Hole Size in a Spherical Resonator

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Abstract

When air is blown strongly through a straw and across a hole in a hollow sphere, a high-pitched whistling sound is heard. This paper tests two models, Helmholtz Resonance and Spherical Harmonics, to determine which most accurately models this phenomenon. This was done by measuring the frequencies produced when air was blown across identical spheres with different hole sizes, as well as across spheres of different volumes with identical holes. The frequencies were found to closely match frequencies predicted by spherical harmonics.

Introduction

Some websites claim that whistling and whistles operate as Helmholtz resonators[1,2]. If a whistle is blown softly then a certain frequency is produced, however if the whistle is blown harder, then a louder, higher frequency sound can be produced. The soft, low frequency, sound produced when air is blown across a hollow sphere with a hole in it has been shown to be accurately modeled by Helmholtz resonance.[3] Here we investigate the high frequency sound produced from blowing hard on a hollow sphere with a hole in it. The effect of the hole and its size on the frequencies produced will be investigated. It will be determined if this higher frequency sound can be modeled by Helmholtz resonance.

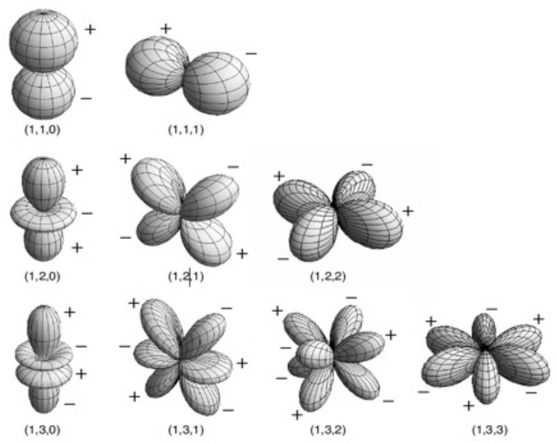

Figure 1 The different shapes of the harmonics of a spherical cavity for the three lowest mode indices are indicated below the figures in (n ,l ,m). Each row represents a different harmonic, and thus each row has a different frequency. The phase of the oscillation is indicated by the plus or minus signs.[5]

An alternate model, the applicability of which will be tested, is spherical harmonics. Spherical harmonics explain the noise created as a standing wave within the sphere. The “white” noise of the air being blown into the hole creates a standing sound wave inside the cavity with the frequencies present depending on the dimensions of the cavity. To test if spherical harmonics govern this phenomenon the frequencies produced by hollow spheres of differing volume will be analyzed.

For this investigation the equations for finding the speed of sound can be used with a known sound velocity to calculate the frequency assuming spherical harmonics.

The simplified equation for finding the speed of sound in a sphere if the volume is known and the frequency is found, is:

$$f_{ln} = z_{ln}\left(\frac{c}{2\pi r}\right)$$ Equation 1[4]

Where f_{ln} is the frequency, z_{ln} represents discrete harmonics created by three-dimensional harmonic shapes, c is the speed of sound, and r is the radius of the spherical cavity. The harmonics recorded are determined by the vector coordinates, and the number of modes in the sphere as in figure 1.

Methods

A circular hole of diameter 6.4 mm was drilled in six hollow plastic spheres with volumes ranging from 26ml to 1135ml and average wall thickness of .0005m ± .0003. A microphone was connected to a computer and set at a data collection rate of 100,000 samples per second for 0.5 seconds. Air was blown strongly through a straw and across the hole in the sphere at an angle such that a high-pitched whistling sound was made. The sound was recorded on the computer and an FFT graph was produced. This was done 6 times for each of the six spheres. It was noted that a lower frequency could be obtained under these conditions by blowing more softly, but this frequency was not studied.

To test the effect of the diameter of the hole on the resonant frequencies produced, six identical hollow, plastic spheres with a volume of 220ml ± 10 and a wall thickness of 0.00035m ± 0.00005 were drilled with hole diameters ranging from 5.0mm ± 0.5 to 13.0mm ± 0.5. The previous method was again used to record the frequency produced by each of the six spheres with differing hole sizes.

The temperature of the air in the room was 27°C ± 1 whereas the temperature of the air inside the sphere while being blown was 32°C ± 2. The speed of sound[6] in the sphere was calculated to be 350m/s ± 3.

Results & Discussion

In figure 2, the relationship between the period of the sound and the volume of the cavity is shown, to test if the relationship predicted by the Helmholtz equation describes this phenomenon.[7] Not only does the best proportional fit not pass through a single point or its error bars, the proportionality constant is very different than

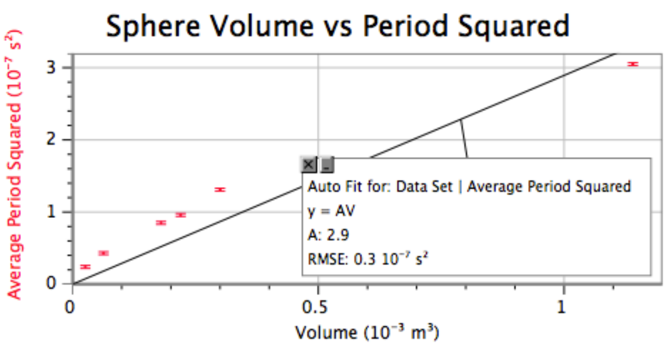

Figure 2 The average period squared of the sound produced as compared to the volume of the sphere producing the sound.

that predicted by the Helmholtz equation. Using the given constants, the predicted slope is 500×10^{-4} s^2m^{-3}, compared to the slope of the best fit line of 2.9×10^{-4} s^2/m^3. This clearly indicates that this phenomenon is not accurately modeled by the Helmholtz equation.

Figure 3 is a sample FFT graph of the recorded sound. The multiple harmonics produced provide further evidence that Helmholtz Resonance does not describe the higher, whistling frequencies, as the Helmholtz model predicts no harmonics. As it has been shown that Helmholtz resonance does not correctly describe the high-pitched whistle phenomenon, spherical harmonics will be tested for validity as a model for the phenomenon investigated.

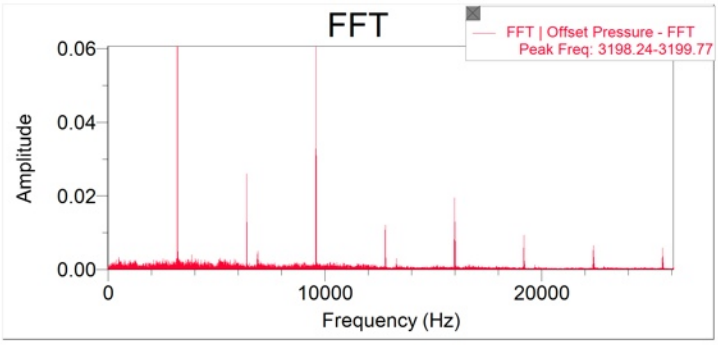

Figure 3 the frequency as compared to the amplitude of the sound made by the 217 ml sphere with the 4mm radius hole

It is known that the spherical harmonic equations describe an inversely proportional relationship between the radius of the sphere and the frequency of the harmonics.[5] Theoretical frequencies for a sphere of radius 0.116 m were obtained[5] and used to determine the theoretical spherical harmonic frequencies for a sphere of radius 0.037 m. The predicted frequencies of all of the possible harmonics produced from the sphere were then compared to the found frequencies. Figure 4 clearly shows that spherical harmonics accurately models the phenomenon investigated, as the points of the measured periods and the predicted periods are very close together. All of the periods experimentally found are slightly lower than their corresponding theoretical frequency.

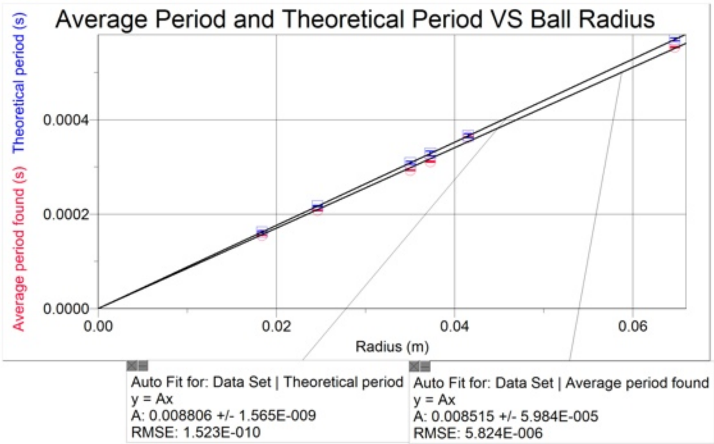

Figure 4 The periods of sound produced by the spheres and the theoretical periods as predicted by spherical harmonics, as compared to the radii of the balls, for the first resonance mode.

Given that the equation used to calculate the frequencies of the spherical harmonics assumed a sealed sphere, it is possible that the slight difference may be due to the hole in the sphere wall. Figure 5 shows that the frequency is not greatly affected by the hole size. Though there is a small increase with increasing hole diameter, the total change in frequency is less than 2%.

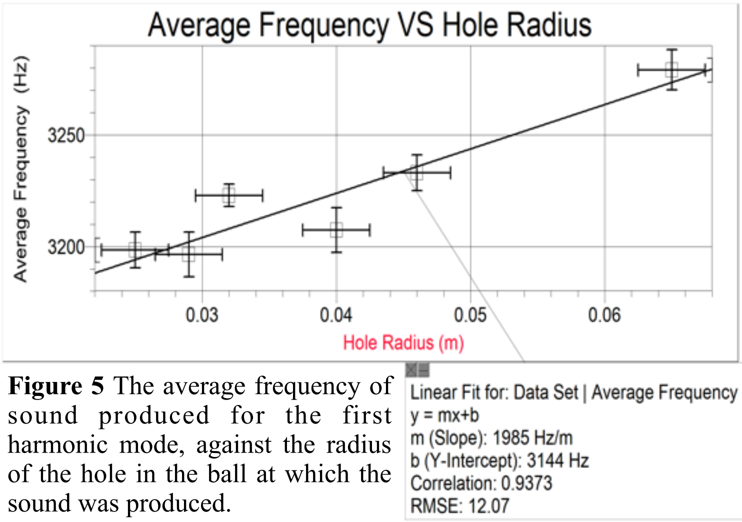

Figure 5 The average frequency of sound produced for the first harmonic mode, against the radius of the hole in the ball at which the sound was produced.

Finally, in figure 6 the recorded frequency is compared to the theoretical frequency for the most prominent four resonant modes of spherical harmonics. It can be seen that the average values are very close to the theoretical values predicted for those harmonics.

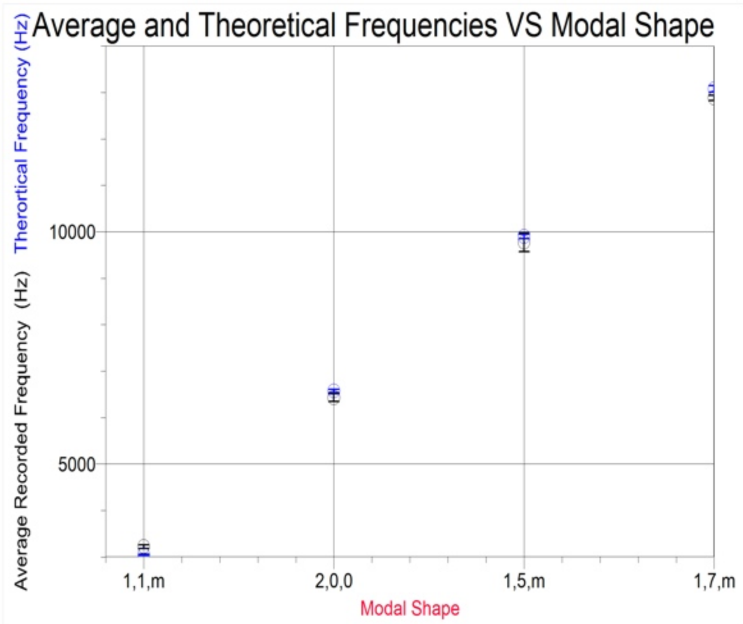

Figure 6 The variation between the recorded value (black) and the theoretical values (blue) of the frequencies for several modal shapes (x-axis). The recorded values represent the average frequency produced by the different hole sizes for the sphere with a volume of 217 ml.

It is interesting to note that the lower frequencies predicted by Helmholtz resonance were occasionally detected in some of the graphs. However few graphs showed a clear peak at the Helmholtz resonance frequency. Also, only for the smallest three hole sizes were peaks detected at the frequency predicted by the Helmholtz equation.

Another issue that should be noted is the fact that each of the trials gave a noticeably different FFT graph. The relative amplitudes of each of the modes of resonance were different each time, and in some cases, some modes were not detected. Since the straw was held by hand, and blown with the mouth, this suggests that the angle and velocity of the airflow could be an important factor in determining which of the resonance modes is present and which has the highest amplitude. It is suggested that a mechanical blowing apparatus be constructed so that the effect of wind angle and velocity could be investigated.

Conclusion

The spherical harmonic theory seems to accurately describe the phenomenon of the high-pitched whistling heard when air is blown strongly across a hole in a hollow sphere. It has been shown that this phenomenon is not modeled by the Helmholtz resonance equation. As all resonant frequencies measured were slightly lower than the predicted values, it appears that the presence of the hole causes the frequency to be lower than theoretical predictions. However, as the diameter of the hole increases it seems that the frequency increases as well, bringing the frequency produced closer to the theoretical values for all but the first harmonic.

References

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