The Effect of the Width of an Aluminum Plate on a Bouncing Steel Ball

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Abstract

The effect of the distance between clamping supports of an aluminum alloy plate on the coefficient of restitution of a bouncing steel ball was investigated. The plate was supported on two wooden blocks with a meter stick secured on either side. A steel ball was dropped from a constant height and a motion detector was used to find the coefficient of restitution. Measurements were made with the wooden blocks at a range of distances. It was found that as the distance between the wooden blocks increased, the coefficient of restitution decreased linearly.

Introduction

When a ball bounces off a solid surface, the kinetic energy is momentarily converted into elastic potential energy, then back into kinetic energy as the ball leaves the surface. The loss in kinetic energy after the bounce can be represented by the coefficient of restitution, which is a fractional value representing the ratio of the incoming and rebound velocities. This value ranges from 0 to 1, with 0 representing a completely inelastic collision (no bounce) and 1 representing a completely elastic collision (the bounce height equals the drop height). The formula for the coefficient of restitution is as follows, where *h* is the bounce height and *H* is the drop height.[1]

$$C_R = \sqrt{\frac{h}{H}} \quad (1)$$

In a paper published in the Journal of Applied Mechanics, R. Sondergaard, K. Chaney, and C. E. Brennen[2] investigated the coefficient of restitution of various steel spheres as a function of the distance of the impact point from the clamping supports when dropped onto a 1.27 cm thick rectangular lucite plate. As shown in Figure 1, it was determined that the coefficient of restitution decreased as the distance of the impact point from the clamped edges increased, until a critical distance at which the coefficient of restitution reached a plateau with negligible change. They theorized that the coefficient of restitution is dependent on the impact distance from the supports when there is sufficient time for the wave generated from the impact to travel through the plate to the support, and then back to the impact point. The shortest impact

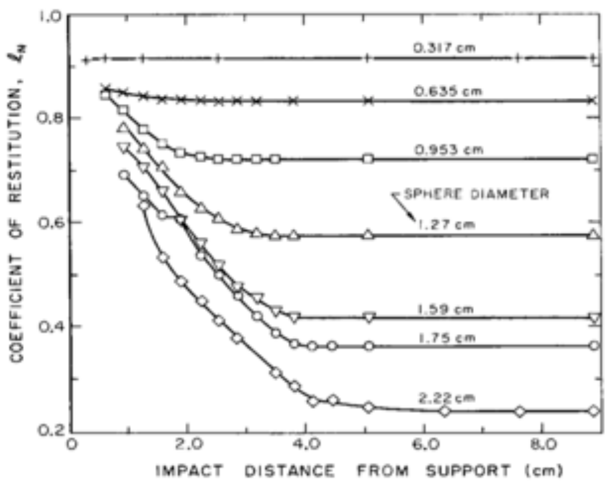

Figure 1 Coefficient of restitution as a function of the distance of the impact point from the clamping support for various steel balls impacting a 1.27 cm thick lucite plate from a drop height of 63.5 cm.[1]

distance that does not allow for this to happen is the critical distance. When the impact distance exceeds the critical distance, the distance of the impact from the supports has no effect on the coefficient of restitution.

This investigation builds on the work of Sondergaard et al. in that, while they manipulated the impact point of the sphere on a lucite plate with fixed distance between clamping supports, in this paper the distance between supports on an aluminum alloy plate was altered. Changing the distance between clamping supports only changed the effective width of the impact area of the plate, not the impact location relative to this area; the impact point remained centered, meaning that the impact produced surface waves that were symmetrical about the impact point. By changing the impact location but keeping the dimensions of the impact area constant, as Sondergaard et al. did, the impact was not always centered, and thus produced surface waves with different patterns. Some waves took less time to travel to the support and back to the impact region, while other waves took longer, due to the unequal distances from the impact point and either of the supports. Due to the complex theoretical basis of this situation, this paper will be empirical, focusing on presenting data and observations.

Method

A motion detector was secured to a stand using clamps so that it was hung at a fixed height above the aluminum alloy plate as shown in Figure 2. Two wooden blocks were placed parallel to each other and along the edges of two tables. The rectangular aluminum alloy plate, whose dimensions are given in Figure 3, was placed on top of the wooden blocks. A meter stick was secured on either side of the plate along the edge of the block and the two sides of the plate were solidly clamped onto the blocks. The free middle portion of the plate, which was the effective width of the plate, was adjusted to 6 different widths ranging from 5.0cm to 28.0cm (±0.2cm) wide. A steel ball with a mass of 44.64±0.01g and diameter of 25.20±0.01 mm was dropped onto the center of the metal plate from a height of 17.0cm (±0.5cm) by means of a string taped onto it. The motion detector recorded heights of the ball before and after the impact, as shown in Figure 4.

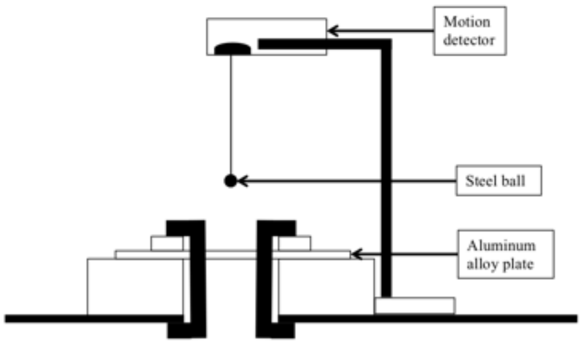

Figure 2 Setup of the investigation.

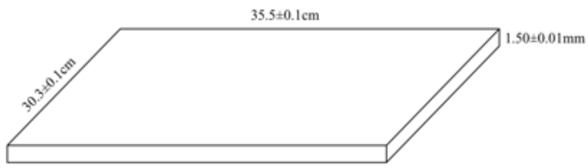

Figure 3 Dimensions of the aluminum alloy plate.

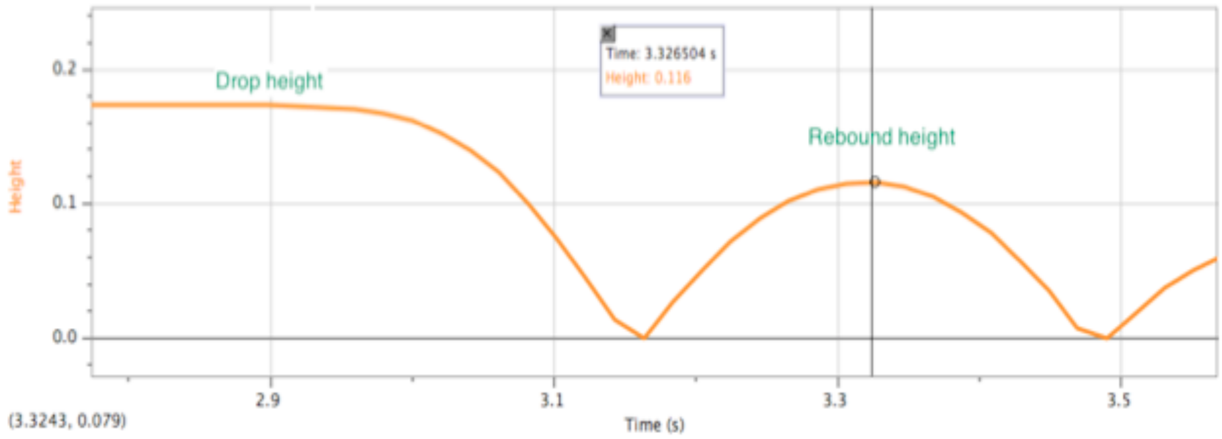

Figure 4 A sample of the height-time graphs used to determine the drop height and rebound height. The drop height is the initial plateau height, while the rebound height is the maximum height of the first parabola.

Results and Discussion

Figure 5 shows the relationship between the distance between the effective width of the aluminum plate and the coefficient of restitution when a steel ball is dropped onto an aluminum alloy plate.

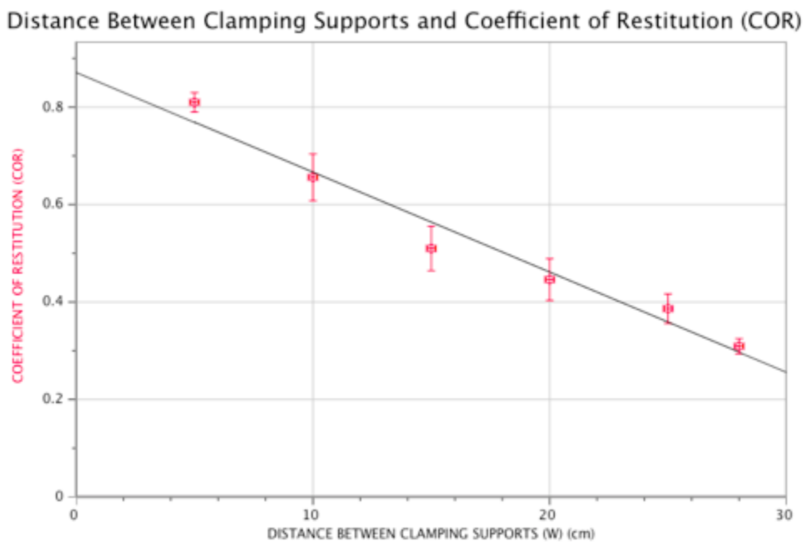

Figure 5 This graph demonstrates the average coefficient of restitution as a function of the width of the plate, modeled by a linear function.

Within the range of values tested and with the particular conditions of the investigation, the relationship can be modeled by a linear equation, despite some fluctuation in the coefficient of restitution values. Analyzing the physical model of the experiment shows that this is a plausible finding. It is clear that a clamped elastic plate experiencing a perpendicular impulse will experience a displacement. The greater the distance between the supports, the lower the effective spring constant, and the greater the expected displacement. Greater displacement of the plate is expected to lead to greater loss of energy within the plate during its displacement and rebound, consequently a lower coefficient of restitution. However, this linear fit is an empirical fit to the measured data, so there is a need for a more complete theoretical model to be developed.

Despite the different plate material and procedures used in the paper by Sondergaard et al., according to Figure 1 there is evidence of a similar linear relationship with impact distances below the critical distance. However, in this investigation, there is no conclusive evidence to support the theory of a critical distance because the range of manipulations was not sufficient.

One of the major issues in the design of this investigation was the fact that clamped wooden meter sticks were used as the two supports. Because wood is not a rigid material, it is possible that the plate material beyond the support was displaced during the bounces as well. In order to reduce this error, it is essential to ensure that the plate is clamped in such a way that only the portion of the plate between the two clamps is displaced during the bounce. Furthermore, because the steel ball was dropped manually, there were inconsistencies in the impact point on the plate. This can be improved by using a mechanical release device, which can allow for more consistent dropping heights and impact points. It is suggested that further research investigate the effect of the plate thickness on the coefficient of restitution. The effect of the material and dimensions of the ball must also be more fully investigated.

Conclusion

As the distance between the clamping supports on an aluminum alloy plate increases, the coefficient of restitution of a steel ball decreases. This can be empirically modeled with a linear relationship. Note that for this situation, the linear relationship is only applicable for distances from 5cm to 28cm between the clamping supports. A wider aluminum alloy plate would have to be used to predict the relationship between the two variables of distances greater than 28cm between the clamping supports. The authors are aware of no theoretical model that predicts this relationship, but it is empirically valid for this situation. When compared to Sondergaard et al.'s results, this investigation provides no evidence for a critical distance, though it is recognized that different procedures and plate materials were used.

References

[1] Bennett, Jamin and Ruwan Meepagala. "Coefficients of Restitution." *The Physics Factbook.* N.p., n.d. Web. 8 Apr. 2013. <http://hypertextbook.com/facts/2006/restitution.shtml>.

[2] Sondergaard, R., K. Chancy, and C.E. Brennen. *Measurements of Solid Spheres Bouncing Off Flat Plates*. Journal of Applied Mechanics 57 (3): 694-695.