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Marine Structures 12 (1999) 425–445
MARINE STRUCTURES
www.elsevier.com/locate/marstruc

Reliability of maintained, corrosion protected plates subjected to non-linear corrosion and compressive loads

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Received 19 June 1998; received in revised form 1 May 1999; accepted 1 July 1999

Abstract

A model is proposed for the non-linear general corrosion wastage in a plate, which is able to describe an initial period without corrosion due to the presence of a corrosion protection system, and an exponential increase in wastage up to a steady-state value. The influence of adopting this model instead of a linear one is demonstrated by studying the reliability of a corrosion-protected plate subjected to compressive loads, and maintenance actions. The paper examines how the plate collapse strength will vary in time as a result of generalised corrosion and plate repair. In the presence of general corrosion, the plate thickness will decrease at a random rate as a function of time and will affect the net plate section resisting the applied load. The collapse strength against compressive loading is described by a function of time and the resulting reliability is quantified. The sensitivity of the reliability estimates with respect to several parameters is also studied. The reliability is predicted by a time-variant formulation and the effects of repair actions in the reliability assessment are shown. © 1999 Elsevier Science Ltd. All rights reserved.

1. Introduction

Plates are the main structural components of several important metal structures such as ships and box girder bridges. Their behaviour under the effect of compressive loads is particularly important because the failure is generally in an unstable mode, which has harmful consequences from the point of view of safety.

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PII: S0951-8339(99)00028-3

Guedes Soares [1], has reviewed several approaches to assess the variability of the predictions of the compressive strength of plate elements and the reliability of structures made with these components, which will not be repeated here. However, an interesting problem not considered in that survey is the change of the compressive strength due to corrosion, which is a potential problem in all metal structures. This problem has attracted the interest of several researchers such as Hart et al. [2], White and Ayyub [3], and Shi [4].

Ship structures operate in a complex environment. Water properties such as salinity, temperature, oxygen content, pH level and chemical composition can vary according to location and water depth. Also the inside face of plates will be exposed to aggressive environments existing in cargo tanks. The structures are often protected, either with paints or with cathodic systems that deliver a current intensity to the protected metal surface inhibiting the corrosion process.

Two main corrosion mechanisms are generally present in steel plates. One is a general wastage that is reflected in a generalised decrease of plate thickness. Another mechanism is pitting which consists of much localised corrosion with very deep holes appearing in the plate. In fact, pitting can lead to leakage but in general, because it is much localised, it does not affect the in-plane stress distribution in plate. Therefore, pitting is not accounted for in this paper and general corrosion is modelled as a monotonic decrease in plate thickness.

Guedes Soares [5], has studied the effect of general corrosion in decreasing the thickness of the plating and considered that repair actions were performed whenever the plate thickness would decrease below a defined limit value. It was shown that in the steady-state situation with frequent inspections the expected value of plate thickness depends on the repair criteria and is independent of the corrosion rate.

The steady-state situation only occurs whenever there are several plate replacements during the ship's life and thus at a random point in time, the plates in the area considered have a thickness that is governed by the replacement criteria rather than by the corrosion rate. Although this situation may occur in some special areas of some ship types, the most common situation is for plating to be replaced only once during the ship's life if at all. Therefore, they will never reach the so-called steady-state situation and a model is required to describe the transient situation of wastage increase until plate replacement.

In many situations, the expected value of the reliability at a random point in time is not enough and information about the time variation of reliability is desired. To achieve this objective, a model has been proposed by Guedes Soares and Garbatov [6], to describe the time-dependent reliability of a ship hull in which the plates are subjected to corrosion and repair actions. However, the effect of corrosion was represented by an uncertain but constant corrosion rate, which resulted in a linear decrease of plate thickness with time. The same corrosion model was adopted in Guedes Soares and Garbatov [7], to study the reliability of a plate element under compressive forces.

The present paper improves those corrosion models in which the corrosion rate is constant and independent of time. A non-linear function of time that describes the growth of corrosion in three different phases is proposed here and the influence of different parameters are examined. In the first phase, it is assumed that there is no corrosion because the corrosion protection system is effective. Failure of the protection system will occur at a random point of time and the corrosion wastage will start a non-linear growing process with time. The formulation presented here uses the asymptotic value of corrosion wastage as input for creating an exponential function of time that describes the effect of corrosion and thus can be applied for the reliability assessment of different plate elements.

The main contribution of this paper is the model of corrosion wastage, which is flexible enough to represent realistic situations. No effort was made here to establish which are the realistic corrosion rates for ship plates as a function of different environmental conditions. Other authors have considered this problem related to different areas of the ship hull [8] as well as of different types of ships [9], depending on the ocean area and steel type [10,11] and as an input for optimal inspection strategies [12,13].

2. Corrosion model

The conventional models of corrosion assume a constant corrosion rate, leading to a linear relationship between the material lost and time. Experimental evidence of corrosion reported by various authors shows that a non-linear model is more appropriate.

Southwell et al. [14], has observed that the wastage thickness increases non-linearly in a period of 2–5 yr of exposure, but afterwards it becomes relatively constant. This means that after a period of initial non-linear corrosion, the oxidised material that is produced remains on the surface of the plate and does not allow the continued contact of the plate surface with the corrosive environment, stopping corrosion. They proposed a linear (Eq. (1)) and a bilinear (Eq. (2)) model, which were considered appropriate for design purposes

$$d = 0.076 + 0.038t, \tag{1}$$

$$d = \begin{cases} 0.090t & 0.00 \leqslant t < 1.46, \\ 0.076 + 0.038t & 1.46 \leqslant t < 16.00. \end{cases} \tag{2}$$

Both models are conservative in the early stages in that they overestimate the corrosion depth, which might occur at the initial phases of the corrosion process.

Melchers and Ahammed [15], suggested a steady-state model for corrosion wastage thickness, which is given by

$$d = \begin{cases} 0.170t & 0 \leqslant t < 1, \\ 0.152 + 0.0186t & 1 \leqslant t < 8, \\ -0.364 + 0.083t & 8 \leqslant t \leqslant 16 \end{cases} \tag{3}$$

and they also proposed a power approximation for the corrosion depth given as

$$d = 0.1207t^{0.6257}. \tag{4}$$

Yamamoto [16], has presented results of the analysis of corrosion wastage in different location of many ships, exhibiting the non-linear dependence of time and a tendency of levelling-off.

The reference to these earlier works shows that the non-linear time dependence of corrosion rate has been already identified experimentally. The model proposed here in addition to being a more flexible alternative to the previous ones also generalise the concept by including an early phase with corrosion protected surface. In fact, the model proposed has free parameters to be adjusted to the data of specific situations.

The time-dependent model of corrosion degradation may be separated into three phases. In the first one there is in fact no corrosion because the protection of the metal surface works properly. The first stage depends on many factors and statistics show that in ships it varies in the range of 1.5–5.5 yr, ([17] $t \in [O', O]$ in Fig. 1). The second phase is initiated when the corrosion protection is damaged and corresponds really to the existence of corrosion, which decreases the thickness of the plate, ($t \in (O, B]$ in Fig. 1). This process was observed to last a period around 4–5 yr in typical ship plating [11]. The third phase corresponds to a stop in the corrosion process and the corrosion rate becomes zero ($t > B$ in Fig. 1). Corroded material stays on the plate surface, protecting it from the contact with the corrosive environment and the corrosion process stops. Cleaning the surface or any involuntary action that removes that surface material originates the new start of the non-linear corrosion growth process.

The model proposed here can be described by the solution of a differential equation of the corrosion wastage

$$d_\infty \dot{d}(t) + d(t) = d_\infty, \tag{5}$$

where d_∞ is the long-term thickness of the corrosion wastage, $d(t)$ is the thickness of the corrosion wastage at time t, and $\dot{d}(t)$ is the corrosion rate.

The solution of Eq. (5) can have the general form

$$d(t) = d_\infty (1 - \mathrm{e}^{-t/\tau_t}) \tag{6}$$

and the particular solution leads to

$$d(t) = d_\infty (1 - \mathrm{e}^{-(t-\tau_c)/\tau_t}), \quad t > \tau_c.$$

$$d(t) = 0, \quad t \leqslant \tau_c, \tag{7}$$

where τ_c is the coating life, which is equal to the time interval between the painting of the surface and the time when its effectiveness is lost, and τ_t is the transition time, which may be calculated as

$$\tau_t = \frac{d_\infty}{tg\alpha}, \tag{8}$$

where α is the angle defined by OA and OB in Fig. 1.

The proposed model is flexible and can be fit to any specific situation, once the long-term corrosion wastage and the duration of the corrosion process is known.

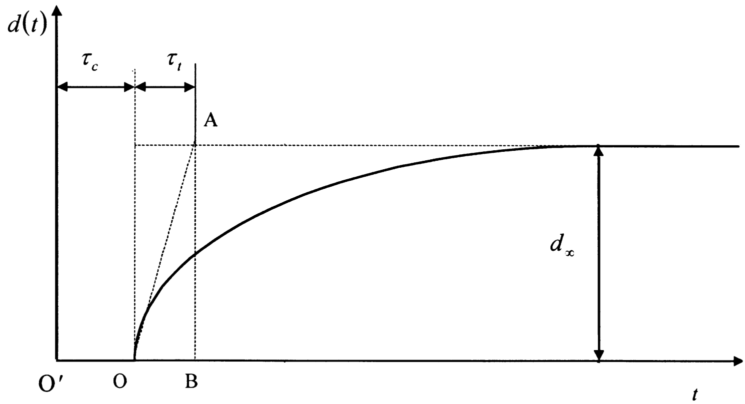

Fig. 1. Thickness of corrosion wastage as a function of time.

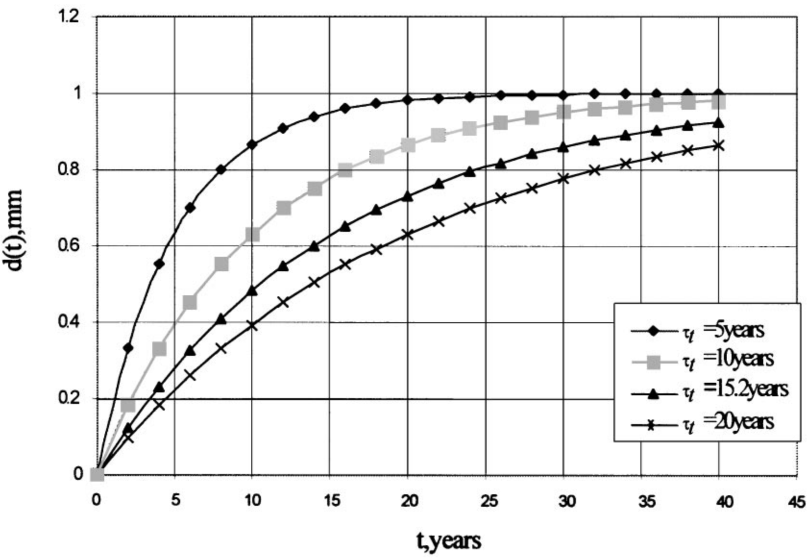

Fig. 2. Influence of the transition time on the non-linear model.

The influence of the pattern of the growth of wastage thickness for various values of the parameters τ_t and d_∞ is shown in Figs. 2 and 3.

For the demonstration of the flexibility of the present model a survey of corrosion data on longitudinal member of bulk carriers presented by Paik et al. [18], is used. It should be noted that some of the data was obtained by measurements of repaired elements. The data has been transformed into piecewise time averages, which were fit to the present model by estimating the model parameters with a least-squares approach.

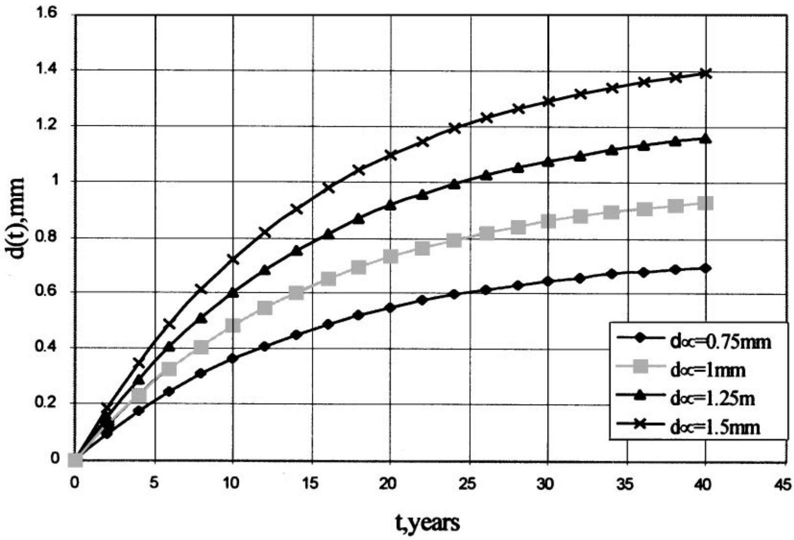

Fig. 3. Influence of the long-term corrosion depth on the non-linear model.

The parameters of the model (Eq. (7)) evaluated for the data presented in Fig. 4 are $d_\infty = 1.6$ mm and $\tau_t = 4.5$ yr and for the data presented in Fig. 5 they are $d_\infty = 1$ mm and $\tau_t = 3.6$ yr.

The average corrosion rate corresponds to the data presented in Figs. 4 and 5 is 0.15 and 0.09 mm per year, respectively. The average corrosion rate is larger for the inner bottom than for the side shell.

There is some variability of the data of Figs. 4 and 5 around the regressed line. The adequacy of the models was judged by the R^2 coefficient and it was observed that R^2 is significantly larger for the non-linear approximation than for the linear one.

However, Figs. 4 and 5 show a clear non-linear trend of the type that is reproduced by the proposed model as shown in Figs. 2 and 3. This trend is also similar to the one presented by Yamamoto [16].

The relative corrosion depth as a function of time under the assumption that corrosion thickness is approximated as a linear and an exponential function is presented in Fig. 6. The linear approximation of corrosion depth is taken as

$$d(t) = 0.12t, \text{ mm} \tag{9}$$

and the exponential approximation as

$$d(t) = 5(1 - e^{-t/15.2}), \text{ mm} \tag{10}$$

where $d_\infty = 5$ mm, $\tau_t = 15.2$ yr. After 38 yr life of the plate element both approximations lead to a reduction of the initial thickness by 5 mm (see Fig. 6).

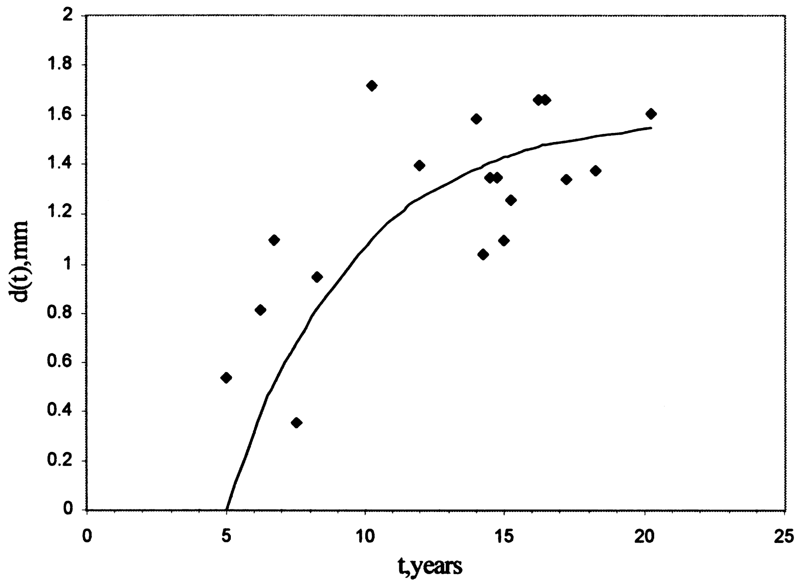

Fig. 4. Time-dependent corrosion wastage for inner bottom plates of bulk carriers.

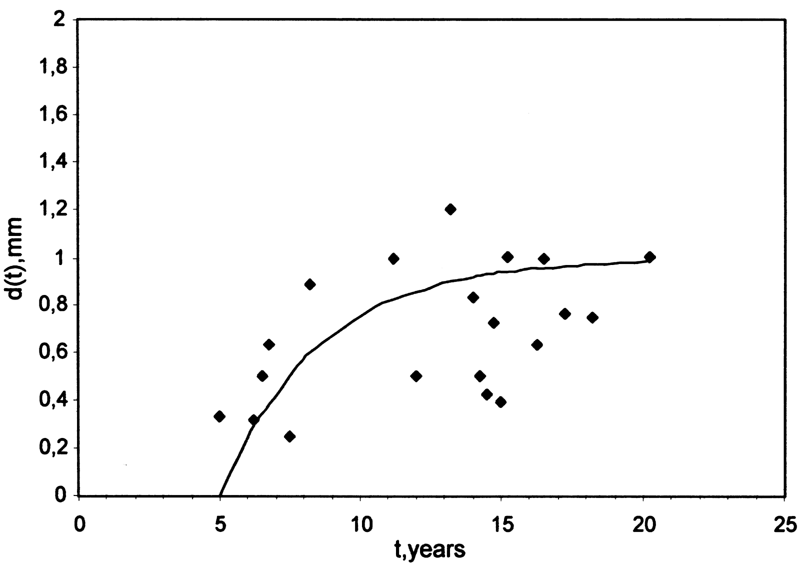

Fig. 5. Time-dependent corrosion wastage for side shells of bulk carriers.

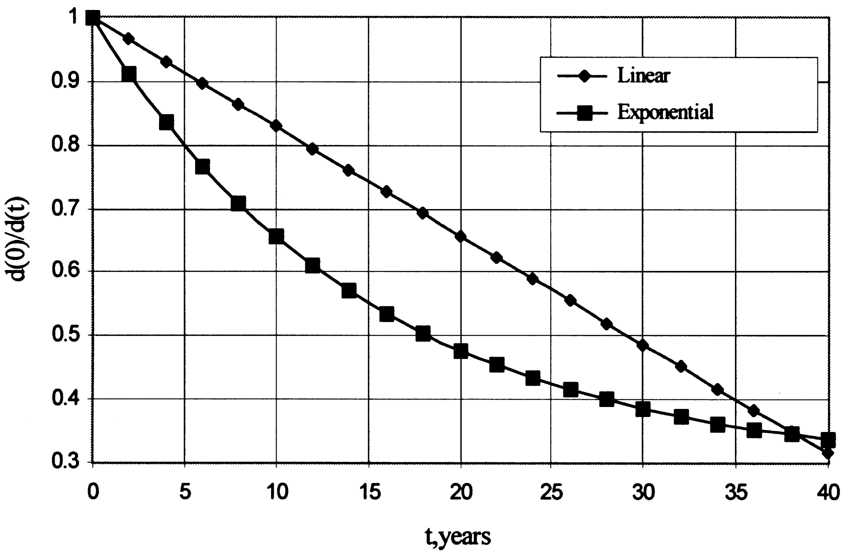

Fig. 6. Corrosion depth approximation as a function of time.

3. Compressive strength of plates under uniaxial compression

The most important parameter that governs the compressive strength of plate elements is the slenderness

$$\lambda = \frac{b}{h}\sqrt{\frac{\sigma_y}{E}}, \tag{11}$$

where b and h are the plate breadth and thickness, respectively, σ_y is the yield stress and E is the Young's modulus of the material. This parameter is included in the classical formula due to Bryan for the critical elastic buckling stress σ_{cr} of infinitely long thin elastic plate with simply supported edges

$$\frac{\sigma_{cr}}{\sigma_y} = \frac{4\pi^2}{12(1-v^2)}\frac{1}{\lambda^2}, \tag{12}$$

where v is the Poisson ratio.

This expression was extended by Faulkner [19], adding one extra term and fitting it to data of ultimate plate strength leading to

$$\frac{\sigma_u}{\sigma_y} \equiv \phi_b = \frac{a_1}{\lambda} - \frac{a_2}{\lambda^2}, \quad \lambda \geqslant 1.0, \tag{13}$$

where the constants a_1 and a_2 are given $a_1 = 2.0$ and $a_2 = 1.0$ for simple supports and $a_1 = 2.5$ and $a_2 = 1.56$ for clamped supports. This equation accounts implicitly for average levels of initial deflection and it can be complemented with others that dealt explicitly with the effect of residual stresses.

Guedes Soares [20], has extended that formulation by deriving a strength assessment expression for the compressive strength of plate elements under uniaxial load, which deals explicitly with initial defects as

$$\phi = (\phi_b B_b)(R_r B_r)(R_\delta B_{r\delta}), \tag{14a}$$

where ϕ_b is given by Eq. (13), B_b, B_r and $B_{r\delta}$ are model uncertainties factors and R_r and R_δ are strength reduction factors which are due to the presence of weld-induced residual stresses and initial distortions, respectively. These expressions are

$$B_b = 1.08, \tag{14b}$$

$$R_r = 1 - \frac{\Delta\phi_b}{1.08\phi_b}, \tag{14c}$$

$$B_r = 1.07, \tag{14d}$$

$$R_\delta = 1 - (0.626 - 0.121\lambda)\frac{\delta_o}{h}, \tag{14e}$$

$$B_{r\delta} = 0.76 + 0.01\eta + 0.24\frac{\delta_o}{h} + 0.1\lambda, \tag{14f}$$

where the strength reduction due to the residual stress σ_r is

$$\Delta\phi_b = \frac{\sigma_r}{\sigma_y}\frac{E_t}{E}. \tag{14g}$$

The residual stress depends on the width ηh of the two strips of tensile yield stress at the edges of the plate

$$\frac{\sigma_r}{\sigma_y} = \frac{2\eta}{b/h - 2\eta}. \tag{15}$$

The tangent modulus of elasticity E_t accounts for the development of plasticity. This modulus can be approximated by the expression which was used by Guedes Soares and Faulkner [21].

$$\frac{E_t}{E} = \frac{\lambda - 1}{1.5}, \quad \text{for} \quad 1 \leqslant \lambda \leqslant 2.5, \tag{16}$$

$$\frac{E_t}{E} = 1, \quad \text{for} \quad \lambda \geqslant 2.5. \tag{17}$$

This formulation was used for the reliability assessment of plates by Guedes Soares and Silva [22], who compared the results with the ones of using other formulations. Guedes Soares and Kmiecik [23], have shown that assessing the plate strength with a more accurate non-linear finite element code would lead to the same general type of results although with different numerical values. Thus, the use of the present formulation in the following analysis can be justified in view of its analytical character.

If should be noticed that expressions resulting from the same type of approach have been derived for plates subjected to transverse [24] and to biaxial loading [25] and thus the approach presented here can also be generalised to those cases.

4. Failure criteria

Applying the exponential approximation for corrosion depth allows two-failure criteria for a plate element to be formulated. Failure is considered to be caused by reaching a specified value of thickness reduction or by the plate ultimate strength. Having a reduction of the original thickness does not mean that the plate will reach the level of ultimate strength and the opposite is also true.

The ultimate strength does depend not only of the thickness but also of many other factors. It is assumed that these two failure modes are independent. Using the limiting thickness as an additional failure criteria accounts for the fact that in addition to ultimate strength there may exist other design or operational considerations that need to be taken care of. The relative ultimate strength $P_{U_r}(t)$ and the relative corrosion depth $C_r(t)$ may be described as (see Fig. 7)

$$P_{U_r}(t) = \frac{P_u(t) - P_T}{P_u(0) - P_T},\ P_{U_r}(t) \in [1, 0], \tag{18}$$

$$C_r(t) = 1 - \frac{A(t)}{A(0)},\ C_r(t) \in [0, 1], \tag{19}$$

where $P_u(t)$ is the ultimate collapse force as a function of time, P_T is the total axial loading, $A(t)$ is the plate cross-sectional area as a function of time.

The relative ultimate strength as a function of time under the assumption that corrosion thickness is approximated as a linear and an exponential function, respectively, is presented in Fig. 7.

The cross-section area is presented as $A(t) = b\,h(t)$ and after transformations, the cross-section area and ultimate strength as a function of a relative corrosion depth are given by

$$A(t) = A(0) - b\,d(t) \equiv A(0) - A(0)C_r(t), \tag{20}$$

$$P_u(t) = \phi(t)\sigma_y A(0) - \phi(t)\sigma_y A(0)C_r(t), \tag{21}$$

where $\phi(t)$ is given by Eq. (14a)

The safety domain described in terms of the relative ultimate strength and the relative corrosion depth may be written

$$\begin{aligned} &\frac{P_{U_r}(t)}{(\phi(t)\sigma_y A(0) - P_T/P_u(t) - P_T)} + C_r(t) = 1. \\ &C_r(t) \leqslant k_c \end{aligned} \tag{22}$$

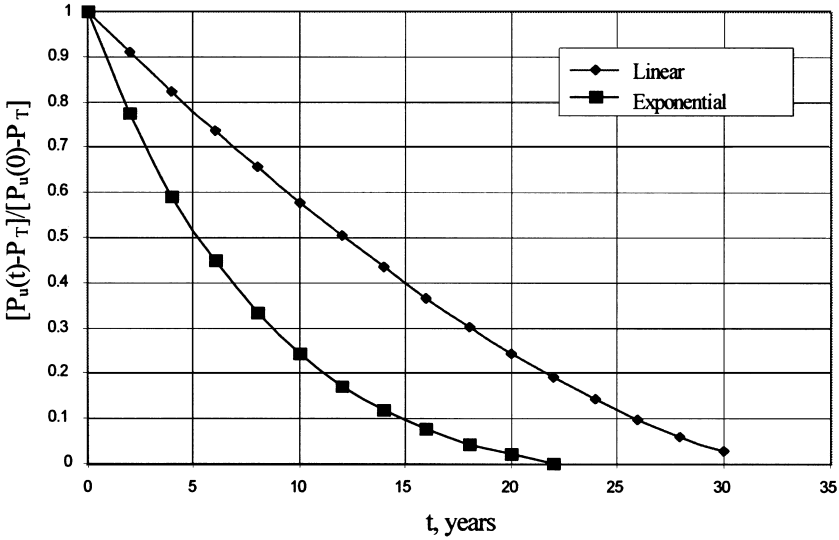

Fig. 7. Relative ultimate strength as a function of time.

Introducing the coefficients $k_\phi(t) = \phi(t)/\phi(0)$ and $k_l = P_u(0) - P_T/P_u(0)$ Eq. (22) can be rewritten as

$$\frac{P_{U_r}(t)}{(k_\phi k_l P_u(0) - (1 - k_\phi)P_T/P_u(t) - P_T)} + C_r(t) = 1, \qquad C_r(t) \leqslant k_c \tag{23}$$

where k_c is a corrosion coefficient and k_l measures the relative initial reserve strength.

Finally, for the relative ultimate strength an expression may be derived

$$P_{U_r}(t) = k_u(t)[1 - C_r(t)], \quad C_r(t) \leqslant k_c, \tag{24}$$

where $k_u = k_\phi k_l P_u(0) - (1 - k_\phi)P_T/P_u(t) - P_T$.

The graphical description of the two-failure criteria is presented in Fig. 8 for various coefficients of loading (k_l). The figure shows that with increasing loading, which is reflected in decreasing k_l, the area of the safety domain is decreased. The allowable space can also be governed by the corrosion coefficient k_c, which here is taken equal of 0.25 and it is shown as a straight line parallel to the ordinate axis.

5. Reliability of a plate element

Reliability assessment requires the modelling of random variables and the assessment of their statistical properties. A variable that governs the resistance of the plate to buckling collapse is its slenderness, which is dependent on the thickness. The

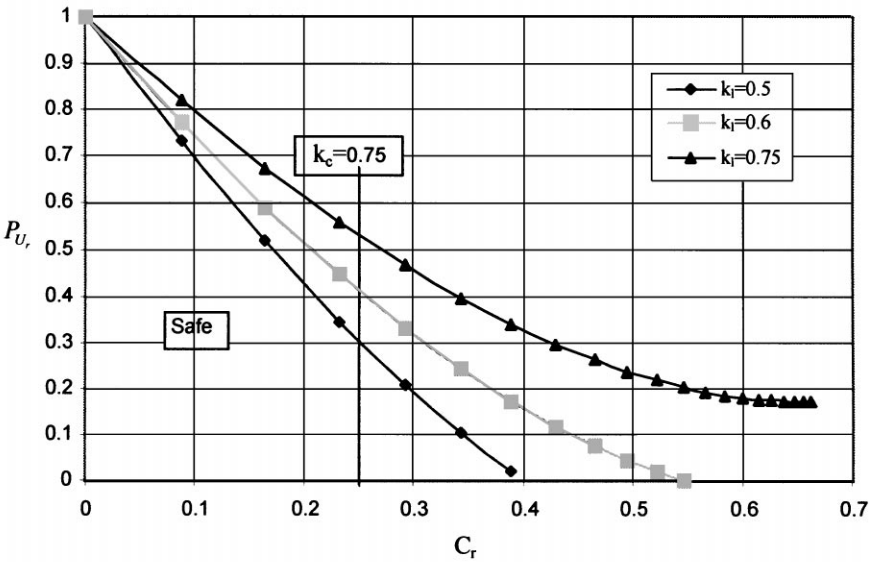

Fig. 8. Two-failure criteria (corrosion and ultimate strength).

thickness also influences other collapse mechanisms and may itself be used as a design variable. It is considered that general corrosion will occur, affecting plate thickness and localised pit corrosion will not be accounted for since its effect on plate collapse is not meaningful.

Guedes Soares [5], showed that the plate collapse is insensitive to the uncertainties in plate dimensions except for the thickness, which has some influence. In the presence of corrosion, the uncertainty in plate thickness will depend mainly on that effect and this can be modelled by a random variable that dominates the uncertainty in plate thickness resulting from the manufacturing process.

In fact, typical values of the coefficient of variation of the thickness of new plate are of the order of 4% and this will be dominated by effect of the uncertainty in the corrosion rates, which can have coefficients of variation in a range of 20–50%.

For a plate element, the cross-sectional area A is given by the product of its breadth b by thickness h. The plate thickness starts from an initial value h_o and decreases with time by a corrosion reduction $d(t)$:

$$h(t) = h_o - d(t). \tag{25}$$

According to Eq. (13) the ultimate compressive force (P_u), which is a function of time, may be written

$$P_u(h_o, b, \sigma_y, E, d_\infty, t) = \phi(h_o, b, \sigma_y, E, d_\infty, t)\sigma_y A(t), \tag{26}$$

which is a product of the ultimate strength by the yield stress of the material and the net section area. For simplification of numerical aspects, this formulation does not consider the effect of initial distortion and residual stresses described in Eq. (14).

In many cases, plates are protected with anti-corrosion paints, which are effective during a limited period of time. This implies that corrosion will only start at the random point in time in which the protection ceases to be effective. Therefore, the effect of corrosion just described is conditional on the initiation of the corrosion process.

The limit state for buckling collapse failure is defined as

$$P_T = P_u(t) = \varsigma(t), \tag{27}$$

where P_T is the total axial loading, $P_u(t)$ is the ultimate collapse force, which has the threshold limit $\varsigma(t)$.

There will be a failure if Eq. (27) is fulfilled and the probability of the load exceeding $\varsigma(t)$ during the period of the time $[0, T]$ is

$$P_f[T] = 1 - \exp\left[-\int_o^T v[\varsigma(t)]\,\mathrm{d}t\right], \tag{28}$$

where $v[\varsigma(t)]$ is the mean upcrossing rate of the threshold $\varsigma(t)$.

If one assumes that the plate studied is an element of a deck of a ship, it will be loaded by wave-induced compressive loads which are often assumed to follow the Weibull distribution and in this case the upcrossing rate is [26]:

$$v[\varsigma(t)] = v_o \exp\left[-\left(\frac{\varsigma(t) - \bar{P}_T}{\gamma_L}\right)^{\alpha_L}\right], \tag{29}$$

where α_L and γ_L are the Weibull parameters and $\bar{P}_T$ is the mean value of the total compressive force.

The scale parameter γ_L is determined from the shape parameter α_L and the force P_{n_o}, determined as the n_0^{-1} probability level

$$\gamma_L = \frac{P_{n_o}}{(Ln(n_o))^{1/\alpha_L}}. \tag{30}$$

Considering that during the plate lifetime $h_o, b, \sigma_y, E, d_\infty$ and P_T are random variables and also that the threshold limit can be described by Eq. (29), the upcrossing rate can be calculated by

$$v(t|h_o, b, \sigma_y, E, d, P_T) = \int_0^t v_o \exp\left[-\left(\frac{\varsigma(\tau|h_o, b, \sigma_y, E, d_\infty) - \bar{P}_T}{\gamma_L}\right)^{\alpha_L}\right]\mathrm{d}\tau. \tag{31}$$

The probability of failure $P_f(t)$ may be obtained by unconditioning

$$P_f(t) = 1 - \int_0^\infty\int_0^\infty\int_0^\infty\int_0^\infty\int_0^\infty\int_0^\infty f_{\bar{P}_T}(\bar{P}_T) f_{d_\infty}(d_\infty) f_E(E) f_{\sigma_y}(\sigma_y) f_b(b) f_{h_o}(h_o)$$
$$\exp[-v(t|h_o, b, \sigma_y, E, d_\infty, \bar{P}_T)]\, d(h_o)\, d(b)\, d(\sigma_y)\, d(E)\, d(d_\infty)\, d(\bar{P}_T). \tag{32}$$

The time-dependent reliability after corrosion has started, implying that the corrosion rate is larger than zero, is denoted as reliability after loss of effectiveness of anticorrosion coating $R_a(t)$:

$$R_a(t) = 1 - P_f(t), \quad t > \tau_c, \tag{33}$$

where τ_c is the time of failure of the coating protection.

For $t \leqslant \tau_c$ the metal plate is still intact and the thickness is equal to its value at $t = 0$:

$$h(t) = h(0) \quad \text{for} \quad t \leqslant \tau_c \tag{34}$$

and the threshold $\varsigma(t)$ becomes a constant reducing the upcrossing rate to

$$v(t|h_o, b, \sigma_o, E, d, \bar{P}_T,) = v_o \exp\left[-\left(\frac{\varsigma(h_o, b, \sigma_o, E, d_\infty) - \bar{P}_T}{\gamma_L}\right)^{\alpha_L}\right]t. \tag{35}$$

Therefore, the reliability before $R_b(t)$ corrosion starts to decrease thickness is modelled substituting Eq. (35) in Eq. (32), which is transformed into the reliability formulation of Eq. (33) when $t \leqslant \tau_c$.

Since the time to loss of effectiveness of the anticorrosion coating is a random variable, the reliability is conditional on the probability of coating time failure, $R(t|\tau_c)$. Therefore, the unconditional reliability of a plate is given by

$$R(t) = \int_o^t R(t|\tau_c) f_{\tau_c}(\tau_c)\, \mathrm{d}\tau_c, \quad t \in [0, T]. \tag{36}$$

The probability of non-occurrence of failure due to corrosion may be expressed as a sum of two terms, which describe the contribution before corrosion action, and after if has started:

$$\begin{aligned} P_{nf} = {} & \{E_{1,1}(t < \tau_c) \cap E_{1,2}(t < \tau_c)\} \cup \{E_{2,1}(t \geqslant \tau_c) \\ & \cap E_{2,2}(t \geqslant \tau_c) \cap E_{2,3}(t \leqslant T - \tau_c)\}, \end{aligned} \tag{37}$$

where T is the lifetime.

The first event is sub-divided into two sub-cases which represent probability that corrosion will not occur during the time t, ($E_{1,1}$) where $t \in [0, T]$ with probability $[1 - F_{\tau_c}(t)]$ and the probability of non-failure $R_b(t)$ under the condition that corrosion does not appear before end of the coating life ($E_{1,2}$).

The second case includes three conditions. The first one ($E_{2,1}$) is when corrosion occurs at time τ_c where $\tau_c \in [0, t]$ with probability $f_{\tau_c}(\tau_c)\, \mathrm{d}\tau_c$. The second sub-case ($E_{2,2}$) represents the probability of non-failure before corrosion starts decreasing the thickness of element at time τ_c and $\tau_c \in [0, t]$, $R_b(\tau_c)$. The third condition ($E_{2,3}$) gives probability of non-failure under condition that the corrosion appears $R_a(t - \tau_c)$.

The total reliability $R(t)$ is given by the reliability of the plate without corrosion plus the reliability of the plate with corrosion:

$$R(t) = [1 - F_{t\tau_c}(t)]R_b(t) + \int_0^t R_b(\tau_c) R_a(t - \tau_c) f_{\tau_c}(\tau_c)\, \mathrm{d}\tau_c, \quad t \in [0, T]. \tag{38}$$

The first term of this equation represents the probability that no corrosion appears and that failure does not occur in time $[0, t]$. The second term represents the probability of non-failure under the condition that the corrosion is initiated.

The reliability of the plate can be related with the generalised index of reliability which is calculated from a multi-normal distribution [27]. Under this assumption the reliability index β can be related with the probability of failure by

$$\beta(t) = -\Phi^{-1}[P_f(t)], \tag{39}$$

where Φ is the standardised normal distribution and $P_f(t)$ can be calculated from Eq. (38).

6. Modelling corrosion inspection and repair

The state of general corrosion in a plate is assessed by measuring the plate thickness at several points. There are two sources of uncertainty in this procedure. One results from the precision of the measuring instrument and the other from sampling variability. Measurements are made at few points of a panel and they are considered to be representative of the thickness in the whole plate.

Inspections are routinely made for structures in service and they may result in detection or no detection of a plate that has a mean thickness smaller than the acceptable value that is a fraction k of the original mean value.

$$h(t) \leqslant kh(0), \quad k \leqslant 1.0, \quad k = 1 - k_c. \tag{40}$$

The uncertainty of the method of detection is considered small and in this work, and it is assumed not to influence plate detection. It is assumed that an element will be inspected every year. It is further assumed that the method of inspection is such that if a plate is smaller than a limit value then it will be replaced and after replacement, their thickness will be h, which is their original value.

The reliability is computed for each period between inspections by using Eq. (38) being a function of the repairing time t_r

$$R(t) = [1 - F_{\tau_c}(t - t_r)]R_b(t - t_r) + \int_0^{t-t_r} R_b(\tau_c)R_a(t - t_r - \tau_c)f_{\tau_c}(\tau_c)\,\mathrm{d}\tau_c \quad \text{for} \quad T_{j-1} \leqslant t < T_j. \tag{41}$$

At each repair operation, a value of t_r must be determined. This value is substituted in Eq. (41) and the reliability can be evaluated for the next interval between inspections. In Eq. (41) the first term denotes the probability of non-failure when corrosion is not initiated in the service interval before $[T_{j-1}, T_j]$ and the second term denotes the probability of non-failure when corrosion appears in the service interval $[T_{j-1}, T_j]$.

7. Numerical example

The proposed approach is applied to assess the reliability of a plate element with corrosion. The plate is simply supported on the edges. The breadth is 0.6 m, and the breadth of tension zone of welding residual stresses $\eta = 5$. Detailed information

Table 1
Probabilistic models of the basic variables

Parameter	Mean value	Cov	Distribution
E	207 GPa	0.04	Log–normal
h_o	7 mm	0.02	Normal
d_∞	5 mm	0.3	Normal
σ_y	245 MPa	0.08	Log–normal
b	0.6 m	0.01	Normal
$\bar{P}_T$	0.265 MPa	0.12	Normal
t_o	5.5 yr	0.4	Normal

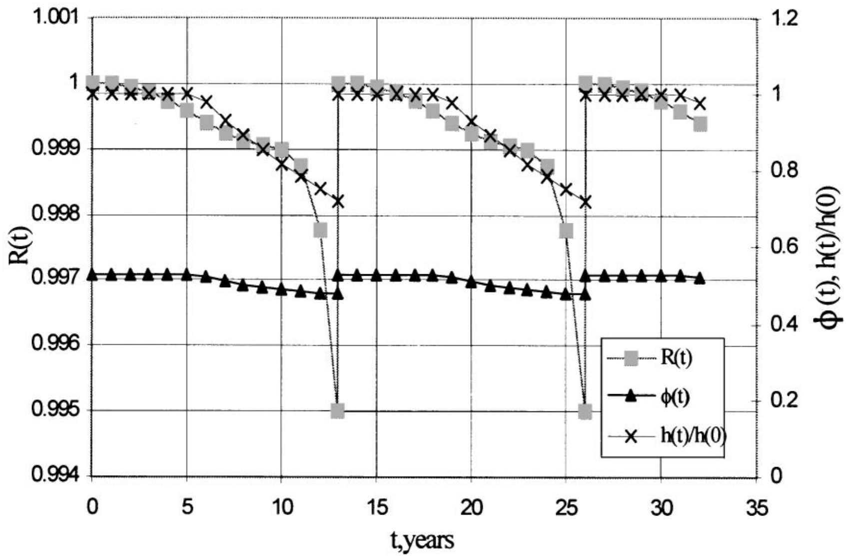

Fig. 9. Reliability of a plate element.

on the probabilistic models adopted to describe the random variables is presented in Table 1.

The compressive force (P) is described by an exponential distribution. The reference probability level is taken as $n_o = 10^{-5}$ with corresponding force $P_{n_o} = 0.183$ MPa. The shape parameter is assumed $\alpha_L = 1$.

The reliability of a plate as given by Eq. (41) is shown in Fig. 9. The restoring action is provided when the thickness of the element is less than 75% of original thickness.

The reliability of the plate element after repair will be equal to the initial value for the new plate. However, in the present example, the plate replacement was made at 13 and 26 yr and this brought the reliability to its initial value. Fig. 10 shows the corresponding results for the reliability index.

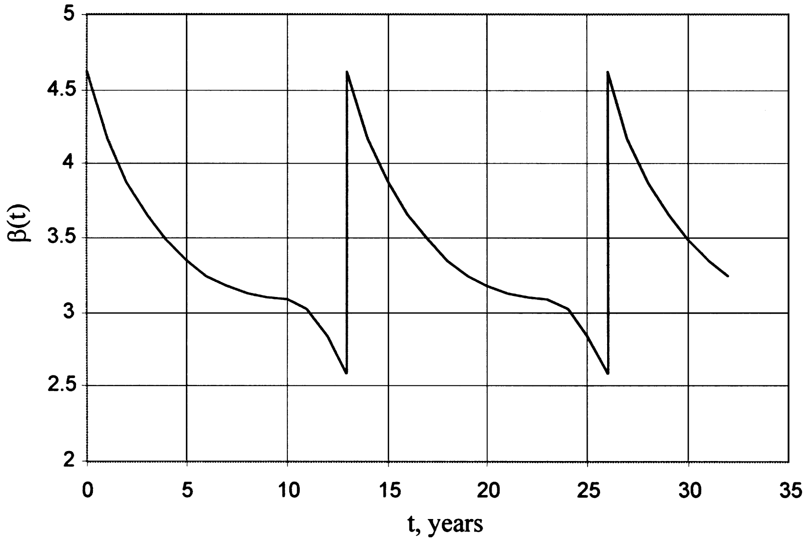

Fig. 10. Reliability index of a plate element ($k_l = 0.75$, $d_\infty = 5$ mm, $\tau_t = 15.2$ yr).

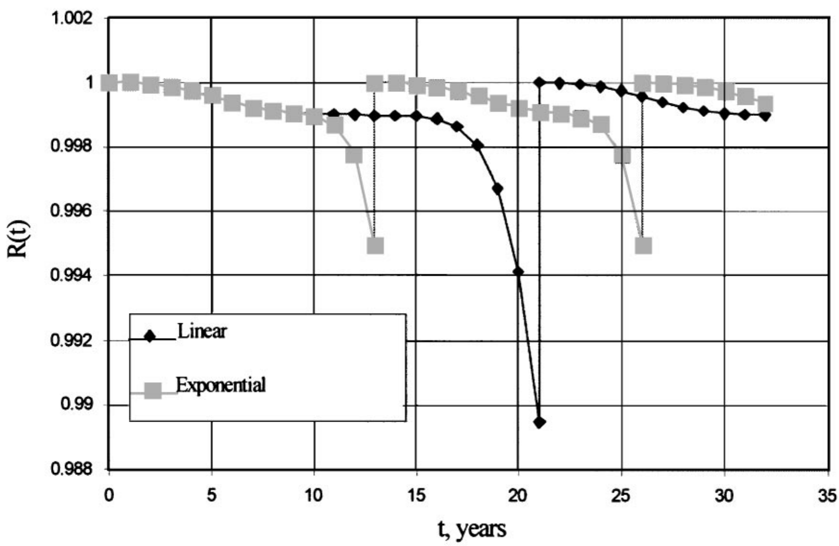

Fig. 11. Influence on the reliability of the approximation adopted for the corrosive depth.

Fig. 11 presents results of the reliability taking into account that a corrosive depth of the element will be approximated by a linear or an exponential relation. The repair is made when the thickness is equal to 75% of the initial value.

The formulation presented can be used to assess the effect of different parameters in the reliability.

The important aspect related with reliability is transition time from the start of corrosion to reaching the steady-state. The effect of different transition time adopted in Eq. (41) is shown in Fig. 12. A relatively large transition time may keep reliability at a relatively higher level and to postpone the necessity of replacement of the corroded element. For example, when the transition time is equal of 10 yr the first replacement is made at 10 yr instead of the cases when transition time is 15.2 and 20 yr the replacements are made at 13 and 15 yr, respectively.

Increasing the parameter d_∞ will decrease the reliability. This can be seen in the result of the calculations shown in Fig. 13 for the values equal to 3, 5 and 7 mm. It is interesting to note that the highest d_∞ leads to plate replacement at 10 yr, while in the basic case (5 mm) it occurs at 13 yr and in the case when $d_\infty = 3$ mm only at 19 yr under consideration that repair is made when the thickness is less or equal of 75% of its original value and the inspection interval is 4 yr.

The initial reserve strength of the plate has also a significant influence as shown Fig. 14. Its change will directly influence the reliability.

The formulation presented here can be used to assess the effect of different parameters such as the repair criteria, the time interval between inspections, the corrosion rate, the coating life and the initial thickness as was done in Guedes Soares and Garbatov [7], for the case of a linear decrease of thickness as a result from a constant corrosion rate.

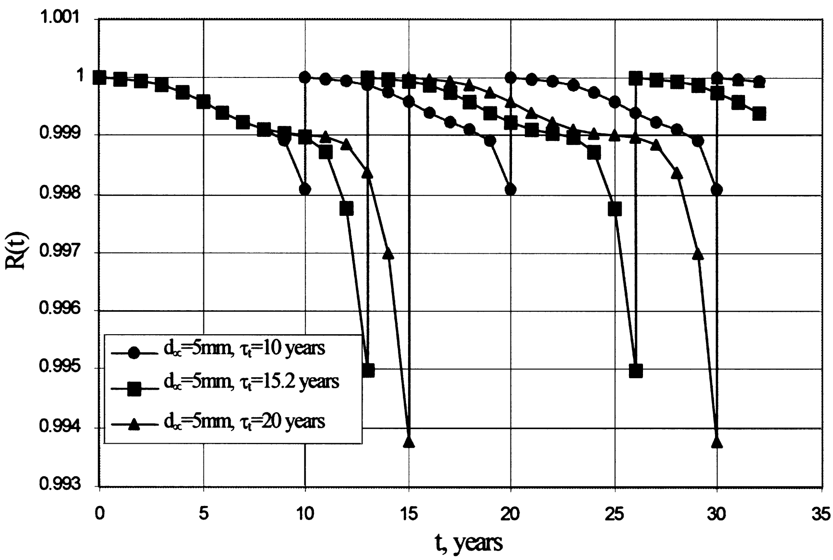

Fig. 12. Influence of the transition time on the reliability.

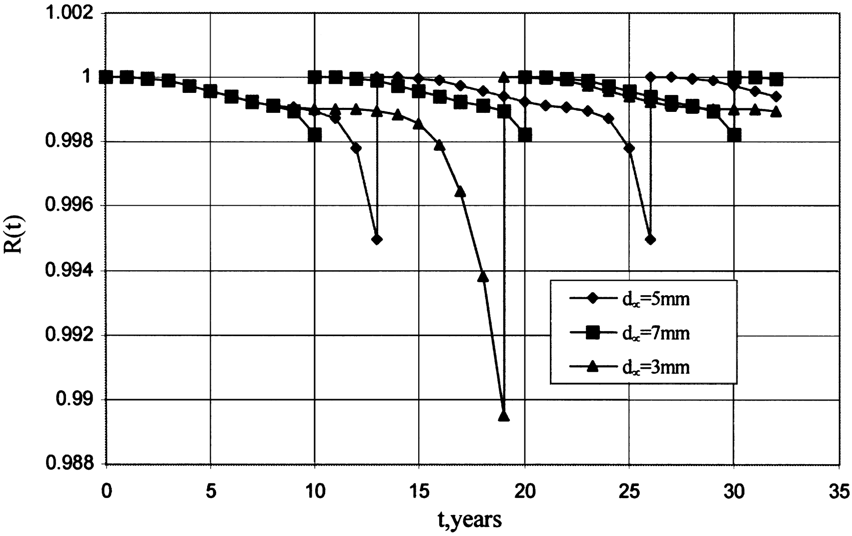

Fig. 13. Influence of the parameter d_∞ on the reliability.

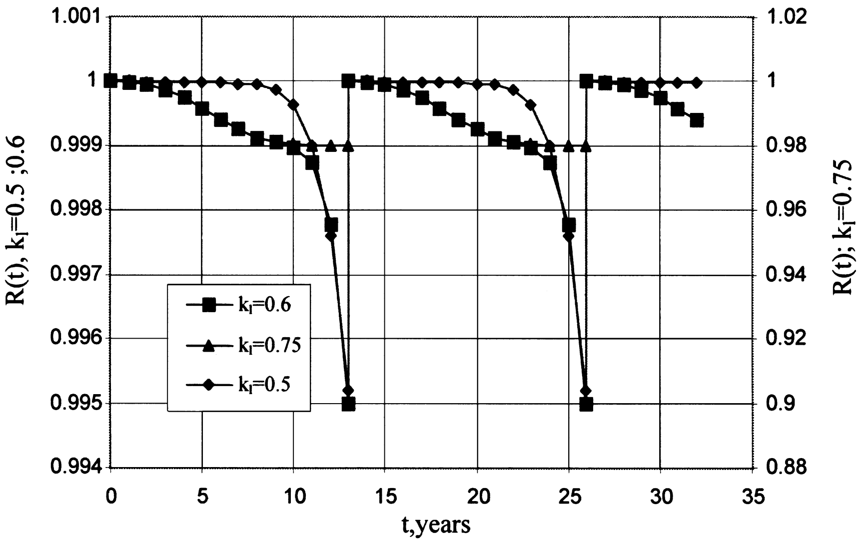

Fig. 14. Influence of the parameter k_l on the reliability.

8. Conclusions

A model is proposed to represent the non-linear time dependence of the corrosion wastage and includes a random initial time of no-corrosion while the protection is active.

A formulation has been presented which can predict the reliability of a plate element subjected to uniaxial compressive load considering the non-linear effect of corrosion on its ultimate strength. The formulation uses the value of the acceptable corrosion depth as an input value to the time-dependent function of corrosion and thus can be applied for the reliability assessment of plate elements with different corrosion rates.

A two-failure criterion is formulated and the influences of different parameters are examined. The sensitivity of the reliability estimates with respect to the transition time, long-term corrosion depth, initial strength reserve and type of approximation of corrosion depth are demonstrated.

The reliability is predicted as a time-variant reliability, which shows step changes whenever a repair is performed. Different criteria for corrosion detection and for the effect of repair can be easily incorporated.

Acknowledgements

The formulation presented here has been developed in the project "Safety of Shipping in Coastal Waters, (SAFECO)", which was partially financed by the European Union through the contract No. PL96-00023 (TRANSPORT).

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