$The\ review\ of\ abstract\ algebra$

$\cdot\ (a\rangle = \bigcap\{left\ ideal\ in\ R\ containing\ a\} = \{na + ra | n \in Z,\ r \in R\}$

$\cdot\ (Z, +, \cdot)\ is\ a\ PID(commutative\ principle\ ideal\ domain)$

$\cdot\ F[x]\ is\ a\ PID(suppose\ F\ is\ a\ field)$

$\cdot\ A\ matrix\ A\ is\ similar\ to\ a\ diagnal\ matrix\ if\ and\ only\ if\ it\ has\ a$
$splitting\ polynomial\ f(x)\ which\ has\ not\ multiplicity\ roots.$

$\cdot\ The\ kernal\ of\ a\ homomorphism\ \psi : R_1 \to R_2\ (ker\psi)\ is\ an\ ideal$

$\cdot\ Suppose\ R\ is\ a\ commutatinve\ ring\ with\ identity\ 1_R$
$then\ M\ is\ a\ maximal\ ideal \Leftrightarrow \frac{R}{M}\ is\ a\ field$

$\cdot\ P\ is\ a\ prime\ ideal\ of\ R(with\ 1_R) \Leftrightarrow \frac{R}{P}\ is\ a\ domain$

$\cdot\ Suppose\ P\ is\ a\ prime\ ideal\ of\ R\ and\ R\ is\ a\ PID,\ then$
$P = 0\ or\ P\ is\ maximal \Leftrightarrow P = pR = (p),\ ab \in P \Rightarrow a \in P\ or\ b \in P$
$(maximal\ ideal \to prime\ ideal,\ but\ the\ reverse\ is\ wrong)$

$\cdot\ Suppose\ R\ is\ a\ PID,\ P\ is\ a\ prime\ ideal \Leftrightarrow R_P\ = (p)\ is\ maximal$

$\cdot\ R\ is\ PID,\ a \in R\ is\ irreducible \Leftrightarrow a\ is\ a\ prime$
$In\ general\ domain,\ prime \to irredubible\ but\ the\ reverse\ is\ wrong$

$\cdot\ 0 \neq f(x) \in Q[x], \exists c \in Q\ s.t.\ f(x) = cf_1(x),\ a_n x^n + ... + a_0 \in Z[x]$
$f_1(x) = a_n x^n + ...a_0 \in Z[x]\ is\ said\ to\ be\ primitive,\ if\ the\ maximal$
$common\ divisor\ of\ a_0...a_n = 1\ or\ a_0...a_n\ are\ coprime$

$\cdot\, If\ g_1(x)\ and\ h_1(x)\ are\ primitive,\ then\ g_1(x)h_1(x)\ is\ primitive$

$\cdot\, Eisenstein's\ irreducible\ criterion:$
$f(x)=a_nx^n+...+a_0\in Z[x],\ p\ is\ a\ prime\ satisfying\ p\nmid a_n,\ p\mid a_i$
$0\leqslant i\leqslant n-1,\ p^2\nmid a_0,\ then\ f(x)\ is\ irreducible$

$\cdot\, The\ ensemble\ of\ nilpotent\ in\ R(commutative)\ constitutes\ an\ ideal$

$\cdot\, A\ ring\ whose\ nonzero\ elements\ are\ idempotents\ is\ commutative$

$\cdot\, A\ ring\ with\ no\ zero\ elements\ and\ with\ some\ idempotents\ has$
$unique\ idempotent\ and\ is\ an\ unitary$

$\cdot\, Suppose\ \psi:R_1\to R_2\ is\ homomorphism,\ ker\psi=\{a\in R_1|\psi(a)=0\}$
$is\ an\ ideal\ of\ R,\ I\subseteq ker\psi\ is\ an\ ideal\ of\ R_1,\ then\ there\ is\ a\ homo$
$\bar{\psi}:\dfrac{R_1}{I}\to R_2\ s.t.\bar{\psi}(a+I)=\psi(a),\ ker\bar{\psi}=\dfrac{ker\psi}{I},\ Im\bar{\psi}=Im\psi$

$\cdot\, The\ first\ homomorphism\ fundemental\ theorem:$
$suppose\ \psi:R_1\to R_2\ is\ homo.,\ then\ \bar{\psi}:\dfrac{R_1}{ker\psi}\to Im\psi\ is\ iso$

$\cdot\, The\ second\ homomorphism\ fundemental\ theorem:$
$suppose\ I,\ J\ are\ ideals\ of\ R\ and\ I\subseteq J,\ then:$
$(1):\dfrac{J}{I}=\{a+I|a\in J\}\ is\ an\ ideal\ of\ \dfrac{R}{I}\quad(2):\dfrac{R/I}{J/I}\simeq\dfrac{R}{J}$

$\cdot\, The\ third\ homomorphism\ fundemental\ theorem:$
$suppose\ S\ is\ a\ subring\ of\ R,\ I\ is\ an\ ideal\ of\ R,\ then:$
$(1):S+I\ is\ a\ subring\ of\ R\ \ (2):I\ is\ an\ ideal\ of\ S+I$
$(3):I\cap S\ is\ an\ ideal\ of\ S\quad(4):\dfrac{S+I}{I}\simeq\dfrac{S}{I\cap S}$

$\cdot\, Suppose\ F\ is\ a\ field,\ f(x)=a_0+a_1x+...+a_{n-1}x^{n-1}+x^n,\ n\in N$

$$\frac{F[x]}{(f(x))} = \{r_0 + r_1x + ... + r_{n-1}x^{n-1} + (f(x))|r_i \in F\}\ is\ a\ vector\ space$$

$over\ F\ with\ basis\{\bar{1}, \bar{x},, ...\overline{x^{n-1}}\},\ \bar{1} = 1 + (f(x)),\ \bar{x} = x + (f(x))..$

$r_0 ... + r_{n-1}x^{n-1} + (f(x))\ is\ invertible \Leftrightarrow (r_0 + ... + r_{n-1}x^{n-1}, f(x)) = 1$

$\cdot\ Suppose\ p(x)\ is\ irreducible,\ f(x) = p(x)^n q(x)\ and\ p(x) \not| q(x),\ then$

$$\frac{F[x]}{(f(x))} \simeq \frac{F[x]}{(p(x)^n)} \oplus \frac{F[x]}{(g(x))} = \{(a + (p(x)^n), b + (g(x)))|a, b \in F[x]\}$$

$\cdot\ Suppose\ p\ is\ a\ prime,\ \mathbb{Z}_P = \{\bar{0}, \bar{1}, ..., \overline{p-1}\} = \dfrac{\mathbb{Z}}{p\mathbb{Z}}\ is\ a\ field$

$|\mathbb{Z}_P| = p,\ \forall p,\ F\ is\ a\ field,\ n \in N^*,\ then\ \forall n, \exists F; \forall F, \exists N : |F| = p^n$

$\cdot\ Hamidton - Caylay\ Theorem :$

$A = (a_{ij})_{n\times n}, \exists f(\lambda) = |\lambda E - A|, f(A) = 0$

$T \to A : (Te_1, Te_2, ..., Te_n) = (e_1, e_2, ..., e_n)A,\ f(T)(\alpha) = 0(\alpha) = 0$

$but\ f(T) \neq 0\ and\ \alpha \neq 0,\ thus\ module\ is\ not\ a\ domain$

$\cdot\ Suppose\ \psi : R^M \to R^{M'}\ mapping,\ \psi(m_1+m_2)=\psi(m_1)+\psi(m_2),$
$\psi(rm)=r\ \psi(m),\ ker\psi=\{m\in M|\psi(m)=0\}\ is\ a\ submodule\ of\ R^M$
$Im\psi=\{\psi(m)|m\in M\}\ is\ a\ submodule\ of\ M',\ consider\ the\ first$
$fundamental\ theorem\ of\ ring\ homomorphism,\ \psi:M\to M'\ is\ homo$
$consider\ M\to^{\psi}M'\Leftrightarrow M\to^{\pi}\dfrac{M}{ker\psi}\to^{\bar\psi}M',\ in\ which\ \pi(m)=m+ker\psi,$
$\bar\psi(m+ker\psi)=\psi(m)\Rightarrow\dfrac{M}{ker\psi}\simeq Im\psi=Im\bar\psi$

$\cdot\ N\leqslant L\leqslant M\Rightarrow\dfrac{M/N}{L/N}\cong\dfrac{M}{L};\ N,L\leqslant M\Rightarrow\dfrac{N+L}{L}\cong\dfrac{N}{N\cap L}$
$notice: if\ R\ is\ a\ field,\ it\ means\ two\ equivalent\ dimension\ formulas$

$\cdot\ Suppose\ M\ is\ a\ finitely\ generated\ R-module,\ then\ there\ is\ an$
$epimorphism\ \psi:R^n\to M,\ satisfying\ M\cong\dfrac{R^n}{ker\psi}$

$\cdot\ Zorn's\ Lemma:$
$\Omega\ is\ a\ nonempty\ partial\ order\ set,\ \forall a_1<...<a_n<...\exists a\in R\ s.t.a_i\leqslant a$
$then\ there\ is\ an\ element\ b\in\Omega\ satisfying\ \forall a\in\Omega,\ b\leqslant a\Rightarrow b=a$

$\cdot\ N\ is\ a\ submodule\ of\ a\ semisimple,\ M=\sum\limits_{i\in I}S_i,\ where\ S_i\ is\ simple$
$then\ there\ is\ subset\ J\ of\ I\ satisfying\ M=N\oplus(\sum\limits_{i\in J}\oplus S_i)$

$\cdot\ Every\ finite\ integer\ domain\ is\ a\ field$

$\cdot\ Suppose\ D\ is\ a\ basis\ of\ M,\ {}_DD^M\simeq\ {}_DD^N\Leftrightarrow m=n,\ diag_DM=|B|$