Non-linear least squares:

$$F(x) = rac{1}{2} \|f(x)\|^2$$
 $J_f = rac{df}{dx^T}, \quad J_F =
abla F^T = rac{dF}{dx^T} = J_f^T f(x)$ $H_F = rac{d^2 F}{dx^T dx} = rac{dJ_F}{dx}$

- ullet Gradient descent: $\Delta x = -lpha J_F^T$
- ullet Newton's method: $\Delta x = -H_F^{-1} J_F$
- ullet Gauss-Newton's method: $\Delta x = -(J_f^T J_f)^{-1} J_f^T f(x)$
- ullet Levenberg-Marquardt: $[J_f^TJ_f+\lambda I]\Delta x=-J_f^Tf(x)$

Outer Product

$$n^{\wedge} = egin{bmatrix} 0 & -n_z & n_y \ n_z & 0 & -n_x \ -n_y & n_x & 0 \end{bmatrix} \ n_1 imes n_2 = n_1^{\wedge} n_2 \ nn^T = I + n^{\wedge 2} \ n^{\wedge} n = 0 \ \end{pmatrix}$$

SO3

$$\mathrm{SO}(3) = \{R \in \mathbb{R}^{3 imes 3} \mid RR^T = I, \det(R) = 1\}$$

- Rotation Angle: θ
- Rotation Axis: $\{n=[n_x,n_y,n_z]^T\in\mathbb{R}^3\mid \|n\|=1\}$
- ullet Lie Algebra: $\phi= heta n\in\mathbb{R}^3$
- Quaternion: $q = [q_w, q_x, q_y, q_z]^T$

Lie Algebra

$$egin{aligned} \exp(\phi^\wedge) &= I &+ \sin heta n^\wedge &+ (1-\cos heta) n^{\wedge 2} \ J_l(\phi^\wedge) &= I &+ rac{1-\cos heta}{ heta} n^\wedge &+ (1-rac{\sin heta}{ heta}) n^{\wedge 2} \ J_r(\phi^\wedge) &= I &- rac{1-\cos heta}{ heta} n^\wedge &+ (1-rac{\sin heta}{ heta}) n^{\wedge 2} \ \exp((\phi + \Delta \phi)^\wedge) &pprox \exp((J_l(\phi^\wedge) \Delta \phi)^\wedge) \exp(\phi^\wedge) \ &pprox \exp(\phi^\wedge) \exp((J_r(\phi^\wedge) \Delta \phi)^\wedge) \end{aligned}$$

Quaternion

Rotate:

$$p^\prime = qpq^{-1}$$

Conjugate:

$$egin{aligned} q^* &
ightarrow R^T \ q^* &= \|q\|^2 q^{-1} \end{aligned}$$

Multiplication:

$$QQ^T = I$$
 $qr = Qr$ $rq = \overline{Q}r$ $Q = L(q) = egin{bmatrix} q_w & -q_x & -q_y & -q_z \ q_x & q_w & -q_z & q_y \ q_y & q_z & q_w & -q_x \ q_z & -q_y & q_x & q_w \end{bmatrix}$ $\overline{Q} = R(q) = egin{bmatrix} q_w & -q_x & -q_y & -q_z \ q_x & q_w & q_z & -q_y \ q_y & -q_z & q_w & q_x \ q_z & q_y & -q_z & q_w & q_x \ q_z & q_y & -q_z & q_w \end{bmatrix}$

Transformation

from \ to	R	ϕ	q
R	_	$ heta = rccos rac{\sqrt{ ext{tr}(R) - 1}}{2} \ n = rac{egin{bmatrix} R_{32} - R_{23} \ R_{13} - R_{31} \ R_{21} - R_{12} \end{bmatrix}}{2\sinrac{ heta}{2}(ext{tr}(R) + 1)} \ n = rac{\left[R_{32} - R_{23} ight]}{2\sinrac{ heta}{2}(ext{tr}(R) + 1)} \ n = rac{\left[R_{32} - R_{23} ight]}{2\sinrac{ heta}{2}(ext{tr}(R) + 1)} \ n = rac{\left[R_{32} - R_{23} ight]}{2\sinrac{ heta}{2}(ext{tr}(R) + 1)} \ n = rac{\left[R_{32} - R_{23} ight]}{2\sinrac{ heta}{2}(ext{tr}(R) + 1)} \ n = rac{\left[R_{32} - R_{23} ight]}{2\sinrac{ heta}{2}(ext{tr}(R) + 1)} \ n = rac{\left[R_{32} - R_{23} ight]}{2\sinrac{ heta}{2}(ext{tr}(R) + 1)} \ n = rac{\left[R_{32} - R_{23} ight]}{2\sinrac{ heta}{2}(ext{tr}(R) + 1)} \ n = rac{\left[R_{32} - R_{23} ight]}{2\sinrac{ heta}{2}(ext{tr}(R) + 1)} \ n = rac{\left[R_{32} - R_{23} ight]}{2\sinrac{ heta}{2}(ext{tr}(R) + 1)} \ n = rac{\left[R_{32} - R_{23} ight]}{2\sinrac{ heta}{2}(ext{tr}(R) + 1)} \ n = rac{\left[R_{32} - R_{23} ight]}{2\sinrac{ heta}{2}(ext{tr}(R) + 1)} \ n = rac{\left[R_{32} - R_{23} ight]}{2\sinrac{ heta}{2}(ext{tr}(R) + 1)} \ n = rac{\left[R_{32} - R_{23} ight]}{2\sinrac{ heta}{2}(ext{tr}(R) + 1)} \ n = rac{\left[R_{32} - R_{23} ight]}{2\sinrac{ heta}{2}(ext{tr}(R) + 1)} \ n = rac{\left[R_{32} - R_{23} ight]}{2\sin\left(ext{tr}(R) + 1 ight)} \ n = rac{\left[R_{32} - R_{23} ight]}{2\sin\left(ext{tr}(R) + 1 ight)} \ n = rac{\left[R_{32} - R_{23} ight]}{2\sin\left(ext{tr}(R) + 1 ight)} \ n = rac{\left[R_{32} - R_{23} ight]}{2\sin\left(ext{tr}(R) + 1 ight)} \ n = rac{\left[R_{32} - R_{23} ight]}{2\sin\left(ext{tr}(R) + 1 ight)} \ n = rac{\left[R_{32} - R_{23} ight]}{2\sin\left(ext{tr}(R) + 1 ight)} \ n = rac{\left[R_{32} - R_{23} ight]}{2\sin\left(ext{tr}(R) + 1 ight)} \ n = rac{\left[R_{32} - R_{23} ight]}{2\sin\left(ext{tr}(R) + 1 ight)} \ n = rac{\left[R_{32} - R_{23} ight]}{2\sin\left(ext{tr}(R) + 1 ight)} \ n = rac{\left[R_{32} - R_{23} ight]}{2\sin\left(ext{tr}(R) + 1 ight)} \ n = rac{\left[R_{32} - R_{23} ight]}{2\sin\left(ext{tr}(R) + 1 ight)} \ n = rac{\left[R_{32} - R_{32} ight]}{2\sin\left(ext{tr}(R) + 1 ight)} \ n = rac{\left[R_{32} - R_{32} ight]}{2\sin\left(ext{tr}(R) + 1 $	$q_w = rac{\sqrt{ ext{tr}(R)+1}}{2} \ egin{bmatrix} q_x \ q_y \ q_z \end{bmatrix} = egin{bmatrix} R_{32} - R_{23} \ R_{13} - R_{31} \ R_{21} - R_{12} \end{bmatrix} / 4q_w$
φ	$R = \exp(\phi^\wedge) = \cos heta I + (1-\cos heta) n n^T + \sin heta n^\wedge$	_	$q_w = \cosrac{ heta}{2} \ [q_x,q_y,q_z]^T = n\sinrac{ heta}{2}$
q	$egin{aligned} p &= [q_x, q_y, q_z]^T \ R &= 2(pp^T + q_w(q_wI + p^\wedge)) - I \end{aligned}$	$egin{aligned} heta &= 2 \arccos q_w \ n &= rac{[q_x, q_y, q_z]^T}{\sin rac{ heta}{2}} \end{aligned}$	_

SE3

$$\mathrm{SE}(3) = \{T = egin{bmatrix} R & t \ 0 & 1 \end{bmatrix} \in \mathbb{R}^{4 imes4} \ | \ R \in \mathrm{SO}(3), t \in \mathbb{R}^3 \}$$

$$T^{-1} = egin{bmatrix} R^T & -R^T t \ 0^T & 1 \end{bmatrix}$$

• Lie Algebra: $\xi = [
ho, \phi]^T \in \mathbb{R}^6$

Epipolar Constraint: $x_2^T E x_1 = p_2^T F p_1 = 0, \quad p = K x$

- ullet Essential Matrix: $E=t^\wedge R$
- Fundamental Matrix: $F = K^{-T}EK^{-1}$
- Homography: $p_2 pprox H p_1$

Lie Algebra

$$\begin{split} \xi^\wedge &= \begin{bmatrix} \phi^\wedge & \rho \\ 0^T & 0 \end{bmatrix} \\ T &= \exp(\xi^\wedge) = \begin{bmatrix} \exp(\phi^\wedge) & J\rho \\ 0^T & 1 \end{bmatrix} \\ J &= \frac{\sin\theta}{\theta} I + (1 - \frac{\sin\theta}{\theta}) n n^T + \frac{1 - \cos\theta}{\theta} n^\wedge \end{split}$$