$p \to q \equiv p$ only if $q \equiv q$ if $p \equiv q$ unless $\neg p \equiv q$ provided that p

- precedence of logical operators : $\neg \ \wedge \ \vee \to \leftarrow$

- $p \to q = \neg p \vee q$. $p \leftrightarrow q = p \wedge q \vee \neg p \wedge \neg q$. $p \uparrow q = \neg(p \wedge q)$. $p \downarrow q = \neg(p \vee q)$

- $(p \to q) \underset{\vee}{\wedge} (p \to r) = p \to (q \underset{\vee}{\wedge} r)$. $(p \to r) \underset{\vee}{\wedge} (q \to r) = (p \underset{\wedge}{\vee} q) \to r$

- CNF : $(A_1 \vee A_2) \wedge (A_3 \vee A_4)$ DNF : $(A_1 \wedge A_2) \vee (A_3 \wedge A_4)$

- $\forall x P(x) \vee A = \forall x (P(x) \vee A)$ $\forall x P(x) \wedge A = \forall x (P(x) \wedge A)$ $\forall x (P(x) \wedge Q(x)) \equiv \forall x P(x) \wedge \forall x Q(x)$

  $\exists x P(x) \vee A = \exists x (P(x) \vee A)$ $\exists x P(x) \wedge A = \exists x (P(x) \wedge A)$ $\exists x (P(x) \vee Q(x)) \equiv \exists x P(x) \vee \exists x Q(x)$

  $\forall x (A \to P(x)) = A \to \forall x P(x)$ $\exists x (A \to P(x)) = A \to \exists x P(x)$

  $\forall x (P(x) \to A) = \exists x P(x) \to A$ $\exists x (P(x) \to A) = \forall x P(x) \to A$

- prove the set of rational numbers is countable

  prove the set of real numbers is uncountable

- $\binom{n+1}{k} = \binom{n}{k} + \binom{n}{k-1}$. $\binom{m+n}{k} = \sum_{r=0}^{k} \binom{m}{k-r} \cdot \binom{n}{r}$. $\binom{n+1}{k+1} = \sum_{r=k}^{n} \binom{r}{k}$

- distribute n distinguishable objects into k indistinguishable boxes has :

  the stirling number of the second kind $S(n,j) = \frac{1}{j!} \sum_{i=0}^{j-1} (-1)^i \binom{j}{i} (j-i)^n$ ways

  the sum of stirling numbers $\sum_{j=1}^{k} S(n,j) = \sum_{j=1}^{k} \frac{1}{j!} \sum_{i=0}^{j-1} (-1)^i \binom{j}{i} (j-i)^n$ total ways

- the number of different permutations of n objects is $\frac{n!}{n_1! n_2! \cdots n_k!}$

- linear homogeneous recurrence relations with constant coefficients :

$$a_n = (\alpha_{1,0} + \alpha_{1,1} n + \cdots + \alpha_{1,m_1-1} n^{m_1-1}) r_1^n + (\alpha_{2,0} + \alpha_{2,1} n + \cdots + \alpha_{2,m_2-1} n^{m_2-1}) r_2^n + \cdots + (\alpha_{t,0} + \alpha_{t,1} n + \cdots + \alpha_{t,m_t-1} n^{m_t-1}) r_t^n$$

$$F(n) = (b_t n^t + \cdots + b_1 n + b_0) s^n$$

$$\Rightarrow a_n^* = n^k (p_t n^t + \cdots + p_1 n + p_0) s^n$$

- increasing function $f(n) = af(\frac{n}{b}) + cn^d$ is $\begin{cases} O(n^d) & \log_b a < d \\ O(n^d \log n) & \log_b a = d \\ O(n^{\log_b a}) & \log_b a > d \end{cases}$
- the number of onto function from n elements' set to m elements' set and $n \geq m$ is $C_m^0 m^n - C_m^1 (m-1)^n + C_m^2 (m-2)^n - \cdots + (-1)^{m-1} C_m^{m-1} 1^n$ (Stirling)
- the number of derangement of n elements' set is $D_n = n! \sum_{i=0}^{n} (\frac{(-1)^i}{i!})$ also. $D_n = n! [1 - \frac{1}{1!} + \frac{1}{2!} - \cdots + (-1)^n \frac{1}{n!}] = (n-1) \cdot (D_{n-1} + D_{n-2})$. $D_1 = 0$. $D_2 = 1$
- reflexive $\forall a : (a,a) \in R$ symmetric $\forall (a,b) \in R : (b,a) \in R$
  irreflexive $\forall a : (a,a) \notin R$ asymmetric $\forall (a,b) \in R : (b,a) \notin R$
  antisymmetric $\forall (a,b) \in R . (b,a) \in R : a = b$ transitive $\forall (a,b) \in R . (b,c) \in R : (a,c) \in R$
- $M_{S \circ R} = M_R \odot M_S$ $R^{-1} = \{(a,b) \mid (b,a) \in R\}$ $\overline{R} = \{(a,b) \mid (a,b) \notin R\}$
- Warshall's algorithm: $W \triangleq M_R$
  for $k=1$ to n : for $i=1$ to n : for $j=1$ to n : $W_{ij} = W_{ij} \vee (W_{ik} \wedge W_{kj})$
- $2|E| = \sum_{v \in V} \deg(v)$. $|E| = \sum_{v \in V} \deg^-(v) = \sum_{v \in V} \deg^+(v)$
- an undirected graph has an even number of vertices of odd degree
- a simple graph is bipartite if and only if it is possible to assign one of two different colors to each vertex of the graph so that no two adjacent vertices are assigned the same color
- Hall's marriage theorem:
  the bipartite graph $G = (V, E)$ with bipartition (V_1, V_2) has a complete matching from V_1 to V_2 if and only if $|N(A)| \geq |A|$. $\forall A \subseteq V_1$
- Havel-Hakimi theorem
  $A = \{s, t_1, \cdots, t_s, d_1, \cdots, d_n\}$ is nonincreasing. $A' = \{t_1 - 1, \cdots, t_s - 1, d_1, \cdots, d_n\}$
  A is graphic if and only if A' is graphic
- Whitney theorem : $\kappa(G) \leq \lambda(G) \leq \min_{v \in V} \deg(v)$

- The number of different paths of length r from v_i to v_j equals $A^r(i,j)$
- Euler circuit if and only if each vertex has even degree
  as for directed graph, if and only if $deg^-(v) = deg^+(v)$ for all vertices
  Euler path if and only if two verteces have odd degree.
  as for directed graph. if and only if two vertices $\begin{cases} deg^-(v_1) - deg^+(v_1) = 1 \\ deg^-(v_2) - deg^+(v_2) = -1 \end{cases}$
- Sufficient condition for Hamilton circuit:
  $deg(u) + deg(v) \geq n$ for every pair of nonadjacent vertices u and v.
  Neccesary condition for Hamilton circuit:
  for any nonempty subset S of V. $G=(V,E)$. $|G-S| \leq |S|$. namely
  the number of connected components in $G-S \leq$ the number of vertices in S
- Dijkstra's Algorithm (Shortest path algorithm)
  divide vertices into two groups. set $L(v_i) = +\infty$. $L(a) = 0$. $S = \phi$
  while $z \notin S$:
  u = a vertex not in S with $L(u)$ minimal ; $S = S \cup \{u\}$
  for all vertices v not in S. $L(v) = \min(L(v), L(u) + w(u,v))$
- Euler's formula: $r = e - v + 2$ for connected plane simple graph
  furthur, suppose the graph has k connected components: $r = e - v + k + 1$
- $2e = \Sigma\, deg(R_i)$; $e \leq 3v - 6$; $\exists V_0$. $deg(V_0) \leq 5$; $\chi(G) \leq 4$. G is planar graph
- a simple graph is nonplannar if and only if it contains a subgraph $\simeq K_{3,3}$ or K_5
- an undirected graph is tree if and only if $\exists$ simple path between any two vertices
- $e(Tree) = v - 1$. $v = l + i$. full m-ary tree $v = mi + 1$
  $\Rightarrow i = \frac{v-1}{m}$. $l = \frac{(m-1)v+1}{m}$; $v = mi + 1$, $l = (m-1)i + 1$; $v = \frac{ml-1}{m-1}$, $i = \frac{l-1}{m-1}$
- every tree is a bipartite

- DMP Algorithm (Planarity judgment algorithm)
- Kruskal Algorithm (Minimum spanning tree algorithm)
- Ford-Fulkerson Algorithm (Maximum flow algorithm)
- Hungarian Algorithm (Bipartite graph matching algorithm)
- Kirchhoff's matrix-tree theorem (Number of minimum spanning trees)