Optimal Approximation of Non-linear Power Flow Problem*

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This paper addresses Optimal Approximation of Non-linear Power Flow Problem. To solve it, the "trust region approach" method is used, which implies a consequent consideration of convex optimization problems. The obtained results are compared with results of well-known methods such as SDP Relaxation, SOC Relaxation etc. (1). The rate of convergence of studied method for large dimensions problems with tens of thousands of nodes is investigated.The results of direct computations confirm a higher rate of convergence for the proposed method and a reduction in the computational resources, necessary for performing each iteration.

Index terms: *optimal power flow, non-convex optimization problem, trust region approach, convex restriction* .

1 Introduction

In this article we explore the problem of optimization of the workload in the energy network in order to reduce the cost of electricity production. This is a very important topic in the power industry (2). Energy flows are huge, so any results, leading to small improvements in the scale of the power grid, lead to a tremendous economic effect [YM: blue]"Energy flows are huge..." – try to rewrite the sentence. Take a look on the paper you cite; what is there motivation?.

The problem is called AC-Optimal Power Flow problem AC-OPF problem) [YM: We have two problems: AC-OPF and DC-OPF, the second one is a linear programming]. In general case this optimization problem with constraints, imposed by nonlinear Kirchhoff's equations and the needs of consumers, has a convex functional and non-convex constraints. A large number of articles offer various solutions. Linear approximation method suggests linearizing the power flow constraints. Local optimization methods are about looking for a local optimum of the AC-OPF problem. And global optimization methods propose convexifying the constraints imposed by the Kirchhoff's laws.

In this paper, we solve the OFP problem using the "trust region approach" method, which implies a consequent consideration of convex optimisation problems. We study the convergence of the method and the rate of convergence in relation to large dimension problems with tens of thousands of nodes.

The obtained results are compared with another methods: DC Approximation, LPAC Approximation, SDP Relaxation, SOC Relaxation, QC Relaxation. The results of direct computations confirm a fast rate of convergence for the proposed method and a reduction in the computational resources, necessary for performing each iteration.

[YM: Nina, Very good! Sorry for the very brief comments, I have a short stopover on my way to Europe. Few comments are below:

- References are missing. I have added a few (2–5), take a look please. In (2) you will find a proper setup and a motivation behind the problem;
- try to support each statement by a citation, e.g., the problem hardness, the problem motivation etc.

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2 Preliminaries

The buses in network can be classified as PQ buses, PV buses and Slack buses. The names of first two correspond to letters p and q, which are standard letter used to denote Active Power Injection and Reactive Power Injection, and letter v - for word Voltage. PQ buses are the buses where Active and Reactive Power Injection are fixed when the voltage phase angle and magnitude are free. Generally Active and Reactive Power of PQ buses are assumed to be zero, so PQ buses are non-generator buses. PV buses are the ones with Active Power Injection and Voltage magnitude fixed while Reactive Power Injection and Voltage phase angle are free, are actually the generator buses. And, finally, Slack bus (usually one, but there can be several in the power grid)- the bus with fixed Voltage magnitude and phase angle, and the power which can swing. Slack bus is used...

3 Problem Statement

Consider a power network modeled by a directed graph $\mathcal{G}(\mathcal{N}, \mathcal{E})$ where buses are represented by nodes in $\mathcal{N}$ and transmission lines by edges in $\mathcal{E} \in \mathcal{N} \times \mathcal{N}$.

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\begin{aligned}
&\text{sets:}\\
&\qquad N \text{ - buses}\\
&\qquad R \text{ - reference buses}\\
&\qquad E, E^R \text{ - branches, forward and reverse orientation}\\
&\qquad G, G_i \text{ - generators and generators at bus } i\\
&\qquad L, L_i \text{ - loads and loads at bus } i\\
&\qquad S, S_i \text{ - shunts and shunts at bus } i\\
&\text{data:}\\
&\qquad S^{gl}_k, S^{gu}_k \;\; \forall k \in G \text{ - generator complex power bounds}\\
&\qquad c_{2k}, c_{1k}, c_{0k} \;\; \forall k \in G \text{ - generator cost components}\\
&\qquad v^l_i, v^u_i \;\; \forall i \in N \text{ - voltage bounds}\\
&\qquad S^d_k \;\; \forall k \in L \text{ - load complex power consumption}\\
&\qquad Y^s_k \;\; \forall k \in S \text{ - bus shunt admittance}\\
&\qquad Y_{ij}, Y^c_{ij}, Y^c_{ji} \;\; \forall (i,j) \in E \text{ - branch pi-section parameters}\\
&\qquad T_{ij} \;\; \forall (i,j) \in E \text{ - branch complex transformation ratio}\\
&\qquad s^u_{ij} \;\; \forall (i,j) \in E \text{ - branch apparent power limit}\\
&\qquad i^u_{ij} \;\; \forall (i,j) \in E \text{ - branch current limit}\\
&\qquad \theta^{\Delta l}_{ij}, \theta^{\Delta u}_{ij} \;\; \forall (i,j) \in E \text{ - branch voltage angle difference bounds}
\end{aligned}
\tag{1}
$$

$$
\begin{aligned}
\text{variables:}\quad & && (2)\\
& S_k^g \ \ \forall k \in G \text{ - generator complex power dispatch} && (3)\\
& V_i \ \ \forall i \in N \text{ - bus complex voltage} && (4)\\
& S_{ij} \ \ \forall (i,j) \in E \cup E^R \text{ - branch complex power flow} && (5)\\
\text{minimize:}\quad & \sum_{k \in G} c_{2k} (\mathrm{Re}(S_k^g))^2 + c_{1k}\,\mathrm{Re}(S_k^g) + c_{0k} && (6)\\
\text{subject to:}\quad & && \\
& \angle V_r = 0 \ \ \forall r \in R && (7)\\
& S_k^{gl} \leqslant S_k^g \leqslant S_k^{gu} \ \ \forall k \in G && (8)\\
& v_i^l \leqslant |V_i| \leqslant v_i^u \ \ \forall i \in N && (9)\\
& \sum_{k \in G_i} S_k^g - \sum_{k \in L_i} S_k^d - \sum_{k \in S_i} Y_k^s |V_i|^2 = \sum_{(i,j) \in E_i \cup E_i^R} S_{ij} \ \ \forall i \in N && (10)\\
& S_{ij} = \left(Y_{ij} + Y_{ij}^c\right)^* \frac{|V_i|^2}{|T_{ij}|^2} - Y_{ij}^* \frac{V_i V_j^*}{T_{ij}} \ \ \forall (i,j) \in E && (11)\\
& S_{ji} = \left(Y_{ij} + Y_{ji}^c\right)^* |V_j|^2 - Y_{ij}^* \frac{V_i^* V_j}{T_{ij}^*} \ \ \forall (i,j) \in E && (12)\\
& |S_{ij}| \leqslant s_{ij}^u \ \ \forall (i,j) \in E \cup E^R && (13)\\
& |I_{ij}| \leqslant i_{ij}^u \ \ \forall (i,j) \in E \cup E^R && (14)\\
& \theta_{ij}^{\Delta l} \leqslant \angle (V_i V_j^*) \leqslant \theta_{ij}^{\Delta u} \ \ \forall (i,j) \in E && (15)
\end{aligned}
$$

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Plan of the text:

1) Describe AC Power Flow Equations and the cost function to mimimize

2) About considering 1) as general representation with control variables $u \in R^m$ and states variables $x \in R^n$

3) Our aim to find convex restriction in R^m (in projection to the u-variables) using base point. The base algorithm is described in (6).

4) Trust region approach.

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Let N_G be vertexes corresponding to generetors buses: PV and Slack buses.

4 *

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