# How to speed up MHEs - project ideas

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This document aims to present the general MHE problem and to describe the main issues regarding the computational burden required by the algorithm. Furthermore, the document proposes some ideas to speed up the MHE implementation. These ideas come from the main topics discussed during the May 2022 EECI Spring School *Sparsity and Big Data in Control, System Identification and Machine Learning*, held in Toulouse. For a more detailed discussion of the topics reported in this document, please refer to *project_discussion.pdf*.

**Problem Statement** The problem tackled by the *Moving Horizon Estimators* (MHEs) is the design of robust observers, intending to reconstruct the plant states that are not directly measurable by sensors. Roughly speaking MHEs handles the estimation process by minimising an optimisation problem, defined as a model matching problem on the system state and exploiting a buffer of output measurements [2, 3, 6–8, 10]. Due to its optimisation-based approach, the main drawback of the MHE is the computational cost. Indeed, the more complex the model and the longer the measurement buffer, the slower the optimisation process. In the following paragraph some ideas on how to speed up the MHE solution are outlined. The potential approaches to tackle the computational burden problem of MHE are the following:

1. **Reduce the buffer length $N$:** by doing so, less model integrations are required (see [4, 5]).
2. **Reduce the model complexity:** this approach aims to simplify the model equations by finding an equivalent or approximated form.
3. **Speeding up the optimisation algorithm:** this approach aims to improve the efficiency of the chosen optimisation algorithm. This could be done by exploiting some structure (e.g. sparsity) in the problem definition.

**Proposed solutions** This paragraph presents three ideas to speed up the solution of the MHE problem, focusing on bullets 2 and 3 above.

- **Convex Envelope:** the MHE problem is highly non convex, due to the need to forward integrate model dynamics to define the optimisation problem behind its structure. A first approach one could take is trying to relax the MHE problem by considering a convex approximation of the cost function by exploiting its convex envelope. The main issue with this approach is that there is no guarantee that the unique minimum of the convex envelope coincides with the global minimum of the initial cost function. The project could study the performance of this solution and potential assumptions behind it.

- **Sparsity in Hessian:** if a gradient-like optimisation algorithm is used, an explicit structure of the Gradient and/or the Hessian of the cost function could speed-up the process. This solution aims to exploit some structure in the Hessian (e.g. Cholesky Factorization) to actually estimate it (see course notes). The project could try to exploit sparsity to improve a built-in optimisation algorithm or develop a custom one.

- **Koompan operator:** As reported above the MHE could be improved by reducing the complexity of the model dynamics. This could be achieved through the Koopman operator [1, 9]. Roughly speaking, the Koopman operator performs a lifting of the system dynamics from the state space to the observable state. In the latter, the model dynamics result linear. The forward integration would benefit in terms of speed by using the linear dynamics obtained through the Koopman operator. The project could consider a toy model, approximate it through the Koopman operator theory, and then run the MHE using the new model.

## References


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