

Impulse in a Double Ball Bounce

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Abstract

A steel ball was dropped onto a handball which was resting on top of a horizontal Vernier force-plate. The relationship between the impact momentum of the steel ball and the impulse on the force-plate was investigated. The measured impulse was found to be proportional to the impact momentum of the falling ball, with the impulse being 2.3 times greater than the momentum. The impact of the metal ball caused the handball to compress and then bounce up following the steel ball's bounce. This contributed to the measured impulse being greater than twice the impact momentum.

Introduction

A bouncing ball will exert an impulse on the surface on which it bounces. Impulse and change in momentum during a collision are equal. Newton's Second Law^[1] describes this relationship with the equation

$$F \cdot dt = m \cdot dv \quad \text{(Equation 1)}$$

where F is force, t is time, m is mass and v is velocity. The impulse on the surface will be one to two times the value of the ball's impact momentum, depending on the elasticity of the bounce. For an inelastic collision, such as a lump of clay hitting the ground, the impulse would equal the change in momentum. In a perfectly elastic collision, such as a ball rebounding at its impact speed, the impulse would be twice the impact momentum. Upward and downward speeds would be equal meaning a coefficient of restitution (e) of one. The e for a ball bounce is the ratio of the rebound speed to the impact speed^[2]. For a constant e , the impulse would be expected to be proportional to the overall velocities. The ratio between impulse and impact momentum would depend on the e of the ball.

Instead of a metal ball bouncing on the ground, a metal ball will be bounced off a handball resting on the ground. What might be expected is that the impulse will be close to twice the impact momentum of the metal ball. If the e is close to 1 for all impact velocities then the impulse would be nearly double the impact momentum in all trials. A proportional relationship between impulse and impact momentum would be expected if the coefficient of restitution is constant – independent of the impact momentum. Impulse as a function of the impact momentum of the metal ball will be investigated in the situation described above.

Methods

A handball, pumped to a gauge pressure of 50 ± 5 kPa, was placed on a force-plate and loosely secured using tape, as shown in Figure 1. This apparatus was placed on the floor beneath a ring-stand. A steel ball of mass 0.537 ± 0.001 kg was attached to a short string using tape. The string was used to eliminate spin in the ball during the fall.

The steel ball was lifted to displacements ranging from 0.20 m to 0.70 ± 0.01 m above the top of the handball, using the clamp as a reference point. Logger Pro was set to record data and the ball was dropped onto the center of the handball on the force-plate. Four measurements were made for each value of displacement.

The force against time graph for each trial was analyzed using Logger Pro. The integral of the first force peak was taken, giving a value for impulse, as shown in Figure 2.



Figure 1: The handball loosely taped to the black force-plate.

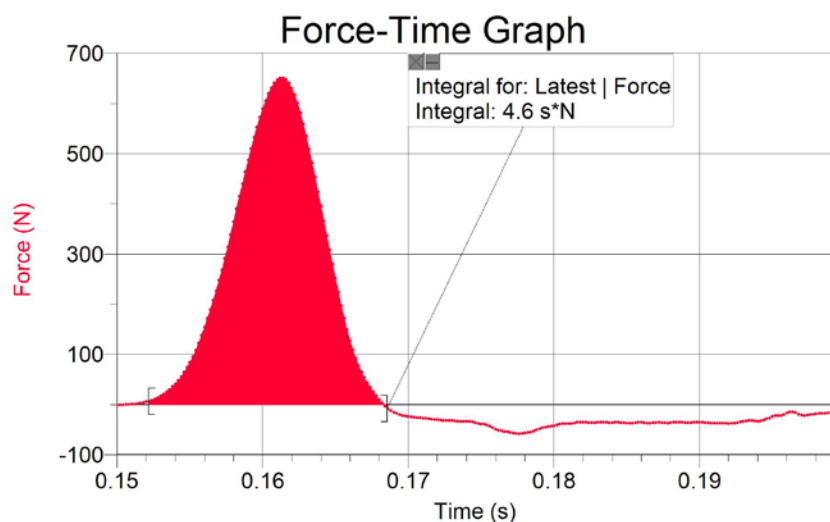


Figure 2: The integral of the force-time graph was taken to determine the impulse on the force-plate.

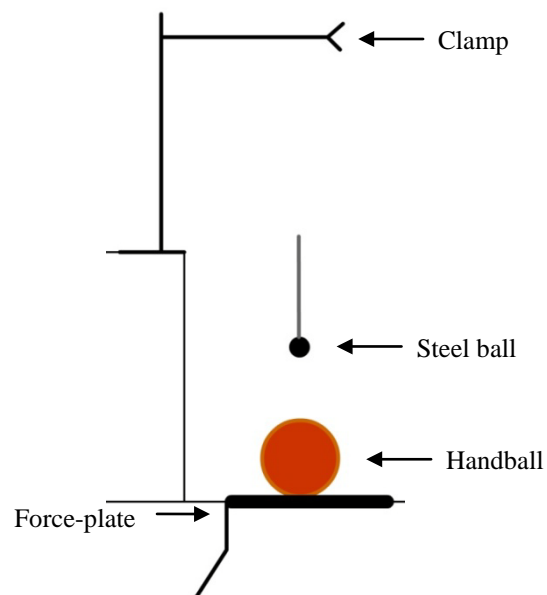


Figure 3: The experimental setup.

Results and Discussion

As shown in Figure 4, the relationship between the impulse on the force-plate and the impact momentum of the ball is proportional. This means the effective coefficient of restitution for this situation with the double ball bounce is constant over the range of velocities tested. The relationship between impulse and impact momentum is shown to be

$$J = 2.3 \cdot p \quad (\text{Equation 2})$$

where J is the impulse exerted on the force-plate and p is the impact momentum of the metal ball. A slope of 2.3 suggests the ball rebounded with a speed greater than the downward speed, this was not the case. The slope being greater than two can be explained by the role of the handball in the situation.

The metal ball compressed the handball, moving the handball downward as shown in Figure 5 frames A to C. The top and sides of the handball moved upward when it expanded back to its original shape. This is shown in frames E to F. The handball had therefore gained upward momentum during the bounce. The force-plate had thus provided an impulse for the bounce of the metal ball and also provided an impulse to the handball as it expanded and sprung off the force-plate. The impulse provided by the force-plate to the bounce of the metal ball would be between one and two times the impact momentum of the metal ball. The handball's momentum registered an additional impulse on the force-plate, resulting in the slope of above 2.0 in Equation 2.

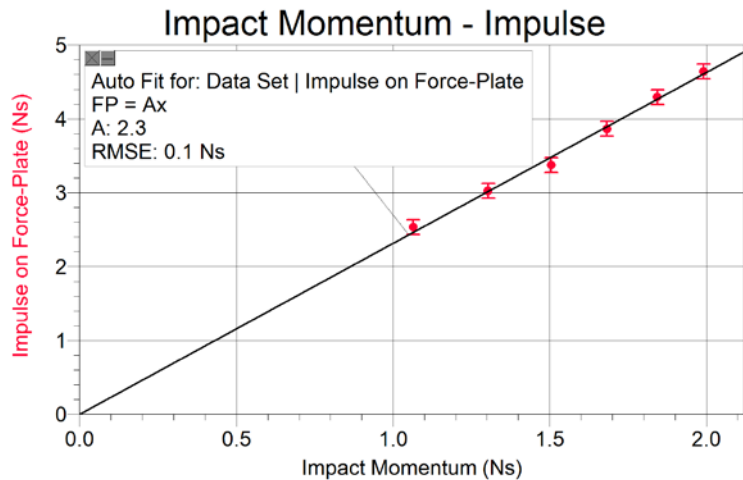


Figure 4: The proportional fit of the graph for impact momentum against impulse.

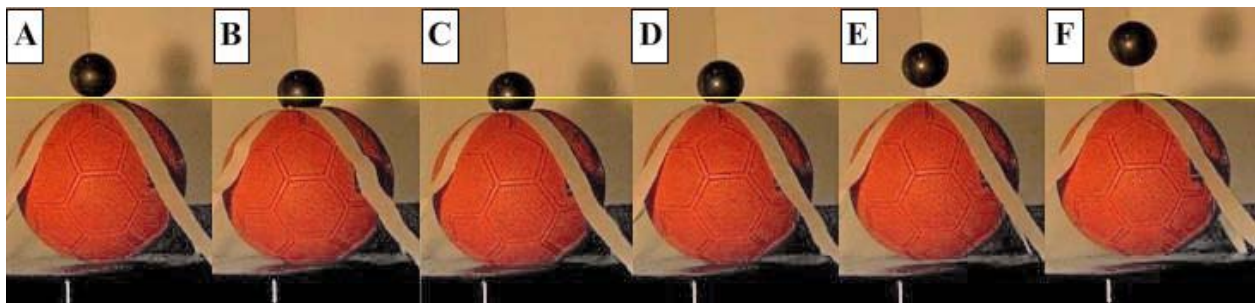


Figure 5: The compression of the handball when the metal ball struck and the subsequent upward velocity of the handball after the impact. Horizontal lines are added to highlight that the handball is bouncing up with the metal ball.

A limiting factor to the confidence of the results is that the metal ball did not hit the handball in the center in each trial. The distribution of force would have been affected. Non-vertical impulse may have been exerted on the force-plate, which cannot be measured by the force-plate. This effect was minimized by using the ring stand to increase the precision of the drop location. Further research is suggested to investigate the division of the impulses. The amount of impulse provided to the metal ball and the handball individually could be determined by using a video analysis of the motion to calculate the momentums and impulses exerted on each ball during the bounce. Whether the division of impulse is dependent upon the drop height of the metal ball or the handball's pressure could be studied. The response time of the force-plate may be investigated as a factor affecting the data.

Conclusions

When a metal ball is dropped on to a handball, the impulse varies proportionally with the impact momentum of the metal ball; the impulse is 2.3 times the impact momentum. A value of 2.0 would be expected for a situation involving a coefficient of restitution of one. The proportion of greater than 2.0 is explained by the additional impulse arising from the handball bouncing up following the metal ball.

References

^[1]Zhang, Y., Finger, S., & Behrens, S. (n.d.). *Introduction to mechanisms*. Retrieved June 2, 2010, from <http://www.cs.cmu.edu/~rapidproto/mechanisms/chpt1.html>

^[2]Weisstein, E. W. (2007). *Coefficient of Restitution*. Retrieved June 25, 2010, from <http://scienworld.wolfram.com/physics/CoefficientofRestitution.html>