Signal Decoding in multicorrelated high-dimensional spaces

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Signal decoding problem

Goal

- Investigate input, latent, and target spaces for signal decoding problem.
- Build a stable model for time series decoding in the case of multicorrelated object description.
- Suggest dimensionality reduction algorithm for signal decoding problem.

Challenge

- Measurements of spatial-temporal data are interacted since sensors are close to each other.
- Target variable is a vector whose elements are dependent.

Solution

Propose to use joint description of independent and target variables. This description allows to reduce the multicorrelation and to build stable adequate model with acceptable accuracy.

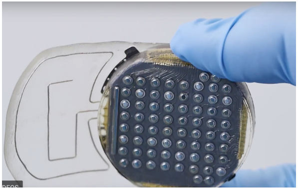
Related works

- Katrutsa A., Strijov V. Comprehensive study of feature selection methods to solve multicollinearity problem according to evaluation criteria. // *Expert Systems with Applications* 76, 2017.
- Li J. et al. Feature selection: A data perspective //*ACM Computing Surveys (CSUR)* 50(6), 2017.
- Eliseyev A. et al. Iterative N-way partial least squares for a binary self-paced brain–computer interface in freely moving animals //*Journal of neural engineering* 4(8), 2011.
- Rodriguez-Lujan I. et al. Quadratic programming feature selection // *Journal of Machine Learning Research* 11(Apr), 2010.
- Motrenko A., Strijov V. Multi-way Feature Selection for ECoG-based Brain-Computer Interface // *Expert Systems with Applications* Submitted to the journal.

Application: Brain Computer Interface (BCI)

Aim

Develop systems to help people with a severe motor control disability recover mobility.

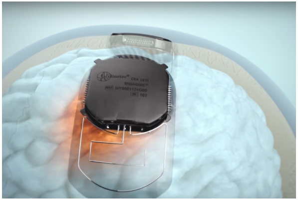


Hypothesis

"When we imagine making a movement, we trigger the same electrical activity in the motor cortex of the brain as when we actually perform that activity."*

Solution

Record electrical signals – electrocorticograms (ECoG), decode them to drive complex objects, for example, to move the limbs of an exoskeleton.



* `http://clinatec.fr`;

Eliseyev A. et al. CLINATEC BCI platform based on the ECoG-recording implant WIMAGINE and the innovative signal-processing: preclinical results, 2014.

Model

Forecast a dependent variable $\mathbf{y} \in \mathbb{R}^r$ from an independent input object $\mathbf{x} \in \mathbb{R}^n$

$$\mathbf{y} = \boldsymbol{\Theta}\mathbf{x} + \boldsymbol{\varepsilon}, \quad \boldsymbol{\Theta} \in \mathbb{R}^{r \times n}$$

Given

Dataset $(\mathbf{X}, \mathbf{Y})$, design matrix $\mathbf{X} \in \mathbb{R}^{m \times n}$, target matrix $\mathbf{Y} \in \mathbb{R}^{m \times r}$,

$$\mathbf{X} = [\mathbf{x}_1, \dots, \mathbf{x}_m]^T = [\boldsymbol{\chi}_1, \dots, \boldsymbol{\chi}_n]; \quad \mathbf{Y} = [\mathbf{y}_1, \dots, \mathbf{y}_m]^T = [\boldsymbol{\nu}_1, \dots, \boldsymbol{\nu}_r].$$

Error function

$$S(\boldsymbol{\Theta}|\mathbf{X}, \mathbf{Y}) = \left\| \underset{m \times r}{\mathbf{Y}} - \underset{m \times n}{\mathbf{X}} \cdot \underset{r \times n}{\boldsymbol{\Theta}}^T \right\|_2^2 \to \min_{\boldsymbol{\Theta}}.$$

$$\boldsymbol{\Theta} = (\mathbf{X}^T\mathbf{X})^{-1}\mathbf{X}^T\mathbf{Y}.$$

The linear dependent columns of the matrix $\mathbf{X}$ leads to an instable solution. To avoid the strong linear dependence, feature selection and dimensionality reduction techniques are used.

Feature selection problem statement

Goal

Find the index set $\mathcal{A} = \{1, \dots, n\}$ of $\mathbf{X}$ columns.

Quality Criteria

To select the set $\mathcal{A}$ among all possible $2^n - 1$ subsets, introduce the feature selection quality criteria

$$\mathcal{A} = \mathop{\arg\max}_{\mathcal{A}' \subseteq \{1,\dots,n\}} Q(\mathcal{A}'|\mathbf{X}, \mathbf{Y}).$$

Once the solution $\mathcal{A}$ is known:

$$S(\mathbf{\Theta}_\mathcal{A}|\mathbf{X}_\mathcal{A}, \mathbf{Y}) = \left\|\mathbf{Y} - \mathbf{X}_\mathcal{A}\mathbf{\Theta}_\mathcal{A}^T\right\|_2^2 \to \min_{\mathbf{\Theta}_\mathcal{A}},$$

where the subscript $\mathcal{A}$ indicates columns with indices from the set $\mathcal{A}$.

$$\|\boldsymbol{\nu} - \mathbf{X}\boldsymbol{\theta}\|_2^2 \to \min_{\boldsymbol{\theta} \in \mathbb{R}^n}.$$

Quadratic Programming Feature Selection

$$(1-\alpha) \cdot \underbrace{\mathbf{a}^T\mathbf{Q}\mathbf{a}}_{\mathrm{Sim}(\mathbf{X})} - \alpha \cdot \underbrace{\mathbf{b}^T\mathbf{a}}_{\mathrm{Rel}(\mathbf{X}, \boldsymbol{\nu})} \to \min_{\substack{\mathbf{a} \geq \mathbf{0}_n \\ \mathbf{1}_n^T\mathbf{a}=1}}.$$

- $\mathbf{a} \in \mathbb{R}^n$ — feature importances;
- $\mathbf{Q} \in \mathbb{R}^{n \times n}$ - pairwise feature similarities;
- $\mathbf{b} \in \mathbb{R}^n$ - feature relevances to the target vector.

$$j \in \mathcal{A}^* \Leftrightarrow a_j > \tau$$

Quality Criteria

$$\mathcal{A} = \operatorname*{arg\,max}_{\mathcal{A}' \subseteq \{1, \dots, n\}} Q(\mathcal{A}' | \mathbf{X}, \boldsymbol{\nu}) \Leftrightarrow \operatorname*{arg\,min}_{\mathbf{a} \geq \mathbf{0}_n, \, \mathbf{1}_n^T\mathbf{a}=1} \left[\mathbf{a}^T\mathbf{Q}\mathbf{a} - \alpha \cdot \mathbf{b}^T\mathbf{a}\right].$$

Quadratic Programming Feature Selection

Quality Criteria

$$\mathcal{A} = \underset{\mathcal{A}' \subseteq \{1,\dots,n\}}{\arg\max}\, Q(\mathcal{A}' | \mathbf{X}, \boldsymbol{\nu}) \Leftrightarrow \underset{\mathbf{a} \geq \mathbf{0}_n,\, \mathbf{1}_n^T \mathbf{a} = 1}{\arg\min} \left[\mathbf{a}^T \mathbf{Q} \mathbf{a} - \alpha \cdot \mathbf{b}^T \mathbf{a} \right].$$

Similarity measure

- Correlation

$$|\mathrm{corr}(\mathbf{x}, \mathbf{y})| = \left| \frac{\mathrm{cov}(\mathbf{x}, \mathbf{y})}{\sqrt{\mathrm{Var}(\mathbf{x}) \mathrm{Var}(\mathbf{y})}} \right|$$

- Mutual information

$$I(\mathbf{x}, \mathbf{y}) = \int \int p(\mathbf{x}, \mathbf{y}) \log \frac{p(\mathbf{x}, \mathbf{y})}{p(\mathbf{x}) p(\mathbf{y})} d\mathbf{x} d\mathbf{y}.$$

$$\mathbf{Q} = \left\{ \left| \mathrm{corr}(\boldsymbol{\chi}_i, \boldsymbol{\chi}_j) \right| \right\}_{i,j=1}^{n}, \quad \mathbf{b} = \left\{ \left| \mathrm{corr}(\boldsymbol{\chi}_i, \boldsymbol{\nu}) \right| \right\}_{i=1}^{n}.$$

Statement

In the case of semidefinite matrix $\mathbf{Q}$ the QPFS problem is convex.

$$\mathbf{Q} \to \mathbf{Q} - \lambda_{\min} \mathbf{I}$$

Relevance aggregation

$$\mathbf{Q} = \left\{ \left| \text{corr}(\boldsymbol{\chi}_i, \boldsymbol{\chi}_j) \right| \right\}_{i,j=1}^{n}, \quad \mathbf{b} = \left\{ \sum_{k=1}^{r} \left| \text{corr}(\boldsymbol{\chi}_i, \boldsymbol{\nu}_k) \right| \right\}_{i=1}^{n}.$$

This approach does not use the dependencies in the columns of the matrix $\mathbf{Y}$.

Example:

$$\mathbf{X} = [\boldsymbol{\chi}_1, \boldsymbol{\chi}_2, \boldsymbol{\chi}_3], \quad \mathbf{Y} = [\underbrace{\boldsymbol{\nu}_1, \boldsymbol{\nu}_1, \dots, \boldsymbol{\nu}_1}_{r-1}, \boldsymbol{\nu}_2],$$

$$\mathbf{Q} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0.8 \\ 0 & 0.8 & 1 \end{bmatrix}, \quad \mathbf{B} = \begin{bmatrix} \underbrace{\begin{matrix} 0.4 & \dots & 0.4 \\ 0.5 & \dots & 0.5 \\ 0.8 & \dots & 0.8 \end{matrix}}_{r-1} & \begin{matrix} 0 \\ 0.8 \\ 0.1 \end{matrix} \end{bmatrix}, \quad \mathbf{b} = \begin{bmatrix} (r-1) \cdot 0.4 + 0 \\ (r-1) \cdot 0.5 + 0.8 \\ (r-1) \cdot 0.8 + 0.1 \end{bmatrix}$$

Best subset: $[\boldsymbol{\chi}_1, \boldsymbol{\chi}_2]$.

QPFS ($r = 2$): $\mathbf{a} = [\mathbf{0.37}, \mathbf{0.61}, 0.02]$.
QPFS ($r = 5$): $\mathbf{a} = [\mathbf{0.40}, 0.17, \mathbf{0.43}]$.

Symmetric Importance

Penalize correlated targets by $\text{Sim}(\mathbf{Y})$

$$\alpha_1 \cdot \underbrace{\mathbf{a}_x^{\mathsf{T}} \mathbf{Q}_x \mathbf{a}_x}_{\text{Sim}(\mathbf{X})} - \alpha_2 \cdot \underbrace{\mathbf{a}_x^{\mathsf{T}} \mathbf{B} \mathbf{a}_y}_{\text{Rel}(\mathbf{X},\mathbf{Y})} + \alpha_3 \cdot \underbrace{\mathbf{a}_y^{\mathsf{T}} \mathbf{Q}_y \mathbf{a}_y}_{\text{Sim}(\mathbf{Y})} \to \min_{\substack{\mathbf{a}_x \geq \mathbf{0}_n,\, \mathbf{1}_n^{\mathsf{T}} \mathbf{a}_x = 1 \\ \mathbf{a}_y \geq \mathbf{0}_r,\, \mathbf{1}_r^{\mathsf{T}} \mathbf{a}_y = 1}}.$$

$$\mathbf{Q}_x = \left\{\left|\text{corr}(\boldsymbol{\chi}_i, \boldsymbol{\chi}_j)\right|\right\}_{i,j=1}^{n}, \ \mathbf{Q}_y = \left\{\left|\text{corr}(\boldsymbol{\nu}_i, \boldsymbol{\nu}_j)\right|\right\}_{i,j=1}^{r}, \ \mathbf{B} = \left\{\left|\text{corr}(\boldsymbol{\chi}_i, \boldsymbol{\nu}_j)\right|\right\}_{\substack{i=1,\ldots,n \\ j=1,\ldots,r}}.$$

$$\alpha_1 + \alpha_2 + \alpha_3 = 1 \quad \alpha_i \geq 0,\ i = 1, 2, 3.$$

Statement

Balance between the terms $\textit{Sim}(\mathbf{X})$ *and* $\textit{Rel}(\mathbf{X}, \mathbf{Y})$ *for the problem is achieved by the following coefficients:*

$$\alpha_1 = \frac{(1 - \alpha_3)\overline{\mathbf{B}}}{\overline{\mathbf{Q}}_x + \overline{\mathbf{B}}}; \quad \alpha_2 = \frac{(1 - \alpha_3)\overline{\mathbf{Q}}_x}{\overline{\mathbf{Q}}_x + \overline{\mathbf{B}}}; \quad \alpha_3 \in [0, 1],$$

where $\overline{\mathbf{Q}}_x$, $\overline{\mathbf{B}}$ *are the mean values of* $\mathbf{Q}_x$ *and* $\mathbf{B}$ *respectively.*

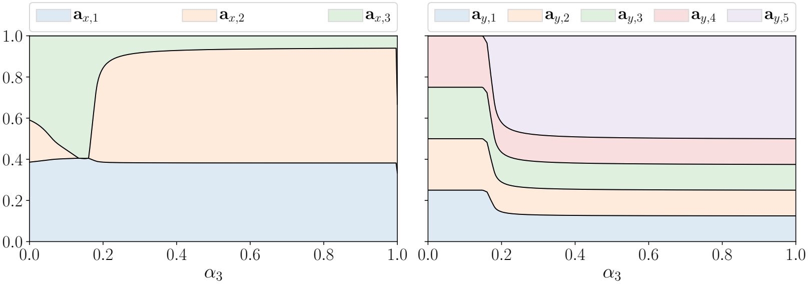
Example:

$$\mathbf{X} = [\chi_1, \chi_2, \chi_3], \quad \mathbf{Y} = [\underbrace{\boldsymbol{\nu}_1, \boldsymbol{\nu}_1, \dots, \boldsymbol{\nu}_1}_{r-1}, \boldsymbol{\nu}_2],$$

$$\mathbf{Q} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0.8 \\ 0 & 0.8 & 1 \end{bmatrix}, \quad \mathbf{B} = \begin{bmatrix} \underbrace{\begin{matrix} 0.4 & \dots & 0.4 \\ 0.5 & \dots & 0.5 \\ 0.8 & \dots & 0.8 \end{matrix}}_{r-1} & \begin{matrix} 0 \\ 0.8 \\ 0.1 \end{matrix} \end{bmatrix}, \quad \mathbf{b} = \begin{bmatrix} (r-1) \cdot 0.4 + 0 \\ (r-1) \cdot 0.5 + 0.8 \\ (r-1) \cdot 0.8 + 0.1 \end{bmatrix}$$

$$\mathbf{Q}_x = \mathbf{Q}; \ \mathbf{Q}_y : \text{corr}(\boldsymbol{\nu}_1, \boldsymbol{\nu}_2) = 0.2 \text{ all others entries } = 1.$$

Best subset: $[\chi_1, \chi_2]$.



Min-max / Max-min

Symmetric Importances penalizes targets that are correlated and are not sufficiently explained by the features.

$$\alpha_1 \cdot \underbrace{\mathbf{a}_x^T \mathbf{Q}_x \mathbf{a}_x}_{\mathrm{Sim}(\mathbf{X})} - \alpha_2 \cdot \underbrace{\mathbf{a}_x^T \mathbf{B} \mathbf{a}_y}_{\mathrm{Rel}(\mathbf{X},\mathbf{Y})} \to \min_{\substack{\mathbf{a}_x \geq \mathbf{0}_n, \\ \mathbf{1}_n^T \mathbf{a}_x = 1}} ; \quad \alpha_3 \cdot \underbrace{\mathbf{a}_y^T \mathbf{Q}_y \mathbf{a}_y}_{\mathrm{Sim}(\mathbf{Y})} + \alpha_2 \cdot \underbrace{\mathbf{a}_x^T \mathbf{B} \mathbf{a}_y}_{\mathrm{Rel}(\mathbf{X},\mathbf{Y})} \to \min_{\substack{\mathbf{a}_y \geq \mathbf{0}_r, \\ \mathbf{1}_r^T \mathbf{a}_y = 1}} .$$

Min-max / Max-min

$$\min_{\substack{\mathbf{a}_x \geq \mathbf{0}_n \\ \mathbf{1}_n^T \mathbf{a}_x = 1}} \max_{\substack{\mathbf{a}_y \geq \mathbf{0}_r \\ \mathbf{1}_r^T \mathbf{a}_y = 1}} \left(\text{or} \max_{\substack{\mathbf{a}_y \geq \mathbf{0}_r \\ \mathbf{1}_r^T \mathbf{a}_y = 1}} \min_{\substack{\mathbf{a}_x \geq \mathbf{0}_n \\ \mathbf{1}_n^T \mathbf{a}_x = 1}} \right) \left[\alpha_1 \cdot \underbrace{\mathbf{a}_x^T \mathbf{Q}_x \mathbf{a}_x}_{\mathrm{Sim}(\mathbf{X})} - \alpha_2 \cdot \underbrace{\mathbf{a}_x^T \mathbf{B} \mathbf{a}_y}_{\mathrm{Rel}(\mathbf{X},\mathbf{Y})} - \alpha_3 \cdot \underbrace{\mathbf{a}_y^T \mathbf{Q}_y \mathbf{a}_y}_{\mathrm{Sim}(\mathbf{Y})} \right] .$$

$$\mathcal{A} = \arg\max_{\mathcal{A}' \subseteq \{1,\dots,n\}} Q(\mathcal{A}' | \mathbf{X}, \mathbf{Y}) \Leftrightarrow \arg\min_{\mathbf{a}_x \geq \mathbf{0}_n,\, \mathbf{1}_n^T \mathbf{a}_x = 1} \left[\max_{\substack{\mathbf{a}_y \geq \mathbf{0}_r \\ \mathbf{1}_r^T \mathbf{a}_y = 1}} f(\mathbf{a}_x, \mathbf{a}_y) \right] .$$

Theorem

For positive definite matrices $\mathbf{Q}_x$ *and* $\mathbf{Q}_y$ *the max-min and min-max problems have the same optimal value.*

Lagrangian for fixed $\mathbf{a}_x$

$$L(\mathbf{a}_x, \mathbf{a}_y, \lambda, \boldsymbol{\mu}) = \alpha_1 \cdot \mathbf{a}_x^T \mathbf{Q}_x \mathbf{a}_x - \alpha_2 \cdot \mathbf{a}_x^T \mathbf{B} \mathbf{a}_y - \alpha_3 \cdot \mathbf{a}_y^T \mathbf{Q}_y \mathbf{a}_y + \lambda \cdot (\mathbf{1}_r^T \mathbf{a}_y - 1) + \boldsymbol{\mu}^T \mathbf{a}_y.$$

Theorem

Min-max problem is equivalent to the following quadratic problem with $n + r + 1$ variables

$$\min_{\substack{\mathbf{a}_x \geq \mathbf{0}_n,\, \mathbf{1}_n^T \mathbf{a}_x = 1 \\ \lambda,\, \boldsymbol{\mu} \geq \mathbf{0}_r}} g(\mathbf{a}_y, \lambda, \boldsymbol{\mu}), \quad \text{where}$$

$$\begin{aligned} g(\mathbf{a}_x, \lambda, \boldsymbol{\mu}) = \max_{\mathbf{a}_y \in \mathbb{R}^r} L(\mathbf{a}_x, \mathbf{a}_y, \lambda, \boldsymbol{\mu}) &= \mathbf{a}_x^T \left(-\frac{\alpha_2^2}{4\alpha_3} \mathbf{B} \mathbf{Q}_y^{-1} \mathbf{B}^T - \alpha_1 \cdot \mathbf{Q}_x \right) \mathbf{a}_x \\ - \frac{1}{4\alpha_3} \lambda^2 \cdot \mathbf{1}_r^T \mathbf{Q}_y^{-1} \mathbf{1}_r &- \frac{1}{4\alpha_3} \boldsymbol{\mu}^T \mathbf{Q}_y^{-1} \boldsymbol{\mu} + \frac{\alpha_2}{2\alpha_3} \lambda \cdot \mathbf{1}_r^T \mathbf{Q}_y^{-1} \mathbf{B}^T \mathbf{a}_x \\ &- \frac{1}{2\alpha_3} \lambda \cdot \mathbf{1}_r^T \mathbf{Q}_y^{-1} \boldsymbol{\mu} + \frac{\alpha_2}{2\alpha_3} \boldsymbol{\mu}^T \mathbf{Q}_y^{-1} \mathbf{B}^T \mathbf{a}_x + \lambda. \end{aligned}$$

The problem is not convex. If we shift the spectrum for the matrix of quadratic form, the optimality is lost.

Drop the term Sim($\mathbf{Y}$).

$$\min_{\substack{\mathbf{a}_x \geq \mathbf{0}_n \\ \mathbf{1}_n^T \mathbf{a}_x = 1}} \max_{\substack{\mathbf{a}_y \geq \mathbf{0}_r \\ \mathbf{1}_r^T \mathbf{a}_y = 1}} \left[(1-\alpha) \cdot \mathbf{a}_x^T \mathbf{Q}_x \mathbf{a}_x - \alpha \cdot \mathbf{a}_x^T \mathbf{B} \mathbf{a}_y \right].$$

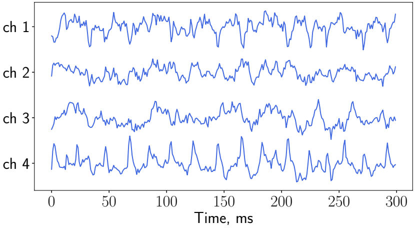
Dual function

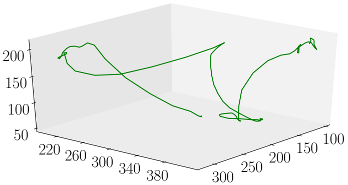
$$g(\mathbf{a}_x, \lambda, \boldsymbol{\mu}) = \begin{cases} (1-\alpha) \cdot \mathbf{a}_x^T \mathbf{Q}_x \mathbf{a}_x - \lambda, & \alpha \cdot \mathbf{B}^T \mathbf{a}_x = \lambda \cdot \mathbf{1}_r + \boldsymbol{\mu}; \\ +\infty, & \text{otherwise.} \end{cases}$$

Statement

For the case $r = 1$ the proposed strategies coincide with the original QPFS algorithm.

Data example

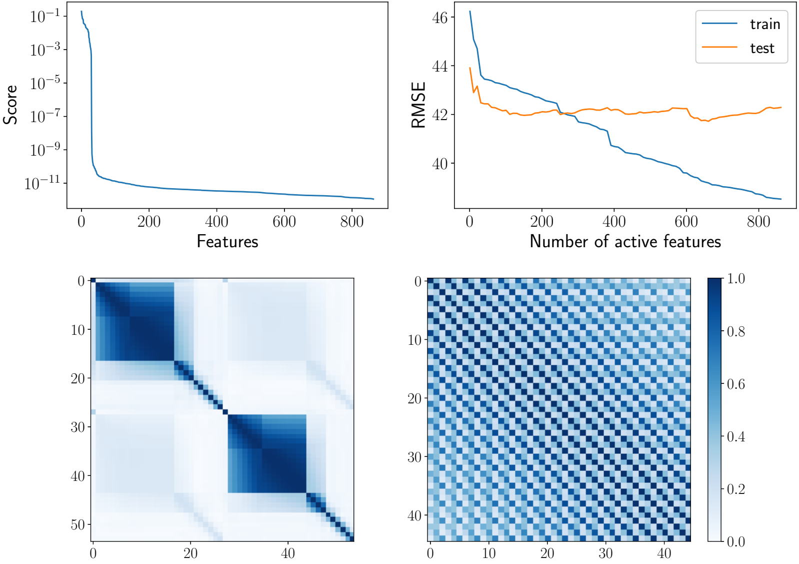




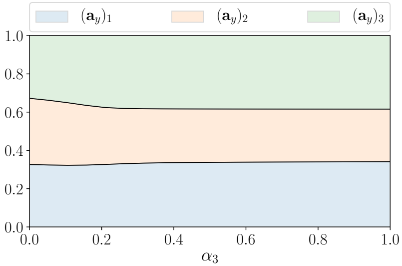
$$\mathbf{X} \in \mathbb{R}^{m\times(32\cdot 27)}; \quad \mathbf{Y} = \begin{pmatrix} x_1 \; y_1 \; z_1 & \dots & x_k & y_k & z_k \\ x_2 \; y_2 \; z_2 & \dots & x_{k+1} & y_{k+1} & z_{k+1} \\ \dots & \dots & & \dots & \\ x_m \; y_m \; z_m & \dots & x_{m+k} & y_{m+k} & z_{m+k} \end{pmatrix} \in \mathbb{R}^{m\times 3k}.$$

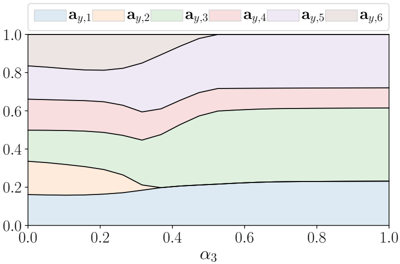
http://neurotycho.org

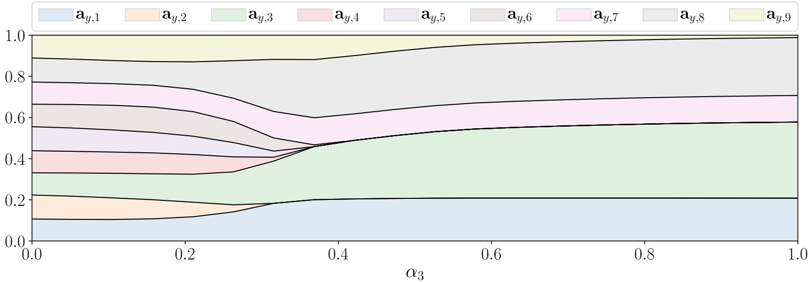
Data redundancy



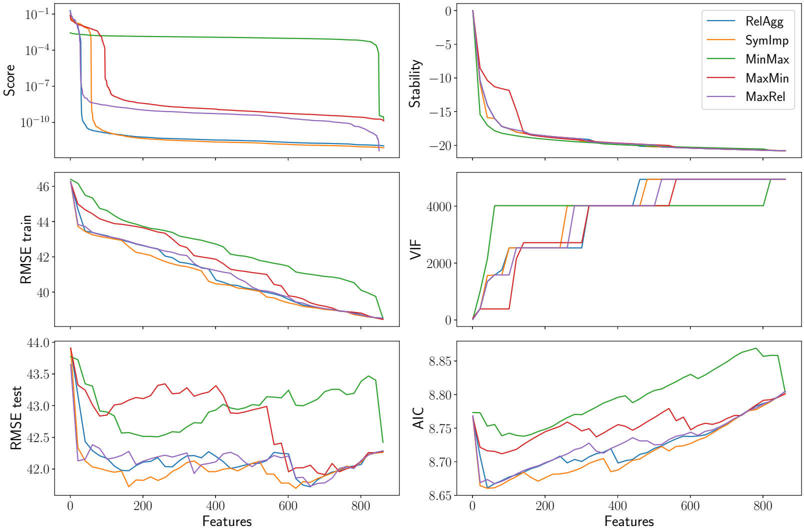
Target importances



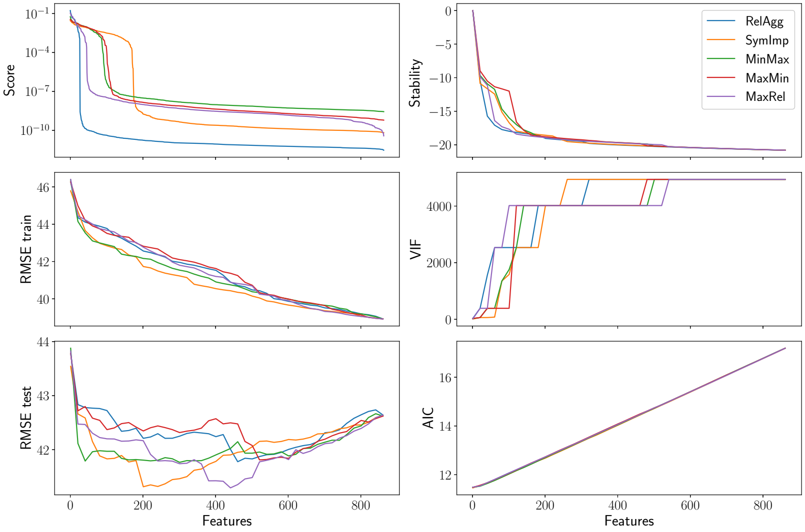




Autoregression step $= 1$



Autoregression step = 30



Conclusion

- BCI signal decoding problem is investigated.
- Feature selection algorithms for spatio-temporal data are proposed.
- Suggested algorithms are explored and compared.

Publications

- Isachenko R. et al. Feature Generation for Physical Activity Classification. *Artificial Intellegence and decision making*, submitted.
- Isachenko R., Strijov V. Quadratic programming optimization for Newton method. *Lobachevskii Journal of Mathematics*, submitted.
- Isachenko R., Vladimirova M., Strijov V. Dimensionality reduction for time series decoding and forecasting problems. Ready for submission.

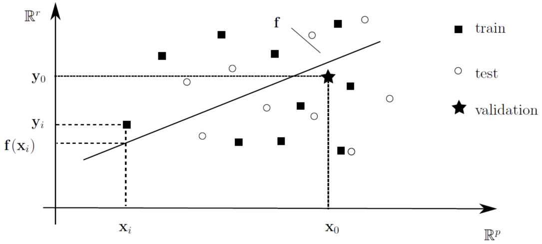
Feature categorization

1. non-relevant features
$$\{j : \text{corr}(\boldsymbol{\chi}_j, \boldsymbol{\nu}_k) = 0, \ \forall k \in \{1, \dots, r\}\};$$
2. non-**X**-correlated features, which are relevant to non-**Y**-correlated targets
$$\{j : \big(\text{VIF}(\boldsymbol{\chi}_j) < 10\big) \text{ and } \big(\text{VIF}(\boldsymbol{\nu}_k) < 10, \ \forall k \in \{1, \dots, r\} : \text{corr}(\boldsymbol{\chi}_j, \boldsymbol{\nu}_k) \neq 0\big)\};$$
3. non-**X**-correlated features, which are relevant to **Y**-correlated targets
$$\{j : \big(\text{VIF}(\boldsymbol{\chi}_j) < 10\big) \text{ and } \big(\exists k \in \{1, \dots, r\} : \text{VIF}(\boldsymbol{\nu}_k) > 10 \ \& \ \text{corr}(\boldsymbol{\chi}_j, \boldsymbol{\nu}_k) \neq 0\big)\};$$
4. **X**-correlated features, which are relevant to non-**Y**-correlated targets
$$\{j : \big(\text{VIF}(\boldsymbol{\chi}_j) > 10\big) \text{ and } \big(\text{VIF}(\boldsymbol{\nu}_k) < 10, \ \forall k \in \{1, \dots, r\} : \text{corr}(\boldsymbol{\chi}_j, \boldsymbol{\nu}_k) \neq 0\big)\};$$
5. **X**-correlated features, which are relevant to **Y**-correlated targets
$$\{j : \big(\text{VIF}(\boldsymbol{\chi}_j) > 10\big) \text{ and } \big(\exists k \in \{1, \dots, r\} : \text{VIF}(\boldsymbol{\nu}_k) > 10 \ \& \ \text{corr}(\boldsymbol{\chi}_j, \boldsymbol{\nu}_k) \neq 0\big)\}.$$

$$r = \text{corr}(\boldsymbol{\chi}, \boldsymbol{\nu}), \quad t = \frac{r\sqrt{m-2}}{1-r^2} \sim \text{St}(m-2).$$
$$\text{VIF}(\boldsymbol{\chi}_j) = \frac{1}{1-R_j^2}, \quad \text{VIF}(\boldsymbol{\nu}_k) = \frac{1}{1-R_k^2},$$

where $R_j^2(R_k^2)$ are coefficients of determination for the regression of $\boldsymbol{\chi}_j(\boldsymbol{\nu}_k)$ on the other features(targets).

Problem Statement



$$\begin{tikzcd} \mathbf{X} \in \mathbb{X} \arrow[r] \arrow[d, leftrightarrow] & \mathbf{Y} \in \mathbb{Y} \arrow[d, leftrightarrow] \\ \mathbf{T} \in \mathbb{Z} \arrow[r, leftrightarrow] & \mathbf{U} \in \mathbb{Z} \end{tikzcd}$$

Partial Least Squares (PLS)

$$\underset{m\times n}{\mathbf{X}} = \underset{m\times l}{\mathbf{T}} \cdot \underset{l\times n}{\mathbf{P}^T} + \underset{m\times n}{\mathbf{F}} = \sum_{k=1}^{l} \underset{m\times 1}{\mathbf{t}_k} \cdot \underset{1\times n}{\mathbf{p}_k^T} + \underset{m\times n}{\mathbf{F}}$$

$$\underset{m\times r}{\mathbf{Y}} = \underset{m\times l}{\mathbf{U}} \cdot \underset{l\times r}{\mathbf{Q}^T} + \underset{m\times r}{\mathbf{E}} = \sum_{k=1}^{l} \underset{m\times 1}{\mathbf{t}_k} \cdot \underset{1\times r}{\mathbf{q}_k^T} + \underset{m\times r}{\mathbf{E}}$$

$$\hat{\mathbf{Y}} = \mathbf{T}\mathrm{diag}(\beta)\mathbf{Q}^T = \mathbf{X}\mathbf{\Theta}.$$

- map $\mathbf{X}$ into low-dimensional $\mathbf{T}$;
- map $\mathbf{Y}$ into low-dimensional $\mathbf{U}$;
- maximize correlation between $\mathbf{t}_k$ and $\mathbf{u}_k$.

PLS Example

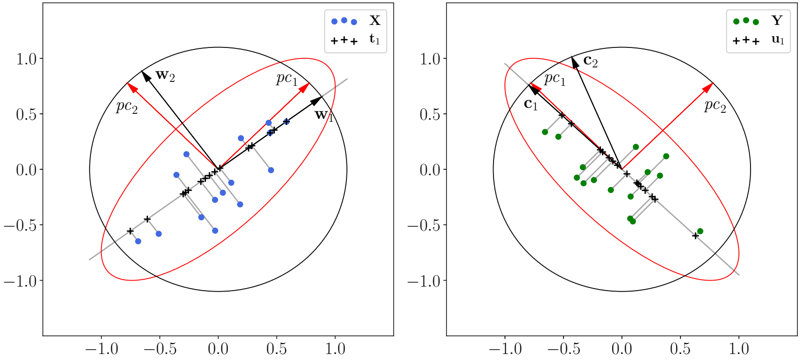


Figure: The result of the PLS algorithm for the case $n = r = l = 2$.

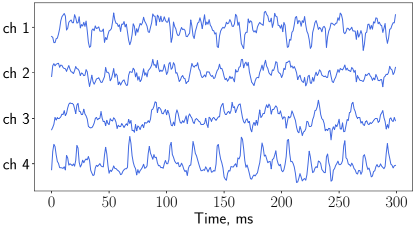
Datasets

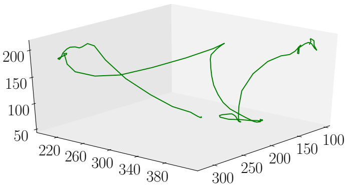
- energy consumption
- electrocorticogram signals (ECoG)

Autoregressive approach

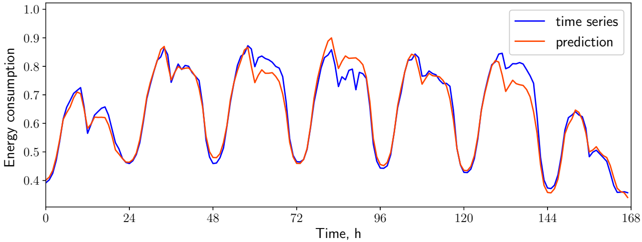
$$\mathbf{X} = \begin{pmatrix} x_1 & x_2 & \dots & x_n \\ x_2 & x_3 & \dots & x_{n+1} \\ \dots & \dots & \dots & \dots \\ x_{T-n+1} & x_{T-n+2} & \dots & x_T \end{pmatrix}$$

ECoG data





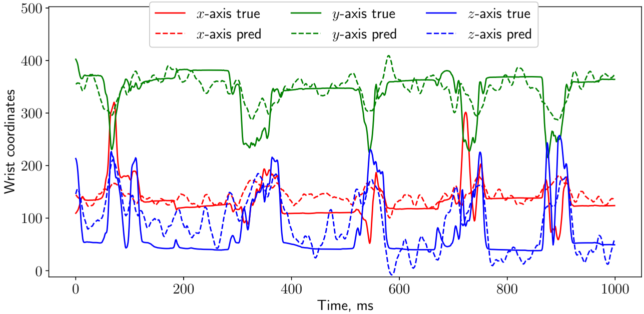
Energy consumption



Results

- Space dimensionalities: $\mathbf{X} = 700 \times (24 \cdot 7)$, $\mathbf{Y} = 700 \times 24$.
- Dimensionality of latent space: 14
- NMSE: 0.047

ECoG



Results

- Space dimensionalities: $\mathbf{X} = 13000 \times (864 \cdot 18)$, $\mathbf{Y} = 13000 \times 3$.
- Dimensionality of latent space: 16
- NMSE: 0.731

Quadratic Programming Model Selection

QPFS

- works for linear problems;
- does not take into account the model;
- ignores the structure of the target space.

Problem

$$\underbrace{(1-\alpha)\mathbf{z}^T\mathbf{Q}\mathbf{z}}_{\text{Sim}} - \underbrace{\alpha\mathbf{b}^T\mathbf{z}}_{\text{Rel}} \to \min_{\substack{\mathbf{z}\in\mathbb{R}^p_+ \\ \|\mathbf{z}\|_1=1}} .$$

- $\mathbf{z} \in \mathbb{R}^p$ — weight importances;
- $\mathbf{Q} \in \mathbb{R}^{p\times p}$ - pairwise weights interactions;
- $\mathbf{b} \in \mathbb{R}^p$ - weight relevances to the target vector.

$$w_j = 0 \Leftrightarrow z_j < \tau.$$