Total No. of Printed Pages:3

T.E(Information Technology) Semester V (Revised Course 2019-20)
EXAMINATION FEBRUARY 2022
Graph Theory

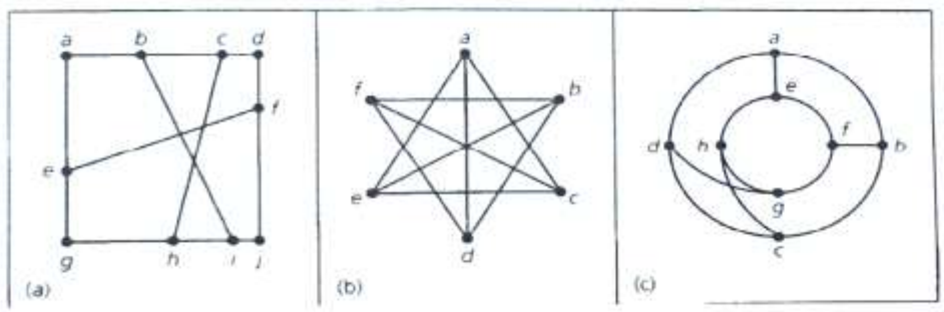
[Duration : Three Hours] **[Total Marks : 100]**

Instructions:-
1) Answer any **5 questions** by selecting **2 questions** from **Part A and 2 questions** from **Part B** and **1 question** from **Part C.**
2) Make necessary assumptions wherever required.
3) Draw neatly labeled diagram wherever necessary.

Part - A

Q.1 a) Define bipartite graph and complete bipartite graph with example. **5**

b) Identify which of the graphs in the figure are planar. If graph is planar, redraw it with no edges overlapping. **6**



c) State and prove the handshaking theorem. A non-directed graph G has 8 edges. Find the number of vertices, if the degree of each vertex in G is 2. **5**

d) Show that graph $K_{3,3}$ and K_5 is non planar and also state Kuratowski's theorem. **4**

Q.2 a) Define n-regular graph. Show for which value of n the following graphs are regular: (i) Kn (ii) Qn **4**

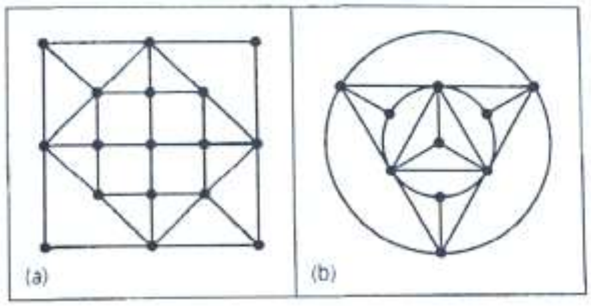
b) Define adjacency matrix representation of the graph. Draw a graph having following matrix as it's adjacency matrix. **6**

$$\begin{matrix} 0 & 1 & 2 & 2 \\ 1 & 0 & 3 & 2 \\ 2 & 3 & 0 & 1 \\ 2 & 2 & 1 & 1 \end{matrix}$$

c) State and prove Euler's Polyhedra formula. **4**

d) Define the following with an example:(i) Simple graph (ii) Finite and infinite graph (iii) Isolated vertex (iv) Subgraph **6**

Q.3 a) List out from the below figure, the number of vertices, number of edges and number of regions for each of the planar graph. Compare your answer to show that it satisfies Euler's theorem for planar graph. **6**



b) Define the concept of a complete graph. Draw complete graph each for the case when number of vertices is given by: n=3, n=4. **4**

c) Define cut-point and bridge with an example. **4**

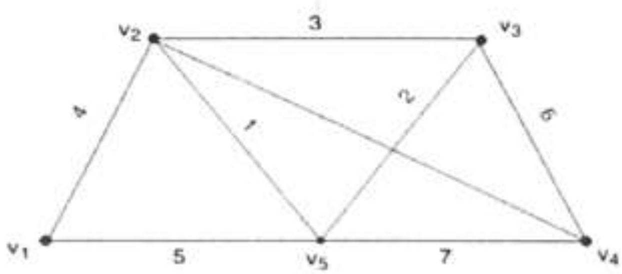
d) Define isomorphism of graphs. Determine whether the graphs are isomorphic. **6**

Part - B

Q.4 a) Discuss about some type of digraphs with suitable example. **6**

b) Explain max-flow min-cut theorem. **6**

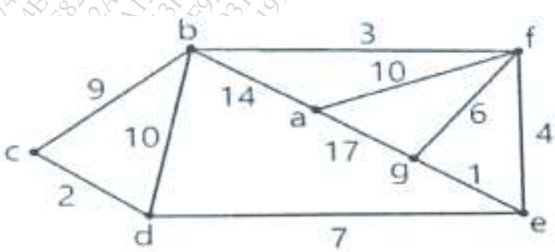
c) Using Prim's algorithm, find the minimum spanning tree for the weighted graph shown Below. **8**



Q.5 a) Show that every complete tournament has a directed Hamiltonian path. **6**

b) Define adjacency matrix of a digraph. List down the properties of the adjacency matrix X of a digraph G. **6**

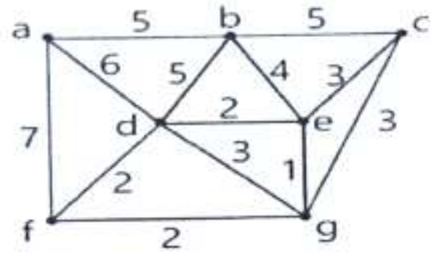
c) Using Kruskal's algorithm, find a minimal spanning tree for the weighted graph shown below. **8**



Q.6 a) Define Incidence matrix of a digraph. Show that the determinant of every square submatrix of A, the incidence matrix of a digraph, is 1,-1, or 0. **6**

b) Define chromatic number. Explain four color problem. **6**

c) Apply Dijkstra's algorithm to the following weighted graph shown below and determine the shortest distance from vertex 'a' to each of the other six vertices in the graph. **8**



Part - C

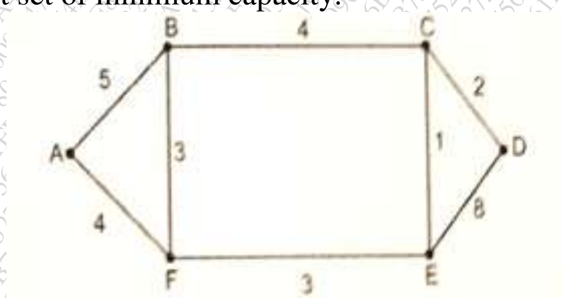
Q.7 a) Define a tree. Prove that tree with n vertices has n-1 edges. **6**

b) Define i) Spanning tree ii) Rooted tree iii) Full binary tree. Give example of each. **6**

c) Define dual of a planar graph. Draw the geometric dual of the given graph. **8**

Q.8 a) Discuss Konisberg bridge problem and the solution of the problem. **6**

b) For the network shown below, determine the maximum flow between the vertex A and D by identifying cut set of minimum capacity. **8**



c) Define connected graph. Give example of connected graph G where removing any edge of G results in a disconnected graph. **6**