Electrical Networks

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1 Introduction to Networks & Circuits

A **Network** is simply a connected combination of several circuit elements.
A **Circuit** is a special case of a network where the combination forms a closed path.
The study of these combinations of circuit elements i.e their analysis and synthesis come under 'Network Theory'.

For a circuit or a network to be useful, it must consist of a source (or excitation) and a load (or response), connected across two different ports of the network.

Analysis problem deals with finding out the response of a network for a given excitation.

Synthesis problem deals with designing an appropriate network that gives desired response for a given excitation.

The analysis problem is fairly straightforward and it has a unique solution regardless of how it is obtained.
On the other hand, the synthesis problem is more complicated since realizability test, design procedure and optimal design are all part of the problem.

1.1 Circuits and Elements

Linear Elements: Elements whose input-output characteristic graph is a straight line passing through the origin.

Linear Circuit: Circuit that is made of only linear elements and sources i.e it must consist only of linear elements under zero initial conditions, current and voltage sources.

Passive Elements: Elements that do not supply power/energy. These elements only dissipate power.
Example: Resistor, Capacitor and Inductor under zero initial conditions.

Active Elements: Elements that are capable of supplying power/energy.
Example: Voltage source, Current source.

Passive elements will have V-I characteristic curve only in first and third quadrants. Active elements will have V-T characteristic curve even in second and fourth quadrants.

Bilateral Elements : Two terminal elements that give same magnitude of response of both polarities when the polarity of the excitation is reversed.
Example: Resistor

Unilateral Elements : Two terminal elements that behave differently for excitations of same magnitude but opposite polarities. Example: Diode

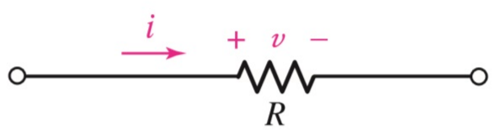
A network consisting only of bilateral elements is always reciprocal. A network consisting of one more more unilateral elements may or may not be reciprocal. More about reciprocity will be explained later.

Lumped Network is a network in which the space variables (i.e distance) can be neglected since dimensions are not comparable to wavelength of signals being operated with.

Distributed Network is a network in which the space variables (i.e distance) have to be considered since dimensions are comparable to wavelength of signals being operated with.

1.1.1 Basic Electrical Components

Resistor
A resistor is a passive two-terminal electrical component that implements electrical resistance as a circuit element.
The resistance offered by a resistor is given by $R = v/i$ where v is the voltage drop across it and i is the current flowing through it.



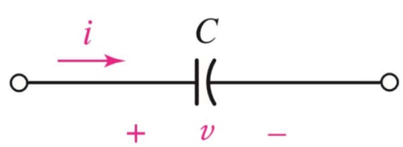
Ideally, a resistor is linear and responds instantaneously. However, practically there will be a limit to how much current a resistor can withstand.

(unit of resistance - Ω [ohm])

The reciprocal of resistance is called Conductance, denoted by G.
(unit of conductance - ℧ [seimen])

A resistor behaves exactly the same for both AC and DC sources.

Capacitor
A capacitor is a passive electronic component with two terminals that stores electrical energy in an electric field.
The charge storage capacity of a capacitor is called capacitance and is given by $C = q/v$ where q is the charge stored and v is the potential developed (voltage drop) across it.



A capacitor is an energy storage device. Hence in the presence of a source, it will store energy and in the absence of a source it will provide energy.

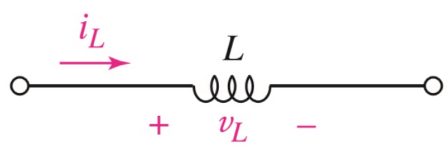
The relation between current and voltage in terms of capacitance is,

$$i(t) = C\frac{dv(t)}{dt}$$

$$v(t) = \frac{1}{C}\int_{0^-}^{t} i(\tau)d\tau + v(0^-)$$

(unit of capacitance - F [farad])

Inductor
An inductor is a passive two-terminal electrical component that stores energy in a magnetic field when electric current flows through it.
The voltage inducing capacity of an inductor is called inductance and is given by $L = \phi/i$ where ϕ is the flux linked and i is the current flowing through it.



An inductor is an energy storage device. Hence in the presence of a source, it will store energy and in the absence of a source it will provide energy.

The relation between current and voltage in terms of inductance is,

$$v(t) = L\frac{di(t)}{dt}$$

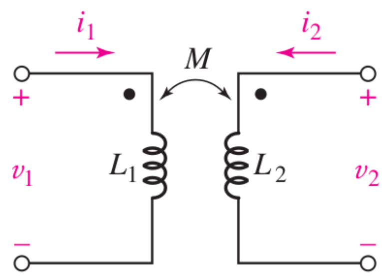
$$i(t) = \frac{1}{L}\int_{0^-}^{t} v(\tau)d\tau + i(0^-)$$

(unit of inductance - H [henry])

A component consisting of resistive, capacitive and inductive properties is called **Impedance** and is denoted by Z. The reciprocal of impedance is called Admittance, denoted by Y. More about these will be explained in frequency domain analysis.

Mutual Inductance
If two inductors are placed close to each other, the flux caused due to one

will interact with the other. The figure below shows two inductors with self-inductances L_1 and L_2 that affect each other.



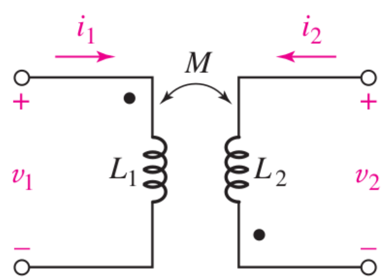
M is called "Mutual Inductance" of the given pair of inductors.
It is given by $M = k\sqrt{L_1 L_2}$ where k is the coefficient of coupling.

Dot convention is used to specify the direction of currents and voltages. Current-voltage relationship for the above combination is as follows.

$$v_1(t) = L_1 \frac{di_1}{dt} + M \frac{di_2}{dt}$$

$$v_2(t) = L_2 \frac{di_2}{dt} + M \frac{di_1}{dt}$$

The following figure shows the same inductors with different dot convention.



Current-voltage relationship for this combination is as follows.

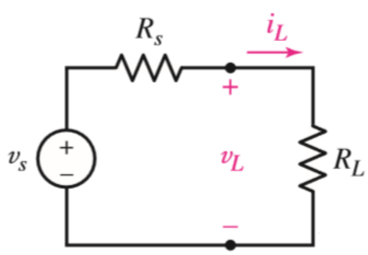
$$v_1(t) = L_1 \frac{di_1}{dt} - M \frac{di_2}{dt}$$

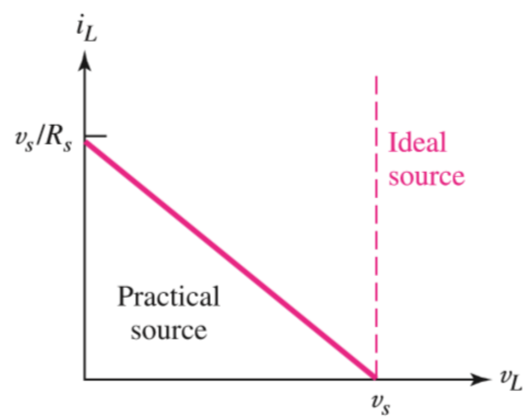
$$v_2(t) = L_2 \frac{di_2}{dt} - M \frac{di_1}{dt}$$

Ideal Voltage Source
A source that provides constant voltage drop across its terminals irrespective of the load connected. Meaning, an ideal voltage source can supply any amount of current.

An ideal voltage source in parallel/shunt with any other element is equivalent to having just the voltage source itself.

Non-ideal Voltage Source
A source such that the voltage drop provided varies with respect to the load connected. This can be modelled using an ideal voltage source in series with an impedance.

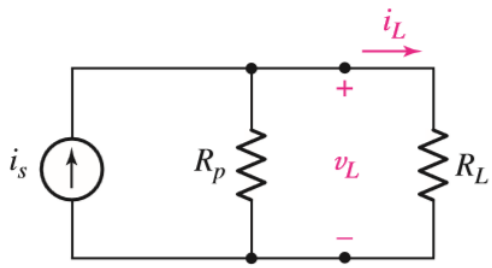


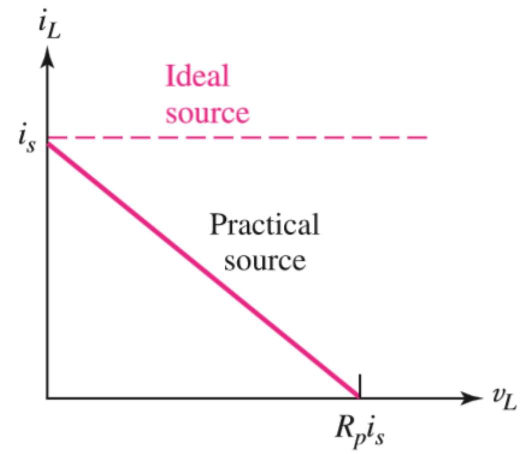


Ideal Current Source
A source that provides constant current flow through the terminals irrespective of the load connected. Meaning, an ideal current source can create any amount of voltage drop.

An current voltage source in series with any other element is equivalent to having just the current source itself.

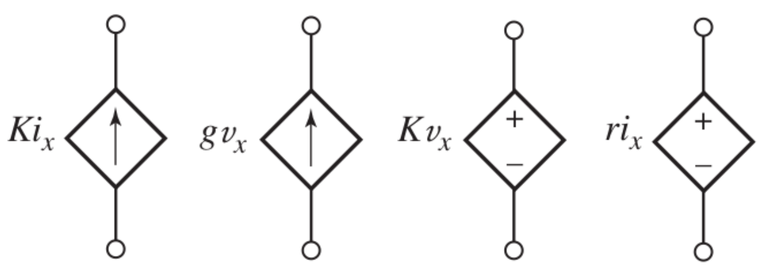
Non-ideal Current Source
A source such that the current provided varies with respect to the load connected. This can be modelled using an ideal current source in parallel (shunt) with an impedance.





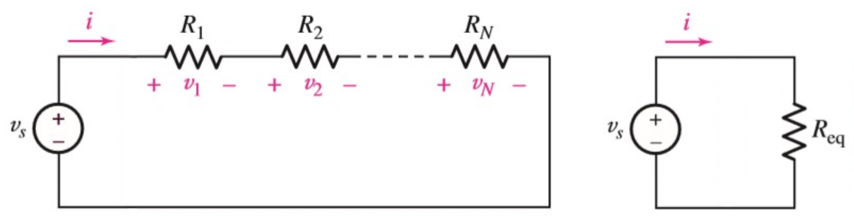
Controlled Source

A controlled source is any source (current or voltage) whose value depends on some current or voltage through or across some other terminals.



1.1.2 Basic combinations of Elements

Resistors in series



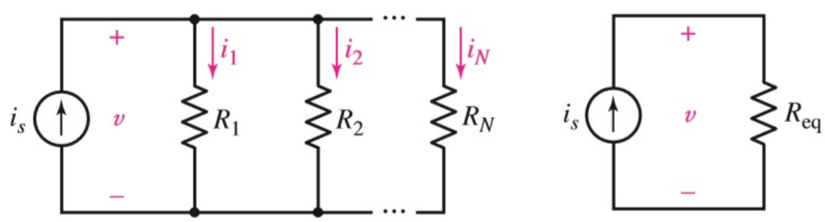
The equivalent resistance is given by $\boxed{R_{eq} = R_1 + R_2 + R_3 + R_4 + \dots}$

If n equal resistors of value R are connected in series, then $R_{eq} = nR$.

The equivalent resistance of a series combination is always greater than the greatest individual resistance.

The current through each of the resistors in series is constant, the voltage drop gets distributed between each of them (depending on the values).

Resistors in parallel



The equivalent resistance is given by $\boxed{\frac{1}{R_{eq}} = \frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3} + \frac{1}{R_4} + \dots}$

If n equal resistors of value R are connected in parallel, then $R_{eq} = R/n$.

The equivalent resistance of a parallel combination is always lesser than the lowest individual resistance.

The voltage drop across the parallel combination of resistors is constant, the current gets divided between each of them (depending on the values).

Capacitors in series
Sum of reciprocals of each capacitance will give reciprocal of equivalent capacitance (like parallel combination of resistors).

Capacitors in parallel
Sum of each capacitance will give equivalent capacitance (like series combination of resistors).

Inductors in series
Sum of each inductance will give equivalent inductance (like series combination of resistors).

Two inductors L_1 and L_2 in series having mutual inductance M will have equivalent inductance as,

$$L_{eq} = \begin{cases} L_1 + L_2 + 2M & : \text{ dots in same direction} \\ L_1 + L_2 - 2M & : \text{ dots in opposite direction} \end{cases}$$

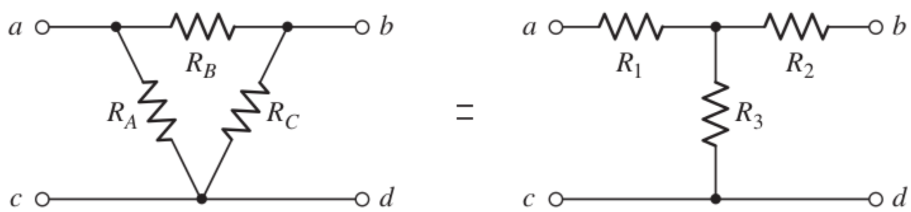
Inductors in parallel
Sum of reciprocals of each inductance will give reciprocal of equivalent inductance (like parallel combination of resistors).
Two inductors L_1 and L_2 in parallel having mutual inductance M will have equivalent inductance as,

$$L_{eq} = \begin{cases} \frac{L_1 L_2 - M^2}{L_1 + L_2 - 2M} & : \text{ dots in same direction} \\ \frac{L_1 L_2 - M^2}{L_1 + L_2 + 2M} & : \text{ dots in opposite direction} \end{cases}$$

1.1.3 Star-Delta Conversion

Star network and Delta network are special types of combinations that can't be directly simplified using series or parallel simplifications.

The networks are illustrated below.



Based on convenience, these networks can be interchanged i.e they are replaceable by each other (in order to make overall simplifications easier).

Delta to Star formula:

$$R_1 = \frac{R_A\ R_B}{R_A + R_B + R_C} \quad R_2 = \frac{R_B\ R_C}{R_A + R_B + R_C} \quad R_3 = \frac{R_C\ R_A}{R_A + R_B + R_C}$$

If resistors are equal, then $R_y = \frac{R_\Delta}{3}$

Star to Delta formula:

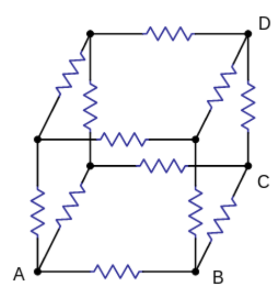
$$R_A = \frac{R_1R_2 + R_2R_3 + R_3R_1}{R_2} \quad R_B = \frac{R_1R_2 + R_2R_3 + R_3R_1}{R_3} \quad R_C = \frac{R_1R_2 + R_2R_3 + R_3R_1}{R_1}$$

If resistors are equal, then $R_\Delta = 3R_y$

Star-Delta conversion of capacitors is reverse as resistors.
Star-Delta conversion of inductors is same as resistors.

1.1.4 Cube of Resistors

Consider the following arrangement of resistors (6 resistors connected like they form the edges of a cube). Let the resistance of each resistor be R.



Equivalent resistance between adjacent terminals: $R_{AB} = \frac{3}{4}R$

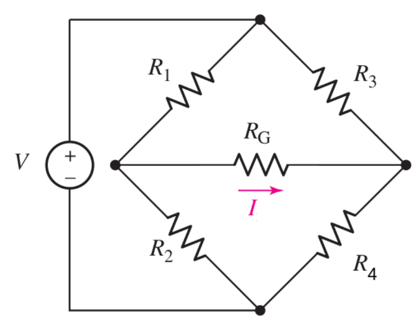
Equivalent resistance between terminals of face diagonal: $R_{AC} = \frac{7}{12}R$

Equivalent resistance between terminals of body diagonal: $R_{AD} = \frac{5}{6}R$

1.1.5 Wheatstone Bridge

A Wheatstone bridge is an electrical circuit used to measure an unknown electrical resistance by balancing two legs of a bridge circuit, one leg of which includes the unknown component.

A typical Wheatstone bridge circuit is shown in the figure.



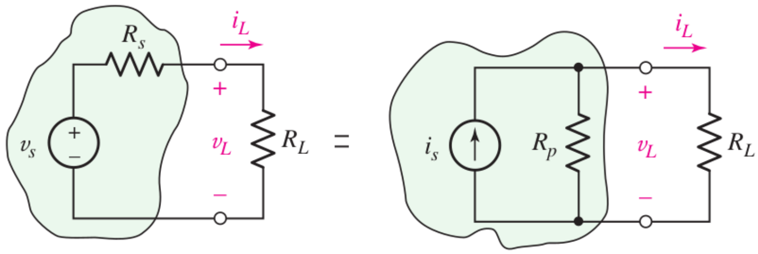
A Wheatstone bridge is said to be in **Balanced** condition when the resistors satisfy the following condition.

$$\frac{R_1}{R_2} = \frac{R_3}{R_4}$$

When the bridge is balanced, the current through the middle resistor R_G (i.e I) will be zero, hence R_G can be removed and the circuit will effectively be the same. This property can be used to simplify some circuits.

1.1.6 Source Transformation

A practical voltage source (ideal voltage source with series resistance) can be converted to a practical current source (ideal current source with shunt resistance) and vice versa to make network simplification more convenient.



1.1.7 Temperature dependent resistors

The resistance of a resistor ideally should not vary with temperature. However, in practical cases, there will be slight variation in the resistance value.

$$R(T_2) = R(T_1)[1 + \alpha\ (T_2 - T_1)]$$

$R(T_1)$ is resistance at temperature T_1
$R(T_2)$ is resistance at temperature T_2
α is temperature coefficient and is constant for the given material

1.2 Energy and Power

Power delivered or dissipated/absorbed by a circuit elements is given by the product of current flowing and voltage drop.

$$\boxed{P = v(t)i(t)}$$

(unit of power - W [watt])

Delivered power and absorbed/dissipated power must be of opposite signs. As a convention, absorbed/dissipated power is taken to be positive and delivered power is taken to be negative.

- Current flowing in the device is from negative polarity of voltage to positive polarity of voltage (i.e from lower to higher potential) means the device is delivering power.
- Current flowing in the device is from positive polarity of voltage to negative polarity of voltage (i.e from higher to lower potential) means the device is absorbing/dissipating power.

Note that a resistor will always dissipate power, whereas a capacitor and an inductor can absorb power in presence of sources and deliver the absorbed power in absence of sources.

Power dissipated by resistor:

$$P = \frac{v(t)^2}{R} = \frac{i(t)^2}{R}$$

The total amount of power delivered/absorbed in a given period of time is the **Energy**.

$$\boxed{E = \int_0^t v(t)i(t)dt}$$

(unit of energy - J [joules])

Energy dissipated by resistor after time t:

$$E = \int_0^t \frac{v(t)^2}{R}dt = \int_0^t \frac{i(t)^2}{R}dt$$

The above expression shows that E can never be negative, and hence a resistor can never delivery energy.

Power absorbed by a inductor:

$$P = Li(t)\frac{di}{dt}$$

- Current is positive and increasing $\implies$ Power is being absorbed
- Current is positive and decreasing $\implies$ Power is being delivered
- Current is negative and decreasing $\implies$ Power is being absorbed
- Current is negative and increasing $\implies$ Power is being delivered

Energy stored in inductor after time t:

$$E = \frac{1}{2}Li^2(t)$$

If initial current of inductor is 0, then it can only absorb energy. It is capable of delivering energy only after absorbing, and hence an inductor is a passive element.

The energy stored in an inductor at any instant of time is equal to the total energy stored from infinite past to that instant of time.

Power absorbed by a capacitor:

$$P = Cv(t)\frac{dv}{dt}$$

- Voltage is positive and increasing $\implies$ Power is being absorbed
- Voltage is positive and decreasing $\implies$ Power is being delivered
- Voltage is negative and decreasing $\implies$ Power is being absorbed
- Voltage is negative and increasing $\implies$ Power is being delivered

Energy stored in capacitor after time t:

$$E = \frac{1}{2}Cv^2(t)$$

If initial voltage of capacitor is 0, then it can only absorb energy. It is capable of delivering energy only after absorbing, and hence a capacitor is a passive element.

The energy stored in a capacitor at any instant of time is equal to the total energy stored from infinite past to that instant of time.

Voltage source and current source can either absorb or deliver power based on the rest of the circuit conditions.

For a positive voltage source v,

- If operating in first quadrant ($v > 0;\ i > 0$) $\implies$ Power is being absorbed
- Operating in fourth quadrant ($v > 0;\ i < 0$) $\implies$ Power is being delivered

(similar condition can be obtained for negative voltage source as well)

For a positive current source i,

- If operating in first quadrant ($v > 0;\ i > 0$) $\implies$ Power is being absorbed
- Operating in second quadrant ($v < 0;\ i > 0$) $\implies$ Power is being delivered

(similar condition can be obtained for negative voltage source as well)

Since there is a possibility for current and voltage sources to deliver power at any instant of time, they are active elements.

1.3 Basic Laws

The base for network analysis is given by two laws, namely KCL and KVL. Before understanding these laws, there are a few terminologies that have to be defined.

Node is a point in the network where two or more elements meet i.e an intersection of terminals occurs.

Branch represents an element i.e the part between two nodes.

Loop is a combination of branches such that they form a closed path i.e the starting and ending node are the same.

Mesh is a loop with no other loops inside it.
Hence, all meshes are loops but all loops are not meshes.

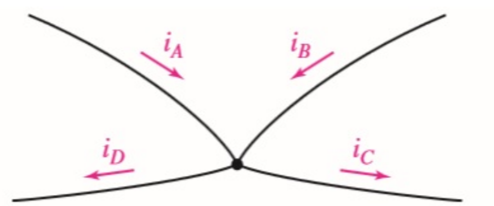
KCL is applied at a node and KVL is applied around a loop.

Both of Kirchhoff's laws can be understood as corollaries of Maxwell's equations in the low-frequency limit. They are accurate for DC circuits, and for AC circuits at frequencies where the wavelengths of electromagnetic radiation are very large compared to the circuit dimensions.

1.3.1 Kirchhoff's Current Law

"The algebraic sum of all currents entering or leaving a particular node is equal to zero"

The KCL is based on law of conservation of charge.
It can also be interpreted as the sum of incoming currents is equal to the sum of outgoing currents (at a node).

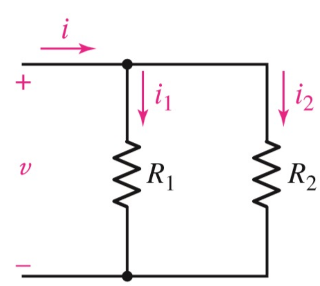


In the given figure, application of KCL would give $i_A + i_B + (-i_C) + (-i_D) = 0$, hence $i_A + i_B = i_C + i_D$.

Note that if entering currents are taken as positive, then leaving currents are taken as negative (and vice versa).

Current Division Rule
If an incoming current path is split into 2 (or more) paths with different resistances, then the current gets distributed.

Consider the given network



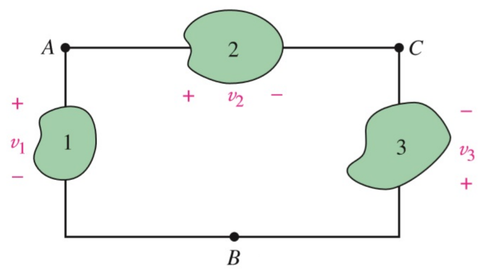
$$i_1 = i\,\frac{R_2}{R_1 + R_2} \qquad i_2 = i\,\frac{R_1}{R_1 + R_2}$$

Note that current division can only be applied when voltage division is not happening.

1.3.2 Kirchhoff's Voltage Law

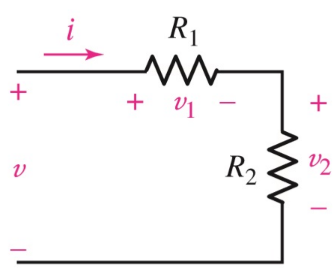
"The algebraic sum of all voltages around any closed loop in a circuit is equal to zero."

The KVL is based on law of conservation of energy.
It can also be interpreted as the sum of all voltage rises is equal to the sum of all voltage drops (in a loop).



In the given loop, application of KVL will result in $v_1 - v_2 + v_3 = 0$ i.e $v_1 + v_3 = v_2$.

Voltage Division Rule
If a voltage is connected across a combination of 2 or more resistances with different resistances, then the voltage gets distributed.
Consider the given network



$$v_1 = v\,\frac{R_1}{R_1 + R_2} \qquad v_2 = v\,\frac{R_2}{R_1 + R_2}$$

Note that voltage division can only be applied when current division is not happening.

1.4 Network Equations

Given an electrical network, its mathematical model has to be obtained using KVL or KCL.

If the number of branches in a circuits is B and the number of nodes is N, then the number of independent equations that can be written for a circuit are as follows (derived using graph theory):

- KVL: $n(E) = B - N + 1$
- KCL: $n(E) = N - 1$

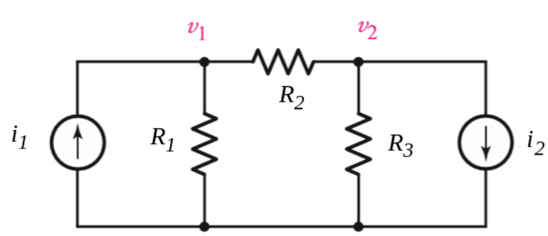
A circuit with N nodes and B branches needs to be solved for B voltage drops and B branch currents, i.e $2B$ variables in total.
Adding the independent equations obtained from KVL and KCL, B equations can be obtained. Then using element relationships (Ohm's Law), B more equations can be obtained and hence $2B$ variables can be solved for using the $2B$ equations.

1.4.1 Nodal Analysis

Nodal analysis is application of KCL on each nodes whose voltages (with respect to ground) are unknown to find out their values.
The node which has most number of branches connected to it is usually taken as the reference node (ground node).
Hence if the circuit has N nodes, KCL is written for $N - 1$ nodes.

KCL is most convenient to use when the circuit consists only of independent current sources and resistors.

Example for basic nodal analysis:



KCL at node 1: $i_1 = v_1/R_1 + (v_1 - v_2)/R_2$
KCL at node 2: $(v_2 - v_1)/R_2 + v_2/R_3 = -i_2$

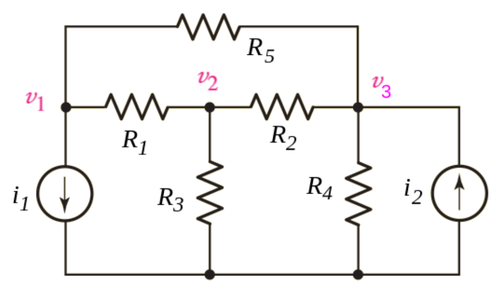
It is generally better to use conductances instead of resistances while performing nodal analysis for obvious reasons.
The conductance of each branch is simply the reciprocal of the resistance.

The equations representing the any circuit can be expressed in matrix form $[G][v] = [i]$ from nodal analysis. $[G]$ is the conductance matrix, $[v]$ is the vector of unknown voltages and $[i]$ is the vector of independent current sources.

$$\begin{bmatrix} G_1 + G_2 & -G_2 \\ -G_2 & G_2 + G_3 \end{bmatrix} \begin{bmatrix} v_1 \\ v_2 \end{bmatrix} = \begin{bmatrix} i_1 \\ -i_2 \end{bmatrix}$$

Using the equations (or matrix) obtained, the unknown node voltages can be found either by solving the set of linear equations or by matrix inversion. Once node voltages are found, voltage across any branch or current through any branch can be calculated.

More complicated circuit:



$$\begin{bmatrix} G_1 + G_5 & -G_1 & -G_5 \\ -G_1 & G_1 + G_2 + G_3 & -G_2 \\ -G_5 & -G_2 & G_2 + G_4 + G_5 \end{bmatrix} \begin{bmatrix} v_1 \\ v_2 \\ v_3 \end{bmatrix} = \begin{bmatrix} -i_1 \\ 0 \\ i_2 \end{bmatrix}$$

Observations:

- Diagonal elements will simply be the sum of conductances connected to the respective nodes. [Eg: $G_{11} = \Sigma\ G_{v1}$].

- Non-diagonal elements will be the negative of the conductances connected between nodes. [Eg: $G_{13} = -G_5$]

- Circuit with only independent sources and resistors will have symmetric conductance matrix. [Eg: $G_{12} = G_{21}$]
- The independent source vector will consist of currents pushed into the respective nodes via independent current sources.

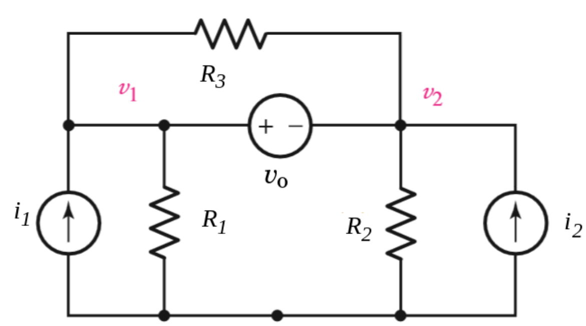
Some modifications are necessary if independent voltage sources are present in the circuit.

Supernode
This is a special case of nodal analysis where a voltage source exists between two nodes, both whose nodal voltages are unknown.
The branch consisting of the voltage source is treated as a single node and nodal analysis is performed with the supernode equation for simplification.

This method is derived by writing KCL equations as usual along with unknown currents that can't be found since they are flowing through voltage sources and adding the equations with unknown currents.
It can be noticed that the unknown currents cancel out and hence an equation with just node voltages, conductances and independent current sources exist will be obtained.
However, this will reduce the number of such effective nodal equations, which is compensated for by writing the voltage constraint equation using the node voltages and the voltage source between them.

In this case, the matrix $[G]$ will no longer just consist of conductances and the vector $[I]$ will have all independent sources (voltage and current).

Example of supernode analysis:



Here, the nodes corresponding to v_1 and v_2 are separated only by a voltage source. Hence the constraint equation is $v_1 - v_2 = v_0$.
The combined KCL equation is $v_1G_1 + v_2G_2 = i_1 + i_2$.
{ the individual equations before adding are $v_1G_1 + i_{12} + (v_1 - v_2)G_3 = i_1$ and $v_2G_2 - i_{12} - (v_1 - v_2)G_3 = i_2$ }

$$\implies \begin{bmatrix} G_1 & G_2 \\ 1 & -1 \end{bmatrix} \begin{bmatrix} v_1 \\ v_2 \end{bmatrix} = \begin{bmatrix} i_1 + i_2 \\ v_0 \end{bmatrix}$$

In case a voltage source is connected between a node and the reference node, the work is easier since the voltage at that node is equal to the voltage source. This will simply reduce the number of equations necessary to solve the circuit.

In the presence of dependent sources, the conduction matrix will not be symmetric because the current or voltage of a dependent source depends on the current or voltage present in some other part of the circuit, which disrupts the symmetry.

However, analysis can be done in similar way. The only extra step will be to express the dependent source in the relevant form.

- VCVS ($k_v v_x$): v_x is the voltage drop between nodes 1 and 2
  $\implies k_v v_x = k_v(v_1 - v_2)$
- CCVS ($R_m i_x$): i_x is the current flowing from node 1 to 2 through G_1
  $\implies R_m i_x = R_m G_1(v_1 - v_2)$
- VCCS ($G_m v_x$): v_x is the voltage drop between nodes 1 and 2
  $\implies G_m v_x = G_m(v_1 - v_2)$
- CCCS ($k_i i_x$): i_x is the current flowing from node 1 to 2 through G_1
  $\implies k_i i_x = k_i G_1(v_1 - v_2)$

Once the dependent sources are expressed as illustrated, then analysis reduces to the generic nodal analysis procedure (supernode treatment may be necessary for controlled voltage sources).
In case the dependent source depends on current through a voltage source, it can't be expressed in the relevant form for nodal analysis so different approach has to be taken only for this scenario.

1.4.2 Mesh Analysis

Loop analysis is application of KVL on all independent loops with loop currents as the unknowns and solving the equations to find the loop currents.

Mesh analysis is a variant of loop analysis applied only to planar circuits.
A planar circuit is a circuit which can be drawn on a plane such that no branches will cross/overlap each other.

If a circuit is planar, then meshes can be chosen without ambiguity. Note that loop analysis can be done for any circuit, but due to possibility of selecting independent loops in multiple ways, mesh analysis is preferred.

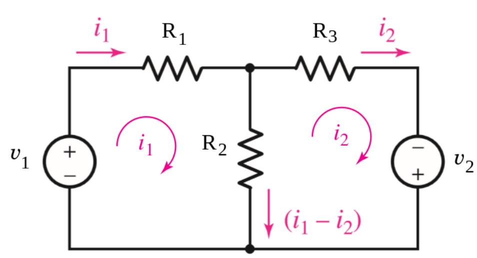
Hence, mesh analysis is the application of KVL to each of the meshes in a circuit whose mesh currents are unknown to find out their values.

KVL is most convenient to use when the circuit consists only of independent voltage sources and resistors.
Mesh currents are the currents assumed to be flowing in a mesh.

- If a branch is part of only 1 mesh, then the current through that branch will be equal to the mesh current.
- If a branch is part of 2 meshes, then the current through that branch will be equal to the difference between the two mesh currents.

(a branch can at most be part of only 2 meshes)

Example for basic mesh analysis:



KVL on mesh 1: $R_1 i_1 + R_2(i_1 - i_2) = v_1$
KVL on mesh 2: $R_3 i_2 + R_2(i_2 - i_1) = v_2$

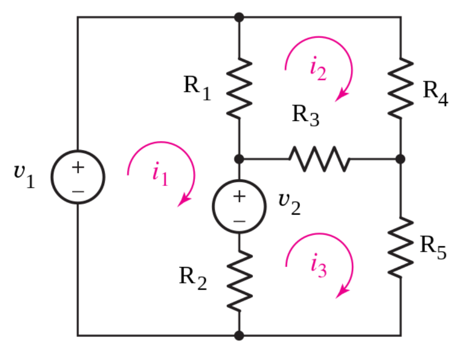
Unlike nodal analysis, it is more convenient to use resistors itself.

The equations representing any planar circuit can be expressed in matrix form $[R][i] = [v]$ from mesh analysis. $[R]$ is the resistance matrix, $[i]$ is the vector of unknown currents and $[v]$ is the vector of independent voltage sources.

$$\begin{bmatrix} R_1 + R_2 & -R_2 \\ -R_2 & R_2 + R_3 \end{bmatrix} \begin{bmatrix} i_1 \\ i_2 \end{bmatrix} = \begin{bmatrix} v_1 \\ v_2 \end{bmatrix}$$

Using the equations (or matrix) obtained, the unknown mesh currents can be found either by solving the set of linear equations or by matrix inversion. Once mesh currents are found, current through any branch or voltage across any branch can be calculated.

More complicated circuit:



$$\begin{bmatrix} R_1 + R_2 & -R_1 & -R_2 \\ -R_1 & R_1 + R_3 + R_4 & -R_3 \\ -R_2 & -R_3 & R_2 + R_3 + R_5 \end{bmatrix} \begin{bmatrix} i_1 \\ i_2 \\ i_3 \end{bmatrix} = \begin{bmatrix} v_1 - v_2 \\ 0 \\ v_2 \end{bmatrix}$$

Observations:

- Diagonal elements will simply be the sum of resistances present in the respective meshes. [Eg: $R_{11} = \Sigma R_{i1}$]
- Non-diagonal elements will be the negative of the resistances that are parts of 2 meshes. [Eg: $R_{12} = -R_1$]
- Circuit with only independent sources and resistors will have symmetric resistance matrix. [Eg: $R_{13} = R_{31}$]

- The independent source vector will consist of voltage rise provided by voltage sources in each mesh.

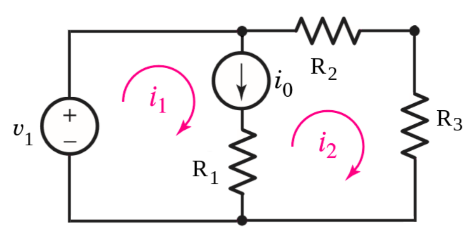
Some modifications are necessary if independent current sources are present in the planar circuit.

Supermesh
This is a special case of mesh analysis where a current source exists between two meshes, both whose mesh currents are unknown.
The entire loop consisting of multiple meshes that have current source between them is treated as a single mesh and mesh analysis is performed on the supermesh, with the common current source equation for simplification.

This method is derived by writing KVL equations as usual along with unknown voltages that can't be found since they are voltage drops across current sources and adding the equations with such unknown voltages.
It can be noticed that the unknown voltages cancel out and hence an equation with just mesh currents, resistances and independent voltage sources exist is obtained.
However, this will reduce the number of such effective mesh equations, which is compensated for by writing the current constraint equation using the mesh currents and the current source between them.

In this case, the matrix $[R]$ will no longer just consist of resistances and the vector $[v]$ will have all independent sources (voltage and current).

Example of supermesh:



Here, the branch with current source i_0 is common to both meshes consisting of currents i_1 and i_2. Hence the constraint equation is $i_1 - i_2 = i_0$.

The combined KVL equation is $R_2 i_2 + R_3 i_2 = v_1$.
{ the individual equations before adding are $v_{i_0} + R_1(i_1 - i_2) = v_1$ and $-v_{i_0} + R_1(i_2 - i_1) + R_2 i_2 + R_3 i_2 = 0$ }

$$\implies \begin{bmatrix} 0 & R_2 + R_3 \\ 1 & -1 \end{bmatrix} \begin{bmatrix} i_1 \\ i_2 \end{bmatrix} = \begin{bmatrix} v_1 \\ i_0 \end{bmatrix}$$

In case a current source is connected to a branch that is part of only one mesh, the work is easier since the mesh current is equal to the current source. This will simply reduce the number of equations necessary to solve the circuit.

In the presence of dependent sources, the resistance matrix will not be symmetric because the current or voltage of a dependent source depends on the current or voltage present in some other part of the circuit, which disrupts the symmetry.

However, analysis can be done in similar way. The only extra step will be to express the dependent source in the relevant form.

- VCVS ($k_v v_x$): v_x is the voltage drop across R_1 while current through it is $i_1 - i_2$
  $\implies k_v v_x = k_v R_1(i_1 - i_2)$
- CCVS ($R_m i_x$): i_x is same as branch current $i_1 - i_2$
  $\implies R_m i_x = R_m(i_1 - i_2)$
- VCCS ($G_m v_x$): v_x is the voltage drop across R_1 while current through it is $i_1 - i_2$
  $\implies G_m v_x = G_m R_1(i_1 - i_2)$
- CCCS ($k_i i_x$): i_x is same as branch current $i_1 - i_2$
  $\implies k_i i_x = k_i(i_1 - i_2)$

Once the dependent sources are expressed as illustrated, then analysis reduces to the generic nodal analysis procedure (supermesh treatment may be necessary for controlled voltage sources).
In case the dependent source depends on voltage across a current source, it can't be expressed in the relevant form for mesh analysis so different approach has to be taken only for this scenario.

2 Network Theorems

Network theorems in general are techniques used for either isolating specific parts of a circuit in order to simplify the circuit analysis or to quickly obtain some key information about a circuit by inspection.

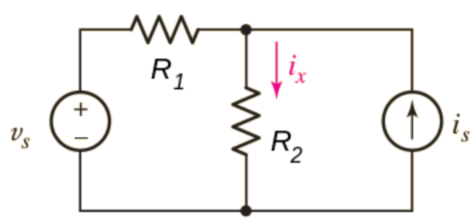
2.1 Superposition Theorem

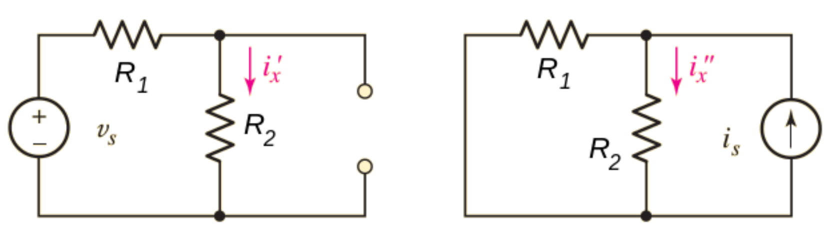
In a linear network with multiple independent sources, the response (i.e any current through a branch or voltage across 2 nodes) when all the sources are acting simultaneously is equal to the sum of individual responses calculated by taking one independent source at a time, while making the others inoperative.

- Ideal voltage source is made inoperative by replacing with short circuit.
- Ideal current source is made inoperative by replacing with open circuit.

Homogeneity Principle
In a linear network, if the excitation is multiplied by a constant factor, then the response is also multiplied by the same constant factor.

The following example illustrates that usage of superposition principle.





$i_x = i'_x + i''_x$ where $i'_x = \frac{v_s}{R_1+R_2}$ and $i''_x = i_s \times \frac{R_1}{R_1+R2}$

Superposition theorem can be applied to find current through any branch or voltage at any node (or across any branch) when multiple sources are present.

To find power dissipated/absorbed by an element, use superposition to find total current and voltage and multiply them.
Do not sum corresponding vi individual products, this will not work since power does not have linear relationship with the circuit elements.

2.2 Thevenin's and Norton's Theorems

Thevenin's Theorem
A linear network consisting of 2 terminals can be effectively replaced by the series combination of a voltage source V_{TH} and a resistance R_{TH}

Procedure to find V_{TH} and R_{TH}

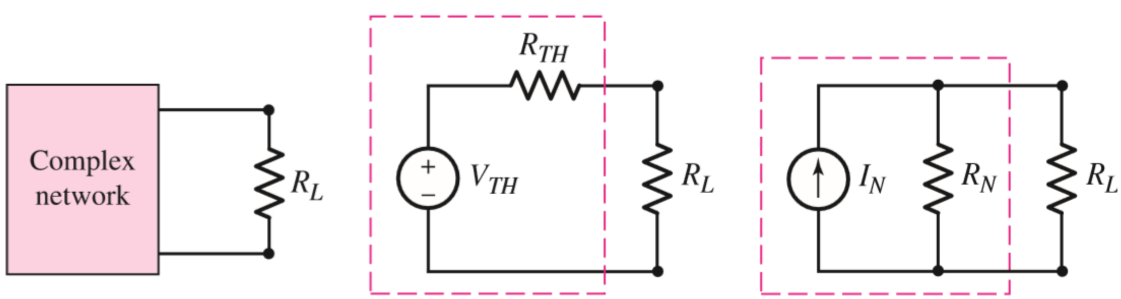
- Only independent sources:
  - Short all voltage sources, open all current sources and find the equivalent resistance across the 2 terminals to find R_{TH}.
  - Replace the load resistance with open circuit and find the open circuit voltage drop across the 2 terminals to find V_{TH}
- Only dependent sources:
  - Connect a voltage source V across the terminals which supplies current I; find $R_{TH} = V/I$
  - Since there is no independent source, there won't be any equivalent voltage source i.e it is a dead circuit so will only have resistance
- Both independent sources and dependent sources:
  - Short all voltage sources, open all current sources. Connect a voltage source V across the terminals which supplies current I; find $R_{TH} = V/I$
  - Replace the load resistance with open circuit and find the open circuit voltage drop across the 2 terminals to find V_{TH}

Norton's Theorem

A linear network consisting of 2 terminals can be effectively replaced by the parallel combination of a current source I_N and a resistance R_N

Procedure to find I_N and R_N

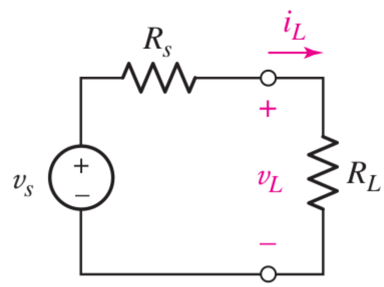
- Only independent sources:
  - Short all voltage sources, open all current sources and find the equivalent resistance across the 2 terminals to find R_N.
  - Replace load resistance with short circuit and find the short circuit current flowing between the 2 terminals to find I_N
- Only dependent sources:
  - Connect a voltage source V across the terminals which supplies current I; find $R_N = V/I$
  - Since there is no independent source, there won't be any equivalent current source i.e it is a dead circuit so will only have resistance
- Both independent sources and dependent sources:
  - Short all voltage sources, open all current sources. Connect a voltage source V across the terminals which supplies current I; find $R_{TH} = V/I$
  - Replace load resistance with short circuit and find the short circuit current flowing between the 2 terminals to find I_N



Note that Norton and Thevenin equivalents can be obtained from one another by using source transformation.

2.3 Maximum Power Transfer Theorem

Maximum power transfer theorem states that the DC voltage source will deliver maximum power to the variable load resistor only when the load resistance is equal to the source resistance.



For a given linear network, to find the load resistance at which maximum power transfer will occur, the Thevenin's equivalent (or Norton but Thevenin is more commonly used) of the network has to be found.

$\Longrightarrow$ For maximum power transfer, $R_L = R_{TH}$

The value of the maximum power is given by,

$$P_{max} = i_L^2 R_L = \frac{v_{TH}^2}{4R_L}$$

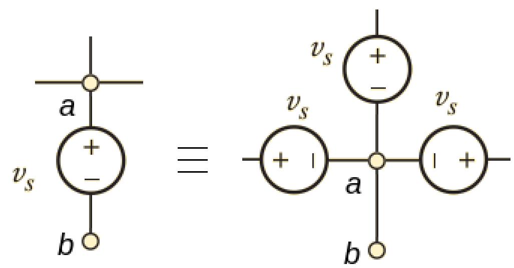
Note that this is applicable only when the load resistor is variable.

If the load is constant and some other resistor is to be varied, then maximum power transfer is obtained by changing the resistor such that maximum current flows through the constant load.

2.4 Other minor theorems

2.4.1 Pushing a voltage source into a node

Consider a case where a voltage source with terminals a and b is connected to multiple branches from the node terminal a.
The voltage source can be pushed into each of these branches without altering any other circuit parameter.

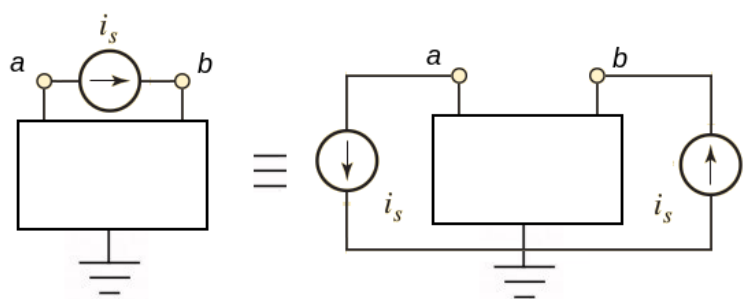


The illustrated systems are equivalent and hence one can replace the other based on convenience.
This works because different nodes that are fixed at the same voltage can be effectively shorted without changing anything else in a circuit.

2.4.2 Splitting a current source

A current source is equivalent to two current sources of the same value in series. If a current source is split in this way and an extra branch is taken from the middle i.e the node connecting the two sources in series, there will be no current flowing through this branch. Hence, it can be connected anywhere in the circuit without causing any changes to the circuit parameters.

Usually, this technique is used when a current source is connected between two non-reference nodes a and b. Here, the source is split and the middle point is connected to the reference node.



Upon rearranging this circuit, it can be observed that the same current source can be equivalently represented as two current sources (running in opposite directions), one between a and the reference node and another between b and the reference node.

2.4.3 Substitution Theorem

Any branch in a circuit can be substituted by either a voltage source or a current source.

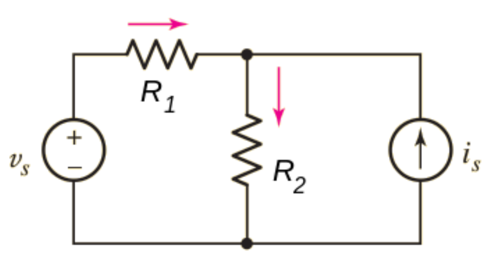
- To substitute a branch with a voltage source, find the voltage drop across the branch and use voltage source of the same value.
- To substitute a branch with a current source, find the current flowing through the branch and use current source of the same value.

A branch can also be replaced with a resistor assuming the branch is dissipating power. The equivalent resistance is found using Ohm's Law.
If power is being delivered in a branch, then it has to be substituted with a negative resistance, which does not directly exist physically.

The substitution theorem can be used to substitute a branch consisting of multiple elements also. Meaning, if the voltage difference across any two terminals is known, then all the circuitry between them can be replaced by a voltage source of that value and if the current flowing from one terminal to another is known, then all the circuitry between them can be replaced by a current source of that value.

2.4.4 Tellegen's Theorem

Tellegen's theorem simply states that the overall power consumed in a circuit is equivalent to the overall power delivered in a circuit.
Therefore, the algebraic sum of all powers in a circuit always equals to 0.

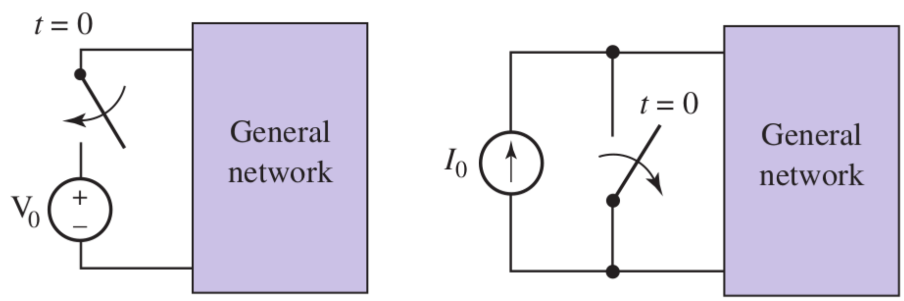


Power delivered by v_s is $v_s i_{R_1}$ and power delivered by i_s is $i_s v_{R_2}$.
Power absorbed by R_1 is $v_{R_1} i_{R_1}$ and power absorbed by R_2 is $v_{R_2} i_{R_2}$.
Hence, $v_s i_{R_1} + i_s v_{R_2} = v_{R_1} i_{R_1} + v_{R_2} i_{R_2}$

3 DC Transient Analysis

Transient condition occurs when switching action takes place in a circuit. A switch is typically connected between 2 nodes and can be either "on" or "off", with "on" indicating the 2 nodes are shorted (closed) and "off" indicating the 2 nodes are not connected (opened). A switch can also be such that it moves from one node to other.

Typically, the time at which switching action takes place is taken as $t = 0$. Just before switching occurs i.e $t = 0^-$, the circuit will be in steady state and just after switching occurs i.e $t = 0^+$, the circuit will be in transient state.



Steady state occurs when the circuit is left unchanged for a long period of time (ideally infinite time) so that the transients have died out. In steady state, the energy stored in the network is maximum and constant.

The behavior of components at transient and steady state have to analyzed in order to solve circuits with switching action.

Transient conditions in a circuit are caused by energy storage elements i.e capacitors and inductors since energy stored can not change instantaneously.

- The current flowing through an inductor can't change instantaneously.
  $\implies i_L(0^-) = i_L(0^+) = I_0$
- The voltage drop across a capacitor can't change instantaneously.
  $\implies v_C(0^-) = v_C(0^+) = V_0$

These can be proved using current-voltage relations, by taking integration from infinite past till just before and just after switching actions.

Note that exception occurs when there is an impulse in the inductor voltage or capacitor current.

The inductor current and capacitor voltage after switching action can be found as follows

$$i_L(t) = I_0 + \frac{1}{L}\int_0^t v_L t dt$$

I_0 is initial current through inductor found before switching action

$$v_C(t) = V_0 + \frac{1}{C}\int_0^t i_C t dt$$

V_0 is initial voltage across capacitor found before switching action

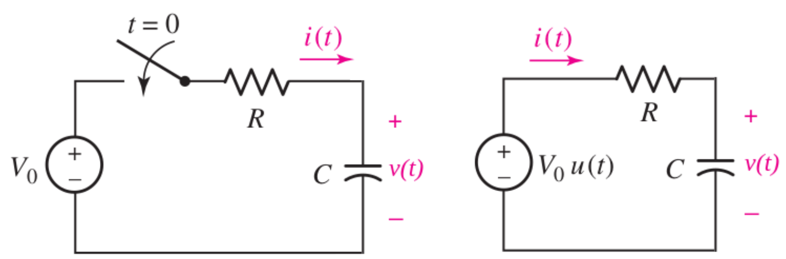
- Inductor in steady state has maximum constant current flowing through it, hence it behaves like a short circuit.
- Capacitor in steady state has maximum constant voltage drop across it, hence it behaves like an open circuit.
- Inductor in transient state has the exact same current flowing through it as before switching.
  - If initial condition is zero, the inductor in transient state behaves like an open circuit.
  - If initial condition is non-zero, the inductor in transient state behaves like a current source
- Capacitor in transient state has the exact same voltage drop across it as before switching.
  - If initial condition is zero, the capacitor in transient state behaves like a short circuit.
  - If initial condition is non-zero, the capacitor in transient state behaves like a voltage source.

The above observations hold true as long as there is no impulse function involved.

3.1 First order switching circuits

First order circuits are those circuits which effectively have one energy storage element.

3.1.1 RC Circuit



Initial condition: $v(0^-) = V_c$

Analysis using differential equation approach:
$V_0 u(t) = i(t)R + v(t)$; $i(t) = C\frac{dv}{dt}$

$$\implies RC\frac{dv}{dt} + v(t) = V_0 u(t)$$

Taking Laplace transform on both sides will give,

$$RC(sV(s) - V_c) + V(s) = \frac{V_0}{s} \qquad \implies (RCs+1)V(s) = \frac{V_0}{s} + RCV_c$$

$$\implies V(s) = \frac{V_0}{s(RCs+1)} + \frac{RCV_c}{RCs+1} = \frac{V_0}{sRC(1+\frac{1}{RC})} + \frac{V_c}{s+\frac{1}{RC}}$$

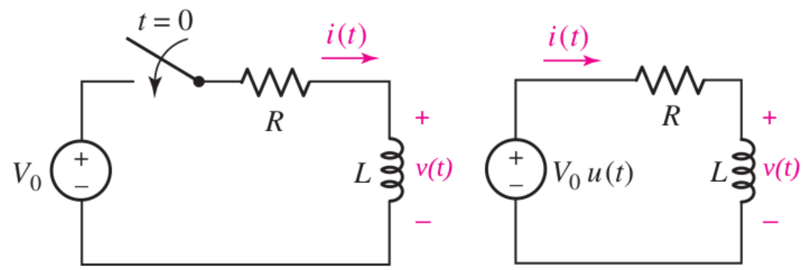
By taking inverse Laplace transform after splitting to partial fractions, the time domain expression for $i(t)$ is obtained as

$$i(t) = V_0(1 - e^{-\frac{t}{RC}})u(t) + V_C\, e^{-\frac{t}{RC}}u(t)$$

Here, $V_0(1 - e^{-\frac{t}{RC}})u(t)$ is called "forced response" because it is caused by the applied input and $V_C\, e^{-\frac{t}{RC}}u(t)$ is called "natural response" because it is caused by initial conditions and circuit behavior alone.

Note that the same expression can be written as $v(t) = V_0 - (V_0 - V_C)e^{-\frac{t}{RC}}u(t)$. where V_0 is the final value $v(\infty)$, V_C is the initial value $v(0)$ and $\frac{1}{RC}$ is the time constant τ.

3.1.2 RL Circuit



Initial condition: $i(t = 0-) = I_l$

Analysis using differential equation approach:
$V_0 u(t) = i(t)R + v(t)$; $v(t) = L\frac{di}{dt}$

$$\implies \frac{L}{R}\frac{di}{dt} + i(t) = \frac{V_0}{R}u(t)$$

Taking Laplace transform on both sides will give,

$$\frac{L}{R}(sI(s) - I_l) + I(s) = \frac{V_0}{Rs} \qquad \implies (\frac{L}{R}s + 1)I(s) = \frac{V_0}{Rs} + \frac{L}{R}I_l$$

$$\implies I(s) = \frac{V_0}{Rs(\frac{L}{R}s + 1)} + \frac{LV_c}{R(\frac{L}{R}s + 1)} = \frac{V_0}{sL(1 + \frac{R}{L})} + \frac{V_c}{s + \frac{R}{L}}$$

By taking inverse Laplace transform after splitting to partial fractions, the time domain expression for $i(t)$ is obtained as

$$i(t) = \frac{V_0}{R}(1 - e^{-\frac{tR}{L}})u(t) + I_l\, e^{-\frac{tR}{L}}u(t)$$

Here, $\frac{V_0}{R}(1 - e^{-\frac{tR}{L}})u(t)$ is called "forced response" because it is caused by the applied input and $I_l\, e^{-\frac{tR}{L}}u(t)$ is called "natural response" because it is caused by initial conditions and circuit behavior alone.

Note that the same expression can be written as $i(t) = I_0 - (I_0 - I_l)e^{-\frac{tR}{L}}u(t)$. where $I_0 = \frac{V_0}{R}$ is the final value $i(\infty)$, I_l is the initial value $i(0)$ and $\frac{R}{L}$ is the time constant τ.

3.1.3 Arbitrary Circuit Procedure

For an arbitrary driven RC or RL or RLC circuit, to find the required current or voltage, the general procedure to be followed is-

- Replace capacitors and inductors with steady state equivalents (open and short circuits respectively) to find the voltage across capacitors and current through inductors at $t = 0^-$. These will give initial conditions after switching.

- Once switching occurs, replace capacitors and inductors with transient equivalents using the initial conditions found in prior step. Find the required voltage or current at $t = 0^+$.
  (if the required current is inductor current or required voltage is capacitor voltage, then the value will be same as initial condition value)

  - In case multiple capacitors exist, then capacitor voltages at $t = 0^+$ can be found by opening all the resistors and using capacitive divider rule.
  - In case multiple inductors exist, then inductor currents at $t = 0^+$ can be found by shorting all the resistors and using inductive divider rule.

- Finally, replace capacitors and inductors with steady state equivalents (open and short circuits respectively) once again to obtain the circuit at $t \to \infty$. Find the final value of required voltage or current.

- In-operate all sources and find the time constant τ of the circuit post switching ($\tau = R_{eq}C_{eq}$ or $\tau = L_{eq}/R_{eq}$).

- Find required voltage or current using the formula:

  $$\boxed{x(t) = x(\infty) - [x(\infty) - x(0^+)]\ e^{-\frac{t}{\tau}}}$$

  where $x(t)$ is the required current or voltage for $t > 0$.

Note that this is applicable only if the components are separable such that the time constant can be found. Otherwise, Laplace Transform approach has to be taken.

4 Sinusoidal Steady State Analysis

So far, circuits have been analysed only for DC excitation (i.e constant sources). Another form of excitation that is widely used is sinusoidal or AC excitation.

AC voltage source is generally of the form $v(t) = V_m cos(\omega t + \phi)$ and AC current source is generally of the form $i(t) = I_m cos(\omega t + \psi)$.

In linear circuits, for a sinusoidal source, the response will also be sinusoidal in nature with the same frequency but different amplitude and phase.

For convenience of notation, $v(t) = Re[V_m e^{j\omega t + \phi}]$ from which, the phasor notation is obtained and hence expressed as $V_m e^{j\phi}$ or $V_m \angle \phi$.

4.0.1 Electrical components for AC input

The basic electrical components discussed at the start are best analysed using the phasor approach if sinusoidal input is provided.
For an input of the form $v(t) = V_m cos(\omega t)$ or $i(t) = I_m cos(\omega t)$, the behaviour of the 3 basic components is illustrated as follows.

First of all, the input is expressed using complex exponentials, whose real part can be extracted to get the desired result.

Resistor
Consider the voltage across a resistor to be $v_r(t) = V_m e^{j\omega t}$ and current through it to be $i_r(t) = I_m e^{j\omega t}$. From Ohm's Law, $v_r(t) = i_r(t)R$.

Hence, the behaviour of resistor (and a conductor) is exactly the same for AC as it was for DC.

Capacitor
Consider the voltage across a capacitor to be $v_c(t) = V_m e^{j\omega t}$ and current through it to be $i_c(t) = I_m e^{j\omega t}$. From Ohm's Law, $i_c(t) = v_c'(t)C$.

$$\frac{dv_c(t)}{dt} = V_m \frac{e^{j\omega t}}{dt} = j\omega V_m e^{j\omega t} = j\omega v_c(t)$$

The above result will give a new linear relation between capacitor voltage and current.

$$v_c(t) = \frac{i_c(t)}{j\omega C} = X_c i_c(t)$$

where X_c is called the 'capacitive reactance'.

Hence, a capacitor behaves like a resistor of value $1/j\omega C$ for sinusoidal excitations. It can also be noted that the capacitor current will lag behind the capacitor voltage by 90^o.

Inductor
Consider the voltage across an inductor to be $v_c(t) = V_m e^{j\omega t}$ and current through it to be $i_c(t) = I_m e^{j\omega t}$. From Ohm's Law, $v_c(t) = i'_c(t)L$.

$$\frac{di_c(t)}{dt} = I_m \frac{e^{j\omega t}}{dt} = j\omega I_m e^{j\omega t} = j\omega i_c(t)$$

The above result will give a new linear relation between inductor voltage and current.

$$v_c(t) = i_c(t) j\omega C = X_l i_c(t)$$

where X_l is called the 'inductive reactance'.

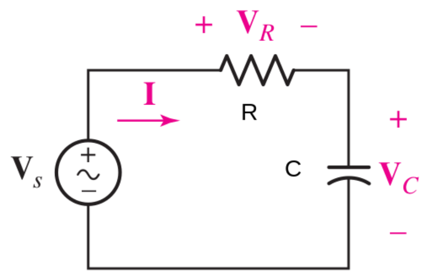
Hence, an inductor behaves like a resistor of value $j\omega L$ for sinusoidal excitations. It can also be noted that the inductor current will lead ahead of the inductor voltage by 90^o.

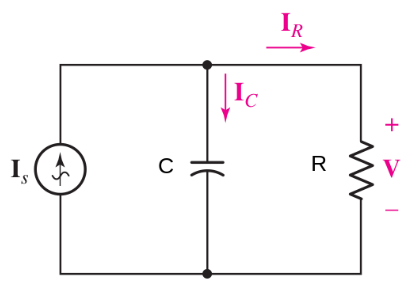
4.1 First order circuits: sinusoidal input

Behaviour of first order circuits for sinusoidal (or complex exponential) inputs is studied in detail.

4.1.1 RC Circuit

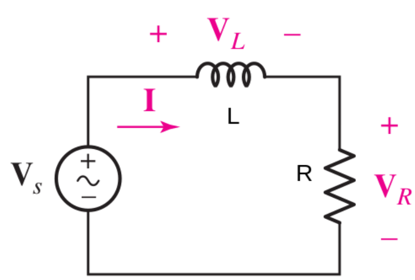
Consider an RC circuit with exponential excitation voltage $V_s = V_p e^{st}$ where s is a complex number and capacitor voltage initially is V_0.

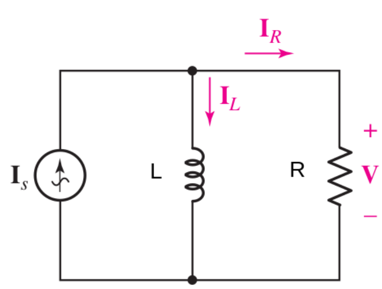




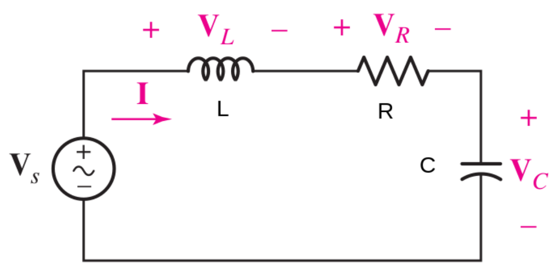
4.1.2 RL Circuit

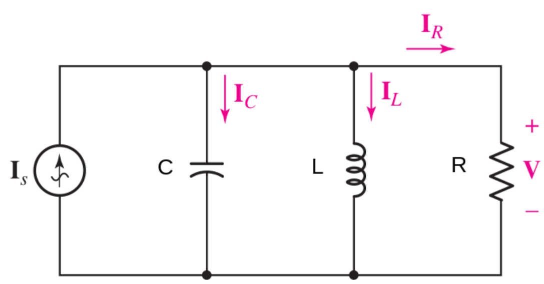
Consider an RC circuit with exponential excitation voltage $V_s = V_p e^{st}$ where s is a complex number.





4.2 Second order circuits: sinusoidal input





4.3 Resonance

4.4 Maximum Power Transfer Theorem

5 Two Port Networks

A pair of terminals at which a signal may enter or leave a network is called a port, and a network having two such pairs of terminals is called a two-port network.

A two-port network can have either 3 or 4 terminals. A 3 terminal two-port network will have 1 common terminal between both the ports (which is usually taken as reference terminal).
Such a network can be characterized by a set of parameters, which define the overall behaviour of the network.

Different possibilities of defining two port network parameters are using terminal voltages and currents as the applied and/or measured quantities are:

Applied quantities	Measured quantities	Parameters name
V_1 V_2	I_1 I_2	Admittance (Y) parameters
I_1 I_2	V_1 V_2	Impedance (Z) parameters
V_1 I_2	I_1 V_2	Inverse Hybrid (g) parameters
I_1 V_2	V_1 I_2	Hybrid (h) parameters
V_1 I_1	V_2 I_2	Inverse Transmission (T^{-1}) parameters
V_2 I_2	V_1 I_1	Transmission (T) parameters

Each of these parameters give the measured quantities as some linear combination of the applied quantities.

There is another different set of parameters called Scattering (S) parameters which is defined using powers rather than voltages and currents (covered in detail at the end).

In a **Reciprocal** two-port network, the response at port 2 due to source at port 1 is the same as the response at port 1 due to same source at port 2. Meaning, the ratio of response to source is the same when their positions are interchanged.

In a **Symmetric** two-port network, the input impedance offered is same as the output impedance offered.

5.1 Z-Parameters

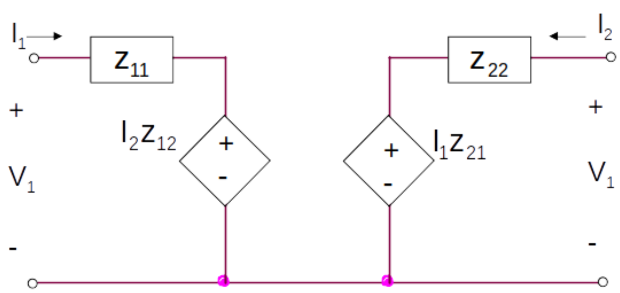
Z-parameters are also called "Open Circuit Impedance Parameters".

The equations used to obtain Z-parameters are:

$$V_1 = z_{11}I_1 + z_{12}I_2$$

$$V_2 = z_{21}I_1 + z_{22}I_2$$

$$\implies \begin{bmatrix} V_1 \\ V_2 \end{bmatrix} = \begin{bmatrix} z_{11} & z_{12} \\ z_{21} & z_{22} \end{bmatrix} \begin{bmatrix} I_1 \\ I_2 \end{bmatrix}$$



Hence, the Z-parameters are defined as:

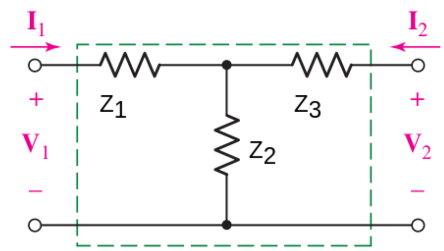
$z_{11} = \frac{V_1}{I_1}|_{I_2=0}$ $z_{12} = \frac{V_1}{I_2}|_{I_1=0}$ $z_{21} = \frac{V_2}{I_1}|_{I_2=0}$ $z_{22} = \frac{V_2}{I_2}|_{I_1=0}$

$z_{11} \rightarrow$ Open Circuit Input Impedance
$z_{12} \rightarrow$ Open Circuit Transfer Impedance from port 1 to port 2
$z_{21} \rightarrow$ Open Circuit Transfer Impedance from port 2 to port 1
$z_{22} \rightarrow$ Open Circuit Output Impedance

Condition for reciprocity: $z_{12} = z_{21}$

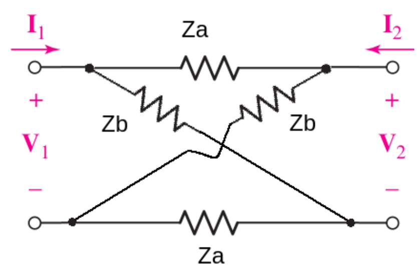
Condition for symmetry: $z_{11} = z_{22}$

T (or Star) Network-



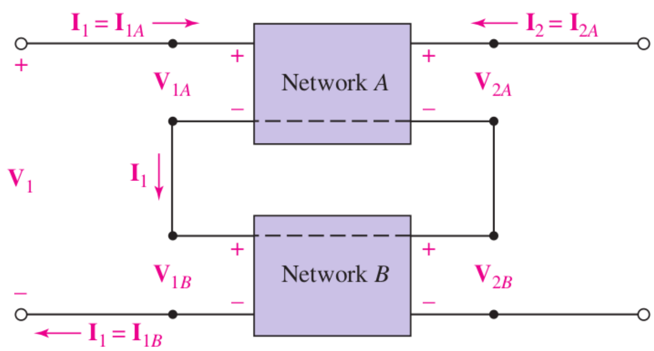
$$Z = \begin{bmatrix} Z_1 + Z_2 & Z_2 \\ Z_2 & Z_2 + Z_3 \end{bmatrix}$$

Cross Network-



$$Z = \begin{bmatrix} (Z_a + Z_b)/2 & (Z_b - Z_a)/2 \\ (Z_b - Z_a)/2 & (Z_a + Z_b)/2 \end{bmatrix}$$

Series Combination of 2 two-port networks-



Z parameters of the overall network is obtained by adding the Z parameters of the individual networks.
$\Longrightarrow$ $[Z] = [Z_A] + [Z_B]$

5.2 Y-Parameters

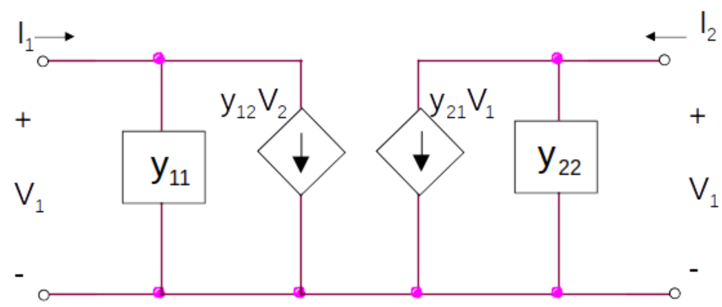
Y-parameters are also called "Short Circuit Admittance Parameters".

The equations used to obtain Y-parameters are:

$$I_1 = y_{11}V_1 + y_{12}V_2$$

$$I_2 = y_{21}V_1 + y_{22}V_2$$

$$\Longrightarrow \begin{bmatrix} I_1 \\ I_2 \end{bmatrix} = \begin{bmatrix} y_{11} & y_{12} \\ y_{21} & y_{22} \end{bmatrix} \begin{bmatrix} V_1 \\ V_2 \end{bmatrix}$$



Hence, the Y-parameters are defined as:

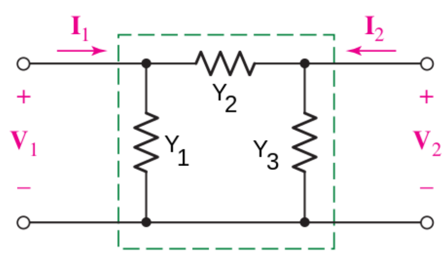
$y_{11} = \frac{I_1}{V_1}|_{V_2=0} \quad y_{12} = \frac{I_1}{V_2}|_{V_1=0} \quad y_{21} = \frac{I_2}{V_1}|_{V_2=0} \quad y_{22} = \frac{I_2}{V_2}|_{V_1=0}$

$y_{11} \rightarrow$ Short Circuit Input Admittance
$y_{12} \rightarrow$ Short Circuit Transfer Admittance from port 1 to port 2
$y_{21} \rightarrow$ Short Circuit Transfer Admittance from port 2 to port 1
$y_{22} \rightarrow$ Short Circuit Output Admittance

Condition for reciprocity: $y_{12} = y_{21}$

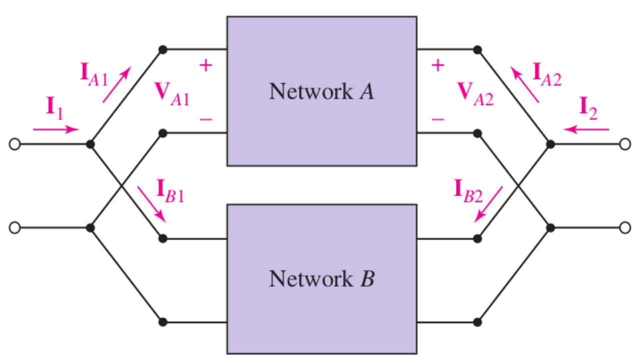
Condition for symmetry: $y_{11} = y_{22}$

Pi (or Delta) Network-



$$Y = \begin{bmatrix} Y_1 + Y_2 & -Y_2 \\ -Y_2 & Y_2 + Y_3 \end{bmatrix}$$

Parallel Combination of 2 two-port networks-



Y parameters of the overall network is obtained by adding the Y parameters of the individual networks.
$\implies [Y] = [Y_A] + [Y_B]$

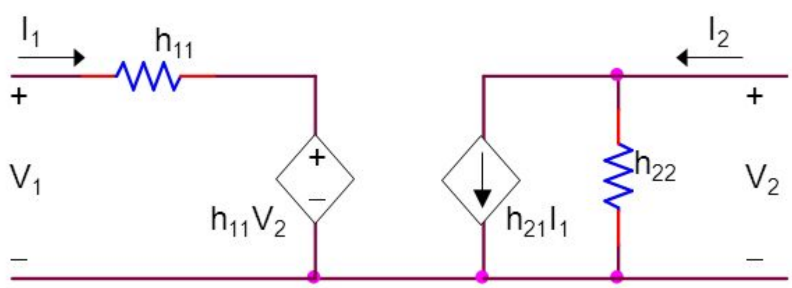
5.3 Hybrid (h) Parameters

hybrid parameters are defined by taking input voltage and output current as the dependent variables.

The equations used to obtain h-parameters are:

$$V_1 = h_{11}I_1 + h_{12}V_2$$
$$I_2 = h_{21}I_1 + y_{22}V_2$$

$$\implies \begin{bmatrix} V_1 \\ I_2 \end{bmatrix} = \begin{bmatrix} h_{11} & h_{12} \\ h_{21} & h_{22} \end{bmatrix} \begin{bmatrix} I_1 \\ V_2 \end{bmatrix}$$



Hence, the h-parameters are defined as:

$h_{11} = \frac{V_1}{I_1}|_{V_2=0}$ $h_{12} = \frac{V_1}{V_2}|_{I_1=0}$ $h_{21} = \frac{I_2}{I_1}|_{V_2=0}$ $h_{22} = \frac{I_2}{V_2}|_{I_1=0}$

$h_{11} \rightarrow$ Short Circuit Input Impedance
$h_{12} \rightarrow$ Open Circuit Reverse Voltage Gain
$h_{21} \rightarrow$ Short Circuit Forward Current Gain
$h_{22} \rightarrow$ Open Circuit Output Admittance

Condition for reciprocity: $h_{12} = -h_{21}$

Condition for symmetry: $h_{11}h_{22} - h_{12}h_{21} = 1$

5.4 Transmission (T) or ABCD-Parameters

Transmission parameters are defined by taking input voltage and input current as the dependent variables.

The equations used to obtain ABCD-parameters are:

$$V_1 = AV_2 - BI_2$$

$$I_1 = CV_2 - DI_2$$

$$\implies \begin{bmatrix} V_1 \\ I_1 \end{bmatrix} = \begin{bmatrix} A & B \\ C & D \end{bmatrix} \begin{bmatrix} V_2 \\ -I_2 \end{bmatrix}$$

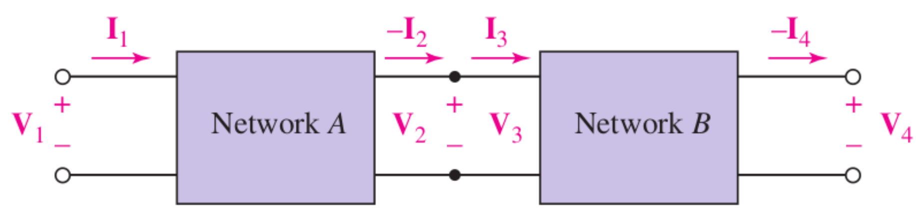
Hence, the ABCD-parameters are defined as:

$$A = \frac{V_1}{V_2}|_{I_2=0} \quad B = -\frac{V_1}{I_2}|_{V_2=0} \quad C = \frac{I_1}{V_2}|_{I_2=0} \quad D = -\frac{I_1}{I_2}|_{V_2=0}$$

Condition for reciprocity: $AD - BC = 1$

Condition for symmetry: $A = D$

Cascade Combination of 2 two-port networks-



ABCD parameters of the overall network is obtained by multiplying the ABCD parameter matrices of the individual networks.
$\implies [T] = [T_A][T_B]$

5.5 Scattering (S) Parameters

All the parameters discussed so far are only applicable to lumped two port networks. In case of distributed networks, the description using current and

voltage is not suitable; electric and magnetic fields have to be used.
Since power is product of the electric and magnetic fields, the parameters obtained using incident power and reflected power are used. These are called Scattering Parameters.

Note that S-parameters are applicable for both lumped and distributed networks and S-parameters are also defined for multi-port networks with more than two ports.

To define S-parameters of a two port network, new variables are defined.

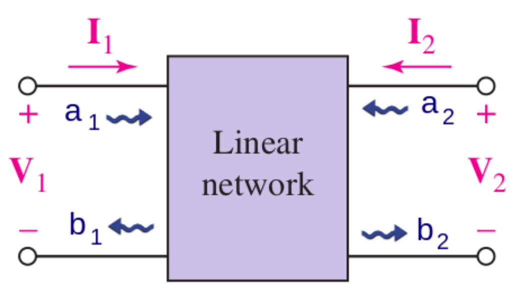
$$a_1, b_1 = \frac{1}{2}[\frac{V_1}{\sqrt{R_{01}}} \pm I_1\sqrt{R_{01}}]$$

where R_{01} is the reference resistance for port 1

$$a_2, b_2 = \frac{1}{2}[\frac{V_2}{\sqrt{R_{02}}} \pm I_2\sqrt{R_{02}}]$$

where R_{02} is the reference resistance for port 2

a_1 is the incident wave on port 1; b_1 is the reflected wave from port 1
a_2 is the incident wave on port 2; b_2 is the reflected wave from port 1



S-parameters are defined by finding b_1 and b_2 in terms of a_1 and a_2 under different conditions.

$$b_1 = s_{11}a_1 + s_{12}a_2$$

$$b_2 = s_{21}a_1 + s_{22}a_2$$

$$\implies \begin{bmatrix} b_1 \\ b_2 \end{bmatrix} = \begin{bmatrix} s_{11} & s_{12} \\ s_{21} & s_{22} \end{bmatrix} \begin{bmatrix} a_1 \\ a_2 \end{bmatrix}$$

Hence, the S-parameters are defined as:

$s_{11} = \frac{b_1}{a_1}|_{a_2=0}$ $\quad s_{12} = \frac{b_1}{a_2}|_{a_1=0}$ $\quad s_{21} = \frac{b_2}{a_1}|_{a_2=0}$ $\quad s_{22} = \frac{b_2}{a_2}|_{a_1=0}$

Terminating conditions:

- $a_2 = 0$ when the terminating resistance at port 2 is equal to reference resistance of port 2 i.e $R_2 = R_{02}$
- $a_1 = 0$ when the terminating resistance at port 1 is equal to reference resistance of port 1 i.e $R_1 = R_{01}$

(hence, the terminating conditions are different from the usual short circuit or open circuit methods)

$s_{11} \rightarrow$ Reflection Coefficient at port 1
$s_{12} \rightarrow$ Forward Transmission Coefficient
$s_{21} \rightarrow$ Reverse Transmission Coefficient
$s_{22} \rightarrow$ Reflection Coefficient at port 2