一、解：1) Jacobi: $X^{(k+1)}=B_J X^{(k)}+f_J$. $B_J=D^{-1}(L+U)$ $f_J=D^{-1}b$

$$\begin{pmatrix}1&2&3\\2&5&3\\-2&-2&3\end{pmatrix}. \quad 则\ D=\begin{pmatrix}1&&\\&5&\\&&3\end{pmatrix} \quad L=\begin{pmatrix}0&&\\-2&0&\\2&2&0\end{pmatrix} \quad U=\begin{pmatrix}0&-2&-3\\&0&-3\\&&0\end{pmatrix}$$

$$则\ B_J=D^{-1}(L+U)=\begin{pmatrix}1&&\\&\frac{1}{5}&\\&&\frac{1}{3}\end{pmatrix}\begin{pmatrix}0&-2&-3\\-2&0&-3\\2&2&0\end{pmatrix}=\begin{pmatrix}0&2&-3\\-\frac{2}{5}&0&-\frac{3}{5}\\\frac{2}{3}&\frac{2}{3}&0.\end{pmatrix}$$

$$f_J=D^{-1}b=\begin{pmatrix}1&&\\&\frac{1}{5}&\\&&\frac{1}{3}\end{pmatrix}\begin{pmatrix}4\\7\\1\end{pmatrix}=\begin{pmatrix}4\\\frac{7}{5}\\\frac{1}{3}\end{pmatrix} \qquad 则\ X^{(k+1)}=\begin{pmatrix}0&-2&-3\\-\frac{2}{5}&0&-\frac{3}{5}\\\frac{2}{3}&\frac{2}{3}&0\end{pmatrix}X^{(k)}+\begin{pmatrix}4\\\frac{7}{5}\\\frac{1}{3}\end{pmatrix}$$

Gauss-seidel: $B_{G\text{-}S}=(D-L)^{-1}U$ $f_{G\text{-}S}=(D-L)^{-1}b$.

$$B_{G\text{-}S}=\begin{pmatrix}1&&\\2&5&\\-2&-2&3\end{pmatrix}^{-1}\begin{pmatrix}0&-2&-3\\&0&-3\\&&0\end{pmatrix}=\begin{pmatrix}0&-2&-3\\0&\frac{4}{5}&\frac{3}{5}\\0&-\frac{4}{5}&-\frac{4}{5}\end{pmatrix}$$

$$f_{G\text{-}S}=\begin{pmatrix}1&&\\2&5&\\-2&-2&3\end{pmatrix}^{-1}\begin{pmatrix}4\\7\\1\end{pmatrix}=\begin{pmatrix}4\\-\frac{1}{5}\\\frac{43}{15}\end{pmatrix} \qquad 则\ X^{(k+1)}=B_{G\text{-}S}X^{(k)}+f_{G\text{-}S}.$$

2).

$$|\lambda E-B_{G\text{-}S}|=\begin{vmatrix}\lambda&2&3\\0&\lambda-\frac{4}{5}&-\frac{3}{5}\\0&\frac{4}{5}&\lambda+\frac{4}{5}\end{vmatrix}=\lambda(\lambda+\frac{2}{5})(\lambda-\frac{2}{5})=0.\ 则\ \lambda=0,-\frac{2}{5}或\frac{2}{5}.$$

则 $\rho(B_{G\text{-}S})=\max_i|\lambda_i|=\frac{2}{5}<1$. 收敛.

二.解：(1) $\varphi(x)=\sqrt[3]{0.8+x^2}$. 则 $\varphi'(x)=\frac{1}{3}(0.8+x^2)^{-\frac{2}{3}}\cdot 2x$. $|\varphi'(1.5)|\approx 0.476<1$. 收敛

(2). $\varphi(x)=\sqrt{x^3-0.8}$. 则 $\varphi'(x)=\frac{1}{2}(x^3-0.8)^{-\frac{1}{2}}\cdot 3x^2$. $|\varphi'(1.5)|\approx 1.4>1$ 发散.

三.解：

$$A=\begin{pmatrix}1&1&3&4\\2&4&1&3\\3&2&1&1\\2&1&2&1\end{pmatrix}\rightarrow\begin{pmatrix}1&1&3&4\\2&2&-5&-5\\3&-\frac{1}{2}&-\frac{21}{2}&-\frac{27}{2}\\2&-\frac{1}{2}&\frac{17}{21}&-\frac{8}{7}\end{pmatrix} \quad 则\ L=\begin{pmatrix}1&&&\\2&1&&\\3&-\frac{1}{2}&1&\\2&-\frac{1}{2}&\frac{17}{21}&1\end{pmatrix} \quad U=\begin{pmatrix}1&1&3&4\\&2&-5&-5\\&&-\frac{21}{2}&-\frac{27}{2}\\&&&-\frac{8}{7}\end{pmatrix}$$

四.解：

$$y^{(1)}=AX^{(0)}=\begin{pmatrix}-4&14&0\\-5&13&0\\-1&0&2\end{pmatrix}\begin{pmatrix}1\\1\\1\end{pmatrix}=\begin{pmatrix}10\\8\\1\end{pmatrix} \quad \lambda^{(1)}=10. \quad x^{(1)}=\begin{pmatrix}1\\0.8\\0.1\end{pmatrix}$$

$$y^{(2)}=AX^{(1)}=\begin{pmatrix}-4&14&0\\-5&13&0\\-1&0&2\end{pmatrix}\begin{pmatrix}1\\0.8\\0.1\end{pmatrix}=\begin{pmatrix}7.2\\5.4\\-0.8\end{pmatrix} \quad \lambda^{(2)}=7.2 \quad x^{(2)}=\begin{pmatrix}1\\\frac{5.4}{7.2}\\\frac{-0.8}{7.2}\end{pmatrix}$$

五.解：设 $P_2(x)=a_0+a_1x+a_2x^2$ $P_2'(x)=a_1+2a_2x$

$$\Rightarrow \begin{pmatrix}1&x_0&x_0^2\\1&x_1&x_1^2\\0&1&2x_0\end{pmatrix}\begin{pmatrix}a_0\\a_1\\a_2\end{pmatrix}=\begin{pmatrix}y_0\\y_1\\y_0'\end{pmatrix} \quad \Rightarrow \begin{pmatrix}a_0\\a_1\\a_2\end{pmatrix}=\begin{pmatrix}\cdots\\\cdots\\\cdots\end{pmatrix}$$

代入有 $P_2(x)$, $R_2(x)=\frac{f'''(\xi)}{3!}(x-x_0)^2(x-x_1)$

六.解 1. ★

2. ① 设 $y(x)=a_0+a_1x$ 设 $\varphi_0(x)=1$ $\varphi_1(x)=x$

$$\Rightarrow \begin{pmatrix}6&28\\28&170\end{pmatrix}\begin{pmatrix}a_0\\a_1\end{pmatrix}=\begin{pmatrix}61\\385\end{pmatrix} \quad \Rightarrow \begin{pmatrix}a_0\\a_1\end{pmatrix}=\begin{pmatrix}-1.7373\\2.5508\end{pmatrix}$$

则 $y(x)=-1.7373+2.5508x$

② 设 $y(x)=a_0+a_1x+a_2x^2$. $\varphi_0(x)=1$ $\varphi_1(x)=x$ $\varphi_2(x)=x^2$

$$\Rightarrow \begin{pmatrix} 6 & 28 & 170 \\ 28 & 170 & 1144 \\ 170 & 1144 & 8066 \end{pmatrix} \begin{pmatrix} a_0 \\ a_1 \\ a_2 \end{pmatrix} = \begin{pmatrix} 61 \\ 385 \\ 1884 \end{pmatrix} \Rightarrow \begin{pmatrix} a_0 \\ a_1 \\ a_2 \end{pmatrix}.$$ 回代.

七.解:

八.解: $y_{n+1}=y_n+hf(x_n,y_n)$ $f(t_n,y_n)=-4t_n^3y_n^2$.

则 $y_{n+1}=y_n+h(-4t_n^3y_n^2)$

| | $t_n$ | $y_n$ |
|---|---|---|
| 0 | -10 | 0.1 |
| 1 | -9.9 | 4.1 |

$y_1=y_0+h(-4t_0^3y_0^2)=0.1+0.1(-4(-10^3)\times 0.1^2)=4.1$

$$\begin{cases} \overline{y_{n+1}}=y_n+hf(x_n,y_n) \\ y_{n+1}=y_n+\frac{h}{2}[f(x_n,y_n)+f(x_{n+1},\overline{y_{n+1}})] \end{cases}$$

则 $$\begin{cases} \overline{y_{n+1}}=y_n+h(-4t_n^3y_n^2) \\ y_{n+1}=y_n+\frac{h}{2}[-4t_n^3y_n^2+(-4t_{n+1}^3\overline{y_{n+1}}^2)] \end{cases}$$

| | $t_n$ | $\overline{y_n}$ | $y_n$ |
|---|---|---|---|
| 0 | -10 | 0.1 | 0.1 |
| 1 | -9.9 | 4.1 | |

$\overline{y_1}=y_0+h(-4t_0^3y_0^2)=4.1$

$y_1=y_0+\frac{h}{2}[-4t_0^3y_0^2-4t_1^3\overline{y_1}^2]=0.1+0.05(-4(-10)^3\times(0.1)^2-4\times(-9.9)^3(4.1)^2)$

$=3262.245$