# אלגברה ב' - גליונות תרגילים $3, 4$

♣ גליון 3: [HK] סעיף 8.2, תרגילים $4, 6, 7, 9$.
<u>תאריך הגשה:</u> 30.03.2000 עד השעה $12:00$.

♣ גליון 4: [HK] תרגילים $8.2.13, 8.3.1, 8.3.11$; [BK], עמוד 216 תרגיל מספר 1. **בנוסף, הגישו פתרונות לתרגילים הבאים מ- Herstein:**

♣ **[H]** 2.5.14: If $H$ is a subgroup of $G$, then by the centralizer $C_G(H)$ of $H$ in $G$ is the set -

$$\{x \in G \,|\, xh = hx \quad for\ all\ h \in H\}.$$

Prove that $C_G(H)$ is a subgroup of $G$.

♣ **[H]** 2.5.15: The center $Z(G)$ of a group $G$ is defined by

$$Z(G) = \{x \in G \,|\, xg = gx\, for\ all\ g \in G\}.$$

Prove that $Z(G)$ is a subgroup of $G$. Can you recognize $Z(G)$ as $C_G(T)$ for some suitable subgroup $T < G$?

♣ **[H]** 2.5.37: If in a group $G$, $a^5 = e$, $aba^{-1} = b^2$ for some $a, b \in G$, find $o(b)$.

♣ **[H]** 2.5.38: Let $G$ be a finite abelian group in which the number of distinct solutions of the equation $x^n = e$ is at most $n$ for every positive integer $n$. Prove that $G$ is a cyclic group. (Hint: count the elements in the sets $U_n = \{x \in G |\, x^n = e\}$ and $V_n = \{x \in G |\, o(x) = n\}$)

<u>**תאריך הגשה לגליון 4:**</u> 6.04.2000 עד השעה $12:00$.

**תרגיל מומלץ (לא להגשה):**

♣ **[H]** 2.5.24: Let $G$ be a finite group whose order is not divisible by 3. Show, that if $(ab)^3 = a^3b^3$ for all $a, b \in G$ then $G$ is abelian. (Hint: recall that $(ab)^2 = a^2b^2$ implies $ab = ba$; use this to find a necessary and sufficient condition for the given group to be abelian)