Date ....

Assigmen I (Solution)

1. $f(0) = -1$, $f(1) = 0.0$, $f(3) = 8.0$ is given.

$\Rightarrow$ $x_0 = 0$; $f_0 = -1$; $x = 1$; $f_1 = 0$

$x_2 = 3$; $f_2 = 8$

$$P_2(x) = \frac{(x-x_1)(x-x_2)f_0}{(x_0-x_1)(x_0-x_2)} + \frac{(x-x_0)(x-x_2)f_1}{(x_1-x_0)(x_1-x_2)}$$

$$+ \frac{(x-x_0)(x-x_1)f_2}{(x_2-x_0)(x_2-x_1)}$$

$$P_2(x) = \frac{(x-1)(x-3)(-1)}{(0-1)(0-3)} + \frac{(x-0)(x-3) \times 0}{(1-0)(1-3)}$$

$$+ \frac{(x-0)(x-1)(8)}{(3-0)(3-1)}$$

$$= \frac{-(x-1)(x-3)}{3} + \frac{8x(x-1)}{6}$$

$$= \frac{-2x^2 - 6 + 8x + 8x^2 - 8x}{6}$$

$$= \frac{6x^2 - 6}{6} = x^2 - 1$$

$$P_2(2) = 2^2 - 1 = 3.0$$

2. $f(1) = 3.0$; $f(0) = 1$, $f(2) = 11.0$ Date ....
$f(-1.0) = -1.0$, $f(4) = 69.0$

x_i	f_i	f_1	f_2	f_3	f_4
1	3	2.	3.0	1.0	0.0
0	1	5	1.0	1.0	
2	11	4	5.0		
-1	-1	14			
4	69				

$a_0 = f(x_0) = f(1) = 3 = f_0$

$a_1 = f(x_0, x_1) = 2.0$; $a_2 = 3.0$; $a_4 = 0.0$

$a_3 = 1.0$

$$P(x) = a_0 + a_1(x - x_0) + a_2(x-x_0)(x-x_1) + a_3(x-x_0)(x-x_1)(x-x_2)$$

$$P(x) = 3 + 2(x-1) + 3(x-1)(x-0) + 1(x-1)(x-0)(x-2)$$

where $x_0 = 1$; $x_1 = 0$; $x_2 = 2$, $x_3 = -1$

$$P(x) = 3 + 2x - 2 + 3x^2 - 3x + x^3 - 3x^2 + 2x$$

$$= 1 + x + x^3$$

Since $f_4 = 0$, this interpolating polynomial is exact (same as the function which created the given data).

3.	x_i	f_0	f_1	f_2
x_0	0	1	1.718	1.4765
x_1	1	2.718	4.671	
x_2	2	7.389		

$\Rightarrow a_0 = 1$; $a_1 = 1.718$; $a_2 = 1.4765$

$P_2(x) = a_0 + a_1(x - x_0) + a_2(x - x_0)(x - x_1)$

$= 1 + 1.718(x - 0) + 1.4765\, x(x-1)$

$= 1.4765x^2 + 1 + 0.2415x$

$P_2(x = 0.15) = 3.3221 + 1 + .36225$

$= 4.6844$

The actual function is exp(x)

exp(1.5) = 4.4817

% error $= \dfrac{4.6844 - 4.4817}{4.6844} \times 100$

error = 4.6844 − 4.4817 = .2027

The actual value and interpolated value are always difference. Here we are using too few points to determine the interpolating polynomial. hence larger the error.

Assignment 2.

Date ....

1. $\frac{dy}{dx} = x + y \quad y(0) = 1 ; h = 0.1 ; y(0.3) =$

$f(x,y) = x + y . \quad ; \quad y_{i+1} = y_i + f(x_i, y_i)h$

x_n	y_n	y_n'	hy_n'
0	1.0	1.0	0.1
0.1	1.1	1.2	.12
0.2	1.22	1.42	.142
0.3	1.362		

$x_{i+1} = x_i + h$

2.

x_n	y_n	y_n'	hy_n'
0	1.0	1.0	0.15
0.15	1.15	1.3	0.195
.30	1.345		

$(h = 0.15)$

$x_{i+1} = x_i + h$

The predicted values are different because we have chosen a different step size. The value 1.362 is more accurate because the step size is smaller for that calculation.

3. $\frac{dy}{dx} = x+y$; $y(0)=1$; $h=0.1$

$f(x,y) = x+y$ $\quad y_{i+1,p} = y_i + h \cdot f(x_i, y_i)$

$y_{i+1} = y_i + h(y'_i + y'_{i+1})/2$

x_i	y_i	hy'_i	$y_{i+1,p}$	$hy'_{i+1,p}$	hy'_{av}	$y_{i+1,c}$
0	1	0.1	1.1	0.12	.11	1.11
0.1	1.11	.121	1.231	.1431	.132	1.242
0.2	1.242	.1442	1.3862	0.169	.1566	1.396
0.3	1.397 (three decimal) accuracy.					

$x_0 = 0$; $y_0 = 1$ are given $\quad y_{1p} = 1 + 0.1(0+1) = 1.1$

$y'_{1p} = f(0.1, y_{1p}) = 0.1 + 1.1 = 1.2$

$hy'_{av} = 0.1 + .12/2 = .11$

$y_{1,c} = 1 + .11 = 1.11$

$x_1 = 0 + .1 = .1$ $\quad y_1 = 1.11$ $\quad y_{2p} = 1.11 + 0.1(.1 + 1.11)$

$y_{2p} = 1.231$; $hy'_{2p} = 0.1(0.2 + 1.231) = .1431$

$hy'_{av} = (.1431 + .121)/2 = .132$

$y_{2c} = 1.11 + .132 = 1.242$

$x_2 = 0.2$; $y_2 = 1.242$ $\quad y_{3p} = 1.242 + 0.1(.2 + 1.242)$

$y_{3p} = 1.242 + .1442 = 1.3862$

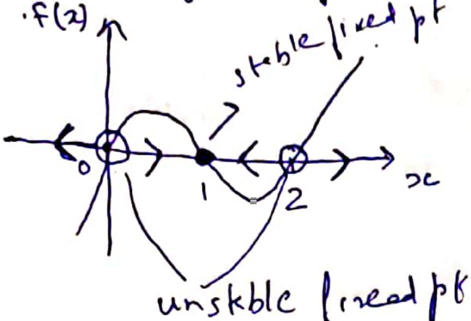
$hy'_{3p} = 0.1 f(x_3, y_{3p}) = 0.1(0.3 + 1.3862) = .1686$

$hy'_{3p} = .169$ (rounding off)

Solution Assignment 3

1. $\frac{dx}{dt} = x(1-x)(2-x) \Rightarrow f(x) = x(1-x)(2-x)$; $f(x^*) = 0$

x^* = fixed pt's. $x^* = 0, 1, 2$



$f'(x) = -3x^2 - 6x + 2$

$f'(0) = 2 > 0$ so $x^* = 0$ unstable

$f'(1) = -7 < 0$ so $x^* = 1$ stable

$f'(2) = 2 > 0$ so $x^* = 2$ unstable

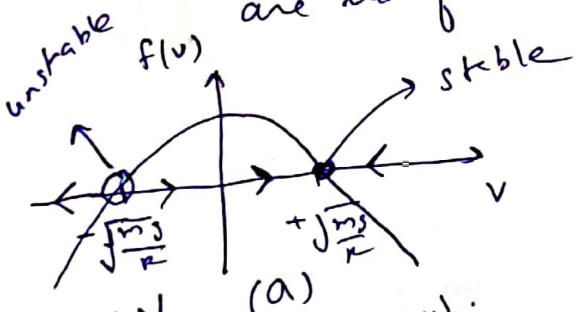
2. $V(t \to \infty) \equiv$ terminal velocity $\equiv V_T$ $(\dot{V} = 0)$

$m\dot{V} = mg - kV^2$

$mg = kV_T^2 \Rightarrow V_T = \sqrt{mg/k}$

$f(v) = mg - kv^2 = 0 \Rightarrow V = +\sqrt{\frac{mg}{k}}$ and $-\sqrt{\frac{mg}{k}}$

are the fixed pt's. graphical $\to$ stable, unstable



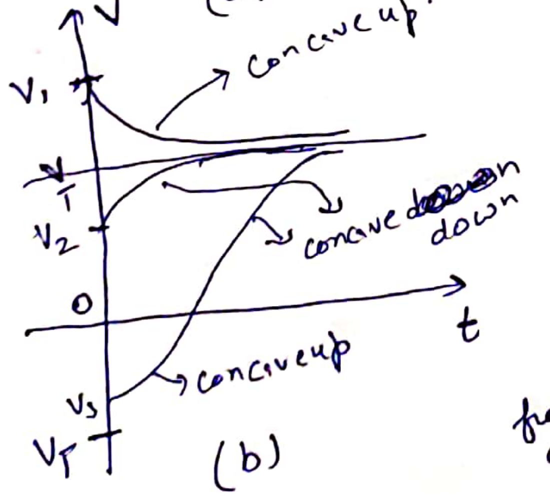
$f'(V) = -2kV$

$f'\left(\sqrt{\frac{mg}{k}}\right) = -2k\sqrt{\frac{mg}{k}} < 0$

so $V = \sqrt{\frac{mg}{k}}$ stable

$f'\left(-\sqrt{\frac{mg}{k}}\right) = +2k\sqrt{\frac{mg}{k}} > 0$

$V = \sqrt{\frac{mg}{k}}$ is unstable



$0 < V_2 < V_T$: $\dot{V}$ is decreasing from the start so the velocity curve is concave down and $V \to V_T$ as $t \to \infty$

$V_3 < 0$: According to fig (a) the velocity is increasing till $V = 0$ (accelerating) and starts to decelerate. So the velocity initially grows in a accelerating fashion (concave up) till it reaches zero

After $v=0$, the $\dot{v}$ is decelerating so the velocity grows like concave down and approaches v_T as $t\to\infty$.

$v > v_T$: the solution of v will again go towards v_T as $t\to\infty$. The v curve is a concave up decreases towards v_T manner.

8. Refer to pg 13 of the nonlinear dynamics ppt to understand how to draw the graphs.

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Solution Assignment 4.

$$I = \int_a^b f(x)\,dx = \int_a^b \left(\frac{f(x)}{p(x)}\right) p(x)\,dx$$

Generate random no's (M) according to probability distribution $p(x)$.

$$I = \frac{1}{M}\left(\sum_{i=1}^{M} \frac{f(x_i)}{p(x_i)}\right)$$

Given : $M = 10{,}000$; $p(x) = \exp(-x)$; $a = 0$; $b = 5$.

10,000 random no's in rn.txt according to the prob. distribution $p(x) = \exp(-x)$. Assuming Normalization was done. $f(x) = \exp(-x^2/2)$

$I = 0$; $M = 10{,}000$; $f(x) = \exp\left(-\frac{x^2}{2}\right)$; $p(x) = c\exp(-x)$

c ↑ normalization constant.

For $i = 1, M$

Read (rn.txt) x

$I = I + \dfrac{f(x)}{p(x)}$

end for

$I = I/M$

Read pg 19 of ppt Monte Carlo Integration.