Home Assignment V

The solution will be published at the net at 04.02

Exercise 1. Let $X_1, X_2, \ldots$ be independent random variables with $P(X_n = \sqrt{n}) =$ $= P(X_n = -\sqrt{n}) = \frac{1}{2}$. Prove that $\{X_n\}$ does not obey the weak low of large numbers; i.e. $P\left(\left|\frac{1}{n}\sum_{k=1}^{n} X_k\right| < \epsilon\right)$ does not converge to 0, although $E\left(\frac{1}{n}\sum_{k=1}^{n} X_k\right) = 0$ for each n.

Exercise 2. Let ρ_n be the distance between two randomly and independently chosen points in the unit cube in $\mathbb{R}^n$. Find $\lim_{n\to\infty} \frac{E(\rho_n)}{\sqrt{n}}$.

Exercise 3. Let $X_\lambda \sim Poiss(\lambda)$. Prove that $Y_\lambda = \frac{X_\lambda - \lambda}{\sqrt{\lambda}}$ is asymptotically normal $N(0,1)$ as $\lambda \to \infty$.

Exercise 4. Let $X_1, X_2, \ldots$ be independent random variables with $P(X_n = n) =$ $= P(X_n = -n) = \frac{1}{4}$, $P(X_n = 0) = \frac{1}{2}$. Check the validity of the central limit theorem for this sequence.

Exercise 5. Let $X_1, X_2, \ldots$ be independent identically distributed random variables with $P(X_n = 1) = p > \frac{1}{2}$, $P(X_n = -1) = 1-p$. Let $S_n = X_1 + \ldots + X_n$, $T_n = \inf\{k > 0 : S_k = n\}$. **(a)** Prove that $\lim_{n\to\infty} P\left(\left|\frac{S_n}{n} - (2p-1)\right| > \epsilon\right) = 0$ for each $\epsilon > 0$. **(b)**** In the class we found $E(T_1)$ assuming that $E(T_1) < \infty$. Define for arbitrary $M > 0$, $T_1 \wedge M = \min\{T_1, M\}$ and apply the same argument as in the class to this variable in order *to prove* that $E(T_1) < \infty$. **(c)*** Prove that $\lim_{n\to\infty} P\left(\left|\frac{T_n}{n} - \frac{1}{2p-1}\right| > \epsilon\right) = 0$ for each $\epsilon > 0$.