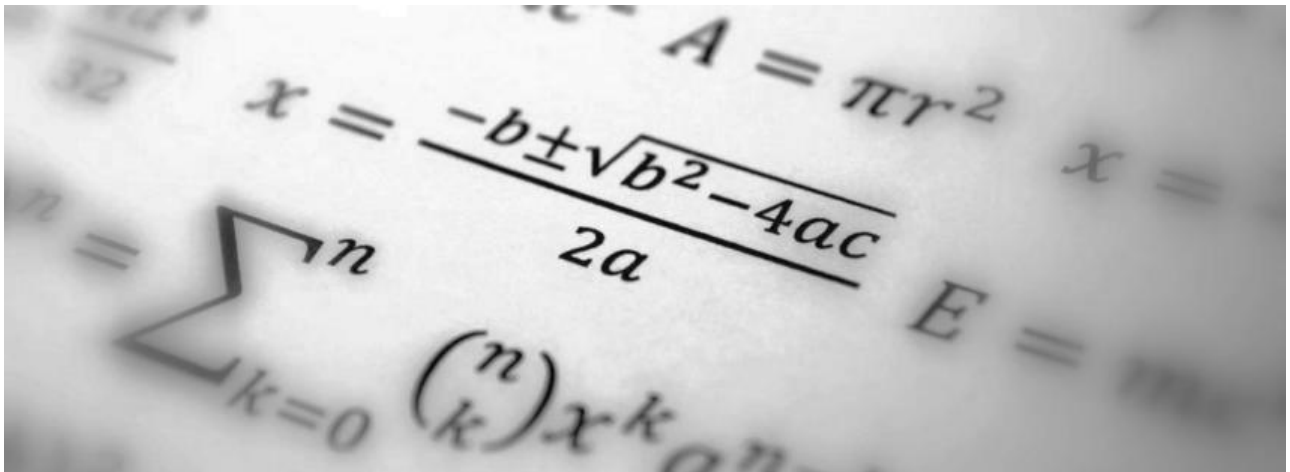




Hills Road  
Sixth Form College  
Cambridge

# A Level Maths Summer Work



Hello!

We're looking forward to welcoming you to the Maths department in September and we hope you're looking forward to joining us too! In order to make your transition to A Level Maths as smooth as possible, it's important that those key algebraic skills you learnt at GCSE are as fresh as possible. To help you keep them sharp, we've put together some questions on key areas of algebra for you to complete over the holidays. You should write up your solutions (showing all your working) on separate lined paper and bring them to your first lesson in September.

There are 3 pages of questions, which will may take around 4 hours to complete so don't feel the need to do it all in one go. If you're a bit rusty, you can find worked examples after the questions that may help you to complete them, but you may not need to look at these at all!

See you in September!

If you're unsure how to do some of the questions, check the corresponding worked examples at the end for help. Although you will have access to a calculator in all of the A-Level papers, you should be able to complete these without one!

## Indices and Surds

1 Evaluate each of these without using a calculator.

a  $49^{\frac{1}{2}}$

b  $27^{\frac{1}{3}}$

c  $5^{-1}$

d  $64^{-\frac{1}{3}}$

e  $9^{\frac{3}{2}}$

f  $16^{\frac{3}{4}}$

g  $125^{\frac{2}{3}}$

h  $\left(\frac{1}{2}\right)^3$

i  $\left(\frac{1}{9}\right)^{-2}$

j  $\left(\frac{4}{9}\right)^{\frac{1}{2}}$

k  $\left(\frac{9}{16}\right)^{-0.5}$

l  $\left(\frac{27}{8}\right)^{\frac{2}{3}}$

2 Simplify these expressions fully without using a calculator.

a  $\sqrt{8}$

b  $\sqrt{75}$

c  $2\sqrt{24}$

d  $3\sqrt{48}$

e  $\sqrt{20} + \sqrt{5}$

f  $\sqrt{27} - \sqrt{12}$

g  $5\sqrt{32} - 3\sqrt{8}$

h  $\sqrt{50} + 3\sqrt{125}$

i  $\sqrt{68} + 3\sqrt{17}$

j  $3\sqrt{72} - \sqrt{32}$

k  $4\sqrt{18} - 2\sqrt{3}$

l  $6\sqrt{5} + \sqrt{50}$

3 Write each of these expressions in simplified index form.

a  $x^3 \times x^7$

b  $7x^5 \times 3x^6$

c  $5x^4 \times 8x^7$

d  $x^8 \div x^2$

e  $8x^7 \div 2x^9$

f  $3x^8 \div 12x^7$

g  $(x^5)^7$

h  $(x^2)^{-5}$

i  $(3x^2)^4$

j  $(6x^5)^2$

k  $\sqrt{x^3}$

l  $\sqrt[4]{x^5}$

m  $\frac{5\sqrt{x}}{x}$

n  $2x\sqrt{x}$

o  $\frac{x^2}{3\sqrt{x}}$

p  $x^3(x^5 - 1)$

q  $x^3(\sqrt{x} + 2)$

r  $\frac{x+2}{x^3}$

s  $\frac{\sqrt{x}+3}{x}$

t  $\frac{(3-x^3)}{\sqrt{x}}$

u  $(\sqrt{x}+3)^2$

v  $\frac{3+\sqrt{x}}{x^2}$

w  $\frac{1-x}{2\sqrt{x}}$

x  $\frac{\sqrt{x}+2}{3x^3}$

## Solving & Rearranging Equations/Inequalities

1 Solve each of these linear equations.

a  $3(2x+9)=7$

b  $7-3x=12$

c  $\frac{x+4}{5}=7$

d  $2x+7=5x-6$

e  $8x-3=2(3x+1)$

f  $\frac{2x+9}{12}=x-1$

g  $2(3x-7)=4x$

h  $7-2x=3(4-5x)$

2 Solve each of these linear inequalities.

a  $\frac{x}{2}+7 \geq 5$

b  $3-4x < 15$

c  $5(x-1) > 12+x$

d  $\frac{x+1}{3} > 2$

e  $8x-1 \leq 2x-5$

f  $3(x+1) \geq \frac{x-3}{2}$

g  $3(2x-5) < 1-x$

h  $x-(3+2x) \geq 2(x+1)$

3 Rearrange each of these formulae to make  $x$  the subject.

a  $2x+5=3A-1$

b  $x+u=vx+3$

c  $\frac{3x-1}{k}=2x$

d  $5(x-3m)=2nx-4$

e  $(1-3x)^2=t$

f  $\frac{1}{x}=\frac{1}{p}+\frac{1}{q}$

g  $\frac{1}{x^2+k}-6=4$

h  $\sqrt{x+A}=2B$

## Algebraic fractions

1 Simplify these fractions.

$$\text{a } \frac{x(x-5)(x+2)}{x^3(x+2)} \quad \text{b } \frac{(x+3)^2}{x(x+3)} \quad \text{c } \frac{(x-4)}{2x(x-4)} \quad \text{d } \frac{x^2(x+5)}{x(x+5)^2}$$

2 Simplify these fractions by first factorising the numerator and the denominator.

$$\begin{array}{lllll} \text{a } \frac{x^2-2x-8}{x^2+4x+4} & \text{b } \frac{x^2-10x+21}{x^2-x-6} & \text{c } \frac{x^2-3x-10}{x^2-10x+25} & \text{d } \frac{x^2+10x+24}{2x+8} & \text{e } \frac{x^2+6x}{x^2-36} \\ \text{f } \frac{3x^2+6x}{x^2-5x-14} & \text{g } \frac{5x^3+15x^2}{x^2+6x+9} & \text{h } \frac{x^2-64}{3x^2-24x} & \text{i } \frac{25-x^2}{45-4x-x^2} & \text{j } \frac{2x^2-x-28}{2x^3+7x^2} \end{array}$$

## Lines

1 Find the gradient of the line through each pair of points.

$$\begin{array}{lll} \text{a } (3, 7) \text{ and } (2, 8) & \text{b } (5, 2) \text{ and } (-4, -6) & \text{c } (1.3, 4.7) \text{ and } (2.6, -3.1) \\ \text{d } \left(\frac{1}{2}, \frac{1}{3}\right) \text{ and } \left(\frac{3}{4}, \frac{2}{3}\right) & \text{e } (\sqrt{3}, 2) \text{ and } (2\sqrt{3}, 5) & \text{f } (3a, a) \text{ and } (a, 5a) \end{array}$$

2 Calculate the exact distance between each pair of points.

$$\begin{array}{lll} \text{a } (8, 4) \text{ and } (1, 3) & \text{b } (-3, 9) \text{ and } (12, -7) & \text{c } (5.9, 6.2) \text{ and } (-8.1, 3.8) \\ \text{d } \left(\frac{1}{5}, -\frac{1}{5}\right) \text{ and } \left(\frac{3}{5}, -\frac{4}{5}\right) & \text{e } (5, -3\sqrt{2}) \text{ and } (2, \sqrt{2}) & \text{f } (k, -3k) \text{ and } (2k, -6k) \end{array}$$

3 Find the coordinates of the midpoint of each pair of points.

$$\begin{array}{lll} \text{a } (3, 9) \text{ and } (1, 7) & \text{b } (2, -4) \text{ and } (-3, -9) & \text{c } (2.1, 3.5) \text{ and } (6.3, -3.7) \\ \text{d } \left(\frac{2}{3}, -\frac{1}{2}\right) \text{ and } \left(-\frac{5}{3}, -\frac{3}{2}\right) & \text{e } (6\sqrt{5}, 2\sqrt{5}) \text{ and } (-\sqrt{5}, \sqrt{5}) & \text{f } (m, 2n) \text{ and } (3m, -2n) \end{array}$$

4 Find the equation of the line through each pair of points.

$$\begin{array}{lll} \text{a } (2, 5) \text{ and } (0, 6) & \text{b } (1, -3) \text{ and } (2, -5) & \text{c } (4, 4) \text{ and } (7, -7) \\ \text{d } (8, -2) \text{ and } (4, -3) & \text{e } (-3, -7) \text{ and } (5, 9) & \text{f } (\sqrt{2}, -\sqrt{2}) \text{ and } (3\sqrt{2}, 4\sqrt{2}) \end{array}$$

5 Use algebra to find the point of intersection between each pair of lines.

$$\begin{array}{lll} \text{a } y=8-3x, y=2-5x & \text{b } y=7x-4, y=3x-2 & \text{c } y=2x+3, y=5-x \\ \text{d } y+5=3x, y=-5x+7 & \text{e } y=\frac{1}{2}x+3, y=5-2x & \text{f } y=3(x+2), y=7-2x \end{array}$$

6 The line  $l_1$  has equation  $2x+6y=5$ . The line  $l_2$  is parallel to  $l_1$  and passes through the point  $(1, -5)$ . Find the equation of  $l_2$  in the form  $ax+by+c=0$  where  $a, b$  and  $c$  are integers.

7 Decide whether or not each line is parallel or perpendicular to the line  $y=4x-1$

$$\text{a } 2x+8y=5 \quad \text{b } 20x+5y=2 \quad \text{c } 16x-4y=5$$

Decide whether or not each line is parallel or perpendicular to the line  $y=4-3x$

$$\text{a } 3x+6y=2 \quad \text{b } 5x-15y=7 \quad \text{c } 18x+6y+5=0$$

8 The line  $l_1$  has equation  $4x+6y=3$ . A second line,  $l_2$  is perpendicular to  $l_1$  and passes through the point  $(-1, 5)$ . Find the equation of  $l_2$  in the form  $ax+by+c=0$  where  $a, b$  and  $c$  are integers.

## Quadratics

1 Fully factorise each of these quadratics.

a  $3x^2+5x$

b  $8x^2-4x$

c  $17x^2+34x$

d  $18x^2-24x$

6 Use factorisation to find the roots of these quadratic equations.

a  $21x^2-7x=0$

b  $x^2-36=0$

c  $17x^2+34x=0$

d  $6x^2+13x+5=0$

e  $4x^2-49=0$

f  $x^2=7x+18$

g  $x^2-7x+6=0$

h  $21x^2=2-x$

7 Sketch each of these quadratic functions, labelling where they cross the x and y axes.

a  $y=x(x-3)$

b  $y=-x(3x+2)$

c  $y=x(3-x)$

d  $y=(x+2)(x-2)$

e  $y=(x+4)^2$

f  $y=15x-10x^2$

g  $y=49-x^2$

h  $y=-x^2+2x+3$

i  $y=x^2-4x+4$

j  $y=-x^2+14x-49$

k  $y=3x^2+4x+1$

l  $y=-2x^2+11x-12$

## Completing the Square

1 Write each of these quadratic expressions in the form  $p(x+q)^2+r$

a  $x^2+8x$

b  $x^2-18x$

c  $x^2+6x+3$

d  $x^2+12x-5$

e  $x^2-7x+10$

f  $x^2+5x+9$

g  $2x^2+8x+4$

h  $3x^2+18x-6$

i  $2x^2-10x+3$

j  $-x^2+12x-1$

k  $-x^2+9x-3$

l  $-2x^2+5x-1$

2 Use completing the square to find the turning point of each of these curves and state whether it is a maximum or a minimum.

a  $y=x^2+14x$

b  $y=x^2-18x+3$

c  $y=x^2-9x$

d  $y=-x^2+4x$

e  $y=x^2+11x+30$

f  $y=-x^2+6x-7$

g  $y=2x^2+16x-5$

h  $y=-3x^2+15x-2$

## Key Points & Examples to jog your memory if needed...

### Indices & Surds

$$x^a \times x^b = x^{a+b} \quad x^a \div x^b = x^{a-b} \quad (x^a)^b = x^{ab}$$

**Key point**

The  $n$ th root of  $x$  is written  $\sqrt[n]{x} = x^{\frac{1}{n}}$ , and this can be raised to a power to give  $\sqrt[n]{x^m} = x^{\frac{m}{n}}$

**Key point**

A power of  $-1$  indicates a reciprocal, so  $x^{-1} = \frac{1}{x}$  and, in general,  $x^{-n} = \frac{1}{x^n}$

**Key point**

#### Example 1

Simplify these expressions.

**a**  $2x^3 \times 3x^5$

**b**  $12x^7 \div 4x^3$

**c**  $(3x^5)^3$

**a**  $2x^3 \times 3x^5 = 6x^{3+5}$

$= 6x^8$

**b**  $12x^7 \div 4x^3 = \frac{12x^7}{4x^3}$

$= 3x$

**c**  $(3x^5)^3 = 3^3(x^5)^3$

$= 27x^{15}$

Since  $(x^a)^b = x^{ab}$

Multiply the coefficients together and use  $x^a \times x^b = x^{a+b}$

Since  $\frac{12}{4} = 3$  and  $x^a \div x^b = x^{a-b}$  so  $\frac{x^7}{x^3} = x^4$  which we just write as  $x$

Both the 3 and the  $x^5$  must be raised to the power 3

#### Example 2

Write these expressions in simplified index form.

**a**  $\sqrt[3]{x}$

**b**  $\frac{2}{x^3}$

**c**  $\frac{2x}{\sqrt{x}}$

**a**  $\sqrt[3]{x} = x^{\frac{1}{3}}$

**b**  $\frac{2}{x^3} = 2x^{-3}$

**c**  $\frac{2x}{\sqrt{x}} = \frac{2x}{x^{\frac{1}{2}}}$

$= 2x^{1-\frac{1}{2}}$

$= 2x^{\frac{1}{2}}$

Since  $\sqrt{x} = x^{\frac{1}{2}}$

Subtract the powers, remembering that  $x = x^1$



### Example 3

Simplify these expressions without using a calculator.

**a**  $\sqrt{18} + 5\sqrt{2}$

**b**  $\frac{6}{\sqrt{3}}$

**c**  $\frac{2}{1-\sqrt{5}}$

**a**  $\sqrt{18} = \sqrt{9 \times 2}$   
 $= 3\sqrt{2}$

Therefore  $\sqrt{18} + 5\sqrt{2} = 3\sqrt{2} + 5\sqrt{2}$   
 $= 8\sqrt{2}$

**b**  $\frac{6}{\sqrt{3}} = \frac{6\sqrt{3}}{\sqrt{3}\sqrt{3}}$   
 $= \frac{6\sqrt{3}}{3}$   
 $= 2\sqrt{3}$

**c**  $\frac{2}{1-\sqrt{5}} = \frac{2(1+\sqrt{5})}{(1-\sqrt{5})(1+\sqrt{5})}$   
 $= \frac{2(1+\sqrt{5})}{-4}$   
 $= -\frac{1}{2}(1+\sqrt{5})$

9 is a square-number factor of 18 so you can simplify  $\sqrt{18}$

Collect like terms.

Rationalise the denominator by multiplying numerator and denominator by  $\sqrt{3}$

Since  $6 \div 3 = 2$

Rationalise the denominator by multiplying numerator and denominator by  $1 + \sqrt{5}$

$(1-\sqrt{5})(1+\sqrt{5}) = 1 - \sqrt{5} + \sqrt{5} - 5$   
 $= 1 - 5 = -4$

## Solving & Rearranging Equations/Inequalities

### Example 1

Solve the inequality  $5(x-2) \leq 2x+1$

$5x-10 \leq 2x+1$

$3x-10 \leq 1$

$3x \leq 11$

$x \leq \frac{11}{3}$

First expand the brackets.

Subtract  $2x$  from both sides.

Add 10 to both sides.

Divide both sides by 3

### Example 2

Rearrange  $Ax-3 = \frac{x+B}{2}$  to make  $x$  the subject.

$2Ax-6 = x+B$

$2Ax-6-x = B$

$2Ax-x = B+6$

$x(2A-1) = B+6$

$x = \frac{B+6}{2A-1}$

Divide both sides by  $(2A-1)$  to make  $x$  the subject.

Multiply both sides by 2

Subtract  $x$  from both sides.

Add 6 to both sides.

Factorise the side involving  $x$

## Algebraic Fractions

### Example

Simplify the fractions **a**  $\frac{x^2+5x+6}{2x^2+6x}$

**b**  $\frac{2x^2-3x-9}{4x^2-9}$

**a**  $x^2+5x+6=(x+2)(x+3)$

$2x^2+6x=2x(x+3)$

So  $\frac{x^2+5x+6}{2x^2+6x} = \frac{(x+2)(x+3)}{2x(x+3)} = \frac{x+2}{2x}$

**b**  $2x^2-3x-9=2x^2-6x+3x-9$

$=2x(x-3)+3(x-3)$

$= (2x+3)(x-3)$

$4x^2-9=(2x+3)(2x-3)$

So  $\frac{2x^2-3x-9}{4x^2-9} = \frac{(2x+3)(x-3)}{(2x+3)(2x-3)} = \frac{x-3}{2x-3}$

Factorise numerator and denominator.

Cancel the common factor of  $x+3$  from numerator and denominator.

Write  $-3x$  as  $-6x+3x$  since  $3 \times (-6) = -18$  and  $2 \times (-9) = -18$

Factorise in pairs.

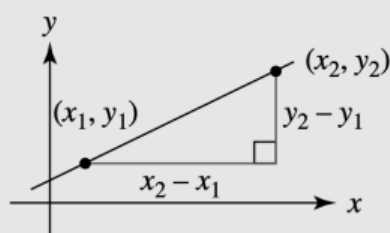
Recognise the denominator as a difference of two squares.

Cancel common factor  $2x+3$  from numerator and denominator.

## Lines

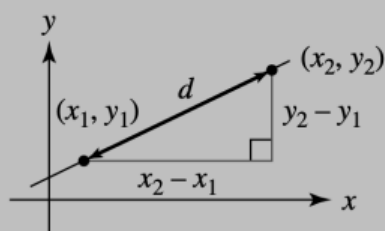
The gradient,  $m$ , of a line between two points  $(x_1, y_1)$  and  $(x_2, y_2)$  is given by  $m = \frac{y_2 - y_1}{x_2 - x_1}$

**Key point**



The length of the line segment,  $d$ , between two points  $(x_1, y_1)$  and  $(x_2, y_2)$  is  $d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$

**Key point**



The midpoint of the line segment from  $(x_1, y_1)$  to  $(x_2, y_2)$  is  $\left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2}\right)$

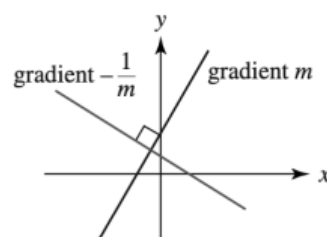
**Key point**

The equation of a straight line is  $y = mx + c$  where  $m$  is the gradient and  $c$  is the y-intercept.

**Key point**

If the gradient of a line is  $m$  then the gradient of a perpendicular line is  $-\frac{1}{m}$  since  $m \times \left(-\frac{1}{m}\right) = -1$

**Key point**





### Example 1

Work out the gradient and the y-intercept of each of these lines.

**a**  $y = \frac{1}{2}x + 4$       **b**  $y + x = 5$       **c**  $-2x + 3y + 7 = 0$

**a** Gradient =  $\frac{1}{2}$  and y-intercept = 4

**b**  $y = 5 - x$

So gradient =  $-1$  and y-intercept = 5

**c**  $3y = -7 + 2x$

$y = -\frac{7}{3} + \frac{2}{3}x$

So gradient =  $\frac{2}{3}$  and y-intercept =  $-\frac{7}{3}$

Since  $y = mx + c$  where  $m$  is the gradient and  $c$  is the y-intercept.

Rearrange to make  $y$  the subject.

Rearrange to make  $y$  the subject.

### Example 2

Solve the simultaneous equations  $5x - 4y = 17$ ,  $3x + 8y = 5$

$15x + 40y = 25$  (1)

$15x - 12y = 51$  (2)

(1) - (2):  $52y = -26$

$y = -\frac{1}{2}$

$5x - 4\left(-\frac{1}{2}\right) = 17$

$5x + 2 = 17$

$5x = 15$

$x = 3$

Solve this equation to find the value of  $x$

Multiply the second equation by 5

Multiply the first equation by 3

Subtract equation (2) from equation (1) to eliminate  $x$

Substitute  $y = -\frac{1}{2}$  into one of the original equations.

### Example 3

Decide whether or not each line is parallel or perpendicular to the line  $y = 4x - 1$

**a**  $2x + 8y = 5$       **b**  $20x + 5y = 2$       **c**  $16x - 4y = 5$

First note that the gradient of  $y = 4x - 1$  is 4

**a**  $2x + 8y = 5 \Rightarrow 8y = 5 - 2x$

$\Rightarrow y = \frac{5}{8} - \frac{1}{4}x$

$4 \times \left(-\frac{1}{4}\right) = -1$  so this line is perpendicular to  $y = 4x - 1$

**b**  $20x + 5y = 2 \Rightarrow 5y = 2 - 20x$

$\Rightarrow y = \frac{2}{5} - 4x$

The gradient is  $-4$  so this line is neither parallel nor perpendicular to  $y = 4x - 1$

**c**  $16x - 4y = 5 \Rightarrow 4y = 16x - 5$

$\Rightarrow y = 4x - \frac{5}{4}$

The gradient is 4 so this line is parallel to  $y = 4x - 1$

Rearrange to make  $y$  the subject.

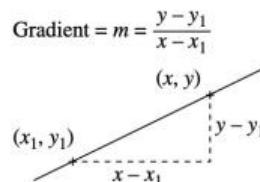
The gradient is  $-\frac{1}{4}$

Since the product of the gradients is  $-1$

Rearrange to make  $y$  the subject.

You can write the gradient of a line in terms of a known point on the line  $(x_1, y_1)$ , the general point  $(x, y)$ , and the gradient,  $m$ .

$$m = \frac{y - y_1}{x - x_1} \text{ or alternatively } y - y_1 = m(x - x_1)$$



The equation of the line with gradient  $m$  through the point  $(x_1, y_1)$  is  $y - y_1 = m(x - x_1)$

**Key point**

#### Example 4

The line  $l_1$  has equation  $7x + 4y = 8$ . The line  $l_2$  is perpendicular to  $l_1$  and passes through the point  $(7, 3)$ . Find the equation of  $l_2$  in the form  $ax + by + c = 0$  where  $a$ ,  $b$  and  $c$  are integers.

$$l_1: 7x + 4y = 8 \Rightarrow 4y = -7x + 8$$

$$\Rightarrow y = -\frac{7}{4}x + 2$$

So the gradient of  $l_1$  is  $-\frac{7}{4}$  and the gradient of  $l_2$  is  $\frac{4}{7}$ .

$$\text{So the equation of } l_2 \text{ is } y - 3 = \frac{4}{7}(x - 7)$$

$$\Rightarrow 7y - 21 = 4(x - 7)$$

$$\Rightarrow 7y - 21 = 4x - 28$$

$$\Rightarrow 4x - 7y - 7 = 0$$

Rearrange to the correct form.

Rearrange to make  $y$  the subject so you can see what the gradient is.

$$\text{Since } \left(-\frac{7}{4}\right) \times \frac{4}{7} = -1$$

Use  $y - y_1 = m(x - x_1)$  to write the equation of  $l_2$ .

Multiply both sides by 7 so that all coefficients are integers.

## Quadratics

When factorising quadratics of the form  $ax^2 + bx + c$  with  $a \neq 1$ , first split the  $bx$  term into two terms where the coefficients multiply to give the same value as  $a \times c$ .

#### Example 1

Factorise each of these quadratics.

**a**  $3x^2 + 11x + 6$

**b**  $2x^2 - 9x + 10$

$$\text{a } 3x^2 + 11x + 6 = 3x^2 + 9x + 2x + 6$$

$$= 3x(x + 3) + 2(x + 3)$$

$$= (3x + 2)(x + 3)$$

$$\text{b } 2x^2 - 9x + 10 = 2x^2 - 4x - 5x + 10$$

$$= 2x(x - 2) - 5(x - 2)$$

$$= (2x - 5)(x - 2)$$

Split  $11x$  into  $9x + 2x$  since  $9 \times 2 = 18$  and  $3 \times 6 = 18$ .

Factorise the first pair of terms and the second pair of terms.

Split  $9x$  into  $-4x - 5x$  since  $-4 \times -5 = 20$  and  $2 \times 10 = 20$ .

Factorise the first pair of terms and the second pair of terms.

## Example 2

Use factorisation to find the roots of these quadratic equations.

**a**  $4x^2 + 12x = 0$

**b**  $5x^2 = 21x - 4$

**a**  $4x^2 + 12x = 4x(x+3)$

$4x(x+3) = 0 \Rightarrow 4x = 0$  or  $x+3 = 0$

If  $4x = 0$  then  $x = 0$  and if  $x+3 = 0$  then  $x = -3$

**b**  $5x^2 - 21x + 4 = 0$

$5x^2 - 21x + 4 = 5x^2 - 20x - x + 4$

$= 5x(x-4) - (x-4)$

$= (5x-1)(x-4)$

$(5x-1)(x-4) = 0 \Rightarrow 5x-1 = 0$  or  $x-4 = 0$

If  $5x-1 = 0$  then  $x = \frac{1}{5}$  and if  $x-4 = 0$  then  $x = 4$

Factorise the quadratic.

One of the factors must be equal to zero.

Solve to find the roots.

Rearrange so you have a quadratic expression equal to zero.

Write  $-21x = -x - 20x$  since  $-20 \times -1 = 20$  and  $5 \times 4 = 20$

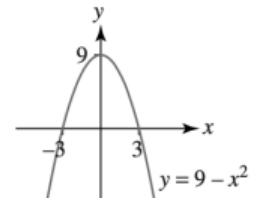
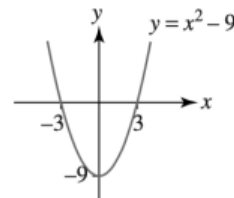
Factorise the quadratic.

The product is zero so one of the factors must be equal to zero.

Solve to find the roots.

A quadratic function has a **parabola** shaped curve.

When you sketch the graph of a quadratic function you must include the coordinates of the points where the curve crosses the  $x$  and  $y$  axes.



## Example 3

Sketch these quadratic functions.

**a**  $y = x^2 + x - 6$

**b**  $y = -x^2 + 4x$

**a** When  $x = 0$ ,  $y = -6$

When  $y = 0$ ,  $x^2 + x - 6 = 0$

$x^2 + x - 6 = (x+3)(x-2)$

$(x+3)(x-2) = 0 \Rightarrow x = -3$  or  $x = 2$

**b** When  $x = 0$ ,  $y = 0$

When  $y = 0$ ,  $-x^2 + 4x = 0$

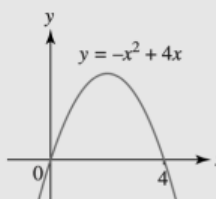
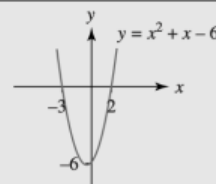
$-x^2 + 4x = -x(x-4)$

$-x(x-4) = 0 \Rightarrow x = 0$  or  $x = 4$

Factorise to find the roots.

Find the  $y$ -intercept by letting  $x = 0$

Find the  $x$ -intercept by letting  $y = 0$



Find the  $y$ -intercept by letting  $x = 0$

Find the  $x$ -intercept by letting  $y = 0$

Factorise to find the roots.

Sketch the parabola and label the  $y$ -intercept of  $-6$  and the  $x$ -intercepts of  $-3$  and  $2$

Sketch the parabola, it will be this way up since the  $x^2$  term in the quadratic is negative. Label the  $x$  and  $y$  intercepts.

**Key point**

The completed square form of  $x^2 + bx + c$  is  $\left(x + \frac{b}{2}\right)^2 - \left(\frac{b}{2}\right)^2 + c$

If you have an expression of the form  $ax^2 + bx + c$  then first factor out the  $a$ , as shown in Example 1

**Example 1**

Write each of these quadratics in the form  $p(x+q)^2 + r$  where  $p$ ,  $q$  and  $r$  are constants to be found.

**a**  $x^2 + 6x + 7$       **b**  $-2x^2 + 12x$

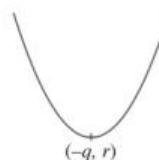
$$\begin{aligned} \text{a } x^2 + 6x + 7 &= \left(x + \frac{6}{2}\right)^2 - \left(\frac{6}{2}\right)^2 + 7 \\ &= (x+3)^2 - 9 + 7 \\ &= (x+3)^2 - 2 \end{aligned}$$

$$\begin{aligned} \text{b } -2x^2 + 12x &= -2[x^2 - 6x] \\ &= -2[(x-3)^2 - 9] \\ &= -2(x-3)^2 + 18 \end{aligned}$$

The constant term in the bracket will be half of the coefficient of  $x$

First factor out the coefficient of  $x^2$  then complete the square for the expression in the square brackets.

The turning point on the curve with equation  $y = p(x+q)^2 + r$  has coordinates  $(-q, r)$ , this will be a minimum if  $p$  is positive and a maximum if  $p$  is negative.

**Key point**

**Example 2**

Find the coordinates of the turning point of the curve with equation  $y = -x^2 + 5x - 2$

$$\begin{aligned} -x^2 + 5x - 2 &= -\left[x^2 - 5x + 2\right] \\ &= -\left[\left(x - \frac{5}{2}\right)^2 - \frac{25}{4} + 2\right] \\ &= -\left[\left(x - \frac{5}{2}\right)^2 - \frac{17}{4}\right] \\ &= -\left(x - \frac{5}{2}\right)^2 + \frac{17}{4} \end{aligned}$$

So the maximum point is at  $\left(\frac{5}{2}, \frac{17}{4}\right)$

First factor out the  $-1$  then complete the square for the expression in the square brackets.

The curve is at its highest point when the bracket is equal to zero:  $x - \frac{5}{2} = 0 \Rightarrow x = \frac{5}{2}$