Electromagnetic Field Theory

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1 Transmission Lines

A transmission line is a medium used to transfer power from one point to another.
Usually, a transmission line consists of two (or more) parallel conductors connecting the source to the load.

Examples: Co-axial line, micro-strip line, two-wire line.

Validity of Kirchhoff laws

The transit time in a circuit is the time that is required for signal to move from one point to another.
Transit time is given by $t_r = l/v$ where l is length of wire and v is velocity of propagation of signal.

- Transit time effect can be ignored if time period of signal is much greater than transit time or if wavelength of signal is much greater than dimensions of elements.
- Transit time effect has to be considered if time period of signal is comparable to the transit time or if wavelength of signal is comparable to the dimensions of elements.

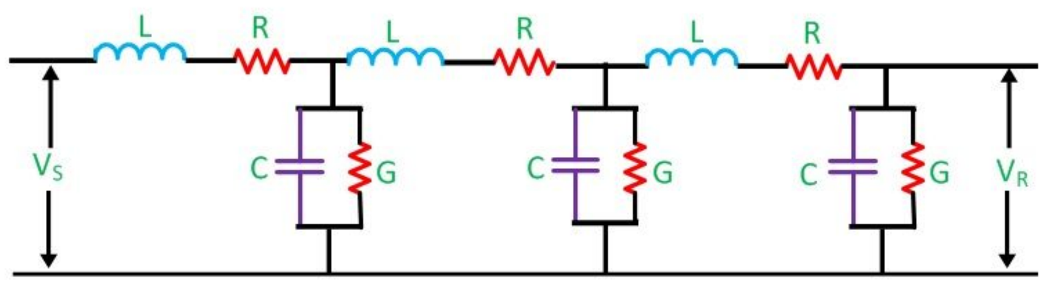
Meaning, if the dimensions are longer or if signals are of high frequencies (such as in transmission lines), the transit time can't be neglected. Hence, the space variable will play a role in transmission line analysis.

Note that KVL and KCL will be valid if voltages and currents are taken as functions of both time and space.

1.1 Transmission Line Parameters

Since currents and voltages are treated as functions of both time and length, the transmission line parameters being defined as quantities per unit length is more appropriate.

The changing currents through the conductors induce magnetic fields and the changing voltages between the conductors induce electric fields.
Meaning, there will be inductance and capacitance elements along the entire structure in a parallel conductor transmission line system.
Also, since no conductor is ideal and no dielectric between the parallel conductors is ideal, there will be resistive and conductive components throughout the structure as well.
These elements are called "Distributed elements".

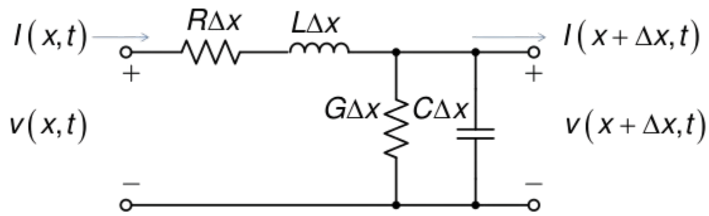


$R \rightarrow$ Resistance per unit length in Ω/m
$L \rightarrow$ Inductance per unit length in H/m
$G \rightarrow$ Conductance per unit length $\mho/m$
$C \rightarrow$ Capacitance per unit length F/m

Consider a sinusoidal voltage V applied at the source of angular frequency ω which causes sinusoidal current flow I.

In order to apply KVL and KCL to get voltage and current equations, a small differential length Δx of the transmission line is considered.

If limit as Δx tends to 0 is taken, then it can be observed that the equations will be differential and not algebraic.



$$\Delta V = -(R\Delta x + j\omega L\Delta x)I \quad \implies \frac{dV}{dx} = -(R + j\omega L)I$$

$$\Delta I = -(G\Delta x + j\omega C\Delta x)V \quad \implies \frac{dI}{dx} = -(G + j\omega C)V$$

Upon differentiation the equations a second time and substitution,

$$\frac{d^2V}{dx^2} = (R + j\omega L)(G + j\omega C)V$$

$(R + j\omega L)(G + j\omega C) = \gamma^2$ where γ is called the "Propagation Constant".

$$\implies \frac{d^2V}{dx^2} = \gamma^2 V; \quad \frac{d^2I}{dx^2} = \gamma^2 I$$

Upon solving the above 2^{nd} order differential equations, the solutions will be of the form

$$V(x) = V^+e^{-\gamma x} + V^-e^{\gamma x}$$

$$I(x) = I^+e^{-\gamma x} + I^-e^{\gamma x}$$

Since γ is a complex quantity, it can be represented as $\gamma = \alpha + j\beta$ where α is called "Attenuation Constant" and β is called "Phase Constant".

If they are to be expressed in terms of both time and space variables,

$$V(x,t) = Re[(V^+e^{-\gamma x}+V^-e^{\gamma x})\, e^{j\omega t}] = V^+e^{-\alpha x}cos(\omega t-\beta x)+V^-e^{\alpha x}cos(\omega t+\beta x)$$

$$I(x,t) = Re[(I^+e^{-\gamma x}+I^-e^{\gamma x})\, e^{j\omega t}] = I^+e^{-\alpha x}cos(\omega t-\beta x)+I^-e^{\alpha x}cos(\omega t+\beta x)$$

Therefore, the voltage or current at any point in the transmission line is a sum of travelling waves.

$e^{-\alpha x}$ indicates exponential decay in positive x direction.
$e^{\alpha x}$ indicates exponential decay in negative x direction.
$cos(\omega t - \beta x)$ indicates travelling wave in positive x direction.
$cos(\omega t + \beta x)$ indicates travelling wave in negative x direction.

β is the phase change per unit length, and hence is in rad/m.
The phase change for 1 wavelength of the wave λ is equal to 2π.
$\implies \beta = 2\pi/\lambda$

α is the distance at which the voltage/current is attenuated to $1/e$ times the initial value (at $x = 0$), and is in Nep/m or in dB/m.
$1\ Nep/m \ = \ -20\log_{10}(1/e)\ dB/m \ = \ 8.68\ dB/m$

Substituting for V and I in the original differential equations, the following relations are obtained.

$$\frac{V^+}{I^+} = \sqrt{\frac{R + j\omega L}{G + j\omega C}} \qquad \frac{V^-}{I^-} = -\sqrt{\frac{R + j\omega L}{G + j\omega C}}$$

A new quantity called "Characteristic Impedance" (Z_0) is defined.
$Z_0 = \sqrt{\frac{R+j\omega L}{G+j\omega C}}$

$$\implies V^+ = Z_0 I^+ \qquad V^- = -Z_0 I^-$$

$$\therefore V(x) = V^+e^{-\gamma x} + V^-e^{\gamma x} \qquad I(x) = \frac{V^+}{Z_0}e^{-\gamma x} - \frac{V^-}{Z_0}e^{\gamma x}$$

Here, the V^+ term represents the transmitted wave travelling in the positive x direction (i.e source to load) and the V^- term represents the reflected wave travelling in the negative x direction (i.e load to source).

At boundary condition where $l = 0 (l = -x)$, the transmission line is terminated by load impedance Z_L.
$Z_L = Z_0 \frac{V^+ + V^-}{V^+ - V^-}$

The "Reflection Coefficient" is defined as the ratio of reflected wave to the transmitted wave.
$\Gamma_L = V^- / V^+$ (at the boundary)

$$\implies Z_L = Z_0 \frac{1 + \Gamma_L}{1 - \Gamma_L}$$

$$\therefore \Gamma_L = \frac{Z_L - Z_0}{Z_L + Z_0}$$

The above equation tells that $\Gamma_L = 0$ if $Z_L = Z_0$. Meaning, when the load impedance is equal to the characteristic impedance, then the reflected wave does not exist and the entire source is transmitted to the load.
$Z_L = Z_0$ is called "Matched Load Condition".

$\Gamma(l)$ is the reflection coefficient at some length l from the load. And the above relations can be used to find this quantity or this quantity can be used to find the impedance at any point on the transmission line.
$\Gamma(l) = \frac{V^- e^{\gamma l}}{V^+ e^{-\gamma l}}$

$\implies Z(l) = Z_0 \frac{1 + \Gamma(l)}{1 - \Gamma(l)}$ and $\Gamma(l) = \frac{Z(l) - Z_0}{Z(l) + Z_0}$

The reflection coefficient in terms of power is given by the square root of ratio of reflected power and incident power. $\Gamma = \sqrt{\frac{P_r}{P_i}}$

Note that $0 \leq |\Gamma_L| \leq 1$

The transmitted signal will not be absorbed by the load and hence the entire signal will be reflected back if $|\Gamma_L| = 1$ which will happen if any of the following conditions are met.

- If $Z_L = 0$ i.e the TL is terminated by a short circuit, then $\Gamma_L = -1$ and hence $|\Gamma_L| = 1$

- If $Z_L = \infty$ i.e the TL is terminated by an open circuit, then $\Gamma_L = 1$ and hence $|\Gamma_L| = 1$
- If $Z_L = jX$ i.e the TL is terminated by a purely reactive load, then $\Gamma_L = \frac{Z_0 - jX}{Z_0 + jX}$ and hence $|\Gamma_L| = 1$

Impedance Transformation Relation:

$$Z(l) = Z_0 \, \frac{Z_L \cosh \gamma l + Z_0 \sinh \gamma l}{Z_0 \cosh \gamma l + Z_L \sinh \gamma l}$$

Normalized impedance is the ratio of any impedance to the characteristic impedance. $\implies \bar{Z} = Z/Z_0$
Using normalization, the impedance transformation relation becomes:

$$\bar{Z}(l) = \frac{\bar{Z}_L \cosh \gamma l + \sinh \gamma l}{\cosh \gamma l + \bar{Z}_L \sinh \gamma l}$$

This means that the impedance at any point on the transmission line is a function of the length at which it is measured.
However if $Z_L = Z_0$, then $\bar{Z}_L = 1$ and hence $\bar{Z}(l) = 1$.
Therefore, under matched load condition, the impedance at any and every point on the transmission line is the same and is equal to the characteristic impedance.

The Characteristic Impedance is defined as that value of load impedance at which the impedance measured at any point on the line is same as the load impedance, independent of the length.

The impedance transformation relation can be generalized for impedances at any two points that are separated by a distance l

$$Z_2 = Z_0 \, \frac{Z_1 \cosh \gamma l + Z_0 \sinh \gamma l}{Z_0 \cosh \gamma l + Z_1 \sinh \gamma l} \quad or \quad Z_1 = Z_0 \, \frac{Z_2 \cosh \gamma l - Z_0 \sinh \gamma l}{Z_0 \cosh \gamma l - Z_2 \sinh \gamma l}$$

1.1.1 Loss-less Transmission Line

A loss-less transmission line will have no power loss during transmission of power.

This occurs when the values of R and G are equal to 0.

$R = 0,\ G = 0 \quad \implies \gamma = \sqrt{j\omega L\ j\omega C} = j\omega\ \sqrt{LC}$
$\therefore \alpha = 0$ and $\beta = \omega\sqrt{LC}$
There is no attenuation in a loss-less transmission line.

The velocity of propagation of wave is given by, $v = \lambda f = \lambda\omega/2\pi = \omega/\beta$
$\implies v = 1/\sqrt{LC}$

$R = 0,\ G = 0 \quad \implies Z_0 = \sqrt{L/C}$
It can be noted that even though the there is no resistive component in the transmission line, the characteristic impedance is purely resistive in nature (since it is a real quantity).

1.1.2 Low-loss Transmission Line

A low-loss transmission line has low values of resistance and conductance when compared to the reactance of inductor and susceptance of capacitor respectively.
$R <<< \omega L$ and $G <<< \omega C$

Using first order approximation, the characteristic parameters can be found.

$$\gamma = j\omega\ \sqrt{LC} + \frac{R}{2}\sqrt{\frac{C}{L}} + \frac{G}{2}\sqrt{\frac{L}{C}}$$

$$Z_0 = \sqrt{\frac{L}{C}}\left[1 - j\frac{R}{2\omega L} + j\frac{G}{2\omega C}\right]$$

Here, the phase constant is same as for loss-less TL and the attenuation constant is a finite non-zero value but negligibly small.

The condition at which a transmission line can be treated as a low-loss TL in terms of α and β can be derived as follows:
$\alpha = \frac{R}{2}\sqrt{\frac{C}{L}} + \frac{G}{2}\sqrt{\frac{L}{C}}$
$\implies \alpha = \beta[\frac{R}{2\omega L} + \frac{G}{2\omega C}]$

Since $\frac{R}{\omega L} <<< 1$ and $\frac{G}{\omega C} <<< 1$ for a low-loss TL, the condition is $\alpha <<< \beta$.

Note that there is no fixed threshold for this condition and it depends on the application necessary.

Unless otherwise specified, a low-loss transmission line can be approximately modelled as loss-less transmission line and the parameters can be found using the loss-less conditions itself.

For loss-less TL, the voltage and current equations can be modified as

$$V(l) = V^+ e^{j\beta l}\left[1 + |\Gamma_L|\ e^{j(\phi_L - 2\beta l)}\right] \qquad I(l) = \frac{V^+}{Z_0} e^{j\beta l}\left[1 - |\Gamma_L|\ e^{j(\phi_L - 2\beta l)}\right]$$

Here, the reflection coefficient has been expressed as $\Gamma_L = |\Gamma_L|\angle\phi_L$

- When $\phi_L - 2\beta l$ is an even multiple of 2π, $V(l)$ is maximum and $I(l)$ is minimum.
- When $\phi_L - 2\beta l$ is an odd multiple of 2π, $V(l)$ is minimum and $I(l)$ is maximum.

$$Z_{max}(= R_{max}) = \frac{|V_{max}|}{|I_{min}|} = Z_0\ \frac{1 + |\Gamma_L|}{1 - |\Gamma_L|}$$

$$Z_{min}(= R_{min}) = \frac{|V_{min}|}{|I_{max}|} = Z_0\ \frac{1 - |\Gamma_L|}{1 + |\Gamma_L|}$$

Another useful parameter to measure for Transmission Line analysis is the "Voltage Standing Wave Ratio".
VSWR is the ratio of the maximum voltage to the minimum voltage.

$$\rho(= VSWR) = \frac{1 + |\Gamma_L|}{1 - |\Gamma_L|}$$

$\implies 1 \leq \rho \leq \infty$

The maximum and minimum impedances can also be expressed in terms of VSWR as $Z_{max} = Z_0\ \rho$ and $Z_{min} = Z_0/\rho$

Ideally, the value of VSWR should be 1 since the value of reflection coefficient should be 0.

Also, the impedance transformation formula will be modified to

$$Z(l) = Z_0 \, \frac{Z_L \cos(\beta l) + jZ_0 \sin(\beta l)}{Z_0 \cos(\beta l) + jZ_L \sin(\beta l)}$$

The normalized impedance will be

$$\bar{Z}(l) = \frac{\bar{Z}_L \cos(\beta l) + j \sin(\beta l)}{\cos(\beta l) + j\bar{Z}_L \sin(\beta l)}$$

- The impedance value repeats itself at every $\lambda/2$ increment in l.
- The normalized impedance value will be reciprocal of itself at every $\lambda/4$ increment in l.

For an increment of $\lambda/8$ in l, the impedance value changes as

$$Z(l) = Z_0 \, \frac{Z_L + jZ_0}{Z_0 + jZ_L}$$

Power Delivered to Load

$V(0) = V^+ \ (1 + \Gamma_L)$ and $I(0) = \frac{V^+}{Z_0} \ (1 - \Gamma_L)$
Using $P = \frac{1}{2} Re[V(0)\bar{I}(0)]$, the expression for power delivered can be found to be

$$P = \frac{|V^+|^2}{2Z_0}[1 - |\Gamma_L|^2]$$

Alternatively, another approach can be taken.
Power carried by forward wave is given by, $P_{for} = \frac{|V^+|^2}{2Z_0}$
Power reflected by backward wave is given by, $P_{back} = \frac{|V^-|^2}{2Z_0}$
The net power delivered to the load is $P = P_{for} - P_{back}$.

$$\implies P = \frac{|V^+|^2 - ||V^-|^2}{2Z_0}$$

Note that this relation is equivalent to the one given earlier.

The power delivered to the load in a loss-less TL is same as the real/actual power calculated at any point on the TL since there is no other element to absorb power in the TL except the load.

Evaluation of V^+

Using impedance transformation, the effective input impedance as seen by the source can be calculated.
Once that is done, the lumped equivalent circuit is used to find the required relation.

$$V_A = V_s \left[\frac{Z_L'}{Z_s + Z_L'} \right]$$

At the input (source) end of the transmission line,

$$V_A = V(l) = V^+ \, e^{j\beta l}(1 + \Gamma_L e^{-2j\beta l}$$

From the above two relations, the expression for V^+ can be obtained.

$$V^+ = \frac{Z_L' V_s e^{-j\beta l}}{(Z_s + Z_L')(1 + \Gamma_L) \; e^{-2j\beta l}}$$

1.1.3 Lossy Transmission Line

A lossy transmission line has high values of resistance and conductance that are comparable to the reactance of inductor and susceptance of capacitor respectively.
$R \approx \omega L$ and $G \approx \omega C$

For a lossy TL, due to non-negligible attenuation, the voltage waves will not be standing i.e the maximum and minimum voltages won't be the same, nor will they always be separated by $\lambda/4$.
Hence, VSRW also becomes a function of length, which is why it is not a useful quantity.

If the values of resistance and conductance are much higher, then the TL is very lossy and it loses it's moving wave properties.
$R >>> \omega L$ and $G >>> \omega C$

$\implies Z_0 = \sqrt{R/G}$ and $\gamma = \sqrt{RG}$
This means $\alpha = \sqrt{RG}$ and $\beta = 0$ which indicates there is only attenuation and no phase change.

1.1.4 Distortionless Transmission Line

A TL is said to be Distortionless when attenuation constant (α) is frequency independent and phase constant (β) varies linearly with frequency.
Condition for TL to be Distortionless: $\frac{R}{L} = \frac{G}{C}$.

$Z_0 = \sqrt{L/C}$
$\gamma = (R/L + j\omega)\sqrt{LC} = (G/C + j\omega)\sqrt{LC}$
$\implies \alpha = R/Z_0 \;= G\ Z_0$ and $\beta = j\omega\sqrt{LC}$

1.2 Smith Chart

The complex $\bar{Z}$-plane graphically represents the normalized impedances.
If $\bar{Z} = r + jx$, it is known that r can only take positive values and hence, the imaginary axis and the entire right hand side of plane will cover all possible passive loads (normalized w.r.t characteristic impedance).
Note that loss-less transmission lines are being considered and hence the characteristic impedance is a real quantity.

There is a one-to-one mapping between impedance and reflection coefficient.
$\Gamma = \frac{\bar{Z}-1}{\bar{Z}+1}$ and $\bar{Z} = \frac{1+\Gamma}{1-\Gamma}$
Using this, Γ can also be expressed as a complex quantity $\Gamma = u + jv$.
Since it is known that $0 \le |\Gamma| \le 1$, any passive load impedance in the complex $\bar{Z}$-plane will map to some reflection coefficient in the complex Γ-plane within the unit circle.
By substituting and manipulating the above expressions to obtain separate real and imaginary parts of Γ in terms of r and x, two different set of circles can be obtained.

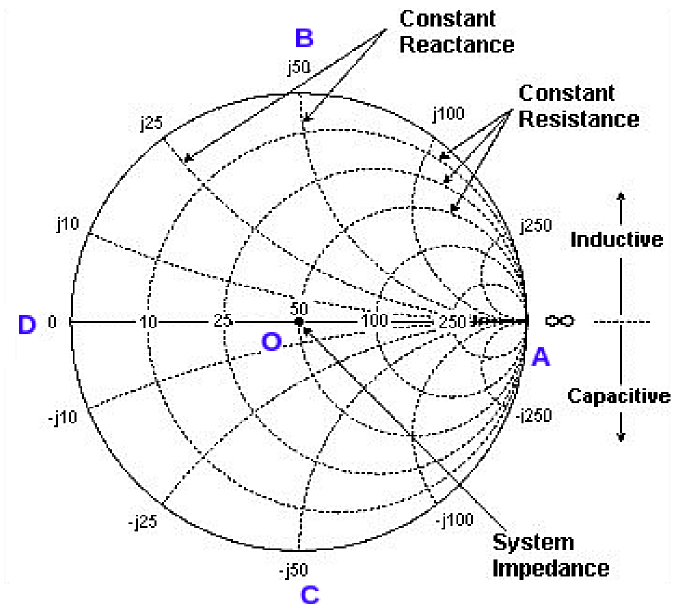
Constant Resistance Circles:
The set of circles which represent all possible reactances for a constant resistance value [changing x for constant r].

$$Centre \to \left(\frac{r}{r+1}, 0\right); \qquad Radius = \frac{1}{r+1}$$

Constant Reactance Circles:
The set of circles which represent all possible resistances for a constant reactance value [changing r for constant x].

$$Centre \to \left(1, \frac{1}{x}\right); \quad Radius = \frac{1}{x}$$

The Smith Chart is the superimposition of the constant resistance and constant reactance circles on the complex Γ-plane.
It gives a convenient way to map between the values of normalized impedance and reflection coefficient.



Special Points on Smith Chart
A $\to r = \infty, x = \infty \implies$ Load is open
B $\to r = 0, x = 1 \implies$ Purely Inductive Load
C $\to r = 0, x = -1 \implies$ Purely Capacitive Load
D $\to r = 0, x = 0 \implies$ Load is short
O $\to r = 1, x = 0 \implies$ Matched Load

Constant VSWR Circles:
Another set of circles can be superimposed on the Smith Chart for calculations, which are the constant VSWR circles.

Since $\Gamma(l) = |\Gamma_L|e^{j(\phi_L - 2\beta l}$ where $|\Gamma_L|$ is constant and $\rho = \frac{1+|\Gamma_L|}{1-|\Gamma_L|}$, for each value of a Reflection coefficient, a circle with centre at origin and radius $|\Gamma_L|$ can be drawn.

- The unit circle represents $\rho = \infty$ i.e where no power is delivered to the load
- The origin represents $\rho = 0$ i.e where the all the power is delivered to the load (matching condition).

Note that ideally $|\Gamma_L| = 0$ and hence, closer a point is to the origin of the Smith chart, better is the matching.

Moving in the clockwise direction of the VSWR circle corresponds to moving towards source from load.
Moving in the anti-clockwise direction of the VSWR circle corresponds to moving from source towards load.
By observing the pattern of movement on the VSWR circle with change in length, the following observations can be made from the voltage wave regarding the type of load.

- If $V_{min} \neq 0$ and V_{min} occurs first while observing the wave from load end, then the load is capacitive and resistive.
- If $V_{min} \neq 0$ and V_{max} occurs first while observing the wave from load end, then the load is inductive and resistive.
- If $V_{min} = 0$ and V_{min} occurs first while observing the wave from load end, then the load is purely capacitive.
- If $V_{min} \neq 0$ and V_{max} occurs first while observing the wave from load end, then the load is purely inductive.
- If V_{min} or V_{max} occur directly at the load end, then the load is purely resistive.

The maximum impedance offered is at the right most end of the VSWR circle drawn. Hence, $\bar{Z}_{max} = \rho$.
The minimum impedance offered is at the left most end of the VSWR circle drawn. Hence, $\bar{Z}_{min} = 1/\rho$.

This observation tells that the movement along a semicircle of the VSWR circle corresponds to $\lambda/4$ change in length and movement along the entire circle corresponds to $\lambda/2$ change in length.

Admittance Smith Chart
In some cases it is more convinient to deal with admittances than impedances. Hence the Smith Chart has to be modified.
The relation between the reflection coefficient and normalized admittance is given by,

$$\Gamma = \frac{\bar{Y} - 1}{\bar{Y} + 1} \, e^{j\pi}$$

It can be observed that the relation between Γ and $\bar{Y}$ is same as that between Γ and $\bar{Z}$ with a phase shift of 180^o.
Therefore, the same Smith Chart can be used, with both the axes pointing at opposite directions.
The special points and regions will vary accordingly.

Smith Chart can also be applied to lossy TL but constant VSWR circle don't exist since VSWR varies. Instead, spiral curves have to be drawn and analyzed.

1.3 Applications of Transmission Lines

Apart from being used to transmit signals over large distances, transmission lines have several applications due to their properties.

1.3.1 Measuring of Unknown Impedance

At high frequencies, measurement of unknown impedance becomes difficult at high frequencies since measurement of phase is unreliable.
Using a loss-less transmission line, by measuring the voltage maximum and minimum and their locations, the unknown impedance can be estimated by using impedance transformation.

The location of minimum or maximum voltages can be reliably found by connection a short or an open circuit at the first and then comparing with their occurrences when connected to the unknown impedance.

1.3.2 Using TL as Circuit Element

Circuit elements such as inductors and capacitors will not behave as desired at very high frequencies due to presence of stray capacitance (in inductors) and stray inductance (in capacitors).

Instead,transmission lines can be used as alternatives to circuit elements to provide the required reactance at high frequencies.
This is done by either short circuiting or open circuiting the TL and using it at some finite length which corresponds to the required reactance.

$Z_{in} = jZ_0 \tan \beta l_{sc}$
$Z_{in} = -jZ_0 \cot \beta l_{oc}$

This means any arbitrary reactance can be realized using a short or open circuited TL by varying the length.

Note that the required length to realize a given reactance can be easily found using the Smith Chart.

- Open circuited TL:
  - $0 < l_{oc} < \lambda/4 \implies$ Inductor
  - $\lambda/4 < l_{oc} < \lambda/2 \implies$ Capacitor
  - $l_{oc} =$ odd multiples of $\lambda/4 \implies$ Infinite reactance i.e parallel resonant circuit
  - $l_{oc} =$ even multiples of $\lambda/4 \implies$ Zero reactance i.e series resonant circuit
- Short circuited TL:
  - $0 < l_{sc} < \lambda/4 \implies$ Capacitor load
  - $\lambda/4 < l_{sc} < \lambda/2 \implies$ Inductor
  - $l_{sc} =$ odd multiples of $\lambda/4 \implies$ Zero reactance i.e series resonant circuit
  - $l_{sc} =$ even multiples of $\lambda/4 \implies$ Infinite reactance i.e parallel resonant circuit

The Quality Factor of resonant circuits is the ratio of energy stored to energy lost per cycle.

$$Q = 2\pi f_0 \ [\frac{Energy\ stored\ in\ circuit}{Power\ lost\ in\ circuit}]$$

where f_0 is the resonant frequency corresponding to the wavelength λ.

Since it is assumed that the TL is loss-less, quality factor will be infinite. However, if losses are considered, then the quality factor can be estimated.

Input impedance for short circuited line at $l_{sc} = \lambda/4$ is given by,
$Z_{insc} = \frac{Z_0}{\alpha\ l} = \frac{4Z_0}{\alpha\ \lambda}$.

Input impedance for open circuited line at $l_{oc} = \lambda/4$ is given by,
$Z_{inoc} = Z_0\ \alpha\ l = \frac{Z_0\ \alpha\ \lambda}{4}$.

Using these, the energy stored and power lost can be calculated, which will give approximate expression for Quality Factor.

$\therefore Q = \beta/2\alpha$

Note that since $\beta >> \alpha$, the value of Q will be very high and hence, resonant circuits realized using TL will be very selective.

1.3.3 Voltage or Current Step Up Transformer

Consider a TL segment of length $\lambda/4$ which is open circuited at one end and short circuited at the other end.
If a voltage is induced in some part of the TL, then it will send travelling waves towards both directions. And both these waves will undergo complete reflection on both ends before reaching back to the induced point, with 180^o phase shift in the short circuited end along with another 180^0 phase shift since the wave has travelled a distance of $\lambda/2$.

This means, overall phase shift of 360^o occurs which causes addition of the induced wave and the wave that travels one cycle. This process keeps happening and hence very high voltage is observed at the open end and high current at the short end.

Hence, a segment of a TL can be used for stepping up action. Ideally due to no losses, this will go on forever but due to losses, the voltage and current growth will stop at some value.

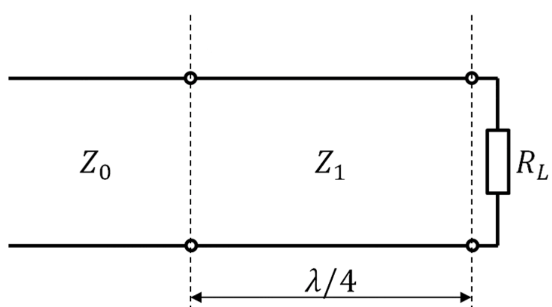
Note that this phenomenon can also be counter-productive if voltage is unknowingly induced at some point, it can give rise to high voltages and currents that can't be handled by the circuitry.

1.3.4 Impedance Matching

The load impedance that needs to be driven can't always be equal to the characteristic impedance of the transmission line. But it has to be so in order to obtain maximum power transfer.
A solution is to match the impedances on both sides using a matching unit which makes the impedance from input side seem like the characteristic impedance even though the load is not equal to it.

Quarter-wave Transformer

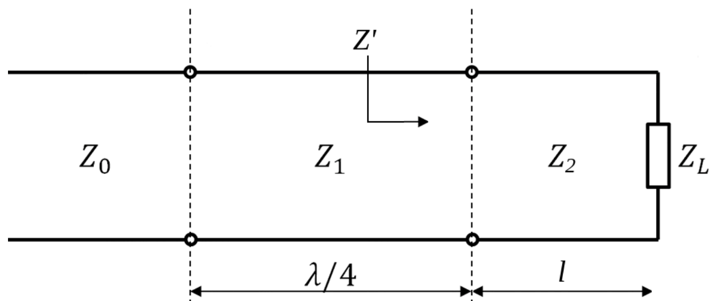
Consider the case where load R_L has to be matched to a TL with characteristic impedance Z_0. As mentioned, the impedance seen from the source side has to be equal to Z_0 (instead of R_L).
Using a new transmission line of arbitrary characteristic impedance Z_1 and length $\lambda/4$, the load can be transformed to another resistive value.



At the load side, normalized impedance is R_L/Z_1.
At a distance $\lambda/4$ from the load, normalized impedance will be inverted i.e it is Z_1/R_L.
Hence, the source will now see an impedance of Z_1^2/R_L and for matching to

occur, this should be equal to Z_0.
Therefore, the characteristic impedance of new TL of length $\lambda/4$ should be $Z_1 = \sqrt{Z_0 R_L}$.

This technique will directly work only if the load is purely real (resistive). If the load is complex, then extra length of TL needs to be added to transform the load impedance to a purely resistive value (R_{max} or R_{min}) and then the same technique can be used for matching.



$l = l_{max} \implies Z' = R_{max} = Z_2\ \rho$
$l = l_{min} \implies Z' = R_{min} = Z_2/\rho$
$\therefore Z_1 = \sqrt{Z_0\ Z'}$

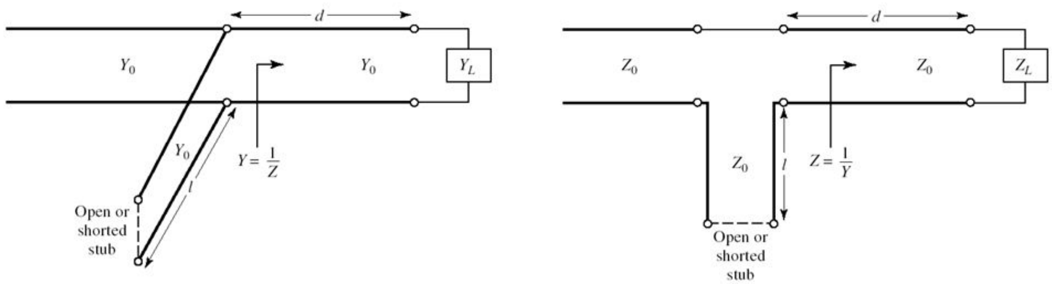
However, it is not easy to get a transmission line of arbitrary characteristic impedance and hence this strategy can't be used if the required impedance values are not available.

Stub Matching

Stub matching is a technique in which a shorted or opened TL segment is connected to the required TL at some point in order to modify the impedance as seen by the input side to the characteristic impedance.

Stub matching can be done using either series stub or shunt stub.

The load impedance is transformed to a different value with R_L matching the characteristic impedance at a distance d from the load. Hence at the junction, impedance seen by the source will be $Z_0 + jX_L$. The reactance part is eliminated by combining (in series or parallel) a shorted or opened stub providing the opposite reactance necessary.



1.3.5 Measurement of Characteristic Impedance

Given a transmission line, the characteristic impedance can be measured by using the open circuited load and short circuited load conditions.
Input impedance as seen by source when the load end of TL is opened is given by Z_{oc} and when the load end of TL is shorted is given by Z_{sc}. These two can be measured using impedance measuring device.

$$Z_{oc} = Z_0 \ \coth\gamma\, l \ \text{ and } \ Z_{sc} = Z_0 \ \tanh\gamma\, l$$
$$\implies Z_0 = \sqrt{Z_{oc}\, Z_{sc}}$$

The propagation constant γ can also be obtained using the above relations.

1.4 Types of Transmission Lines

Co-axial line

Parallel wire line

Micro-strip line

2 Electromagnetics

Before going to the laws and equations that govern electromagnetic theory, understanding or defining a few important terms and concepts is necessary.

2.0.1 Basic terms and definitions

Electric Field is defined as the force experienced by a unit positive charge. It is a vector quantity denoted as $\vec{E}$ measured in V/m.

Permittivity is the property of medium which measures the ability of a substance to store electrical energy in an electric field.
Permittivity of free space (or air) is given by ϵ_o and is approximately equal to $\frac{10^{-9}}{36\pi}\ F/m$

Relative Permittivity or Dielectric Constant of a medium is the ratio of permittivty in that medium to the permittivty of free space. $\implies \epsilon_r = \epsilon/\epsilon_o$

If the value of ϵ does not change with space and direction, then the medium is called a Homogeneous Isotropic medium. In such a medium, ϵ is a scalar quantity.

Electric Displacement Vector or **Electric Flux Density** is the product of permittivity of the medium and the electric field, measured in C/m^2.
$\vec{D} = \epsilon\ \vec{E}$

Magnetic Field is a vector field that arises due to moving charge i.e current element.
It is denoted by $\vec{H}$ and is measured in A/m.

Permeability is the measure of the resistance of a material against the formation of a magnetic field.
Permeability of free space (or air) is given by μ_o and is approximately equal to $4\pi \times 10^{-7}\ H/m$

Relative Permeability of a medium is the ratio of permeability in that medium to the permeability of free space. $\implies \mu_r = \mu/\mu_o$

Magnetic Flux Density is the vector field that arises due to magnetic properties of the material along with the effect of current element, measured in Wb/m or T. $\vec{B} = \mu\vec{H}$

2.1 Electrostatics

Electrostatics is the study of electrical phenomena that occurs due to static charges.

Types of charges (Q is the overall charge)-

- Point charge: $Q = q$ where q is the charge on the point charge in C.
- Line charge: $Q = \int \rho_l \, dl$ where ρ_l is the line charge density in C/m.
- Surface charge: $Q = \int \rho_s \, ds$ where ρ_s is the surface charge density in C/m^2.
- Volume charge: $Q = \int \rho_v \, dv$ where ρ_v is the volume charge density in C/m^3.

2.1.1 Coulomb's Law

Coulomb's Law quantifies the force $\vec{F}$ between two stationary, electrically charged particles Q_1 and Q_2 separated by a finite distance r.

$$\boxed{\vec{F}_{21} = \frac{1}{4\pi\epsilon}\frac{Q_1\,Q_2}{r^2}\,\hat{r}_{21}} \qquad \hat{r}_{21} = \frac{\vec{r}_{21}}{|r_{21}|}$$

Here, $\vec{F}_{21}$ is the force exerted on Q_1 by Q_2 and $\hat{r}_{21}$ is the unit vector pointing in the direction of the line joining Q_1 to Q_2.
$\implies \vec{F}_{21} = -\vec{F}_{12}$

The expression for electric field can be derived can be derived from the above equation if one of the charges is assumed to be a unit positive charge.

$$\vec{E} = \frac{1}{4\pi\epsilon}\frac{Q}{r^2}\,\hat{r}$$

Electric field lines are lines drawn from a charge indicating the direction of electric field around it. This means, the electric field line at a location shows the path taken by a unit positive charge that is placed at that location.

A positive charge will have field lines diverging away from it since it will repeal a unit positive charge.
A negative charge will have field lines converging towards it since it will attract a unit positive charge.

The electric displacement vector is given by

$$\vec{D} = \frac{1}{4\pi}\frac{Q}{r^2}\ \hat{r}$$

Hence, electric displacement vector is a quantity that is independent of the nature of the medium.

2.1.2 Gauss's Law

Gauss's Law states that the net electric flux lines coming out of a closed surface is equal to the total charge enclosed by the surface.

$$Q = \oiint_s \vec{D}.\vec{ds}$$

The charge enclosed can be expressed as the volume integral of the volume charge density.

$$\implies \iiint_v \rho_v\ dv = \oiint_s \vec{D}.\vec{ds}$$

From Divergence theorem,

$$\oiint_s \vec{D}.\vec{ds} = \iiint_v \nabla.\vec{D}\ dv$$

By comparing the above two equations, it can be deduced that

$$\nabla.\vec{D} = \rho_v \qquad \implies \nabla.\vec{E} = \frac{\rho_v}{\epsilon}$$

Applications of Gauss's Law

Electric field intensity at a point due to line charge of infinite length having uniform line charge density ρ_l is given by

$$\vec{E} = \frac{\rho_l}{2\pi\epsilon r}\hat{r}$$

where $\vec{r}$ is the perpendicular distance vector joining the line charge and the point.

Electric field intensity at a point due to surface charge of infinite area having uniform surface charge density ρ_s is given by

$$\vec{E} = \frac{\rho_s}{2\epsilon}\hat{n}$$

where $\hat{n}$ is unit normal vector from the surface charge to the point.

Consider a uniformly charged sphere of volume charge density ρ_v and radius a. If the Gaussian surface enclosing this volume charge is considered as another sphere of radius r, the electric field intensity is given by

$$\vec{E} = \begin{cases} \frac{\rho_v}{3}\frac{r}{\epsilon}\hat{r} & r < a \\ \frac{\rho_v}{3}\frac{a^3}{\epsilon\, r^2}\hat{r} & r > a \end{cases}$$

The field strength is maximum at $r = a$.

2.1.3 Electric Potential

Electric Potential (also called electric field potential) is the amount of work needed to move a unit of charge from a reference point to a specific point inside the electric field without producing an acceleration, measured in V.

Electric field is the negative gradient of the electric potential. $\vec{E} = -\nabla V$.

If charge is moving against the electric field, then the potential increases.
If charge is moving with the electric field, then the potential decreases.

The potential difference between two points A and B is the work needed to move a unit positive charge from A to B against the electric field.

$$V_{AB} = -\int_A^B \vec{E}.\vec{dl}$$

Note that if a charge is moved to a different point and then moved back to the same point, the net work done is zero. Hence, electric field is a conservative field, so the work done is independent of the path taken.

Poisson's Equation
Laplacian of electric potential at a point is equal to the ratio of the volume charge density to the absolute permittivity of the medium.

$$\nabla^2 V = -\frac{\rho_v}{\epsilon}$$

In a source free region i.e where there are no charges, since $\rho_v = 0$, $\nabla^2 V = 0$.

2.2 Magnetostatics

Magnetostatics is the study of magnetic phenomena that occurs due to current element.

Types of current elements/distributions:

- Line current I in A
- Sheet current K in A/m
- Volume current J in A/m^2

2.2.1 Biot-Savart's Law

The differential magnetic flux intensity $d\vec{H}$ at a point due to current I flowing through a small current element $\vec{dl}$ is given by the following relation.

$$\boxed{d\vec{H} = \frac{1}{4\pi}\frac{I\vec{dl} \times \hat{r}}{r^2}}$$

where $\vec{r}$ is the vector joining the current element to the point

Hence, the magnetic flux intensity will be perpendicular to the directions of both the current element and the vector joining them.

Magnetic Field Intensity due to a finite line current at a point P which is at a perpendicular distance r from the line current, which subtends angles of α_1 and α_2 between the perpendicular and the line joining the point to the ends of the line current is given by,

$$\vec{H} = \frac{I\vec{dl} \times \hat{r}}{4\pi\ r} \left(\sin\alpha_1 + \sin\alpha_2\right)$$

If the line current is infinitely long, then the angles will both be 90^o and hence the above equation will reduce to,

$$\vec{H} = \frac{I\vec{dl} \times \hat{r}}{2\pi r}$$

2.2.2 Ampere's Circuital Law

The line integral of a tangential component of the magnetic flux intensity around a closed path is equal to the next current enclosed by the area of the path.

$$\oint_c \vec{H}.\vec{dl} = I; \qquad I = \vec{J}.\vec{ds}$$

From Stokes' theorem,

$$\oint_c \vec{H}.\vec{dl} = \iint_s \left(\nabla \times \vec{H}\right).\vec{ds}$$

Hence, it can be concluded that

$$\nabla \times \vec{H} = \vec{J}$$

Consider an infinitely long cylindrical wire of radius a carrying current I. The magnetic flux intensity at an arbitrary distance r from the axis of the wire is given by

$$\vec{H} = \begin{cases} \frac{I}{2\pi}\frac{r}{a^2}\ \hat{\phi} & r < a \\ \frac{I}{2\pi\ r}\ \hat{\phi} & r > a \end{cases}$$

The field strength is maximum at $r = a$.

2.2.3 Gauss's Law for Magnetostatics

Magnetic mono-poles do not exist i.e there is always a north pole and a south pole.
Hence, the net magnetic field lines going out of any surface is equal to the net field lines coming into the surface.

$$\oiint_s \vec{B}.\vec{ds} = 0 \qquad \implies \nabla.\vec{B} = 0$$

Since the divergence of $\vec{B}$ is 0, it means magnetic field lines are solenoidal.

2.3 Maxwell's Equations

Maxwell's set of equations is the compilation of 4 laws that govern all electromagnetic phenomena.

Before getting into Maxwell's Equations, it is important to understand a few more concepts and laws that Maxwell used to correctly give the universal equations.

It is known that static charges give rise to electric field and moving charges give rise to magnetic field.
Accelerated charges give rise to what is known as electro-magnetic field.
In an electro-magnetic field, the electric and magnetic fields will essentially interact with each other, hence modifications some of the previously discussed laws are necessary.

2.3.1 Faraday's Law & Lenz's Law

Faraday's law states that a current will be induced in a conductor which is exposed to a changing magnetic field (i.e changing flux linkage).

Lenz's law states that the direction of this induced current will be such that the magnetic field created by the induced current opposes the initial changing magnetic field which produced it.

This phenomenon is called "Electromagnetic Induction".

$$\oint_c \vec{E}.\vec{dl} = -\frac{\partial}{\partial t}\left(\iint_s \vec{B}.\vec{ds}\right) = -\iint_s \left(\frac{\partial \vec{B}}{\partial t}.\vec{ds}\right)$$

Applying Stokes' theorem,

$$\oint_c \vec{E}.\vec{dl} = \iint_s (\nabla \times \vec{E}).\vec{ds}$$

It can be deduced that

$$\nabla \times \vec{E} = -\frac{\partial \vec{B}}{\partial t}$$

2.3.2 Continuity Equation

The net outward current flow through a closed surface is equal to the total decrease in charge enclosed in the volume of the closed surface.

$$\oiint_s \vec{J}.\vec{ds} = -\iiint_v \frac{\partial \rho_v}{\partial t} dv$$

From Divergence theorem,

$$\oiint_s \vec{J}.\vec{ds} = \iiint_v (\nabla.\vec{J}) dv$$

Hence, it can be concluded that

$$\nabla.\vec{J} = -\frac{\partial \rho_v}{\partial t}$$

But from Ampere's Circuital Law,

$$\nabla.(\nabla \times \vec{H}) = \nabla.\vec{J} = 0$$

This means the Ampere's Law does not satisfy the continuity equation and hence requires modification.

Using Gauss's Law to substitute for ρ_v,

$$\nabla.\vec{J} = -\frac{\partial(\nabla.\vec{D})}{\partial t} = -\nabla.\left(\frac{\partial \vec{D}}{\partial t}\right)$$

By reverse engineering, this can be rewritten as

$$\oiint_s (\vec{J} + \frac{\partial \vec{D}}{\partial t}).\vec{ds} = 0$$

$\frac{\partial \vec{D}}{\partial t}$ is called "Displacement Current Density" since it has the same dimensions as current density and is caused due to rate of change of electric displacement vector, which is independent of the medium.

Hence, $\vec{J}$ is called "Conduction Current Density" since it is caused by conductive action i.e movement of charges.

$$\nabla \times \vec{H} = \vec{J} + \frac{\partial \vec{D}}{\partial t}$$

The total current caused by both the above phenomena which effectively modifies Ampere's Law to satisfy the Continuity Equation.

Maxwell's Equations

$$\boxed{\nabla . \vec{D} = \rho_v \quad \leftrightarrow \quad \oiint_s \vec{D}.\vec{ds} = \iiint_v \rho_v \; dv}$$

$$\boxed{\nabla . \vec{B} = 0 \quad \leftrightarrow \quad \oiint_s \vec{B}.\vec{ds} = 0}$$

$$\boxed{\nabla \times \vec{E} = -\frac{\partial \vec{B}}{\partial t} \quad \leftrightarrow \quad \oint_c \vec{E}.\vec{dl} = -\iint_s \frac{\partial \vec{B}}{\partial t}.\vec{ds}}$$

$$\boxed{\nabla \times \vec{H} = \vec{J} + \frac{\partial \vec{D}}{\partial t} \quad \leftrightarrow \quad \oint_c \vec{H}.\vec{dl} = \iint_s \left(\vec{J} + \frac{\partial \vec{D}}{\partial t} \right).\vec{ds}}$$

2.3.3 Boundary Conditions

Boundary conditions are useful in analyzing behaviour of electromagnetic waves especially in practical scenarios where the waves might be moving from one medium to another medium with different material properties.

When the media changes, due to difference in properties, the differential forms of Maxwell's equations can not be used. This is why integral forms are used to analyze what happens at the interfaces in order to relate the fields travelling through different media.

Dielectric-Dielectric Interface

Consider two different dielectric media (medium 1 and medium 2) having material properties $\sigma_1, \epsilon_1, \mu_1$ and $\sigma_2, \epsilon_2, \mu_2$ respectively.

Let the electric fields in medium 1 and medium 2 be $\vec{E_1}$ and $\vec{E_2}$, with electric displacement vectors being $\vec{D_1}$ and $\vec{D_2}$ respectively.

All the vectors can be decomposed into component normal to the interface and component tangential to the interface.
$\vec{E_1} \rightarrow E_{t1}, E_{n1}\ \ ;\ \vec{E_2} \rightarrow E_{t2}, E_{n2}$
$\vec{D_1} \rightarrow D_{t1}, D_{n1}\ \ ;\ \vec{D_2} \rightarrow D_{t2}, D_{n2}$

Assume a Gaussian surface enclosing some length of the interface, with width tending to 0. By applying Gauss Law, it can be observed that the normal components of the electric displacement vector will be equal if there is no charge enclosed by the Gaussian surface.

If the interface has surface charge ρ_s, then the difference between the normal components of the displacement vectors will be equal to ρ_s.

Assume a loop encircles some length of the interface, with width tending to 0. By applying Faraday's Law, it can be found that the algebraic sum of tangential components of the electric field vectors multiplied by length of the loop equals 0.
This simplifies to the tangential components of the electric field vectors on both sides of the interface being equal.

The above two observations give two boundary conditions for electric fields between two dielectric media.

- $D_{n1} - D_{n2} = \rho_s \implies$ if $\rho_s = 0$, then $D_{n1} = D_{n2}$
- $E_{t1} = E_{t2}$

Let the magnetic fields in medium 1 and medium 2 be $\vec{H_1}$ and $\vec{H_2}$, with magnetic flux densities being $\vec{B_1}$ and $\vec{B_2}$ respectively.

All the vectors can be decomposed into component normal to the interface and component tangential to the interface.
$\vec{H_1} \to H_{t1}, H_{n1}$; $\vec{H_2} \to H_{t2}, H_{n2}$
$\vec{B_1} \to B_{t1}, B_{n1}$; $\vec{B_2} \to B_{t2}, B_{n2}$

Similar application of Gauss' Law for magneto statics here as well gives the normal components of magnetic flux densities being equal.

Assuming the interface is enclosed by an area of finite length and width tending to 0, Ampere's Law can be applied. The difference between the tangential components of the magnetic field strengths will be equal to the sum of displacement current, conduction current and the current component through the interface.
Since the width of the area tends to 0, the conduction and displacement currents will be 0. Hence, the difference between the tangential components of the magnetic field strengths will be equal to the current through the interface only.

The above two observations give two boundary conditions for magnetic fields between two dielectric media.

- $B_{n1} = B_{n2}$
- $H_{t1} - H_{t2} = \vec{J_s}$ i.e $\hat{n} \times (\vec{H_1} - \vec{H_2}) = \vec{J_s}$
  $\implies$ if $\vec{J_s} = 0$, then $H_{t1} = H_{t2}$

Thus, the 4 boundary conditions on the electric and magnetic fields between dielectric-dielectric interface are obtained.

Conductor-Dielectric Interface

An ideal conductor will have $\sigma \to \infty$ and hence any charges or currents will only be surface phenomena. This means that the electric field and magnetic field components inside the conducting media must be 0.

If medium 1 is a dielectric and medium 2 is a conductor, then
$\vec{E}_2 = \vec{0}$ and $\vec{H}_2 = \vec{0}$.

Now the 4 boundary conditions will reduce to:

- $D_{n1} = \rho_s$
- $E_{t1} = 0$
- $B_{n1} = 0$
- $\hat{n} \times \vec{H}_1 = \vec{J}_s$

Thus, the 4 boundary conditions on the electric and magnetic fields between conductor-dielectric interface are obtained.

2.3.4 Time Harmonic Fields

A time harmonic field is a field that varies sinusoidally with time.
Such fields are very useful for analysis because any periodic signal can be represented as a sum of sinusoids using Fourier analysis.

In general, the time factor used to represent time harmonic fields is the complex exponential $e^{j\omega t}$.

Hence, time harmonic electric field can be expressed as $E(x, y, z, t) = E(x, y, z)e^{j\omega t}$ and time harmonic magnetic field can be expressed as $H(x, y, z, t) = H(x, y, z)e^{j\omega t}$.

Since the differential of $Ae^{j\omega t}$ w.r.t t is $j\omega Ae^{j\omega t}$, the Maxwell's equations in differential form for time harmonic fields can be modified as follows.

$$\nabla.\vec{D} = \rho_v \ ; \quad \nabla.\vec{B} = 0 \ ; \nabla \times \vec{E} = -j\omega\vec{B} \ ; \quad \nabla \times \vec{H} = \vec{J} + j\omega\vec{D} \ ;$$

For further analysis, time harmonic fields will be considered (unless mentioned otherwise).

3 Uniform Plane Waves

Consider an unbounded isotropic homogeneous medium.
Maxwell's second and fourth equations can be written as,

$$\nabla \times \vec{E} = -\frac{\partial \vec{B}}{\partial t} = -\mu \, \frac{\partial \vec{H}}{\partial t}$$
$$\nabla \times \vec{H} = \vec{J} + \frac{\partial \vec{D}}{\partial t} = \epsilon \, \frac{\partial \vec{E}}{\partial t}$$

From these equations, it can be concluded that Time varying Magnetic Field cannot exist without corresponding Electric Field and vice-versa.
The Electric and Magnetic Fields have to co-exist whenever they are time varying and such fields are caused by accelerated charges.

The phenomena governed by these equations is called 'Electromagnetic Wave'.

Now assuming time harmonic fields and applying curl on both the equations, they can be further modified as,

$$\nabla^2 \vec{E} = \gamma^2 \vec{E} \qquad\qquad \nabla^2 \vec{H} = \gamma^2 \vec{H}$$

where $\gamma^2 = j\omega\mu(\sigma + j\omega\epsilon)$

These equations are called "Helmholtz's equations" or "Wave equations".
The simplest possible solution for the wave equations has to be found by trial and error from the most basic case.

- A time varying electric or magnetic field which is uniform in the three-dimensional space can not exist.
- A time varying field which is constant in a plane perpendicular to the field direction also can not exist.
- The simplest form of field which can exist is the field which is constant in a plane containing the field vector, and consequently has spatial variation along the direction perpendicular to the constant field plane.

This solution is called 'Uniform Plane Wave' solution.

3.1 Uniform Plane Wave Propagation

An electric field being directed along x direction and varied with respect to z only i.e is fixed in the $x-y$ plane, then the electric field is a uniform wave propagating in the z direction.

Similarly, a magnetic field being directed along y direction and varied with respect to z only i.e is fixed in the $x-y$ plane, then the magnetic field is a uniform wave propagating in the z direction.

The system of time-varying EM waves that are fixed in the $x-y$ plane with electric field being directed in x direction, magnetic field being directed in y direction and the entire wave propagating in z direction is the standard uniform plane wave system that is usually assumed/considered.

Propagation of EM waves has to be analyzed in different media. They are,

- Free space
- Loss-less dielectrics
- Lossy dielectrics
- Good conductors

3.1.1 Wave Propagation in Lossy Dielectrics

A lossy dielectric is a medium in which an EM wave, as it propagates, loses power owing to imperfect dielectric.

The medium parameters of a lossy dielectric are:
$\sigma \neq 0$, $\epsilon = \epsilon_o \epsilon_r$, $\mu = \mu_o \mu_r$

Considering a linear, homogeneous, lossy dielectric that is charge free, Maxwell's equations for time-harmonic fields are reduced to Helmholtz's equations.

$$\nabla^2 \vec{E} = \gamma^2 \vec{E} \qquad\qquad \nabla^2 \vec{H} = \gamma^2 \vec{H}$$

where $\gamma^2 = j\omega\mu(\sigma + j\omega\epsilon)$ is called "Propagation constant".

Each of these equations is equivalent to three scalar wave equations, one of each component of $\vec{E}$ and $\vec{H}$.

Since γ is a complex quantity, it can be written as, $\gamma = \alpha + j\beta$ where α is called "Attenuation constant" and β is called "Phase constant".

(note that these equations are parameters are similar to voltage and current equations in a transmission line)

$$\alpha = \omega \sqrt{\frac{\mu\epsilon}{2}\left[\sqrt{1 + (\frac{\sigma}{\omega\epsilon})^2} - 1\right]}$$

$$\beta = \omega \sqrt{\frac{\mu\epsilon}{2}\left[\sqrt{1 + (\frac{\sigma}{\omega\epsilon})^2} + 1\right]}$$

As mentioned before, it is generally assumed that the electric field propagates in the x direction only in uniform plane wave propagation. $\implies E = E_x(z)\hat{a_x}$

Solution to Helmhoultz's equations will give, $E_x(z) = E_o e^{-\gamma z}$
(the other exponential term has to have constant of 0 since otherwise that will indicate field travelling in $-x$ direction).

$$\implies E(z,t) = \mathrm{Re}[E_x(z)e^{j\omega t}\hat{a_x}] = \mathrm{Re}[E_o e^{-\gamma z}e^{j\omega t}\hat{a_x}] = \mathrm{Re}[E_o e^{-\alpha z}e^{j\omega t - \beta z}\hat{a_x}]$$

$$\therefore E(z,t) = E_o e^{-\alpha z}\cos(\omega t - \beta z)\ \hat{a_x}$$

Similarly, by assuming that the magnetic field propagates in the y direction only, expression for $H(z,t)$ can also be derived.

$$\therefore H(z,t) = H_o e^{-\alpha z}\cos(\omega t - \beta z)\ \hat{a_y}$$

A new complex quantity called "Intrinsic Impedance" of the medium is defined as the ratio of the Electric field to the Magnetic field magnitudes.
$\eta = |\vec{E}|/|\vec{H}|$

Expression for intrinsic impedance in terms of the medium parameters is,

$$\eta = \sqrt{\frac{j\omega\mu}{\sigma + j\omega\epsilon}}$$

Since η is complex, it can be expressed as $\eta = |\eta|\angle\theta_\eta = |\eta|e^{j\theta_\eta}$.

$$|\eta| = \frac{\sqrt{\mu/\epsilon}}{\left[1 + (\sigma/\omega\epsilon)^2\right]^{1/4}}$$

$$\theta_\eta = \frac{1}{2}\tan^{-1}\left(\frac{\sigma}{\omega\epsilon}\right)$$

Using the intrinsic impedance in the equation for magnetic field, the magnetic field can be expressed in terms of the same constant used for electric field as follows.

$$\vec{H} = \frac{E_o}{|\eta|}e^{-\alpha z}\cos\left(\omega t - \beta z - \theta_\eta\right)\,\hat{a_y}$$

The attenuation constant α is measured in Np/m where 1 $Np = 20\log_{10} e = 8.686\ dB$ and the phase constant β is measured in rad/m.

The phase velocity of the wave is obtained by setting the phase to some constant and differentiation it w.r.t t . Let $\omega t - \beta z - \theta_\eta = k$
$\implies v_p = \frac{dz}{dt} = \frac{\omega}{\beta}$

Since wavelength is given by $\lambda = v_p f$, it can also be expressed as $\lambda = \frac{2\pi}{\beta}$.

Note that due to the complex nature of the intrinsic impedance, $\vec{E}$ leads $\vec{H}$ by angle θ_η.

The loss tangent of a medium is defined as the ratio of magnitude of conduction current density and displacement current density, and is denoted by $\tan\theta$. $\implies \tan\theta = \frac{\sigma}{\omega\epsilon}$

- $\tan\theta << 1 \implies$ Displacement current density is dominating, hence the medium is a good dielectric.
- $\tan\theta >> 1 \implies$ Conduction current density is dominating, hence the medium is a good conductor.

The above result indicates that a medium being a good conductor or not also depends on the frequency of EM wave.

The loss angle of the medium θ is related to the phase difference (i.e angle of attenuation constant) θ_η as, $\theta = 2\theta_\eta$.

3.1.2 Wave Propagation in Loss-less Dielectrics

A loss-less dielectric is a medium in which an EM wave, as it propagates, does not lose power owing to perfect dielectric.

The medium parameters of a loss-less dielectric are:
$\sigma \approx 0$, $\epsilon = \epsilon_o \epsilon_r$, $\mu = \mu_o \mu_r$
$\implies \sigma << \omega\epsilon$ or $\frac{\sigma}{\mu\epsilon} >> 1$

Hence, this is a special case of lossy dielectric with no loss. Upon substituting the above parameters in the general expressions derived earlier, it is found that

- $\alpha = 0$
- $\beta = \omega\sqrt{\mu\epsilon} = \frac{\omega}{c}$
- $v_p = \frac{1}{\sqrt{\mu\epsilon}}$
- $\eta = \sqrt{\frac{\mu}{\epsilon}} \angle 0^o$

Note that there is no attenuation in a loss-less dielectric, meaning there is no power loss as an EM wave propagates deeper into the medium.

The phase velocity is a function of the material properties i.e the permittivity and permeability. Hence, any EM wave travelling in a particular loss-less dielectric will have the same phase velocity.

Since the phase angle of intrinsic impedance is 0^o, there is no phase difference between $\vec{E}$ and $\vec{H}$. Hence, the fields are in time phase with each other.

3.1.3 Wave Propagation in Free Space

Free space is simply absence of any medium. In general, even air can be approximated as free space.

The medium parameters of free space are:
$\sigma = 0$, $\epsilon = \epsilon_o$, $\mu = \mu_o$

Hence, free space a special case of perfect dielectric. Upon substituting the above parameters in the general expressions derived earlier, it is found that

- $\alpha = 0$ (no attenuation)
- $\beta = \omega\sqrt{\mu_o \epsilon_o}$
- $v_p = \frac{1}{\sqrt{\mu_o \epsilon_o}}$
- $\eta = \sqrt{\frac{\mu_o}{\epsilon_o}} \angle 0^o \implies |\eta| = 120\pi \approx 377\Omega$

The phase velocity calculated for free space gives the constant c i.e velocity of light in vacuum, which is a useful quantity.
$c = 3 \times 10^8 m/s$

3.1.4 Wave Propagation in Good Conductors

A good conductor is a medium in which an EM wave, as it propagates, loses power almost instantly owing to bad dielectric.

The medium parameters of a good conductor are:
$\sigma \approx \infty$, $\epsilon = \epsilon_o \epsilon_r$, $\mu = \mu_o \mu_r$
$\implies \sigma >> \omega\epsilon$ or $\frac{\sigma}{\mu\epsilon} >> 1$

Hence, this is another special case of lossy dielectric (the opposite extreme). Upon substituting the above parameters in the general expressions derived earlier, several results are obtained.

The attenuation and phase constants are as follows,

$$\alpha = \beta = \sqrt{\frac{\omega\mu\sigma}{2}} = \sqrt{\pi f \mu\sigma}$$

Since the attenuation and phase constants are equal, it proves that the EM waves attenuate rapidly inside a good conductor.

The intrinsic impedance is given by,

$$\eta = \sqrt{\frac{j\omega\mu}{\sigma}} = \sqrt{\frac{\omega\mu}{\sigma}} \angle 45^o$$

The above expression shows that in a good conductor, the electric field leads magnetic field by a phase of $\pi/4$.

The phase velocity will be,

$$v_p = \sqrt{\frac{2\omega}{\mu\sigma}}$$

In a good conductor, the phase velocity of the EM wave varies with frequency i.e difference frequency components move at different velocities.
Hence, a good conductor is a "dispersive medium".

Skin Depth
As the EM wave travels in a conducting medium, the amplitude is attenuated by a factor $e^{\alpha z}$. The distance through which the wave amplitude decreases to about 37% of the original value (i.e by a factor of e^{-1}) is called "Skin Depth" or "Penetration Depth" of the medium and is denoted by δ.
$E_o e^{-\alpha\delta} = E_o e^{-1}$

$$\therefore \ \delta = \frac{1}{\alpha} = \frac{1}{\sqrt{\pi f \mu\sigma}} = \sqrt{\frac{2}{\omega\mu\sigma}}$$

Hence, the skin depth is a measure of the depth to which an EM wave can penetrate the medium.

Typically for a good conductor, since α is significantly large, the skin depth will be very small (few microns at radio frequencies). This means that beyond that the EM waves are almost non-existent in the inner bulk of the conductor, and hence the travelling of EM waves in a conductor is mostly a surface phenomena.

3.2 Poyting Theorem

Energy can be transported from one point to another by means of EM waves. Poyting theorem states that the net power flowing out of a given volume is equal to the time rate of decrease in the energy stored within the volume minus ohmic losses.

The above statement can be mathematically derived upon manipulation of Maxwell's equations and using a few vector properties.

$$\int_v \nabla.(\vec{E} \times \vec{H}) dv = -\int_v \frac{\mu}{2} \frac{\partial |\vec{H}|^2}{\partial t} dv - \int_v \frac{\epsilon}{2} \frac{\partial |\vec{E}|^2}{\partial t} dv - \int_v \vec{E}.\vec{J} dv$$

$$\implies \int_v \nabla.(\vec{E}\times\vec{H})dv = -\frac{\partial}{\partial t}\int_v \frac{\mu}{2}|\vec{H}|^2 dv - \frac{\partial}{\partial t}\int_v \frac{\epsilon}{2}|\vec{E}|^2 dv - \int_v \vec{E}.\vec{J}dv$$

In the RHS of the equation, the first term corresponds to the power (rate of change of energy) due to magnetic field, the second term corresponds to the power (rate of change of energy) due to magnetic field and the third term corresponds to ohmic losses.
Hence by law of conversation of energy, the LHS must represent the net power flow through the volume.

The net power flowing out of a surface is given by the integration of the Poyting vector over the closed surface (which is obtained by converting the volume integral in LHS of the equation to surface integral).
The Poyting vector gives the instantaneous power density and is given by,

$$\vec{P} = \vec{E}\times\vec{H}$$

Since the direction of power flow is the cross produce of $\vec{E}$ and $\vec{H}$, it will be perpendicular to both the electric and magnetic fields.

The average Poyting vector can be obtained by integrating the instantaneous Poyting vector over a time period of the EM wave.

$$\vec{P}_{ave} = \frac{1}{2}\ \text{Re}\ [\vec{E}\times\bar{\vec{H}}]$$

Upon substituting for $\vec{E}$ and $\bar{\vec{H}}$, the following expression can be derived, which is applicable for any media.

$$\vec{P}_{ave} = \frac{1}{2}\ \frac{E_o^2}{|\eta|}\ e^{-2\alpha z}\cos\theta_\eta\ \hat{a_z}$$

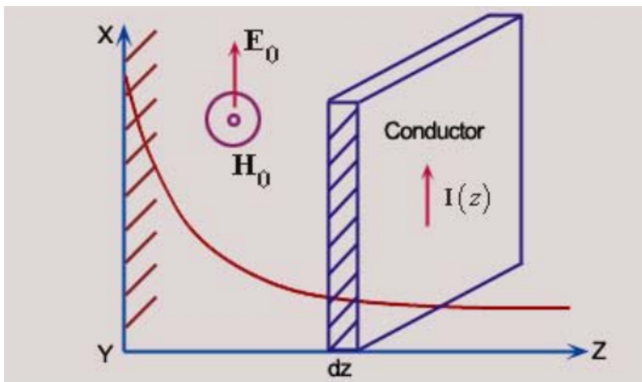
For a loss-less medium, since $\alpha = 0$ and $\theta_\eta = 0$,

$$\vec{P}_{ave} = \ \frac{1}{2}\ \frac{E_o^2}{\eta}\ \hat{a_z} = \ \frac{1}{2}\ H_o^2\ \eta\ \hat{a_z}$$

3.2.1 Surface Current in a Conductor

Let the electric field be E_o at the surface of the conductor.

The field at any distance z inside the conductor is given as
$\vec{E}(z) = E_o e^{-\gamma z} = E_o e^{-\alpha z} e^{-j\beta z} \hat{a_x}$;
and hence the conduction current density at z will be
$J(\vec{z}) = \sigma E(z) = \sigma E_o e^{-\alpha z} e^{-j\beta z} \hat{a_x}$.



The total current density under unit width of the surface is given by,

$$\vec{J_s} = \int_0^\infty E_o \sigma e^{\gamma z} dz \hat{a_x} = \frac{E_o \sigma}{\gamma} \hat{a_x}$$

Since for a good conductor, the current is confined to a very thin region below the surface, the current density $\vec{J_s}$ can be treated as a surface current density.

For non-ideal conductors, a parameter called surface impedance is defined as the ratio of magnitudes of tangential component of electric field and the surface current density.

$$Z_s = \frac{|\vec{E}_{tan}|}{|J_s|} = \frac{E_o}{J_s} = \frac{\gamma}{\sigma} = \eta = \sqrt{\frac{\omega\mu}{2\sigma}} + j\sqrt{\frac{\omega\mu}{2\sigma}}$$

3.2.2 Power Loss in a Conductor

The resistance of a slab of a conductor along the direction of the current is, $dR = \rho l / A = 1/(\sigma dz)$.

The ohmic loss in this slab is $dW = |I(z)|^2 dR = \sigma |E_o|^2 e^{-2\gamma z} dz$.

Total ohmic loss per unit area of the conductor can be obtained by integrating over ohmic losses for such slabs which will give,

$$W = \int_0^\infty \sigma |E_o|^2 e^{-2\gamma z} dz = \frac{\sigma |E_o|^2}{2\alpha} = R_s |J_s|^2$$

The power loss is proportional to the surface resistance which increases with frequency and decreases with conductivity. Higher the conductivity lesser the loss and for ideal conductor when the conductivity is infinite the ohmic loss is zero.

3.3 Wave Vector at Arbitrary Direction

Let a wave be moving in some direction making angles ϕ_x, ϕ_y and ϕ_z with the x, y and z axes respectively as shown.

The unit vector in the direction of the wave propagation is $\hat{n} = \cos\phi_x \hat{a_x} + \cos\phi_y \hat{a_y} + \cos\phi_z \hat{a_z}$.
Here, $\cos\phi_x$, $\cos\phi_y$ and $\cos\phi_z$ are called the 'direction cosines' of the vector $\hat{n}$.

The equation of a constant phase plane (the phase front) is given as $\hat{n}.\vec{r} = l$.
$\implies$ The phase of this constant phase plane is $\beta \hat{n}.\vec{r}$.

Electric & Magnetic fields for wave moving in arbitrary direction

The electric field of a plane wave travelling in direction $\hat{n}$ can be written as $\vec{E} = \vec{E_o} e^{-j\beta \hat{n}.\vec{r}}$ where $\vec{r} = x\hat{a_x} + y\hat{a_y} + z\hat{a_z}$.
Let $\vec{k} = \beta \hat{n} = k_x \hat{a_x} + k_y \hat{a_y} + k_z \hat{a_z}$.

The electric field then is $\vec{E} = \vec{E_o} e^{-j(k_x x + k_y y + k_z z)}$ where:
$k_x = \beta \cos\phi_x$, $k_y = \beta \cos\phi_y$ and $k_z = \beta \cos\phi_z$.

From Maxwell's equations, the magnetic field is then obtained as
$\vec{H} = -\frac{1}{j\omega\mu} \nabla \times \vec{E} = \frac{1}{\omega\mu} \vec{k} \times \vec{E} = \vec{H_o} e^{-j\vec{k}.\vec{r}}$.

Note that $\vec{E_o}$, $\vec{H_o}$ and $\vec{n}$ are all perpendicular to each other.

3.3.1 Phase Velocity and Group Velocity

The electric field of a uniform plane wave travelling in a direction which makes angles ϕ_x, ϕ_y and ϕ_z with the x, y and z axes respectively can be expressed as,

$$\vec{E} = \vec{E_o} e^{-j\beta \cos\theta_x} e^{-j\beta \cos\theta_y} e^{-j\beta \cos\theta_z}$$

If the actual velocity of the wave is v_o, then the phase velocities are given by:

- Phase velocity in x direction is $v_{px} = \frac{\omega}{k_x} = \frac{v_o}{\cos\phi_x}$
- Phase velocity in x direction is $v_{py} = \frac{\omega}{k_y} = \frac{v_o}{\cos\phi_y}$
- Phase velocity in x direction is $v_{pz} = \frac{\omega}{k_z} = \frac{v_o}{\cos\phi_z}$

It can be noted that the phase velocities in the 3 directions are all greater than the actual velocity. This is because they are just quantities obtained by analysis and they are not physical velocities of the wave.

The velocity with which the energy of the wave actually travels in each direction is called group velocity, and this is a physical quantity.
Group velocities in each direction are basically components of the actual velocity and hence, they are given by:

- Group velocity in x direction is $v_{gx} = v_o \cos\phi_x$
- Group velocity in x direction is $v_{gy} = v_o \cos\phi_y$
- Group velocity in x direction is $v_{gz} = v_o \cos\phi_z$

Obviously, the group velocities in the 3 directions are smaller than the actual velocity.
$\implies \; v_o < v_p < \infty;\; 0 < v_g < v_o;\; v_g < v_o < v_p$

Also, another useful relation between the three velocities is,

$$v_p v_g = v_o^2 \qquad \implies \; v_o = \sqrt{v_p v_g}$$

3.4 Wave Polarization

The wave polarization is the time behaviour of the electric field of a transverse EM wave at a given point in space.
In other words, the state of polarization of a wave is described by the geometrical shape which the tip of the electric field vector draws as a function of time at a given point in space.
Polarization is a fundamental characteristic of a wave, and every wave has a definite state of polarization.

Consider electric field in a loss-less medium given by $E(z,t) = E_x \cos{(\omega t - \beta z)}\hat{a_x} + E_y \cos{(\omega t - \beta z + \phi)}\hat{a_y}$. Here, z is the direction of propagation and ϕ is the phase difference between the x and y components of the electric field.

To obtain the polarization (which is time behaviour), the space co-ordinate z can be fixed, let it be 0.
Hence, the electric field will be $E(t) = E_x \cos{(\omega t)}\hat{a_x} + E_y \cos{(\omega t + \phi)}\hat{a_y}$.

This expression can be analyzed for different conditions to give different polarization patterns.
The most general case will give elliptical polarization with the major and minor axes not inclined to the co-ordinate axes.

1. Case 1 - $\phi = 0^o$:
   $E(t) = (E_x\hat{a_x} + E_y\hat{a_y})\cos{\omega t}$
   The electric field with respect to time will trace a line of slope 45^o in the $x - y$ plane.
   $\implies$ **Linear Polarization**

2. Case 2 - $\phi = 180^o$ (π):
   $E(t) = (E_x\hat{a_x} - E_y\hat{a_y})\cos{\omega t}$
   The electric field with respect to time will trace a line of slope -45^o in the $x - y$ plane.
   $\implies$ **Linear Polarization**

3. Case 3 - $\phi = 90^o$ $(\pi/2)$ and $E_x = E_y = E_o$:
   $E(t) = E_o \cos{\omega t}\hat{a_x} - E_o \sin{\omega t}\hat{a_y}$ The electric field with respect to time traces a circle in the $x - y$ plane in the clock-wise direction.
   $\implies$ **Left circular polarization**

4. Case 4 - $\phi = -90^o$ $(3\pi/2)$ and $E_x = E_y = E_o$:
   $E(t) = E_o \cos\omega t\hat{a_x} + E_o \sin\omega t\hat{a_y}$
   The electric field with respect to time traces a circle in the $x-y$ plane in the anti clock-wise direction.
   $\Longrightarrow$ **Right circular polarization**

5. Case 5 - $0 < \phi < \pi$ (between 0^o and 180^o) and $E_x \neq E_y$:
   $E(t) = E_x \cos\omega t\hat{a_x} - E_y \sin\omega t\hat{a_y}$
   The electric field with respect to time traces an ellipse in the $x-y$ plane in the clock-wise direction.
   $\Longrightarrow$ **Left elliptical polarization**

6. Case 6 - $\pi < \phi < 2\pi$ (between 180^o and 360^o) and $E_x \neq E_y$:
   $E(t) = E_x \cos\omega t\hat{a_x} + E_y \sin\omega t\hat{a_y}$
   The electric field with respect to time traces an ellipse in the $x-y$ plane in the anti clock-wise direction.
   $\Longrightarrow$ **Right elliptical polarization**

The following conclusions can be made from the above cases.

- If $\phi = n\pi$ (integral multiple of 180^o), it means the electric field will be linearly polarized.

- If $\phi = \pm(2n+1)\pi/2$ (odd multiple of 90^o) and the x and y components are equal, it means the electric field will be circularly polarized.

- For any ϕ, if the x and y components are not equal, it means the electric field will be elliptically polarized.

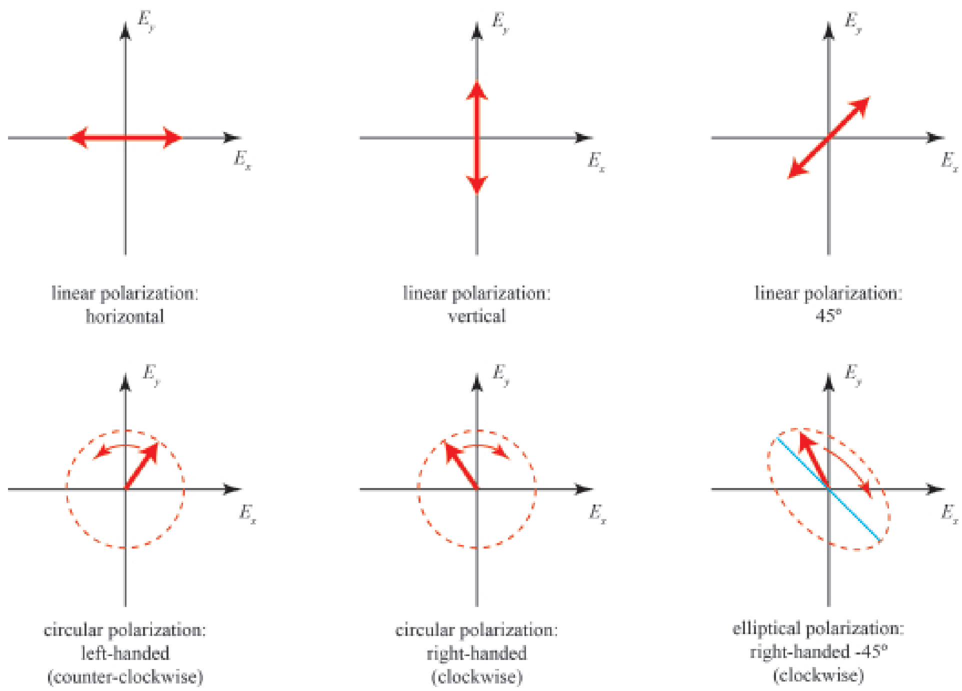
All these are valid if the direction of propagation is fixed in the positive z direction (which is observed from the wave equation given at the start).
If the direction of propagation is in the negative z direction, i.e the field is $E(z,t) = E_x \cos(\omega t + \beta z)\hat{a_x} + E_y \cos(\omega t + \beta z + \phi)\hat{a_y}$, then the left and right polarization will be swapped.

- A wave is said to be Un-polarized if the variation of electric field is not restricted to a plane i.e it varies in all 3 directions.

- If the electric field has only x component, then it is said to be in Horizontal Polarization.

- If the electric field has only y component, then it is said to be in Vertical Polarization.

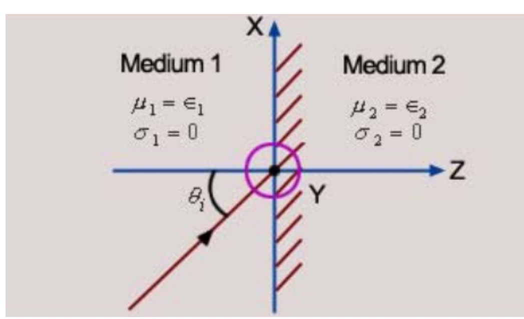
A few examples of polarization patterns are illustrated.



3.5 Plane Wave at Dielectric Interface

Consider the propagation of a plane wave across two dielectric media interface. Assume that the media are loss-less i.e their conductivities are zero.

Orient the co-ordinate system as shown in the figure below for convenience.



The line perpendicular to media interface is called normal to the interface (z-axis in this case).

Let the wave be incident from medium 1 such that the wave vector lies in the xz- plane making an an angle θ_i with respect to the interface normal. The angle θ_i is called the "Angle of Incidence".

The plane containing the interface normal and the wave vector (i.e the xz-plane in this case) is called the 'Plane of Incidence'.

$$\implies \theta_x = \frac{\pi}{2} - \theta_i; \; \theta_y = \frac{\pi}{2}; \; \theta_z = \theta_i$$

From the angles obtained, any generic electric or magnetic field is given by $\vec{F_i} = F_o e^{-j\beta_1(x\cos\theta_x + y\cos\theta_y + z\cos\theta_z)}$
Upon substituting for the angles, the above expression can be simplified to

$$\vec{F_i} = F_o e^{-j\beta_1(x\sin\theta_i + z\cos\theta_i)}$$

where $\beta_1 = \omega\sqrt{\mu_1\epsilon_1}$

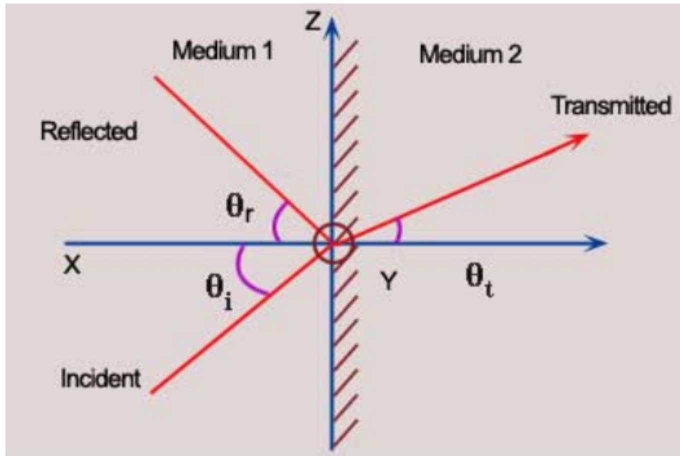
The incident wave will create a phase variation at the interface which is $\beta_1 x \sin\theta_i$ along x-direction and no variation in y-direction.

When this wave is incident at the interface, on the other side of the interface similar phase variation will be induced to maintain continuity of the fields. It can also be shown that the continuity for both electric and magnetic field cannot be achieved without altering the fields in medium 1.

Hence, there will be two induced fields which constitute waves in both the media going away from the interface. Overall all the fields present will be,

- Combination of incident field and the induced field in medium 1.
- Induced field in medium 2.

The induced field in medium 1 are called the 'Reflected Field' and the induced field in medium 2 are called the 'Transmitted Field'.



Laws of Reflection

First Law of Reflection:
Since the phase is constant in y-direction the reflected and transmitted wave have wave vectors in the xy-plane i.e the plane of incidence, it can be concluded that the incident, reflected and transmitted wave vectors lie in the same plane.

Second Law of Reflection:
If the reflected wave vector makes an angle θ_r with respect to the interface normal as shown in the figure, the reflected field can be written as,
$\vec{F_r} = F_o e^{-j\beta_1(x\sin\theta_r - z\cos\theta_r)}$

At the interface i.e $z = 0$, continuity of the fields demands that $\beta_1 x \sin\theta_i = \beta_1 x \sin\theta_r \implies \theta_i = \theta_r$
Hence, the second law is derived which states that "The Angle of Reflection is always equal to the Angle of Incidence"

Law of Refraction
If the transmitted (or refracted) wave vector makes an angle θ_t with respect to the interface normal as shown in the figure, the refracted field can be written as,
$\vec{F_t} = F_o e^{-j\beta_2(x\sin\theta_t + z\cos\theta_t)}$

At the interface i.e $z = 0$, continuity of the fields demands that

$\beta_1 x \sin\theta_i = \beta_2 x \sin\theta_t \quad \implies \sqrt{\mu_1\epsilon_1}\sin\theta_i = \sqrt{\mu_2\epsilon_2}\sin\theta_t$
This is called "Snell's Law".

For ideal dielectrics, $\mu_1 = \mu_2 = \mu_o$. Hence, Snell's Law will be reduced to $\sqrt{\epsilon_1}\sin\theta_i = \sqrt{\epsilon_2}\sin\theta_t$ which is same as $n_1 \sin\theta_i = n_2 \sin\theta_t$ where n_1 and n_2 are refractive indices of medium 1 and medium 2.

3.5.1 Reflection and Refraction from Dielectric Interface

After applying the law of reflection and Snell's law, the problem of wave propagation across the dielectric interface reduces to finding the amplitudes of the fields of three waves at the interface.
Once the fields are known at the interface, the field in region 1 can be obtained by super-position of F_i and F_r whereas in medium 2 the field is just F_t.

For an arbitrary polarization of the wave, handling waves across the media interface is quite complicated.
The problem is elegantly handled by decomposing the fields into its components i.e one in the plane of incidence and other perpendicular to it.
Therefore analysis can be done using the following two cases:

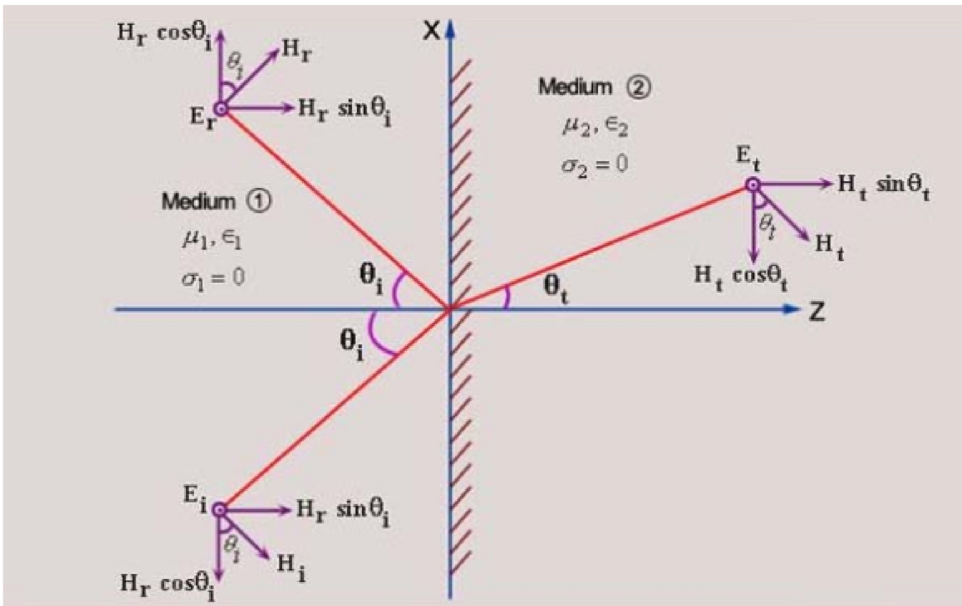
1. Field in the plane of incidence called Parallel Polarization.
2. Field perpendicular to the plane of incidence called Perpendicular Polarization.

For each case, the reflection coefficient and transmission coefficient can be found using the following definitions.
Reflection coefficients : $\Gamma_\perp = \frac{E_{r_\perp}}{E_{i_\perp}}$ and $\Gamma_{||} = \frac{E_{r_{||}}}{E_{i_{||}}}$
Transmission coefficients : $\tau_\perp = \frac{E_{t_\perp}}{E_{i_\perp}}$ and $\tau_{||} = \frac{E_{t_{||}}}{E_{i_{||}}}$

Reflection and Refraction with Perpendicular Polarization

The electric field component of the EM wave incident on the interface is perpendicular to the plane of incidence. Since the plane of incidence is the xz-plane, the electric field vector will be pointing in y-direction.

Since the Poyting vector denotes the direction of power flow, using the direction of the EM wave (i.e Poyting vector) and the electric field direction, the magnetic field direction can be obtained (the 3 must be perpendicular to each other).



The figure above shows the electric fields that are perpendicular to the plane of incidence and the corresponding magnetic fields that are decomposed to horizontal and vertical components in the plane.

Media constants here are $\beta_1 = \sqrt{\mu_1 \epsilon_1}$, $\beta_2 = \sqrt{\mu_2 \epsilon_2}$, $\eta_1 = \sqrt{\frac{\mu_1}{\epsilon_1}}$ and $\eta_2 = \sqrt{\frac{\mu_2}{\epsilon_2}}$

Electric field waves:

- Incident wave: $\vec{E_i} = |E_i| e^{-j\beta_1(x\sin\theta_i + z\cos\theta_i)} \hat{a_y}$
- Reflected wave: $\vec{E_r} = |E_r| e^{-j\beta_1(x\sin\theta_i - z\cos\theta_i)} \hat{a_y}$
- Incident wave: $\vec{E_t} = |E_t| e^{-j\beta_2(x\sin\theta_t + z\cos\theta_t)} \hat{a_y}$

Corresponding magnetic field waves:
$|H_i| = |E_i|/\eta_1$; $|H_r| = |E_r|/\eta_1$; $|H_t| = |E_t|/\eta_2$;

Applying boundary condition that the tangential components must be continuous (since there is no surface current),

$\vec{E}_i + \vec{E}_r = \vec{E}_t$ and $\vec{H}_i \cos\theta_i - \vec{H}_r \cos\theta_i = \vec{H}_t \cos\theta_t$.

Upon substituting for the magnetic fields in terms of electric fields and solving the above two equations, the reflection and transmission coefficients are obtained as:

$$\Gamma_\perp = \frac{\eta_2 \cos\theta_i - \eta_1 \cos\theta_t}{\eta_2 \cos\theta_i + \eta_1 \cos\theta_t}$$

$$\tau_\perp = \frac{2\eta_2 \cos\theta_i}{\eta_2 \cos\theta_i + \eta_1 \cos\theta_t}$$

It can also be deduced that $\tau_\perp = 1 + \Gamma_\perp$

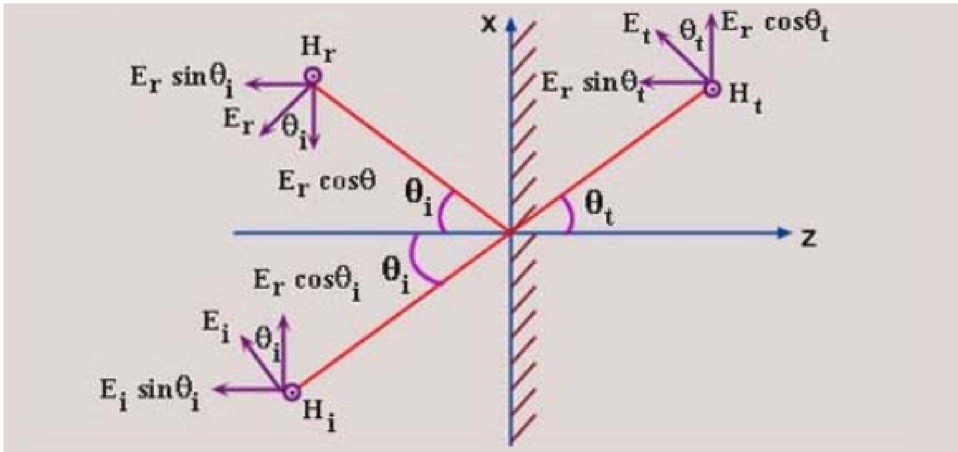
From the derived relations, the following conclusions can be drawn regarding perpendicular polarization incidence on a dielectric interface.

- The reflection and transmission coefficients are real and hence the phase differences between incident wave and the transmitted or reflected wave is either 0 or $\pi/2$.
- The magnitude of the reflection coefficient is always less than unity.
- The transmission coefficient could be greater or less than unity. However, a transmission coefficient greater than unity does not mean higher transmitted power compared to the incident wave.

Reflection and Refraction with Parallel Polarization

The electric field component of the EM wave incident on the interface is in the plane of incidence. Since the plane of incidence is the xz-plane, the electric field vector will be pointing at an arbitrary direction in the xz-plane.

Since the Poyting vector denotes the direction of power flow, using the direction of the EM wave (i.e Poyting vector) and the electric field direction, the magnetic field direction can be obtained (the 3 must be perpendicular to each other).



The figure above shows the electric fields that are decomposed to horizontal and vertical components in the plane of incidence and the corresponding magnetic fields that are perpendicular to the plane.
Media constants are same as before.

Magnetic field waves in terms of electric field magnitudes:

- Incident wave: $\vec{H}_i = \frac{|E_i|}{\eta_1} e^{-j\beta_1(x\sin\theta_i + z\cos\theta_i)} \hat{a_y}$
- Reflected wave: $\vec{H}_r = \frac{|E_r|}{\eta_1} e^{-j\beta_1(x\sin\theta_i - z\cos\theta_i)} \hat{a_y}$
- Incident wave: $\vec{H}_t = \frac{|E_t|}{\eta_2} e^{-j\beta_2(x\sin\theta_t + z\cos\theta_t)} \hat{a_y}$

Applying boundary condition that the tangential components must be continuous (since there is no surface current),
$\vec{H}_i + \vec{H}_r = \vec{H}_t$ and $\vec{E}_i \cos\theta_i - \vec{E}_r \cos\theta_i = \vec{E}_t \cos\theta_t$.

Upon substituting for the magnetic fields in terms of electric fields and solving the above two equations, the reflection and transmission coefficients are obtained as:

$$\Gamma_{||} = \frac{\eta_1 \cos\theta_i - \eta_2 \cos\theta_t}{\eta_1 \cos\theta_i + \eta_2 \cos\theta_t}$$

$$\tau_{||} = \frac{2\eta_2 \cos\theta_i}{\eta_1 \cos\theta_i + \eta_2 \cos\theta_t}$$

It can also be deduced that $\tau_{||} = 1 + \frac{\eta_1}{\eta_2}\Gamma_{||}$
Similar conclusions can be drawn regarding parallel polarization incidence on a dielectric interface.

Reflection and Refraction for Normal Incidence

Normal incidence can be treated as a special case of oblique incidence discussed earlier where the angle of incidence is equal to 0.
Since $\theta_i = 0$, it means $\theta_r = 0$ and $\theta_t = 0$ by laws of reflection and refraction.

Hence upon substituting the angles in expressions for perpendicular polarization case, the reflection and transmission coefficients will be,

$$\Gamma = \frac{\eta_2 - \eta_1}{\eta_2 + \eta_1}$$

$$\tau = \frac{2\eta_2}{\eta_2 + \eta_1}$$

Note that the sign for reflection coefficient will be reversed if substituted in expressions for parallel polarization case, this is because in parallel case it can be observed that E_i and E_r are point in opposite directions (whereas in perpendicular case both point in same direction).

Lossy Media Interface
The analysis of the lossy media interface can be carried out on the lines similar to that of loss less interface with appropriate changes needed for the propogation constant and intrinsic impedance of the lossy media.

For a lossy medium the conductivity is not zero and hence the expressions for intrinsic impedance and the propagation constant will be complex.

It should be noted that for lossy media the wave amplitude does not remain constant over the distance and therefore the transmitted and reflected wave decay exponentially as they travel away from the interface.

When the conductivity becomes infinite, only surface current exists and the skin depth is zero giving no propagation of EM wave inside the conductor.

3.5.2 Total Internal Reflection

From Snell's Law, $\beta_1 \sin\theta_i = \beta_2 \sin\theta_t$.
If $\frac{\beta_1}{\beta_2}\sin\theta_i > 1$, it means that $\sin\theta_t > 1$ and consequently the angle of transmission can not physically exist.

That means if a wave is launched at an angle which satisfies the above condition, there is no transmitted wave.

The incidence angle for which the angle of transmission is 90^o is called the "Critical Angle" (θ_c).
$\implies \theta_c = \sin^{-1}\frac{\beta_2}{\beta_1}$

It can be observed that if the angle of incidence is greater than the critical angle, then the transmission coefficient (for both perpendicular and parallel polarization cases) will be a complex quantity but with unit magnitude.
This also implies there will be a phase change between the incident and reflected waves.

Since the reflection coefficient magnitude is 1, it means that the entire power in the incident wave will be reflected back to the same medium and there is no transmitted wave. This phenomena is called therefore called Total Internal Reflection.

The following points are to be noted regarding TIR.

- Since $\sin\theta_i \leq 1$, TIR can take place only if $\mu_2\epsilon_2 \leq \mu_1\epsilon_1$. In case of ideal dielectric where $\mu_1 = \mu_2 = \mu_0$, then TIR can take place only if $\epsilon_2 \leq \epsilon_1$ which is equivalent to saying $n_2 \leq n_1$ where n_1 and n_2 are refractive indices of medium 1 and medium 2.
  This means for TIR to take place, medium 2 should be rarer than medium 1 (or medium 1 should be denser than medium 2)

- The wave undergoes a phase change when TIR occurs. The phase change depends upon the media parameters as well as the angle of incidence. For a given angle of incidence, waves with parallel polarization and perpendicular polarization undergo different phase change.

3.5.3 Wave Polarization at Media Interface

Media interface can be used to change the state of polarization of an EM Wave.
The electric field for any state of polarization can be resolved into two orthogonal components one parallel to the plane of incidence and other perpendicular to it. Therefore the incident electric field can be written as: $\vec{E_i} = \vec{E_i}_{||} + \vec{E_{i\perp}} e^{j\phi}$.
The reflected and transmitted fields can be written as:
$\vec{E_r} = \Gamma_{||}\vec{E_r}_{||} + \Gamma_{\perp}\vec{E_{r\perp}} e^{j\phi}$ and $\vec{E_t} = \tau_{||}\vec{E_t}_{||} + \tau_{\perp}\vec{E_{t\perp}} e^{j\phi}$.
Note that the reflection coefficients are real for ordinary scenario and complex for total internal reflection scenario.

Linearly Polarized Incident Wave:
For a linearly polarized wave, $\phi = 0$. Then,

- For ordinary reflection, since Γ and τ are real, the reflected and transmitted fields also remain linearly polarized. However since in general $|\Gamma_{\perp}| \neq |\Gamma_{||}|$ and $|\tau_{\perp}| \neq |\tau_{||}|$, the plane of linear polarization will change.
- For total internal reflection, since Γ and τ are complex, the reflected field will become elliptically polarized due to induced non-zero phase difference. Note that here, $|\Gamma_{\perp}| = |\Gamma_{||}| = 1$.

Hence, linearly polarized wave remains linearly polarized at ordinary reflection but becomes elliptically polarized at Total Internal Reflection.

Circularly Polarized Incident Wave:
For a circularly polarized wave, $\phi = \pm\pi/2$ and $E_{i||} = E_{i\perp}$. Then, the reflected and transmitted wave will in general be elliptically polarized since $\Gamma_{\perp} \neq \Gamma_{||}$ and $\tau_{\perp} \neq \tau_{||}$.

Hence, circularly polarized wave becomes elliptically polarized with change in axial ratio and the tilt angle.

Brewster Angle
Breswter angle is the angle of incidence for which there is no reflection from the media interface i.e it is the angle of incidence for which the reflection coefficient is zero.

Hence if the angle of incidence of an EM wave is equal to the Brewster angle, then the entire EM wave will be transmitted from medium 1 to medium 2.

For an ideal dielectric interface, the permeablilities of both the media's are same as that of the free space. In such a case,

- Brewster Angle does not exist for Perpendicular polarization. Complete transmission occurs only if $\epsilon_1 = \epsilon_2$.
- Brewster Angle for Parallel polarization is given by, $\tan\theta_B = \sqrt{\frac{\epsilon_2}{\epsilon_1}}$.

If an arbitrary polarized wave is incident at the Brewster angle, then parallel polarization is completely transmitted but the perpendicular polarization is only partially transmitted.
Hence, the reflected wave then will only have perpendicular polarization irrespective of the polarization of the incident wave. In other words, the reflected wave is linearly polarized (perpendicular polarization) irrespective of the state of polarization of the incident wave.
Because of this effect, the Brewster angle is also called "Polarizing angle".

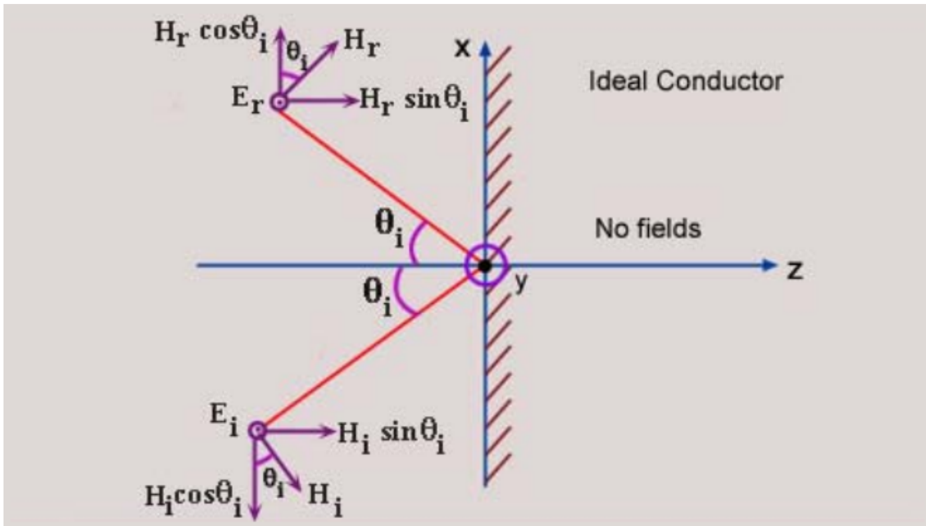
Although the Brewster angle exists for only parallel polarization at the ideal dielectric interface (i.e both media have permeability of free space), there could be different Brewster angles for both polarizations if the permeablilities of two media are not same.

3.5.4 Reflection from a Conducting Boundary

Consider a dielectric conductor-interface with a wave incident from the dielectric side.
Since no time varying fields exists inside an ideal conductor there is no question of transmitted wave in this case. The wave is therefore completely reflected from a conducting boundary.

Again, investigation can be done using two different cases of Perpendicular polarization and Parallel polarization.

Perpendicular Polarization:



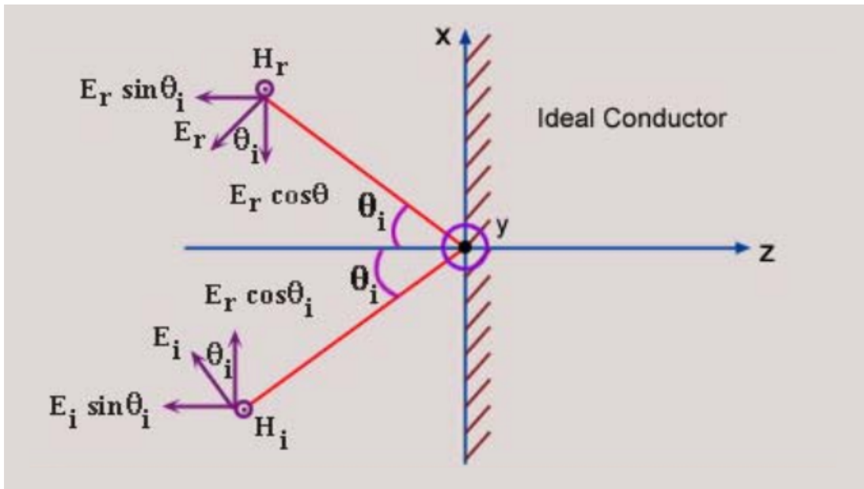
The electric and magnetic fields for incident wave for the geometry given in the above figure can be written as:
$\vec{E_i} = |E_i| e^{-j\beta(x\sin\theta_i + z\cos\theta_i)} \hat{a_y}$
$\vec{H_i} = \frac{|E_i|}{\eta} e^{-j\beta(x\sin\theta_i + z\cos\theta_i)} (-\cos(\theta_i)\hat{a_x} + \sin(\theta_i)\hat{a_z})$

Similarly, the electric and magnetic fields for reflected wave will be:
$\vec{E_r} = |E_r| e^{-j\beta(x\sin\theta_i - z\cos\theta_i)} \hat{a_y}$
$\vec{H_r} = \frac{|E_r|}{\eta} e^{-j\beta(x\sin\theta_i - z\cos\theta_i)} (\cos(\theta_i)\hat{a_x} + \sin(\theta_i)\hat{a_z})$

Making the tangential component of total electric field (incident and reflected) at the interface as zero, the reflection coefficient can be obtained as $\Gamma_\perp = -1$.

Recall that for a Transmission Line, the voltage reflection coefficient is -1 for a short circuit load. The ideal conducting boundary for a perpendicularly polarized EM wave is therefore is analogous to a short circuit impedance of a transmission line.

Parallel Polarization:



The magnetic and electric fields for incident wave for the geometry given in the above figure can be written as:
$\vec{H_i} = \frac{|E_i|}{\eta} e^{-j\beta(x\sin\theta_i + z\cos\theta_i)} \hat{a_y}$
$\vec{E_i} = |E_i| e^{-j\beta(-x\sin\theta_i - z\cos\theta_i)} (-\cos(\theta_i)\hat{a_x} + \sin(\theta_i)\hat{a_z})$

Similarly, the magnetic and electric fields for reflected wave will be:
$\vec{H_r} = \frac{|E_r|}{\eta} e^{-j\beta(x\sin\theta_i - z\cos\theta_i)} \hat{a_y}$
$\vec{E_r} = |E_r| e^{-j\beta(x\sin\theta_i - z\cos\theta_i)} (-\cos(\theta_i)\hat{a_x} - \sin(\theta_i)\hat{a_z})$

Making the tangential component of total electric field (incident and reflected) at the interface as zero, the reflection coefficient can be obtained as $\Gamma_\perp = 1$.

Since the reflection coefficient is +1 in this case, it can't be concluded that for Parallel polarization, the conducting boundaries behaves like the open circuit impedance of transmission line.
The reflection coefficient becomes +1 because the electric field directions have already been taken such that the tangential components for the incident and reflected waves cancel each other at the interface. Hence irrespective of the polarization, a conducting boundary is always equivalent to short circuit impedance of transmission line.

Normal Incidence at Conducting Boundary

As usual, normal incidence can be treated like a special case of oblique incidence discussed earlier. For Normal Incidence, $\theta_i = 0$ and hence the incident fields are expressed as
$\vec{E}_i = |E_i|e^{-j\beta z}\hat{a_x}$ and $\vec{H}_i = \frac{|E_i|}{\eta}e^{-j\beta z}\hat{a_y}$
and the reflected fields are expressed as
$\vec{E}_r = |E_r|e^{j\beta z}\hat{a_x}$ and $\vec{H}_r = \frac{|E_r|}{\eta}e^{j\beta z}\hat{a_y}$

The reflection coefficient is evidently $\Gamma = -1$.

This case is exactly identical to transmission line with short circuited load. Since magnitude of reflection coefficient is equal to one, the entire power is reflected from the conducting boundary.
Therefore, there are two waves with equal amplitude travelling in opposite directions in the dielectric medium, which leads to standing waves.

4 Waveguides

In transmission line analysis, a time varying voltage voltage source created a potential difference between two conductors which caused time varying currents to flow through them.
Since there will be flow of charges on the two conductors, there will be induced electric field and magnetic field.
If direction of propagation is taken as z direction, and x is the direction pointing from upper conductor to lower conductor, then an electric field is induced in the x direction whereas a magnetic field is induced around the conductors in y direction (both of which are functions of space and time).
$\vec{E} = E(z,t)\hat{x}$ and $\vec{H} = H(z,t)\hat{y}$.

Hence, it can be noted that energy flow in transmission lines can also be described using electric and magnetic fields, which forms a plane wave. Therefore, transmission lines are also guiding structures for plane waves.
In fact, transmission lines on support the propagation of plane waves i.e Transverse Electro-Magnetic waves. This means the component of electric and magnetic fields along the direction of propagation must be zero.
$E_z = 0,\ H_z = 0 \implies$ TEM mode.

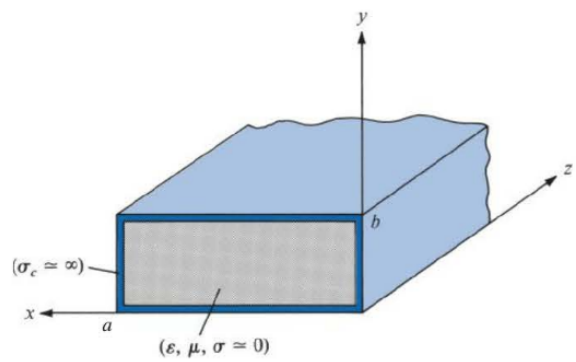
However, transmission lines will not be able to guide waves at high frequencies. They work well upto several MHz but when the required frequency is in GHz, then transmission lines become ineffective due to skin effect and dielectric losses.

Waveguides are structures that can guide plane waves at significantly high frequencies (even upto GHz and THz), but they do not work for lower frequencies.
Structurally, waveguides are made up of only a single conductor (as opposed to 2 conductors in transmission lines).
And waveguides support propagation of waves that are not plane waves, i.e either E_z or H_z or both can be non-zero (more on these modes later) but they do not support TEM mode.

Common waveguides are either rectangular or circular. Both types of waveguides will be discussed in detail.

4.1 Rectangular Waveguides

A rectangular waveguide of width a and height b is shown in the figure, positioned such that width is along x-direction, height is along y-direction and direction of wave propagation along z.



It is assumed that the walls are perfect conductors and it is filled with source-free lossless dielectric ($\rho_v = 0, \vec{J} = 0$).

In general, $a \geq 2b$. The ratio of width to height (a/b) is called 'aspect ratio' of the waveguide.

For a lossless medium, Maxwell's equations in phasor form are:

$$\nabla^2 E_s - \gamma^2 E_s = 0 \qquad\qquad \nabla^2 H_s - \gamma^2 H_s = 0$$

where $\gamma = j\omega\sqrt{\mu\epsilon}$ and time factor $e^{j\omega t}$ is assumed.

In general, the fields will obviously have 3 components each.
$E_s = (E_{xs}, E_{ys}, E_{zs})$ and $H_s = (H_{xs}, H_{ys}, H_{zs})$.

The field patterns or configurations come in different types. Each of these distinct field patterns is called a mode. Four different mode categories can exist, namely:

1. $E_{zs} = 0 = H_{zs}$ (TEM mode):
   In the transverse electromagnetic mode, both the E and H fields are transverse to the direction of wave propagation. A hollow rectangular waveguide cannot support TEM mode.

2. $E_{zs} \neq 0, H_{zs} = 0$ (TM modes):
   In this case, the remaining components (H_{xs} and H_{ys}) of the magnetic field are transverse to the direction of propagation.

3. $E_{zs} = 0, H_{zs} \neq 0$ (TE modes):
   In this case, the remaining components (E_{xs} and E_{ys}) of the electric field are transverse to the direction of propagation.

4. $E_{zs} \neq 0, H_{zs} \neq 0$ (HE modes):
   In this case neither the E nor the H field is transverse to the direction of wave propagation. These modes are referred to as hybrid modes.

TE and TM modes are explored in more detail.

4.1.1 Transverse Magnetic Modes

As mentioned earlier, in TM mode, $E_{zs} \neq 0$ and $H_{zs} = 0$.
Upon applying boundary conditions that tangential components of E_s must be continuous, it can be deduced that

$$E_{zs} = E_0 \sin\left(\frac{m\pi x}{a}\right) \sin\left(\frac{n\pi y}{b}\right) e^{-\gamma z}$$

m is half cycle variation in x-direction ($m = 0, 1, 2, ...$)
n is half cycle variation in y-direction ($n = 0, 1, 2, ...$)

If $m = 0$ or $n = 0$, then E_{zs} will become 0 and it will no longer be TM mode.
Hence, TM_{01} and TM_{10} modes don't exist.
Which is why the lowest order TM mode that can exist is TM_{11} mode.

Each mode is characterized by values of m,n and cut-off frequency.

The expression for propagation constant can be found to be

$$\gamma^2 = \left(\frac{m\pi}{a}\right)^2 + \left(\frac{n\pi}{b}\right)^2 - \omega^2\mu\epsilon$$

From the above expression, three different possibilities are present.

1. Cut-off:
$$\left(\frac{m\pi}{a}\right)^2 + \left(\frac{n\pi}{b}\right)^2 = \omega^2\mu\epsilon; \qquad \Longrightarrow \gamma = \alpha = \beta = 0$$
The value of ω that causes this condition is called 'cut-off' frequency.
$$\omega_c = \frac{1}{\sqrt{\mu\epsilon}}\sqrt{\left(\frac{m\pi}{a}\right)^2 + \left(\frac{n\pi}{b}\right)^2}$$
No propagation takes place at this frequency

2. Evanescent:
$$\left(\frac{m\pi}{a}\right)^2 + \left(\frac{n\pi}{b}\right)^2 < \omega^2\mu\epsilon; \qquad \Longrightarrow \gamma = \alpha \quad \beta = 0$$
No propagation takes place in this mode either

3. Propagation:
$$\left(\frac{m\pi}{a}\right)^2 + \left(\frac{n\pi}{b}\right)^2 > \omega^2\mu\epsilon; \qquad \Longrightarrow \gamma = j\beta \quad \alpha = 0$$
Propagation takes place with attenuation.

Thus, the cut-off frequency is the frequency above which the waveguide operates (which proves it is a high pass structure).

$$f_c = \frac{1}{2\pi\sqrt{\mu\epsilon}}\sqrt{\left(\frac{m\pi}{a}\right)^2 + \left(\frac{n\pi}{b}\right)^2} = \frac{u'}{2}\sqrt{\left(\frac{m}{a}\right)^2 + \left(\frac{n}{b}\right)^2}$$
where $u' = \frac{1}{\sqrt{\mu\epsilon}}$ is the phase velocity of uniform plane wave in the medium.

Hence, the cut-off wavelength is given by,
$$\lambda_c = \frac{2}{\sqrt{\left(\frac{m}{a}\right)^2 + \left(\frac{n}{b}\right)^2}}$$
Dominant Mode is the mode with lowest cut-off frequency (or highest cut-off wavelength).
In TM modes, the dominant mode is obviously TM_{11} mode.

4.1.2 Transverse Electric Modes

As mentioned earlier, in TE mode, $H_{zs} \neq 0$ and $E_{zs} = 0$.
Upon applying boundary conditions that tangential components of E_s must be continuous, it can be deduced that

$$H_{zs} = H_0 \cos\left(\frac{m\pi x}{a}\right) \cos\left(\frac{n\pi y}{b}\right) e^{-\gamma z}$$

m is half cycle variation in x-direction ($m = 0, 1, 2, ...$)
n is half cycle variation in y-direction ($n = 0, 1, 2, ...$)

For TE modes, (m,n) may be (0, 1) or (1, 0) but not (0, 0). Both m and n cannot be zero at the same time because this will force all field components apart from H_{zs} to vanish.
Hence, the lowest mode can be TE_{10} or TE_{01} depending on the values of a and b, the dimensions of the waveguide.

Note that the cut-off frequency and cut-off wavelength expressions in TE mode will be the same as the ones described in TM mode.

Since usually $a \geq 2b$, $f_c(\mathrm{TE}_{10}) = \frac{u'}{2a} < f_c(\mathrm{TE}_{01}) = \frac{u'}{2b}$.
Therefore, TE_{10} mode is the dominant mode among TE modes.

4.1.3 Wave propagation inside rectangular waveguides

If f is the operating frequency and f_c is the cut-off frequency (both in Hz), then the tilt angle is given by,

$$\sin\theta = \frac{f_c}{f} \quad \Longrightarrow \quad \theta = \sin^{-1}\left(\frac{f_c}{f}\right)$$

Phase constant along the waveguide is given by β_g,

$$\beta_g = \beta\cos\theta = \beta\sqrt{1 - \left(\frac{f_c}{f}\right)^2}$$

Wavelength along the waveguide is given by λ_g, obtained from $\beta = 2\pi/\lambda$.

$$\lambda_g = \frac{\lambda}{\cos\theta} = \frac{\lambda}{\sqrt{1 - \frac{f_c^2}{f^2}}}$$

Phase velocity along waveguide is given by v_p, obtained from $v = \omega/\beta$.

$$v_p = \frac{v}{\cos\theta}$$

If the dielectric is free space, then $v = c$ and hence $v_p = \frac{c}{\cos\theta}$. $\implies v_p \geq c$.

Group velocity along waveguide is given by v_g, obtained by differentiation ω w.r.t β.

$$v_g = v\cos\theta$$

If the dielectric is free space, then $v = c$ and hence $v_g = c\cos\theta$. $\implies v_g \leq c$.

Hence, relation between velocity of propagation, phase velocity and group velocity can be obtained.

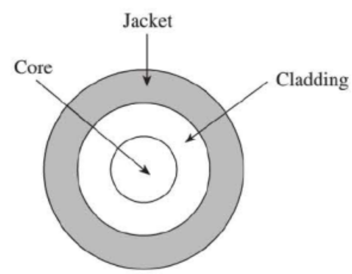
$$v_p v_g = v^2 \qquad\qquad \implies v = \sqrt{v_p v_g}$$

4.2 Circular Waveguides

4.3 Optical Fibers

Optical fibers are dielectric waveguides operating at optical frequencies (i.e frequencies of the order 100 THz).

An optical fiber consists of three concentric cylindrical sections: the core, the cladding, and the jacket.



A ray of light entering the core will be internally reflected when incident in the denser medium and the angle of incidence is greater than a critical value. Thus a light ray is reflected back into the original medium and the process is repeated as light passes down the core.

Numerical Aperture:
The most important parameter of an optical fiber is its numerical aperture (NA). The value of NA is dictated by the refractive indices of the core and cladding. It is given by,

$$\text{NA} = \sin\theta_a = \sqrt{n_1^2 - n_2^2}$$

where n_1 is the refractive index or the core, n_2 is the refractive index of the cladding and θ_a is the "acceptance angle". It is the maximum angle over which light rays entering the fiber will be trapped in its core.

Note that the "critical angle" is the minimum angle of incidence of light rays for total internal reflection to take place and is given by $\theta_c = \sin^{-1}(\frac{n_2}{n_1})$.

5 Antenna

Electric charges are the sources of EM fields. If the sources are time varying, EM waves propagate away from the sources and radiation is said to have taken place. Radiation may be thought of as the process of transmitting electric energy. The radiation or launching of the waves into space is efficiently accomplished with the aid of conducting or dielectric structures called antennas.

An antenna may also be viewed as a transducer used in matching the transmission line or as a waveguide (used in guiding the wave to be launched) to the surrounding medium, or vice versa.

The transmitter antenna is responsible for converting V-I waves to E-M waves i.e radiation.
The receiver antenna is responsible for converting E-M waves to V-I waves i.e induction.

Basic radiation equation:

$$Q\frac{dv}{dt} = l\frac{di}{dt}$$

where $Q \rightarrow$ charge, $\frac{dv}{dt} \rightarrow$ acceleration, $l \rightarrow$ length of radiative element and $\frac{di}{dt} \rightarrow$ rate of current flow

5.1 Basic Antenna

5.1.1 Hertzian Dipole

5.1.2 Half-wave Dipole

5.2 Monopole Antenna

5.3 Linear Antenna Arrays