Eduard Vladimirov<sup>1\*</sup>, Vadim Strijov<sup>1†</sup> and Roman Isachenko<sup>1†</sup>

<sup>1</sup>Moscow Institute of Physics and Technology, Inststitutskii per. 9, Dolgoprudny, 141700, Moscow Region, Russia.

\*Corresponding author(s). E-mail(s):
 vladimirov.ea@phystech.edu;
Contributing authors: strijov@phystech.edu;
 roman.isachenko@phystech.edu;

†These authors contributed equally to this work.

#### Abstract

In this paper, we consider the problem of forecasting a time series with a complex structure. The complex structure means the presence of non-linear dependencies and a varying period. We must to find causal relationships between time series. In order to do this, we reduce the dimension of trajectory spaces. The paper introduces a new way for the consistent dimensional reduction of time series. The proposed method combines the partial least squares method and convergent cross mapping method. To demonstrate the results of the work we solve the problem of hand trajectory reconstruction with video.

**Keywords:** Pose estimation, Time series, Phase trajectory, Trajectory subspace, Convergent cross mapping, Partial least squares

## 1 Introduction

In this paper, we solve the problem of forecasting a time series based on other time series. One of the challenges of the task is to detect relationships between time series and exclude unrelated time series from the predictive model. Solving this problem improves its quality.

In this paper, we apply the convergent cross mapping method (CCM) or the Sugihara method [?, ?], which is effective for time series generated by a dynamic system. It is based on a comparison of the nearest neighbors in the trajectory space of the time series y obtained from the time series x.

When constructing a predictive model, we use a trajectory matrix (or shift matrix) that describes the phase space of a time series. For example, in the method of singular spectral analysis (SSA) [?, ?, ?], the time series prediction is based on the spectral decomposition of the covariance matrix obtained from the trajectory matrix. In CCM, shift matrices are used for checking the presence of a Lipschitz mapping between trajectory spaces.

However, the dimension of the trajectory space may be extremely high, which leads to instability of the predictive model. In this case, it is necessary to reduce the dimension of the trajectory space by constructing a projection of the phase trajectory into some subspace. There is no specific way for CCM to select a subspace in which the phase trajectory is approximated. In the paper [?] this problem is solved using spherical regression. According to this method, information about the desired subspace is extracted from the set of empirical directions  $\{\mathbf{x}_i - \mathbf{x}_j \mid i < j\}$ , where  $\mathbf{x}_i$  — elements of the trajectory space in spherical coordinates. In the work [?] automatic selection of a pair of main components is used. The main idea is to compare the spectral densities of the principal components. A simple iteration over the main components [?] is also used.

The partial least squares method (PLS) [?, ?] selects the most significant features and builds new ones as their linear combination. This allows us to obtain a simple, accurate and stable predictive model. Along with PLS, the canonical correlation analysis (CCA) [?] method is used. It is similar to PLS except that the former method maximizes the covariance between projections, and the latter — correlation. The disadvantage of these models is their low accuracy in estimating nonlinear dependencies between data. The nonlinear extensions of PLS [?] and CCA [?] have been developed. This article uses the PLS-Autoencoder model [?], which converts the source data using autoencoders.

The theoretical part of the paper shows how the Sugihara method can be applied to reduce the dimension of the trajectory space and how to combine the ideas of the PLS and CCM methods. To achieve the latter goal, a new metric for the consistency of latent projections has been introduced.

The algorithm of sequential locally weighted global linear map (SMAP) [?] is used as a model for predicting time series from a set of time series.

The experiment is carried on a set of manually collected data. It is a collection of key points obtained from a video of a person's movement, as well as accelerometer and gyroscope readings taken from a person's hand. In the experiment, a time series forecast is constructed using the detected related components of the time series.

### 2 Results

Sample body text. Sample body text. Sample body text. Sample body text. Sample body text.

# 3 This is an example for first level head—section head

# 3.1 This is an example for second level head—subsection head

# 3.1.1 This is an example for third level head—subsubsection head

Sample body text. Sample body text. Sample body text. Sample body text. Sample body text.

# 4 Equations

Equations in IATEX can either be inline or on-a-line by itself ("display equations"). For inline equations use the  $\dots$  commands. E.g.: The equation  $H\psi = E\psi$  is written via the command  $H = E\psi$  is written via the command  $H = E\psi$ .

For display equations (with auto generated equation numbers) one can use the equation or align environments:

$$\|\tilde{X}(k)\|^{2} \leq \frac{\sum_{i=1}^{p} \|\tilde{Y}_{i}(k)\|^{2} + \sum_{j=1}^{q} \|\tilde{Z}_{j}(k)\|^{2}}{p+q}.$$
 (1)

where,

$$D_{\mu} = \partial_{\mu} - ig \frac{\lambda^a}{2} A^a_{\mu}$$

$$F^a_{\mu\nu} = \partial_{\mu} A^a_{\nu} - \partial_{\nu} A^a_{\mu} + g f^{abc} A^b_{\mu} A^a_{\nu}$$
(2)

Notice the use of \nonumber in the align environment at the end of each line, except the last, so as not to produce equation numbers on lines where no equation numbers are required. The \label{} command should only be used at the last line of an align environment where \nonumber is not used.

$$Y_{\infty} = \left(\frac{m}{\text{GeV}}\right)^{-3} \left[1 + \frac{3\ln(m/\text{GeV})}{15} + \frac{\ln(c_2/5)}{15}\right]$$
 (3)

The class file also supports the use of  $\mathbb{R}$ ,  $\mathcal{R}$  and  $\mathcal{R}$  produces  $\mathbb{R}$ , and  $\mathcal{R}$  respectively (refer Subsubsection 3.1.1).

### 5 Tables

Tables can be inserted via the normal table and tabular environment. To put footnotes inside tables you should use \footnotetext[]{...} tag. The footnote appears just below the table itself (refer Tables 1 and 2). For the corresponding footnotemark use \footnotemark[...]

Table 1 Caption text

Column 1	Column 2	Column 3	Column 4
row 1 row 2	data 1 data 4	$\begin{array}{c} \text{data 2} \\ \text{data 5}^1 \end{array}$	data 3 data 6
row 3	data 7	data 8	data $9^2$

Source: This is an example of table footnote. This is an example of table footnote.

The input format for the above table is as follows:

```
\begin{table}[<placement-specifier>]
\begin{center}
\begin{minipage}{<preferred-table-width>}
\caption{<table-caption>}\label{<table-label>}%
\begin{tabular}{0{}11110{}}
\toprule
Column 1 & Column 2 & Column 3 & Column 4\\
\midrule
row 1 & data 1 & data 2 & data 3 \\
row 2 & data 4 & data 5\footnotemark[1] & data 6 \\
row 3 & data 7 & data 8 & data 9\footnotemark[2]\\
\botrule
\end{tabular}
\footnotetext{Source: This is an example of table footnote.
This is an example of table footnote.}
\footnotetext[1]{Example for a first table footnote.
This is an example of table footnote.}
\footnotetext[2]{Example for a second table footnote.
This is an example of table footnote.}
\end{minipage}
\end{center}
\end{table}
```

<sup>&</sup>lt;sup>1</sup>Example for a first table footnote. This is an example of table footnote.

 $<sup>^2</sup>$ Example for a second table footnote. This is an example of table footnote.

 $1239 \pm 100$ 

 $1092 \pm 40$ 

#### Reconstruction of hand trajectory with video

780 A

900 A

1166

1268

	_					
		Element 1 <sup>1</sup>	1		Element 2 <sup>2</sup>	
Project	Energy	$\sigma_{calc}$	$\sigma_{expt}$	Energy	$\sigma_{calc}$	$\sigma_{expt}$

 ${\bf Table~2}~~{\bf Example~of~a~lengthy~table~which~is~set~to~full~textwidth}$ 

1168

961

Note: This is an example of table footnote. This is an example of table footnote this is an example of table footnote this is an example of table footnote.

 $1547 \pm 12$ 

 $922 \pm 10$ 

Element 3

Element 4

990 A

500 A

In case of double column layout, tables which do not fit in single column width should be set to full text width. For this, you need to use \begin{table\*} ... \end{table\*} instead of \begin{table} ... \end{table} environment. Lengthy tables which do not fit in textwidth should be set as rotated table. For this, you need to use \begin{sidewaystable} ... \end{sidewaystable} instead of \begin{table\*} ... \end{table\*} environment. This environment puts tables rotated to single column width. For tables rotated to double column width, use \begin{sidewaystable\*} ... \end{sidewaystable\*} ... \end{sidewaystable\*}.

## 6 Figures

As per the LATEX standards you need to use eps images for LATEX compilation and pdf/jpg/png images for PDFLaTeX compilation. This is one of the major difference between LATEX and PDFLaTeX. Each image should be from a single input .eps/vector image file. Avoid using subfigures. The command for inserting images for LATEX and PDFLaTeX can be generalized. The package used to insert images in LaTeX/PDFLaTeX is the graphicx package. Figures can be inserted via the normal figure environment as shown in the below example:

```
\begin{figure}[<placement-specifier>]
\centering
\includegraphics{<eps-file>}
\caption{<figure-caption>}\label{<figure-label>}
\end{figure}
```

In case of double column layout, the above format puts figure caption-s/images to single column width. To get spanned images, we need to provide \begin{figure\*} ... \end{figure\*}.

For sample purpose, we have included the width of images in the optional argument of \includegraphics tag. Please ignore this.

<sup>&</sup>lt;sup>1</sup>Example for a first table footnote.

<sup>&</sup>lt;sup>2</sup>Example for a second table footnote.

**Table 3** Tables which are too long to fit, should be written using the "sidewaystable" environment as shown here

		Element 1 <sup>1</sup>			Element <sup>2</sup>	
$\operatorname{Projectile}$	Energy	$\sigma_{calc}$	$\sigma_{expt}$	Energy	$\sigma_{calc}$	$\sigma_{expt}$
Element 3	990 A	1168	$1547 \pm 12$	780 A	1166	$-1239 \pm 100$
Element 4	500  A	961	$922 \pm 10$	900 A	1268	$1092 \pm 40$
Element 5	990 A	1168	$1547 \pm 12$	780 A	1166	$1239 \pm 100$
Element 6	500  A	961	$922 \pm 10$	900 A	1268	$1092 \pm 40$
Note: This is an example		footnote this is ar	of table footnote this is an example of table footnote this is an example of table footnote this is an	tnote this is an ex	ample of table for	otnote this is an

<sup>1</sup>This is an example of table footnote.

example of table footnote this is an example of table footnote.



Fig. 1 This is a widefig. This is an example of long caption this is an example of long caption this is an example of long caption

# 7 Algorithms, Program codes and Listings

Packages algorithm, algorithmicx and algorithms in LATEX using the format:

```
\begin{algorithm}
\caption{<alg-caption>}\label{<alg-label>}
\begin{algorithmic}[1]
. . .
\end{algorithmic}
\end{algorithm}
```

You may refer above listed package documentations for more details before setting algorithm environment. For program codes, the "program" package is required and the command to be used is \begin{program} ... \end{program}. A fast exponentiation procedure:

```
begin
  for i := 1 to 10 step 1 do
      expt(2, i);
      newline() od
                                  Comments will be set flush to the right margin
where
proc expt(x, n) \equiv
  z := 1:
  do if n = 0 then exit fi;
      do if odd(n) then exit fi;
         comment: This is a comment statement;
         n := n/2; \ x := x * x \text{ od};
      {n > 0};
      n := n - 1; \ z := z * x \text{ od};
  print(z).
end
```

Similarly, for listings, use the listings package. \begin{lstlisting} ... \end{lstlisting} is used to set environments similar to verbatim environment. Refer to the lstlisting package documentation for more details.

```
Algorithm 1 Calculate y = x^n
```

```
Require: n > 0 \lor x \neq 0
Ensure: y = x^n

 u ← 1

 2: if n < 0 then
         X \Leftarrow 1/x
         N \Leftarrow -n
 4:
 5. else
         X \Leftarrow x
 7.
         N \Leftarrow n
 8: end if
     while N \neq 0 do
         if N is even then
10:
              X \Leftarrow X \times X
11:
              N \Leftarrow N/2
19.
         else[N \text{ is odd}]
13:
              y \Leftarrow y \times X
14:
              N \Leftarrow N - 1
15:
          end if
17: end while
```

```
for i:=maxint to 0 do
begin
{ do nothing }
end;
Write('Case_insensitive_');
Write('Pascal_keywords.');
```

## 8 Cross referencing

Environments such as figure, table, equation and align can have a label declared via the \label{#label} command. For figures and table environments use the \label{} command inside or just below the \caption{} command. You can then use the \ref{#label} command to cross-reference them. As an example, consider the label declared for Figure 1 which is \label{fig1}. To cross-reference it, use the command Figure \ref{fig1}, for which it comes up as "Figure 1".

To reference line numbers in an algorithm, consider the label declared for the line number 2 of Algorithm 1 is \label{algln2}. To cross-reference it, use the command \ref{algln2} for which it comes up as line 2 of Algorithm 1.

#### 8.1 Details on reference citations

Standard IATEX permits only numerical citations. To support both numerical and author-year citations this template uses natbib IATEX package. For style guidance please refer to the template user manual.

# 9 Examples for theorem like environments

For theorem like environments, we require amsthm package. There are three types of predefined theorem styles exists—thmstyleone, thmstyletwo and thmstylethree

thmstyleone	Numbered, theorem head in bold font and theorem
	text in italic style
thmstyletwo	Numbered, theorem head in roman font and theorem
	text in italic style
thmstylethree	Numbered, theorem head in bold font and theorem
	text in roman style

For mathematics journals, theorem styles can be included as shown in the following examples:

**Theorem 1** (Theorem subhead) Example theorem text. Example theorem text.

Sample body text. Sample body text. Sample body text. Sample body text. Sample body text. Sample body text.

**Proposition 2** Example proposition text. Example proposition text.

Sample body text. Sample body text. Sample body text. Sample body text. Sample body text. Sample body text.

Example 1 Phasellus adipiscing semper elit. Proin fermentum massa ac quam. Sed diam turpis, molestie vitae, placerat a, molestie nec, leo. Maecenas lacinia. Nam ipsum ligula, eleifend at, accumsan nec, suscipit a, ipsum. Morbi blandit ligula feugiat magna. Nunc eleifend consequat lorem.

Sample body text. Sample body text.

Remark 1 Phasellus adipiscing semper elit. Proin fermentum massa ac quam. Sed diam turpis, molestie vitae, placerat a, molestie nec, leo. Maecenas lacinia. Nam ipsum ligula, eleifend at, accumsan nec, suscipit a, ipsum. Morbi blandit ligula feugiat magna. Nunc eleifend consequat lorem.

Sample body text. Sample body text.

**Definition 1** (Definition sub head) Example definition text. Example definition text.

Additionally a predefined "proof" environment is available: \begin{proof} ... \end{proof}. This prints a "Proof" head in italic font style and the "body text" in roman font style with an open square at the end of each proof environment.

*Proof* Example for proof text.  $\Box$ 

Sample body text. Sample body text. Sample body text. Sample body text. Sample body text.

Proof of Theorem 1 Example for proof text.  $\Box$ 

For a quote environment, use \begin{quote}...\end{quote}

Quoted text example. Aliquam porttitor quam a lacus. Praesent vel arcu ut tortor cursus volutpat. In vitae pede quis diam bibendum placerat. Fusce elementum convallis neque. Sed dolor orci, scelerisque ac, dapibus nec, ultricies ut, mi. Duis nec dui quis leo sagittis commodo.

Sample body text. Sample body text.

## 10 Methods

Topical subheadings are allowed. Authors must ensure that their Methods section includes adequate experimental and characterization data necessary for others in the field to reproduce their work. Authors are encouraged to include RIIDs where appropriate.

Ethical approval declarations (only required where applicable) Any article reporting experiment/s carried out on (i) live vertebrate (or higher

invertebrates), (ii) humans or (iii) human samples must include an unambiguous statement within the methods section that meets the following requirements:

- 1. Approval: a statement which confirms that all experimental protocols were approved by a named institutional and/or licensing committee. Please identify the approving body in the methods section
- 2. Accordance: a statement explicitly saying that the methods were carried out in accordance with the relevant guidelines and regulations
- 3. Informed consent (for experiments involving humans or human tissue samples): include a statement confirming that informed consent was obtained from all participants and/or their legal guardian/s

If your manuscript includes potentially identifying patient/participant information, or if it describes human transplantation research, or if it reports results of a clinical trial then additional information will be required. Please visit (https://www.nature.com/nature-research/editorial-policies) for Nature Portfolio journals, (https://www.springer.com/gp/authors-editors/journal-author/journal-author-helpdesk/publishing-ethics/14214) for Springer Nature journals, or (https://www.biomedcentral.com/getpublished/editorial-policies#ethics+and+consent) for BMC.

### 11 Discussion

Discussions should be brief and focused. In some disciplines use of Discussion or 'Conclusion' is interchangeable. It is not mandatory to use both. Some journals prefer a section 'Results and Discussion' followed by a section 'Conclusion'. Please refer to Journal-level guidance for any specific requirements.

## 12 Conclusion

Conclusions may be used to restate your hypothesis or research question, restate your major findings, explain the relevance and the added value of your work, highlight any limitations of your study, describe future directions for research and recommendations.

In some disciplines use of Discussion or 'Conclusion' is interchangeable. It is not mandatory to use both. Please refer to Journal-level guidance for any specific requirements.

**Supplementary information.** If your article has accompanying supplementary file/s please state so here.

Authors reporting data from electrophoretic gels and blots should supply the full unprocessed scans for key as part of their Supplementary information. This may be requested by the editorial team/s if it is missing.

Please refer to Journal-level guidance for any specific requirements.

**Acknowledgments.** Acknowledgments are not compulsory. Where included they should be brief. Grant or contribution numbers may be acknowledged.

Please refer to Journal-level guidance for any specific requirements.

## **Declarations**

Some journals require declarations to be submitted in a standardised format. Please check the Instructions for Authors of the journal to which you are submitting to see if you need to complete this section. If yes, your manuscript must contain the following sections under the heading 'Declarations':

- Funding
- Conflict of interest/Competing interests (check journal-specific guidelines for which heading to use)
- Ethics approval
- Consent to participate
- Consent for publication
- Availability of data and materials
- Code availability
- Authors' contributions

If any of the sections are not relevant to your manuscript, please include the heading and write 'Not applicable' for that section.

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Springer journals and proceedings: https://www.springer.com/gp/editorial-policies

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Scientific Reports:

https://www.nature.com/srep/journal-policies/editorial-policies

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https://www.biomedcentral.com/getpublished/editorial-policies

## Appendix A Section title of first appendix

An appendix contains supplementary information that is not an essential part of the text itself but which may be helpful in providing a more comprehensive understanding of the research problem or it is information that is too cumbersome to be included in the body of the paper.