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Pipe Diameter and End Correction of a Resonant Standing Wave

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Abstract

To investigate the relationship between the diameter of a pipe and the end correction of a standing wave created in the pipe, five plastic pipes with different diameters were cut to 0.402 m in length. A speaker emitting a single frequency was held near one end of the pipe and the frequency adjusted until the loudest resonance was heard. It was found that there is a proportional relationship between the diameter (*D*) and end correction (*C*), as modeled by the equation $C = 0.33D$.

Introduction

When air resonates in a tube a standing wave is formed in the air column, as shown in figure 1. In order to accurately determine the wavelength of a wave at the first harmonic with an anti-node at each end, the end correction must be taken into account. The equation for the wavelength of the longest possible standing wave in a tube open at both ends is

$$\lambda = 2L + 4C \qquad \text{(Equation 1)}$$

where λ is wavelength, L is the length of the tube, and C is the end correction.

While it is widely acknowledged that the relationship between end correction and diameter of tubing can be illustrated by the equation

$$C = xD \qquad \text{(Equation 2)}$$

where D is the tube diameter, there is confusion over the value of the constant, x. In a paper published in the Physical Review, Herbert Anderson and Floyd Ostensen review work done on end corrections and present their own results. Rayleigh theorized, based on both his work and that of Bosanquet, that the end correction of a cylinder can be found with x = 0.3, if the pipe has two open ends and a high λ/D value is employed. This value is not always consistent with experimental results.

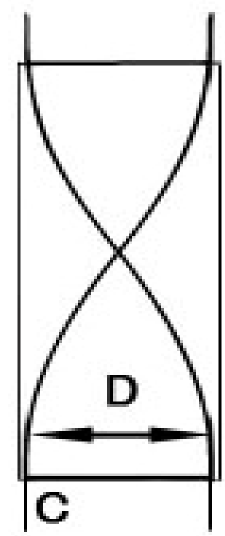

Figure 1 End correction (C) in the first harmonic for a tube with two open ends.

Wertheim found that x = 0.332, while Bosanquet's conclusion was that x = 0.318 for λ/D = 6, and x = 0.272 for λ/D = 15. According to Anderson and Ostensen, these experiments cannot be considered very reliable. They also leave a range of λ/D values to be tested. Ostensen and Anderson found that for pipes open at one end, with a range of λ/D values from 9 to 30, x = 0.30.

In this research, end correction will be determined for resonating pipes of different diameters with two open ends. It is expected that the results will confirm the proportional relationship between the tube diameter and the end correction. It is also hoped that the proportionality constant can be more precisely determined, thereby settling the disagreements in values presented in Anderson and Ostensen's paper.

Method

Five plastic pipes with diameters ranging from 0.018 m to 0.075 m were cut to 0.402 ± 0.003 m in length. In order to find the frequency at which the first harmonic is created, each pipe was set on top of two supports, as shown in figure 2. A speaker was connected to a laptop with a frequency generator program and the speaker was held near one end of the pipe. One person placed their ear near the other end of the pipe and the frequency emitted by the speaker was adjusted until the resonance was at its loudest point. A range of 5 Hz was recorded because the listener was unable to pinpoint the exact frequency of the loudest resonance. Each diameter was tested five times. The room temperature was 25.0 ± 0.5°C for the duration of the investigation.

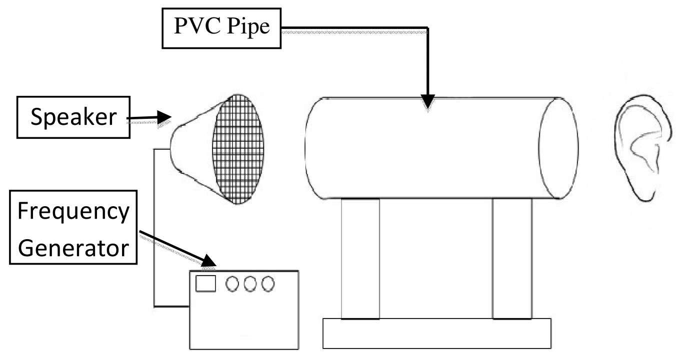

Figure 2 Setup used during the investigation

Results and Discussion

Figure 3 shows the relationship between pipe diameter and end correction is

$$C = 0.33D \qquad \text{(Equation 3)}$$

The relationship is proportional, as expected. The proportionality constant is in agreement with Wertheim's results. Anderson and Ostensen's result of 0.30 is not supported by these results, although it must be noted that their results were for a tube open on one end only.

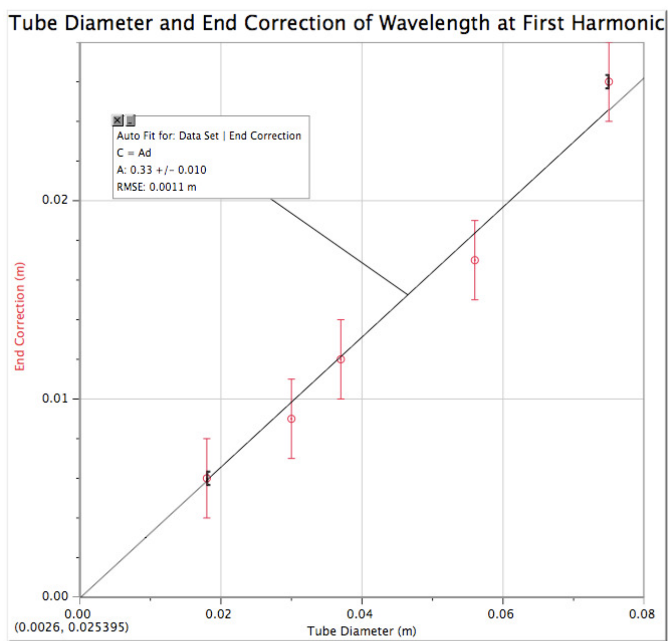

Figure 3 A proportional relationship between end correction and tube diameter is shown, with a proportionality constant of 0.33.

Table 1 illustrates the range of λ/D ratio values for which the proportionality constant x= 0.33 applies. This is a wider range than any of the previous studies mentioned in Anderson and Ostensen's paper. It also includes many of the λ/D values they, and others, have tested.

Pipe Diameter (m)	λ/D ratio
0.018	45
0.030	27
0.037	22
0.056	14
0.075	11

Table 1 The λ/D ratio for pipes of differing diameters

One of the major issues in the design of this investigation was using the ear to determine the frequency at which the loudest resonance occurred. This affects the precision of the data collected, and could possibly be improved by using a decibel meter. Another issue was the relative position of the speaker near the end of the tube. While this seemed to have no noticeable effect on the resonant frequency, more precise measurements should be made to confirm this. Further research could also be done in different mediums. In this research, only air columns were tested, but other gases could also be investigated. Finally, only the first harmonic was investigated in this research. The end correction produced by different harmonics in the same pipe could also be tested to further the understanding of this topic.

Conclusion

The end correction of a standing wave in a cylindrical pipe is proportional to the diameter of the pipe and can be modeled by equation 3 for λ/D ratios ranging from 11 to 45.

Equation 3 indicates that Wertheim was the most accurate in his conclusions. The conclusions of this research are not in agreement with those drawn by Rayleigh, Bosanquet, and Anderson and Ostensen. In addition to settling these conflicting results, this research has determined the value of x for the previously unreported range of λ/D ratios from 31-45.

References

[1]Anderson, S. H., and Floyd C. Ostensen. *Effect of Frequency on the End Correction of Pipes.* Tech. Vol. 31. 1928. Print. Physical Review