Phase detection of human motions with signals of wearable devices

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Phase detection of human motion

The current generation of portable mobile devices incorporates various types of sensors, that open up new areas for the analysis of human behavior.

Goal

To construct a model that determines the phase of the time series trajectory.

Problem

Find the phase of a quasi-periodic time series.

Method

Reduce the dimensionality of the trajectory space. The criterion of dimensionality is the phase trajectory does not have self-intersections. Extract the phase from the obtained trajectory.

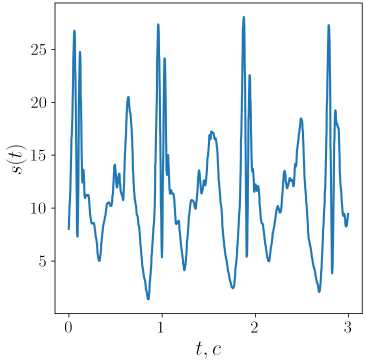
$\{s_i\}_{i=1}^{N}$ original time series

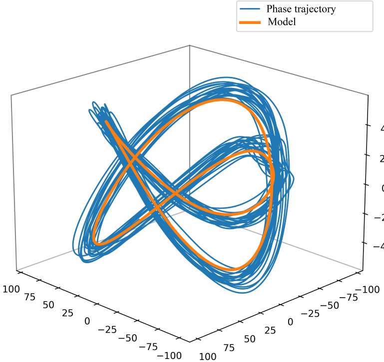
$\mathbf{H} = \begin{bmatrix} \mathbf{s}_1 & \dots & \mathbf{s}_m \end{bmatrix}$, $m = N - n + 1$ trajectory matrix

$\mathbf{s}_k \in \mathbb{R}^n$ form the phase trajectory

$\mathbf{X} = \mathbf{HW} = \begin{bmatrix} \mathbf{x}_1 & \dots & \mathbf{x}_m \end{bmatrix}^{\mathsf{T}}$ dimension reduction (PCA)

$\mathbf{x}_k \in \mathbb{R}^p$ phase trajectory in the lower dimensional space





Literature

1. Motrenko A., Strijov V. Extracting fundamental periods to segment biomedical signals //IEEE journal of biomedical and health informatics, 2015.
2. Ignatov A. D., Strijov V. V. Human activity recognition using quasiperiodic time series collected from a single tri-axial accelerometer //Multimedia tools and applications, 2016.
3. Grabovoy A. V., Strijov V. V. Quasi-Periodic Time Series Clustering for Human Activity Recognition //Lobachevskii Journal of Mathematics, 2020.

Dimension reduction of trajectory space

Trajectory matrix

$$\mathbf{H} = \begin{bmatrix} s_1 & \dots & s_n \\ s_2 & \dots & s_{k+1} \\ \dots & \dots & \dots \\ s_{N-n+1} & \dots & s_N \end{bmatrix}^{\mathsf{T}}$$

Singular value decomposition of matrix $\mathbf{H}$

$$\frac{1}{n}\mathbf{H}^{\mathsf{T}}\mathbf{H} = \mathbf{V}\Lambda\mathbf{V}^{\mathsf{T}}, \qquad \Lambda = \mathrm{diag}(\lambda_1, \dots, \lambda_n)$$

Select principal components $\mathbf{y}_1, \dots, \mathbf{y}_p$, where

$$\mathbf{y}_k = \mathbf{H}\mathbf{v}_k,\ k \in \overline{1, n}$$

Reconstructed part of the trajectory matrix $\mathbf{H}$

$$\widehat{\mathbf{H}} = \mathbf{H}_1 + \dots + \mathbf{H}_p, \quad \mathbf{H}_j = \sqrt{\lambda_j}\mathbf{v}_j\mathbf{y}_j^{\mathsf{T}},\ j \in 1, p.$$

Dimension reduction $\mathbf{X} = \widehat{\mathbf{H}}\mathbf{W} = \begin{bmatrix}\mathbf{x}_1 & \dots & \mathbf{x}_m\end{bmatrix}^{\mathsf{T}}$

Approximation of the phase trajectory

The model $g : \varphi \to \mathbf{x}$ set the point of the average trajectory $\mathsf{E}(\mathbf{x}|\varphi)$ and the value of the variance $\mathsf{D}(\mathbf{x}|\varphi)$ in accordance with the phase $\varphi \in [0, 2\pi)$, where $\mathbf{x} \in \boldsymbol{X}$ is a point of the trajectory space.

The Nadaraya-Watson Regression

$$\mathsf{E}(\boldsymbol{x}|\varphi) = \frac{\sum\limits_{\boldsymbol{x}_k \in X} \boldsymbol{x}_k K\left(\frac{\rho(\varphi'_k, \varphi)}{h}\right)}{\sum\limits_{\boldsymbol{x}_k \in X} K\left(\frac{\rho(\varphi'_k, \varphi)}{h}\right)},$$

where φ'_i is a value of phase assigned to the point $\boldsymbol{x}_i$ with assumption about the period T: $\varphi'_i = \frac{2\pi}{T} \cdot i \mod 2\pi,\ i \in \overline{1, m}$.

Introduce a metric

$$\rho(\varphi', \varphi) = \frac{1 - \cos(\varphi' - \varphi)}{2}, \quad \varphi', \varphi \in [0, 2\pi).$$

Dimensionality reduction

Minimal dimension

Trajectory subspace have minimal dimension if the model of the phase trajectory does not have any self-intersections.

Self-intersections

Points with significantly different phases that are close in phase space

$$\text{there exist } i, j \in \overline{1, m}:$$

$$\|\mathbf{x}_i - \mathbf{x}_j\|^2 < \mathsf{D}(\mathsf{x}_i|\varphi) + \mathsf{D}(\mathsf{x}_j|\varphi), \quad \|\boldsymbol{\varphi}_i - \boldsymbol{\varphi}_j\|_1 > \frac{\pi}{4}.$$

Phase detection model

The points $\boldsymbol{x}_i \rightsquigarrow \boldsymbol{x}_j$ are neighbouring if $\|\boldsymbol{x}_i - \boldsymbol{x}_j\|_2^2 < \varepsilon$,
ε – hyperparameter.

Assumptions

1. A point with a large index corresponds to a large phase

$$j > i \ \rightarrow \ \varphi_j > \varphi_i \quad i, j \in \overline{1, m}.$$

2. The phases of neighboring points are close

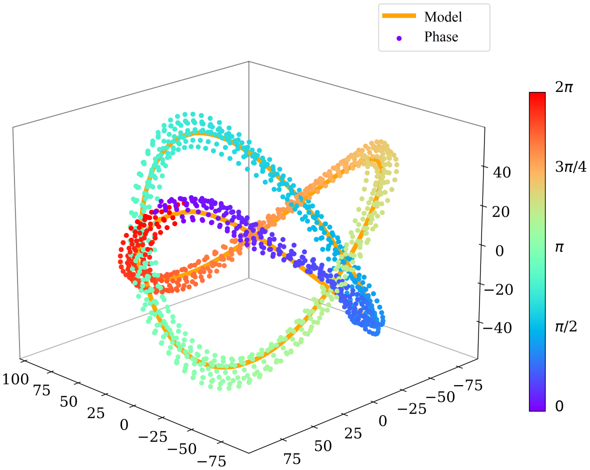
$$\boldsymbol{x}_i \rightsquigarrow \boldsymbol{x}_j \quad \rightarrow \quad \|\varphi_i - \varphi_j\| < \delta \quad i, j \in \overline{1, m}.$$

The desired phase value

$$\widehat{\varphi}_i = \arg\min_{\varphi_j} L(\varphi_j).$$

$$L(\varphi_j) = \lambda_1 \sum_{i<j} |\varphi_i - \varphi_j|_+ + (1 - \lambda_1) \sum_{\|\mathrm{x}_i - \mathrm{x}_j\| < \varepsilon} \rho(\varphi_i, \varphi_j).$$

The phase of a human motion



Approximation of the phase trajectory. Phase values for the points of the phase trajectory.

Conclusions

- The algorithm for phase extraction of a quasi-periodic time series is developed
- The phase trajectory approximation problem for quasi-periodic time series is solved
- The criterion for detecting self-intersections of the average trajectory is proposed