Optimal Approximation of Non-linear Power Flow Problem

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Power Flow Optimisation: non-convex feasible set

AC-OPF Problem

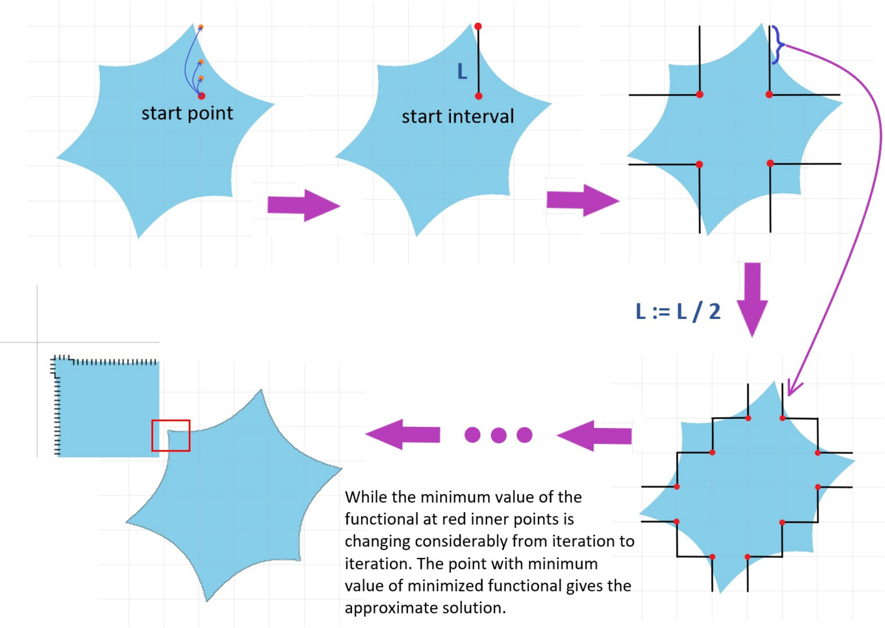
Alternating Current - Optimal Power Flow Problem is the problem of optimization of the workload in the energy network in order to reduce the cost of electricity production.

Main difficulty

AC-OPF Problem with constraints, imposed by nonlinear Kirchhoff's equations and the needs of consumers, has non-convex feasible set. A special case of this optimization problem, Quadratically Constrained Quadratic Program, is NP-hard.

Objective

The goal of research is to introduce economical in computational resources algorithm to highlight the borders of non-convex feasible set. It takes one point inside feasible set and and provides with two, inside one and outside one, knowingly tight sets of points which are knowingly close to the boundaries.



While the minimum value of the functional at red inner points is changing considerably from iteration to iteration. The point with minimum value of minimized functional gives the approximate solution.

- **Intoduction to AC-OPF Problem and its solutions**
  Carleton Coffrin and Line Roald, *Convex Relaxations in Power System Optimization: A Brief Introduction*, 2018
- **Summary on convex relaxations for AC-OPF Problem**
  Steven H. Low, *Convex Relaxation of Optimal Power Flow Par I, Par II*, 2014
- **Linear approximation method for solving AC-OPF Problem**
  Sidhant Misra, Daniel K. Molzahn, and Krishnamurthy Dvijotham, *Optimal Adaptive Linearizations of the AC Power Flow Equations*, 2018
- **Convex restriction algorithm**
  Dongchan Lee, Hung D. Nguyen, Krishnamurthy Dvijotham, and Konstantin Turitsyn, *Convex Restriction of Power Flow Feasible Sets*, 2019

AC-OPF Problem statement

Model of power network

$\mathbf{G}(\mathcal{N}, \mathcal{E})$ - directed graph of power network
$\mathcal{N}$ - nodes that is buses
$\mathcal{E} \subset \mathcal{N} \times \mathcal{N}$ - edges that is transmission lines

Denominations

$\mathcal{N}_{slack}$ - the set of slack(reference) buses
$\mathcal{N}_{ns} = \mathcal{N} \backslash \mathcal{N}_{slack}$ - the set of non-slack buses
$\mathcal{N}_{pv}, \mathcal{N}_{pq}$ - the sets of PV and PQ buses respectively
$\mathcal{N}_G = \mathcal{N}_{pv} \bigcup \mathcal{N}_{slack}$ - all generators

Variables

$p_k \quad \forall k \in \mathcal{N}_{slack}$ - the generator powers, control variables
$\theta_k \quad \forall k \in \mathcal{N}_{slack}$ - voltage phases, state variables
$\theta_i - \theta_j$ - voltage phase difference between i-th and j-th nodes
$\theta_l^{from} - \theta_l^{to}$ voltage phase difference between *from* and *to* ends of the transmission lines
$v_k \quad \forall k \in \mathcal{N}_{pq}$ - voltage magnitudes, state variables

AC-OPF Problem is to minimize

$$\sum_{k \in \mathcal{N}_G} (c_{2k}((p_k^g)^2 + c_{1k}(p_k^g) + c_{0k}) \longrightarrow \min_{p^g}$$

Data

$Y_{ik} = G_{ik} + jB_{ik}$ - nodal admittance matrix
p_i^{min}, p_i^{max} - generators active power lower and upper bounds
q_i^{min}, q_i^{max} - generators reactive power lower and upper bounds
v_i^{min}, v_i^{max} - voltage magnitude lower and upper bounds
$\phi_l^{min}, \phi_l^{max}$ - voltage phase lower and upper bounds
$c_{2k}, c_{1k}, c_{0k} \quad \forall k \in \mathcal{G}$ - generator cost components

Power balance equations in polar form

$$p_i^{inj} = \sum_{k \in \mathcal{N}} v_i v_k \left(G_{ik} \cos(\theta_{ik}) + B_{ik} \sin(\theta_{ik})\right)$$

$$q_i^{inj} = \sum_{k \in \mathcal{N}} v_i v_k \left(G_{ik} \sin(\theta_{ik}) - B_{ik} \cos(\theta_{ik})\right)$$

Operational constraints

$$p_i^{min} \leq p_i^{inj} \leq p_i^{max}, \quad i \in \mathcal{N}_G$$

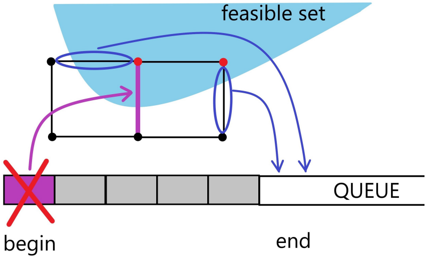
$$q_i^{min} \leq q_i^{inj} \leq q_i^{max}, \quad i \in \mathcal{N}_G$$

$$v_i^{min} \leq v_i^{inj} \leq v_i^{max}, \quad i \in \mathcal{N}_{pq}$$

$$\phi_l^{min} \leq \theta_l^{from} - \theta_l^{to} \leq \phi_l^{max}, \quad l \in \mathcal{E}$$

Iteration of the algorithm: highlight the borders of non-convex feasible set with intervals of given length

Input: one interval of given length with one end inside feasible set and one outside
Output: the tight set of intervals of given length with one end inside feasible set and one outside



Actions

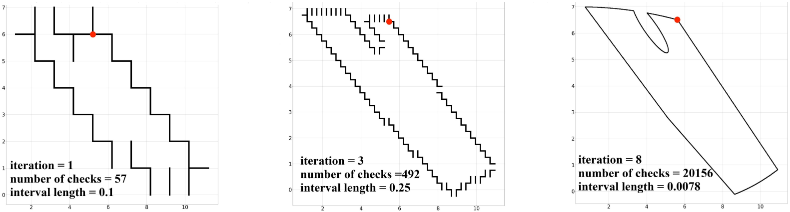
1. Put the input interval to the queue.
2. Take the interval from the beginning of the queue is there is one.
3. Consider the adjacent 4 points. Check weather they are feasible.
4. Put to the queue the obtained intervals, which ends are one inside and another outside the feasible set.

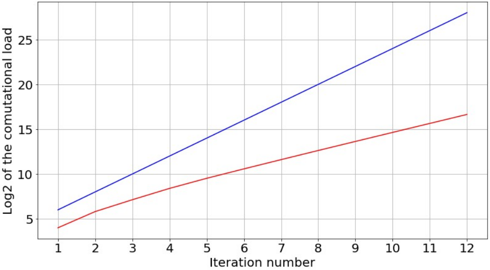
Important

- Never put the same interval to the queue for the second time. Just skip the action if the obtained interval has already been put in queue. It guarantees that method won't be infinite.
- Never check the feasibility of the same point for the second time - reuse the result of the first check.

The description of the algorithm is given for a 2D case. It is generalized for multidimensional case by considering multidimensional cubes instead of intervals.

Computational load in comparison with grid-search on IEEE 39 bus system





The lines denote the logarithm of computational complexity of introduces algorithm (red line) and grid search algorithm with step equal to interval length in introduced algorithm (blue line).

iteration index	time benefit
#4	12.2 times
#8	66.3 times
#12	2619.9 times

The computational load is counted as the number of point feasibility checks. Due to the specifics of Power Flow Problem, this operation is complex as requires to find out weather there are states variables, which, joined with control variables that are checked for feasibility, represent a feasible point.

Bottom line: advantages, applications and plans

Benefits and strengths of the algorithm, which highlights the borders

- In addition to highlighting the borders, the algorithm gets a considerable amount of tightly nestled points lying closely to the borders. So that the approximate solution to the AC-OPF problem is found.
- Only one point to start the iterations of the algorithm is required.
- No need to know the boundaries limiting the feasible set, like it is, when using grid-search approach.
- The computational complexity benefit is amounted to one dimension reduction in comparison to grid-search.

Applications

- Finding reference solution to compare with, when developing another method to solve AC-OPF Problem.
- Visualization of the restrictions, relaxations and other constructions in different approaches to AC-OPF Problem in relation to the non-convex set for research purposes.

Possibilities for future improvements

- After calculating the value of the minimized functional at inner points during each iteration, reduce the step length for the next iteration only in areas with potentially small values.