Non-linear least squares:

$$F(x) = \frac{1}{2}\|f(x)\|^2$$

$$J_f = \frac{df}{dx^T}, \quad J_F = \nabla F^T = \frac{dF}{dx^T} = J_f^T f(x)$$

$$H_F = \frac{d^2F}{dx^T dx} = \frac{dJ_F}{dx}$$

- Gradient descent: $\Delta x = -\alpha J_F^T$
- Newton’s method: $\Delta x = -H_F^{-1} J_F$
- Gauss-Newton’s method: $\Delta x = -(J_f^T J_f)^{-1} J_f^T f(x)$
- Levenberg-Marquardt: $[J_f^T J_f + \lambda I]\Delta x = -J_f^T f(x)$

Outer Product

$$n^\wedge = \begin{bmatrix} 0 & -n_z & n_y \\ n_z & 0 & -n_x \\ -n_y & n_x & 0 \end{bmatrix}$$

$$n_1 \times n_2 = n_1^\wedge n_2$$

$$nn^T = I + n^{\wedge 2}$$

$$n^\wedge n = 0$$

SO3

$\mathrm{SO}(3) = \{R \in \mathbb{R}^{3\times 3} \mid RR^T = I, \det(R) = 1\}$

- Rotation Angle: θ
- Rotation Axis: $\{n = [n_x, n_y, n_z]^T \in \mathbb{R}^3 \mid \|n\| = 1\}$
- Lie Algebra: $\phi = \theta n \in \mathbb{R}^3$
- Quaternion: $q = [q_w, q_x, q_y, q_z]^T$

Lie Algebra

$$\begin{aligned} \exp(\phi^\wedge) &= I & +\sin\theta n^\wedge & \quad +(1-\cos\theta)n^{\wedge 2} \\ J_l(\phi^\wedge) &= I & +\frac{1-\cos\theta}{\theta}n^\wedge & \quad +(1-\frac{\sin\theta}{\theta})n^{\wedge 2} \\ J_r(\phi^\wedge) &= I & -\frac{1-\cos\theta}{\theta}n^\wedge & \quad +(1-\frac{\sin\theta}{\theta})n^{\wedge 2} \end{aligned}$$

$$\begin{aligned} \exp((\phi+\Delta\phi)^\wedge) &\approx \exp((J_l(\phi^\wedge)\Delta\phi)^\wedge)\exp(\phi^\wedge) \\ &\approx \exp(\phi^\wedge)\exp((J_r(\phi^\wedge)\Delta\phi)^\wedge) \end{aligned}$$

Quaternion

Rotate:

$$p' = qpq^{-1}$$

Conjugate:

$$q^* \to R^T$$
$$q^* = \|q\|^2 q^{-1}$$

Multiplication:

$$QQ^T = I$$

$$qr = Qr$$

$$rq = \overline{Q}r$$

$$Q = L(q) = \begin{bmatrix} q_w & -q_x & -q_y & -q_z \\ q_x & q_w & -q_z & q_y \\ q_y & q_z & q_w & -q_x \\ q_z & -q_y & q_x & q_w \end{bmatrix}$$

$$\overline{Q} = R(q) = \begin{bmatrix} q_w & -q_x & -q_y & -q_z \\ q_x & q_w & q_z & -q_y \\ q_y & -q_z & q_w & q_x \\ q_z & q_y & -q_x & q_w \end{bmatrix}$$

Transformation

from \ to	R	ϕ	q
R	$-$	$\theta = \arccos \frac{\sqrt{\mathrm{tr}(R) - 1}}{2}$ $n = \frac{\begin{bmatrix} R_{32} - R_{23} \\ R_{13} - R_{31} \\ R_{21} - R_{12} \end{bmatrix}}{2 \sin \frac{\theta}{2} (\mathrm{tr}(R) + 1)}$	$q_w = \frac{\sqrt{\mathrm{tr}(R) + 1}}{2}$ $\begin{bmatrix} q_x \\ q_y \\ q_z \end{bmatrix} = \begin{bmatrix} R_{32} - R_{23} \\ R_{13} - R_{31} \\ R_{21} - R_{12} \end{bmatrix} / 4q_w$
ϕ	$R = \exp(\phi^\wedge) = \cos\theta I + (1 - \cos\theta) nn^T + \sin\theta n^\wedge$	$-$	$q_w = \cos\frac{\theta}{2}$ $[q_x, q_y, q_z]^T = n \sin\frac{\theta}{2}$
q	$p = [q_x, q_y, q_z]^T$ $R = 2(pp^T + q_w(q_w I + p^\wedge)) - I$	$\theta = 2 \arccos q_w$ $n = \frac{[q_x, q_y, q_z]^T}{\sin\frac{\theta}{2}}$	$-$

SE3

$\mathrm{SE}(3) = \{T = \begin{bmatrix} R & t \\ 0 & 1 \end{bmatrix} \in \mathbb{R}^{4\times 4} \mid R \in \mathrm{SO}(3), t \in \mathbb{R}^3\}$

$$T^{-1} = \begin{bmatrix} R^T & -R^T t \\ 0^T & 1 \end{bmatrix}$$

- Lie Algebra: $\xi = [\rho, \phi]^T \in \mathbb{R}^6$

Epipolar Constraint: $x_2^T E x_1 = p_2^T F p_1 = 0, \quad p = Kx$

- Essential Matrix: $E = t^\wedge R$
- Fundamental Matrix: $F = K^{-T} E K^{-1}$
- Homography: $p_2 \approx H p_1$

Lie Algebra

$$\xi^\wedge = \begin{bmatrix} \phi^\wedge & \rho \\ 0^T & 0 \end{bmatrix}$$

$$T = \exp(\xi^\wedge) = \begin{bmatrix} \exp(\phi^\wedge) & J\rho \\ 0^T & 1 \end{bmatrix}$$

$$J = \frac{\sin\theta}{\theta} I + (1 - \frac{\sin\theta}{\theta}) nn^T + \frac{1 - \cos\theta}{\theta} n^\wedge$$