Fast algorithm for approximating the inner and the outer boundaries of the iris

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Problem

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Describe and implement a fast algorithm that determines the inner and the outer boundaries of the iris in the image of the eye.

Applications

- Biometrics
- Medicine

Literature

Basic algorithm

- Iris border detection using a method of paired gradients
  Y. S. Efimov, I. A. Matveev, 2015

Related to this paper

- Use of the Hough transformation to detect lines and curves in pictures
  R. O. Duda, P. E. Hart, 1972
- Learning using privileged information
  R.Neychev, 2018

Data description

Input data

Monochromatic raster graphic image $\mathbf{I}_0$ size of $W \times H$, obtained by photographing wide-open eye located approximately on the optical axis.

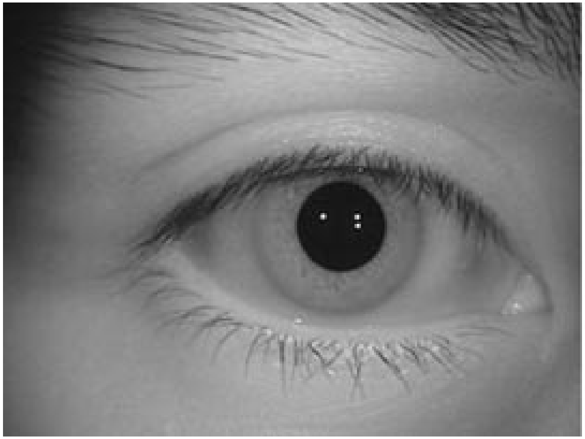
Output data

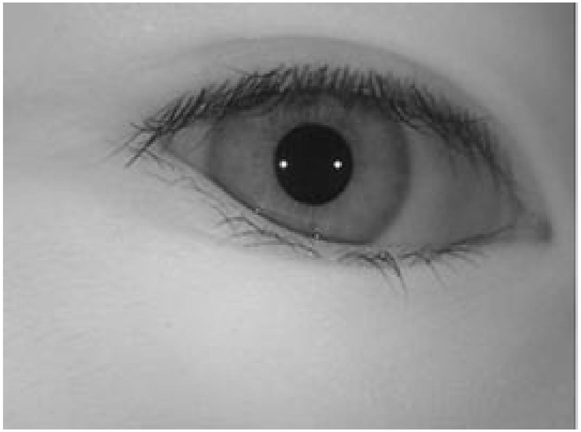
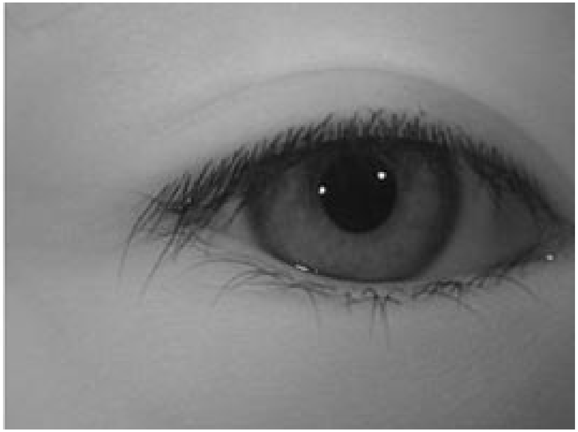
The radiuses and the coordinates of the centers of the circles approximating the inner and the outer borders of the iris:
$\{i_{\text{iris}}, j_{\text{iris}}, r_{\text{iris}}\}$, $\{i_{\text{pupil}}, j_{\text{pupil}}, r_{\text{pupil}}\}$

Expert data

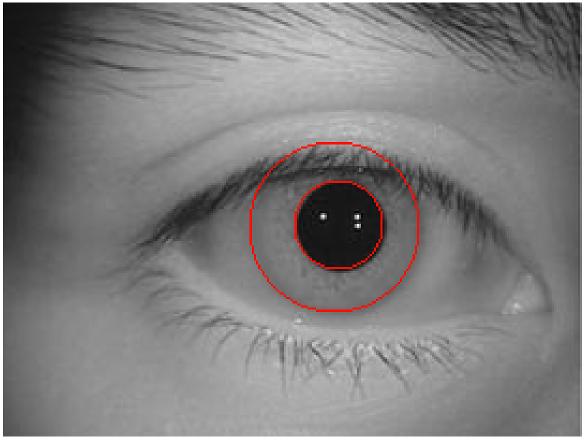
True parameters are provided by an expert for each image:
$\{\widetilde{i}_{\text{iris}}, \widetilde{j}_{\text{iris}}, \widetilde{r}_{\text{iris}}\}$, $\{\widetilde{i}_{\text{pupil}}, \widetilde{j}_{\text{pupil}}, \widetilde{r}_{\text{pupil}}\}$

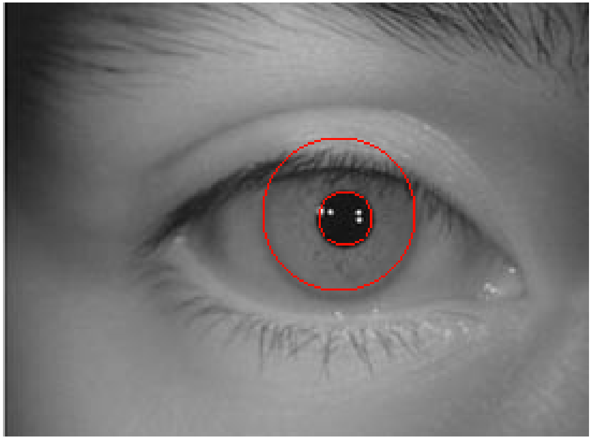
Input data example

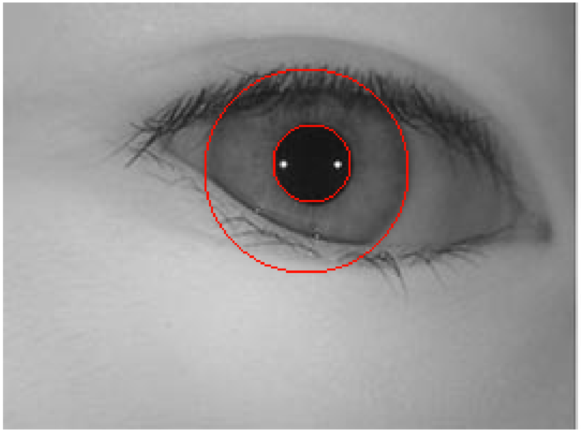


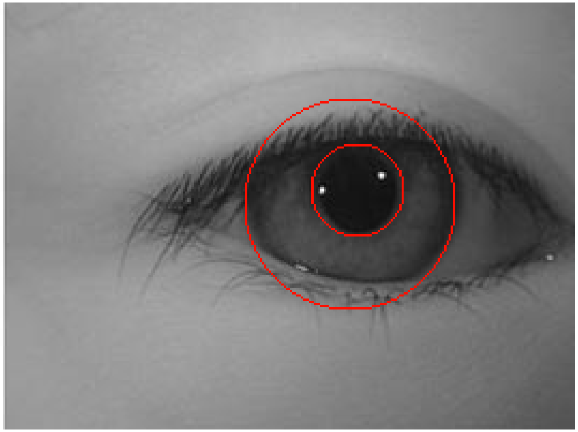



Output data example









Quality criterion

Absolute error

The maximum deviation from expert values among all parameters:

$$\Delta := \max\{|\widetilde{i}_{\text{iris}} - i_{\text{iris}}|, |\widetilde{j}_{\text{iris}} - j_{\text{iris}}|, |\widetilde{r}_{\text{iris}} - r_{\text{iris}}|,$$
$$|\widetilde{i}_{\text{pupil}} - i_{\text{pupil}}|, |\widetilde{j}_{\text{pupil}} - j_{\text{pupil}}|, |\widetilde{r}_{\text{pupil}} - r_{\text{pupil}}|\}$$

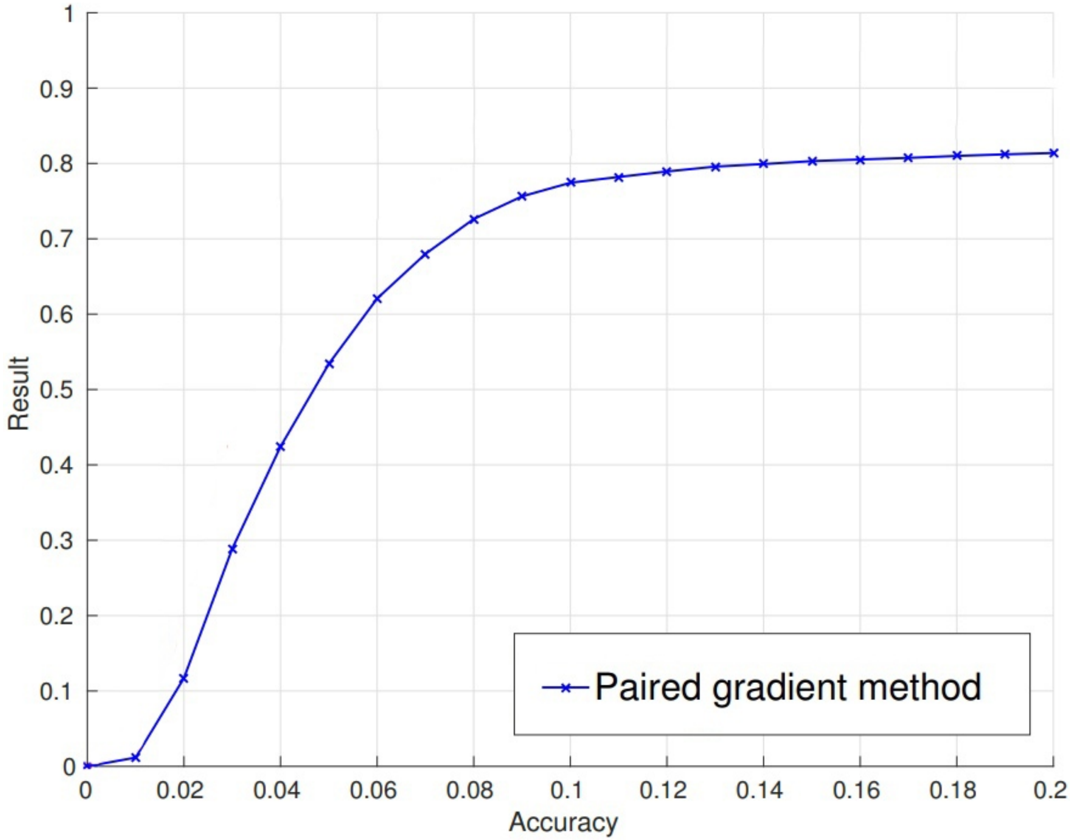
Relative error

The ratio of the absolute error to the outer circle's radius: $\varepsilon := \frac{\Delta}{\widetilde{r}_{\text{iris}}}$

Quality

The percentage of images on which the absolute error does not exceed the threshold δ set by the expert.

Basic algorithm results



Algorithm flowchart

Figure 1: Algorithm stages

Space transformation

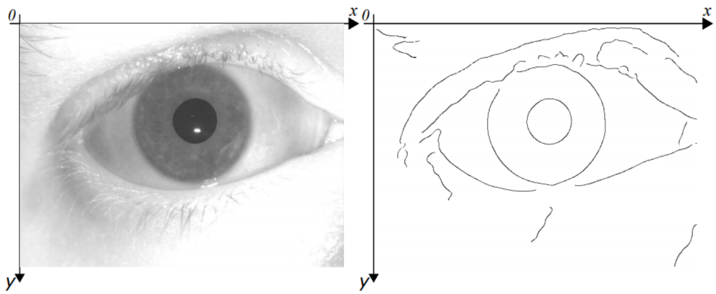


Figure 2: Detection of boundary points

Let $\rho(i, j) \to \{0, 1\}$ be the result of the binarization stage.

Consider the following space transformation $T : \mathbb{R}^2 \to \mathbb{R}^5$:
$T(x, y) \triangleq \left(x^2, x, y^2, y, 1\right)$.

Let $\mathcal{X}$ be the result of transformation T applied to the boundary points: $\mathcal{X} = \{T(i, j) : \rho(i, j) = 1\}$.

Multimodel optimization problem

Let $\mathbf{w_j} = \left(x_j^2, x_j, y_j^2, y_j, 1\right), j = 1, 2$ denote the approximating circles.

Let $\pi_{j,\mathbf{z}_i}$ be the indicator of point $\mathbf{z}_i$ belonging to class j.
The classes are:

- 1. The inner boundary
- 2. The outer boundary
- 3. Noise, which is assumed to be normally distributed

Multimodel optimization problem

$$\mathbf{w_1}, \mathbf{w_2} = \underset{\mathbf{w_1}, \mathbf{w_2}}{\operatorname{argmin}} \sum_{\mathbf{z_i} \in \mathcal{X}} \pi_{1,i} \left(\mathbf{z_i}^\top \mathbf{w_1}\right)^2 + \pi_{2,i} \left(\mathbf{z_i}^\top \mathbf{w_2}\right)^2 + \pi_{3,i} \frac{(z_i - \mathrm{E}\mathbf{z})^2}{\mathrm{Var}\mathbf{z}}$$

Multimodel optimization problem

Additional constraints

The inner and the outer circles have a common center, so we can assume that the corresponding lines in the transformed space have the same angle of inclination:

$$\mathbf{w_1}, \mathbf{w_2} = \underset{\mathbf{w_1}, \mathbf{w_2}}{\operatorname{argmin}} \sum_{\mathbf{z_i} \in \mathcal{X}} \pi_{1,i} \left(\mathbf{z_i}^\top \mathbf{w_1} \right)^2 + \pi_{2,i} \left(\mathbf{z_i}^\top \mathbf{w_2} \right)^2 + \pi_{3,i} \frac{(z_i - \mathrm{E}\mathbf{z})^2}{\mathrm{Var}\mathbf{z}}$$

$$\text{s.t. } w_{1,j} = w_{2,j} \quad j \in \{2, 4\}$$

Summary

- The basic experiment has been conducted
- The problem is formulated in terms of conditional optimization