BSc project proposals

in

Computational Complexity and Game Theory

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The following pages contain potential topics for BSc projects, but other projects are also possible. The final topic and direction of the project is settled under guidance by the advisor. In particular the balance between theory and practical implementations is done on an individual basis, and is often adjusted during the BSc project process. Please do not hesitate to pass by our offices on or send us an e-mail for setting up a meeting to discuss potential BSc projects.

Gradient Descent: Theory and Practice

The aim of this project is to study first-order methods in convex optimization. The project should be a mix of theory and practice. The theoretical part of the project consists presenting the mathematical foundations and of analyzing convergence rates and complexity of several of the main methods used for solving large-scale problems. The practical part of the project consists of implementing and comparing these methods.

References

- To be determined based upon scope of project in terms of theory and practice.

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Linear Complementarity Problems

A linear complementarity problem (LCP) in n variables is given by an $n \times n$ matrix M and a vector $q \in \mathbb{R}^n$ and the task is simply to find a non-negative vector $z \in \mathbb{R}^n$ such that the vector $Mz + q$ is both non-negative and orthogonal to z.

LCPs generalize Linear Programming (LP) problems (a major topic in the course Optimization). In fact, the larger class of convex quadratic programs can be formulated as LCPs. These are both classes of problems that can be solved in polynomial time with interior point algorithms. Several other algorithms exist for LCPs, for instance Lemke's algorithm that is closely related to the Simplex algorithm for Linear Programming.

For other classes of LCPs we have no known polynomial time algorithms. The problem of computing a Nash equilibrium (NE) in 2-player (general sum) games can also be formulated as LCPs. While the task of computing a NE is generally viewed as a computationally hard problem (hard for a complexity class called PPAD), Lemke's algorithm will often do well in practice for concrete games. Several other important problems from algorithmic game theory can likewise be formulated as LCPs and Lemke's algorithm or a variant of Lemke's algorithm can be used to solve these. Moving to general LCPs, it turns out that solving them is actually an NP-hard problem!

The project would typically be a mix of theory and practice, involving the analysis of algorithms and experimentation with concrete implementations and problems.

References

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Fair Division Algorithms for Indivisible Items

Fair division is the task of allocating items to agents so that some fairness criterion is satisfied. The problem finds applications in every setting where some kind of allocation of resources is required. In the very popular variations with indivisible items, problem instances consist of a set of n agents and a collection of m items. Agents have valuations for the items: agent i has a valuation $v_i(j)$ for item j. Valuations for sets (or bundles) of items are then additive, with the valuation of agent i for the bundle of items S being $v_i(S) = \sum_{j \in S} v_i(j)$. An allocation of items to agents is simply a partition $A = (A_1, A_2, ..., A_n)$ of the items to the agents so that agent i gets the bundle of items A_i.

An important fairness criterion is envy-freeness. We say that an allocation A is envy-free if for every pair of agents i and j, it holds that $v_i(A_i) \geq v_i(A_j)$. In words, every agent is happy with his/her bundle of items and does not envy the bundle allocated to some other agent. It can be easily seen that envy-freeness is rarely feasible. Indeed, consider an instance with two agents, who have positive valuations for a single item. Then, any allocation that gives the item to one of the agents is not envy-free. For this reason, several relaxations of envy-freeness have been considered in the literature.

Among them, the notion of envy-freeness up to any item (or EFX) is the most appealing. An allocation A is EFX, if for every pair of agents i and j, it holds that $v_i(A_i) \geq v_i(A_j \setminus \{g\})$, for any item $g \in A_j$. In words, an allocation is EFX if the possible envy of an agent for another is eliminated by removing any item from the bundle of items allocated to the latter. Unfortunately, until now, it is not known whether all instances have EFX instances or not. Fortunately, there are polynomial-time algorithms to compute partial EFX allocations (i.e., by discarding some of the items), simultaneously providing additional efficiency guarantees (e.g., they make sure that all agents get a bundle of items, for which they have a high value). The objective of this project is to study the recent literature on fair division of indivisible items, with a particular focus on the proposed algorithms for computing partial EFX allocations. Part of the work will include the implementation of these algorithms (and of variations of them) and their testing on real and synthetic instances.

References

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Algorithmic Aspects of Participatory Budgeting

Participatory budgeting is a relatively recent trend used by municipalities and regional governments worldwide to decide the distribution of funding to public projects. The main idea is to let the citizens express their opinion through voting over spending priorities and then implement the outcome of the vote as a public decision.

The details of voting are rather complicated (compared to a typical government election) and include a series of steps and features. We briefly present three components that are interesting from an algorithmic viewpoint. The first is the selection of the available alternatives and their nature. For example, alternatives could be discrete such as the construction of a bridge or continuous such as the length of bicycle paths. Second, how are the actual preferences of the citizens will be structured in the ballots? This step may result in a loss of information. Third, how should the votes be aggregated into a collective decision? This part is the most crucial algorithmically but strongly depends and interplays with the previous two.

The interdisciplinary field of computational social choice (lying at the intersection of social choice theory and theoretical computer science), which traditionally studies the computational complexity of voting rules, has extensively considered participatory budgeting recently, together with other modern trends of participatory democracy. The focus has been on axiomatic properties, and on quantifying the lack of efficiency of the voting process, due to the loss of information when expressing preferences.

The aim of the project is to survey the recent approaches in participatory budgeting, focusing on theoretical developments and analysis, case studies from its application to municipalities around the world, and available online tools. The project will involve the implementation of representative aggregation methods and experiments/simulations with synthetic voting scenarios.

References

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- *Interactive Democracy*. Markus Brill. In *Proceedings of the 17th International Conference on Autonomous Agents and Multi-Agent Systems (AAMAS)*, 1183–1187, 2018. dl.acm.org/doi/10.5555/3237383.3237873.

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Financial Networks and Systemic Risk

The last major financial crisis and its aftermath have revealed the systemic risks and hazards for the society that can arise in financial markets. These are mainly due to the complicated structure of these markets, with many different financial institutions (e.g., banks or firms) that are highly interconnected and interact with each other.

Over the last decade, substantial research has been undertaken to analyse, understand, and manage systemic risks in financial markets. This originates from the seminal work of Eisenberg and Noe, who proposed a graph-theoretic model that is considered the standard today. Interconnections between financial institutions are represented by a directed graph $G = (V, E)$. The node set V contains the financial institutions. A weighted directed edge $e \in E$ expresses a debt relation between two institutions. In addition, each institution has non-negative external assets, which capture the value of property rights (such as real estate, gold, business and mortgage loans, etc.) that the firm has acquired from non-financial institutions. Eisenberg and Noe discuss a clearing mechanism for such a market in which every institution v uses its available assets to pay its debt.

This BSc thesis aims to survey recent work on the model of Eisenberg and Noe and its extensions. Two kinds of questions will be considered. First, we will study computational problems related to identifying risks on the graph representing the interactions between financial institutions. Second, we will study simplified strategic games played by the several institutions. The work will mainly focus on mathematical proofs and analysis of algorithms in the graph-theoretic model above, but it may include implementations and simulations of strategic play by the financial institutions and experiments on these dynamics.

References

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Reinforcement Learning in Prisoner's Dilemma

The Prisoner's Dilemma (PD) is a fundamental setting in game theory illustrating how two rational strategic agents may fail to cooperate, even when cooperation would yield a better outcome for both. In PD, two prisoners are separately offered a deal: betray the other for a reduced sentence, or stay silent. If both betray, they both receive moderate sentences; if both remain silent, they receive minimal sentences. However, if one betrays while the other stays silent, the betrayer goes free while the silent prisoner receives a harsh sentence. While mutual silence leads to the best collective outcome, betrayal is known to be the dominant strategy. The Prisoner's Dilemma captures various real-world situations, from pricing strategies in economics to behaviors in animal species, highlighting the tension between individual gain and mutual cooperation.

Classical game theory predicts that mutual cooperation is unlikely to arise in a single, one-shot version of the game. However, recent studies indicate that Machine Learning algorithms can exhibit cooperative behaviors when the game is played repeatedly over time. This project aims to explore whether Reinforcement Learning (RL) algorithms, such as Q-Learning, can foster cooperative behavior in the context of the Prisoner's Dilemma. The project will begin with an experimental phase, evaluating various RL algorithms, followed by a mathematical analysis based on experimental findings.

References

- *Artificial Intelligence, Algorithm Design, and Pricing.* Larry Eisenberg and Thomas Noe. AEA Papers and Proceedings, 452–56, 2022. Available from: https://www.aeaweb.org/articles?id=10.1257/pandp.20221059
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Online Learning and Assembly of Calculus

Online Learning (OL) is a crucial subfield of Machine Learning that addresses decision-making problems. Recently, Online Learning has gained significant importance in game theory, as OL algorithms provide formal optimality guarantees on an agent's payoff regardless of the behavior of other agents over time. Over the years, various OL algorithms have been proposed, and recent studies reveal that the behavior of human agents aligns closely with the behavior of these OL algorithms.

The goal of this project is to investigate whether Online Learning can explain the behavior of real human agents in competitive game-theoretic settings. A key challenge is that current OL algorithms rely on complex mathematical operations that do not align with human and animal brain. However, recent work introduces a computational model called Assembly Calculus, which models fundamental brain operations. In this project, we will explore classical OL settings (e.g., the Expert Problem, Multi-Armed Bandits etc.) and examine whether biological learning processes, such as Hebbian Plasticity, can inspire new OL algorithms compatible with Assembly Calculus. Ultimately, the goal is to develop novel OL algorithms that, while forgoing complex mathematical operations, are compatible with the fundamental operations of the brain.

References

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