

Air Pressure and the Coefficient of Restitution of a Ball

Kareem Osman and Boree Sophie Kim

Abstract

The relationship between the internal air pressure and the coefficient of restitution was investigated for an inflated rubber ball. The ball was dropped and its position was tracked with a motion detector. The velocity, before and after impact, and the internal pressure were determined. It was found that the pressure and the coefficient of restitution were exponentially related, with the coefficient of restitution approaching a maximum value at higher pressures.

Introduction

The coefficient of restitution is defined as the ratio of relative velocities in a two-body collision. When one body is static, for instance the Earth, it simplifies to the ratio of the initial velocity to the final velocity of the object after impact. The coefficient of restitution is given by:

$$C_R = \frac{-v_f}{v_i} \quad (\text{Equation 1})$$

where C_R is the coefficient of restitution, v_f is the velocity of the object after the collision, and v_i is the velocity before the collision.^[1]

In this investigation, the effect of the internal air pressure on the coefficient of restitution when an inflated rubber ball bounces on a hard floor will be determined. It is predicted that the value of C_R will approach a maximum at high pressures because the collision will never be perfectly elastic due to hysteresis in the rubber.

The ideal gas law is a model of real gases at low pressure and thus defines an ideal gas. An ideal gas obeys the equation:

$$PV = nRT \quad (\text{Equation 2})$$

where P is the absolute pressure of the gas in Pascals, V is the volume in cubic meters, n is the moles of gas, R is the gas constant ($8.31 \text{ m}^3\text{PaK}^{-1}\text{mol}^{-1}$), and T is the absolute temperature of the gas in Kelvin.^{[3][4]}

Although air is not an ideal gas, at temperatures close to 300 K and pressures between one and three atmospheres, it approximates an ideal gas. For this investigation, air is assumed to be an ideal gas, and thus can be modeled by the ideal gas law.



Figure 1 The ball used in the investigation.

Methods

A rubber handball was inflated to atmospheric pressure. At an internal pressure of 1atm, the ball had a diameter of 0.018 m. It was weighed on a balance and the circumference was measured with a measuring tape. The ball was held beneath a fixed motion detector with a single finger on each side, and the motion detector was started. The ball was released from a height of 0.160 ± 0.001 m below the motion detector.

The position of the ball was tracked as it was released and bounced off the floor. After five trials at each pressure, air was added, the weight and diameter were measured again, and the bounce was recorded five times. This was done for a total of seven pressures.

Two quadratic functions were fitted to the position-time graph of each ball bounce: one before the bounce and one after the bounce (see figure 2). In order to accurately find the velocities before and after the impact, the derivatives of the position-time function were evaluated for the instant before the bounce and the instant after the bounce.

In order to determine the internal pressure of the ball, the volume of air, moles of air, and temperature and humidity were measured. The volume was calculated using the circumference of the ball at each pressure, accounting for thickness of the skin of the ball. The number of moles of air in the ball was determined using the mass of the ball when inflated, the mass of the empty ball, and the molar mass of air, while accounting for the buoyant force on the ball in air.

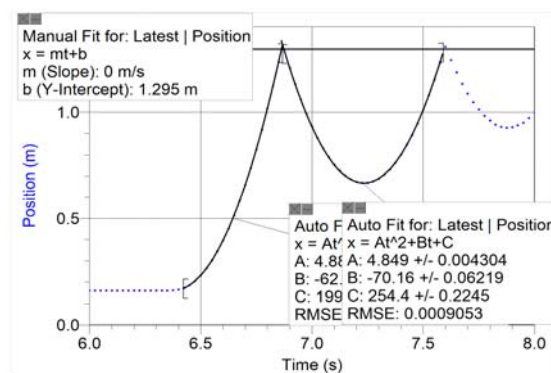


Figure 2 The derivative of each of the two quadratic functions was calculated, at the time when the ball was just beginning to touch the floor and the time right after the bounce. The position is the distance from the motion detector to the top of the ball.

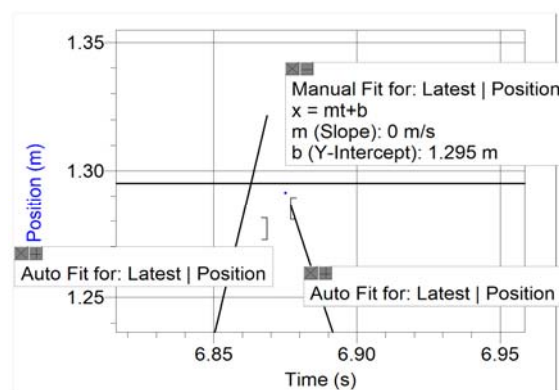


Figure 3 Within the uncertainty of 2 mm, the two curves intersect at the position of the floor, which was 1.295 m away from the motion detector.

Results and Discussion

From figure 4, the relationship between the internal air pressure and the coefficient of restitution is given by the equation:

$$C_R = (-40 \pm 10) * e^{(-4.5 \pm 0.4)P} + (0.89 \pm 0.01) \quad (\text{Equation 3})$$

There is an inverse exponential relationship between the pressure and the coefficient of restitution.

For the ball used in this experiment, bounces could not be obtained at pressures below 1.0 atm. At the internal gas pressure of 1.0 atm, the coefficient of restitution was 0.45. This was due to the momentary compression of the ball, which caused an increase in the internal pressure during the bounce. The maximum value of gas pressure investigated in this experiment was 2.5 atm, as the ball was incapable of withstanding higher pressures. The maximum limit of the coefficient of restitution for the ball is shown to be 0.9. This is less than 1.0, as expected. Even at very high pressures, there will be inevitable loss of energy due to compression of the gas in the ball and hysteresis in the rubber walls of the ball

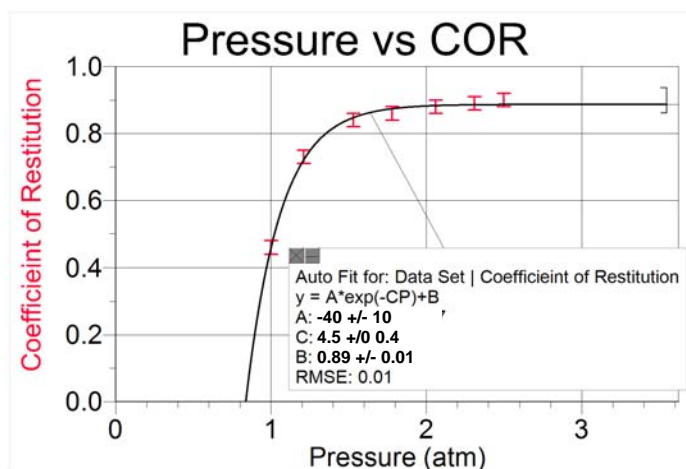


Figure 4 The internal pressure of the ball is plotted against the coefficient of restitution. An inverse exponential relationship is shown, within the limits of uncertainty.

loss of energy due to compression of the gas in the ball and hysteresis in the rubber walls of the ball

Experiments using a different ball or surface are predicted to show a similar inverse exponential relationship, with the asymptote shifting up if the collision is more elastic. Tests conducted at different temperatures or with different gases are expected to show the same general trend, but again with different asymptotes, depending on the conditions. The actual value for the maximum coefficient of restitution would depend on the ball, the surface, the temperature, the gas in the ball, and the gases present in the atmosphere.

One weakness to be addressed relates to the procedure used to measure the circumference of the ball. For each pressure, the circumference was measured at only one location on the ball with a measuring tape. Even though three trials were conducted and the average value was recorded, there was difficulty ensuring that the tape went around the exact middle of the ball. Additionally, the ball might not have been truly spherical, so the combination of these two factors might have resulted in error in the volume calculation. While this would not have changed the trend of the results, it would affect the constants derived in equation 3.

Further research is suggested for a variety of inflated ball types and sizes, to increase the level of confidence in the relationship. By using a sturdier ball, the effect of gas pressure on the coefficient of restitution at internal pressures lower than 1.0 atm or greater than 2.5 atm could be determined. Additionally, tests could be conducted with internal pressures between 1.0 and 1.4 atm, to confirm the nature of the relationship in that pressure range. Research could also be conducted to determine the effect of the speed of impact on the coefficient of restitution.

Conclusion

The relationship between the internal gas pressure of a ball and the coefficient of restitution in collision with a static body was explored. According to the results, there is an inverse exponential relationship between the pressure of the air in the ball and the coefficient of restitution, approaching a maximum of 0.9 at high pressures.

References

[1] Coefficient of Restitution. Retrieved May 31, 2009, from Wikipedia:
http://en.wikipedia.org/wiki/Coefficient_of_restitution

[2] Elastic Collision. Retrieved May 31, 2009, from Wikipedia:
http://en.wikipedia.org/wiki/Elastic_collision

[3] Ideal Gas Law. May 31, 2009, from Wikipedia:
http://en.wikipedia.org/wiki/Ideal_gas_law

[4] Ideal Gas Law. May 31, 2009, from Wikipedia:
http://en.wikipedia.org/wiki/Gas_constant