SED Equations

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1 Apparent Magnitude

We are primarily concerned with calculating magnitudes from the SED of objects in a lightcone. In order to get started I will focus on AB magnitudes. The AB magnitude of an object is defined as

$$m_{ab} = -2.4 \log_{10} F_\nu - 48.6 \ ,$$

where F_ν is the spectral flux density in $erg \cdot s^{-1} \cdot cm^{-1} \cdot Hz^{-1}$. We will be defining F_ν as

$$F_\nu = \frac{I}{R} \ ,$$

where I is the intensity of the object and R is the normalisation with respect to the bandpass filter used. I and R are defined as

$$\begin{aligned} I &= \int_\nu f_\nu r \, \mathrm{d}\nu \\ R &= \int_\nu r \, \mathrm{d}\nu \end{aligned}$$

where f_ν is the sepctral energy density of the object in $erg \cdot s^{-1} \cdot cm^{-2} \cdot Hz^{-1}$ and r is a unitless transmission response filter.

The spectral energy densitiy will be given to us not as a function of frequency, as we need, but instead as a function of wavelength and in units of $erg \cdot s^{-1} \cdot cm^{-2} \cdot {}^{-1}$. We refer to this quantity as f_λ. In general $f_\lambda \neq f_\nu$, so we will need to find f_ν in terms of f_λ. We know that the energy of these values must be equal for any arbitrary subdomain,

$$f_\nu \, \mathrm{d}\nu = f_\lambda \, \mathrm{d}\lambda \ ,$$

which can be rearranged to produce

$$f_\nu = f_\lambda \frac{\mathrm{d}\lambda}{\mathrm{d}\nu} \ .$$

Now, we only need an expression for $\frac{\mathrm{d}\lambda}{\mathrm{d}\nu}$ to complete the transformation. We can use

$$\begin{aligned} \lambda &= \frac{c}{\nu} \\ \therefore \quad \frac{\mathrm{d}\lambda}{\mathrm{d}\nu} &= -\frac{c}{\nu^2} \ , \end{aligned}$$

and substituting into the previous equation yields

$$f_\nu = -f_\lambda \frac{c}{\nu^2} \ .$$

Substitution into the equation for intensity gives

$$I = -\int_\nu f_\lambda r \frac{c}{\nu^2} \, \mathrm{d}\nu \ .$$

We would prefer to integrate over λ, for conenience, so we will transform the integral from ν using the relationship between ν and λ,

$$\begin{aligned} I &= -\int_\lambda f_\lambda r \frac{c}{\nu^2} \left(-\frac{\nu^2}{c} \right) \mathrm{d}\lambda \\ &= \int_\lambda f_\lambda r \, \mathrm{d}\lambda \ . \end{aligned}$$

This result is as expected; the energy of the filtered SED is the same irrespective of whether considered on the wavelength or the frequency domain. It is also very convenient, as we have no need to transform any quantites to frequency.

While we could leave the expression for R in terms of frequency, it's more convenient for the code to integrate it in terms of wavelength. To do this we can simply perform a variable substitution in the integral,

$$\begin{aligned} R &= \int_\nu r \, \mathrm{d}\nu \\ &= \int_\lambda r \left(-\frac{c}{\lambda^2} \right) \mathrm{d}\lambda \\ &= -\int_\lambda r \frac{c}{\lambda^2} \, \mathrm{d}\lambda \\ &= \int_\lambda r \frac{c}{\lambda^2} \, \mathrm{d}\lambda \ . \end{aligned}$$

Note that we are able to eliminate the negation of the RHS because it appears only as a result of reversing the integration direction.

Now we have all the necessary expressions to define F_ν in terms of wavelength,

$$\begin{aligned} F_\nu &= \frac{I}{R} \\ &= \frac{\displaystyle\int_\lambda f_\lambda r \, \mathrm{d}\lambda}{\displaystyle\int_\lambda r \frac{c}{\lambda^2} \, \mathrm{d}\lambda}, \end{aligned}$$

allowing us to provide the full equation for the AB magnitude of an object:

$$m_{ab} = -2.5 \log_{10} \frac{\displaystyle\int_\lambda f_\lambda r \, \mathrm{d}\lambda}{\displaystyle\int_\lambda r \frac{c}{\lambda^2} \, \mathrm{d}\lambda} - 48.6$$