet weld. After finding the results the thickness of the weld was verified using the dimensional method explained in EC3[2].

4.2.3 Part 2 - Bolted joints

For the design of the bolted joints, the internal loads in correspondence of the joints are taken into account. Starting from them it is possible to determine the values of the different actions (shear action, separating action ...) on the bolts, in both non preloaded or preloaded scenario.
Once such kind of actions are determined, starting from the characteristics of the chosen bolts it

is possible to calculate the resistance of the system to the various types of sollecitation and, in the end, if the verification is satisfied or not.
In case the verification is not satisfied, it is necessary to redesign the system and perform again the verification.

4.3 Loads analysis and verification

4.3.1 Support reactions calculation

The free body diagram of the structure is reported in Figure 4.2.

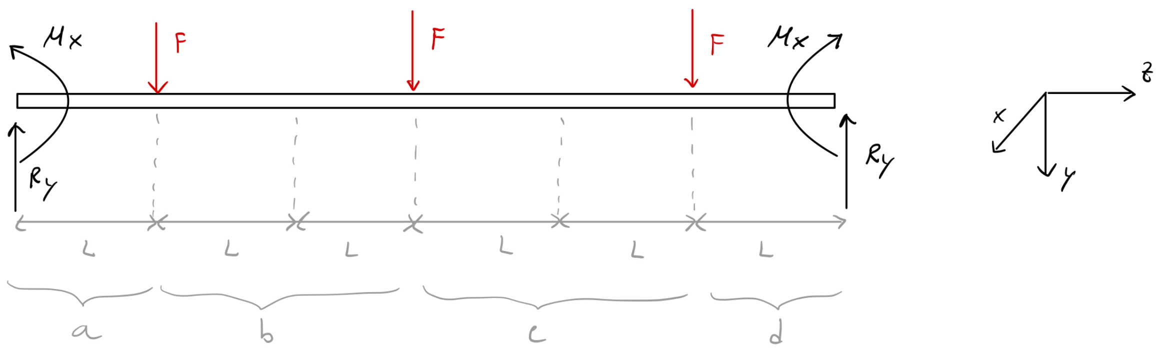


Figure 4.2: Free body diagram of the structure.

Considering the system it is possible to say that many of the support reactions are equal to zero, since they do not correspond to any external action. The only support reactions are represented by a vertical reaction and a bending moment along the x axis for both the extremities of the beam (in total 4 reactions).
For symmetry reasons it is clear that the two vertical reactions are equal and they have to counterbalance the vertical forces, therefore:

$$2R_y = 3F \tag{4.4}$$

$$R_y = 60\ kN$$

For what concerns the two bending moments, they have to be equal in value (also in this case for symmetry reasons) but it is not possible to determine them from the equilibrium equations since they disappear.
So, to determine the value of the bending moments it is necessary to use another calculation. In this case it has been decided to apply the Castigliano's theorem, imposing the displacement equal to zero at the extremities.
Internal actions are calculated by considering the value of a bending moment as an unknown value:

$$V_a = R_y$$

$$V_b = R_y - F$$
$$V_c = R_y - 2F$$
$$V_d = R_y - 3F$$
$$M_{xa} = V_a\, z - X$$
$$M_{xb} = V_b\, z - X + FL$$
$$M_{xc} = V_c\, z - X + FL + 3FL$$
$$M_{xd} = V_d\, z - X + FL + 3FL + 5FL$$

In the application of the Castigliano's theorem it is possible to neglect the shear actions since, for the system under consideration, they are negligible. The expression of the theorem is the following:

$$\eta = \int_0^L \frac{M_{xa}}{IE} \frac{d}{dX}[M_{xa}]dz + \int_L^{3L} \frac{M_{xb}}{IE} \frac{d}{dX}[M_{xb}]dz + \int_{3L}^{5L} \frac{M_{xc}}{IE} \frac{d}{dX}[M_{xc}]dz + \int_{5L}^{6L} \frac{M_{xd}}{IE} \frac{d}{dX}[M_{xd}]dz = 0 \tag{4.5}$$

By solving the equation in function of the parameter X it is possible to determine the value of the bending moments (that are equal):

$$M_x = 63.333\ kN\ m$$

4.3.2 Internal loads diagrams

By substituting the value of X in the equations of the internal loads it is possible to obtain the following diagrams:

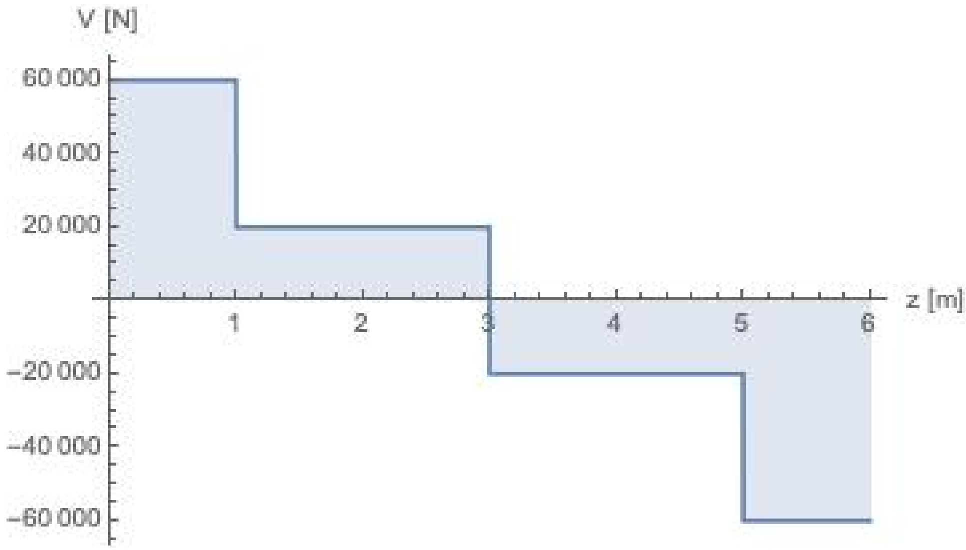


Figure 4.3: Diagram of the shear actions along the beam.

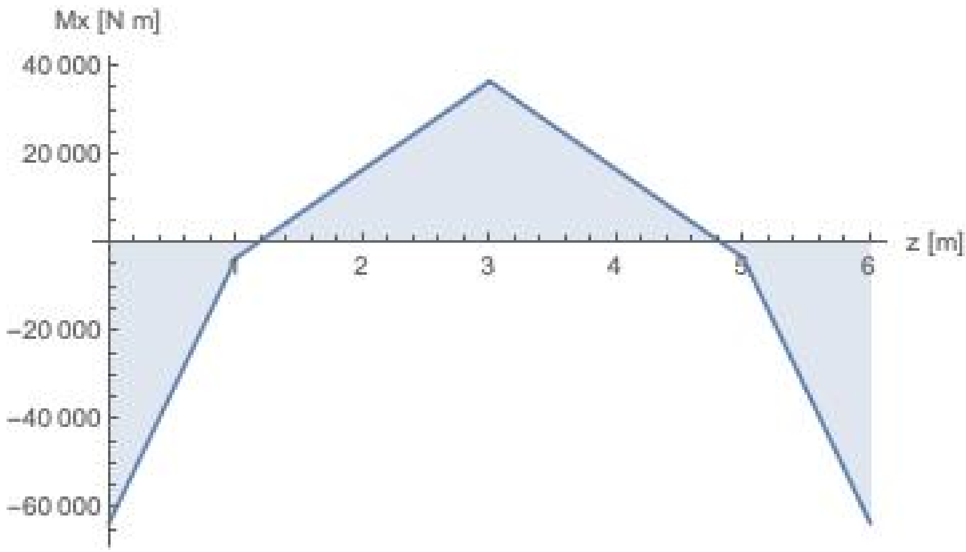


Figure 4.4: Diagram of the bending moment along the beam.

For the next calculations it is fundamental to determine the values of shear action and bending moment in correspondence of the welded and bolted joints. In particular we have:

$$Extremities \longrightarrow V = 60\ kN, M_x = 63.333\ kN\ m$$
$$Internal\,joints \longrightarrow V_{2L} = 20\ kN, M_{x2L} = 16.667\ kN\ m$$

4.3.3 Fillet welding design clamping the two beam extremities

In order to dimension the fillet welds we take into account the force and the moment applied at the extremities of the beam connected with the wall.

$$F = 60\ kN \tag{4.6}$$

$$M_b = 63,3\ kNm \tag{4.7}$$

$$h = 190\ mm \tag{4.8}$$

$$B = 120\ mm \tag{4.9}$$

$$H = 240\ mm \tag{4.10}$$

$$y = \frac{H + 2a}{2} \tag{4.11}$$

Beta parameter depends on the steel in use. In this case we are employing a S235 steel.

$$\beta = 0.8 \tag{4.12}$$

$$\gamma = 1.25 \tag{4.13}$$

$$\sigma_r = 360MPa \tag{4.14}$$

$$I_w = \frac{ah^3}{6} + \frac{B}{12}((H + 2a)^3 - H^3) \tag{4.15}$$

$$\sigma_{orthogonal} = \frac{M_b}{I_w} y \tag{4.16}$$

$$\tau_{parallel} = \frac{F}{2ah} \tag{4.17}$$

$$\sqrt{\tau_{parallel}^2 + \sigma_{orthogonal}^2} \leq \frac{\sigma_R}{\sqrt{3}\beta\gamma} \tag{4.18}$$

$$a \geq 8.1\ mm \tag{4.19}$$

$$a \rightarrow 9\ mm \tag{4.20}$$

The final result is a fillet weld with a throat height of 9 mm.

Weld verification EC3 DM

Using the throat height found using Eq. 4.18 weld verification is conducted using the EC3 Dimensional Method.

$$M_z = 63,3\ kNm \tag{4.21}$$

$$a = 9\ mm \tag{4.22}$$

$$\sigma_{orthogonal} = \frac{M_z}{I_w} y \tag{4.23}$$

Vertical beads

$$\sigma_{thv} = \sigma_{orthogonal} \cos\frac{\pi}{4} + \tau_{parallel} \sin\frac{\pi}{4} \tag{4.24}$$

$$\tau_{thv} = -\sigma_{orthogonal} \sin\frac{\pi}{4} + \tau_{parallel} \cos\frac{\pi}{4} \tag{4.25}$$

$$\sigma_{thv} \leq 0.9 \frac{\sigma_R}{\gamma} \tag{4.26}$$

$$136\ MPa\ \leq 259\ MPa \tag{4.27}$$

Verified.

$$\sqrt{3\tau_{thv}^2 + \sigma_{thv}^2} \leq \frac{\sigma_R}{\beta\gamma}$$

(4.28)

$$261\ MPa \leq 360\ MPa \tag{4.29}$$

Verified.
The equations are both satisfied.

Horizontal beads

$$\sigma_{thv} = \sigma_{orthogonal} \cos\frac{\pi}{4} \tag{4.30}$$

$$\tau_{thv} = -\sigma_{orthogonal} \sin\frac{\pi}{4} \tag{4.31}$$

$$\sqrt{3\tau_{thv}^2 + \sigma_{thv}^2} \leq \frac{\sigma_R}{\beta\gamma} \tag{4.32}$$

$$263\ MPa\ \leq 360\ MPa\ Verified \tag{4.33}$$

$$\sigma_{thv} \leq 0.9\frac{\sigma_R}{\gamma} \tag{4.34}$$

$$131.9\ MPa\ \leq 259\ MPa\ Verified \tag{4.35}$$

The fillet welds in location A has a throat height of 9 mm and a thickness of 12.7 mm.

4.3.4 Welding design of beam segment to the members

$$F = 20\,000\ N \tag{4.36}$$

$$M_b = 16,7\ kNm \tag{4.37}$$

$$\beta = 0.8 \tag{4.38}$$

$$\gamma = 1.25 \tag{4.39}$$

$$\sigma_R = 360\ MPa \tag{4.40}$$

$$I_w = \frac{ah^3}{6} + \frac{B}{12}((H+2a)^3 - H^3) \tag{4.41}$$

$$\sigma_{orthogonal} = \frac{M_b}{I_w}y \tag{4.42}$$

$$\tau_{parallel} = \frac{F}{2ah} \tag{4.43}$$

$$\tau_{parallel} = \frac{F}{2ah} \tag{4.44}$$

$$\sigma_{orthogonal} \leq \frac{\sigma_R}{\sqrt{3}\beta\gamma} \tag{4.45}$$

$$\sqrt{\tau^2_{parallel} + \sigma^2_{orthogonal}} \leq \frac{\sigma_R}{\sqrt{3}\beta\gamma} \tag{4.46}$$

$$a \geq 2.11\ mm \tag{4.47}$$

$$a \rightarrow\ 3mm \tag{4.48}$$

Welding verification EC3 SM

$$\sigma_{orthogonal} = \frac{M_z}{I_w}y \tag{4.49}$$

$$\tau_{parallel} = \frac{F}{2ah} \tag{4.50}$$

$$\sqrt{\tau^2_{parallel} + \sigma^2_{orthogonal}} \leq \frac{\sigma_R}{\beta\gamma\sqrt{3}} \tag{4.51}$$

$$146.87\ MPa\ \leq 207.8\ MPa\ Verified \tag{4.52}$$

The fillet weld in location B has a throat height of 3 mm and a thickness of 4.24 mm.

4.3.5 Selection of number and position of the bolts

A configuration of 8 bolts, distributed like shown in Annexes, was selected.

$$n_b = 8$$

4.3.6 Shear and separating actions evaluation

Shear action

To evaluate the sliding action it is possible to calculate the shear force that is sustained by each bolt, considering bolts with equal cross-section:

$$V_i = \frac{V_{2L}}{n_b} = 2.5\ kN \tag{4.53}$$

Separating action (no preload)

Taking into account the bolts configuration, it is possible to assign a y coordinate to each row of bolts, the result is the following:

$$y_1 = 20\ mm$$
$$y_2 = 140\ mm$$
$$y_3 = 180\ mm$$
$$y_4 = 300\ mm$$

Very often the assumption of rigid members is not realistic in this case, therefore it has been decided to consider the semi-rigid member scenario.
A linear variation of the stresses was defined as:

$$\sigma = -\sigma_{max,co}\left(\frac{1-y}{y_n}\right) \tag{4.54}$$

Where $sigma_{max,co}$ and y_n (the position of the neutral axis) are the unknown values.
To determine them, it is possible to write equilibrium equations of translation and rotation, considering that the portion in compression will resist wit the whole cross section (this results in the integral in the formula) while the portion in tension will resist only through the cross section of the bolts.

$$\int_0^{A(y_n)} \sigma(y)dA + \sum_{i=1}^{n_b} -\sigma_{max,co}\left(1-\frac{y_i}{y_n}\right)A_{b,j} = N \tag{4.55}$$

$$\int_0^{A(y_n)} \sigma(y)(y-y_n)dA + \sum_{i=1}^{n_b} -\sigma_{max,co}\left(1-\frac{y_i}{y_n}\right)(y_i-y_n)A_{b,j} = M_b + N(y_g-y_n) \tag{4.56}$$

Since in the formulas the area of the resisting cross section of the bolts appears, it is necessary to select a specific dimension and category of bolts. In this case bolts M8 of class 8.8 were selected ($A = 36.6\ mm^2$).
In order to know which part of the cross section is in compression and which is in tension it is necessary to impose a position for the neutral axis and then check if the hypothesis was correct. In this case it was supposed that the neutral axis is found between the first and the second row. Resulting formulas are the following:

$$\int_0^{y_n} \sigma(y)120dy - 2\sigma_{max,co}\left(1-\frac{y_2}{y_n}\right)36.6 - 2\sigma_{max,co}\left(1-\frac{y_3}{y_n}\right)36.6 - 2\sigma_{max,co}\left(1-\frac{y_4}{y_n}\right)36.6 = 0 \tag{4.57}$$

$$\int_0^{y_n} \sigma(y)(y-y_n)120dy - 2\sigma_{max,co}\left(1-\frac{y_2}{y_n}\right)(y_2-y_n)36.6 - 2\sigma_{max,co}\left(1-\frac{y_3}{y_n}\right)(y_3-y_n)36.6$$
$$-2\sigma_{max,co}\left(1-\frac{y_4}{y_n}\right)(y_4-y_n)36.6 = M_{x2L} \tag{4.58}$$

Solving the two equations, it is possible to determine the unknown values:

$$y_n \longrightarrow 25.734\ mm$$
$$\sigma_{max,co} \longrightarrow 48.269\ MPa$$

The initial assumption was correct, the first line of bolts lies below the neutral axis. So it is possible to use the values obtained to calculate the maximum separating action. Considering the model of linear variation, the maximum separating action is in correspondence of the maximum y coordinate (y_4):

$$N_{max} \longrightarrow 18.829\ kN$$

Separating action (with preload)

In the case of preloaded bolts of equal cross section, the maximum separating action can be determined with the following formula. In this case the first row of bolts is considered as pivot.

$$N_{max} = \frac{M_b}{\sum_i^{n_b} y_i^2}\ y_{max} \tag{4.59}$$

In this case it was found that the nominal force in the most stressed bolt is:

$$N_{max,p} \longrightarrow 19.708\ kN$$

4.3.7 Verification of non preloaded bolts

For this verification, category A+D must be considered. Verification against shear, crush, tension and punch must be performed. A thickness of 8 *mm* of the member and 360 *MPa* as UTS of S235 steel are considered.
In this case bolts M8 of class 8.8 were considered, with the configuration described in the previous section.

Shear verification

$$R_V = \frac{0.6\ \sigma_U\ A_b}{\gamma_{M2}} = \frac{0.6\ 800\ 50.3}{1.25} = 19.315\ kN \tag{4.60}$$
$$Vi \leq R_V$$

Crush verification

$$R_C = \frac{2.5\ \sigma_{U,m}\ d\ t}{\gamma_{M2}} = \frac{2.5\ 360\ 6.826\ 8}{1.25} = 39.320\ kN \tag{4.61}$$
$$Vi \leq R_C$$

Tension verification

$$R_T = \frac{0.9\ \sigma_U\ A_{bt}}{\gamma_{M2}} = \frac{0.9\ 800\ 36.6}{1.25} = 21.081\ kN \tag{4.62}$$

$$N_{max} \leq R_T$$

Punch verification

$$R_P = \frac{0.6\ \pi\ d_0\ t\ \sigma_{U,m}}{\gamma_{M2}} = \frac{0.6\ 360\ 13\ 8\ \pi}{1.25} = 56.458\ kN \tag{4.63}$$

$$N_{max} \leq R_P$$

Tension and shear resistance verification

$$\frac{V_i}{R_V} + \frac{N_i}{1.4\ R_T} = 0.767 \leq 1 \tag{4.64}$$

4.3.8 Verification of preloaded bolts

For this verification, category C+E is considered; verification against sliding and tension and shear must be performed. After a preliminar calculation it was found that M8 bolts of class 8.8 don't satisfy the verification (see appendix), therefore a re-design was required. Keeping constant the bolts number and position, new M10 bolts of class 10.9 were considered ($A = 58.8\ mm^2$).

Sliding resistance

$$N_0 = \frac{0.7\ \sigma_U\ A_{bt}}{\gamma_{M7}} = 37.418\ kN \tag{4.65}$$

$$R_S = \frac{k_s\ n\ f}{\gamma_{M3}}\ N_0 = \frac{1\ 1\ 0.2}{1.25}\ N_0 = 5.987\ kN \tag{4.66}$$

$$V_i \leq R_S$$

Shear and tension resistance

$$R_{S,s-t} = \frac{k_s\ n\ f}{\gamma_{M3}}(N_0 - 0.8N_{max}) = 3.463\ kN \tag{4.67}$$

$$V_i \leq R_{S,s-t}$$

It is important to notice that also in this case all the other verifications are satisfied, since we are using bolts with better properties.

4.4 Results

Welded joints

The results after the design of fillet welded joints are reported in Table 4.1.

Location A	
Throat height, a	9 *mm*
Minimum height,a_{min}	8.1 *mm*
Thickness fillet weld	12.7 *mm*
Location B	
Throat height, a	3 *mm*
Minimum height,a_{min}	2.11 *mm*
Thickness fillet weld	4.25 *mm*

Table 4.1: Data for the design of welded joints.

Bolted joints

The most important results for the design of the bolted joints in the two cases are reported in Table 5.9 and 4.3.

Number of bolts (n_b)	8
Position of bolts	*see Annexes*
Height of the members (H)	320 *mm*
Thickness of the members (t)	8 *mm*
Type of bolts	$M8,\ class\ 8.8$

Table 4.2: Data for the design of non preloaded bolted joints.

Number of bolts (n_b)	8
Position of bolts	*see Annexes*
Height of the members (H)	320 *mm*
Thickness of the members (t)	8 *mm*
Type of bolts	$M10,\ class\ 10.9$

Table 4.3: Data for the design of preloaded bolted joints.

4.5 Conclusions

Welded joints

The results of the design of the welded joints present a very high safety factor, especially in the design of the fillet weld in location B. This is due to high rounding on the minimum throat height of the weld. The fillet weld is still verified until the value of the throat height equal to 2.5 mm using EC3 SM.

Bolted joints

It is possible to conclude that this configuration, with M8 bolts of class 8.8 and 320 *mm* height and 8 *mm* thick members, satisfies the verification for non preloaded scenario.
For the preloaded scenario, the same configuration and the same bolts do not satisfy the requirements and therefore it is necessary to re-design the system. A possible solution is to keep the same configuration but changing the bolts from M8 8.8 to M10 10.9; in this case all the requirements are satisfied.

4.6 Annexes

4.6.1 Welded and bolted joints drawing

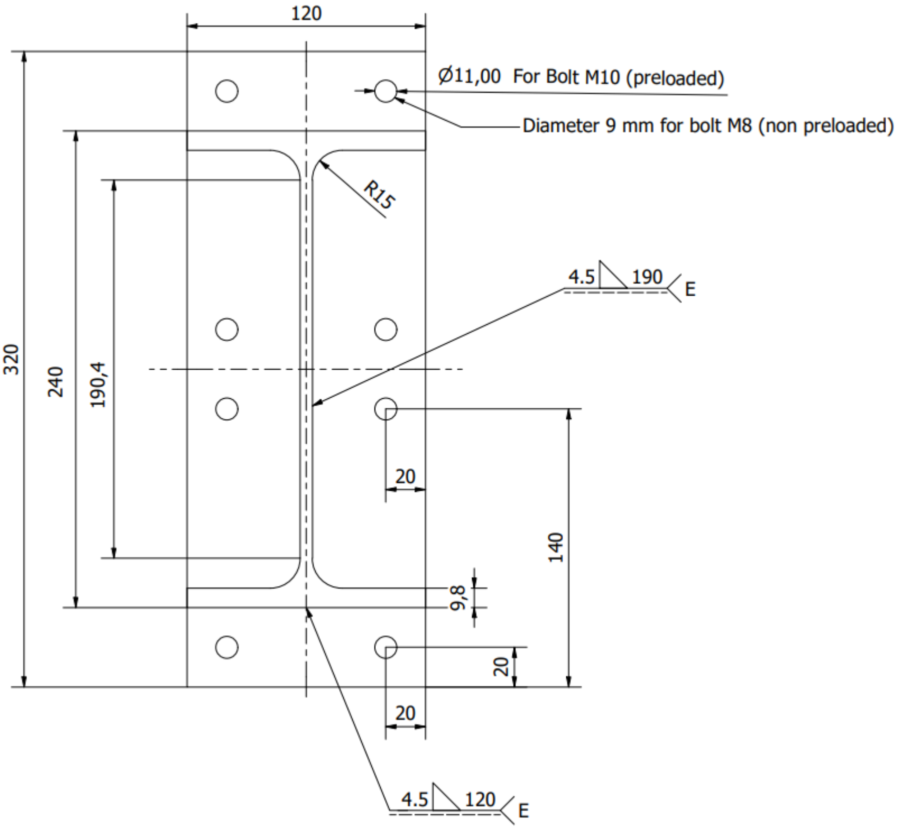


Figure 4.5: Welded and bolted joint drawing.

4.7 Appendix

4.7.1 Preliminar not satisfied analysis for preloaded bolted joints

Sliding resistance

$$N_0 = \frac{0.7\ \sigma_U\ A_{bt}}{\gamma_{M7}} = 18.633\ kN \tag{4.68}$$

$$R_S = \frac{k_s\ n\ f}{\gamma_{M3}}\ N_0 = \frac{1\ 1\ 0.2}{1.25}\ N_0 = 2.981\ kN \tag{4.69}$$

$$V_i \leq R_S$$

This condition is satisfied, but it is possible to notice that it is not the same for what concerns the following one.

Shear and tension resistance

$$R_{S,s-t} = \frac{k_s \; n \; f}{\gamma_{M3}}(N_0 - 0.8N_{max}) = 0.458 \; kN \tag{4.70}$$

$$V_i \nleq R_{S,s-t}$$

5 Design of a powertrain

5.1 Introduction

Figure 5.1 illustrates the layout of a wear-testing machine. The disc sample (5) is mounted on a shaft (4) driven through a spur gear drive (3) by an electric motor (2) and slides against a fixed counterpart (6) pressed by a radial load F_p with peripheral velocity v_p. The shaft (4) is supported by rolling bearings (1). Service factor $K_s = 1.1$.

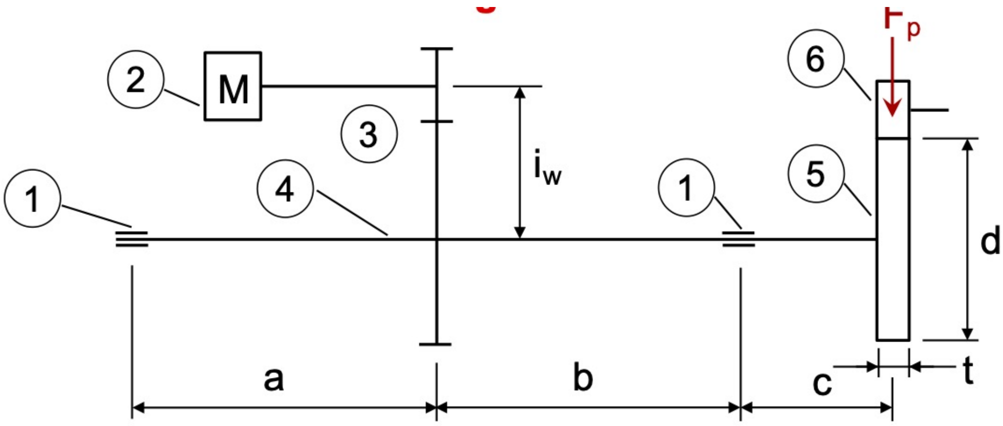


Figure 5.1: Powertrain scheme.

F_p	2500 N
$a+b$	300 mm
a	$100\ mm \leq a \leq 200\ mm$
c	100 mm
D_s	250 mm
Friction coefficient f_p	0.35
t	20 mm
v_p	3 m/s
i_w	140 mm
n_G	0.98
Motor angular speed n_m	750 rev/min
Gear working centre distance i_W	140 mm
ϕ_{max}	0.2°

Table 5.1: Data of the problem.

The selection of low cost shaft and gear materials is a preferential design guideline. It is requested to:

1. Prepare the product design specification

2. Design the pair of spur gears, including lubrication oil
3. Design the constraining system of the main shaft (4) and select the rolling bearings
4. Design the main shaft (4) for strength and stiffness
5. Design the interface fit for gear/shaft coupling
6. Design the key as mechanical interface between shaft (4) and sample (5)
7. Prepare a detailed drawing of the main shaft (4), including indicators about tolerances, surface finish, material and heat treatment

Additional data are that the operating temperature is $T_{op} = 60\,°C$, the bearing service life is 15000 h and the reliability has to be of 95% in moderate contamination conditions.

5.2 Loads analysis and verification

5.2.1 Support reaction calculation

$$v_p = 3000\ \frac{mm}{s} \tag{5.1}$$

$$D_s = 250\ mm \tag{5.2}$$

$$n_s = n_2 = \frac{v_p\ 60}{r\ 2\ P_i} = 229\ \frac{rev}{min} \tag{5.3}$$

$$n_m = 750\frac{rev}{min} \tag{5.4}$$

$$\tau = \frac{n_s}{n_m} = 0.3053 \tag{5.5}$$

$$\begin{cases} D_1 + D_2 = 2i_w \\ \tau = \frac{D_1}{D_2} \end{cases}$$

The solution of the system are $D_1 = 65.53\ mm$ and $D_2 = 214.46\ mm$

$$F_{TN} = F_p \cdot f_p = 875N \tag{5.6}$$

$$T_{FT} = F_{TN} \cdot \frac{D_s}{2} = 109375Nm \tag{5.7}$$

$$P = \frac{T_{FT} n_s 2\pi}{60} = 2623\ W \tag{5.8}$$

$$P_m = \frac{PK_s}{n_g} = 2944W \tag{5.9}$$

In case of overload at 3000 W, a = 200 mm and b= 100 mm where selected in order to minimize bending moment and the length on which the torque is applied.

$$F_{BTMAX} = \frac{T_{MAX}}{\frac{D_2}{2}} = 1166\ N \tag{5.10}$$

$$F_{BRMAX} = F_{BTMAX} \cdot \tan\alpha_n = 425.51\ N \tag{5.11}$$

$$F_p = 2857.14\ N \tag{5.12}$$

$$F_{TN} = F_p \cdot f_p = 1000\ N \tag{5.13}$$

$$T_{MAX} = F_{TN} \cdot r = 125.1\ Nm \tag{5.14}$$

where r is the radius of the gear that oppose the F_p force. Knowing all the forces present the support reaction are calculated :

$$\begin{cases} R_{Ay} - F_{BRMAX} + R_{Cy} - F_p = 0 \\ R_{Ay} \cdot 300 - F_{BRMAX} \cdot 100 + F_p \cdot 100 = 0 \end{cases}$$

$$\begin{cases} R_{Az} - F_{BTMAX} + R_{Cz} + F_{TN} = 0 \\ R_{Az} \cdot 300 - F_{BTMAX} \cdot 100 - F_{TN} \cdot 100 = 0 \end{cases}$$

$$R_{Ay} = -811.5N \tag{5.15}$$

$$R_{Cy} = 4095N \tag{5.16}$$

$$R_{Cz} = -557.1N \tag{5.17}$$

$$R_{Az} = 722.3N \tag{5.18}$$

5.2.2 Internal loadings calculation

The free body diagram of the system is shown in Figure 5.2.

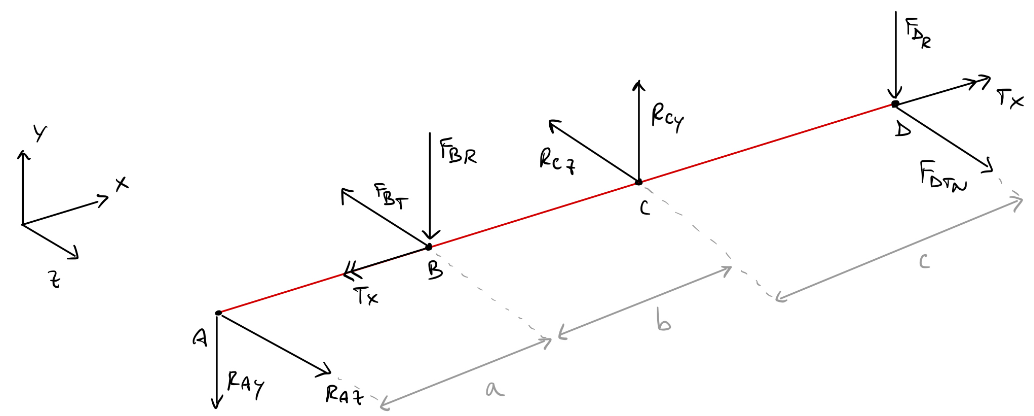


Figure 5.2: Free body diagram of the structure.

The subsequent step is the determination of the expressions of the internal loadings, making reference to this diagram.

For the calculations the following values of the parameters were considered:

$$a = 200\ mm$$
$$b = 100\ mm$$
$$F_{DP} = 2859\ N \longrightarrow \text{max value}$$
$$F_{DTN} = 1000.8\ N \longrightarrow \text{max value}$$
$$T_x = 125.1\ Nm \longrightarrow \text{max value}$$
$$F_{BT} = 1166\ N \longrightarrow \text{max value}$$
$$F_{BR} = 424.4\ N \longrightarrow \text{max value}$$

Shear action along y coordinate

$$\begin{cases} V_y = R_{ay}, & 0 \leq x \leq a \\ V_y = R_{ay} + F_{BR}, & a < x \leq a+b \\ V_y = R_{ay} + F_{BR} - R_{cy}, & a+b < x \leq 400 \end{cases}$$

In Figure 5.3 the correspondent internal loading diagram is shown.

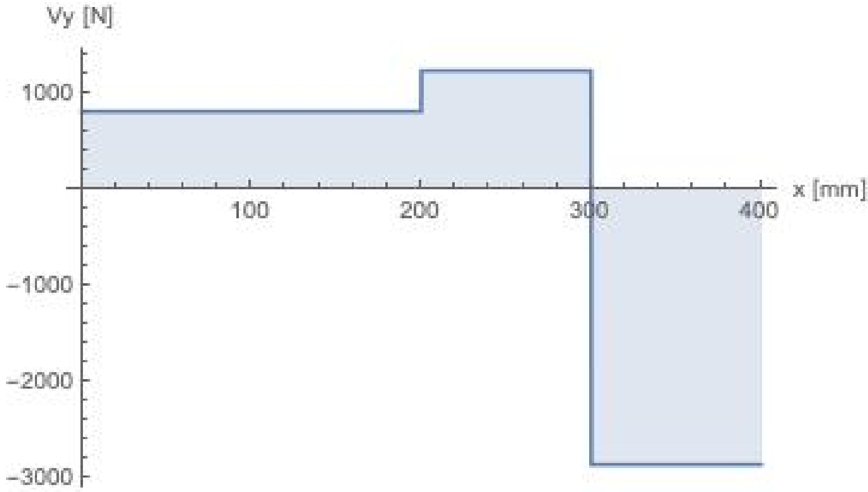


Figure 5.3: Internal loading diagram of V_y.

Shear action along z coordinate

$$\begin{cases} V_z = -R_{az}, & 0 \leq x \leq a \\ V_z = -R_{az} + F_{BT}, & a < x \leq a+b \\ V_z = -R_{az} + F_{BT} + R_{cz}, & a+b < x \leq 400 \end{cases}$$

In Figure 5.4 the correspondent internal loading diagram is shown.

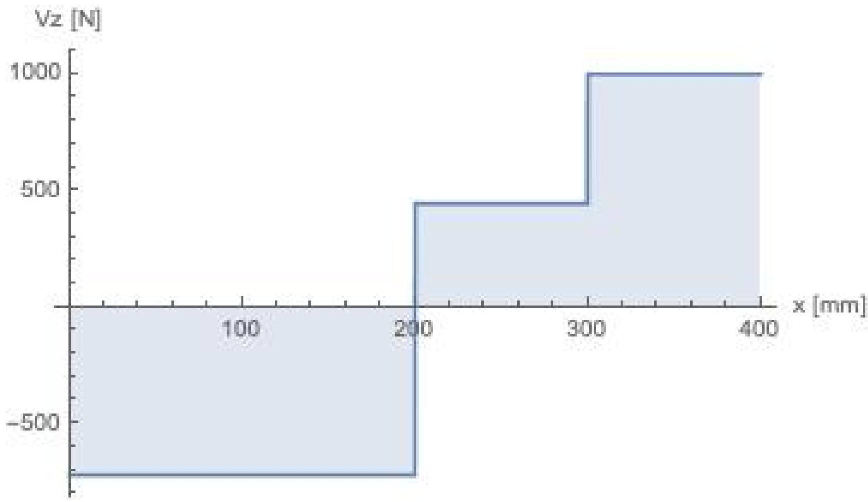


Figure 5.4: Internal loading diagram of V_z.

Torque along x coordinate

$$\begin{cases} T = 0, & 0 \le x \le a \\ T = T_x, & a < x \le a+b \\ T = T_x, & a+b < x \le 400 \end{cases}$$

In Figure 5.5 the correspondent internal loading diagram is shown.

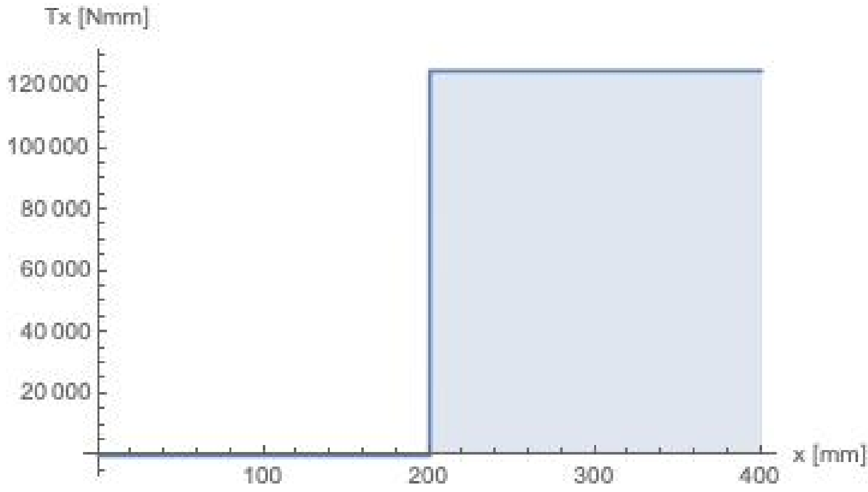


Figure 5.5: Internal loading diagram of T.

Bending moment along the y coordinate (with respect to point A)

$$\begin{cases} M_y = V_z\, x, & 0 \le x \le a \\ M_y = V_z\, x - F_{BT}\, a, & a < x \le a+b \\ M_y = V_z\, x - F_{BT}\, a - R_{cz}\, (a+b), & a+b < x \le 400 \end{cases}$$

In Figure 5.6 the correspondent internal loading diagram is shown.

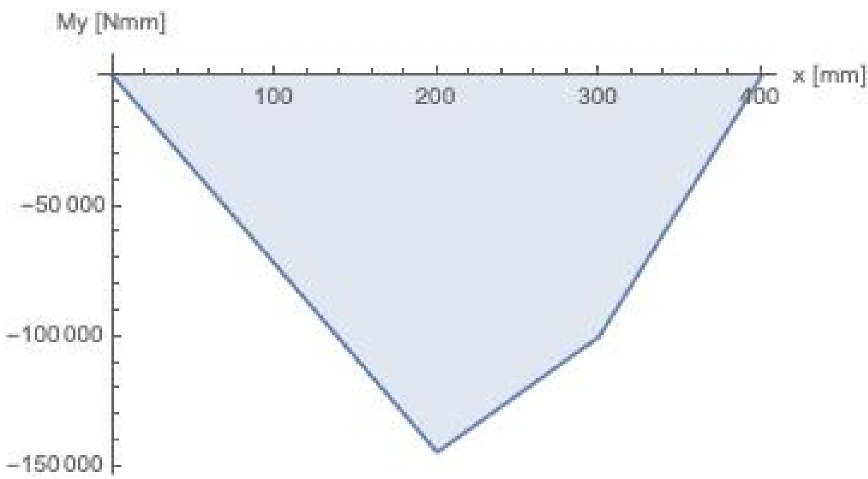


Figure 5.6: Internal loading diagram of M_y.

Bending moment along the z coordinate (with respect to point A)

$$\begin{cases} M_z = -V_y\, x, & 0 \le x \le a \\ M_z = -V_y\, x + F_{BR}\, a, & a < x \le a+b \\ M_z = -V_y\, x + F_{BR}\, a - R_{cy}\,(a+b), & a+b < x \le 400 \end{cases}$$

In Figure 5.7 the correspondent internal loading diagram is shown.

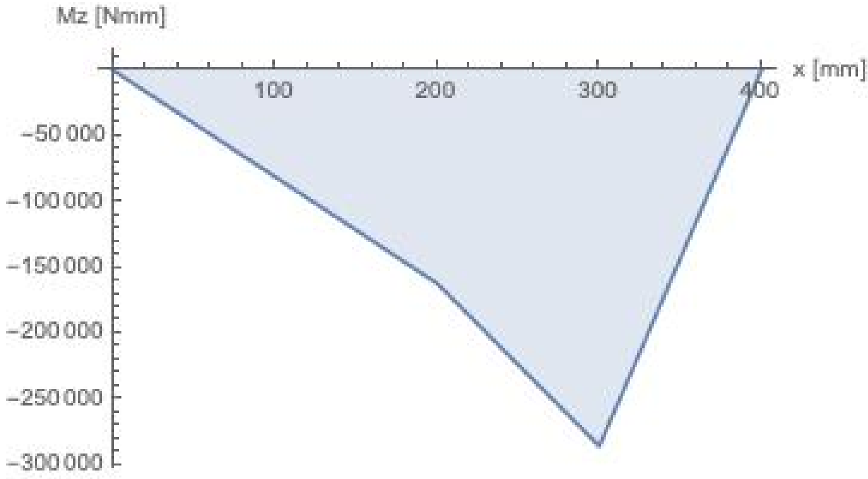


Figure 5.7: Internal loading diagram of M_z.

5.2.3 Shaft design for strength and stiffness

Preliminar design for stress

As fisrt point it is important to define the value of ultimate tensile strength and yield stress of the material (steel C45):

$$S_u \longrightarrow 630\ MPa$$
$$S_y \longrightarrow 370\ MPa$$

Analysing the internal loading diagrams it is possible to understand that the most critical point is the point C. Since a circular cross section is considered, it is possible to simplify the analysis by considering one unique total bending moment $M_{b,tot}$.

$$M_{b,tot} = \sqrt{M_{by}^2 + M_{bz}^2} \tag{5.19}$$

Where

$$M_{by} \longrightarrow M_y(c) \longrightarrow -99969.4\ N\ mm$$
$$M_{bz} \longrightarrow M_z(c) \longrightarrow -285600\ N\ mm$$
so:
$$M_{b,tot} \longrightarrow 302591\ N\ mm$$

The equivalent bending moment can be calculated, taking into account also the torque and a coefficient $\alpha = 0.25$ since a constant torque is considered:

$$T = T_x \longrightarrow 125100\ N\ mm$$

$$M_{b,eq} = \sqrt{M_{b,tot}^2 + \alpha\, T^2} = 308988\ N\ mm \tag{5.20}$$

For shafts under bending and torsion it is possible to carry out the verification by using the following formulas:

$$\sigma = \frac{M_{b,eq}}{Z} \leq \sigma_{all} \tag{5.21}$$

$$Z = \frac{\pi\, d^3}{32} \tag{5.22}$$

$$d_{min} \geq \sqrt[3]{\frac{32\, M_{b,eq}}{\pi\, \sigma_{all}}} \tag{5.23}$$

For the calculation of σ_{all}:

$$\sigma_{all} = \frac{\sigma_a}{\phi} = \frac{315}{7} = 45\ MPa \tag{5.24}$$

Where the value σ_a is the fatigue resistance of the material (calculated as $S_u/2 \longrightarrow 630/2 = 315\ MPa$ and the value $\phi = 7$ is selected as conservative safety factor.
Now the value of the minimum diameter can be determined:

$$d_{min} \geq 41.2012\ mm$$

As first attempt a value of $d = 45\ mm$ was selected, but after the following verification, it was not suitable for the maximum deflection allowed in point D. After various iterations the minimum suitable diameter was found to be $d = 65\ mm$. (For details of the iterations see the Appendix)
Going on with the preliminar design for stresses it is possible to conclude that, once the minimum diameter was set to be $d = 65\ mm$:

$$Z = 26961.2\ mm^3$$
$$\sigma = \tfrac{M_{b,eq}}{Z} \leq \sigma_{all} \longrightarrow 11.4605\ MPa \leq 45\ MPa$$

Since in C the most critical conditions are found, it is possible to increase the diameter in that point to have a better resistance and guarantee a better mounting of other elements. Therefore it was decided to increase the diameter of the cross section for the part from B to C of the shaft as follows:

$$D1 = 1.2\ d \longrightarrow 78\ mm$$

Also for this diameter the preliminar stress verification was carried out:

$$Z_1 = 46589\ mm^3$$
$$\sigma = \tfrac{M_{b,eq}}{Z_1} \leq \sigma_{all} \longrightarrow 6.63221\ MPa \leq 45\ MPa$$

5.2.4 Design for stiffness

In general stiffness requirements are more demanding then strength requirements, so it has been decided to design first for stiffness and then verify the strength.

Requirements

Stiffness requirements for maximum deflection and maximum torsional deformation in this case are:

$$f_{max} \leq \frac{l}{3000} = \frac{400}{3000} = 0.1333 \tag{5.25}$$

$$\theta_{max} \leq 0.044\, l = 0.00176\ rad \tag{5.26}$$

The maximum rotation of the shaft where the sample is mounted is:

$$\phi_{max} = 0.00349\ rad \tag{5.27}$$

Conditions for deflection and slope of the gears are (considering gears of module between 3 and 6 mm:

$$f_{max,g} = 0.125\ mm \tag{5.28}$$

$$\phi_{max,g} = 0.01\ rad \tag{5.29}$$

Slope conditions for bearings are the following:

$$\phi_{max,b} = 0.003\ rad \tag{5.30}$$

Useful values

Some useful values in the following calculation are the moduli of inertia of the two cross sections:

$$I_1 = \frac{d^4\,\pi}{64} \longrightarrow 876241\ mm^4 \tag{5.31}$$

$$I_2 = \frac{D1^4\,\pi}{64} \longrightarrow 1.81697x10^6\ mm^4 \tag{5.32}$$

Elastic modulus and shear modulus that will be considered are the following:

$$E = 210000\ MPa$$
$$G = 80000\ MPa$$

Specific points of the reference system will be considered:

$$x_a = 0\ mm$$
$$x_b = 200\ mm$$
$$x_c = 100\ mm$$
$$x_d = 400\ mm$$

Equations of the elastic line

The analysis of stiffness is carried out with the method of the equations of the elastic line, in which it is possible to write down the equations of the second derivative and then by integration determine the first derivative (that describes the slope) and the primitive function (zero derivative, that determines the deflection). The resolution of the problem requires the presence of boundary conditions that are determined by imposing the continuity in the first derivative or by imposing fixed points along the reference system.

Although two equations of the elastic line are required for the solution of the problem, because one describes the deflection (and slope) making reference to the y coordinate and the other making reference to the z equation, in a preliminar analysis it is possible to make reference to one general resolution, that splits in two once the values of the external loads are substituted inside the equation (solution for y when the loads acting on y coordinate are substitute and for z when loads acting on z coordinate are substituted).
The general second derivative equations are the following:

$$\xi''_{AB} = \frac{F_{\xi,AB}}{E\ I_1}\ x \tag{5.33}$$

$$\xi''_{BC} = \frac{F_{\xi,AB}}{E\ I_2}\ x + \frac{F_{\xi,BC}}{E\ I_2}\ (x - x_b) \tag{5.34}$$

$$\xi''_{CD} = \frac{F_{\xi,AB}}{E\ I_1}\ x + \frac{F_{\xi,BC}}{E\ I_1}\ (x - x_b) + \frac{F_{\xi,CD}}{E\ I_1}\ (x - x_c) \tag{5.35}$$

The general first derivative equations can be calculated as:

$$\xi'_{AB} = \int \xi''_{AB} dx + Cost1 \tag{5.36}$$

$$\xi'_{BC} = \int \xi''_{BC} dx + Cost2 \tag{5.37}$$

$$\xi'_{CD} = \int \xi''_{CD} dx + Cost3 \tag{5.38}$$

The general primitive functions can be calculated as:

$$\xi_{AB} = \int \xi'_{AB} dx + Cost4 \tag{5.39}$$

$$\xi_{BC} = \int \xi'_{BC} dx + Cost5 \tag{5.40}$$

$$\xi_{CD} = \int \xi'_{CD} dx + Cost6 \tag{5.41}$$

The values of the various constants can be determined by setting the proper boundary conditions, that in this case are:

$$\begin{gathered}
\xi_{AB}(x_a) = 0 \\
\xi_{AB}(x_b) = \xi_{BC}(x_b) \\
\xi'_{AB}(x_b) = \xi'_{BC}(x_b) \\
\xi_{CD}(x_c) = 0 \\
\xi_{BC}(x_c) = \xi_{CD}(x_c) \\
\xi'_{BC}(x_c) = \xi'_{CD}(x_c)
\end{gathered}$$

By substituting the values of the external forces along coordinates y and z it is possible to determine the value of the constants of y (C constants) and z (E constants) equations:

$$\begin{gathered}
C1 \longrightarrow 0.0000117719 \\
C2 \longrightarrow 0.000202437 \\
C3 \longrightarrow 0.000303476 \\
C4 \longrightarrow 0
\end{gathered}$$

$$C5 \longrightarrow -0.0111189$$
$$C6 \longrightarrow -0.0166053$$
$$E1 \longrightarrow -0.0000192081$$
$$E2 \longrightarrow -0.00113977$$
$$E3 \longrightarrow -0.0000938539$$
$$E4 \longrightarrow 0$$
$$E5 \longrightarrow 0.0106923$$
$$E6 \longrightarrow 0.00973553$$

Now it is possible to associate the equations of the elastic line to deflections and slopes. Deflections and slopes in y are obtained by substituting the C constants and the external forces acting along y coordinates:

$$y_{AB} = \xi_{AB} \longrightarrow -3.84398\ 10^{-10}\ x^3 + 0.0000117719\ x$$
$$y'_{AB} = \xi'_{AB} \longrightarrow 0.0000117719 - 1.1532\ 10^{-9}\ x^2$$
$$y_{BC} = \xi_{BC} \longrightarrow 1.60252\ 10^{-9}\ x^3 - 1.07274\ 10^{-6}\ x^2 + 0.000202437\ x - 0.0111189$$
$$y'_{BC} = \xi'_{BC} \longrightarrow 4.80757\ 10^{-9}\ x^2 - 2.14548\ 10^{-6}\ x + 0.000202437$$
$$y_{CD} = \xi_{CD} \longrightarrow 7.33464\ 10^{-10}\ x^3 - 1.44757\ 10^{-6}\ x^2 + 0.000303476\ x - 0.0166053$$
$$y'_{CD} = \xi'_{CD} \longrightarrow 2.20039\ 10^{-9}\ x^2 - 2.89515\ 10^{-6}\ x + 0.000303476$$

Deflections and slopes in z are obtained by substituting the E constants and the external forces acting along z coordinates:

$$z_{AB} = \xi_{AB} \longrightarrow -0.0000192081\ x - 1.0561\ 10^{-9}\ x^3$$
$$z'_{AB} = \xi'_{AB} \longrightarrow -0.0000192081 - 3.1683\ 10^{-9}\ x^2$$
$$z_{BC} = \xi_{BC} \longrightarrow 0.0106923 - 0.000113977\ x + 1.45769\ 10^{-7}\ x^2 - 7.52256\ 10^{-10}\ x^3$$
$$z'_{BC} = \xi'_{BC} \longrightarrow -0.000113977 + 2.91539\ 10^{-7}\ x - 2.25677\ 10^{-9}\ x^2$$
$$z_{CD} = \xi_{CD} \longrightarrow 0.00973553 - 0.0000938539\ x + 3.03265\ 10^{-8}\ x^2 - 6.53408\ 10^{-10}\ x^3$$
$$z'_{CD} = \xi'_{CD} \longrightarrow -0.0000938539 + 6.06531\ 10^{-8}\ x - 1.96023\ 10^{-9}\ x^2$$

5.2.5 Bearing slope verification (positions A and C)

It is now possible to use the equations of the deflection and its first derivative to check for the deflection and the slope of specific points in which elements are mounted on the shaft. For what concerns the bearings the slope is considered for the verification. For position A:

$$\phi_{ya} = y'_{AB}(x_a) \longrightarrow 0.0000117719\ rad$$
$$\phi_{za} = z'_{AB}(x_a) \longrightarrow -0.0000192081\ rad$$

$$\phi_{tot,a} = arctg\left(\sqrt{(tg^2(\phi_{ya}) + tg^2(\phi_{za})}\right) \tag{5.42}$$

$$\phi_{tot,a} \leq \phi_{max,b} \longrightarrow 0.0000225284\ rad \leq 0.003\ rad \tag{5.43}$$

For position C:

$$\phi_{yc} = y'_{BC}(x_c) \longrightarrow 0.0000359651\ rad$$
$$\phi_{zc} = z'_{BC}(x_c) \longrightarrow -0.000107391\ rad$$

$$\phi_{tot,c} = arctg\left(\sqrt{(tg^2(\phi_{yc}) + tg^2(\phi_{zc})}\right) \tag{5.44}$$

$$\phi_{tot,c} \leq \phi_{max,b} \longrightarrow 0.000113253\ rad \leq 0.003\ rad \tag{5.45}$$

The slope verification is okay for both positions A and C.

5.2.6 Gear slope and deflection verification (position B)

The first verification in this case is made on the deflection:

$$f_{yb} = y_{AB}(x_b) \longrightarrow -0.000720802\ mm$$
$$f_{zb} = z_{AB}(x_b) \longrightarrow -0.0122904\ mm$$

$$f_{tot,b} = \sqrt{f_{yb}^2 + f_{zb}^2} \tag{5.46}$$

$$f_{tot,b} \leq f_{max,g} \longrightarrow 0.0123115\ mm \leq 0.125\ mm \tag{5.47}$$

Verification for the slope is the following:

$$\phi_{yb} = y'_{AB}(x_b) \longrightarrow -0.0000343559\ rad$$
$$\phi_{zb} = z'_{AB}(x_b) \longrightarrow -0.00014594\ rad$$

$$\phi_{tot,b} = arctg\left(\sqrt{(tg^2(\phi_{yb}) + tg^2(\phi_{zb})}\right) \tag{5.48}$$

$$\phi_{tot,b} \leq \phi_{max,g} \longrightarrow 0.000149929\ rad \leq 0.01\ rad \tag{5.49}$$

5.2.7 Sample deflection and slope verification (position D)

Verification for the deflection in position of the sample is the following:

$$f_{yd} = y_{CD}(x_d) \longrightarrow -0.0798852\ mm$$
$$f_{zd} = z_{CD}(x_d) \longrightarrow -0.0647719\ mm$$

$$f_{tot,d} = \sqrt{f_{yd}^2 + f_{zd}^2} \tag{5.50}$$

$$f_{tot,d} \leq f_{max} \longrightarrow 0.102845\ mm \leq 0.133333\ mm \tag{5.51}$$

Verification for the slope is the following:

$$\phi_{yd} = y'_{CD}(x_d) \longrightarrow -0.000502521\ rad$$
$$\phi_{zd} = z'_{CD}(x_d) \longrightarrow -0.000383229\ rad$$

$$\phi_{tot,d} = arctg\left(\sqrt{(tg^2(\phi_{yd}) + tg^2(\phi_{zd})}\right) \tag{5.52}$$

$$\phi_{tot,d} \leq \phi_{max} \longrightarrow 0.000631975\ rad \leq 0.00349\ rad \tag{5.53}$$

5.2.8 Torsional stiffness verification

For this verification it has been decided to consider only the lower diameter d in order to have a more conservative approach. The polar moment of inertia is equal to 2 times the second moment of inertia of the cross section, since the cross section is circular:

$$J_p = 2\, I_1 \longrightarrow 1.75248\ 10^6\ mm^4 \tag{5.54}$$

The axial distance in which the torque acts must be taken in consideration, that is 200 mm (the part from B to D).

$$\theta_{BD} = \frac{T_x\ 200}{G\ J_p} \tag{5.55}$$

$$\theta_{BD} \leq \theta_{max} \longrightarrow 0.000178461\ rad \leq 0.00176\ rad \tag{5.56}$$

5.2.9 Design for stress - fatigue verification

For this part the Gough Pollard method is used, since it was studied specifically for shafts and notch effects can be included. The critical point for fatigue resistance can be taken as the point C, where we have the most critical stress condition and where also a shoulder is located. Reference values are those of C45 quenched and tempered steel:

$$S_u \longrightarrow 630\ MPa$$
$$S_y \longrightarrow 370\ MPa$$
$$S_e = S_u/2 \longrightarrow 315\ MPa$$

Other fundamental values for the fatigue analysis have been already calculated and are reported here below:

$$M_{b,tot} = 302591\ N\ mm$$
$$T_x = 125100\ N\ mm$$
$$I_1 = 876241\ mm^4$$
$$J_p = 1.75248\ 10^6\ mm^4$$

The nominal bending normal stress and the nominal torsional shear stress are calculated as:

$$\sigma = \frac{32\ M_{b,tot}}{\pi\ d^3} \longrightarrow 11.2232\ MPa \tag{5.57}$$

$$\tau = \frac{16\ T_x}{\pi\ d^3} \longrightarrow 2.32\ MPa \tag{5.58}$$

A fillet radius of 1 mm is selected to have some margin but also for design reasons that will be clarified in the following parts.

$$\tfrac{r}{d} = \tfrac{1}{65} \longrightarrow 0.0153846$$
$$\tfrac{D1}{d} = 1.2$$

Knowing such values it is possible to determine useful values for the fatigue analysis:

$$K_t = 2.35$$
$$q = 0.83$$
$$K_\tau = 1.85$$

$$K_f = 1 + q\ (K_t - 1) \longrightarrow 2.1205 \tag{5.59}$$

Fatigue analysis according to Gough-Pollard criterion

It has been decided to use ground conditions for the surface finish:

$$C_s = 1.58\ (S_u)^{-0.085} \longrightarrow 0.91351 \tag{5.60}$$

The condition for bending and torsion is $C_{load} = 1$, then it is necessary to add the condition for diameters lower than 250 *mm* and higher than 8 *mm*:

$$C_d = 1.189\ d^{-0.097} \longrightarrow 0.793102 \tag{5.61}$$

At this point it is possible to calculate the Gough-Pollard equivalent stresses:

$$\sigma_{cr} = C_s\ C_d\ S_e \longrightarrow 228.22\ MPa \tag{5.62}$$

$$\tau_{cr} = \frac{S_y}{\sqrt{3}} \longrightarrow 213.62\ MPa \tag{5.63}$$

$$H = \tfrac{\sigma_{cr}}{\tau_{cr}}$$

$$\sigma_{eq} = \sqrt{(K_f\ \sigma)^2 + (H^2)(K_\tau\ \tau)^2} \longrightarrow 24.2365\ MPa \tag{5.64}$$

$$\phi = \frac{\sigma_{cr}}{\sigma_{eq}} \longrightarrow 9.41637 >> 1 \tag{5.65}$$

It is possible to see that in such conditions the verification is largely satisfied.

5.2.10 Bearing selection

The selection of the bearings had to be made in section A, where the shaft maintains the minimum diameter of 65 mm and in location C, where the section was raised to a diameter of 75mm to allow the choice of a standardized bearing. The configuration of choice was constituted by a non-locating deep groove single raw ball bearing in location A, and a locating deep groove single raw ball bearing in location C.
Firstly the radial loads (F_A) where calculated in the two positions:

$$R_a = \sqrt{R_{ay}^2 + R_{az}^2} \tag{5.66}$$

$$R_c = \sqrt{R_{cy}^2 + R_{cz}^2} \tag{5.67}$$

The equivalent load on the bearings is expressed as such:

$$F_{R,eq} = X\,F_R + Y\,F_A \tag{5.68}$$

No axial forces (F_A) were present on the bearings. For a ratio ($F_A/F_R < e$) the coefficients are: X = 1 and Y = 0. The life of the bearings expressed in working hours was calculated according to the following formula:

$$L_{5h} = \frac{10^6}{60\,n_s}\,a_1\,a_{skf}\,(\frac{C}{F_{R,eq}})^n \tag{5.69}$$

where:
n_s is the rotational speed of the shaft equal to 229 [rev/min];
C is the dynamic load rating of the bearing, obtained from the spec sheet

of the bearing;

n = 3 for ball bearings.

The coefficient a_1 was obtained from a table in the SKF catalogue, for a reliability of 95% it corresponds to: $a_1 = 0.64$. The coefficient a_{skf} is obtained graphically from a plot in the SKF catalogue. To derive it, a series of coefficients need to be obtained first from a series of graphs and tables:

ν is the rated viscosity of the oil at operating temperature. A VG100 oil was chosen, so the resulting viscosity at T = 60°C is ν = 40 [mm^2/s].

ν_1, the rated viscosity at operating temperature, was obtained knowing the mean bearing diameter $d_m = (D+d)/2$. The knowledge of d_m also allows to derive the parameter η_c under the specified hypothesis of moderate contamination.

$K = \nu/\nu_1$ is a coefficient that is necessary to obtain a_{skf} from the graph, for which it is also necessary to know the fatigue load limit of the bearing P_u. The life verification is performed as such:

$$L_{5h} \geq 15000\ h \tag{5.70}$$

For the position A, the 61813 bearing was selected. The relative data is reported in table 5.2:

$F_{R,eq}$ $[N]$	d $[mm]$	D $[mm]$	B $[mm]$	η_c	ν_1 $[mm^2/s]$	K	P_u $[N]$	a_{skf}
1085.4	65	85	10	0.5	40	1	540	6

Table 5.2: Specifications of bearing 61813.

The bearing service life was 416717 hours, well above the required value.

For the position C, the 61815 bearing was initially selected, but the service life did not satisfy the requirements. The 61915 bearing was thus chosen as a replacement. The data of the bearing is displayed in table 5.3:

$F_{R,eq}$ $[N]$	d $[mm]$	D $[mm]$	B $[mm]$	η_c	ν_1 $[mm^2/s]$	K	P_u $[N]$	a_{skf}
4130.8	75	105	16	0.5	44	0.91	965	1.9

Table 5.3: Specifications of bearing 61915.

The bearing service life was 17794 hours.

5.2.11 Gear design and verification

The gears studied are in position B of the wear-testing machine. The module assumed was $m = 3$. The following equations were used in order to find the main parameters of the gears and the contact ration between the teeth. $\alpha = 20°$.

$$z_1 = \frac{d_1}{m} = 21 \tag{5.71}$$

$$z_2 = \frac{z_1}{\tau} = 69 \tag{5.72}$$

$$z_{imin} = \frac{2}{\sqrt{u^2 + (1+2u)\sin^2\alpha - u}} = 18.47 \tag{5.73}$$

$$m_i \geq \sqrt[3]{\frac{2T}{\lambda z \sigma_{am} y}} = 2.55 < 3 \quad (5.74)$$

$$\sigma_{am} = \frac{S_u}{GR}\frac{A}{A+V} = 220\ MPa \quad (5.75)$$

$$D_1 = z_1 m = 63\ mm \quad (5.76)$$

$$D_2 = z_2 m = 207\ mm \quad (5.77)$$

$$i = \frac{D_1 + D_2}{2} = 134\ mm \quad (5.78)$$

$$r_{e1} = \frac{D_1 + 2m}{2} = 34.5\ mm \quad (5.79)$$

$$r_{e2} = \frac{D_2 + 2m}{2} = 106\ mm \quad (5.80)$$

$$r_{b1} = \frac{D_1 - 2 \cdot 1.25m}{2} = 27.75\ mm \quad (5.81)$$

$$r_{b2} = \frac{D_2 - 2 \cdot 1.25m}{2} = 99.75\ mm \quad (5.82)$$

$$\alpha_{wt} = \arccos \frac{i}{i_w}(\cos\alpha) = 26.106° \quad (5.83)$$

$$v_T = 2.57\ \frac{m}{s} \quad (5.84)$$

$$x_1 + x_2 = \frac{inv\alpha_w t - inv\alpha}{2\tan\alpha} = 0.4618$$

(5.85)

$$x_1 = \frac{x_1 + x_2}{2} + (\frac{1}{2} - \frac{x_1 + x_2}{2})\frac{\log u}{\log \frac{z_1 z_2}{100}} = 0.3504 \quad (5.86)$$

$$x_2 = (x_1 + x_2) - x_1 = 0.111 \quad (5.87)$$

$$\varepsilon_\alpha = \frac{\sqrt{r_{e1}^2 - r_{b1}^2} + \sqrt{r_{e2}^2 - r_{b2}^2} - i_w \sin\alpha_{wt}}{p\cos\alpha} = 1.32 \quad (5.88)$$

module m	3 mm	
α	$20°$	
b	30 mm	
addendum (m) dedenum (1.25 m)	3 mm	3.75 mm
z	21	69
i_w	140 mm	
α_{wt}	$26.106°$	
v_T	2.57 m/s	
τ	0.3055	
F_T	1166 N	
ε_α	1.32	

Table 5.4: Specifications of gear designed.

Design choice

The material chosen for the gears is 34CrMo4. The material is a structural hardened steel. The hardness of the surface 50-60 HRC. The material is the same for both gears. The gears have the following characteristics:

Rockwell hardness	50 HRC
$\sigma_{fatiguelimit}$	380 MPa
$\sigma_{contactpressurelimit}$	1100 MPa

The lubricant in use for the bearing does not satisfy the minimum viscosity obtained from the chart for oil selection. VG220 satisfy it with viscosity at 40 ° C equal to $220\frac{mm^2}{s}$. The working temperature is equal to 60° C, the viscosity value can be find assuming a linear relation between the known value of viscosity found on data chart from oil supplier. For simplicity the value at 40°C is assumed as the right value. The precision grade was assumed 7 and roughness $0.1\mu m$.

Gear verification

Two verifications need to be satisfied :

$$\sigma_F = (\frac{F_T}{bm})K_A K_V K_{F\alpha} K_{F\beta} Y_{Fa} Y_{Sa} Y Y_B \leq \sigma_{Fall} = \frac{\sigma_{fatiguelimit} Y_{ST} Y_{NT} Y_{\delta relT} Y_{RrelT} Y_X}{\varphi} \tag{5.89}$$

$$\sigma_H = \sqrt{\frac{F_T}{bd_1}\frac{u+1}{u}}\sqrt{K_A K_V K_{H\gamma} K_{H\rho}} z_H z_E z_\varepsilon \leq \sigma_{contactpressure,allowable} = \frac{\sigma_{contactpressurelimit} Z_{NT} Z_L Z_V Z_R Z_W z_X}{\varphi} \tag{5.90}$$

$$K_A = 1 \tag{5.91}$$

$$K_V = 1 + (\frac{K_1}{K_A \frac{F_T}{b} + K_2}) \cdot K_3 = 1.223 \tag{5.92}$$

$$f_{ma} = 4.16 \cdot b^{0.44} \cdot q_H = 8.071 \tag{5.93}$$

$$f_{sh} = 0.023(\frac{F_T}{b} \cdot K_V) \cdot [0.7 + K' \frac{ls}{d_1^2} \frac{d_1}{d_s h}^4 + 0.3] \cdot (\frac{b}{d_1})^2 = 0.39 \tag{5.94}$$

$$K_{H\beta} = 1 + \frac{10 \cdot F_{By}}{K_V \cdot \frac{F_T}{b}} = 4.52 \tag{5.95}$$

$$K_{F\beta} = K_{H\beta f}^N = 3.24 \tag{5.96}$$

$$K_{F\alpha} = K_{H\alpha} = 1.38 \tag{5.97}$$

$$Y_{FA} = 2.07 \tag{5.98}$$

$$Y_{SA} = 2.05 \tag{5.99}$$

$$Y = 0.81 \tag{5.100}$$

$$Y_{ST} = 2 \tag{5.101}$$

$$Y_x = 1 \tag{5.102}$$

$$Y_{NT} = 2 \tag{5.103}$$

$$Y_{\delta relT} = 1 \tag{5.104}$$

$$Y_{RrelT} = 1.1 \tag{5.105}$$

$$Z_H = 2.25 \tag{5.106}$$

$$Z_L = 0.903 \tag{5.107}$$

$$Z_V = 0.95 \tag{5.108}$$

$$Z_R = 1.011 \tag{5.109}$$

$$Z_{NT} = 1 \tag{5.110}$$

$$Z_X = 1 \tag{5.111}$$

$$Z_E = 189.8 \tag{5.112}$$

$$Z_W = 1 \tag{5.113}$$

$$Z_\varepsilon = 0.80 \tag{5.114}$$

$$\sigma_F = 243\ MPa \leq \sigma_{Fall} = \frac{834\ MPa}{\phi} \tag{5.115}$$

$$\phi = 3.43 \tag{5.116}$$

$$\sigma_H = 833\ MPa \leq \sigma_{contact pressure,allowable} = \frac{1082\ Mpa}{\phi} \tag{5.117}$$

$$\phi = 1.30 \tag{5.118}$$

The gears have a safety factor against fatigue of 3.43 and 1.3 against wear. The safety factors need to be at least 1, the gear verification can be considered satisfied.

5.2.12 Sample-gear press fit

The necessary parameters were chosen as follows:
f = 0.2, as the interface is constituted by steel against steel;
ϕ_s = 1.25 is the coefficient against slipping;
ϕ_y = 1.5 is the coefficient against yielding;
T = 125100 Nmm is the torque to be transmitted;
D_0 = 212.7 is the pitch diameter of the gear;
b = 30 is the width of the gear;
D = 65 mm is the nominal diameter of the shaft in this section;
E = 210000 MPa is the young modulus of the shaft material (C45);
σ_y = 370 MPa is the yield strength of the shaft material (C45).
The lowest allowable pressure necessary to transmit the torque is determined by the slipping condition, and thus it was calculated with the following equation:

$$P_{c,min} = \frac{2T\,\phi_s}{f\pi\,b\,D^2} = 3.93\ MPa \tag{5.119}$$

The minimum allowable interference was then calculated:

$$i_{min} = \frac{P_{c,min}\,D}{E}\left(\frac{1}{1-\frac{D^2}{D_0^2}}\right) = 0.0013\ mm \tag{5.120}$$

The highest allowable pressure necessary is determined by the yielding condition of the shaft, and it was calculated with the following relation:

$$P_{c,max} = \frac{\sigma_y}{2\,\phi_y}\left(1-\frac{D^2}{D_0^2}\right) = 111.82\ MPa \tag{5.121}$$

There is a linear relationship between the pressure and the interference, so the minimum allowable interference was simply scaled up to find the maximum:

$$i_{max} = \frac{P_{c,max}}{P_{c,min}}\,i_{min} = 0.0382\ mm \tag{5.122}$$

The condition to be satisfied is:

$$i_{min} < \frac{\delta_{min,tol}}{2} < \frac{\delta_{max,tol}}{2} < i_{max} \tag{5.123}$$

The condition was satisfied by a $\phi\,65\,F7t5$ coupling:

$$0.0013 < \frac{0.0060}{2} < \frac{0.0490}{2} < 0.0382 \tag{5.124}$$

This specific coupling was applied only in the section where the gear resides. On the rest of the section A-B of the shaft, the chosen tolerance was $\phi\,65\,k5$. This should allow the easy passage of the gear during mounting, without subtracting material to the minimum cross section.

5.2.13 Key coupling between sample and shaft

The key selected was of the type UNI 6604 B. The choice was made to allow a lower key length, since the type B can transmit torque throughout all of its length. The material chosen for the key was C20, since it has lower strength than the shaft material. This ensures that the key will fail before the shaft. The minimum shaft dimension in point D is d = 65 mm. To allow the manufacturing of the key seat without disrupting the minimum cross section of the shaft the diameter was raised to d = 75 mm. This value is compatible with the available bearings. For this dimension, the key has the following parameters:

b $[mm]$	h $[mm]$	t_1 $[mm]$	p_{all} $[MPa]$	τ_{all} $[MPa]$
20	12	7.5	100	50

Table 5.5: Key dimensions.

The calculation of the minimum length was firstly done considering the crushing caused by compression:

$$L_{min,c} = \frac{2T}{d\,(h - t_1)\,P_{all}} = 7.62\ mm \tag{5.125}$$

The calculation of the minimum length was then done considering the shearing scenario:

$$L_{min,s} = \frac{2T}{d\,b\,\tau_{all}} = 3.43\ mm \tag{5.126}$$

T corresponds to the torque to be transmitted, in this case 125100 Nmm.
The minimum key length found to be commercially available was 28 mm, so the final key is a UNI 6604 B 20x12x28.

5.2.14 Axial constraints

To fix the sample to the shaft impeding axial movement, we opted for a locking nut with integral locking, chosen from the SKF website[3]. This was to allow an easy changing of the samples, and avoid stress risers on the loaded section of the shaft. The characteristics of the nut are reported in table 5.6 and figure 5.8:

$d3$ $[mm]$	B $[mm]$	G
80	14	M60x2

Table 5.6: Dimensions of locking nut KMFE 12.

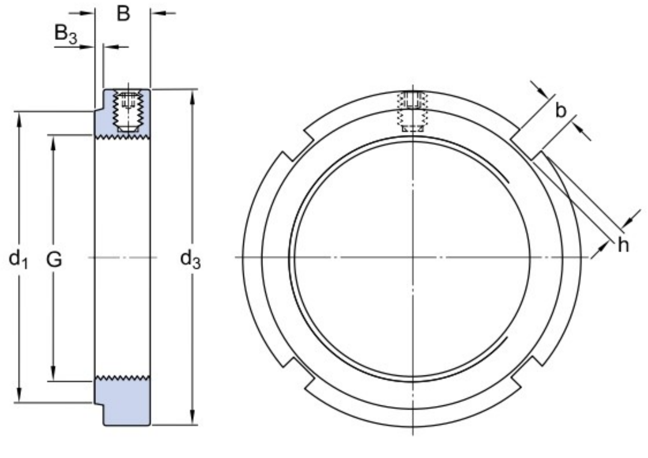


Figure 5.8: Drawing of the locking nut.

To block the axial movement of the sample in the other direction and at the same time constrain the bearing in position C, a spacer was designed. The spacer has the following dimensions:

D_i $[mm]$	D_o $[mm]$	l $[mm]$	t $[mm]$
75	80	82	2.5

Table 5.7: Dimensions of the spacer between sample and bearing C.

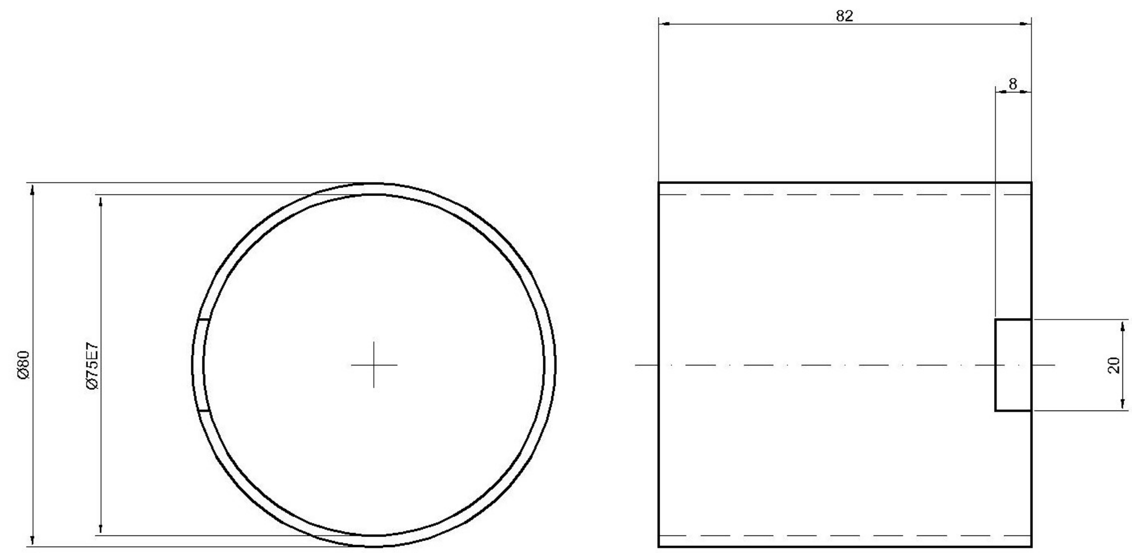


Figure 5.9: Drawing of the spacer between sample and bearing C.

The spacer has to be machined on one side to allow space for the key.
The section of the shaft was then raised to 80 mm between the bearing in C and the gear to have the bearing completely constrained. The section change has a fillet radius of 1 mm, since it is the maximum value allowed by the bearing.
Finally, another spacer was designed to fit between the gear and the bearing in position A. the second spacer has the following dimensions:

D_i $[mm]$	D_o $[mm]$	l $[mm]$	t $[mm]$
65	70	180	2.5

Table 5.8: Dimensions of the spacer between the gear and bearing A.

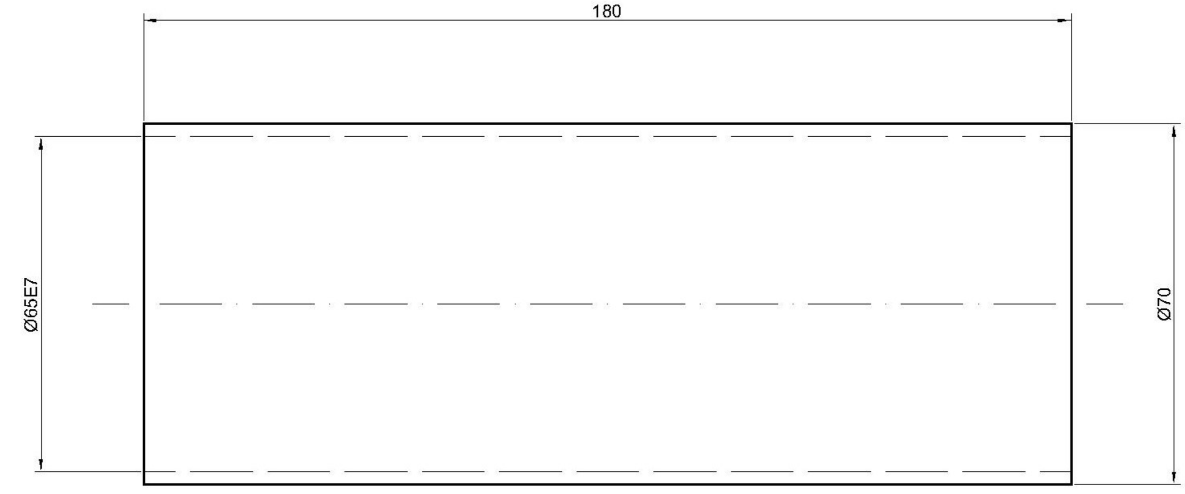
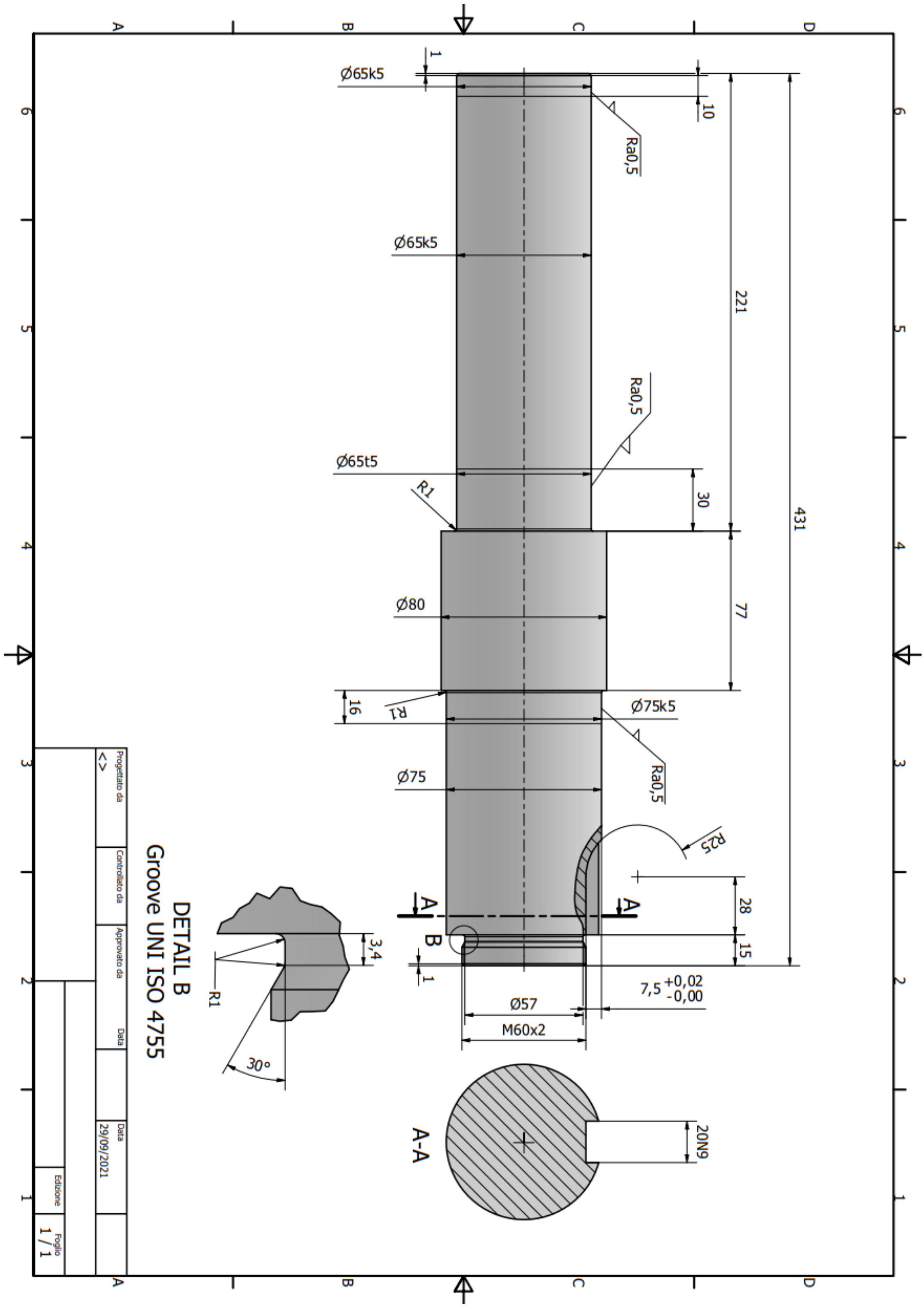


Figure 5.10: Drawing of the spacer between the gear and bearing A.

5.3 Conclusions

After the design of the shaft for stresses and stiffness it is possible to say that the minimum diameter that verifies all the conditions is $d = 65$ mm (diameter found after a cycle of iterations). The fatigue analysis, performed through the Gough-Pollard criterion, shows that the shaft is largely verified under fatigue conditions with a safety coefficient of $\phi = 9.41637 >> 1$.
The bearing selection consists of a 61813 ball bearing in location A and a 61915 ball bearing in location C, both satisfying the service life requirements.
The gears are made of 34CrMo4 and are constituted by 21 e 69 teeth. The diameters of the gears is equal to D_1 =63 mm and $D_2 = 207$ mm. The lubricant oil chosen is VG220. The gear are verified against fatigue and against wear with safety factor respectively of $\phi = 3.43$ and $\phi = 1.30$.
The transmission of the torque is ensured by a 6604B 20x12x28 key, for which the diameter is raised in section C-D to 75 mm. The whole assembly is axially constrained by a locking nut in position D and two spacers. One is located between the first bearing and the gear, and the other between the second bearing and the sample. Between the gear and the second bearing, the diameter is raised to 80 mm to act as a shoulder.
The drawing of the shaft is present in the annexes.

5.4 Annexes



Ø65k5
Ø65k5
Ø65t5
Ø80
Ø75k5
Ø75
Ø57
M60x2
Ra0,5
Ra0,5
Ra0,5
R1
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R25
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5.5 Appendix

5.5.1 Iterations to determine the minimum diameter

During the calculation of the minimum diameter in the preliminar design for stresses the first allowable value was 45 *mm*, but after the verification for stiffness it was seen that this diameter satisfied all the verifications except for that of deflection of the sample in position D (that was the maximum deflection). For this reason some iterations were performed by considering increasing diameter (as multiple of 5) until the chosen one, that is 65 *mm*. Here below the results of iterations dealing with that particular verification are reported:

Iteration	Diameter [*mm*]	Verification condition	Verified
1	45	$0.447698 \nleq 0.133333$	No
2	50	$0.293735 \nleq 0.133333$	No
3	55	$0.200625 \nleq 0.133333$	No
4	60	$0.141655 \nleq 0.133333$	No
5	65	$0.102845 \leq 0.133333$	Yes

Table 5.9: Results of iterations dealing with the increase in diameter and the deflection verification in position D.

References

[1] *M Major et al 2019 IOP Conf. Ser.: Mater. Sci. Eng. 585 012007.*

[2] *EN 1993-1-8 (2005) (English): Eurocode 3: Design of steel structures - Part 1-8: Design of joints, The European Union Per Regulation 305/2011.*

[3] SKF. Ghiere di bloccaggio con fissaggio integrato. https://www.skf.com/it/products/rolling-bearings/accessories/lock-nuts/integral-locking, 2021.