

1. Consider  $G(E_n)$  - geometric algebra upon  $n$ -dimensional euclidean vector space.

Consider 2 linear subspaces:

columns of matrices

$$X = [\vec{x}_1 \dots \vec{x}_m]$$

and

$$Y = [\vec{y}_1 \dots \vec{y}_p]$$

and

$$\vec{x}_1 \wedge \dots \wedge \vec{x}_m \neq 0 \quad (\text{assuming linear independency of } \vec{x}_1 \dots \vec{x}_m)$$

$$\vec{y}_1 \wedge \dots \wedge \vec{y}_p \neq 0 \quad (\text{assuming linear independency of } \vec{y}_1 \dots \vec{y}_p)$$

~~Let  $p = m$  for definiteness~~

2. Consider a vector  $\vec{c} \in \text{lin}\{\vec{x}_1 \dots \vec{x}_m\}$  and

a vector  $\vec{d} \in \text{lin}\{\vec{y}_1 \dots \vec{y}_p\}$ . In terms of outer product, this can be written as:

$$\vec{c} \wedge (\vec{x}_1 \wedge \dots \wedge \vec{x}_m) = 0$$

$$\vec{d} \wedge (\vec{y}_1 \wedge \dots \wedge \vec{y}_p) = 0$$

3. For a given  $\vec{c}$ , we can find such  $\vec{d}$  that it would ~~max~~ minimize the normalized inner product:

$$\arg \max_{\substack{\vec{d} \in \text{lin}\{\vec{y}_1 \dots \vec{y}_p\} \\ \vec{d}^2 = 1}} \frac{\vec{c} \cdot \vec{d}}{\sqrt{\vec{c}^2}} = \frac{P_{\vec{y}_1 \wedge \dots \wedge \vec{y}_p}(\vec{c})}{\sqrt{(P_{\vec{y}_1 \wedge \dots \wedge \vec{y}_p}(\vec{c}))^2}}$$

where  $P_{\vec{y}_1 \wedge \dots \wedge \vec{y}_p}(\vec{c}) = (\vec{c} \lrcorner (\vec{y}_1 \wedge \dots \wedge \vec{y}_p)) \lrcorner (\vec{y}_1 \wedge \dots \wedge \vec{y}_p)^{-1}$  - orthogonal projection of  $\vec{c}$  onto  $\text{lin}\{\vec{y}_1 \dots \vec{y}_p\}$



4. Finally, the goal is to obtain  $\vec{c}$  in  $\text{lin}\{\vec{x}_1, \dots, \vec{x}_m\}$ , such that it would maximize the normalized inner product with its own orthogonal projection:

$$c_1 = \underset{\substack{\vec{c} \in \text{lin}\{\vec{x}_1, \dots, \vec{x}_m\} \\ \vec{c}^2 = 1}}{\text{argmax}} \frac{\vec{c} \cdot P_{\vec{y}_1, \dots, \vec{y}_p}(\vec{c})}{\sqrt{(P_{\vec{y}_1, \dots, \vec{y}_p}(\vec{c}))^2}}, \quad \vec{d}_1 = P_{\vec{y}_1, \dots, \vec{y}_p}(\vec{c}_1) / \sqrt{P_{\vec{y}_1, \dots, \vec{y}_p}(\vec{c}_1)^2}$$

or, alternatively,

$$d_1 = \underset{\substack{\vec{d} \in \text{lin}\{\vec{y}_1, \dots, \vec{y}_p\} \\ \vec{d}^2 = 1}}{\text{argmax}} \frac{\vec{d} \cdot P_{\vec{x}_1, \dots, \vec{x}_m}(\vec{d})}{\sqrt{(P_{\vec{x}_1, \dots, \vec{x}_m}(\vec{d}))^2}}, \quad \vec{c}_1 = P_{\vec{x}_1, \dots, \vec{x}_m}(\vec{d}_1) / \sqrt{P_{\vec{x}_1, \dots, \vec{x}_m}(\vec{d}_1)^2}$$

5. Upon obtaining a pair  $(\vec{c}_1, \vec{d}_1)$ , we can repeat steps 2-4

changing  $\vec{x}_1 \wedge \dots \wedge \vec{x}_m$  to  $\vec{c}_1 \perp (\vec{x}_1 \wedge \dots \wedge \vec{x}_m)$  - orthogonal complement of  $\vec{c}_1$  in  $\vec{x}_1 \wedge \dots \wedge \vec{x}_m$ ; and  $\vec{y}_1 \wedge \dots \wedge \vec{y}_p$  to

$$\vec{d}_1 \perp (\vec{y}_1 \wedge \dots \wedge \vec{y}_p).$$

$\vec{c}_2$  is orthogonal to  $\vec{c}_1$

As a result, we will obtain a pair  $(\vec{c}_2, \vec{d}_2)$ ;  $\vec{c}_1 \vec{c}_2 = \vec{c}_1 \wedge \vec{c}_2$ ,

$\vec{d}_1 \vec{d}_2 = \vec{d}_1 \wedge \vec{d}_2$ . Steps 2-4 then can be repeated once again,

and again, up to  $\min\{m, p\}$  times in total.

6. Statement: this process is equivalent to Canonical Correlation Analysis,

more precisely,  $\vec{c}_i = X\vec{a}_i$ ,  $\vec{d}_i = Y\vec{b}_i$  ( $i = 1 \dots \ell \leq \min\{m, p\}$ ),

where pairs  $(\vec{a}_i, \vec{b}_i)$  are obtained in CCA.