List of open problems related to Working Group 2 – ML for CT

December 2025

T2.1 Addressing the curse of dimensionality with ML tools

1. **Learn set-valued maps related to control problems with machine learning tools**

   - *Contact:* Francisco Periago. Email: f.periago@upct.es
   - *Required skills:* Good command of Python and a basic knowledge of control theory

2. **Regularity theory for PDEs in high dimensions**

   - *Contact:* Francisco Periago. Email: f.periago@upct.es
   - *Required skills:* Good command of functional analysis and PDEs

T2.2 Solving parameterised optimal control problems

1. **Nonlinear and transport-dominated problems**

   - *Goal:* Solve optimal control problems where the governing dynamics are parametric and nonlinear or transport-dominated, for instance

   $$\frac{\partial y}{\partial t} + \mu \frac{\partial y^2}{\partial x} = u.$$

   - *Contact:* Martin Lazar. Email: mlazar@unidu.hr
   - *Required skills:* Good command in numerics of PDEs and in control theory
   - *Some details/related questions:*
     - Nonlinear problems pose difficulties for traditional, linear approximation schemes.
     - General question for model order reduction.
     - What is possible in the context of (optimal) control of such systems?
     - Where can machine learning help to overcome these issues (nonlinear strategies such as autoencoders, etc.)?

2. **Control of flow problems such as the Navier-Stokes equations**

   - *Goal:* Solve optimal control problems for flows governed by parametric Navier-Stokes equations, for instance

   $$\frac{\partial y}{\partial t} - \mu \Delta y + (y \cdot \nabla)y + \nabla p = u,$$
   $$\operatorname{div} y = 0.$$

   - *Contact:* Maria Strazzullo. Email: maria.strazzullo@polito.it
   - *Required skills:* Good command in numerics of PDEs and in control theory
   - *Some details/related questions:*
     - Navier-Stokes equations are an important model for (viscous) flow in real-world applications.

– Depending on the Reynolds number (roughly the parameter μ in the formulation above), the solution behavior can change completely.
– Can machine learning help in order to deal with the turbulent regime?

3. **General convex objective functionals**

- *Goal:* Induce sparsity in the control by solving a problem of the form

$$u_\mu^* = \arg\min_{u \in G} \|u\|_{L^1([0,T];U)} + \frac{\alpha}{2}\|u\|_{L^2([0,T];U)}^2 + h(x_\mu)$$

- *Contact:* Cesare Molinari. Email: cesare.molinari@edu.unige.it
- *Required skills:* Good command in (convex) optimization; Python programming
- *Some details/related questions:*
  - Which algorithm is suited best to solve this OCP?
  - How to deal with the parameter dependence?
  - Can reduced order modeling be applied here in a suitable manner?
  - If so, how to combine it in a reasonable way with machine learning?

4. **Optimization in the parameter space**

- *Goal:* Solve problems of the form

$$\mu^* = \arg\min_{\mu \in \mathcal{P}} \ F(\mu; u_\mu)$$

  where $u_\mu \in G$ solves an optimal control problem for the parameter $\mu \in \mathcal{P}$.
- *Contact:* Hendrik Kleikamp. Email: hendrik.kleikamp@uni-graz.at
- *Required skills:* Knowledge in optimization and control theory; Python programming
- *Some details/related questions:*
  - Optimal control problem for a fixed parameter as an "inner" problem.
  - Derivatives with respect to the parameter are typically required.
  - Optimizer usually moves outside ot the range of training data points.
    $\longrightarrow$ How to extrapolate properly?

5. **Small data regime**

- *Goal:* How to deal with relatively small amount of available data?
- *Contact:* Hendrik Kleikamp. Email: hendrik.kleikamp@uni-graz.at
- *Required skills:* Good command of machine learning and control theory
- *Some details/related questions:*
  - Training data (at least using the FOM) is costly to obtain.
  - Which quantities are easiest to learn when only a small amount of data is available?
    * Optimal control $\longrightarrow$ How to obtain performance guarantees?
    * Reduced quantities $\longrightarrow$ Combination with MOR techniques often allows to collect more training data and to make use of their error estimates.
    * Open loop vs. closed loop systems $\longrightarrow$ Feedback control requires different architectures and learning techniques.

6. **Applications in uncertainty quantification**

- *Goal:* Make use of the derived surrogates in *multilevel Monte Carlo methods*:

$$\mathbb{E}[M_L] = \mathbb{E}[M_0] + \sum_{\ell=0}^{L} \mathbb{E}[M_\ell - M_{\ell-1}].$$

- *Contact:* Hendrik Kleikamp. Email: hendrik.kleikamp@uni-graz.at
- *Required skills:* Machine learning and surrogate modeling; a bit of probability theory and statistics; Python programming
- *Some details/related questions:*
  - Consider different applications in which we want efficient estimates of unknown quantities.
  - Interactions of the different models?
  - Strategies to select the models and the number of evaluations on different levels?
  - Can we derive probabilistic guarantees that this works?

T2.3 Construction of control Lyapunov functions using ML methods

T2.4 Developing ML-based approaches for the life-cycle-optimisation in materials

1. **Control of spatially weighted wave equations**

   - *Contact:* Peter Kogut. Email: p.kogut@i.ua
   - *Required skills:* Good command of control theory and PDEs

2. **Properties of weighted Sobolev spaces**

   - *Contact:* Peter Kogut. Email: p.kogut@i.ua
   - *Required skills:* Good command of functional analysis and PDEs

T2.5 Exploiting PINNs for solving complex free boundary problems