Solution Tut 9.

1. $\dot{y} = x + y + xy$; $y(0) = 1$ $h = .01$ $y(.03) = ?$

$y(0.1) =$ $y_1 = y_0 + h f(x_0 y_0) = 1 + .01 (0 + 1 + 0) = 1.01$

$y(0.2)$ $y_2 = y_1 + h f(x_1 y_1) = 1.01 + .01 (.01 + 1.01 + 1.01 \times .01)$

$= 1.02$

$y(0.3) = y_3 = y_2 + h f(x_2 y_2) = 1.02 + .01 (.02 + 1.02 + .02 \times 1.02)$

$= 1.031$

2. $\frac{dy}{dx} = -2xy^2$; $y(0) = 1$ $h = .2$ $y(0.6) =$

$h = 0.2$

$\dot{y} = -2xy^2$ $y(0) = 1.0$

$y_{i+1} = y_i + \frac{h}{2} (y_i' + y_{i+1}')$

$y_{i+1} = y_i + \frac{h}{2} (f(x_i, y_i) + f(x_{i+1}, y_{i+1}))$

$y_{i+1} = y_i + \frac{h}{2} (f_i + f(x_{i+1}, \underbrace{y_i + h f_i}_{y_i^p}))$

$y_1^p = y_0 + h y_0' = 1 + h \times 0 = 1$ $y_0' = f(0, 1) = 0$

$f(x_1, y_1) = f(.2, 1) = -2 \times .2 \times (1)^2 = -.4$

$y(0.2) = y_1 = y_0 + \frac{h}{2} (f(x_0, y_0) + f(x_1, y_1^p))$

$y_1 = 1 + \frac{.2}{2} (0 + (-.4)) = .96$

Similarly find $y(.4)$ and $y(.6)$

3. $x = 0$ Step = +1 or −1

iseed = −23417 $N \equiv$ no. of steps

```
Do i = 1, N
c = ran2(iseed)
If (c .le. 0.5) then
x = x + 1
else
x = x − 1
endif.
enddo.
Print *, x, 'distance covered.'
```

For different probabilities ($p_{forward} = .6$) ?

4. Use the algo in question 3 as a subroutine.

dist (x, iseed, N).

```
iseed = −23171
Xav = 0.0; Xsqav = 0.0
Do k = 1, n
Call dist (x, iseed, N)
X(k) = x.
Xav = Xav + x.
Xsqav = Xsqav + x^2
enddo.
Xav = Xav / n.
Xsqav = Xsqav / n.
6 = (Xsqav − Xav^2) ** 1/2.
end
```

$X(k) \equiv$ distance travelled in k^{th} walk.

$$\bar{X} = \frac{1}{n} \sum_{k=1}^{N} X(k)$$

$$\overline{X^2} = \frac{1}{n} \sum_{k=1}^{N} X^2(k)$$

$$6 = (\overline{X^2} - \bar{X}^2)^{1/2}$$

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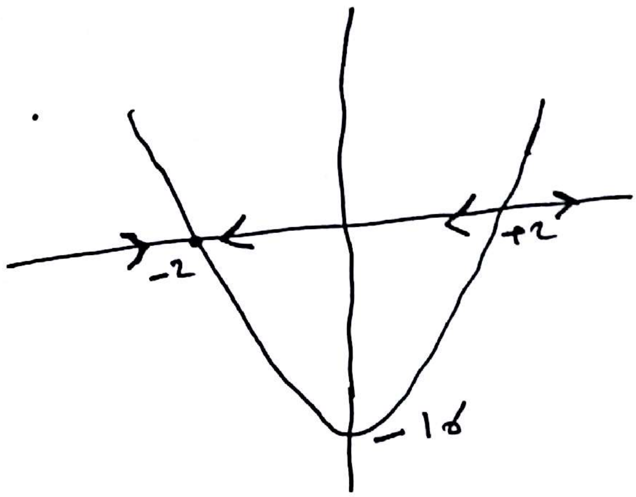
5) $y' = y$ is linear and $y' = \sin y$ is non linear

6). $\frac{dy}{dx} = 4(x^2 - 4)$.



R.H.S = 0 at $x = \pm 2$.

So $x = -2$ is stable fixed pt and $x = +2$ is unstable fixed pt

Stability analysis :

$f'(x^*) > 0$; x^* is unstable fixed pt

$f'(x^*) < 0$; x^* is Stable fixed pt.

$f'(x) = 8x$.

$f'(x=2) = 16 > 0 \longrightarrow$ unstable

$f'(x=-2) = -16 < 0 \longrightarrow$ stable.

7. The logistic equation is $\dot{N} = rN\left[1 - \frac{N}{K}\right]$

Fixed pt $\equiv$ $N^* = K$, $N^* = 0$

$N = 0$ is the unstable fixed pt and $N = K$ is the stable fixed pt.

$N(t = \infty) \cong K$.

8. Refer to Notes