Resonance in a Cone-Topped Tube

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Abstract

The relationship between ratio of the upper opening diameter of a cone-topped cylinder to the cylinder diameter, and the ratio of the length of the air column to resonant period was examined. Plastic cones with upper openings ranging from 1.3 cm to 3.6 cm and tuning forks with frequencies ranging from 261.6 Hz to 523.3 Hz were used. The transition from a standing wave in a cylindrical column to a Helmholtz-type resonance in a resonant cavity with a narrow opening was observed.

Introduction

In this investigation, a tuning fork was held over a cylindrical pipe to create resonance in the air column as shown in figure 1. Plastic cones with varying upper openings were placed on top of the pipe and tuning forks with four different frequencies were used. The nature of the resonance under various situations was studied.

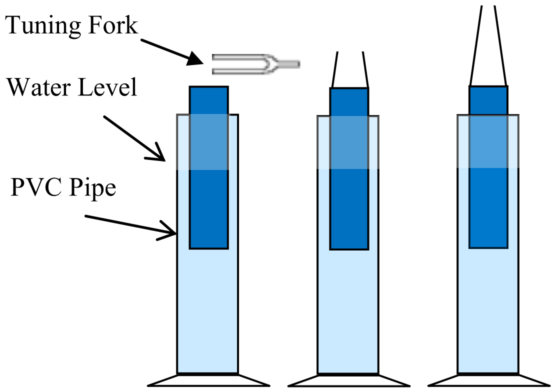

Figure 1 The range of situations that was investigated: from a cylindrical air column, to a cone topped tube with a small opening.

For the situation with no cone or short cones, a quarter-wavelength standing wave may be formed in a cylindrical column of air with one open end. Given that frequency is the inverse of the period and wavelength equals 4(L + c) (end correction) as shown in figure 2, the equation can be derived as

$$L = \frac{v}{4} \times T - c \qquad \text{[Equation 1]}$$

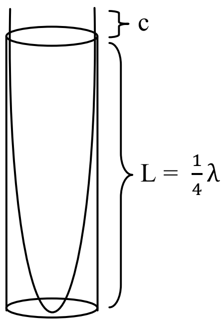

Figure 2 The fundamental standing wave in a tube with one open end.

Figure 3 A classic Helmholtz resonator[1].

For situations with long cones and small upper openings, it is expected that a Helmholtz resonance may form in the air column. A Helmholtz resonator is a container of gas with an open hole and a neck containing an oscillating air column. The equation used to find the frequency, f, of Helmholtz resonance in an ideal Helmholtz resonator (figure 3) is

$$f_H = \frac{v}{2\pi}\sqrt{\frac{A}{V_0 L}} \qquad \text{[Equation 2]}$$

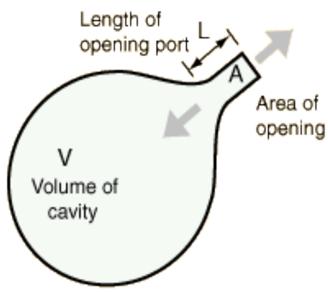

Figure 4 The length of a narrow opening of a classic Helmholtz resonator [2].

where *v* is the speed of sound in the gas, *A* is the cross-sectional area of the neck, *V* is the volume of the cavity, and, as shown in figure 4, *L* is the length of the neck [1]. For a Helmholtz resonator without a neck, a volume of air in and near the opening vibrates up and down forming an effective neck as shown in figure 5.

In the situations examined here, the containers taper to a circular opening at the upper end with no long cylindrical neck, unlike in a classic Helmholtz resonator. In a spherical cavity with a circular opening it has been found that the effective neck length is 1.5 times the radius of the opening [3]. A similar relationship may exist for these tapered containers that model bottles of different neck lengths and different top diameters, if Helmholtz is an appropriate description for resonance in bottles as is commonly assumed.

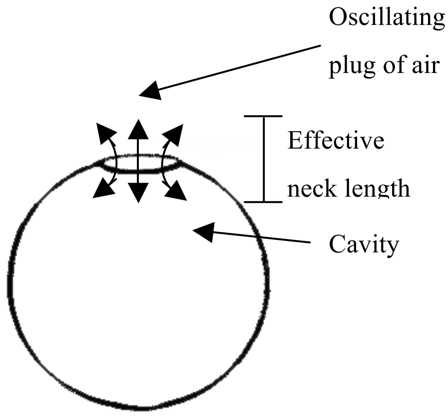

Figure 5 Effective neck length.

It is predicted that cones with larger upper openings will demonstrate a standing wave resonance and thus may be modeled by equation 1 while the cones with smaller upper openings will follow the Helmholtz resonance, equation 2. As the upper opening to tube diameter ratio decreases, a transition from standing wave to Helmholtz resonance is expected.

Methods

A 1000 ml plastic graduated cylinder was filled with tap water. A PVC pipe with an inner diameter of 3.6 cm was placed inside the graduated cylinder. A 14.9 cm tall plastic cone was taped to the top of the PVC pipe. A section of the tip of the plastic cone was cut off and the diameter of the opening was measured. Tuning forks of four different frequencies ranging from 261.6 Hz to 523.3 Hz were used to cause resonance inside the pipe. Each tuning fork was struck and held above the upper opening of the cone. The length of the air column in the tube was adjusted until it resonated. The distance from the surface of the water to the top of the plastic cone was measured.

Three trials were conducted for each tuning fork. The plastic cone was then cut again and the process was repeated for four different cone heights until the cone had been completely cut away. The temperature inside the air column was measured at 26 ± 0.5°C throughout the experiment.

Results and Discussion

Assuming that a standing wave is formed inside the air column, the slope of the period to length graph is expected to be one-fourth the speed of sound according to equation 2. When no cone was placed on top of the PVC pipe, the ratio of speed of sound over slope was 3.9 with a ratio of 4.0 for the shortest cone, as can be seen in Table 1. Based on the data, as the height of the cone increases, the hole to tube diameter ratio decreases, and the ratio between the speed of sound and the resonance length becomes much greater. This indicates that a

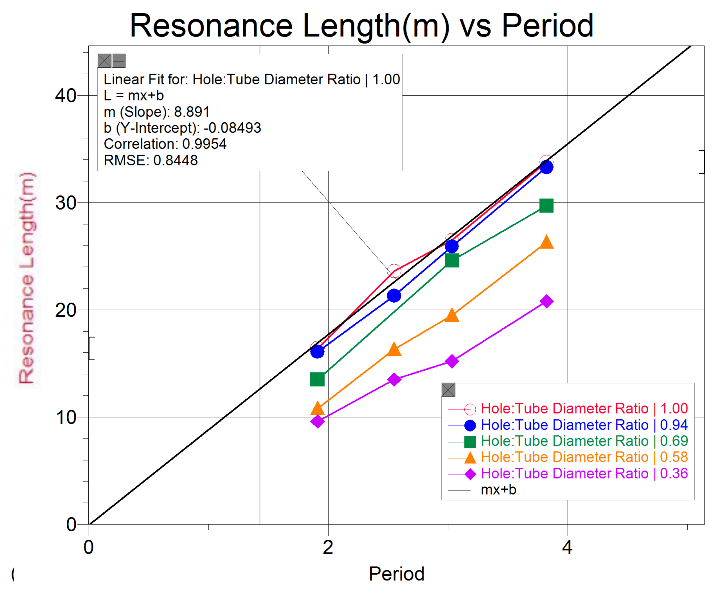

Figure 6 The hole/tube diameter ratio vs. period with four different frequency forks at each cone length.

standing wave was formed when no cone or a short cone was placed on the pipe, but as the cone height increases, the standing wave model no longer fits the data.

Equation 2 has been applied to the data to determine whether there is agreement between the data and the predictions of Helmholtz resonance for a neck-less cavity.

Cone height (m)	Ratio: Hole/Tube Diameter	Slope of Period vs. Length Graphs (m/s)	Ratio: Speed of sound/ Slope
0.000	1.000	89	3.9
0.022	0.944	86	4.0
0.055	0.694	80	4.4
0.087	0.583	65	5.3
0.120	0.361	53	6.6

Table 1: Speed of sound to slope ratio for the 5 hole/tube diameter ratios.

As can be seen from Table 2, the ratios of the effective neck length to hole radius for the longest cone range from 3.1 to 1.4, and the range is greater for all the other cases. For the shorter cones and no cone, the ratios of the effective neck length to hole radius are much larger than 1.5, the ratio predicted if the Helmholtz model applies. Helmholtz resonance clearly is not a good model for these. Even for the longest cone, the ratio of the effective neck length to hole radius is dependent on frequency and ranges from 1.4 to 3.1. While the 1.4 is close to the accepted ratio of 1.5, the dependence of the ratio on frequency indicates that ideal Helmholtz resonance is not a good model for this situation.

Helmholtz resonance with the accepted effective neck length has been shown to not be a good model in any of the cases studied. Given that a tube with a cone on top is very similar in shape to a bottle, it is unlikely that blowing on a bottle causes ideal Helmholtz resonance, as is commonly assumed. Further research should be conducted to determine the nature of resonance in a bottle.

Cone height (m)	Frequency (Hz)	Ratio: Effective Neck Length/ Radius
0.120	261.6	3.1
	329.6	2.5
	392.0	1.8
	523.3	1.4
0.087	261.6	4.0
	329.6	3.2
	392.0	2.6
	523.3	2.0
0.055	261.6	4.3
	329.6	3.2
	392.0	2.4
	523.3	2.0
0.022	261.6	5.4
	329.6	4.2
	392.0	3.5
	523.3	2.4
0.000	261.6	5.6
	329.6	4.3
	392.0	3.4
	523.3	2.5

Table 2: Ratio of effective neck length to hole diameter, assuming ideal Helmholtz resonance.

Conclusion

For an air column in a cylindrical tube, and for air in a tube with a slightly constricted neck, standing waves are formed at resonance. The tubes with extended cone tops studied have resonances that are in a transition region between standing wave resonance and Helmholtz resonance.

References

[1] "Helmholtz Resonator." *Wikipedia, The Free Encyclopedia*. Wikipedia, The Free Encyclopedia, 24 March, 2006. Web. 24 May. 2011.

[2] "Herman von Helmholtz Resonator". *Rexresearch*. Web. 24 May, 2011.

[3] Raichel, Daniel R. *The Science and Applications of Acoustics.*2nd ed. Moore Lane, Fort Collins: Springer, 2006. Print.