Introduction to Quantitative Geology

Lecture 10

Viscous flow down an inclined plane

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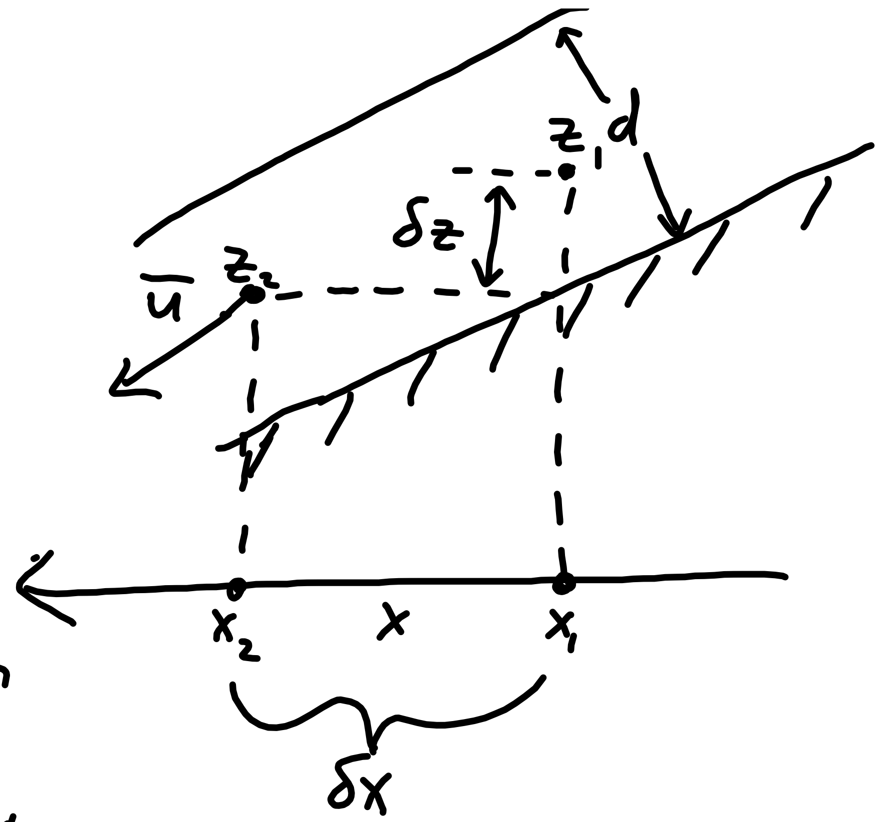
Potential energy of the fluid

$$\Delta E_p = \rho g d \, \delta z$$

$$= \rho g d (\bar{u} S)$$

for a given δx



ρ = density

g = gravity

d = flow thickness

δz = elevation change

$\bar{u}$ = average velocity

S = slope

Bed resistance

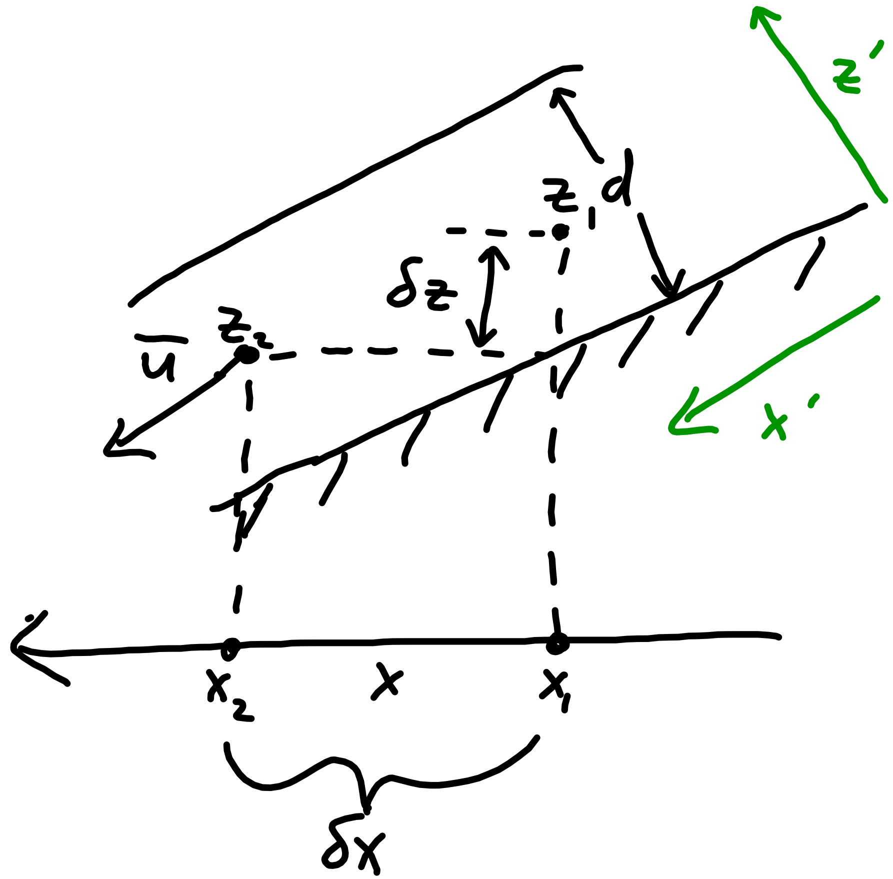
$$\tau_0 = \rho g d S$$

shear stress (drag) at base of fluid

$$\tau(z) = \rho g S (d - z)$$

shear stress at z

$$\tau(z) = \tau_0 \left(1 - \frac{z}{d}\right)$$



Connection to viscous flow

$$\tau = \eta \frac{du}{dz}$$ Linear visosity

$$\tau(z) = \eta \frac{du}{dz}$$

$$= \rho g S (d - z)$$

$$\eta \frac{du}{dz} = \rho g S (d - z)$$

$$\frac{du}{dz} = \frac{\rho g S}{\eta} (d - z)$$

$$\int \frac{du}{dz} dz = \frac{\rho g S}{\eta} \int (d - z) dz$$

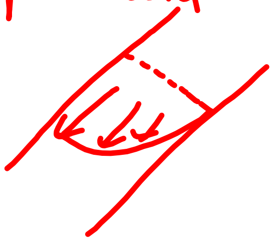
$$u(z) = \frac{\rho g S}{\eta} \left(dz - \frac{z^2}{2} \right) + C_1$$

Assume $u = 0$ at $z = 0$

$\hookrightarrow C_1 = 0$

$$\boxed{u(z) = \frac{\rho g S}{\eta} \left(zd - \frac{\boxed{z^2}}{2} \right)}$$

parabola



Take-home messages

1. Flow is a balance between the gravitational force on the fluid and the resistance (drag) the base

2. Flow velocity increases following a parabolic geometry from u = 0 to u = (rho g S) / eta * d^2 / 2

Caveats:

- Steady state
- 1D
- Laminar flow
- Constants do not vary
- No temperature dependence

An example from Hawai'i

$$u_{max} = \frac{\rho g s}{\eta}\left(\frac{d^2}{2}\right)$$

Mauna Loa is ~4 km high and located ~40 km from the coast

$$s = 0.1$$

Assuming a rock density of 2700 kg/m^3, viscosity of 100 Pa s and flow thickness of 0.3 m, what is the maximum flow velocity?

$$u_{max} = 1.19 \text{ m/s}$$