Formation of Craters in Sand

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Abstract: The diameter of craters formed by spheres of varying mass dropped into sand at low speed was studied. The relationship between the diameter of the crater formed and the kinetic energy of the projectile at impact was found to be of the same general form as that for planetary meteor craters. The relationship is shown to be a power law with exponent 0.17.

Introduction

When a ball is dropped into sand, a circular crater is formed due to the impact of the ball. The size of the crater varies with the kinetic energy of the ball at impact. In this research, the relationship between the kinetic energy of a modified table tennis ball, dropped into sand from a fixed height, and the diameter of the crater formed will be investigated.

The size of a meteor crater is related to the amount of energy the meteor possesses as it strikes the ground. The more energy it possess, the larger the crater diameter. According to the Palaeontology Department, University of Bristol website, the relationship between the kinetic energy of a meteor at impact and diameter of the resulting crater on earth is:

$$D = 0.07 Cf (E \cdot \frac{\rho_a}{\rho_t})^{1/3.4}, \tag{1}$$

where D is the crater diameter, Cf is the crater collapse factor, ρ_a is the density of the projectile, ρ_t is the density of the target rock, and E is the total energy of the meteor. *(http://palaeo.gly.bris.ac.uk/communication/Brana/impact.html)*

Equation 1 describes craters formed by meteors that collide at high speed with the surface of the earth. Given that Cf and ρ_t are constant (and unknown) in our research, and that both E and ρ_a are proportional to the mass, m, of our projectile, equation 1 can be simplified to the form:

$$D = k(E \cdot m)^{1/3.4} \tag{2}$$

where k and n are unknown constants. For the purpose of our research, E is defined as the kinetic energy at the instant the ball impacts the sand, and since the ball is being dropped from constant height, it is assumed that the ball is impacting with constant velocity. It follows that:

$$D = k(0.5 \cdot m \cdot v^2 \cdot m)^{1/3.4}$$

$$D \propto E^{2/3.4}$$

$$D \propto E^{0.59}$$

$$D = k \cdot E^{0.59} \tag{3}$$

According to equation 3, it is predicted that the diameter of a meteor crater is proportional to the energy of the meteor raised to the power of 0.59.   It is not known if the relationship based on meteor craters given in equation 3 will be applicable to our situation, which involves table tennis balls filled with lead shot and latex glue dropped into sand at low speeds. It is expected that the relationship between the kinetic energy of the projectile and the resulting crater diameter will be a power law, but with exponent different from that predicted by equation 3.

This research will attempt to determine the relationship between the kinetic energy at impact of a modified table tennis ball dropped from a fixed height into sand, and the diameter of the crater formed. It will also be determined whether it is the same as the relationship for meteor craters.
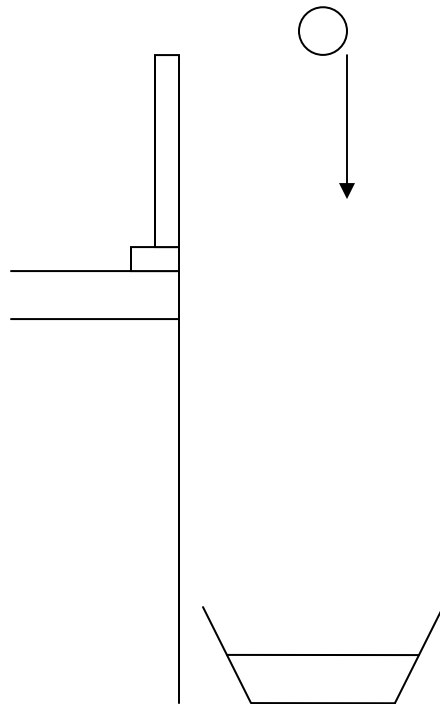
Methods

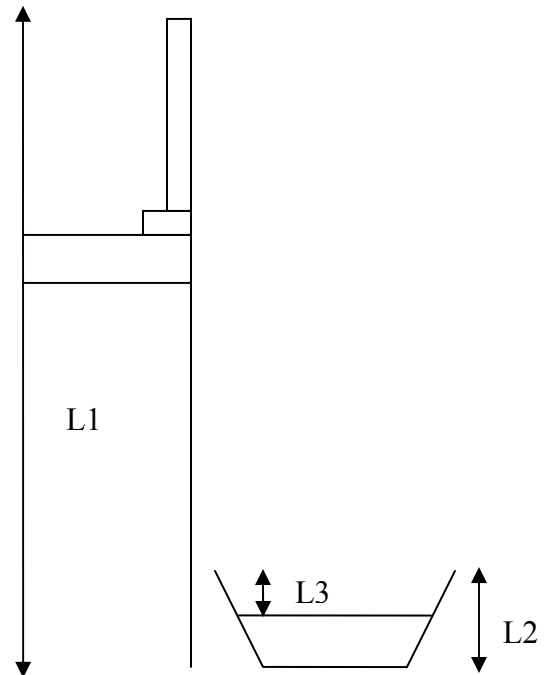Figure 1: The experimental set-up.   Figure 2: Lengths measured

Seven table tennis balls of the same volume and different masses were prepared as follows. Tiny holes were drilled in each ball to insert lead shot and glue. Different amounts of lead shot were put in to each ball to vary the mass of the balls. Latex glue mixed with a small amount of water was inserted through the hole of the ball using a syringe until the ball was completely filled. Glue was used to spread the lead shots evenly inside the ball to make its density uniform. After filling each ball with glue and lead shot, clay was used to cover the hole. A minimal amount of clay was used in order not to change the spherical shape of the balls. The balls were massed using an electronic mass balance.

As seen above in figure 1, a basin filled with a large amount of sand was prepared. A circular basin was prepared so that any wall effect stays constant during the research. The sand was filtered using a sieve with small holes. After that the sand was poured into a basin which was placed next to a table. On the table, a stand was placed. As seen in figure 2, three lengths were measured; L1 (top of stand to the floor), L2 (height of basin), and L3 (top of basin to sand).

Before dropping the balls, the sand was loosened by mixing the sand with the hands. The sand was flattened by running a straight ruler across the top of the loosened sand. Each ball was dropped from the top of the stand (see figure 1). The ball was dropped using two fingers. Each finger was released simultaneously so that the ball did not spin as it fell. After the ball impacted the sand, the diameter of the crater was measured using a ruler. The diameter of the outer edge of the crater was measured as seen in figure 3.

After measuring the diameter of the crater, the sand was loosened again and the same procedure was repeated. For each ball, a total of 5 trials was conducted to quantify random errors. Throughout the experiment, the same type of sand was used, and the whole experiment was conducted in the laboratory room in one hour.
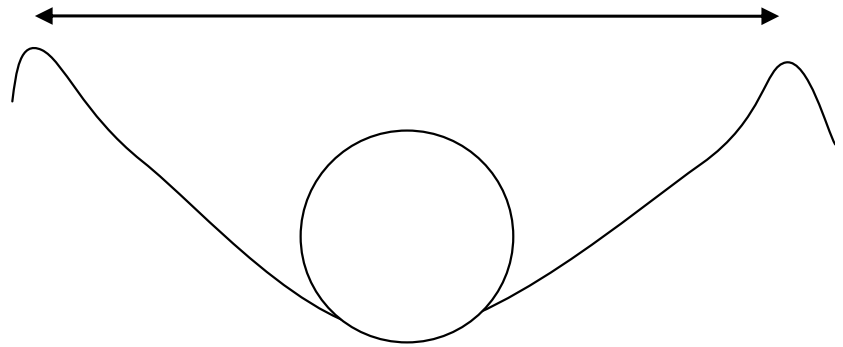
Figure 3: Measuring the diameter of the crater.

Results

Table 1: Lengths measured and height of ball

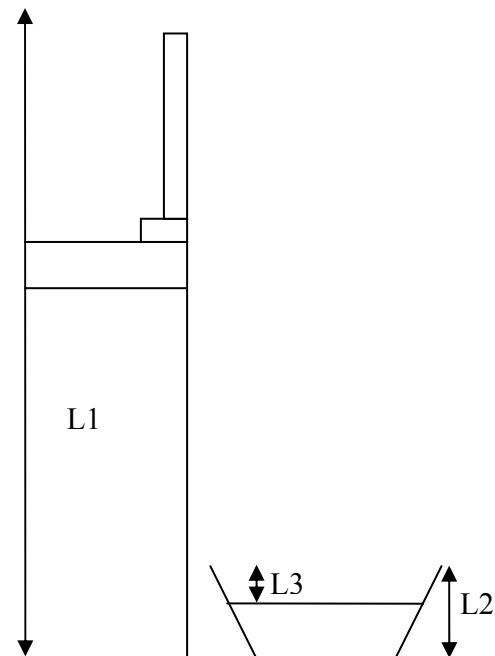|	Length (± 0.001m)
L1: floor to stand	1.465
L2: height of rim of basin	0.148
L3: top of the basin to the sand	0.020
Height of ball above surface of sand	1.337 ± 0.003m

L1

L3

L2

Figure 2 (repeated)

3

Table 2: Mass of balls and diameters of crater formed.

	Mass (± 0.01g)	Diameter of crater (5 Trials) (± 0.1cm)					Average diameter (± 0.2cm)
ball1	27.92	8.0	8.0	8.3	8.4	8.3	8.2
ball2	46.53	9.3	9.4	9.3	9.1	9.4	9.3
ball3	65.37	9.7	9.4	9.5	9.5	9.7	9.6
ball4	105.44	9.9	10.5	10.2	9.9	10.4	10.2
ball5	112.01	10.6	10.6	10.8	10.5	10.6	10.6
ball6	136.74	10.7	10.7	10.9	10.9	11.4	10.9
ball7	174.45	11.2	11.4	11.6	11.2	11.2	11.3

Sample Calculations:

i. Uncertainty of average diameter (ball 4)
    =average deviation of values for crater diameter
    =0.0022 $\approx$ 0.002m

ii. Height of ball above the surface of the sand (refer to figure 2)
    =L1 - L2 + L3=1.465 m - 0.148 m + 0.02 m = 133.7 m $\pm$ 0.003m

iii. Energy of ball (ball 1):
    =mass$\times$gravity$\times$height
    =0.02792kg $\times$ 9.79m/s^2 $\times$ 1.337m =0.36545$\approx$0.365J
        Note that the Kinetic Energy of the ball at impact (the surface of the sand) is calculated, assuming no energy loss to air resistance. The depth of the craters were not measured, therefore the total energy to the bottom of the crater was not calculated. Since the ball ended up between 4 and 6 cm below the surface of the sand, this might cause an error of 3-5 percent in our results.

iv. Uncertainty of energy of ball
    =(%uncertainty of mass + %uncertainty of height) $\times$Energy of ball
    =(0.01g/27.92g+0.3cm/133.7cm) $\times$0.36545J
    =0.00096$\approx$0.001

Table 3: Mass and energy of the ball with average diameter of the crater

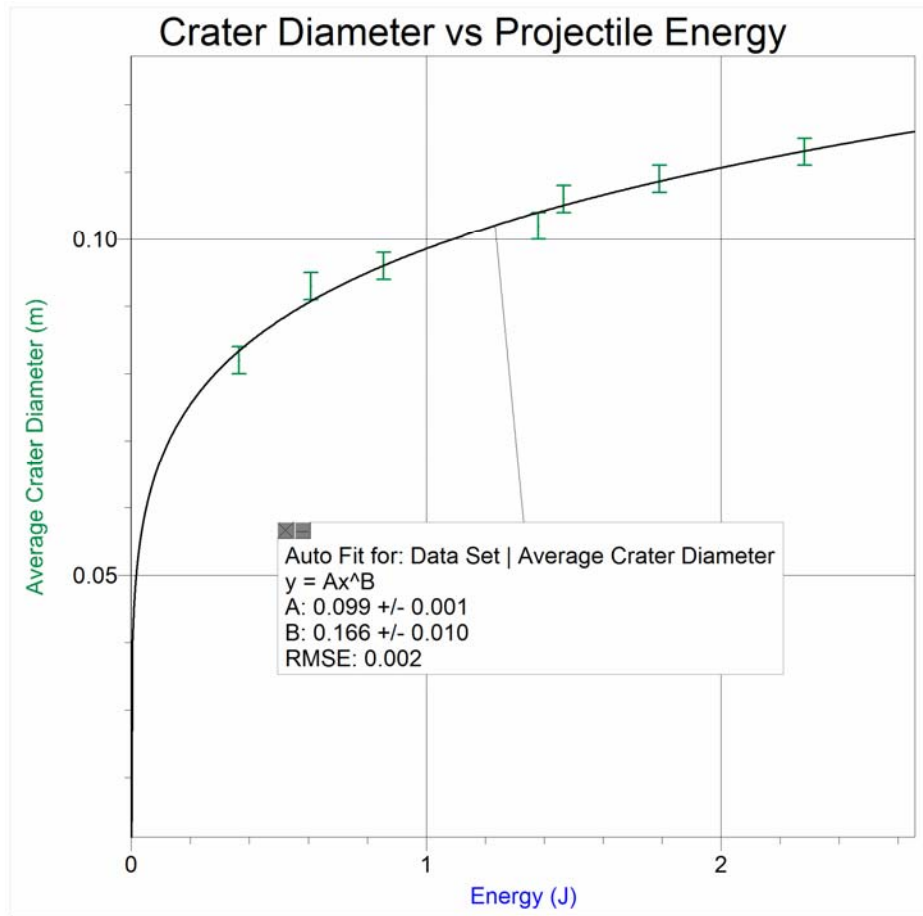| Mass (± 0.00001 kg)	Energy (± 0.001 J)	Average Diameter of Crater (± 0.002 m)
0.02792	0.365	0.082
0.04653	0.609	0.093
0.06537	0.856	0.096
0.10544	1.380	0.102
0.11201	1.466	0.106
0.13674	1.790	0.109
0.17445	2.283	0.113

Figure 4: Diameter of the crater as a function of the kinetic energy of the ball
with a fitted power law curve

Conclusion and Evaluation

As seen in figure 4, an equation of the form $D = k(E)^n$ relating the diameter of the crater as a function of the kinetic energy of the table tennis ball being dropped in sand from a fixed height was fit to the data. The value of n is 0.17 ± 0.01 and the value of k is 0.099 ± 0.001 m/J. The results give the relationship:

$$D = (0.098 \pm 0.002 m/J) \cdot E^{0.17 \pm 0.01} \qquad (4)$$

The curve fit in figure 4 is within uncertainty bars of all data points, suggesting relatively high confidence in the results. The validity of the relationship shown is limited to the situation investigated in this research: a small sphere ranging in mass from 20 g to 200 g being dropped into sand from about 1.3 meters. While it is expected that the relationship for similar situations would be the same, further research must be done to confirm this.

Comparing equation 3 which predicts a relationship of power 0.59 with the result of 0.17 found in equation 4 confirms that the situation here is significantly different from meteors striking earth. This is not entirely unexpected since the magnitude of projectile energies and the nature of the target in this research is very different from meteors striking the earth.

There were a number of weaknesses in the research. The method of measurement of the crater diameter resulted in high uncertainty. Using a ruler held just above the rim of the crater to measure the diameter made it difficult measure accurately without disturbing the rim of the crater. This could be improved by sprinkling bright colored sand or salt on top of the sand before dropping the balls and then photographing the craters. The layer of colored sand would allow the crater to be easily seen and measured photographs.

Another random source of error was the small amount of clay stuck on the balls**.** The clays stuck on the balls slightly changed the shape of the balls, thus changing the nature of the ball. While it was attempted to release the balls with no spin, it is possible that the balls rotated while falling causing the clay part to be on the bottom or sides when the ball impacted, causing the crater to change shape.

The method of preparing the sand also led to errors in the data. During the experiment, several trials were repeated because the diameter was significantly smaller than other trials of the same ball. When the sand was not loosened up enough, the diameter of the crater decreased. Although many trials were conducted to avoid this source of error, it still may have had an effect on our data. For example, in some of the data one or two of the trials are smaller than others. This was probably due to the non-uniformly dense sand. To avoid this source of error in the future, the sand should be mixed from the top to the bottom in a more consistent and thorough manner.

An enclosed space (basin) was used to hold the sand, so there might have been a wall effect. While it is not known what effect the size of the basin might have on crater formation, it would be better if this experiment was conducted in a larger basin of sand to minimize any possible wall effect.

Further research could be conducted increasing the range of projectile masses used, varying the impact speed and size of the projectile, and varying the nature of the target material.