Class overview today - March 21, 2016

- Part I - Common statistical methods in geoscience
  - The many challenges of geological data samples
  - Uncertainty in Earth science data
  - Basic mathematical representations of measurement uncertainty
- Part II - What do geochronological ages mean?
  - Geochronological ages and their meaning
  - Comparing predicted and measured ages
  - Quantifying the fit of predicted and measured ages

Introduction to Quantitative Geology
Lecture 3
Common statistical methods in geoscience

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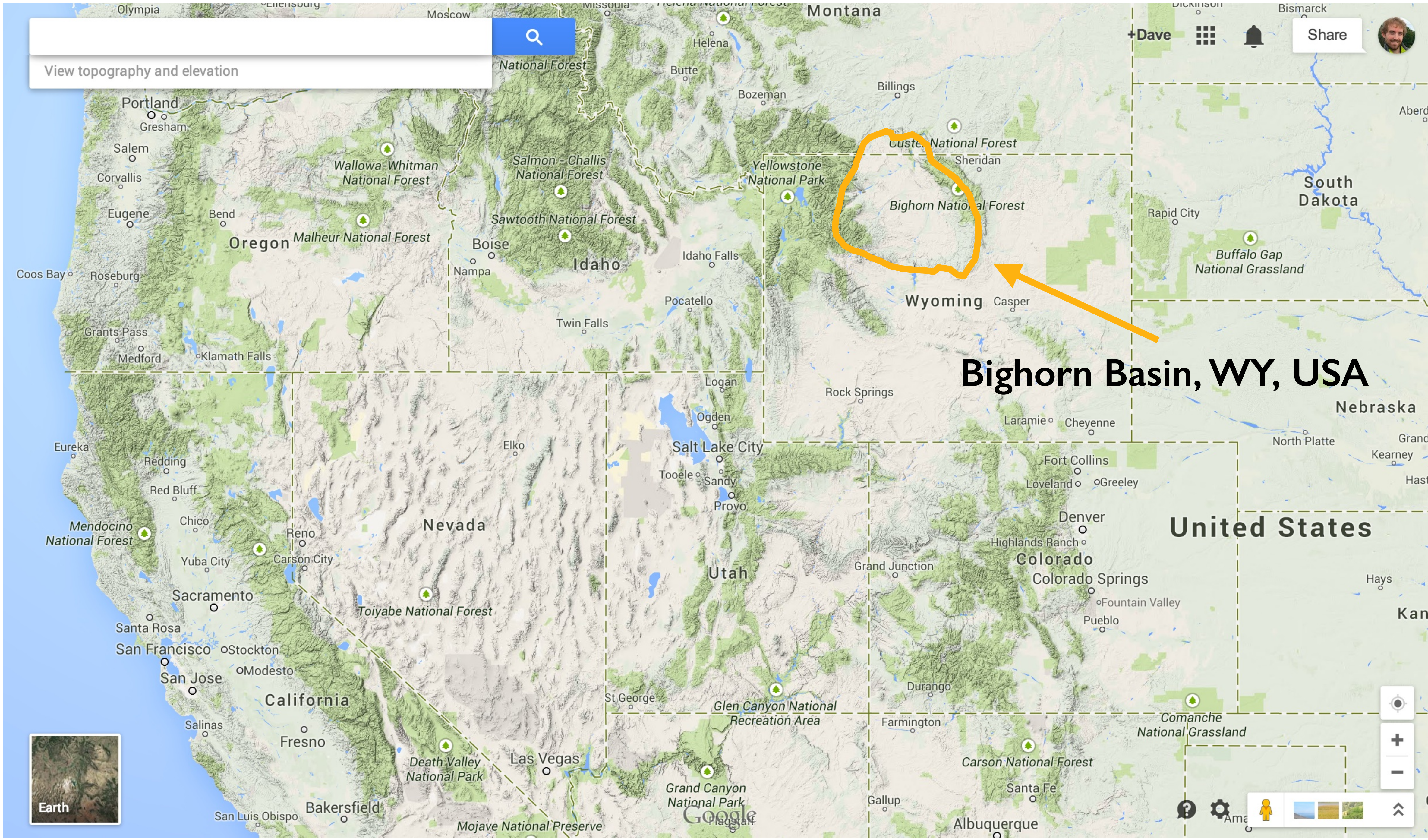
Goals of this lecture

- Address the many challenges of **geological data samples**
- Discuss **uncertainty** in Earth science data
- Review the basic mathematical representations of **measurement uncertainty**

Why are statistics important for us?



Bighorn Basin, Wyoming, USA


Bighorn Mtns, Wyoming, USA

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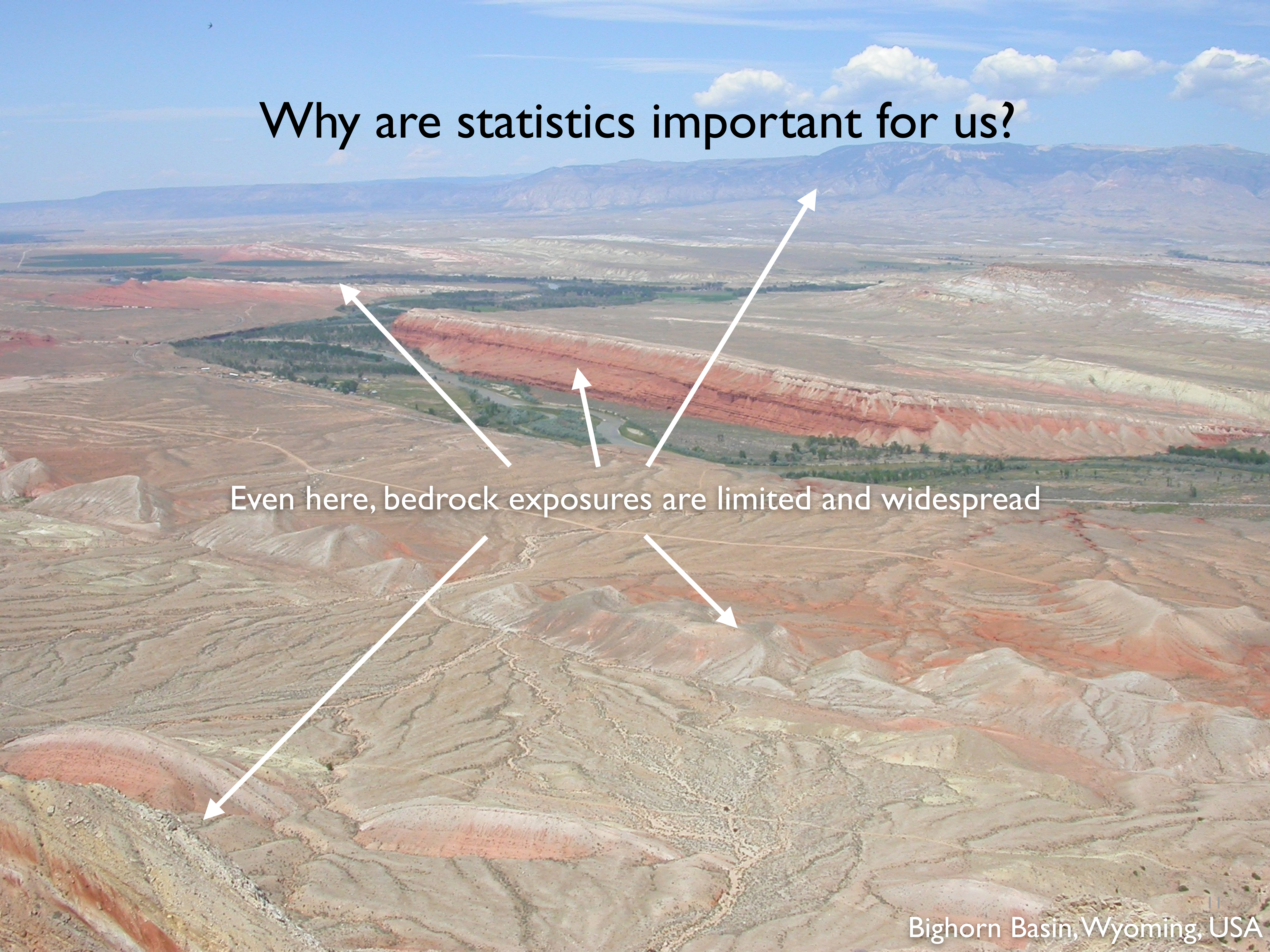
Why are statistics important for us?
Even here, bedrock exposures are limited and widespread
Bighorn Basin, Wyoming, USA

Why are statistics important for us?

- We can directly observe only a tiny fraction of most geological features
  - Bedrock exposure is limited
  - Many rock units are very large and/or widely dispersed
  - Sometimes, even well-exposed units can be hard to access

Bighorn Basin, Wyoming, USA

A few statistical definitions

- **Population**: The total number of occurrences of a particular thing present in a defined area
    - We basically cannot access this geologically

A few statistical definitions

- **Population**: The total number of occurrences of a particular thing present in a defined area
  - We basically cannot access this geologically
- **Subset**: A collected portion of the population
- **Sampling units**: The collected material

A representative sample

- Our goals, as Earth scientists is to collect data that forms a **representative sample**
  - **Representative sample**: A sample that can be used to infer the characteristics of the population

- The best way to get a representative sample is to collect sampling units at **random**, without bias

A representative sample

- Our goals, as Earth scientists is to collect data that forms a **representative sample**
    - **Representative sample**: A sample that can be used to infer the characteristics of the population

- The best way to get a representative sample is to collect sampling units at **random**, without bias
    - **What challenges might we face in collecting a random geological sample?**

The trouble with "Random" sampling

Population

- Making our best effort to collect a "random" sample does not guarantee it is a representative sample

Figure 1.1, McKillup and Dyar, 2010

The trouble with "Random" sampling

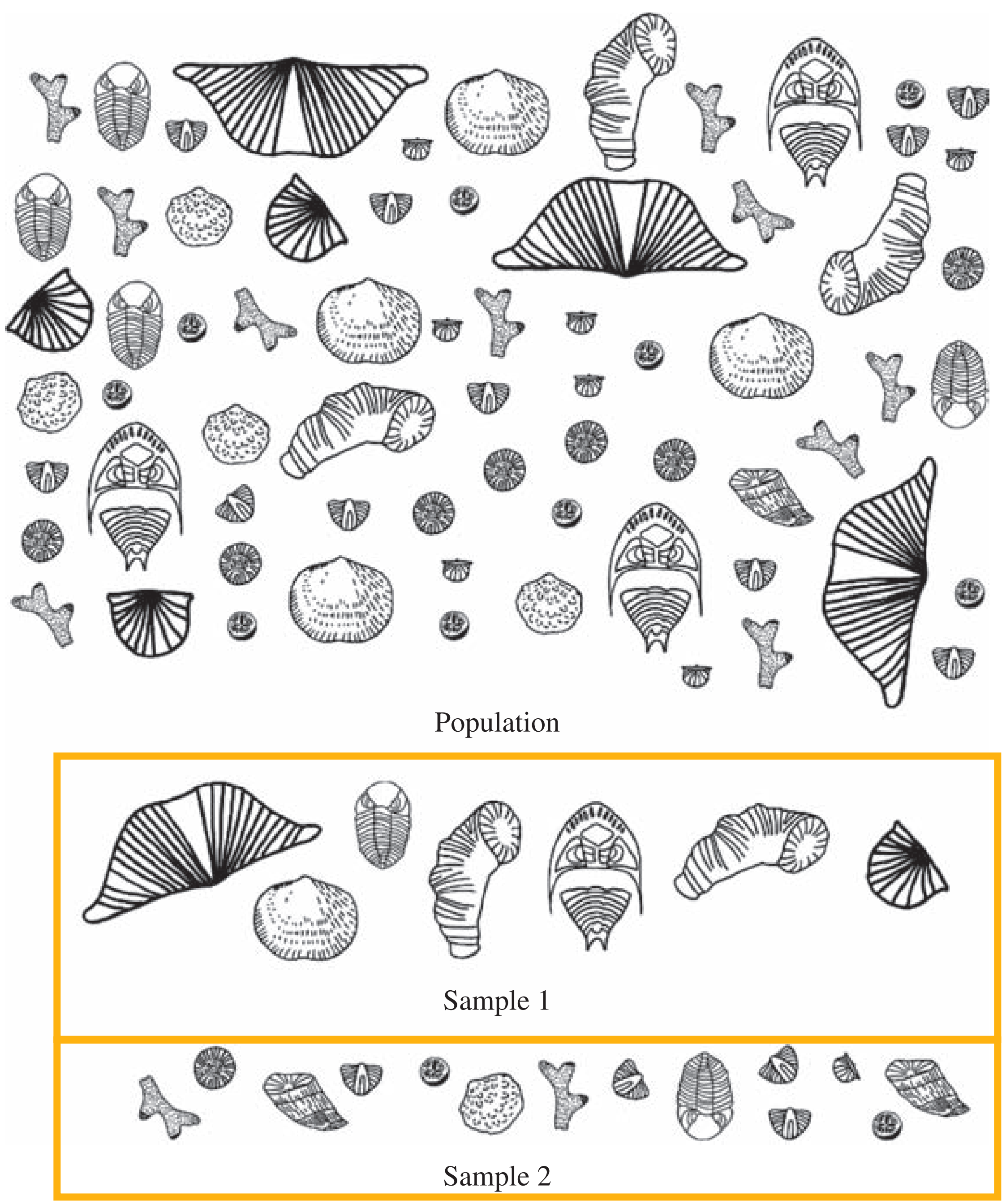

Figure 1.1, McKillup and Dyar, 2010

- Making our best effort to collect a "random" sample does not guarantee it is a representative sample

1. Samples taken from the same population may be very different from one another

Large fossils

Small fossils

The trouble with “Random” sampling

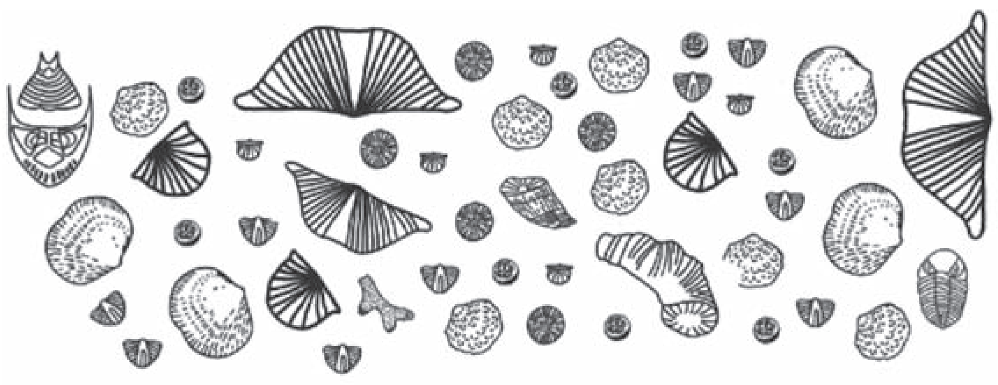

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Figure 1.2, McKillup and Dyar, 2010

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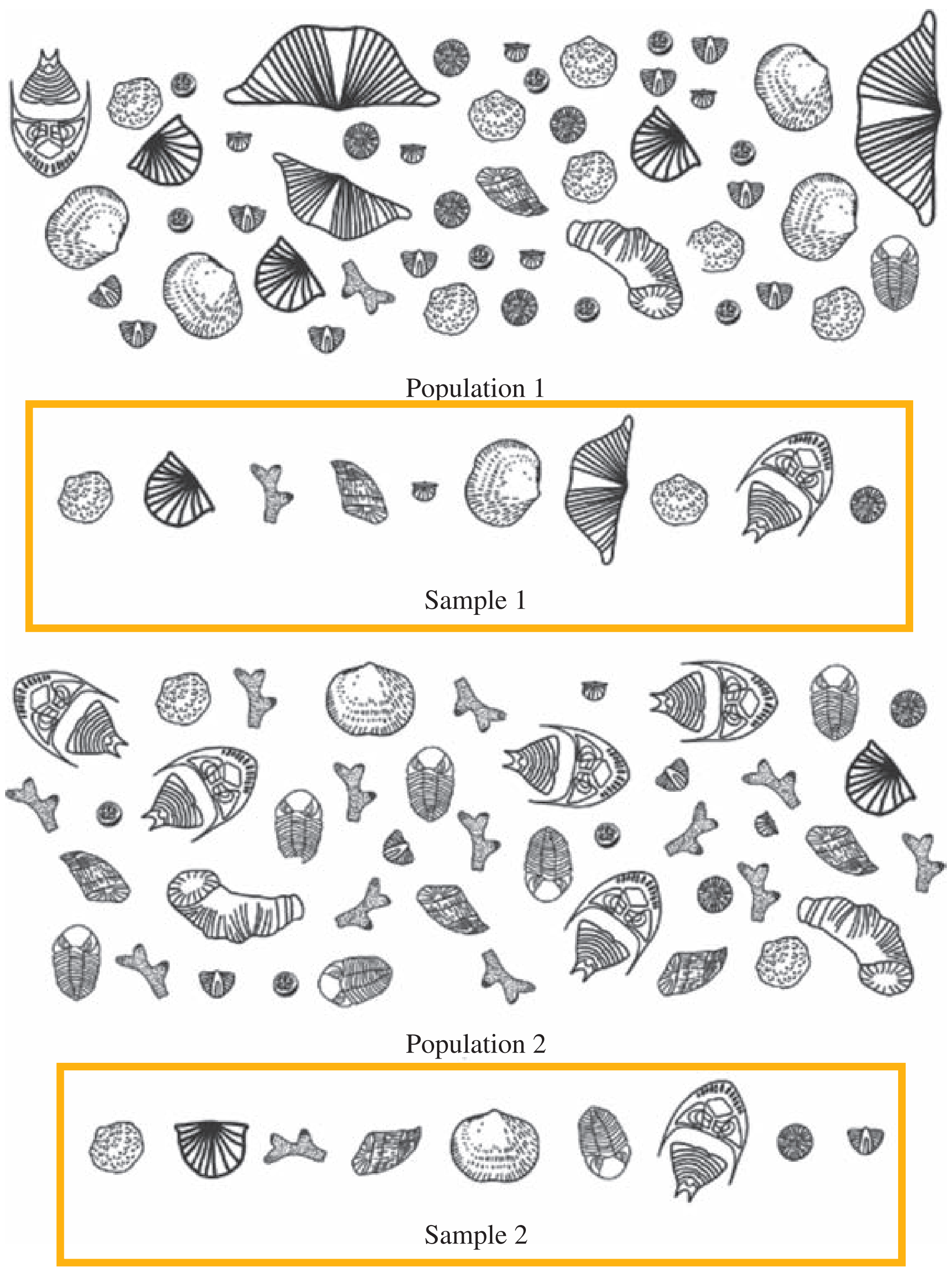

Figure 1.2, McKillup and Dyar, 2010

- Making our best effort to collect a "random" sample does not guarantee it is a representative sample

2. Samples taken from very different populations may be quite similar

Using a sample to infer something about a population

- Collecting a representative sample is a clear challenge and needed to be able to make inferences about a population
  - Every precaution must be taken to collect a representative sample
    - Collecting a sufficient number of measurements
    - Making measurements or collecting samples in the most logical locations
    - Carefully preparing samples for analysis or measurement

Uncertainty

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Know your uncertainties!

Uncertainty in Earth science

- In addition to challenges collecting a representative sample, all geological measurements have inherent **uncertainty**
    - It is not possible to make an exact measurement
    - We may use tools with very high precision, but measurements are still uncertain

- A scientist's goal is to reduce uncertainty when taking measurements by making them as **accurate** and **precise** as possible
- Our goal should be to always include that uncertainty when comparing measured values to predictions

Precision versus accuracy

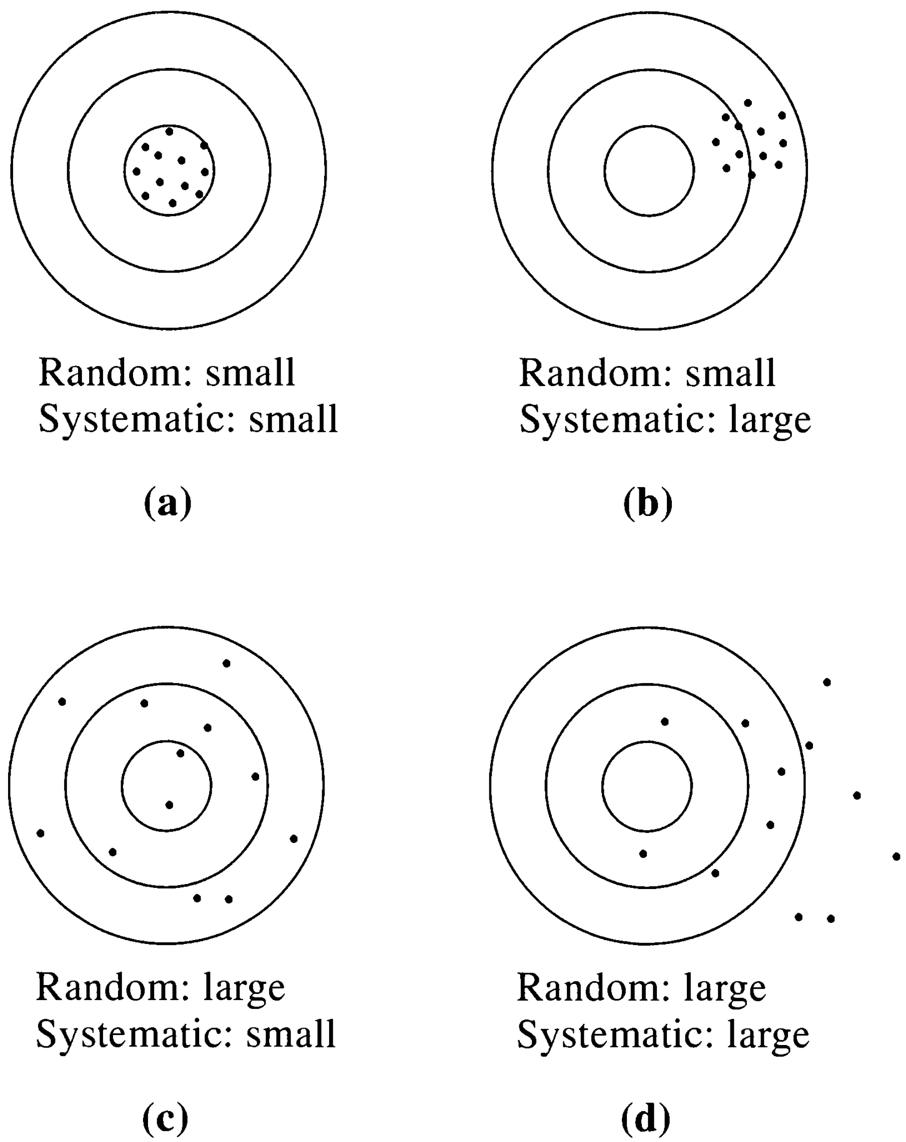
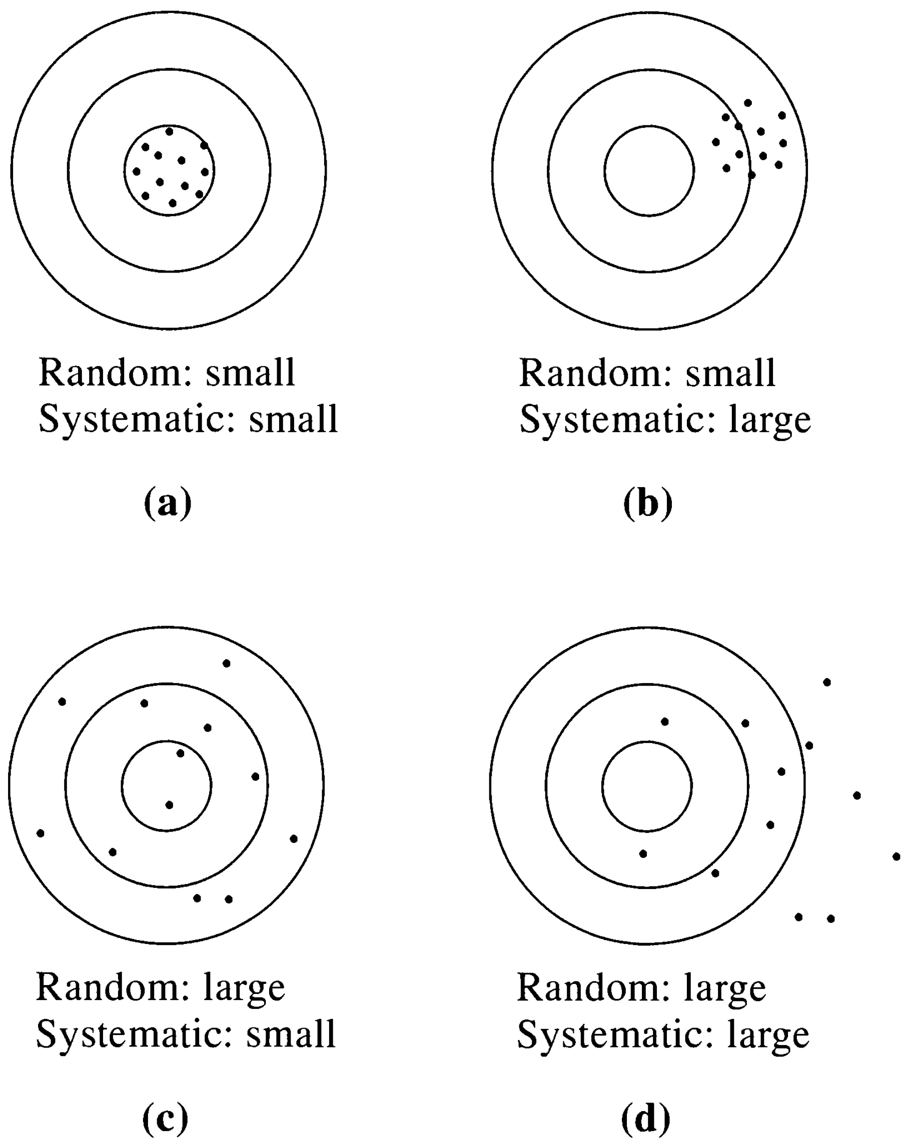


Fig. 4.1, Taylor, 1997

- **Random error**: Experimental uncertainty revealed by repeated measurements
  - Small random error = **Precise**

Precision versus accuracy



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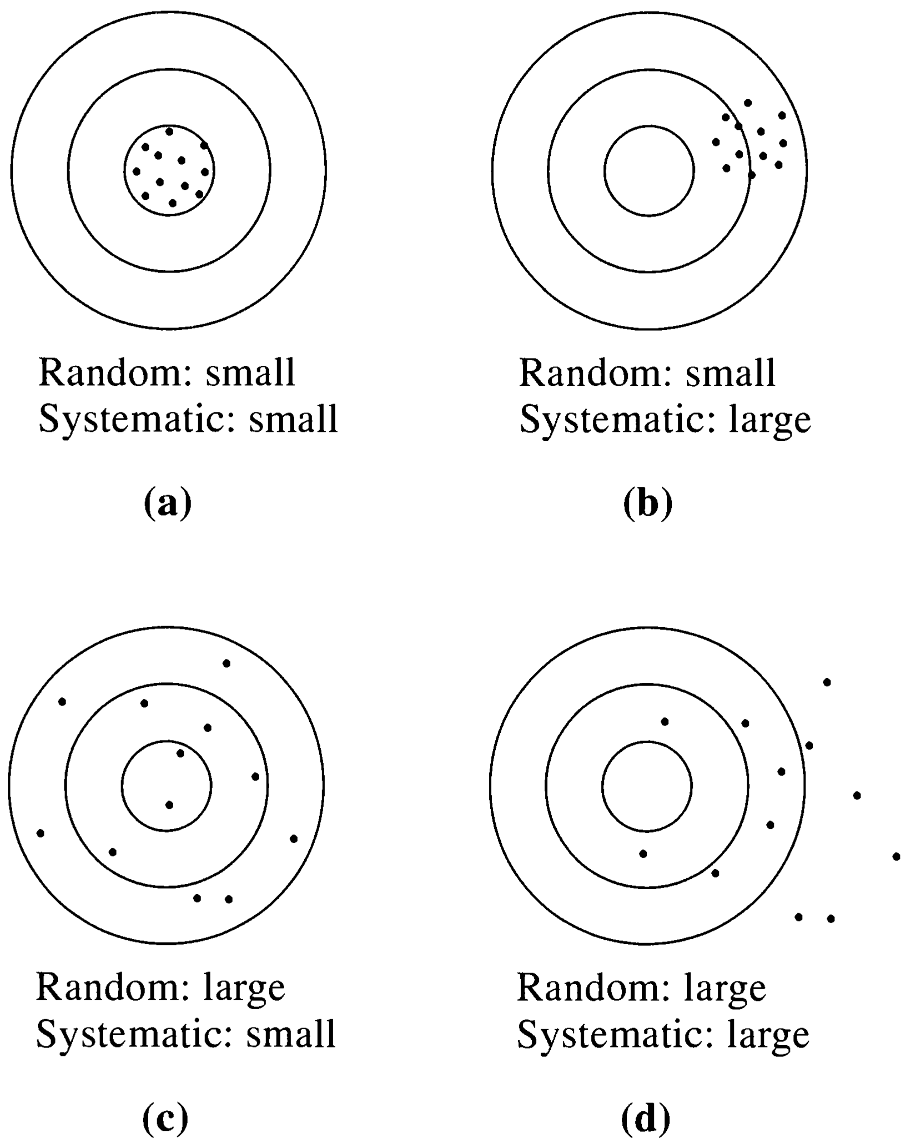

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- **Random error**: Experimental uncertainty revealed by repeated measurements
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  - Small systematic error = **Accurate**
- **How might we detect systematic error?**

Precision versus accuracy

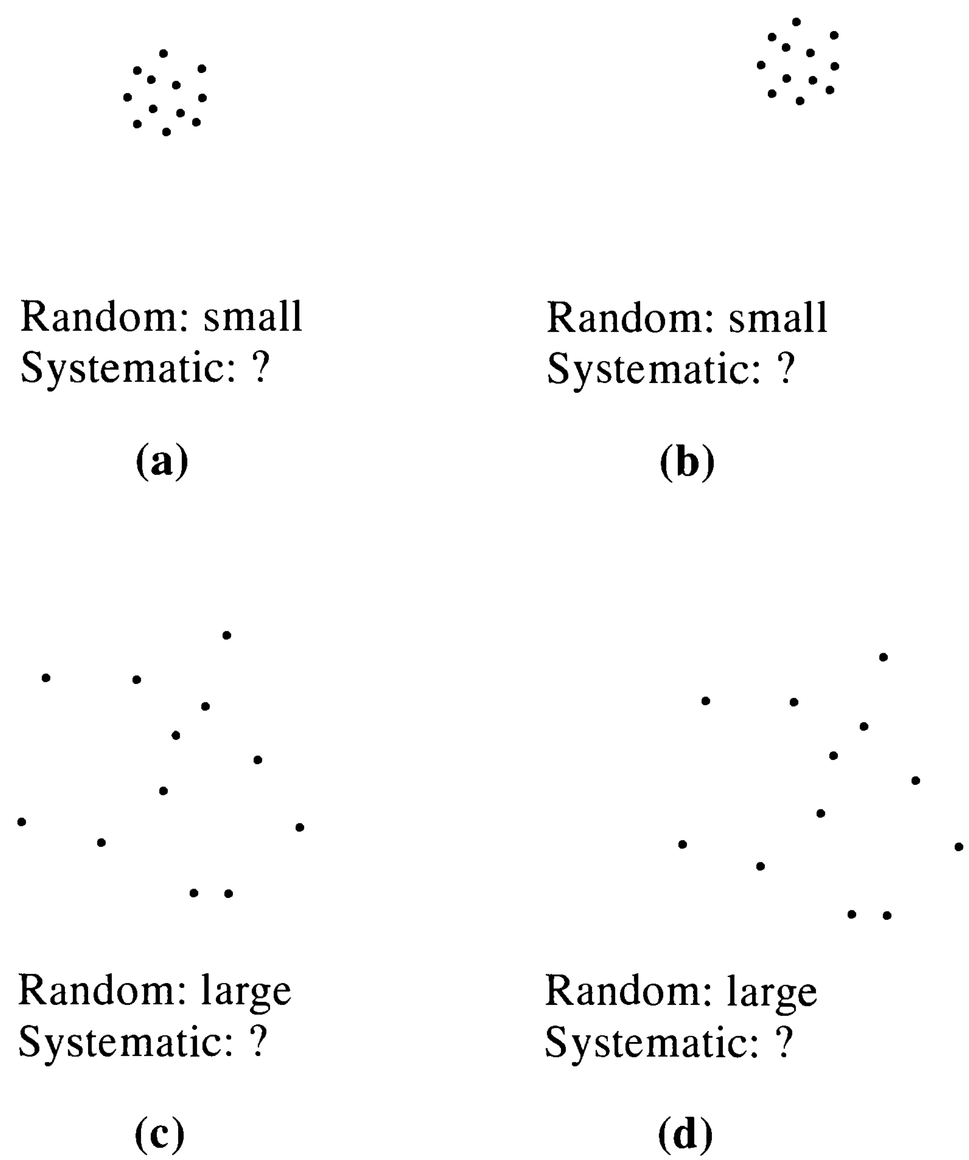
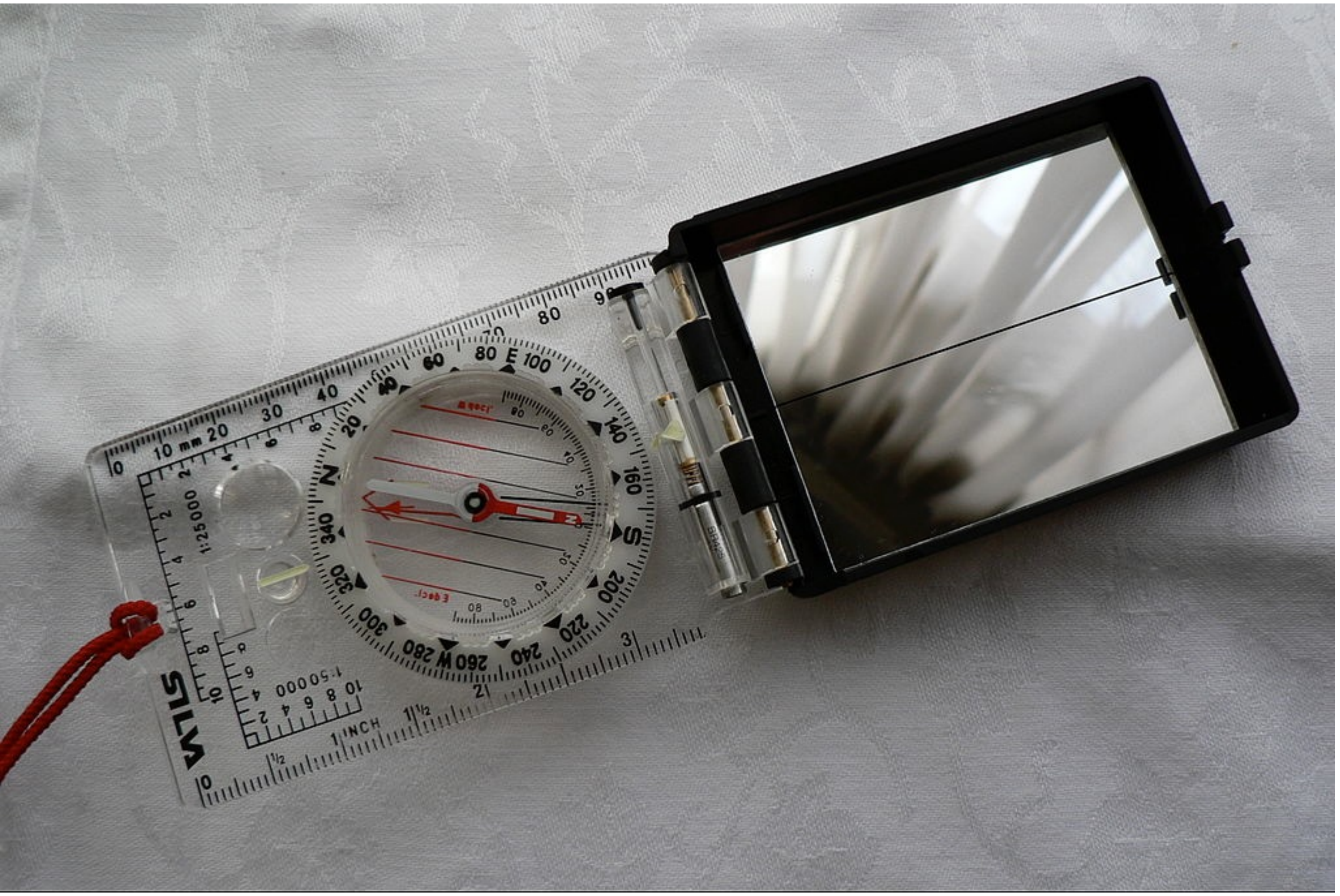


Fig. 4.2, Taylor, 1997

- As it turns out, detecting systematic error is difficult
- Scatter might reveal large random errors, but without a reference for the true value, the systematic error is unclear
    - This is one reason that blanks and standards are used for measuring isotope concentrations when measuring geochronometer ages, for example

Sources of uncertainty

- If we think about it, uncertainty is everywhere



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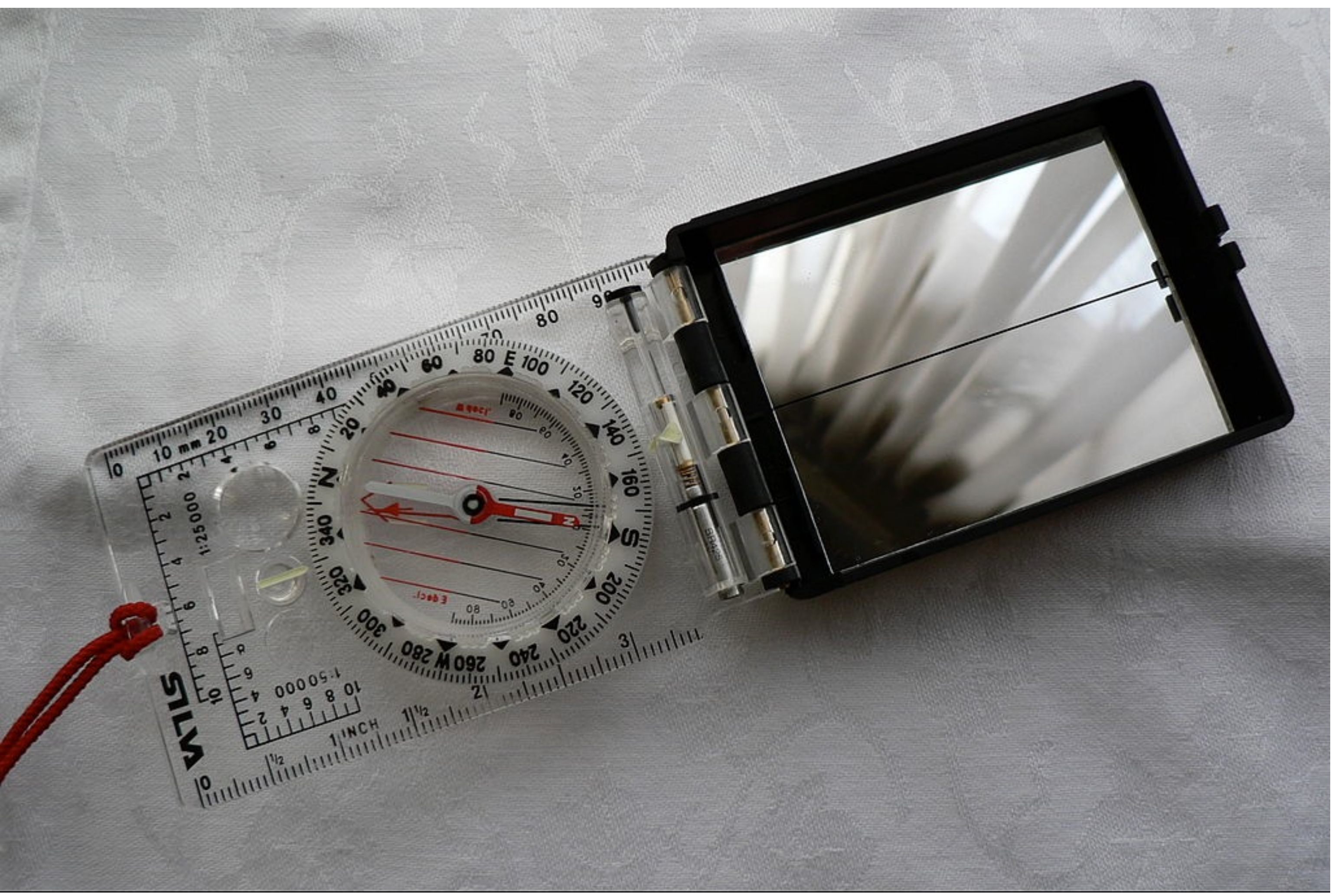

- If we think about it, uncertainty is everywhere
  - A typical field compass is graduated in 1°, so we can't expect any higher precision
  - Magnetic declination can also only be set to within 1°, and the uncertainty larger if not set correctly
  - Outcrops generally don't provide perfect exposure, and repeated measurements vary often by 3-5° (at least)

Reported measurements

- Ignoring the declination, the most precise measurement we can make with our compass is

$$x_{\text{best}} \pm 0.5°$$

- Imagine we take 5 measurements of the strike of a rock unit and find: 33°, 36°, 32°, 35°, 34°
  - Based on these numbers alone, we could state the strike is 34±2°
  - Including the uncertainty in reading the compass, we would say the strike is 34±2.5°
- Generally, any measurement should be reported in the form

$$x_{\text{best}} \pm \delta x$$

Mean and standard deviation

- For the compass measurements, it is quite natural to take the average value, or **mean**, for the best estimate of the strike
- The **mean**, $\bar{x}$, is simply the sum of the measured values divided by the number of measurements

$$\bar{x} = \frac{x_1 + x_2 + ... + x_N}{N}$$
$$= \frac{\Sigma x_i}{N}$$

where x_i is the ith measured value, N is the total number of measurements and Σ represents sigma notation of a sum

$$\sum_i x_i = \sum_{i=1}^{N} x_i = x_1 + x_2 + ... + x_N$$

Mean and standard deviation

- For our example, calculating the mean is easy

$$\bar{x} = \frac{33^\circ + 36^\circ + 32^\circ + 35^\circ + 34^\circ}{5}$$
$$= 34^\circ$$

- If we consider the **mean** as the best estimate of the quantity, a natural value to consider as well is the **deviation** (or **residual**) for each measurement

$$d_i = x_i - \bar{x}$$

- If the **deviation** is small, the measurements are **precise**; if it is large, the measurements are **not so precise**

Mean and standard deviation

- It might be tempting to average the **deviation** to determine how much measurements deviate from the mean on average
  - **What is the average deviation in our dataset?**

Measured value	Mean	Deviation
33	34	-1
36	34	2
32	34	-2
35	34	1
34	34	0

Mean and standard deviation

- It might be tempting to average the **deviation** to determine how much measurements deviate from the mean on average
  - **What is the average deviation in our dataset?**
- If we square the **deviation**, all values will be positive, and then we can average and take the square root of the result

$$\sigma_x = \sqrt{\frac{1}{N-1}(d_i)^2} = \sqrt{\frac{1}{N-1}\sum_{i=1}^{N}(x_i - \bar{x})^2}$$

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- This is the **sample standard deviation**, σ_x

Mean and standard deviation

Measured	Mean	Deviation	Deviation squared
33	34	-1	1
36	34	2	4
32	34	-2	4
35	34	1	1
34	34	0	0

- If we plug in our values we see

$$\sigma_x = \sqrt{\frac{1}{N-1}(d_i)^2}$$

$$= \sqrt{\frac{1}{4}(1+4+4+1+0)}$$

$$= 1.6$$

What does the standard deviation tell us?

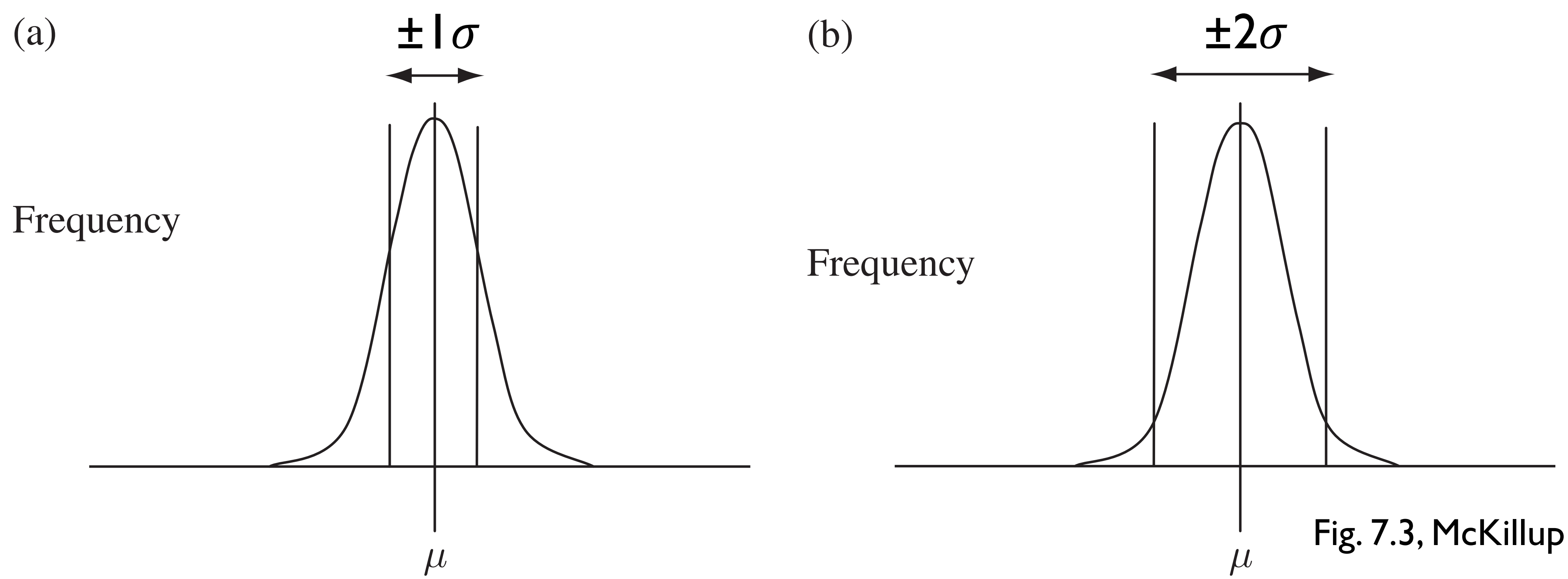
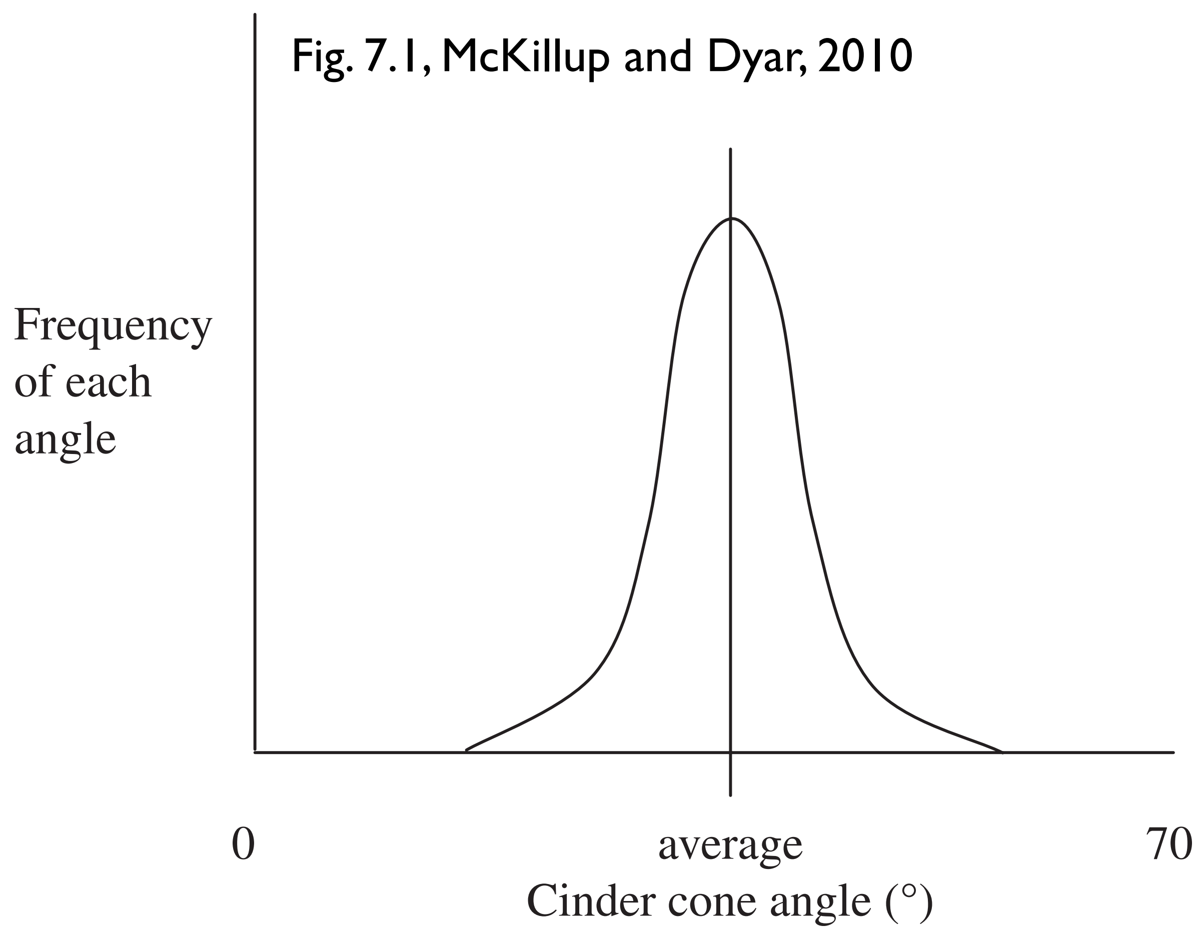


Fig. 7.3, McKillup and Dyar, 2010

~68% of measurements within ±1σ ~95% of measurements within ±2σ

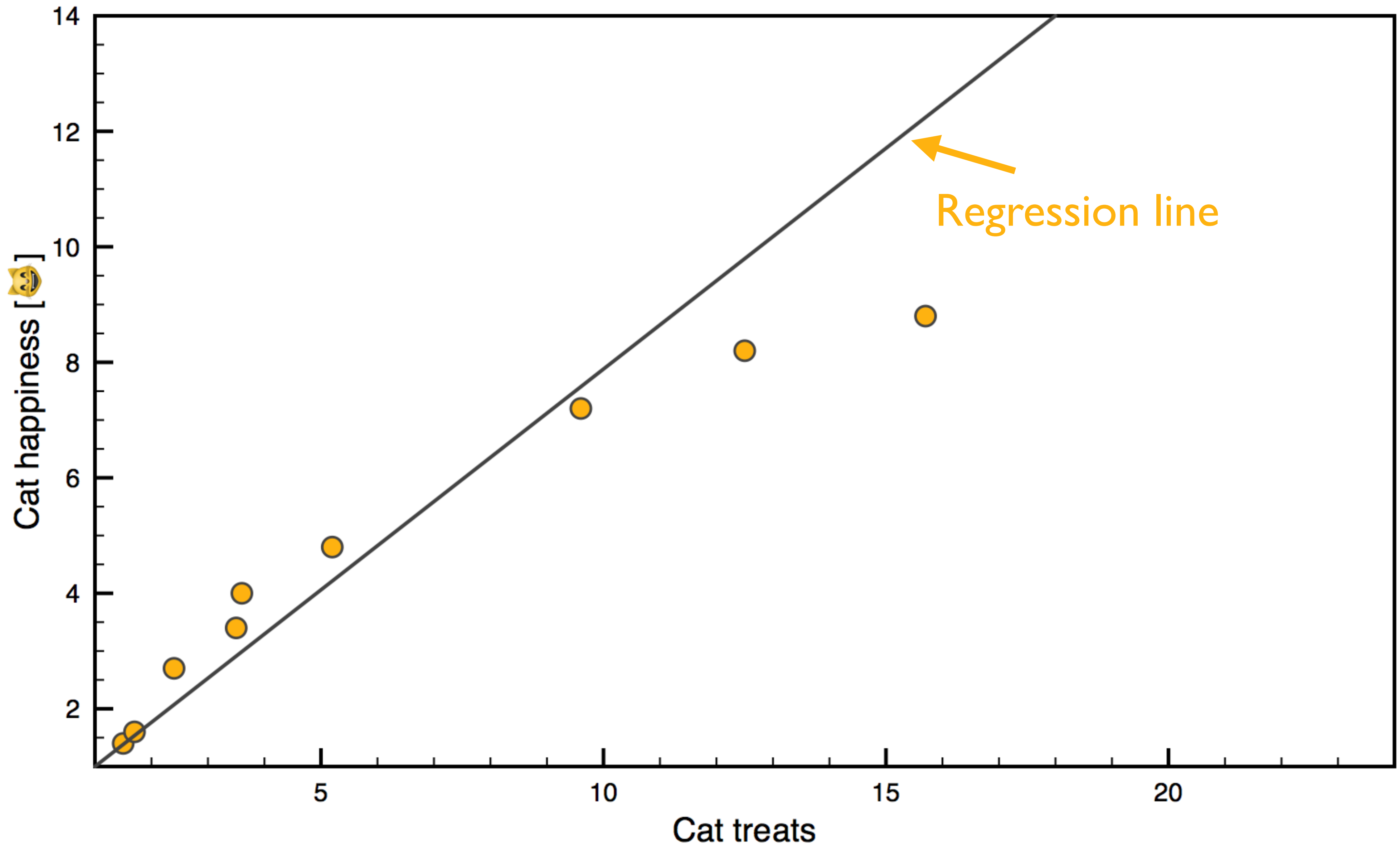
- The 2σ uncertainty is often reported for geological measurements
  - For the measured values, ~95% of the measurements are within ±2σ of the mean μ (μ is the population mean, $\bar{x}$ is the sample mean)

Normal distribution



- The **normal distribution** refers to a bell-shaped curve that mathematically describes the frequency that a measured value is observed
    - This is also known as a **Gaussian curve**
- Many geological variables follow a **normal distribution**
- We will look at more on Wednesday

A silly example of why uncertainties matter



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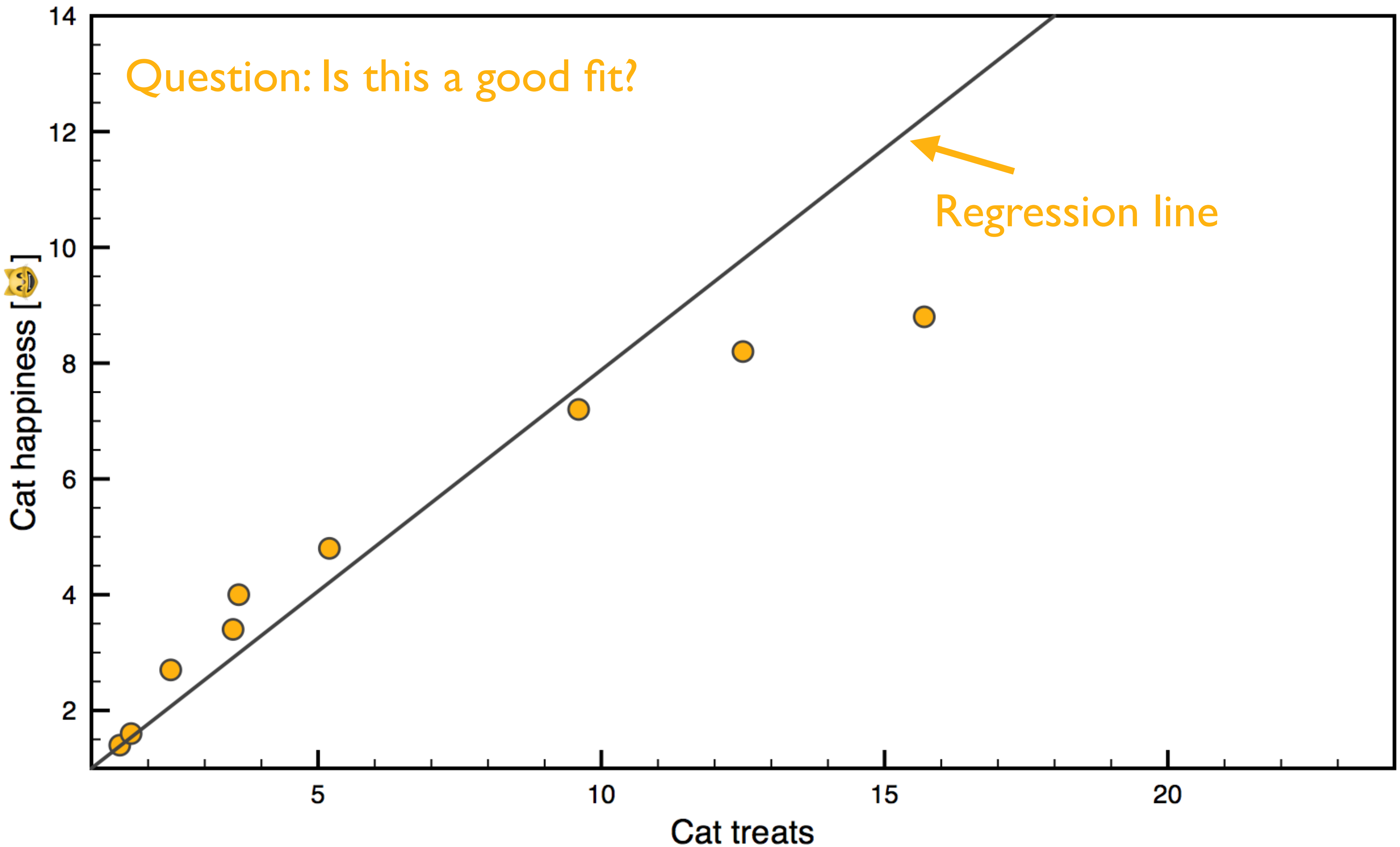

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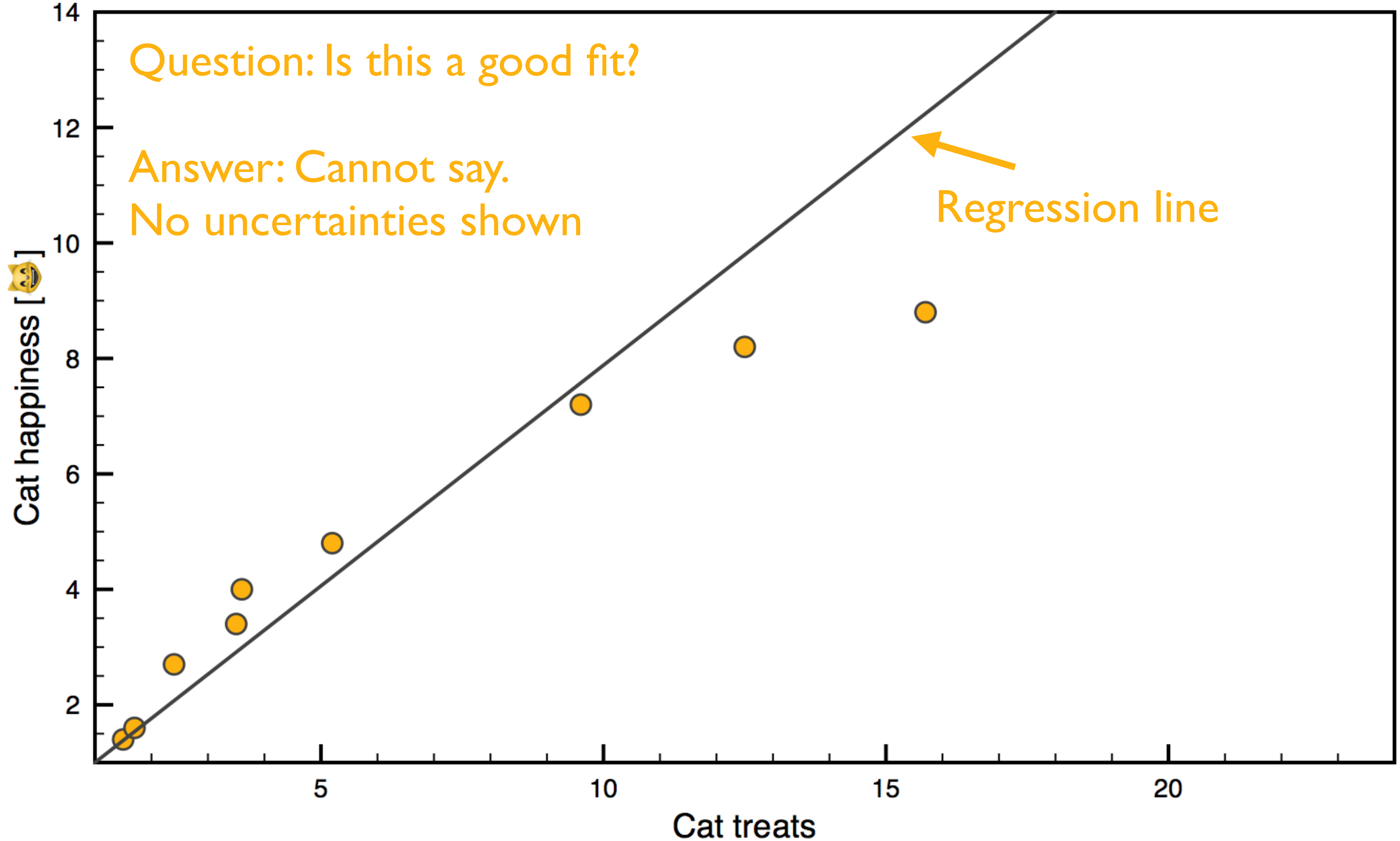

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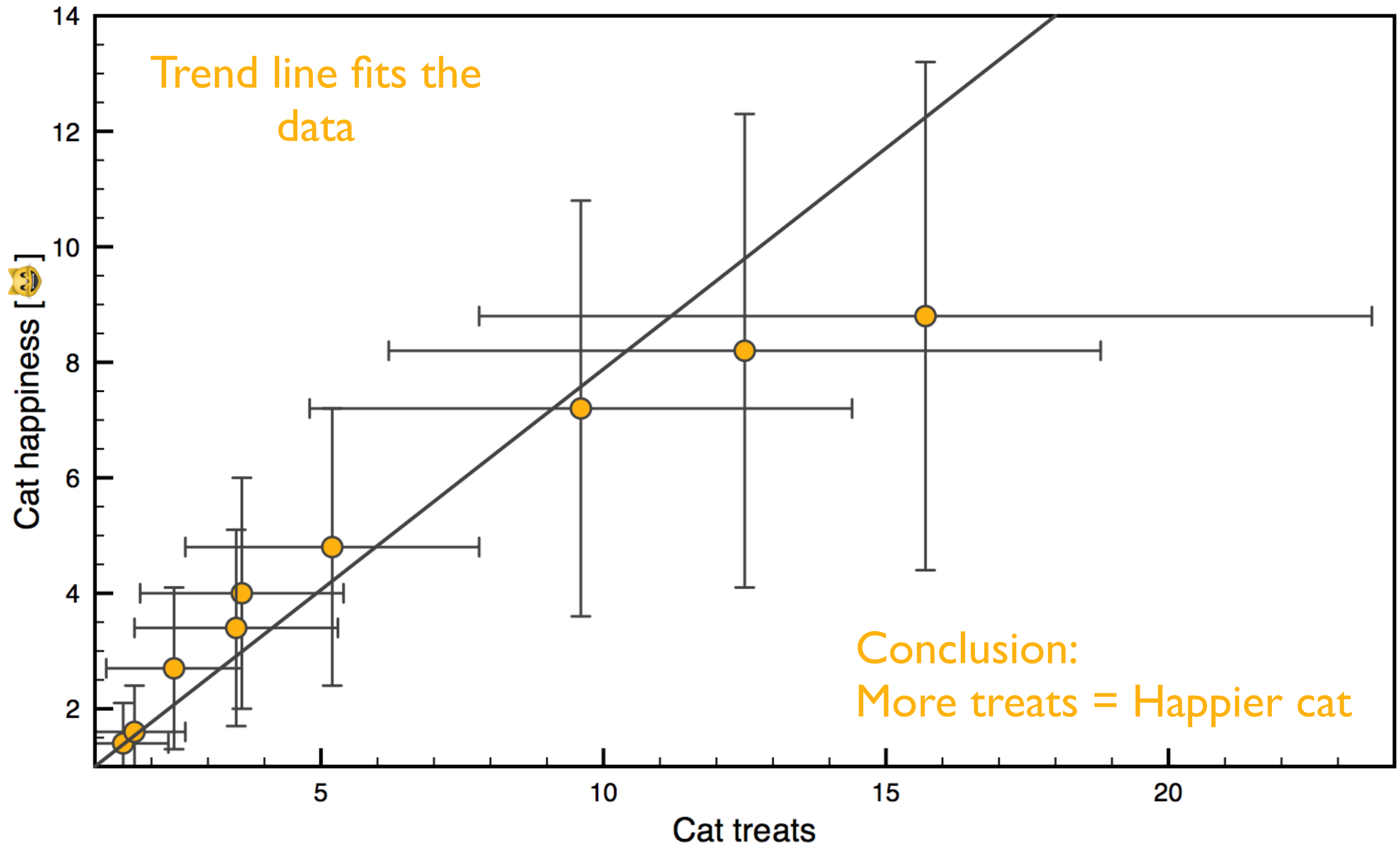

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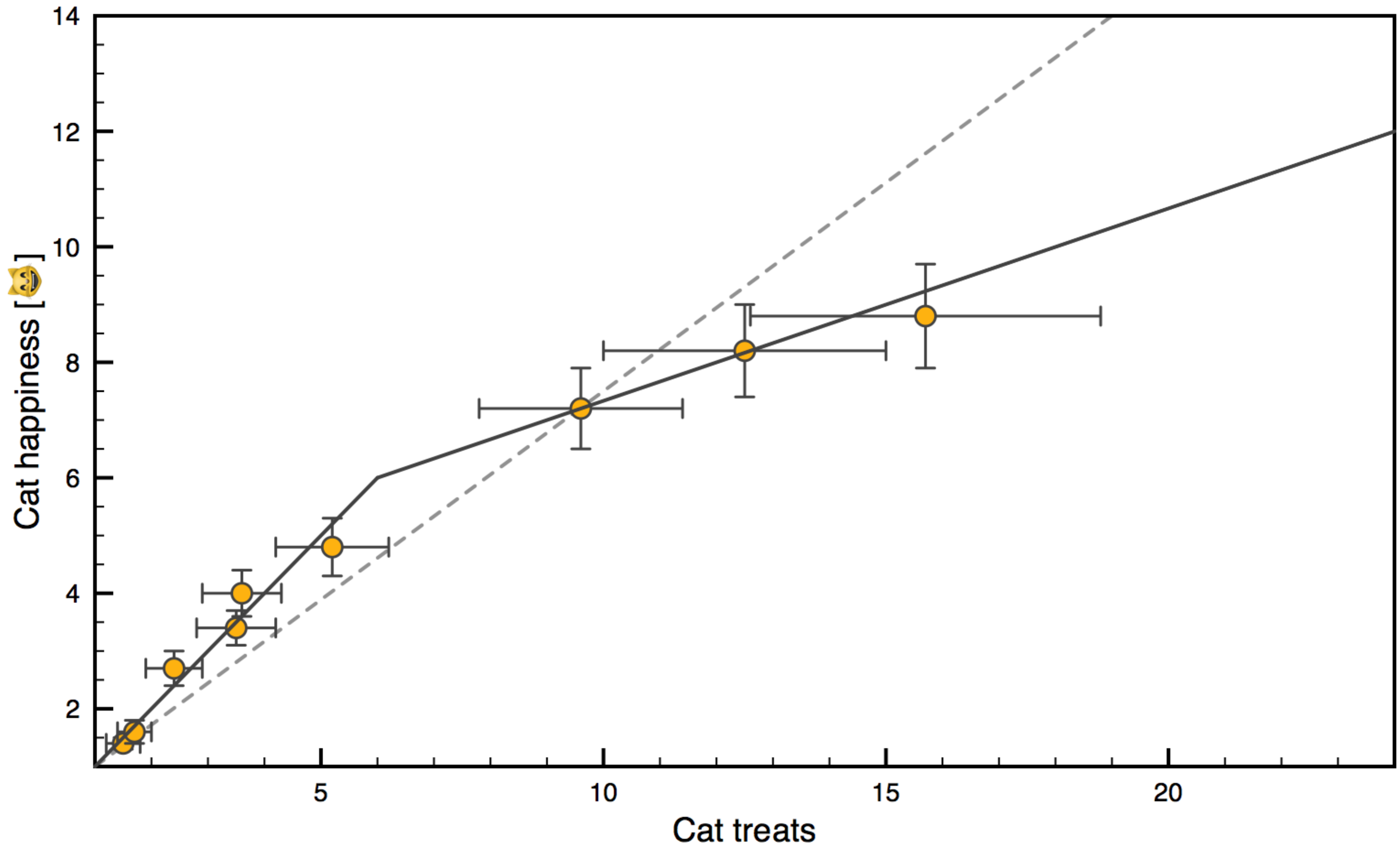

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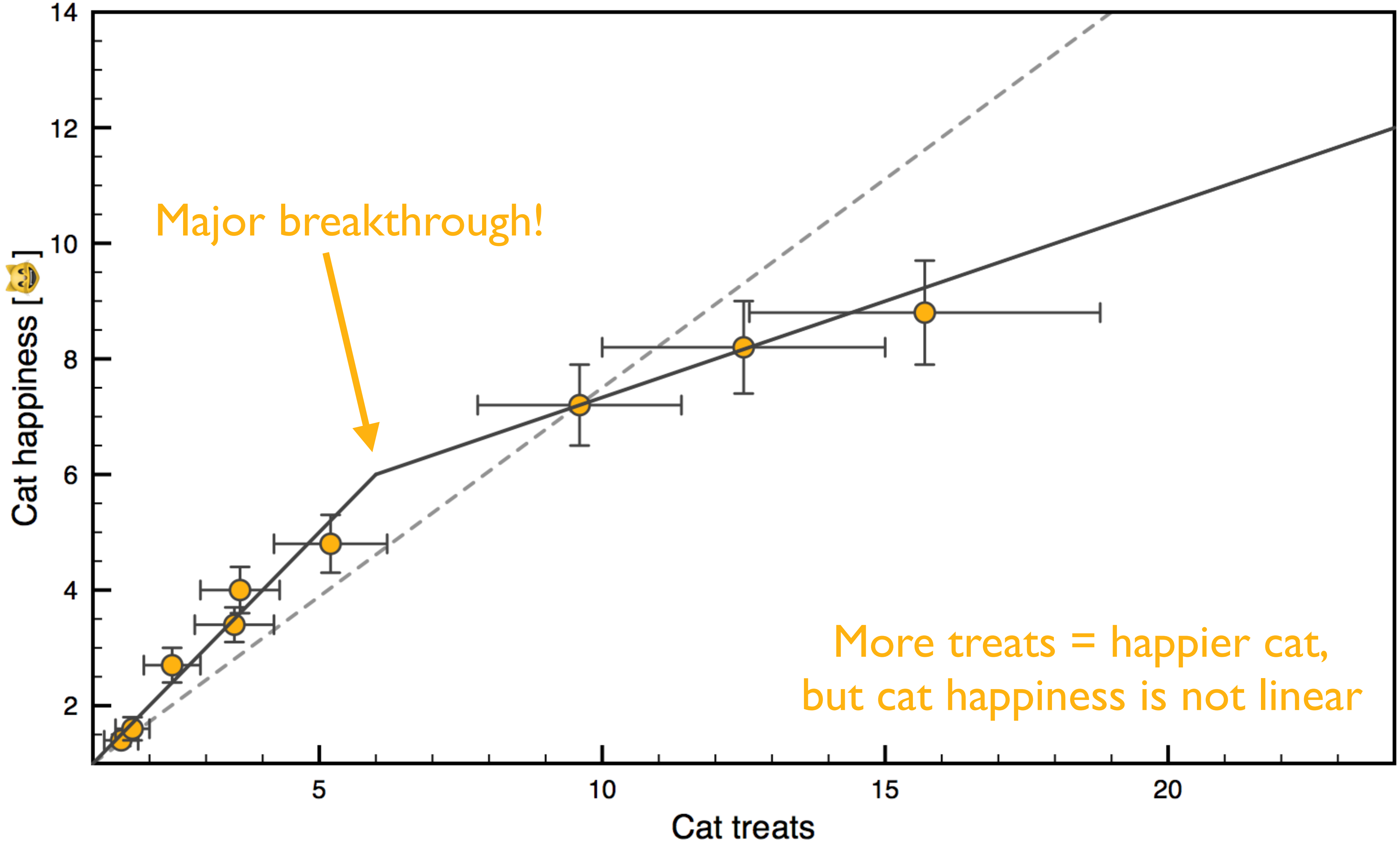

Reporting uncertainties

- The point is uncertainties matter
  - **What our data can say is a function of the uncertainty**
  - A model fit to data is meaningful only if the predictions lie within the uncertainty in the data

- A geological age of 1.88 Ga for an event might be interesting, but an age of 1.88±0.02 Ga is far more powerful

Recap

- What are some of the challenges collecting geological data samples?
- What are some common sources of uncertainty in Earth science data?
- How should you mathematically represent uncertainty in any measurement?

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References

McKillup, S., & Dyar, M. D. (2010). *Geostatistics explained: an introductory guide for earth scientists*. Cambridge University Press.

Taylor, J. R. (1997). *An introduction to error analysis: the study of uncertainties in physical measurements*. Sausalito, CA, USA: University science books.