Optimal Approximation of Non-linear Power Flow Problem

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AC-OPF Problem

Consider the problem of optimization of the workload in the energy network in order to reduce the cost of electricity production.

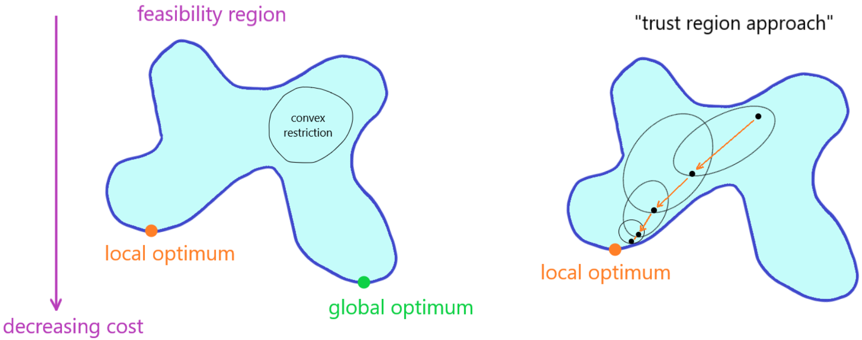
Difficulty

In general case this optimization problem with constraints, imposed by nonlinear Kirchhoff's equations and the needs of consumers, has a convex functional and non-convex constraints.

Objective

The goal of research is to introduce new approximate solution to the AC-OPF Problem based on restrictions idea and trust region approach.

Restrictions and trust region approach for AC-OPF Problem approximation



- **Intoduction to AC-OPF Problem and its solutions**
  .[1] Carleton Coffrin and Line Roald, *Convex Relaxations in Power System Optimization: A Brief Introduction*, 2018
- **Summary on convex relaxations for AC-OPF Problem**
  .[2] Steven H. Low, *Convex Relaxation of Optimal Power Flow Par I, Par II*, 2014
- **Linear approximation method for solving AC-OPF Problem**
  .[3] Sidhant Misra, Daniel K. Molzahn, and Krishnamurthy Dvijotham, *Optimal Adaptive Linearizations of the AC Power Flow Equations*, 2018
- **Base convex restriction algorithm**
  .[4] Dongchan Lee, Hung D. Nguyen, Krishnamurthy Dvijotham, and Konstantin Turitsyn, *Convex Restriction of Power Flow Feasible Sets*, 2019

Model of power network

- $\mathbf{G}(\mathcal{N}, \mathcal{E})$ - directed graph of power network
  $\mathcal{N}$ - nodes that is buses
  $\mathcal{E} \subset \mathcal{N} \times \mathcal{N}$ - edges that is transmissions
- $\mathcal{R} \subset \mathcal{N}$ - the set of reference (slack) buses
  $\mathcal{G}$ and $\mathcal{G}_i$ - all generators and the generators at bus i
  $\mathcal{L}$ and $\mathcal{L}_i$ - all loads and the loads at bus i
  $\mathcal{S}$ and $\mathcal{S}_i$ - all shunts and the shunts at bus i

Data

$S_k^{gl}, S_k^{gu}\ \ \forall k \in \mathcal{G}$ - generator complex power lower and upper bounds

$v_i^l, v_i^u\ \ \forall i \in \mathcal{N}$ - voltage lower and upper bounds

$S_k^d\ \ \forall k \in \mathcal{L}$ - load complex power consumption

$c_{2k}, c_{1k}, c_{0k}\ \ \forall k \in \mathcal{G}$ - generator cost components

Variables

$$S_k^g \ \ \forall k \in \mathcal{G} \text{ - generator complex power dispatch}$$
$$V_i \ \ \forall i \in \mathcal{N} \text{ - bus complex voltage}$$
$$S_{ij} \ \ \forall (i,j) \in \mathcal{E} \text{ - branch complex power flow}$$

Constraints

$$\angle V_r = 0 \ \ \forall r \in \mathcal{R} \tag{1}$$
$$S_k^{gl} \le |S_k^g| \le S_k^{gu} \ \ \forall k \in \mathcal{G} \tag{2}$$
$$v_i^l \le |V_i| \le v_i^u \ \ \forall i \in \mathcal{N} \tag{3}$$
$$\sum_{k \in \mathcal{G}_i} S_k^g - \sum_{k \in \mathcal{L}_i} S_k^d - \sum_{k \in \mathcal{S}_i} Y_k^s |V_i|^2 = \sum_{(i,j) \in \mathcal{E}} S_{ij} \ \ \forall i \in \mathcal{N} \tag{4}$$

AC-OPF Problem is to minimize

$$\sum_{k \in \mathcal{G}} \left(c_{2k} (Re(S_k^g))^2 + c_{1k} Re(S_k^g) + c_{0k} \right)$$

Restrictions obtained by using base algorithm

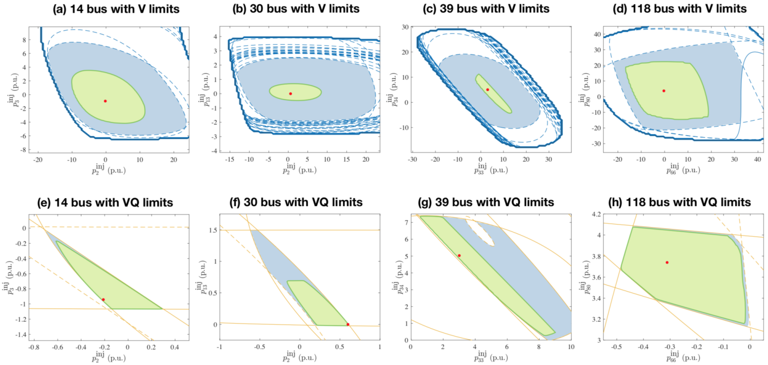


Fig. 7. Convex restrictions of feasible active power injection set in 14, 30, 39 and 118 bus system are shown. Figure (a) to (d) only considers the voltage magnitude limits, and Figure (e) to (h) considers both voltage magnitude and reactive power limits. Thick blue lines show the solvability boundary. Solid yellow lines show reactive power upper limit and dashed yellow lines show reactive power lower limits.

The experiment described in [4] showed that constructions are very close to the true feasible region along some of the boundaries.

The base algorithm gives restrictions which are close to feasibility regions along some boundaries. There is scheme to get improved restrictions. And, using them in "trust region approach", get better results in finding AC-OPF Problem approximation in comparision with existing algorithms. Also there is theoretical basis to believe that the proposed method will have higher rates of convergence and a reduction in the computational resources, necessary for performing each iteration.