Probability integral transformation

Let X have continuous cumulative distribution function $F_X(x)$ and define a random variable Y as $Y = F_X(X)$. Then Y is uniformly distributed on $(0,1)$, that is, $P(Y \leq y) = y,\ 0 < y < 1$.
Proof : We know,

$$\begin{aligned} F_Y(y) &= P(Y \leq y) \\ &= P(F_X(X) \leq y) \\ &= P(X \leq F_X^{-1}(y)) \\ &= P(X \leq F_X^{-1}(F_X(x)) \\ &= P(X \leq x) \\ &= y \end{aligned}$$

* If F_X is monotonic, then F_X^{-1} is well defined by

$$F_X^{-1}(y) = x \quad \Leftrightarrow \quad F_X(x) = y.$$

Example: Consider X is a continuous random variable with PDF and CDF is defined as follows:

$$\begin{aligned} &f_X(x, \alpha, \lambda) = \alpha\lambda x^{\alpha-1} e^{-\lambda x^{\alpha}}, \quad x > 0, \quad \alpha, \lambda > 0, \\ &F_X(x, \alpha, \lambda) = 1 - e^{-\lambda x^{\alpha}}. \end{aligned}$$

This is Weibull distribution with shape parameter α and scale parameter λ, respectively.
Let $u \sim U(0,1)$, then to generate the sample for x, we first define as

$$F_X(x) = u \implies 1 - e^{-\lambda x^{\alpha}} = u \implies x = \Big[-\frac{1}{\lambda} \ln(1-u) \Big]^{1/\alpha}$$