Fast algorithm for approximating the inner and the outer boundaries of the iris

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April 25, 2019

Problem

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Describe and implement a fast algorithm that determines the inner and the outer boundaries of the iris in the image of the eye.

Applications

- Biometrics
- Medicine

Literature

Basic algorithm

- Iris border detection using a method of paired gradients
  Y. S. Efimov, I. A. Matveev, 2015

Related to this paper

- Use of the Hough transformation to detect lines and curves in pictures
  R. O. Duda, P. E. Hart, 1972
- Learning using privileged information
  R.Neychev, 2018

Data description

Input data

Monochromatic raster graphic image $\mathbf{I}_0$ size of $W \times H$, obtained by photographing wide-open eye located approximately on the optical axis.

Output data

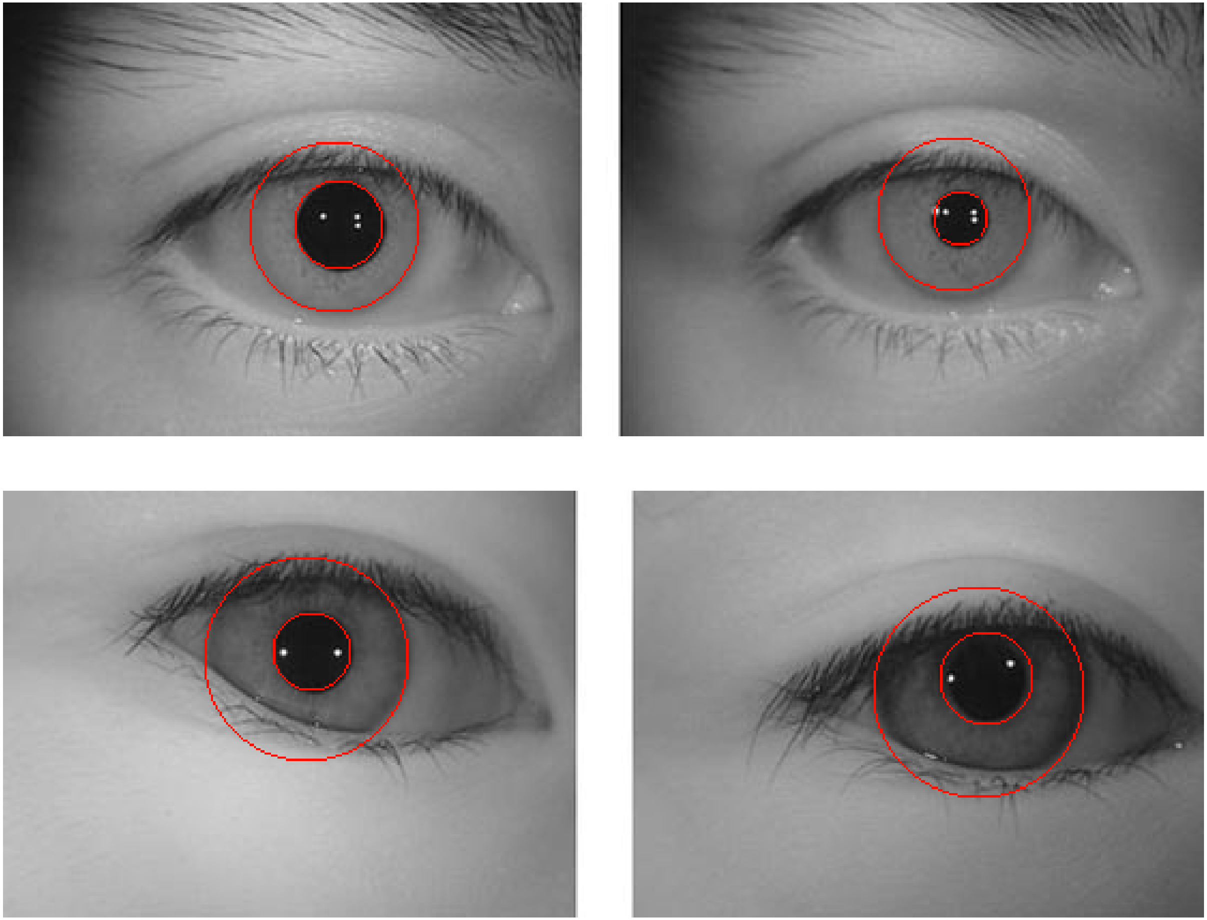
The radiuses and the coordinates of the centers of the circles approximating the inner and the outer borders of the iris:
$\{i_{\text{iris}}, j_{\text{iris}}, r_{\text{iris}}\}$, $\{i_{\text{pupil}}, j_{\text{pupil}}, r_{\text{pupil}}\}$

Expert data

True parameters are provided by an expert for each image:
$\{\widetilde{i}_{\text{iris}}, \widetilde{j}_{\text{iris}}, \widetilde{r}_{\text{iris}}\}$, $\{\widetilde{i}_{\text{pupil}}, \widetilde{j}_{\text{pupil}}, \widetilde{r}_{\text{pupil}}\}$

Input data example

Output data example



Quality criterion

Absolute error

The maximum deviation from expert values among all parameters:

$$\Delta := \max\{|\widetilde{i}_{\text{iris}} - i_{\text{iris}}|, |\widetilde{j}_{\text{iris}} - j_{\text{iris}}|, |\widetilde{r}_{\text{iris}} - r_{\text{iris}}|, \\ |\widetilde{i}_{\text{pupil}} - i_{\text{pupil}}|, |\widetilde{j}_{\text{pupil}} - j_{\text{pupil}}|, |\widetilde{r}_{\text{pupil}} - r_{\text{pupil}}|\}$$

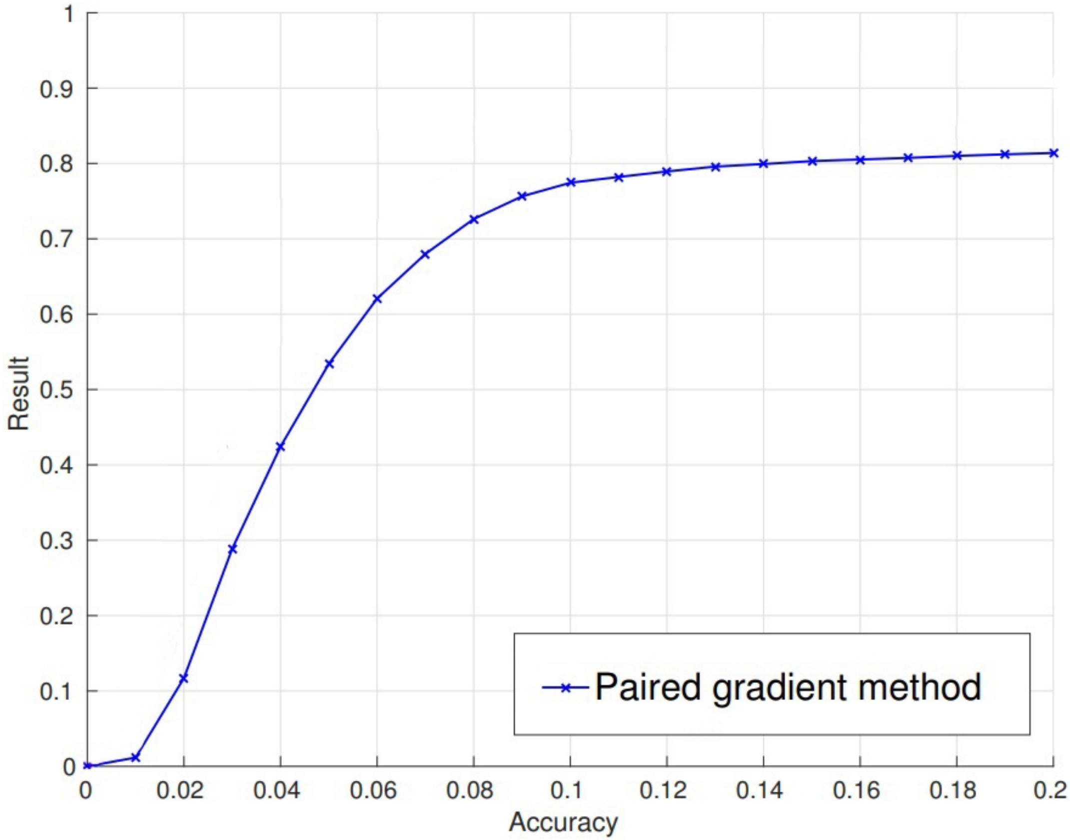
Relative error

The ratio of the absolute error to the outer circle's radius: $\varepsilon := \frac{\Delta}{\widetilde{r}_{\text{iris}}}$

Quality

The percentage of images on which the absolute error does not exceed the threshold δ set by the expert.

Basic algorithm results



Algorithm flowchart

Figure: Algorithm stages

Space transformation

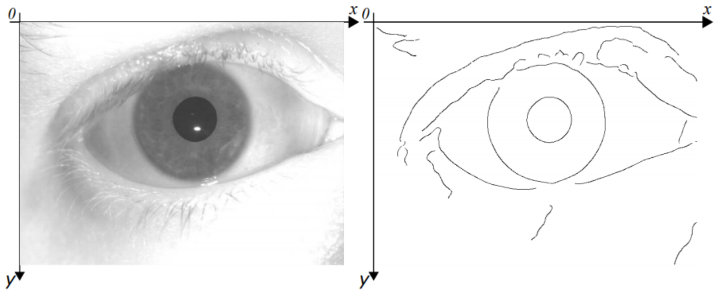


Figure: Detection of boundary points

Let $\rho(i, j) \to \{0, 1\}$ be the result of the binarization stage.

Consider the following space transformation $T : \mathbb{R}^2 \to \mathbb{R}^5$:
$T(x, y) \triangleq \left(x^2, x, y^2, y, 1\right)$.

Let $\mathcal{X}$ be the result of transformation T applied to the boundary points: $\mathcal{X} = \{T(i, j) : \rho(i, j) = 1\}$.

Multimodel optimization problem

Let $\mathbf{w_j} = \left(x_j^2, x_j, y_j^2, y_j, 1\right), j = 1, 2$ denote the approximating circles.

Let $\pi_{j,\mathbf{z}_i}$ be the indicator of point $\mathbf{z}_i$ belonging to class j.
The classes are:

- 1. The inner boundary
- 2. The outer boundary
- 3. Noise, which is assumed to be normally distributed

Multimodel optimization problem

$$\mathbf{w_1}, \mathbf{w_2} = \underset{\mathbf{w_1}, \mathbf{w_2}, \pi}{\operatorname{argmin}} \sum_{\mathbf{z_i} \in \mathcal{X}} \pi_{1,i} \left(\mathbf{z_i}^\top \mathbf{w_1}\right)^2 + \pi_{2,i} \left(\mathbf{z_i}^\top \mathbf{w_2}\right)^2 + \pi_{3,i} \frac{\left(z_i - \mathrm{E}\mathbf{z}\right)^2}{\mathrm{Var}\mathbf{z}}$$

Linear Expert for approximating one circle

Let point (x, y) belong to circle with center (c_1, c_2) and radius r.
Equation for this circle:

$$(x - c_1)^2 + (y - c_2)^2 = r^2$$

$$2x \cdot c_1 + 2y \cdot c_2 + 1 \cdot (\underbrace{r^2 - c_1^2 - c_2^2}_{c_3}) = x^2 + y^2$$

Linear expert for approximating one circle

$$2x \cdot c_1 + 2y \cdot c_2 + 1 \cdot c_3 = x^2 + y^2$$

Let $P = \{(x_i, y_i)\}_{i=1}^{m}$ be the set of points we want to approximate with a circle.

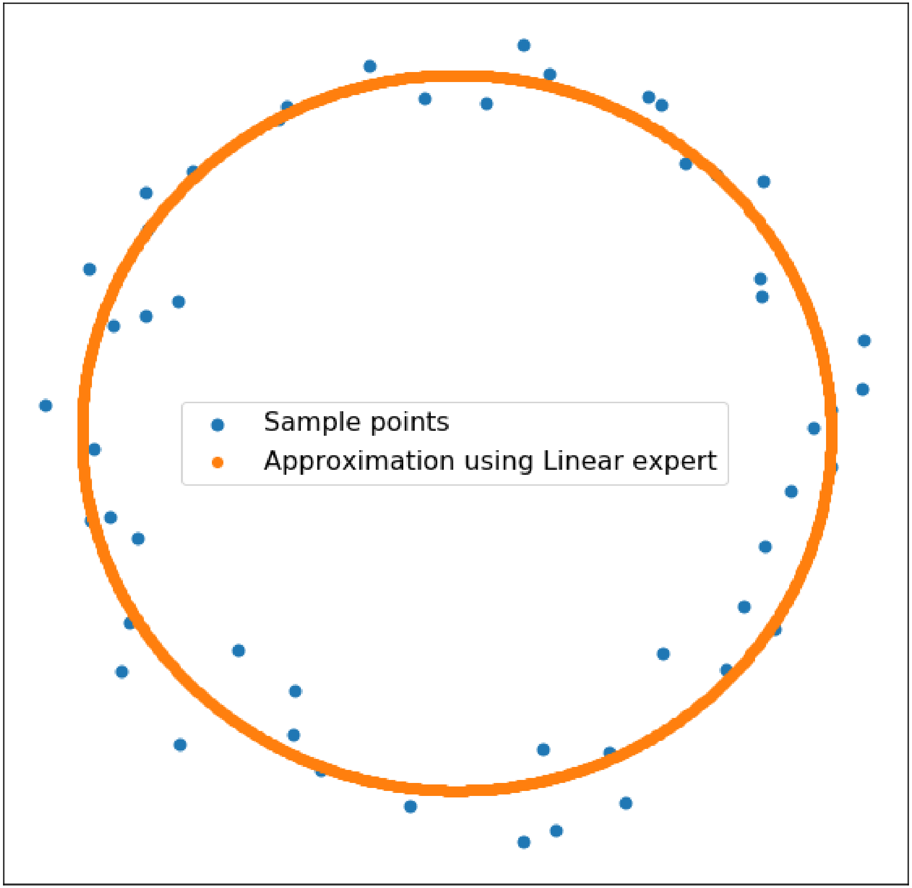
Consider the following system of linear equations:

$$\underbrace{\begin{pmatrix} 2x_1 & 2y_1 & 1 \\ 2x_2 & 2y_2 & 1 \\ \vdots & \vdots & \vdots \\ 2x_m & 2y_m & 1 \end{pmatrix}}_{\mathbf{X}} \underbrace{\begin{pmatrix} c_1 \\ c_2 \\ c_3 \end{pmatrix}}_{\mathbf{c}} = \underbrace{\begin{pmatrix} x_1^2 + y_1^2 \\ x_2^2 + y_2^2 \\ \vdots \\ x_m^2 + y_m^2 \end{pmatrix}}_{\mathbf{Y}}$$

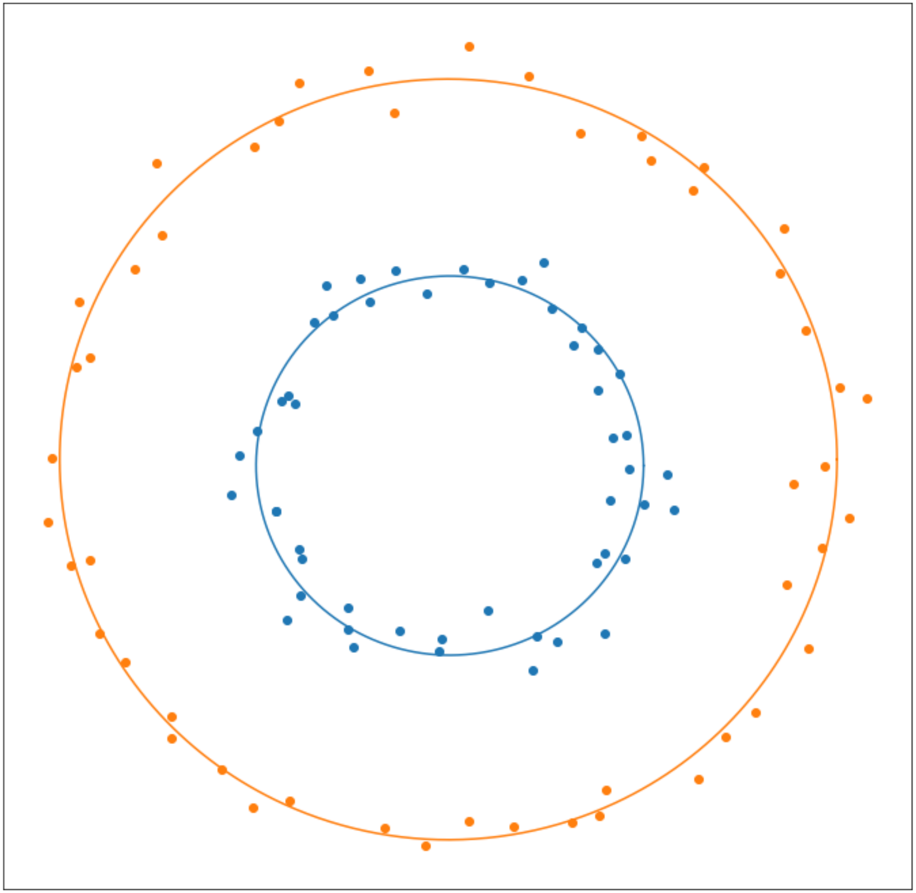
Least Squares

$$\hat{\mathbf{c}} = \underset{\mathbf{c}}{\operatorname{argmin}} \left(\mathbf{X}\mathbf{c} - \mathbf{Y}\right)^\top \mathbf{A} \left(\mathbf{X}\mathbf{c} - \mathbf{Y}\right) + \left(\mathbf{c} - \mathbf{c_0}\right)^\top \mathbf{B} \left(\mathbf{c} - \mathbf{c_0}\right)$$

Linear expert example



MoE example



Summary

- The basic experiment has been conducted
- The problem has been formulated in terms of conditional optimization
- MoE has been implemented