Optimisation based observer and system identifier

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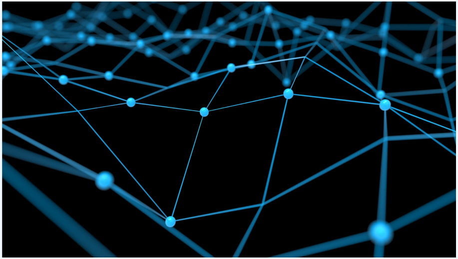

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Adaptive observer design to estimate state and parameters of a nonlinear dynamical system. The main characteristics are the following:

- optimisation based approach
- offline analysis

Inspired by [1], this approach could be used to set initial estimates on local observers (KF) or to identify models.

General model:

- State vector: x
- Measures: y
- Input: u
- parameters: θ

$$\delta x = f(x, \theta, u)$$
$$y = h(x, \theta)$$

Starting from measurements y, the observer aims to both estimate the state x and a set of parameters θ either of the model $f(\cdot)$ or the output mapping $h(\cdot)$.

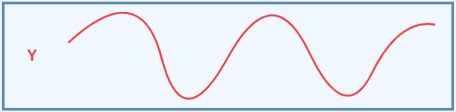

Figure: measured signal Y

Algorithm rationale:

1. Down sampling: N_w
2. Window size: N_{T_s}

e.g. $(N_w = 6,\ N_{T_s} = 4)$

The measurements Y are assumed to be collected every N_{Ts} sampling times and stored in a buffer of dimension N_w.

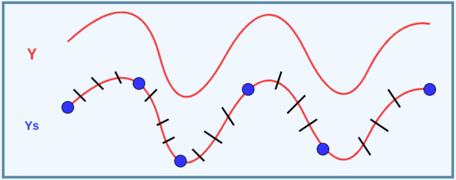

Figure: down-sampled signal Y_s

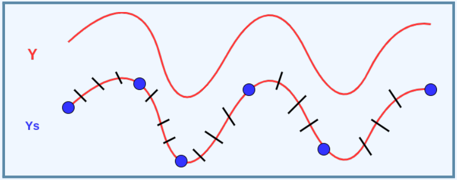

Figure: examples of estimated real signals

Algorithm rationale:

1. Down sampling: N_w
2. Window size: N_{T_s}
3. Optimisation: find best $\xi = [\hat{x}, \hat{\theta}]$ minimising a cost function J evaluated on the current data window

$$e_i = Y_i - H_i(\hat{\xi}_1) = \begin{bmatrix} y_i^1 \\ \vdots \\ y_i^p \end{bmatrix} - \begin{bmatrix} h_1(\hat{\xi}_i) \\ \vdots \\ h_p(\hat{\xi}_i) \end{bmatrix} \in \mathbb{R}^p$$

$$Y_1 = Y(t_k - N_w \cdot N_{T_s})$$

$$\mathbf{E} = [e_1, e_2, \dots, e_{N_w}] \in \mathbb{R}^{p \times N_w}$$

$$J = J(\mathbf{E})$$

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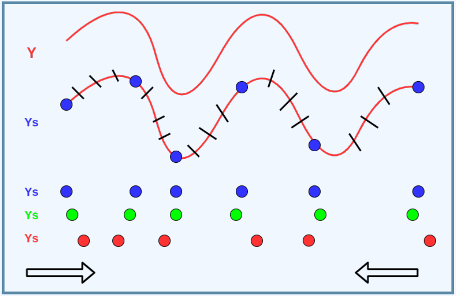

Figure: this approach works both forward and backward in time

Augmented state:

$$\hat{\xi} = \begin{bmatrix} \hat{x}_1 \\ \vdots \\ \hat{x}_n \\ \hat{\theta}_1 \\ \vdots \\ \hat{\theta}_m \end{bmatrix} \implies \min_{\hat{\xi}} J(E)$$

Optimisation run on $\hat{\xi} \implies$ state and parameters estimation. The minimisation algorithm considers the augmented state as the argument.

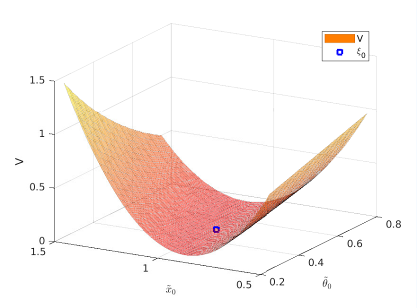

Figure: This figure shows the cost function V($\hat{\xi}$)

Mockup system:

$$\dot{\xi}_1(t) = -\xi_2(t)\xi_1(t), \tag{1a}$$
$$\dot{\xi}_2(t) = 0, \tag{1b}$$
$$y(t) = \xi_1(t). \tag{1c}$$

Evaluation set:

$$\Omega \subset \mathbb{R}^n \text{ and } \xi_0 = [1, 0.5]. \tag{2}$$

Region of Attraction of an equilibrium point ξ_0:

$$\text{RoA}(\xi) \subset \Omega \text{ s.t. } \lim_{t\to\infty} \xi(t) = \xi_0. \tag{3}$$

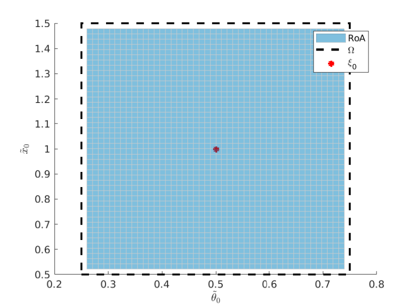

Figure: This figure shows the RoA(ξ) for 6a on Ω. The point ξ_0 is highlighted in red.

Mockup system:

$$\dot{\xi}_1(t) = -|\xi_2(t)|\xi_1(t), \quad (1a)$$
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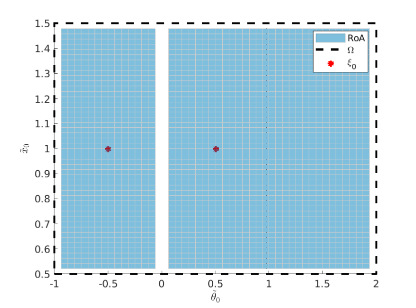

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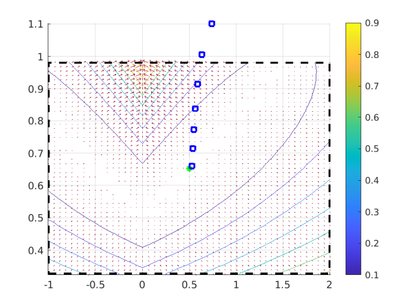

Figure: Contour plot estimation starting from $\hat{\xi}_0 = (1.1, 0.7)$. Optimisations are the blue squares while the green one is ξ_0.

Mockup system:

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$$\dot{\xi}_2(t) = 0, \tag{1b}$$
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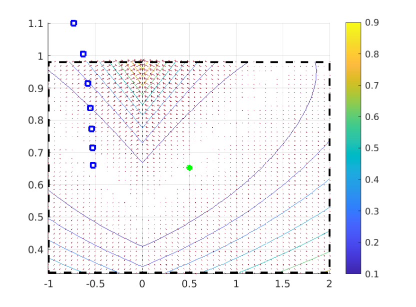

Figure: Same as before but with $\hat{\xi}_0 = (1.1, -0.7)$.

This section goes through some preliminary results on the state and parameters estimation capabilities of the proposed algorithm. The paragraph unfolds as follows:

- Cost function discussion
- Down sampling

Each optimisation step is assumed to have a maximum number of iteration allowed of $Max_{iter} = 100$ and a down-sampling of $(N_w = 5, N_{T_s} = 3)$.

Cost function:

$$J(\mathbf{E}) = \sum_{i=1}^{N_w} \left[e_i^T W_1(i) e_i \right] + (\theta_0 - \theta)^T W_2 (\theta_0 - \theta), \tag{4}$$

where $W_1, W_2 \in \mathbb{R}^{n+m}$ are weight matrices and θ_0 is the initial estimate before the optimisation process.

Down sampling selection:

1. Model: double pendulum
2. Estimation: state + friction coefficients
3. $N_w = 5$ and $T_s = 0.05$s

Examples:

```
simulations/lecture/pendulum_friction_correct_sampling_0503
simulations/lecture/pendulum_friction_wrong_sampling_0503
```

W3 term in J:

1. Model: double pendulum
2. Estimation: state + friction coefficients
3. $N_w = 5$ and $T_s = 0.05$s

Examples:

```
simulations/lecture/pendulum_friction_correct_sampling_0503
simulations/lecture/pendulum_friction_spring_term_0503
```

This section describes some improvements of the algorithm, with the main goal to speed up the estimation process.

Next covered topics:

- Filters on cost function: how act on J in order to both speed up the estimation and increase the system observability.
- Adaptive sampling: how to speed up the process by implementing self tuning capabilities

How to speed up the computational time?

- error definition: $e_i = Y_i - H_i(\hat{\xi}_1)$
- N_w tuning: from [1] a good choice is $N_w \geq 2n + 1$ with $n = \dim \xi$
- proposed improvement: add filtered errors in the cost function J. By adding information, N_w decreases, i.e.

$$J = J(\mathbf{E}, \dot{\mathbf{E}}, \int \mathbf{E}). \tag{5}$$

By reducing N_w state propagation will take lesser time, speeding up the process.

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- N_w tuning: from [1] a good choice is $N_w \geq 2n + 1$ with $n = \dim \xi$
- proposed improvement: add filtered errors in the cost function J. By adding information, N_w decreases, i.e.

$$J(\mathbf{E}) = \sum_{i=1}^{N_w} \left[e_i^T W_1(i) e_i + \dot{e}_i^T W_2(i) \dot{e}_i + \int e_i^T W_3(i) e_i \right] + (\theta_0 - \theta)^T W_5 (\theta_0 - \theta), \quad (5)$$

Mockup system:

$$\dot{\xi}_1(t) = -\xi_2(t)\xi_1(t), \tag{6a}$$
$$\dot{\xi}_2(t) = 0, \tag{6b}$$
$$y(t) = \xi_1(t). \tag{6c}$$

Cost function :

$$\mathcal{V}(\tilde{\xi}) : \mathbb{R}^n \implies \mathbb{R} \tag{7}$$

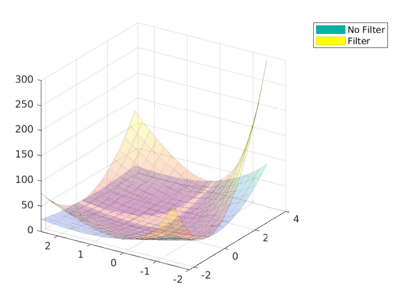

Figure: Comparison between cost function with different down-sampling, namely with $N = 3$ and $N = 3$ with filter.

Mockup system:

$$\dot{\xi}_1(t) = -\xi_2(t)\xi_1(t), \tag{6a}$$
$$\dot{\xi}_2(t) = 0, \tag{6b}$$
$$y(t) = \xi_1(t). \tag{6c}$$

Convex region:

$$\mathcal{H} \subset \mathrm{RoA}(\xi_0) \tag{7}$$

in which $\triangle f(\xi)|_{\xi^*} > 0$

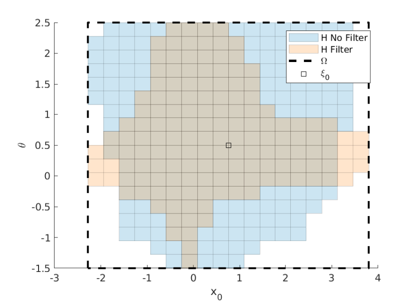

Figure: Comparison between $\mathcal{H}$ with different down-sampling, namely with $N = 3$ with the derivative filter and $N = 3$. The point ξ_0 is highlighted as a black square.

Refine - Filters action on J

- $N_w = 5$,no filtering: 29s
- $N_w = 3$,yes filtering: 65s

Examples:

```
simulations/lecture/pendulum_friction_correct_sampling_0503
simulations/lecture/pendulum_friction_derivative_term_0303
```

The sampling rate should be chosen depending on the signal richness. Consider the time evolution of the estimation error:

$$e_i = Y_i - H_i(\hat{\xi}_1) \in \mathbb{R}^p, i \in \{1, \ldots, N_w\} \tag{6}$$

The goodness of the estimation is gauged through the index ν, namely:

$$\nu = \sum_{i=1}^{N_w} ||e_i|| \tag{7}$$

Refining - adaptive sampling

The richer the signal, the higher ν gets, as smaller model differences are enhanced during the dynamics integration. Therefore the estimation process can be triggered at runtime thresholding ν:

Policy (Adaptive sampling)

```
if (nu < nu_bar)
    measure();
else
    measure();
    estimate();
end
```

- $N_w = 5$, yes filtering, no adaptive: 29s
- $N_w = 3$, yes filtering, yes adaptive: 24.6s

Examples:

```
simulations/lecture/pendulum_friction_derivative_term_0303
simulations/lecture/pendulum_friction_derivative_term_0303_ad
```

Refining - recap parameters

Convergence:

- RoA
- $\mathcal{H}$

Optimisation parameters:

- J filters
- $\mathtt{Max}_{\mathtt{iter}}$
- Adaptive ON/OFF

Sampling parameters:

- N_w
- N_{T_s}

Thanks for the attention