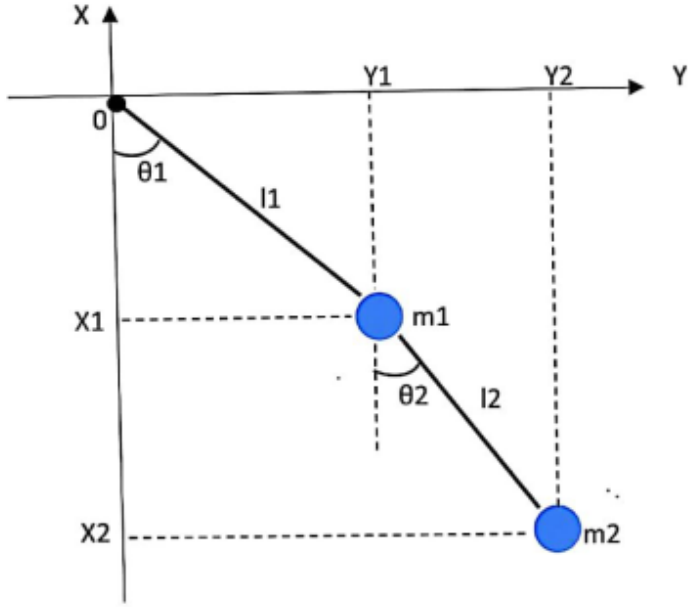
Penurunan Persamaan Lagrange Double Pendulum

Gambar Pendulum



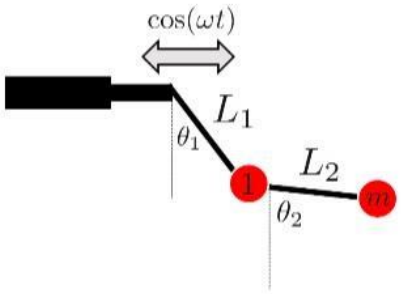
Kelompok 12 :
1. Faris Haidar
2. Intan Opialisti
3. Nurlaila Sari

$$L = \frac{1}{2}m_1(\dot{x}_1^2 + \dot{y}_1^2) + \frac{1}{2}m_2(\dot{x}_2^2 + \dot{y}_2^2) - m_1 g y_1 - m_2 g y_2$$

$$\frac{L}{A^2 m_1} = \frac{1}{2}\left(\left(\frac{\dot{x}_1}{A}\right)^2 + \left(\frac{\dot{y}_1}{A}\right)^2\right) + \frac{1}{2}\frac{m_2}{m_1}\left(\left(\frac{\dot{x}_2}{A}\right)^2 + \left(\frac{\dot{y}_2}{A}\right)^2\right) - \frac{g}{A}\frac{y_1}{A} - \frac{m_2}{m_1}\frac{g}{A}\frac{y_2}{A}$$

$$\frac{L}{A^2 m_1} = \frac{1}{2}(\dot{x'}_1^2 + \dot{y'}_1^2) + \frac{1}{2}m(\dot{x'}_2^2 + \dot{y'}_2^2) - g' y'_1 - m g' y'_2$$

Soal Double Pendulum :



and thus

- $x_1 = \cos(\omega t) + L_1 \sin(\theta_1)$
- $x_2 = \cos(\omega t) + L_1 \sin(\theta_1) + L_2 \sin(\theta_2)$
- $y_1 = -L_1 \cos(\theta_1)$
- $y_2 = -L_1 \cos(\theta_1) - L_2 \cos(\theta_2)$

$$L_1 \cos\left(\theta_1(t)\right)\frac{d}{dt}\theta_1(t) - \omega \sin\left(\omega t\right) \qquad \frac{\partial L}{\partial \theta} - \frac{d}{dt}\frac{\partial L}{\partial \dot{\theta}} = 0$$

$$L_1\left(-L_1 m\frac{d^2}{dt^2}\theta_1(t) - L_1\frac{d^2}{dt^2}\theta_1(t) - L_2 m \sin\left(\theta_1(t) - \theta_2(t)\right)\left(\frac{d}{dt}\theta_2(t)\right)^2 - L_2 m \cos\left(\theta_1(t) - \theta_2(t)\right)\frac{d^2}{dt^2}\theta_2(t)\right.$$
$$\left. + \omega^2 m \cos\left(\omega t\right)\cos\left(\theta_1(t)\right) + \omega^2 \cos\left(\omega t\right)\cos\left(\theta_1(t)\right) - gm \sin\left(\theta_1(t)\right) - g \sin\left(\theta_1(t)\right)\right)$$

$$L_2 m\left(L_1 \sin\left(\theta_1(t) - \theta_2(t)\right)\left(\frac{d}{dt}\theta_1(t)\right)^2 - L_1 \cos\left(\theta_1(t) - \theta_2(t)\right)\frac{d^2}{dt^2}\theta_1(t) - L_2\frac{d^2}{dt^2}\theta_2(t)\right.$$
$$\left. + \omega^2 \cos\left(\omega t\right)\cos\left(\theta_2(t)\right) - g \sin\left(\theta_2(t)\right)\right)$$

$$\frac{L_2 m\left(-L_1 \sin\left(\theta_1(t) - \theta_2(t)\right)\left(\frac{d}{dt}\theta_1(t)\right)^2 - \omega^2 \cos\left(\omega t\right)\cos\left(\theta_2(t)\right) + g \sin\left(\theta_2(t)\right)\right)\cos\left(\theta_1(t) - \theta_2(t)\right)}{-L_1 L_2 m \cos^2\left(\theta_1(t) - \theta_2(t)\right) - L_2\left(-L_1 m - L_1\right)}$$

$$-\frac{L_2\left(L_2 m \sin\left(\theta_1(t)-\theta_2(t)\right)\left(\frac{d}{dt}\theta_2(t)\right)^2-\omega^2 m\cos\left(\omega t\right)\cos\left(\theta_1(t)\right)-\omega^2\cos\left(\omega t\right)\cos\left(\theta_1(t)\right)+gm\sin\left(\theta_1(t)\right)+g\,\mathrm{s}\sin\left(\theta_1(t)\right)\right)}{-L_1L_2m\cos^2\left(\theta_1(t)-\theta_2(t)\right)-L_2\left(-L_1m-L_1\right)}$$

$$L_1\left(-L_1m\frac{d^2}{dt^2}\theta_1(t)-L_1\frac{d^2}{dt^2}\theta_1(t)-L_2m\sin\left(\theta_1(t)-\theta_2(t)\right)\left(\frac{d}{dt}\theta_2(t)\right)^2-L_2m\cos\left(\theta_1(t)-\theta_2(t)\right)\frac{d^2}{dt^2}\theta_2(t)\right.$$
$$\left.+\omega^2m\cos\left(\omega t\right)\cos\left(\theta_1(t)\right)+\omega^2\cos\left(\omega t\right)\cos\left(\theta_1(t)\right)-gm\sin\left(\theta_1(t)\right)-g\sin\left(\theta_1(t)\right)\right)$$