

## Wavelength and End Correction in a Recorder

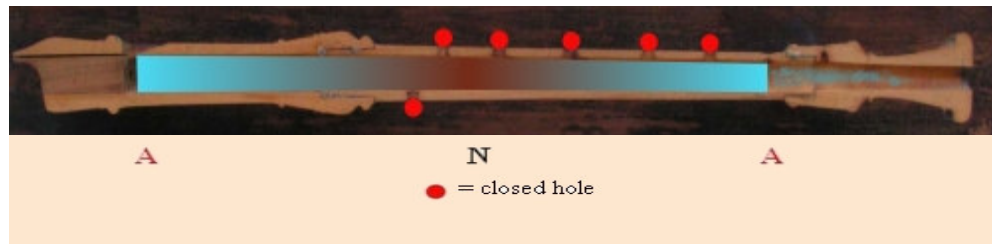
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### Abstract

The wavelength and end correction was investigated as a function of the tube length of a recorder over a range of frequencies. It was found that the period of the sound produced varies linearly with the recorder's tube length, as expected. It was also found that the end correction does not vary as a function of frequency. However, the end correction at the hole was found to be much greater than the end correction at the end of a resonating tube.

### Introduction

The manufacturers of all musical instruments test an instrument's attack time, tone quality and tune before it is released from the factory.<sup>1</sup> Since only certain tones are commonly used in melodies, it is crucial for musicians to tune their instruments. Most wind instruments are tuned by varying the length of the instruments. For a recorder, the tube length is the length between the bore and the nearest open hole. By changing the tube length, different notes can be played.



**Figure 1** The colored strip inside the recorder shows the effective air column.<sup>2</sup>  
("N" stands for Node, "A" stands for Anti-Node)

In a recorder, sound is produced by creating standing waves between the bore on the head joint and the first open hole. The nodes of these standing waves are situated in the center of the tube length and the antinodes are approximately, but not exactly, at the ends. The reflection point at the open hole is beyond the hole. The end correction for still air resonating in a tube is defined as the distance between the anti-node and the end of the tube. This is given as<sup>3</sup>,

$$C_o = 0.6 d \quad (\text{Equation 1})$$

where  $d$  is the diameter of the tube, and  $C_o$  is the end correction at one end. The diameter at each end of the middle section of the recorder was measured as 0.0137 m (diameter towards the head joint) and 0.0097 m (diameter towards the foot joint). Assuming the standard model for end correction at the end of a resonating tube gives predicted end corrections of 0.0082 m and 0.0058 m at the bore and the hole respectively. The inner diameter at the end of the recorder is 0.010m, giving the predicted end correction for the frequency recorded with all 6 holes covered to be 0.0060m.

The wavelength of the standing wave in the recorder is expected to be the sum of twice the length of the tube and the end correction at both ends.

$$\lambda = 2 (L + C_b + C_h) \quad (\text{Equation 2})$$

where  $\lambda$  is the wavelength of the standing waves,  $L$  is the length of the tube, and  $C_b$  and  $C_h$  are the end corrections at the bore and hole.

Since the wavelength of such a standing wave is often hard to measure directly, a formula is used that relates the speed of sound in the air in the tube and the wavelength of the standing waves,

$$\lambda = v/f \quad (\text{Equation 3})$$

where  $\lambda$  is the wavelength of the standing waves,  $v$  is the velocity of sound waves inside the recorder, and  $f$  is the frequency of the standing waves.

Equations 2 and 3 can be combined to give

$$L = (v/2)(1/f) - C \quad (\text{Equation 4})$$

or

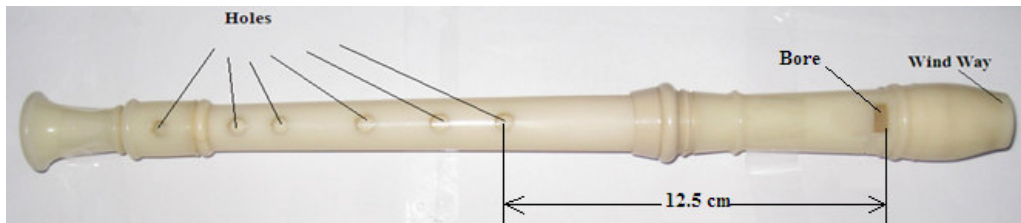
$$L = (v/2)(T) - C \quad (\text{Equation 5})$$

where  $T$  is the period of the wave, and  $C$  is the sum of the end corrections.

The velocity of sound in air varies with temperature. The air blown inside a recorder was measured to be 30°C. The velocity of sound in open air at this temperature is approximately 349 m/s.<sup>4</sup> For most instruments, different frequencies (different harmonics) of waves can be produced using the same fingering. In this research, only the fundamental frequency (first harmonic) is investigated.

## Methods

A recorder was fixed with tape on a piece of paper so that the two ends, the holes and the bore could be marked. A ruler was used to keep the marked lines vertical to the recorder. With a meter rule, the distance between the near edge of the bore (the edge closest to the mouthpiece) and the near edge of each hole was measured. These distances were defined as the lengths of the tube for the various notes.



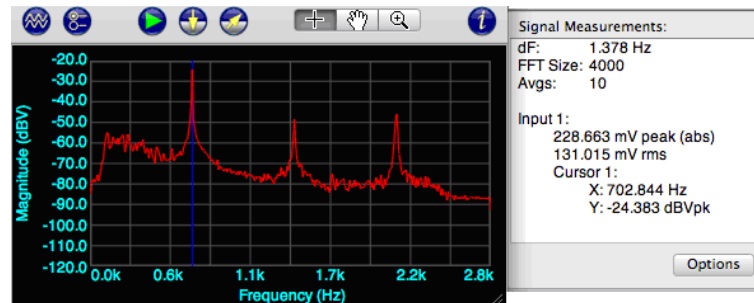
**Figure 2** Measurement of the distance between the bore and the holes.

The sound wave was recorded in SignalScope while the recorder was played. For each

trial, the pressure of the air was controlled, in order to keep the frequency as consistent as possible. The hole on the bottom of the recorder was always covered. The six holes aligned on the top of the recorder allowed seven notes (lengths) to be investigated; the frequency measurement was repeated five times for each note. Note that for the final note (all holes covered), the end of the standing wave occurred at the end of the recorder, rather than at a hole. The end correction at a hole is expected to be different than at the end of the recorder

## Results

A Fast Fourier Transform of the recorded sound wave was performed, as shown in figure 3. The fundamental frequency was measured for each note played.

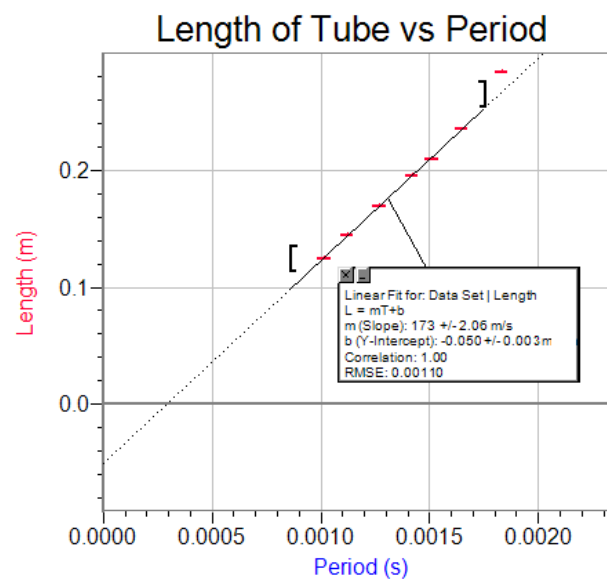


**Figure 3** The fundamental frequency, 703 Hz, of the wave produced when the length between the bore and the first open hole was 0.196 m.

The relationship between the length of the tube and the period of the sound is shown to be linear for the first 6 data points in figure 4. These correspond to the standing wave ending at each of the 6 holes. The final data point is significantly above the linear fit.

Based on equation 4, it can be concluded that, since the graph is linear, both the velocity of the sound in the tube and the end correction of the standing wave are constant across the range of frequencies investigated. The slope of the graph is half the speed of sound in the tube, giving a velocity of  $347 \pm 4 \text{ m/s}$ . This value is equal, within uncertainties to the  $349 \text{ m/s}$  speed of sound in open air at  $30^\circ\text{C}$ .

The y-intercept of the linear fit indicates that the sum of the end corrections of the standing wave in the tube was  $0.050 \text{ m}$ , for notes played with an open hole. This gives an average end correction of  $0.025 \pm 0.02 \text{ m}$ . The average end correction was predicted to be  $0.007 \text{ m}$ . The experimental results are significantly greater than the



**Figure 4** The relationship between the effective tube length and the period of the standing waves is linear. The slope of the graph equals half the speed of sound, and the y-intercept equals the end correction.

predicted values. In predicting the end corrections, it was assumed that the bore and the hole behaved similarly to the open end of a tube. Due to the small size of the hole and the bore, there may be significant variation in pressure there, compared to the pressure nodes which would exist near the end of an open tube. It can be concluded that the end correction for a standing wave ending at a small opening in a tube is significantly greater than the end correction at the end of a tube.

The end correction for the note with all six holes covered is the end correction at the open end of the recorder. This end correction is much smaller than that for the notes with an open hole, since this data point lies above the linear fit for the other 6 notes (see figure 4). Using the linear fit, the end correction is calculated to be  $0.037 \pm 0.004\text{m}$ . The end correction at the end of the tube is expected to be smaller than that at a hole, since the pressure node is able to form at or near the end of an open end, while the pressure will vary significantly due to the constricted movement of air at a small hole. The calculated end correction at the end of the recorder is, however, much larger than the end correction predicted at the end of an open resonating tube ( $0.006\text{ m}$ ).

## Conclusion and Evaluation

The effective tube length in a recorder was found to be linearly related to the period of the sound produced for all notes. The velocity of the sound in the tube was found to be constant over the range of notes investigated, within uncertainties, and was found to be equal to the speed of sound in open air, within uncertainties. The end correction of the standing wave was found to be constant for the 6 notes played with an open hole, within uncertainties.

The theoretical value of the end correction for notes with an open hole ( $0.0070\text{m}$ ) is smaller than the measured end correction ( $0.050 \pm 0.003\text{m}$ ) at the holes and bore since the hole and bore are small openings. The end correction at the end of the tube is  $0.037 \pm 0.004\text{m}$ ; the predicted end correction is  $0.0060\text{m}$ . The measured end correction for the end of the recorder is greater than the theoretical end correction at the end of a tube by a factor of 6. The reasons for this are unclear.

It is recommended that further work be conducted to determine the relationship between hole-size and end correction for a recorder-like instrument. A simplified recorder made from a tube of constant diameter which could be drilled with holes of different diameter could be used to investigate this.

It is also recommended that this work be repeated with a recorder to confirm these results due to difficulties encountered in obtaining reliable data. The embouchure was difficult to control for each trial. This may have affected the pressure of the air column against the bore, thereby affecting the frequency of the sound produced. It is hard to reduce this effect, due to the nature of wind instruments. Further study could also be conducted on the behavior of other wind instruments like flute and clarinet.

**References**

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