

File created: 16-May-90 11:41:16 {DSK}<usr>local>lde>lispcore>sources>AARITH.;2

changes to: (VARS AARITHCOMS)

previous date: 22-Oct-86 19:52:52 {DSK}<usr>local>lde>lispcore>sources>AARITH.;1

Read Table: XCL

Package: INTERLISP

Format: XCCS

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(RPAQQ AARITHCOMS

```
((FNS LOG ANTILOG SIN ARCSIN COS ARCCOS TAN ARCTAN ARCTAN2 ATAN FEXPT \\SIN-FLOAT \\TAN-FLOAT \\SIN.OLD
  \\COS.OLD \\TAN.OLD)
(VARS \\ANTILOGARRAY \\ANTILOGCARRAY \\ARCTANARRAY \\LOGARRAY \\SIN-PPOLY \\SIN-QPOLY \\TAN-PPOLY
  \\TAN-QPOLY \\SINARRAY1 \\SINARRAY2 \\TANARRAY \\ATANARRAY)
(DECLARE\:: EVAL@COMPILE DONTCOPY (MACROS (* |now| |obsolete| - |use| POLYEVAL |instead|)
  HORNERIFY FLEQ FGEQ)
  (FILES (LOADCOMP)
    LLFLOAT)
  (CONSTANTS (\\SIN-EPSILON 2.441406E-4)
    (\\TAN-EPSILON 2.441406E-4)
    (\\EXPONENT.BIAS 127)
    (2PI 6.283185)
    (CL:PI 3.141593)
    (-PI -3.141593)
    (-PI/2 -1.570796)
    (PI/2 1.570796)
    (4/PI 1.273239)
    (3PI/2 4.712389)
    (PI/4 0.7853982)
    (-PI/4 -0.7853982)
    (PI/180 0.01745329)
    (180/PI 57.29578)
    (-PI/2 -1.570796)
    (LN2 0.6931472)
    (2^-126 1.17549407E-38))))))
```

(DEFINEQ

(LOG

```
(LAMBDA (X) (* |hdj| "11-Feb-85 17:14")
  (DECLARE (GLOBALVARS \\LOGARRAY))
  (PROG ((SX (OR (FLOATP X)
    (FLOAT X)))
    (EXP 0)
    SSUM)
    (|if| (NOT (FGREATERP SX 0.0))
      |then| (ERROR "LOG OF NON-POSITIVE NUMBER:" X))
    (|if| (EQ 0 (|fetch| (FLOATP EXPONENT) |of| SX))
      |then|
        (* * |Don't| |really| |need| |to| |consider| |unnormalized| |numbers,| |but| |there| |is| |a| |bug| |in| |Interlisp-D's| |floating| |point|
          |arithmetic| |as| |of| |3/17/84| |regarding| |zero| |exponent.|)
        (SETQ EXP (|while| (FLESSP SX 2^-126) |count| (SETQ SX (FTIMES SX 2.0))))
        (|if| (EQ SX X)
          |then| (* * |Need| |smashable| |copy|)
            (SETQ SX (\\FLOAT.BOX X)))
        (SETQ EXP (IDIFFERENCE (IDIFFERENCE (|fetch| (FLOATP EXPONENT) |of| SX)
          \\EXPONENT.BIAS)
          EXP))
        (* * |Depends| |on| |Interlisp-D's| |use| |of| |IEEE 32 |bit| |float| |format| |internally| |and| |smashes| |the| |number| |to| |the|
          |range| |1| |to| |2| |and| |saves| |the| |exponent|)
        (|replace| (FLOATP EXPONENT) |of| SX |with| \\EXPONENT.BIAS)
        (SETQ SX (FDIFFERENCE SX 1.0))
        (SETQ SSUM (POLYEVAL SX \\LOGARRAY 8))
        (* * |Polynomial| |from| |Handbook| |of| |Mathematical| |Functions|
          (|edited| |by| |Aramowitz|) |page| 69 |accuracy| 28 |bits| (|of| |the| 24 |available|!))
        (RETURN (FPLUS SSUM (FTIMES LN2 EXP))))))
```

(ANTILOG

```
(LAMBDA (X) (* JAS "19-Jul-85 11:55")
  (DECLARE (GLOBALVARS \\ANTILOGARRAY \\ANTILOGCARRAY))
  (PROG ((XX (FLOAT X))
    FRAC IP SSUM YY)
    (DECLARE (TYPE FLOATING XX FRAC SSUM YY))
    (SETQ YY (FABS XX))
    (COND
      ((GREATERP YY 88.7)
```

```

(COND
  ((LESSP XX 0)
   (RETURN 0.0))
  (T (ERROR "FLOATING OVERFLOW" X))))
(SETQ FRAC (FDIFFERENCE YY (FTIMES (CONSTANT (LOG 2))
                                     (SETQ IP (FIX (FTIMES YY (CONSTANT (FQUOTIENT 1.0 (LOG 2))))))))))
(SETQ SSUM (POLYEVAL FRAC \ANTILOGARRAY 7))

(* * |Polynomial| |from| |Handbook| |of| |Mathematical| |Functions|
|edited| |by| |Aramowitz| |page| 71 |accuracy| 32 |bits| |of| |the| 24 |available|)
(* SSUM |is| |in| |the| |range| 0.5 |to| 2
|and| |series| |produced| 0.5 |to| 1)

(SETQ SSUM (QUOTIENT SSUM (ELT \ANTILOGCARRAY (IPLUS IP 127))))
(RETURN (|if| (FGREATERP XX 0.0)
          |then| (FQUOTIENT 1.0 SSUM)
          |else| SSUM))))

```

(SIN

```

(LAMBDA (X RADIANSFLG) (* FS "15-Oct-86 19:56")
  (PROG ((XX X))
    (DECLARE (TYPE FLOATP XX))
    (|if| RADIANSFLG
      |then| (RETURN (\SIN-FLOAT XX))
      |else| (RETURN (\SIN-FLOAT (FTIMES PI/180 XX))))))

```

(ARCSIN

```

(LAMBDA (X RADIANSFLG) (* |JonL| "30-Mar-84 23:59")
  (PROG ((XX (OR (FLOATP X)
                  (FLOAT X)))
        (SSUM NEGP REDUCED Z Q1 Q2)
        (|if| (OR (FLESSP XX -1.0)
                  (FGREATERP XX 1.0))
          |then| (ERROR "ARCSIN: arg not in range" XX)
          |elseif| (FLESSP XX 0.0)
          |then| (SETQ NEGP T)
                  (SETQ XX (FDIFFERENCE 0.0 XX)))
        (|if| (FGREATERP XX 0.5)
          |then| (SETQ REDUCED T)
                  (SETQ XX (SQRT (FTIMES 0.5 (FDIFFERENCE 1.0 XX))))))

    (* |Special| |case| |for| |small| |magnitude| |arguments| |from| |Computer| |Evaluation| |of| |Mathematical| |Functons|
    |by| C. T. |Fike| |page| 57)

    (SETQ Z (FTIMES XX XX))
    (SETQ Q1 (FTIMES 0.5315066 Z))
    (SETQ Q2 (FTIMES (SETQ Q2 (FDIFFERENCE Q1 0.08982446))
                      Q2))
    (SETQ Q2 (FPLUS 0.3697723 (FTIMES Q2 (FPLUS 0.4918762 Q2))))
    (SETQ SSUM (FTIMES XX (FPLUS 0.7533057 (FTIMES Q2 (FPLUS 0.6599526 Q1)))))
    (|if| REDUCED
      |then| (SETQ SSUM (FDIFFERENCE PI/2 (FTIMES 2.0 SSUM)))
    (|if| NEGP
      |then| (SETQ SSUM (FDIFFERENCE 0.0 SSUM)))
    (RETURN (|if| RADIANSFLG
                |then| SSUM
                |else| (FTIMES SSUM 180/PI))))))

```

(COS

```

(LAMBDA (X RADIANSFLG) (* FS "15-Oct-86 19:58")
  (PROG ((XX X))
    (DECLARE (TYPE FLOATP XX))
    (|if| RADIANSFLG
      |then| (RETURN (\SIN-FLOAT XX T))
      |else| (RETURN (\SIN-FLOAT (FTIMES PI/180 XX)
                                  T))))))

```

(ARCCOS

```

(LAMBDA (X RADIANSFLG) (* |JonL| "30-Mar-84 20:21")
  (PROG ((XX (OR (FLOATP X)
                  (FLOAT X)))
        (RETURN (FDIFFERENCE (|if| RADIANSFLG
                                   |then| PI/2
                                   |else| 90.0)
                              (ARCSIN XX RADIANSFLG))))))

```

(TAN

```

(LAMBDA (X RADIANSFLG) (* FS "17-Oct-86 18:20")
  (PROG ((XX X))
    (DECLARE (TYPE FLOATP XX))
    (|if| RADIANSFLG
      |then| (RETURN (\TAN-FLOAT XX))
      |else| (RETURN (\TAN-FLOAT (FTIMES PI/180 XX))))))

```

(ARCTAN

```

(LAMBDA (X RADIANSFLG)
  (DECLARE (GLOBALVARS \\ARCTANARRAY))
  (PROG ((XX (FPLUS X 0.0))
    (SSUM 0.002866226)
    X2 FLIPPED)

    (|if| (OR (FLESSP XX -1.0)
      (FGREATERP XX 1.0))
      |then| (SETQ FLIPPED (|if| (FLESSP XX 0.0)
        |then| -PI/2
        |else| PI/2))
        (SETQ XX (FQUOTIENT 1.0 XX)))
    (SETQ X2 (FTIMES XX XX))
    (SETQ SSUM (FTIMES XX (POLYEVAL X2 \\ARCTANARRAY 8)))
    (|if| FLIPPED
      |then| (SETQ SSUM (FDIFFERENCE FLIPPED SSUM)))
    (RETURN (|if| RADIANSFLG
      |then| SSUM
      |else| (FTIMES SSUM 180/PI))))))

```

(* |hdj| "11-Feb-85 17:24")

(* POLYNOMIAL FROM HANDBOOK OF MATHEMATICAL
FUNCTIONS (EDITED BY ARAMOWITZ) PAGE 81
ACCURACY 28 BITS)

(ARCTAN2

```

(LAMBDA (Y X RADIANSFLG)
  (OR (FLOATP Y)
    (SETQ Y (FLOAT Y)))
  (OR (FLOATP X)
    (SETQ X (FLOAT X)))
  (PROG ((ANGLE (ARCTAN (ABS (FQUOTIENT Y X))
    T)))
    (SETQ ANGLE (|if| (FLESSP X 0.0)
      |then| (|if| (FLESSP Y 0.0)
        |then|
          (FPLUS -PI ANGLE)
          |else|
            (FDIFFERENCE CL:PI ANGLE))
        |else| (|if| (FLESSP Y 0.0)
          |then|
            (FDIFFERENCE 0.0 ANGLE)
            |else|
              ANGLE)))
    (RETURN (|if| RADIANSFLG
      |then| ANGLE
      |else| (FTIMES ANGLE 180/PI))))))

```

(* |JonL| "17-Mar-84 21:41")

(* |Quadrant| 3)

(* |Quadrant| 2)

(* |Quadrant| 4)

(* |Quadrant| 1)

(ATAN

```

(LAMBDA (Y X RADIANSFLG)

  (* |version| |of| |arctan| |which| |returns| |value| |in| |radians| |between| 0 |and| 2 PI.
  |Copied| |from| |the| PDP-10 |MacLisp| |machine| |language| |code|.|)

  (OR (FLOATP Y)
    (SETQ Y (FLOAT Y)))
  (OR (FLOATP X)
    (SETQ X (FLOAT X)))
  (DECLARE (GLOBALVARS \\ATANARRAY))
  (PROG ((Y.NEGP (FLESSP Y 0.0))
    (X.NEGP (FLESSP X 0.0))
    ATAN.Y ATAN.X T. TT D R (ANS -0.004054058))
    (SETQ ATAN.Y (|if| Y.NEGP
      |then| (FDIFFERENCE 0.0 Y)
      |else| Y))
    (SETQ ATAN.X (|if| X.NEGP
      |then| (FDIFFERENCE 0.0 X)
      |else| X))
    (SETQ T. (FQUOTIENT (FDIFFERENCE ATAN.Y ATAN.X)
      (FPLUS ATAN.Y ATAN.X)))
    (SETQ R (FTIMES T. T.))
    (SETQ D (FTIMES T. (POLYEVAL R \\ATANARRAY 7)))
    (SETQ TT (|if| (FLESSP D 0.0)
      |then| (FDIFFERENCE 0.0 D)
      |else| D))
    (SETQ D (|if| (OR (FGEQ TT 0.7855)
      (FLESSP TT 0.7853))
      |then| (FPLUS D 0.7853982)
      |elseif| (FLESSP D 0.0)
      |then|
        (FPLUS D 0.7853982))))

```

(* |hdj| "11-Feb-85 17:26")

(* |When| |the| |rational| |approximation| |is| |not| |very| |good|, |we| |can| |patch| |it| |up| |by| |using| Y/X |and| |an|
|approximation| |for| (ARCTAN Y/X))

|else| (FQUOTIENT ATAN.Y ATAN.X)

(* |Corresponds| |to| |label| ATAN.2)

```

(FPLUS PI/2 (FQUOTIENT (FDIFFERENCE 0.0 ATAN.X)
                      ATAN.Y)))
ATAN.4
(* We now have \a quadrant-1 result;
   patch it up to get other quadrant values.)

(SETQ D (|if| X.NEGP
            |then| (|if| Y.NEGP
                        |then| (FPLUS CL:PI D) (* |Quadrant| 3)
                        |else| (FDIFFERENCE CL:PI D) (* |Quadrant| 2)
                    )
            |else| (|if| Y.NEGP
                        |then| (FDIFFERENCE 2PI D) (* |Quadrant| 4)
                        |else| D) (* |Quadrant| 1)
        ))
(RETURN (|if| RADIANSFLG
            |then| D
            |else| (FTIMES D 180/PI))))))

```

(FEXPT

```

(LAMBDA (A N) (* JAS "29-Jul-85 15:13")

(* \n addition| to| coercing| the| args| to| floating-point,| this| handles| the| case| of| negative| values| for| N)

(COND
  ((EQP A 0.0)
   0.0)
  (T (ANTILOG (FTIMES (OR (FLOATP N)
                          (FLOAT N))
                      (LOG (OR (FLOATP A)
                              (FLOAT A))))))))

```

(\SIN-FLOAT

```

(LAMBDA (X COS-FLAG) (* FS "15-Oct-86 19:52")

(* * SIN of| \a FLOAT X| calculated| via| SIN 3374| rational| approximation| of| Harris| et| al.
*)

(PROG (R SIGN R2 ANSWER)
  (DECLARE (GLOBALVARS \SIN-PPOLY \SIN-QPOLY)
    (TYPE FLOATP X R SIGN R2 ANSWER))

  (* * If| this| function| called| by| COS| then| use| (COS X) = (SIN
  (|minus| PI/2 X)) = (SIN (+ PI/2 X))| Case| out| on| sign| of| X| for| improved| numerical| stability.
  | Avoids| unnecessary| rounding| and| promotes| symmetric| properties.
  (COS X) = (COS (|minus| X))| is| guaranteed| by| this| strategy.
  *)

  (SETQ R (COND
    ((NOT COS-FLAG)
     X)
    ((> X 0)
     (- PI/2 X))
    (T (+ PI/2 X))))

  (* * |First| range| reduce| to| (0 infinity)| by| (SIN (|minus| X)) =
  (|minus| (SIN X))| This| strategy| guarantees| (SIN (|minus| X)) =
  (|minus| (SIN X)) *)

  (COND
    ((< R 0)
     (SETQ SIGN -1.0)
     (SETQ R (- R)))
    (T (SETQ SIGN 1.0)))

  (* * |Next| range| reduce| to| interval| (0 2PI)| by| (SIN X) = (SIN
  (MOD X 2PI)) . *)

  (SETQ R (CL:REM R 2PI))

  (* * |Next| range| reduce| to| interval| (0 CL:PI)| by| (SIN (+ X CL:PI)) =
  (|minus| (SIN X)) *)

  (COND
    ((> R CL:PI)
     (SETQ SIGN (- SIGN))
     (SETQ R (- R CL:PI))))

  (* * |Next| range| reduce| to| interval| (0 PI/2)| by| (SIN (+ X PI/2)) =
  (SIN (|minus| PI/2 X)) *)

  (COND
    ((> R PI/2)
     (SETQ R (- CL:PI R))))

```

```

(COND
  ((< R \\SIN-EPSILON)

  (* * If R |is| |in| |the| |interval| (0 %SIN-EPSILON) |then| (SIN R) = R |to| |the| |precision| |that| |we| |can| |offer.|
  |Return| R |because| (1) |it| |is| |desirable| |that| (SIN R) = R |exactly| |for| |small| R |and|
  (2) |microcode| POLYEVAL |will| |underflow| |on| |sufficiently| |small| |positive| R.
  *)

  (RETURN (FTIMES SIGN R))))

(* * |Now| |use| SIN 3374 |rational| |approximation| |of| |Harris| |et| |al.|
|which| |works| |on| |interval| (0 %PI/2) *)

(SETQ R2 (FTIMES R R))
(SETQ ANSWER (FTIMES SIGN R (/ (POLYEVAL R2 \\SIN-PPOLY 5)
                                (POLYEVAL R2 \\SIN-QPOLY 5))))
(RETURN ANSWER)))

```

(\\TAN-FLOAT

(LAMBDA (X)

(* FS "17-Oct-86 20:29")

```

(* * TAN |of| |a| FLOAT X |calculated| |via| TAN 4288 |rational| |approximation| |of| |Harris| |et| |al.|
*)

(PROG (R SIGN RECIPFLG R2 ANSWER)
  (DECLARE (GLOBALVARS \\TAN-PPOLY \\TAN-QPOLY)
    (TYPE FLOATP X R R2 ANSWER))
  (SETQ R X)

  (* * |First| |range| |reduce| |to| (0 |infinity|) |by| (TAN (|minus| X)) =
  (|minus| (TAN X) *)

  (COND
    ((< R 0)
      (SETQ SIGN -1.0)
      (SETQ R (- R)))
    (T (SETQ SIGN 1.0)))

  (* * |Next| |range| |reduce| |to| (0 CL:PI) *)

  (SETQ R (CL:REM R CL:PI))

  (* * |Next| |range| |reduce| |to| (-PI/4 PI/4) |using| (TAN X) = (TAN
  (|minus| X CL:PI)) |to| |get| |into| |interval| (-PI/2 PI/2) |and| |then|
  (TAN X) = (/ (TAN (|minus| PI/2 X))) |to| |get| |into| |interval| (-PI/4 PI/4) *)

  (COND
    ((> R PI/2)
      (SETQ R (- R CL:PI))
      (COND
        ((< R -PI/4)
          (SETQ RECIPFLG T)
          (SETQ R (- -PI/2 R))))
    (T (COND
      ((> R PI/4)
        (SETQ RECIPFLG T)
        (SETQ R (- PI/2 R))))))

  (COND
    ((< (ABS R)
      \\TAN-EPSILON)

    (* * If R |is| |in| |the| |interval| (0 %TAN-EPSILON) |then| (TAN R) = R |to| |the| |precision| |that| |we| |can| |offer.|
    |Return| R |because| (1) |it| |is| |desirable| |that| (TAN R) = R |exactly| |for| |small| R |and|
    (2) |microcode| POLYEVAL |will| |underflow| |on| |sufficiently| |small| |positive| R.
    *)

    (SETQ ANSWER (FTIMES SIGN R))
    (COND
      (RECIPFLG (SETQ ANSWER (/ ANSWER))))
    (RETURN ANSWER)))

  (* * |Now| |use| TAN 4288 |rational| |approximation| |of| |Harris| |et| |al.|
  |which| |works| |on| |interval| (0 PI/4) *)

  (SETQ R2 (FTIMES R R))
  (SETQ ANSWER (FTIMES SIGN R (/ (POLYEVAL R2 \\TAN-PPOLY 4)
                                  (POLYEVAL R2 \\TAN-QPOLY 5))))

  (COND
    (RECIPFLG (SETQ ANSWER (/ ANSWER))))
  (RETURN ANSWER)))

```

(\\SIN.OLD

(LAMBDA (X RADIANSFLG)

(* FS "15-Oct-86 19:35")

(DECLARE (GLOBALVARS \\SINARRAY1 \\SINARRAY2))

(* * |Old| |version,| |claimed| |less| |accurate| -FS)

```

(PROG ((XX X)
  X2)
(DECLARE (TYPE FLOATP XX X2))
(if| RADIANSFLG
  |then| (if| (OR (FGEQ XX 2PI)
    (FLEQ XX (CONSTANT (MINUS 2PI))))
    |then| (SETQ XX (FREMAINDER XX 2PI)))
  |else| (if| (OR (FGEQ XX 360.0)
    (FLEQ XX -360.0))
    |then| (SETQ XX (FREMAINDER XX 360.0)))
    (SETQ XX (FTIMES PI/180 XX)))
(if| (FLESSP XX -PI/2)
  |then| (SETQ XX (FPLUS XX 2PI)))
(if| (FGREATERP XX 3PI/2)
  |then| (SETQ XX (FDIFFERENCE XX 2PI))
  |elseif| (FGREATERP XX PI/2)
  |then| (SETQ XX (FDIFFERENCE CL:PI XX)))
(RETURN (if| (FGEQ XX PI/4)
  |then| (SETQ X2 (FTIMES XX XX))
  (SETQ X2 (FTIMES XX (POLYEVAL X2 \\SINARRAY1 5)))
  |else| (SETQ XX (FTIMES 4/PI XX))
  (SETQ X2 (FTIMES XX XX))
  (SETQ X2 (FTIMES XX (POLYEVAL X2 \\SINARRAY2 3)))))))

```

(* |Range-reduce| |to| |between| 0 |and| PI/2)

(* |Polynomial| |from| |Handbook| |of| |Mathematical| |Functions|
|edited| |by| |Aramowitz|) |page| 76 |accuracy| 26 |bits|
|of| |the| 24 |available|!))

(* |Chebyshev| |approximation| |from| |Computer| |Evaluation|
|of| |Mathematical| |Functions| |by| C.
T. |Fike|) |Page| 117)

\\COS.OLD

(* FS "15-Oct-86 19:57")

```

(LAMBDA (X RADIANSFLG)
  (PROGN (DECLARE (GLOBALVARS \\SINARRAY1 \\SINARRAY2))
    (PROG ((XX X)
      X2)
      (DECLARE (TYPE FLOATP XX X2))
      (if| RADIANSFLG
        |then| (if| (OR (FGEQ XX 2PI)
          (FLEQ XX (CONSTANT (MINUS 2PI))))
          |then| (SETQ XX (FREMAINDER XX 2PI)))
        |else| (if| (OR (FGEQ XX 360.0)
          (FLEQ XX -360.0))
          |then| (SETQ XX (FREMAINDER XX 360.0)))
          (SETQ XX (FTIMES PI/180 XX)))
      (SETQ XX (FDIFFERENCE PI/2 XX))
      (if| (FLESSP XX -PI/2)
        |then| (SETQ XX (FPLUS XX 2PI)))
      (if| (FGREATERP XX 3PI/2)
        |then| (SETQ XX (FDIFFERENCE XX 2PI))
        |elseif| (FGREATERP XX PI/2)
        |then| (SETQ XX (FDIFFERENCE CL:PI XX)))
      (RETURN (if| (FGEQ XX PI/4)
        |then| (SETQ X2 (FTIMES XX XX))
        (SETQ X2 (FTIMES XX (POLYEVAL X2 \\SINARRAY1 5)))
        |else| (SETQ XX (FTIMES 4/PI XX))
        (SETQ X2 (FTIMES XX XX))
        (SETQ X2 (FTIMES XX (POLYEVAL X2 \\SINARRAY2 3)))))))

```

(* |Range-reduce| |to| |between| 0 |and| PI/2)

(* |Polynomial| |from| |Handbook| |of| |Mathematical| |Functions|
|edited| |by| |Aramowitz|) |page| 76 |accuracy| 26 |bits|
|of| |the| 24 |available|!))

(* |Chebyshev| |approximation| |from| |Computer| |Evaluation|
|of| |Mathematical| |Functions| |by| C.
T. |Fike|) |Page| 117)

\\TAN.OLD

(* FS "17-Oct-86 18:19")

```

(LAMBDA (X RADIANSFLG)
  (DECLARE (GLOBALVARS \\TANARRAY))
  (PROG ((XX X)
    FLIPPED Y X2)
    (DECLARE (TYPE FLOATP XX Y X2))
    (SETQ XX (if| RADIANSFLG
      |then| (FREMAINDER XX CL:PI)
      |else| (FTIMES (FREMAINDER XX 180.0)
        PI/180)))
    (DECLARE (TYPE FLOATP XX Y X2))
    (if| (FGREATERP XX PI/2)
      |then| (SETQ XX (FDIFFERENCE XX CL:PI))
      |elseif| (FLESSP XX -PI/2)
      |then| (SETQ XX (FPLUS XX CL:PI)))
    (SETQ Y (if| (FGREATERP XX PI/4)
      |then| (SETQ FLIPPED T)
      (FDIFFERENCE PI/2 XX)
      |elseif| (FLESSP XX -PI/4)
      |then| (SETQ FLIPPED T)
      (FDIFFERENCE -PI/2 XX)
      |else| XX))
    (SETQ X2 (FTIMES Y Y))

```

(* |First,| |normalize| |to| |between| -PI |and| CL:PI)

(* |Then| |normalize| |to| |between| -PI/2 |and| PI/2)

```
(SETQ Y (FTIMES Y (POLYEVAL X2 \TANARRAY 6)))

(* POLYNOMIAL APPROXIMATION FROM HANDBOOK OF MATHEMATICAL FUNCTIONS
(EDITED BY ABRAMOWITZ) PAGE 76 GOOD TO ALMOST 26 BITS)

(RETURN ((if| FLIPPED
          |then| (SETQ Y (FQUOTIENT 1.0 Y))
          |else| Y))))))

)

(RPAQ \ANTILOGARRAY (READARRAY-FROM-LIST 8 'FLOATP 0 '(-1.413161E-4 0.001329882 -0.00830136 0.04165735
-0.1666653 0.4999999 -1.0 1.0 NIL)))

(RPAQ \ANTILOGCARRAY
(READARRAY-FROM-LIST 255 'FLOATP 0
' (5.87747456E-39 1.17549407E-38 2.350992E-38 4.70198E-38 9.40396E-38 1.880791E-37 3.761582E-37
7.523175E-37 1.504633E-36 3.009266E-36 6.018532E-36 1.203706E-35 2.407413E-35 4.814832E-35
9.629655E-35 1.92593E-34 3.85186E-34 7.70372E-34 1.540746E-33 3.081488E-33 6.162979E-33
1.232595E-32 2.465191E-32 4.930382E-32 9.860764E-32 1.972152E-31 3.944305E-31 7.888613E-31
1.577722E-30 3.155448E-30 6.310891E-30 1.262178E-29 2.524355E-29 5.04871E-29 1.009743E-28
2.019484E-28 4.038968E-28 8.077936E-28 1.615587E-27 3.231174E-27 6.462349E-27 1.29247E-26
2.58494E-26 5.169879E-26 1.033976E-25 2.067952E-25 4.135903E-25 8.271806E-25 1.654361E-24
3.308724E-24 6.617445E-24 1.323489E-23 2.646978E-23 5.293956E-23 1.058791E-22 2.117583E-22
4.235165E-22 8.47033E-22 1.694066E-21 3.388132E-21 6.776264E-21 1.355253E-20 2.710506E-20
5.421011E-20 1.084202E-19 2.168405E-19 4.336809E-19 8.67363E-19 1.734724E-18 3.469447E-18
6.938904E-18 1.387779E-17 2.775558E-17 5.551115E-17 1.110223E-16 2.220446E-16 4.440892E-16
8.881784E-16 1.776359E-15 3.552714E-15 7.105429E-15 1.421086E-14 2.842171E-14 5.684342E-14
1.136868E-13 2.273737E-13 4.547474E-13 9.094948E-13 1.818989E-12 3.637979E-12 7.275959E-12
1.455192E-11 2.910383E-11 5.820766E-11 1.164153E-10 2.328307E-10 4.656613E-10 9.31324E-10
1.862645E-9 3.72529E-9 7.450583E-9 1.490116E-8 2.980232E-8 5.960465E-8 1.192093E-7
2.384186E-7 4.768372E-7 9.536744E-7 1.907349E-6 3.814697E-6 7.629397E-6 1.525879E-5
3.051759E-5 6.103516E-5 1.220703E-4 2.441406E-4 4.882813E-4 9.765626E-4 0.001953125
0.00390625 0.0078125 0.015625 0.03125 0.0625 0.125 0.25 0.5 1.0 2.0 4.0 8.0 16.0 32.0 64.0
128.0 256.0 512.0 1024.0 2048.0 4096.0 8192.0 16384.0 32768.0 65536.0 131072.0 262144.0
524288.0 1048576.0 2097152.0 4194304.0 8388608.0 1.677722E+7 3.355443E+7 6.710887E+7
1.342177E+8 2.684355E+8 5.368709E+8 1.073742E+9 2.147484E+9 4.294968E+9 8.589936E+9
1.717987E+10 3.435974E+10 6.871948E+10 1.37439E+11 2.748779E+11 5.497558E+11 1.099512E+12
2.199023E+12 4.398047E+12 8.796094E+12 1.759219E+13 3.518437E+13 7.036876E+13 1.407375E+14
2.81475E+14 5.6295E+14 1.1259E+15 2.2518E+15 4.5036E+15 9.0072E+15 1.80144E+16 3.60288E+16
7.20576E+16 1.441152E+17 2.882304E+17 5.764609E+17 1.152922E+18 2.305843E+18 4.611686E+18
9.223372E+18 1.844675E+19 3.689349E+19 7.378698E+19 1.47574E+20 2.951479E+20 5.902958E+20
1.180592E+21 2.361183E+21 4.722367E+21 9.444734E+21 1.888947E+22 3.777893E+22 7.555786E+22
1.511157E+23 3.022315E+23 6.044629E+23 1.208926E+24 2.417852E+24 4.835703E+24 9.671406E+24
1.934281E+25 3.868563E+25 7.737125E+25 1.547425E+26 3.09485E+26 6.1897E+26 1.23794E+27
2.47588E+27 4.95176E+27 9.90352E+27 1.980704E+28 3.961408E+28 7.922816E+28 1.584563E+29
3.169126E+29 6.338253E+29 1.267651E+30 2.535301E+30 5.070602E+30 1.01412E+31 2.028241E+31
4.056482E+31 8.112964E+31 1.622593E+32 3.245186E+32 6.490371E+32 1.298074E+33 2.596148E+33
5.192297E+33 1.038459E+34 2.076919E+34 4.153837E+34 8.307675E+34 1.661535E+35 3.32307E+35
6.64614E+35 1.329228E+36 2.658456E+36 5.316912E+36 1.063382E+37 2.126765E+37 4.25353E+37
8.50706E+37 1.701412E+38 NIL)))

(RPAQ \ARCTANARRAY (READARRAY-FROM-LIST 9 'FLOATP 0 '(0.002866226 -0.01616574 0.04290961 -0.07528964 0.1065626
-0.142089 0.1999355 -0.3333315 1.0 NIL)))

(RPAQ \LOGARRAY (READARRAY-FROM-LIST 9 'FLOATP 0 '(-0.006453544 0.03608849 -0.0953294 0.1676541 -0.2407338
0.331799 -0.4998741 0.9999964 0.0 NIL)))

(RPAQ \SIN-PPOLY (READARRAY-FROM-LIST 6 'FLOATP 0 '(-1.312516E-9 5.565546E-7 -8.703754E-5 0.005830397 -0.1509093
1.0 NIL)))

(RPAQ \SIN-QPOLY (READARRAY-FROM-LIST 6 'FLOATP 0 '(3.535755E-12 1.995733E-9 6.131296E-7 1.232982E-4 0.01575741
1.0 NIL)))

(RPAQ \TAN-PPOLY (READARRAY-FROM-LIST 5 'FLOATP 0 '(8.443456E-8 -3.939664E-5 0.004337587 -0.140375 1.0 NIL)))

(RPAQ \TAN-QPOLY (READARRAY-FROM-LIST 6 'FLOATP 0 '(-1.539329E-9 2.275635E-6 -4.822159E-4 0.02890704 -0.4737084
1.0 NIL)))

(RPAQ \SINARRAY1 (READARRAY-FROM-LIST 6 'FLOATP 0 '(-2.39E-8 2.7526E-6 -1.98409E-4 0.008333332 -0.1666667 1.0
NIL)))

(RPAQ \SINARRAY2 (READARRAY-FROM-LIST 4 'FLOATP 0 '(-3.59544E-5 0.002490007 -0.08074545 0.7853982 NIL)))

(RPAQ \TANARRAY (READARRAY-FROM-LIST 7 'FLOATP 0 '(0.00951681 0.002900525 0.02456509 0.05337406 0.1333924
0.3333314 1.0 NIL)))

(RPAQ \ATANARRAY (READARRAY-FROM-LIST 8 'FLOATP 0 '(-0.004054058 0.02186123 -0.05590989 0.09642004 -0.1390853
0.1994654 -0.3332986 0.9999994 NIL)))

(DECLARE\ : EVAL@COMPILE DONTCOPY

(DECLARE\ : EVAL@COMPILE

(PUTPROPS HORNERIFY MACRO (X (PROG ((INITIAL (CAR X))
(VARNAME (CADR X))
(COEFFICIENTS (CDDR X))
```

```

      TERM)
      (OR COEFFICIENTS (SHOULDN'T))
      (OR (AND (LITATOM VARNAME)
                VARNAME
                (NEQ T VARNAME))
           (\\ILLEGAL.ARG VARNAME))
      (SETQ TERM (LIST 'FPLUS (LIST 'FTIMES INITIAL VARNAME)
                       (CAR COEFFICIENTS)))
      (OR (CONSTANTEXPRESSIONP (CAR COEFFICIENTS))
          (ARGS.COMMUTABLEP (CAR COEFFICIENTS)
                           (CADR TERM))
          (LISPERROR X "Can't hack non-commutable coefficient expressions"))
      (RETURN (COND
                ((NULL (CDR COEFFICIENTS))
                 TERM)
                (T (CONS 'HORNERIFY (CONS TERM (CONS VARNAME (CDR COEFFICIENTS))
                                                  ))))))))

```

```

(PUTPROPS FLEQ MACRO ((X Y)
                      (NOT (FGREATERP X Y))))

```

```

(PUTPROPS FGEQ MACRO ((X Y)
                      (NOT (FLESSP X Y))))
)

```

```

(FILELOAD (LOADCOMP)
           LLFLOAT)

```

```

(DECLARE\ : EVAL@COMPILE

```

```

(RPAQQ \\SIN-EPSILON 2.441406E-4)

```

```

(RPAQQ \\TAN-EPSILON 2.441406E-4)

```

```

(RPAQQ \\EXPONENT.BIAS 127)

```

```

(RPAQQ 2PI 6.283185)

```

```

(RPAQQ CL:PI 3.141593)

```

```

(RPAQQ -PI -3.141593)

```

```

(RPAQQ -PI/2 -1.570796)

```

```

(RPAQQ PI/2 1.570796)

```

```

(RPAQQ 4/PI 1.273239)

```

```

(RPAQQ 3PI/2 4.712389)

```

```

(RPAQQ PI/4 0.7853982)

```

```

(RPAQQ -PI/4 -0.7853982)

```

```

(RPAQQ PI/180 0.01745329)

```

```

(RPAQQ 180/PI 57.29578)

```

```

(RPAQQ -PI/2 -1.570796)

```

```

(RPAQQ LN2 0.6931472)

```

```

(RPAQQ 2^-126 1.17549407E-38)

```

```

(CONSTANTS (\\SIN-EPSILON 2.441406E-4)
            (\\TAN-EPSILON 2.441406E-4)
            (\\EXPONENT.BIAS 127)
            (2PI 6.283185)
            (CL:PI 3.141593)
            (-PI -3.141593)
            (-PI/2 -1.570796)
            (PI/2 1.570796)
            (4/PI 1.273239)
            (3PI/2 4.712389)
            (PI/4 0.7853982)
            (-PI/4 -0.7853982)
            (PI/180 0.01745329)
            (180/PI 57.29578)
            (-PI/2 -1.570796)
            (LN2 0.6931472)
            (2^-126 1.17549407E-38))
)
)

```

```

(PUTPROPS AARITH COPYRIGHT ("Venue & Xerox Corporation" 1981 1983 1984 1985 1986 1990))

```


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-PI/28	2^-1268	CL:PI8	\\EXPONENT.BIAS8
-PI/48	3PI/28	PI/1808	\\SIN-EPSILON8
180/PI8	4/PI8	PI/28	\\TAN-EPSILON8

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\\ARCTANARRAY7	\\SIN-PPOLY7	\\SINARRAY27	\\TANARRAY7

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