calculating value for one state using Bellman Equation

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State s: Consider a particular grid cell where fire may spread.

Action a: Assume the action "spread fire" is taken.

Transition Probabilities: The fire can spread to two neighboring states:

$$P(s_1|s,a) = 0.7 \quad \text{and} \quad P(s_2|s,a) = 0.3.$$

Reward: Let the immediate reward for spreading fire be $R(s,a) = -2$ (representing the cost associated with fire risk).

Value of Next States: Assume:

$$V(s_1) = 10 \quad \text{and} \quad V(s_2) = 5.$$

Discount Factor: Let $\gamma = 0.9$.

Bellman Equation: The Bellman equation for state s is:

$$V(s) = \max_a \sum_{s'} P(s'|s,a) \left[R(s,a) + \gamma V(s')\right].$$

For the chosen action "spread fire," we have:

$$V(s) = 0.7\left[-2 + 0.9(10)\right] + 0.3\left[-2 + 0.9(5)\right].$$

Calculation:

- For s_1:
$$-2 + 0.9 \times 10 = -2 + 9 = 7.$$
- For s_2:
$$-2 + 0.9 \times 5 = -2 + 4.5 = 2.5.$$
- Thus:
$$V(s) = 0.7(7) + 0.3(2.5) = 4.9 + 0.75 = 5.65.$$

Conclusion: The estimated value $V(s)$ for the state s under the action "spread fire" is **5.65**. This value represents the expected cumulative impact of fire spread from that cell, taking into account both the immediate risk and future propagation.