Infi II–104281 *Due on Thursday, April 2, 2015*

Homework #1

1. Evaluate the following integrals:

i. $\int x^2 \arcsin x^3 dx$

ii. $\int \frac{\sin 2x}{2+\cos x} dx$

iii. $\int e^x \sin 5x dx$

iv. $\int e^{2x} \sin e^{2x} dx$

v. $\int \frac{\ln x}{x} dx$

vi. $\int \frac{x}{\sqrt{x^2-4}} dx$

vii. $\int \frac{x^2}{x^2-3x-10} dx$

viii. $\int \frac{3x-4}{x^2(x^2+3x+6)} dx$

ix. $\int \frac{1}{(x^2+4)^3} dx$

x. $\int (e^x+1)^{\frac{1}{3}} dx$

2. The substitution $t = \tan \frac{x}{2}$ allows one to convert a rational function of $\sin x$ and $\cos x$ to a standard rational function. One needs to use the following trigonometric identities:

$\sin x = \frac{2\tan \frac{x}{2}}{\tan^2 \frac{x}{2}+1}$

$\cos x = \frac{1-\tan^2 \frac{x}{2}}{1+\tan^2 \frac{x}{2}}$.

Use this method to calculate the following integral:

$\int \frac{1}{2+\cos x+\sin x} dx$

3. i. Let f be a function defined on an interval (a,b) and assume that f is differentiable at every point of (a,b). Show that f' cannot have a discontinuity of the first kind. That is, there cannot be a point $x_0 \in (a,b)$ such that $\lim_{x \to x_0^+} f'(x)$ and $\lim_{x \to x_0^-} f'(x)$ both exist but are not equal. (Hint: Use Lagrange's theorem: *If* f *is continuous on* $[l_1,l_2]$ *and differentiable on* (c,d)*, then* $\frac{f(l_2)-f(l_1)}{l_2-l_1} = f'(c)$*, for some* $c \in (l_1,l_2)$.

ii. Let $g(x) = \begin{cases} 1, x > 0; \\ 0, x \leq 0. \end{cases}$

What is $\int g(x)dx$?