Class overview today - April 4, 2016

- **Part I - Natural diffusion**
    - Introduction to the diffusion process
    - Mathematical description of diffusion
    - Hillslope diffusive processes
- **Part II - Solving the diffusion equation**
    - Components of the diffusion equation
    - General form of the diffusion equation
    - Solution of the diffusion equation for hillslope diffusion

Introduction to Quantitative Geology
Lecture 5
Natural diffusion:
Hillslope sediment transport, Earth's thermal field

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4.4.2016

Goals of this lecture

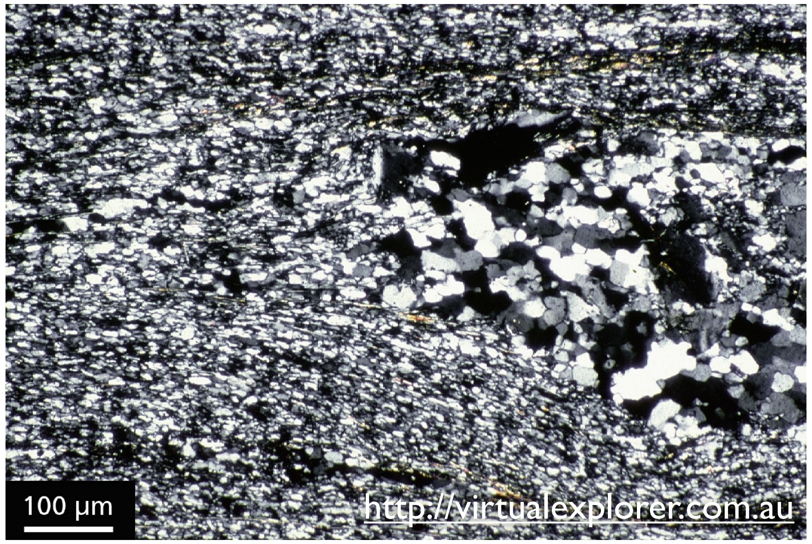
- Introduce the **diffusion process**
- Present some examples of **hillslope diffusive processes** (heave/creep, solifluction, rain splash)

Natural diffusion

- **What are some examples of diffusion processes in nature?**

Diffusion as a geological process

Grain boundary sliding

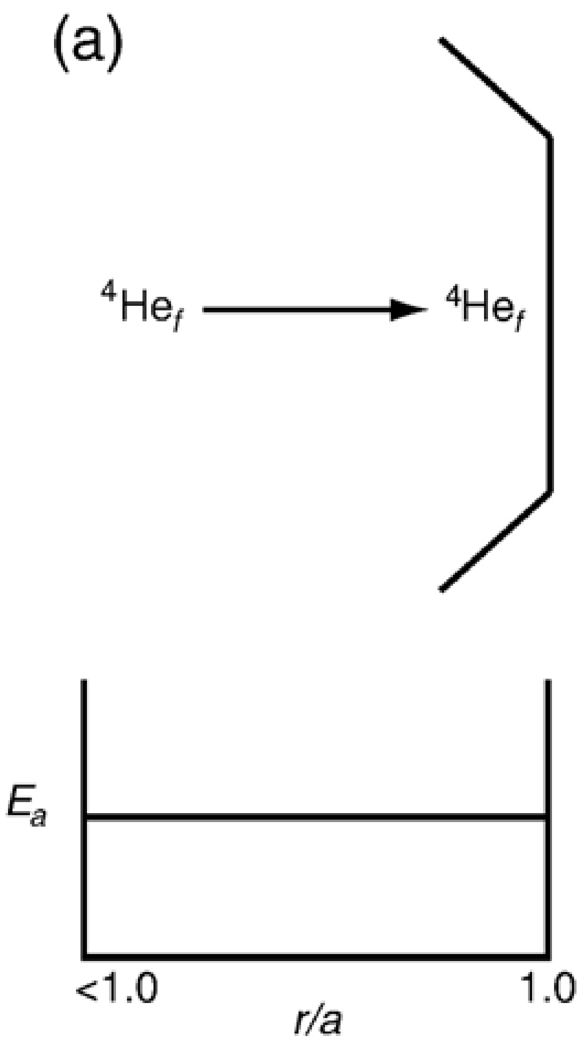


Rock rheology

Rain splash

Hillslope erosion

^{4}He diffusion in apatite



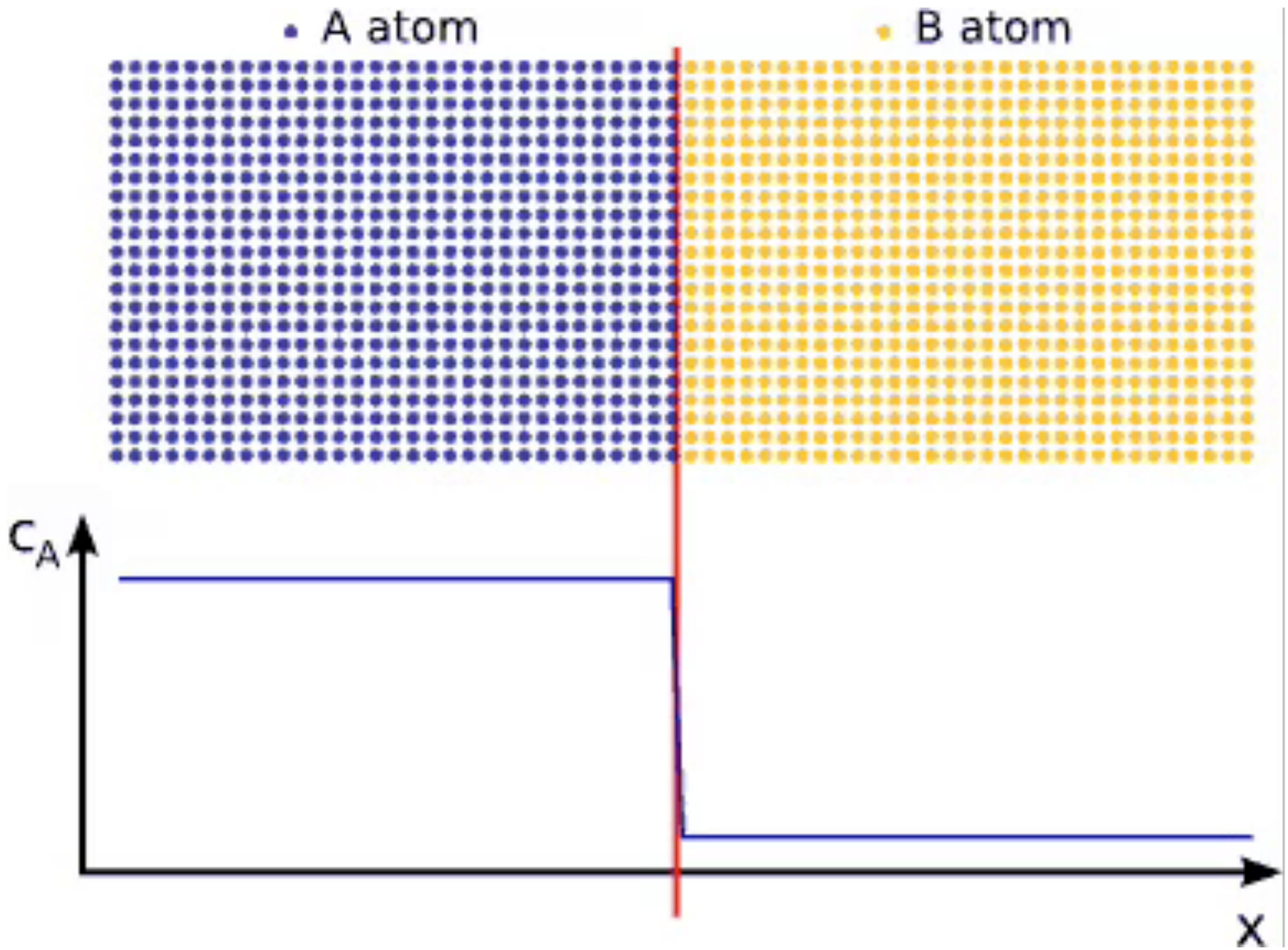
Shuster et al., 2006

Thermochronology

General concepts of diffusion

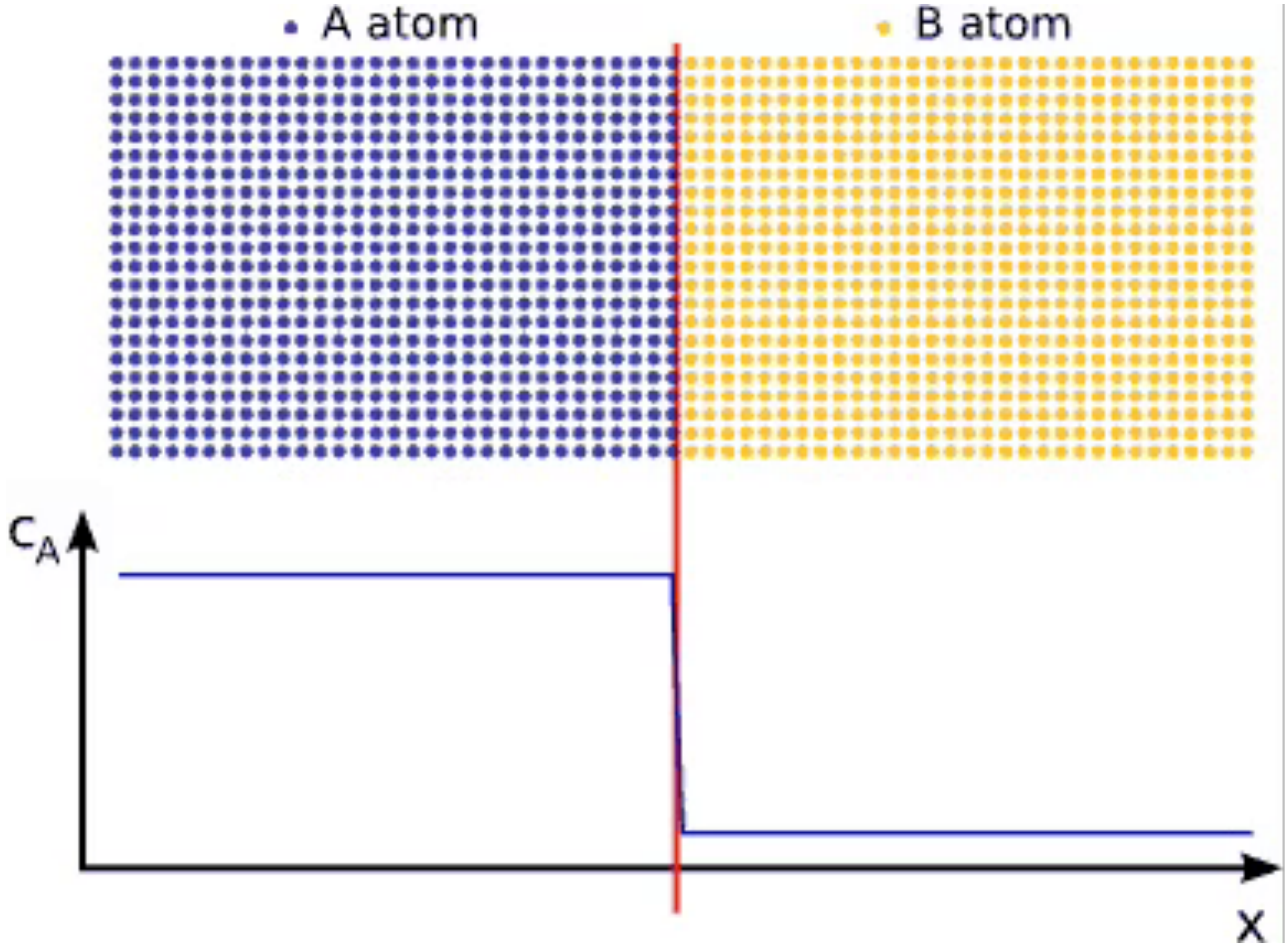
- **Diffusion** is a process resulting in mass transport or mixing as a result of the random motion of diffusing particles

The diffusion process



http://web.unideb.hu/zerdelyi/

The diffusion process



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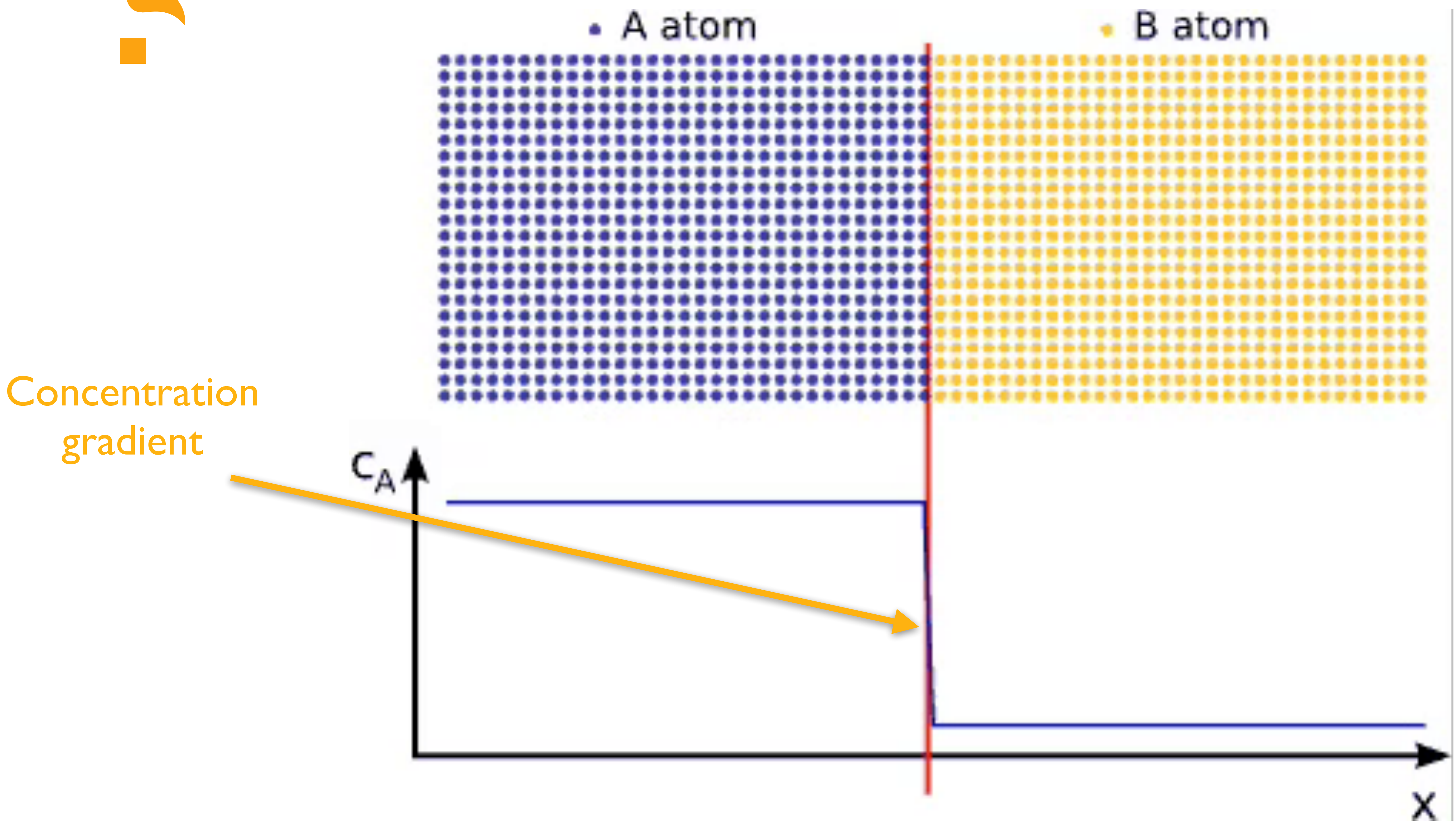

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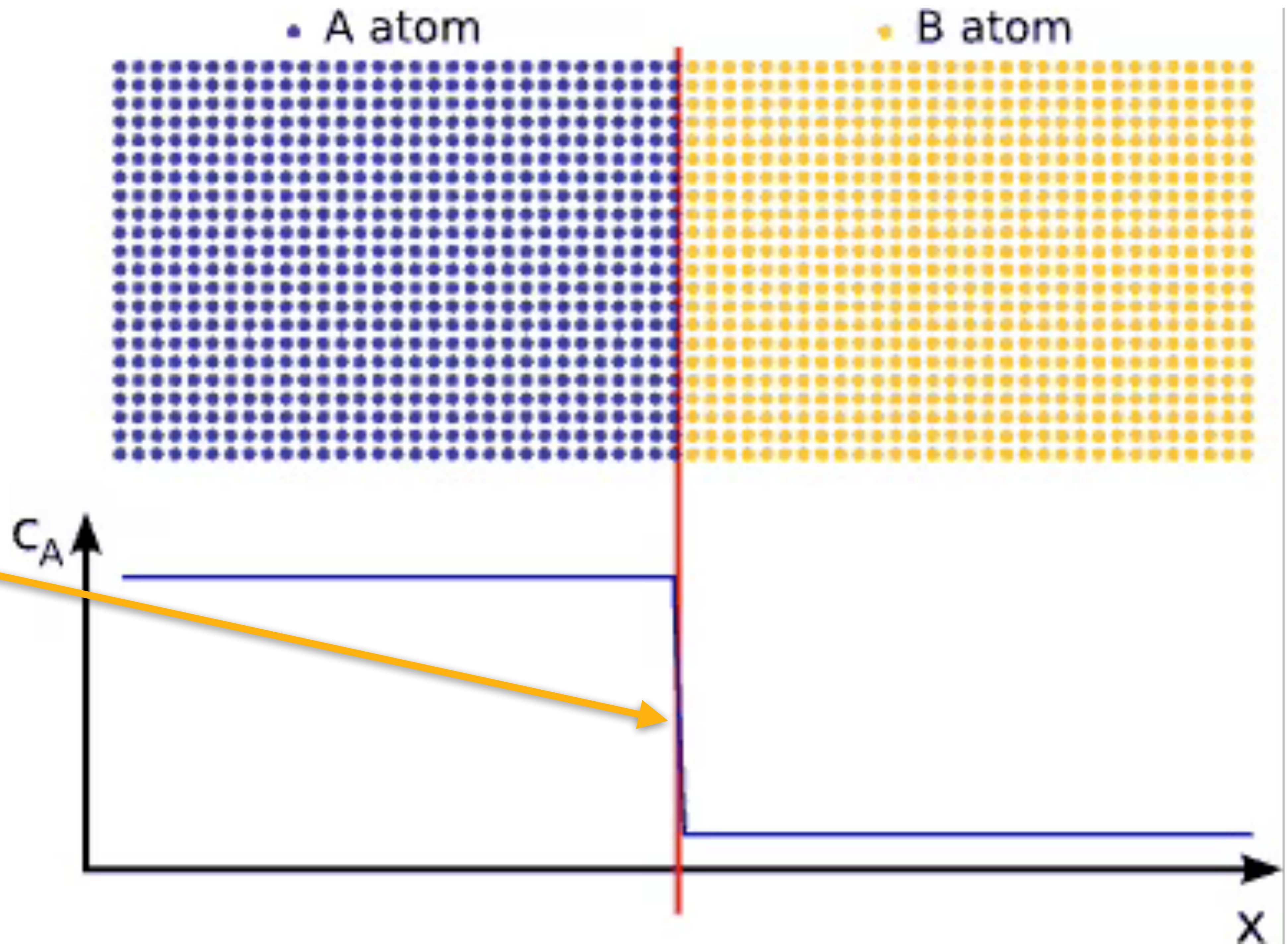

Concentration gradient

http://web.unideb.hu/zerdelyi/

General concepts of diffusion

- **Diffusion** is a process resulting in mass transport or mixing as a result of the random motion of diffusing particles
  - Net motion of mass or transfer of energy is from regions of high concentration to regions of low concentration
  - Diffusion reduces concentration gradients

A more quantitative definition

- **Diffusion** occurs when a **conservative property** moves through space at a **rate proportional to a gradient**

- **Conservative property**: A quantity that must be conserved in the system (e.g., mass, energy, momentum)
- **Rate proportional to a gradient**: Movement occurs in direct relationship to the change in concentration
    - Consider a one hot piece of metal that is put in contact with a cold piece of metal. Along the interface the change in temperature will be most rapid when the temperature difference is largest

A mathematical definition

- We can now translate the concept of diffusion into mathematical terms.
  - We've just seen "**Diffusion** occurs when a (1) **conservative property** moves through space at a (2) **rate proportional to a gradient**"
- If we start with part 2, we can say in comfortable terms that
  [transportation rate] is proportional to [change in concentration over some distance]

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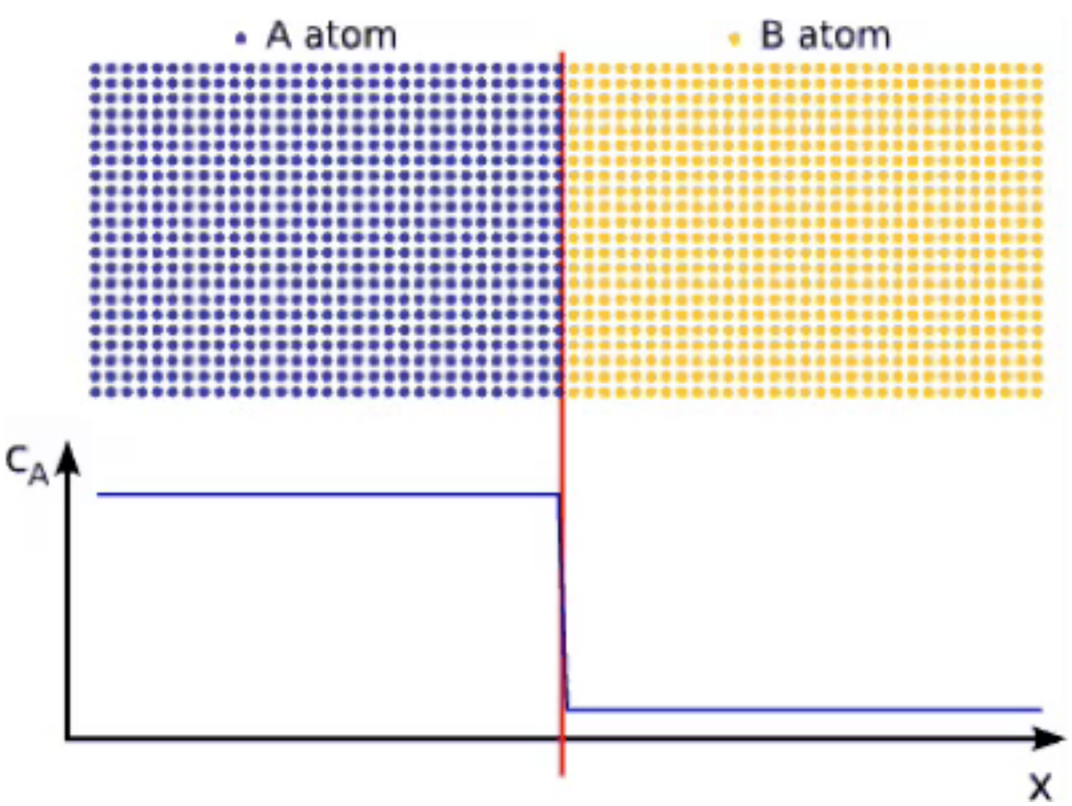
[flux] is proportional to [concentration gradient]
- Finally, in symbols we can say

$$q \propto \frac{\Delta C}{\Delta x}$$

where q is the mass flux, $\propto$ is the "proportional to" symbol, Δ indicates a change in the symbol that follows, C is the concentration and x is distance

A mathematical definition

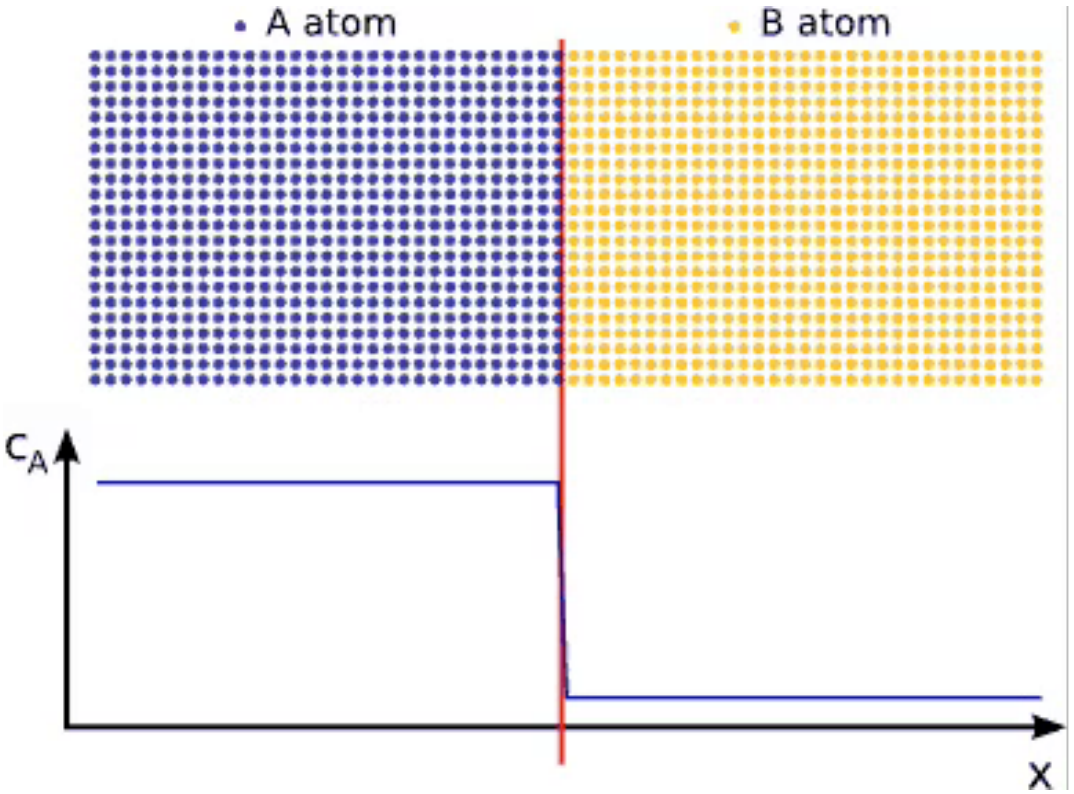


- If transport is directly proportional to the gradient, we can replace the proportional to symbol with a constant
- We can also replace the finite changes *Δ* with infinitesimal changes *∂*
- Keeping the same colour scheme, we see

$$q \propto \frac{\Delta C}{\Delta x} \longrightarrow q = -D\frac{\partial C}{\partial x}$$

where D is a constant called the **diffusion coefficient** or **diffusivity**

A mathematical definition

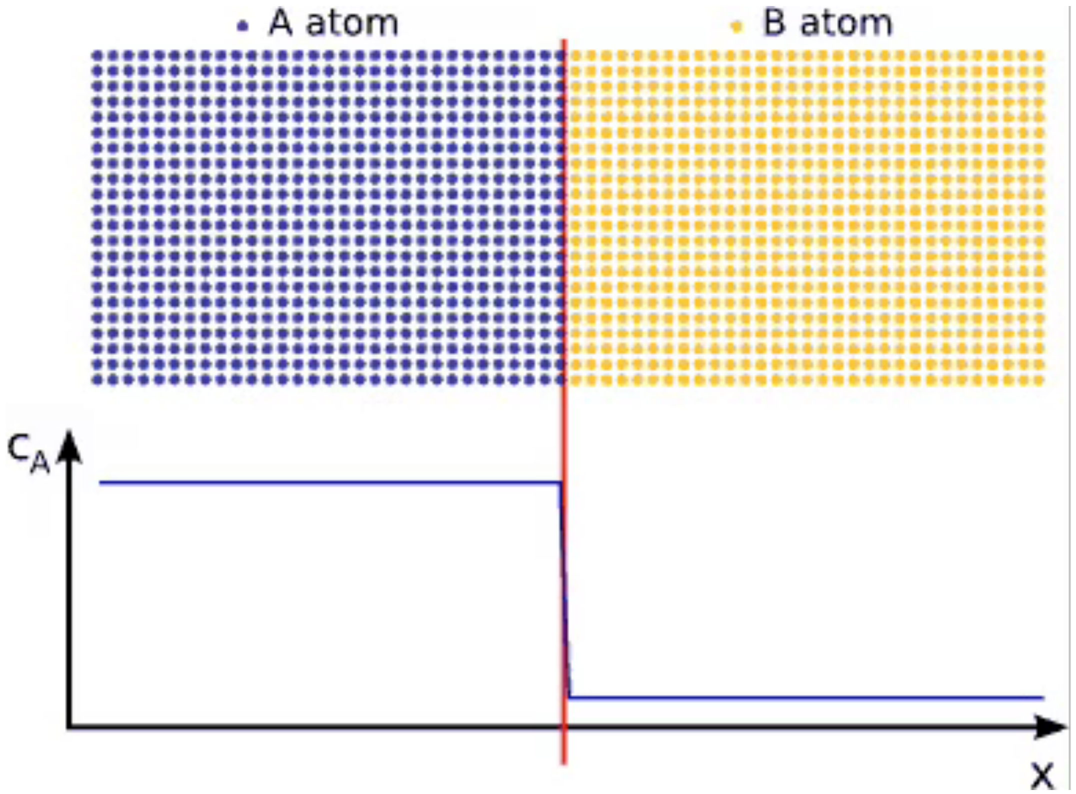


- Consider the example to the left of the concentration of some atoms A and B
- Here, we can formulate the diffusion of atoms of A across the red line with time as

$$q = -D \frac{\partial C_{\mathrm{A}}}{\partial x}$$

where C_{A} is the **concentration** of atoms of A

A mathematical definition



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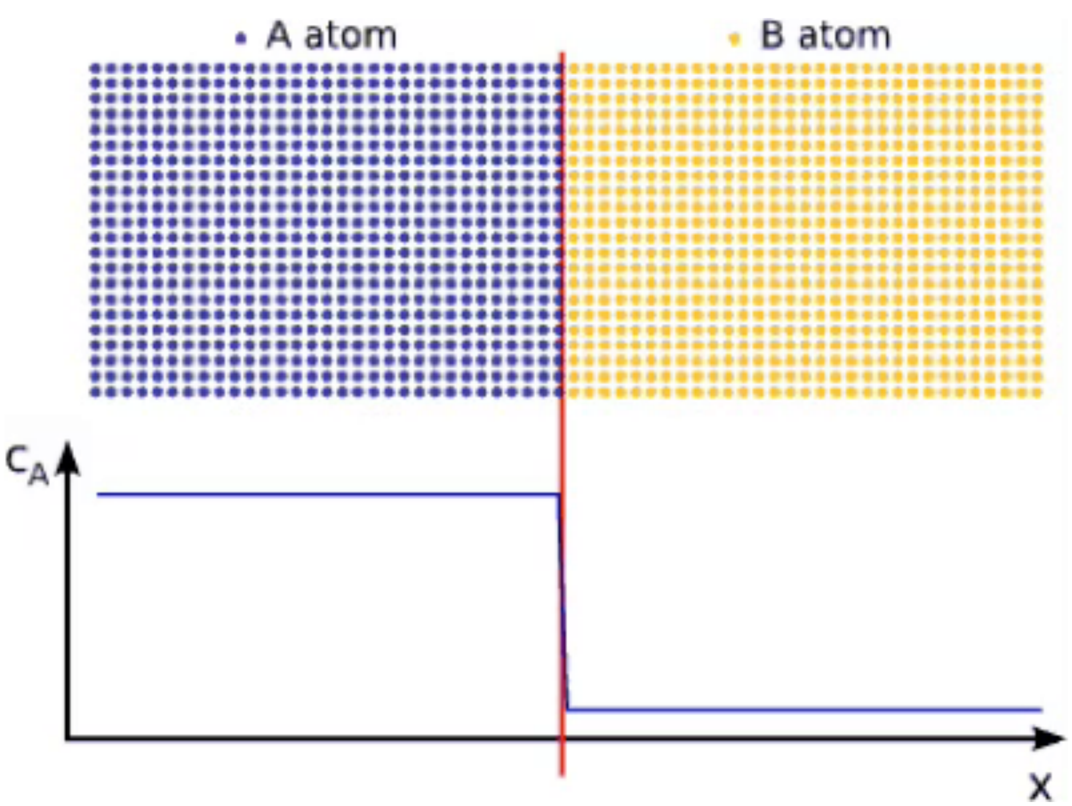
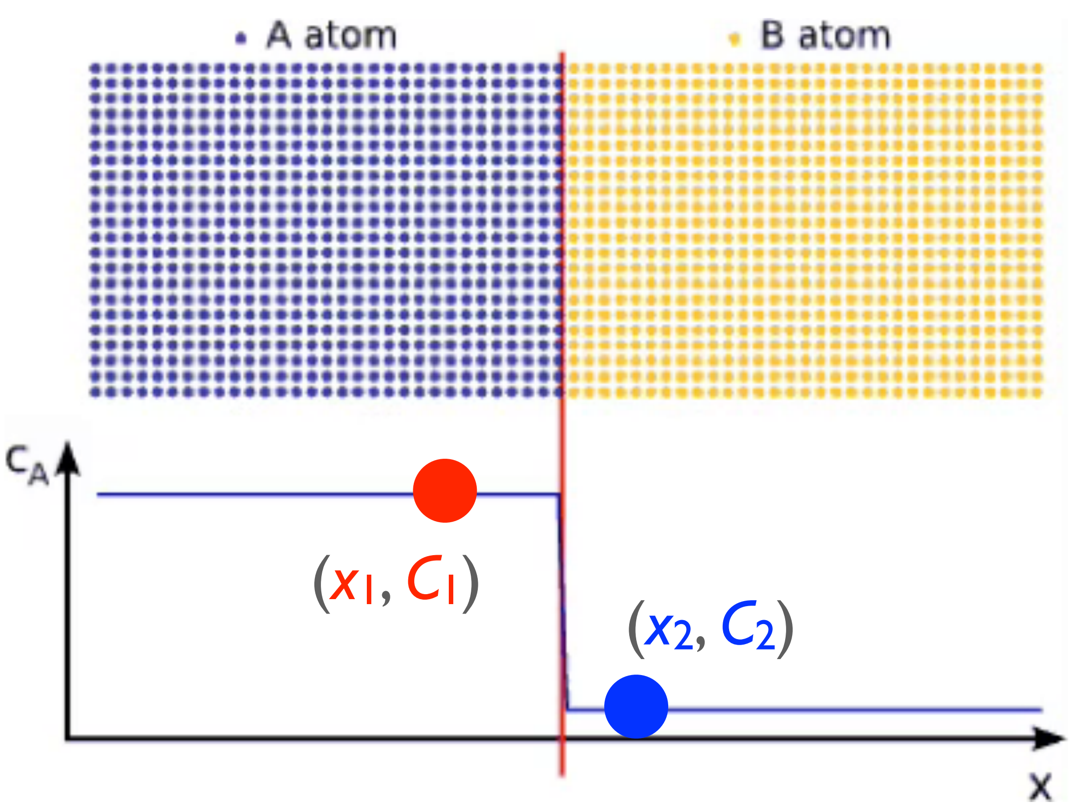


- OK, but **why is there a minus sign?**

$$q = -D\frac{\partial C_{\mathrm{A}}}{\partial x}$$

A mathematical definition



- OK, but **why is there a minus sign?**

$$q = -D\frac{\partial C_{\mathrm{A}}}{\partial x}$$

- We can consider a simple case for finite changes at two points: (x_1, C_1) and (x_2, C_2)
- At those points, we could say

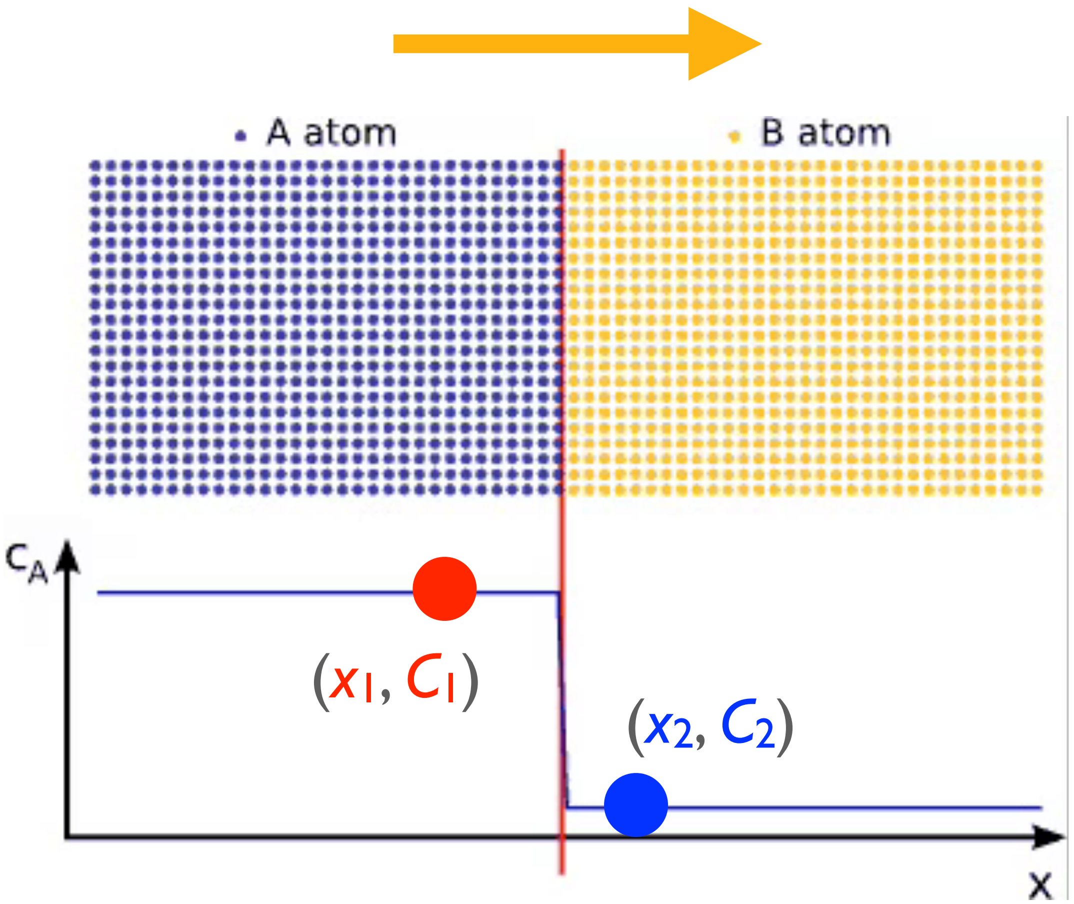
$$q = -D\frac{\Delta C}{\Delta x}$$

$$q = -D\frac{C_2 - C_1}{x_2 - x_1}$$

- As you can see, ΔC will be negative while Δx is positive, resulting in a negative gradient

A mathematical definition

Positive flux of A



- OK, but **why is there a minus sign?**

$$q = -D\frac{\partial C_{\mathrm{A}}}{\partial x}$$

- Multiplying the negative gradient by $-D$ yields a positive flux q along the x axis, which is what we expect

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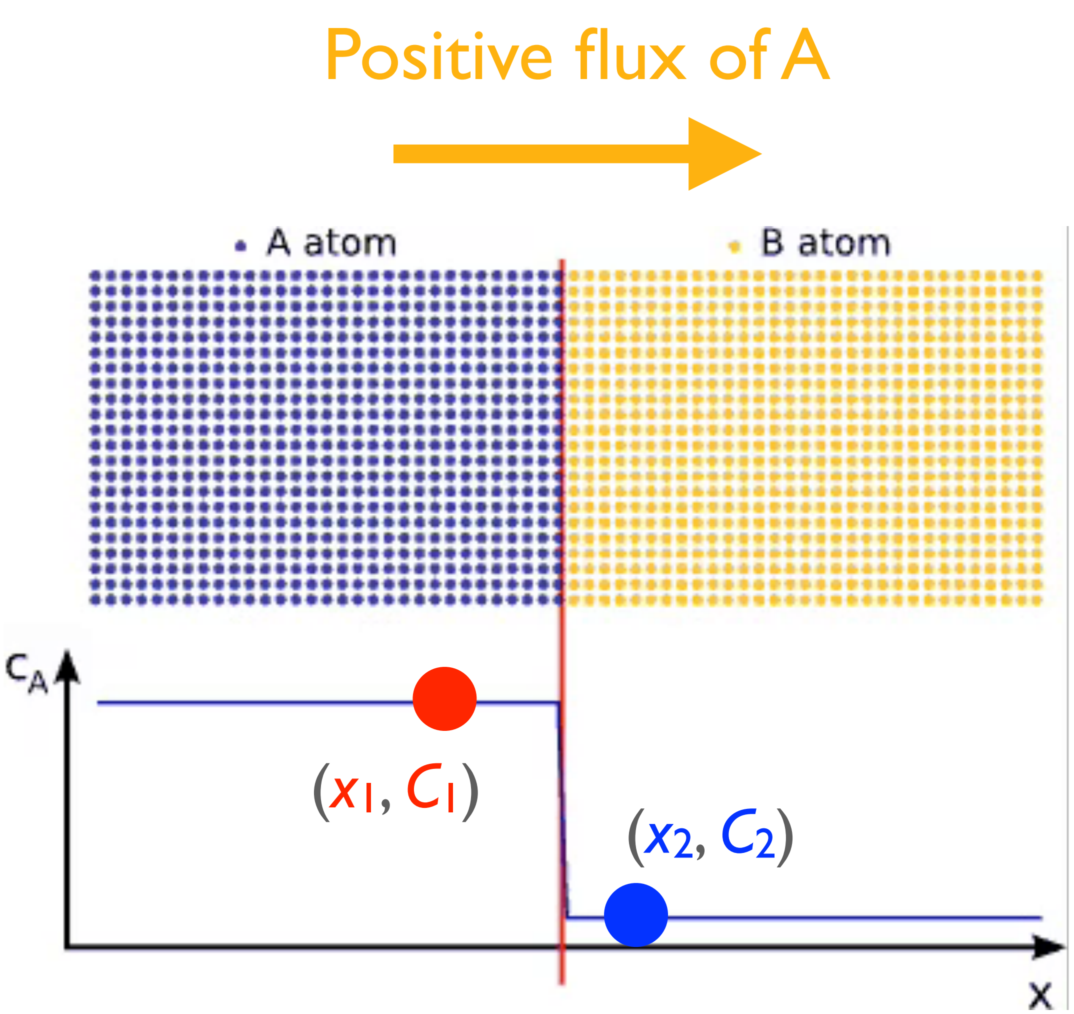
A mathematical definition



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$$\frac{\Delta C}{\Delta t} = -\frac{\Delta q}{\Delta x}$$

where **t** is time

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$$\frac{\Delta C}{\Delta t} = -\frac{\Delta q}{\Delta x} \longleftarrow \text{Conservation of mass/energy}$$

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A mathematical definition

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- So, **how is this a conservation of mass/energy equation?**

A mathematical definition

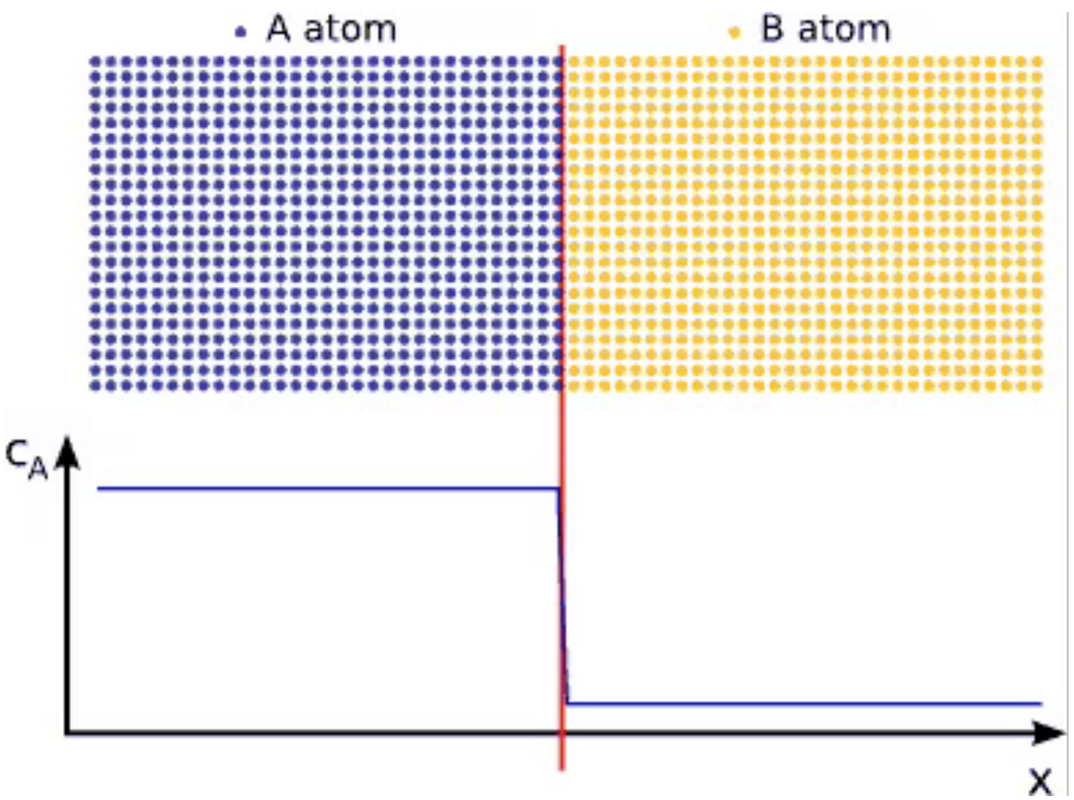
$$\frac{\Delta C}{\Delta t} = -\frac{\Delta q}{\Delta x}$$

- So, **how is this a conservation of mass/energy equation?**

$$\frac{\Delta C}{\Delta t} = -\frac{q_2 - q_1}{x_2 - x_1}$$

- Consider the fluxes q_1 and q_2 at two points, x_1 and x_2
  - What happens when the flux of mass q_2 at x_2 is larger than the flux q_1 at x_1?

A mathematical definition

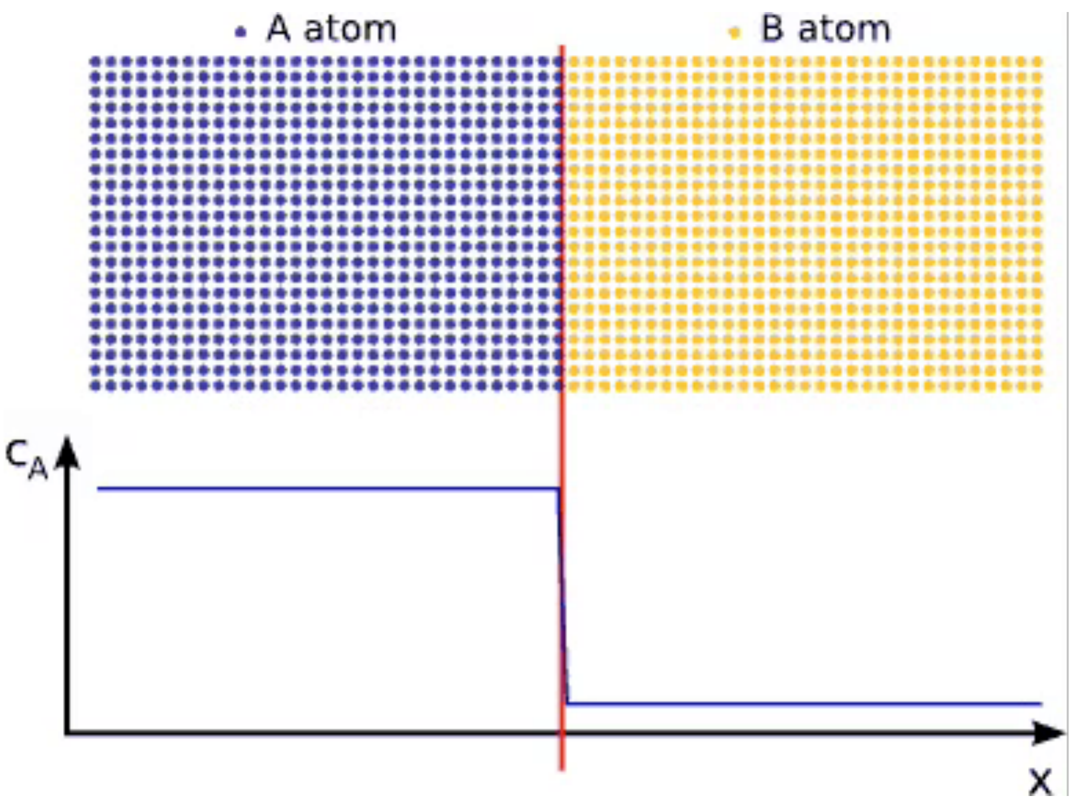


- If we again replace the finite changes Δ with infinitesimal changes ∂, we can describe our example on the left

$$\frac{\partial C_\mathrm{A}}{\partial t} = -\frac{\partial q}{\partial x}$$

  - Essentially, all this says is that the concentration of A will change based on the flux across a reference face at position x minus the flux across a reference face at position $x + dx$

A mathematical definition



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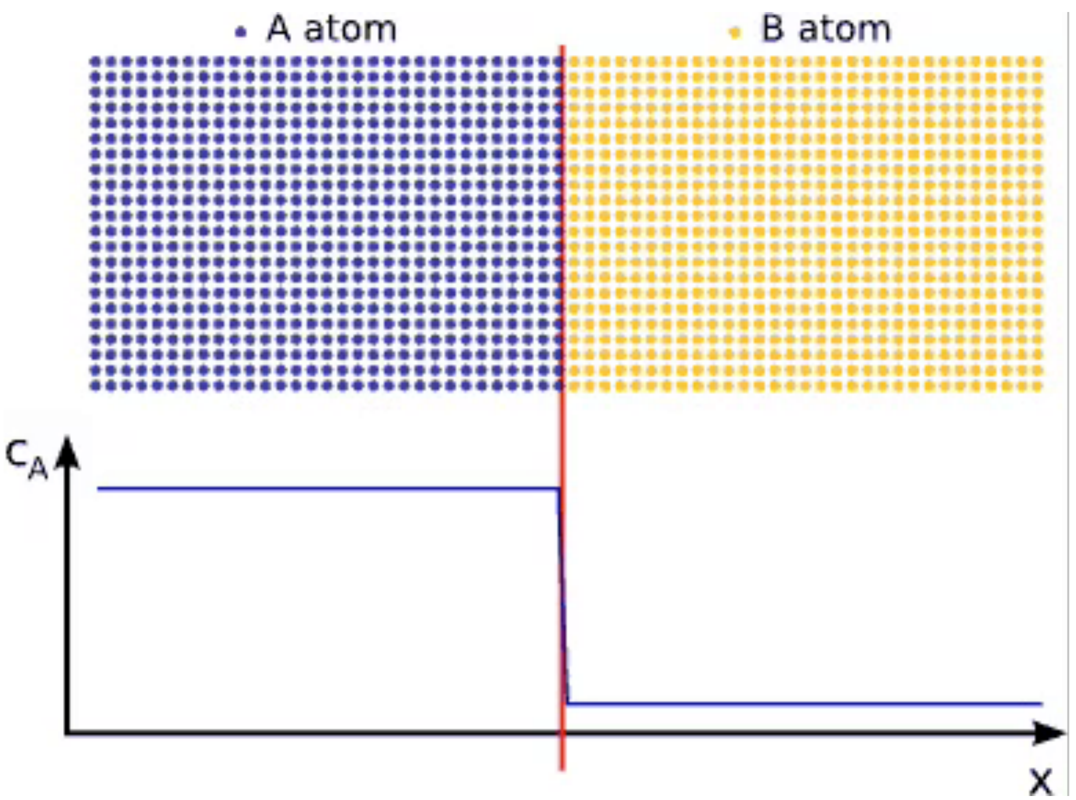
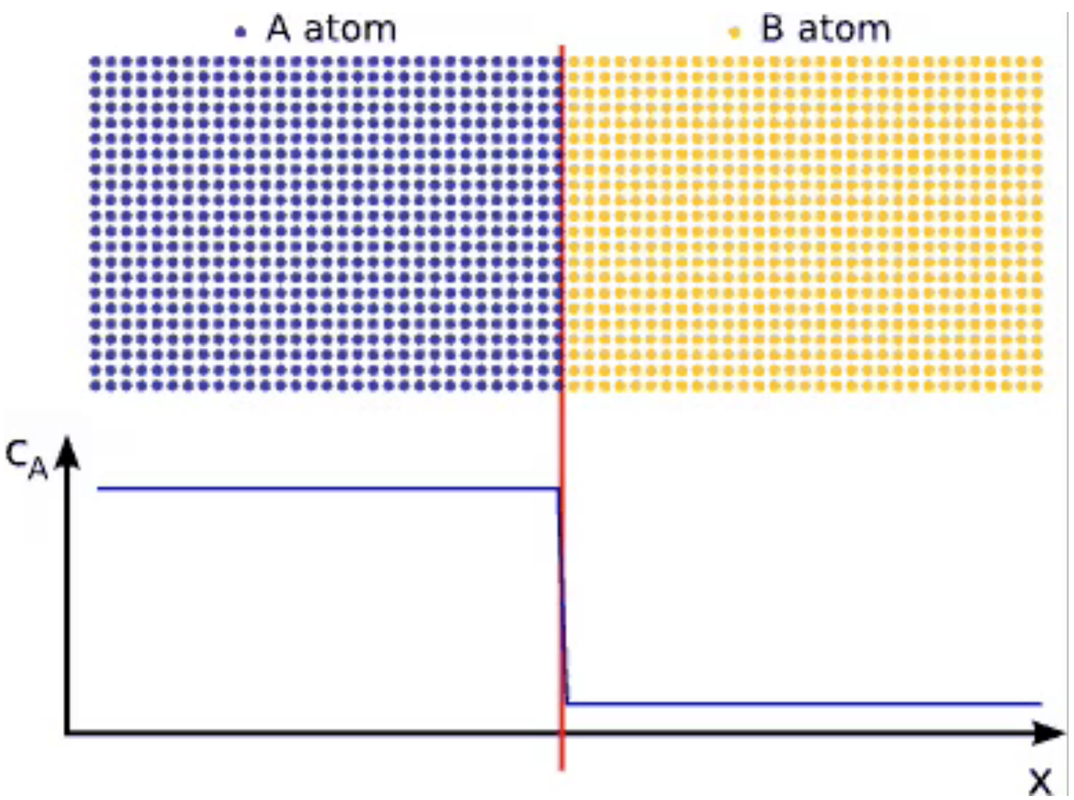


- In part 2 of today's lectures (Lecture 6), we'll see how to combine the two equations we've just seen into the diffusion equation and how to solve that equation

$$q = -D\frac{\partial C_{\mathrm{A}}}{\partial x} + \frac{\partial C_{\mathrm{A}}}{\partial t} = -\frac{\partial q}{\partial x} = \text{diffusion}$$

Note: This is not the actual mathematical relationship for diffusion

A mathematical definition



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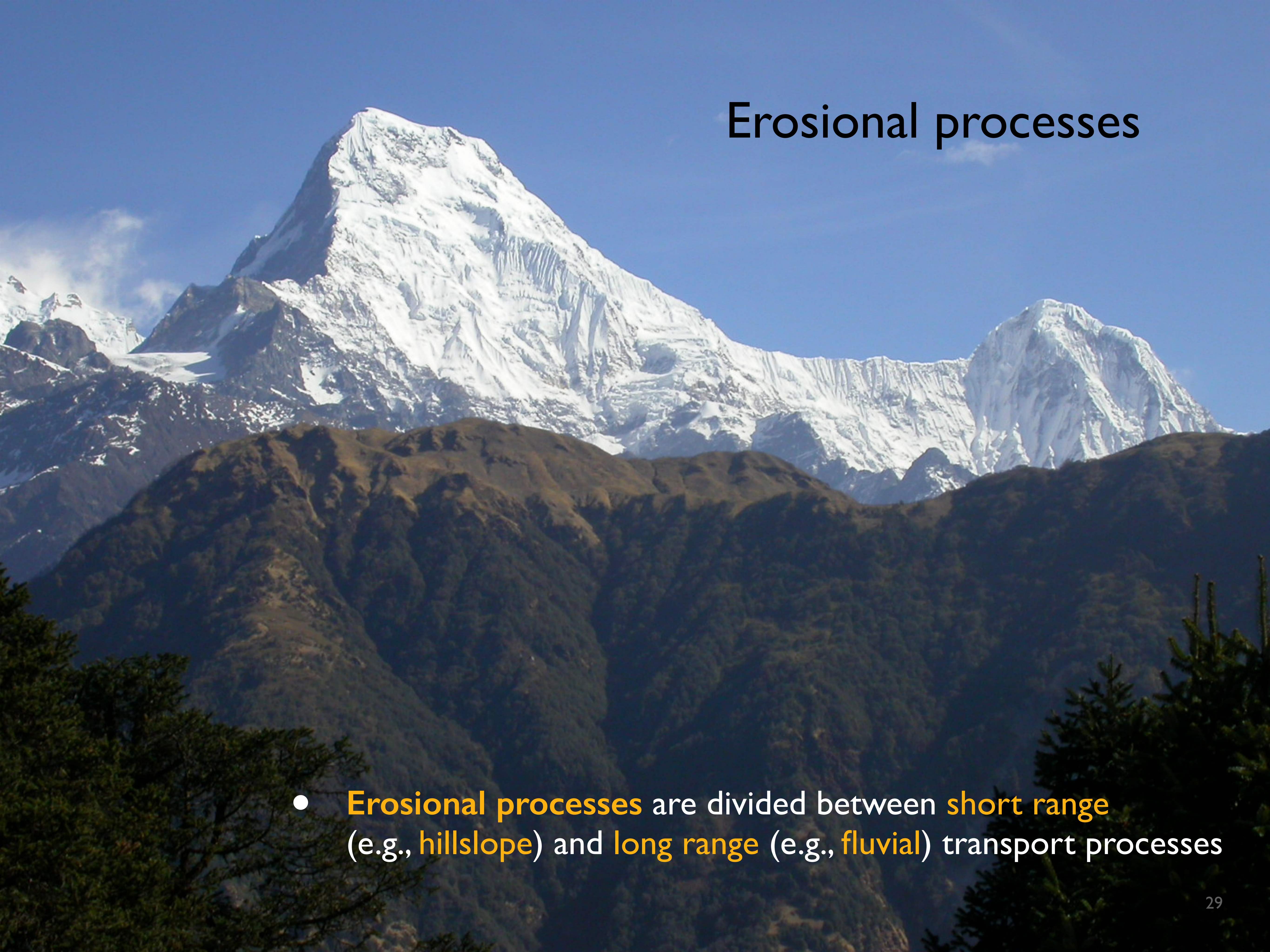
General concepts of diffusion

- So our definitions of diffusion to this point are OK for true diffusion processes, but there are also numerous geological processes that are not themselves diffusion processes, but result in diffusion-like behaviour
    - **Hillslope diffusion** is a name given to the overall behaviour of various surface processes that transfer mass on hillslopes in a diffusion-like manner



Erosional processes

- **Erosional processes** are divided between **short range** (e.g., **hillslope**) and **long range** (e.g., **fluvial**) transport processes

Hillslope processes

- **Hillslope processes** comprise the different types of <u>mass movements that occur on hillslopes</u>
  - **Slides** refer to <u>cohesive blocks</u> of material moving on a <u>well-defined surface of sliding</u>
  - **Flows** move entirely by <u>differential shearing within the transported mass</u> with <u>no clear plane</u> at the base of the flow
  - **Heave** results from disrupting forces acting perpendicular to the ground surface by expansion of the material

Hillslope processes

- Hillslope processes comprise the different types of mass movements that occur on hillslopes
  - Slides refer to cohesive blocks of material moving on a well-defined surface of sliding
  - Flows move entirely by differential shearing within the transported mass with no clear plane at the base of the flow

Our focus

  - Heave results from disrupting forces acting perpendicular to the ground surface by expansion of the material

Mass movement processes

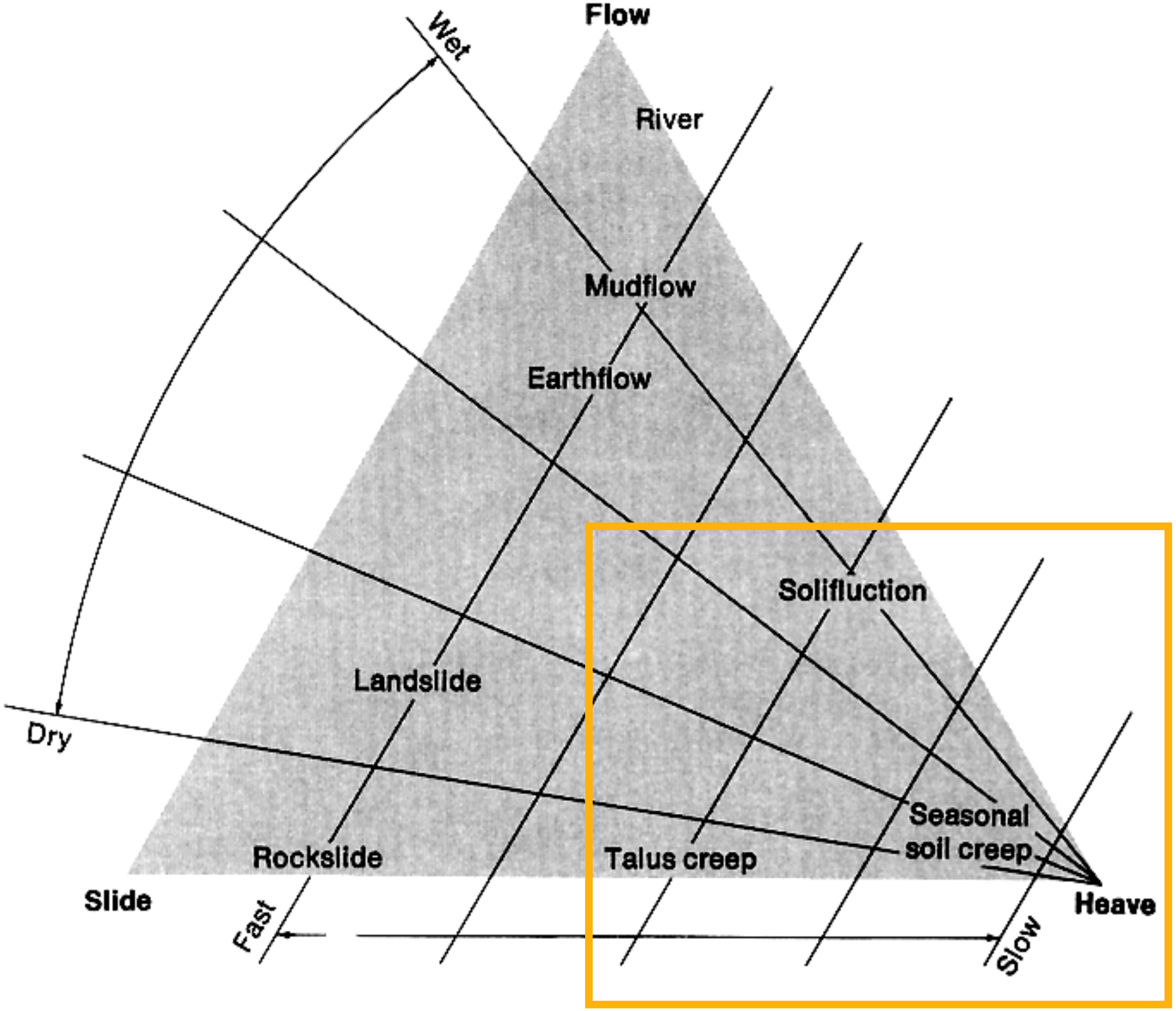


Fig. 4.27, Ritter et al., 2002

- Creep is almost too slow to monitor

Heave and creep

- Creep: The extremely slow movement of material in response to gravity
  - Heave: The vertical movement of unconsolidated particles in response to expansion and contraction, resulting in a net downslope movement on even the slightest slopes
  - Seasonal creep or soil creep is periodically aided by heaving

Heave and creep

Nearly vertical Romney shale displaced by seasonal creep

Fig. 4.28, Ritter et al., 2002

Heave and creep

Fig. 4.29, Ritter et al., 2002

How does heaving work?

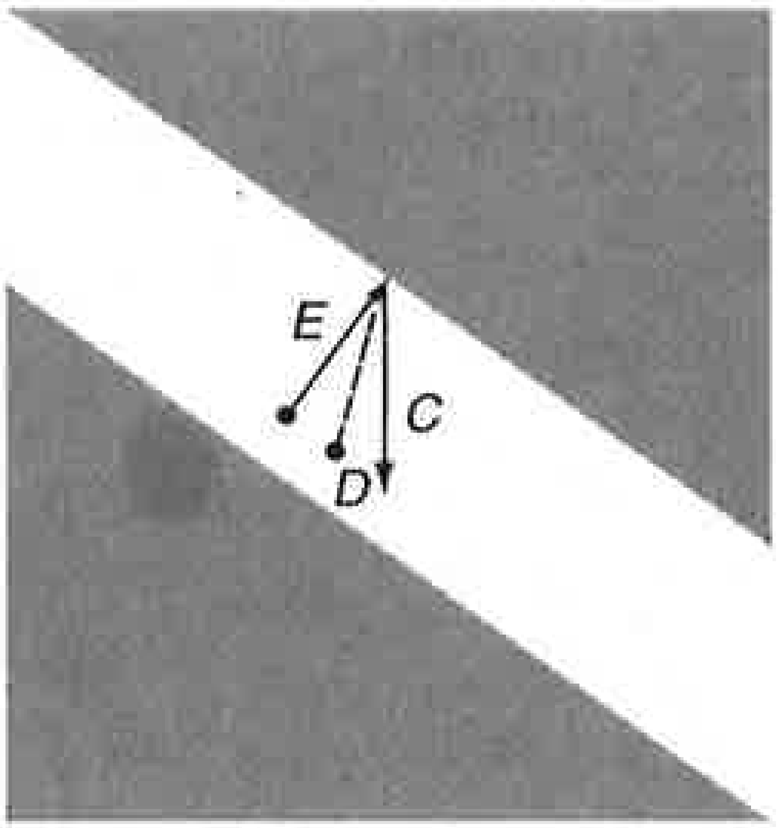
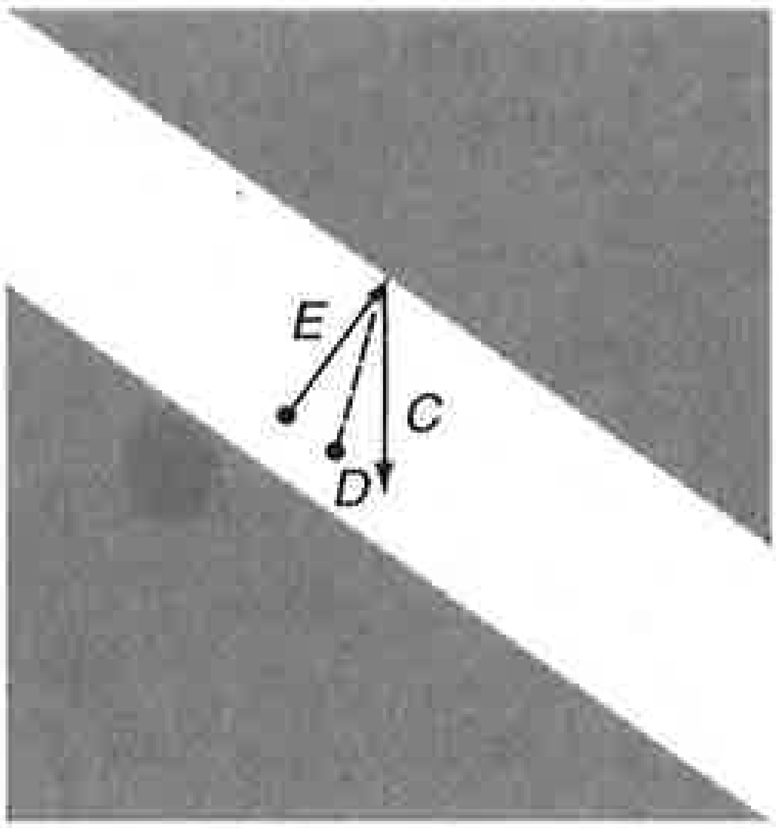


Fig. 4.30, Ritter et al., 2002

- Near-surface material moves perpendicular to the surface during expansion (E)
    - Expansion can result from swelling or freezing
- In theory, particles settle vertically downward during contraction (C)
- In reality, particle settling is not vertical, but follows a path closer to D

How does heaving work?



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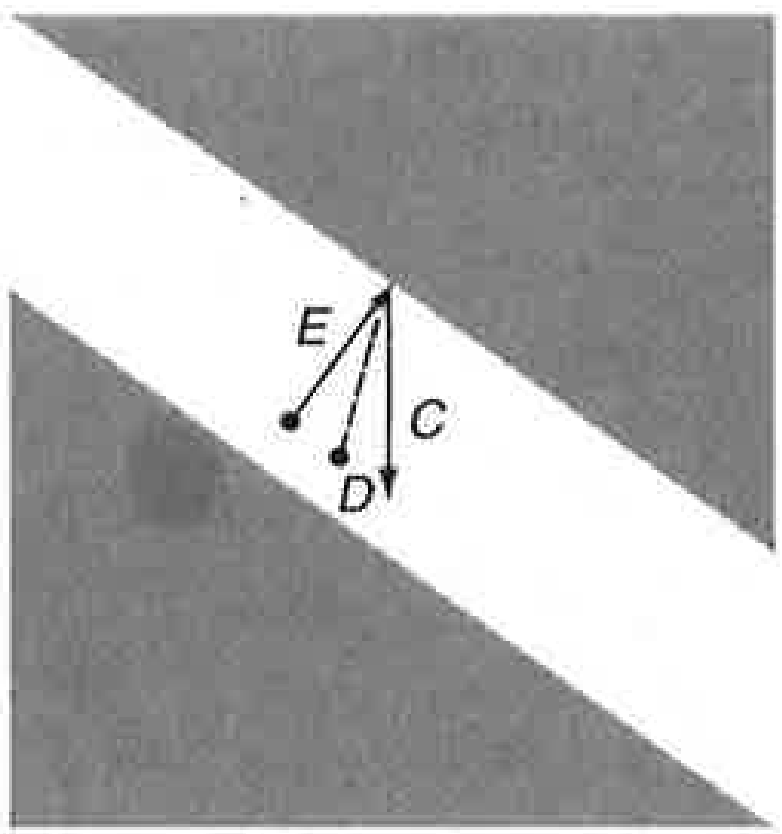
How does heaving work?



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- In theory, particles settle vertically downward during contraction (C)
- In reality, particle settling is not vertical, but follows a path closer to D
- Based on this concept, what do you think will influence the rates of creep?
  Slope angle, soil/regolith moisture, particle size/ composition

Mass movement by heaving

Creep rates vary with depth and soil/ regolith composition

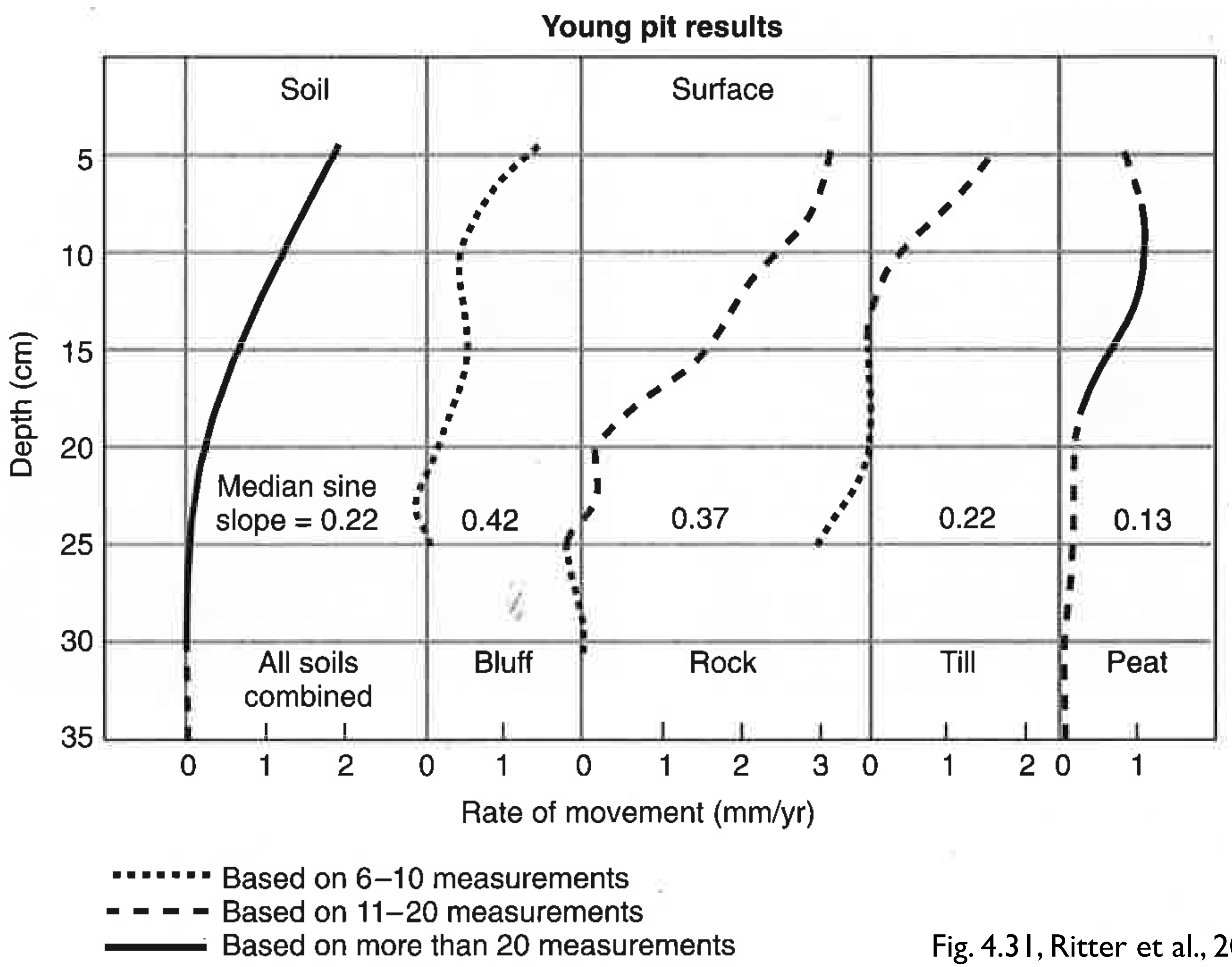


Fig. 4.31, Ritter et al., 2002

Frost creep and solifluction

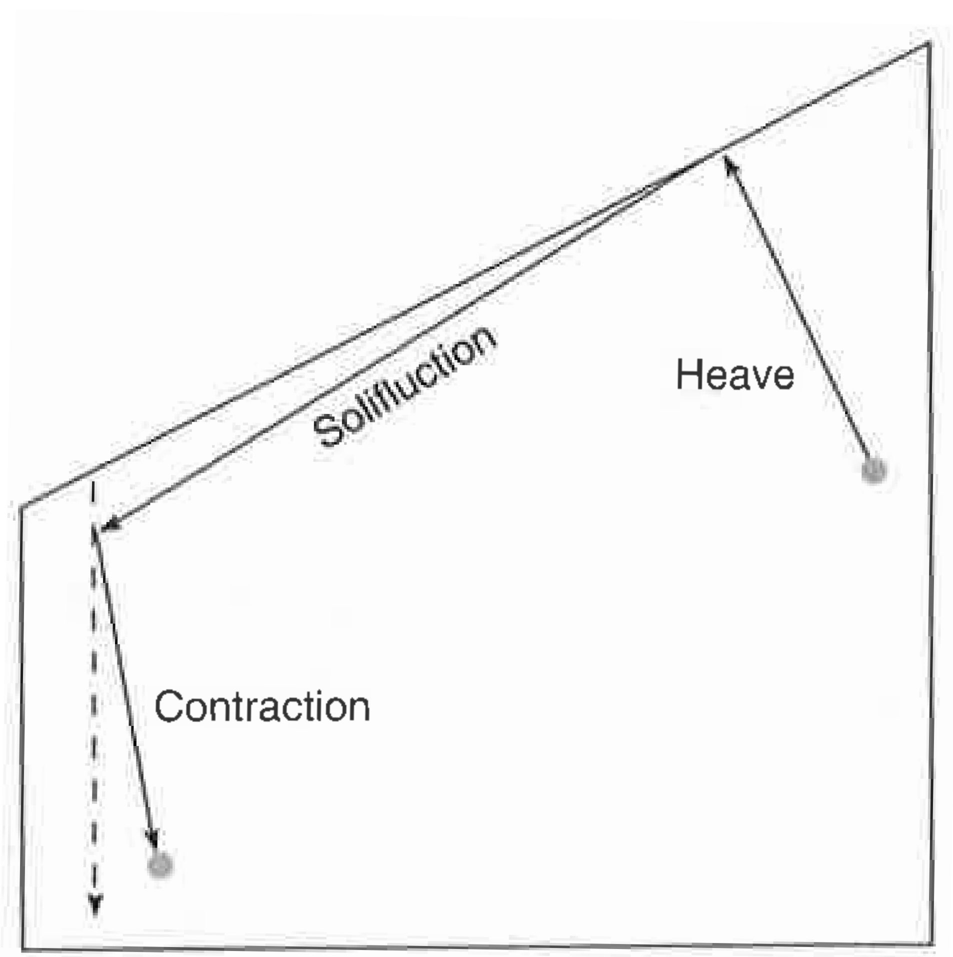
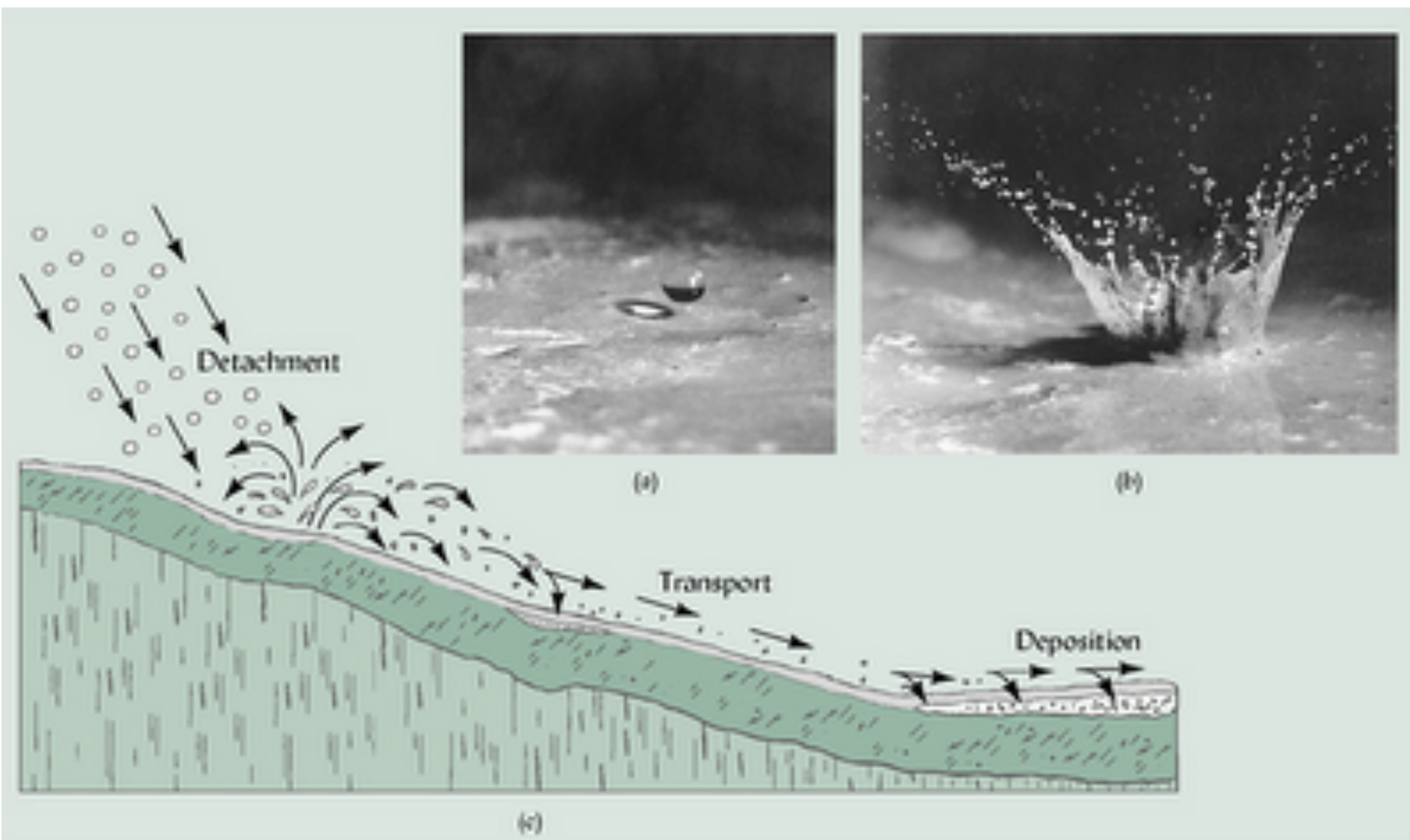


Fig. 11.14b, Ritter et al., 2002

- **Solifluction** occurs in saturated soils, often in periglacial regions
  - In periglacial settings, **frost heave** leads to expansion of the near-surface material
  - During warm periods, saturated material at the surface flows downslope above the impermeable permafrost beneath

Rain splash

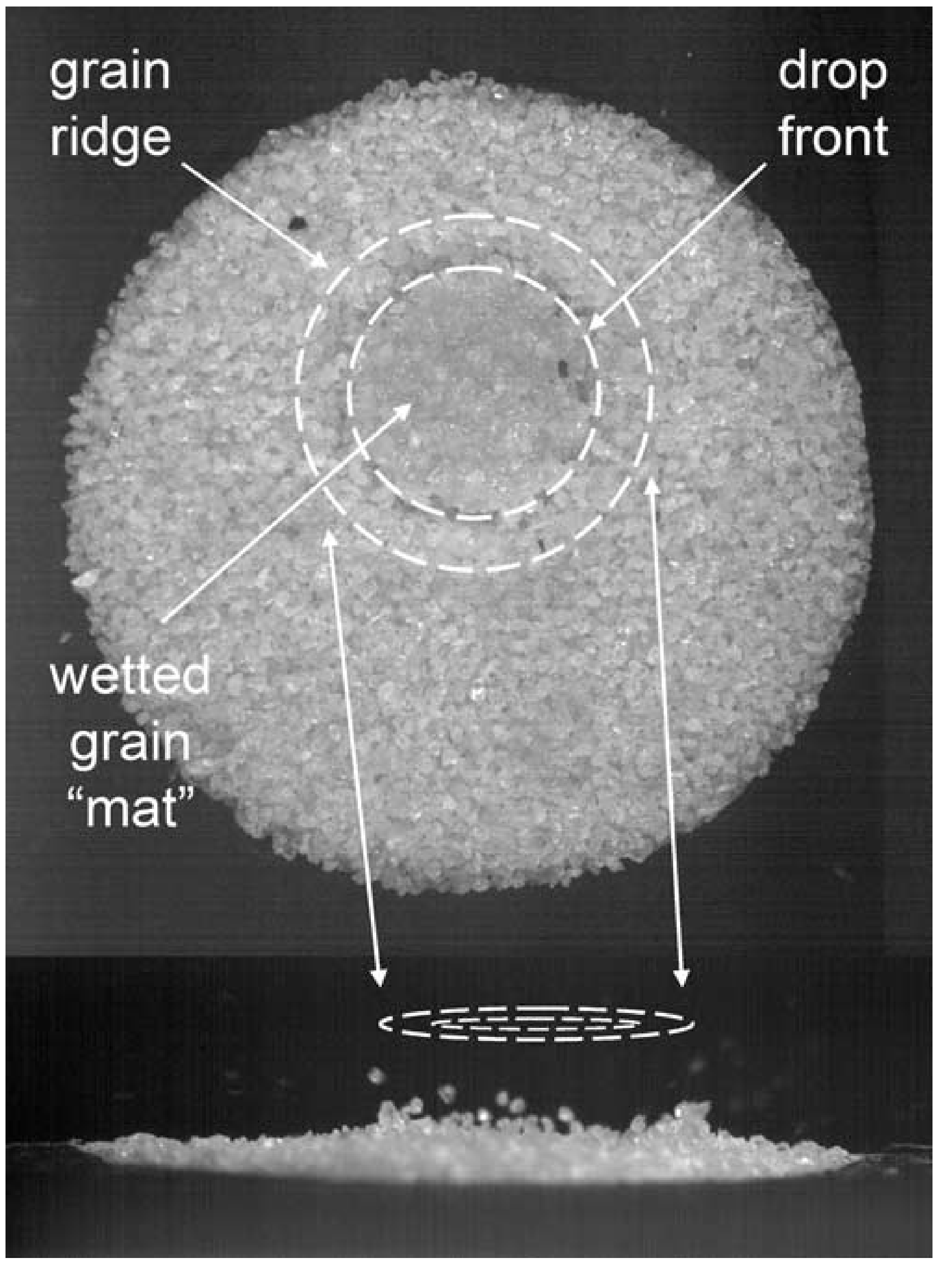


http://geofaculty.uwyo.edu/neil/

- **Rain splash transport** refers to the downslope drift of grains on a sloped surface as a result of displacement by raindrop impacts

- Although this process can produce significant downslope mass transport, it is generally less significant than heave

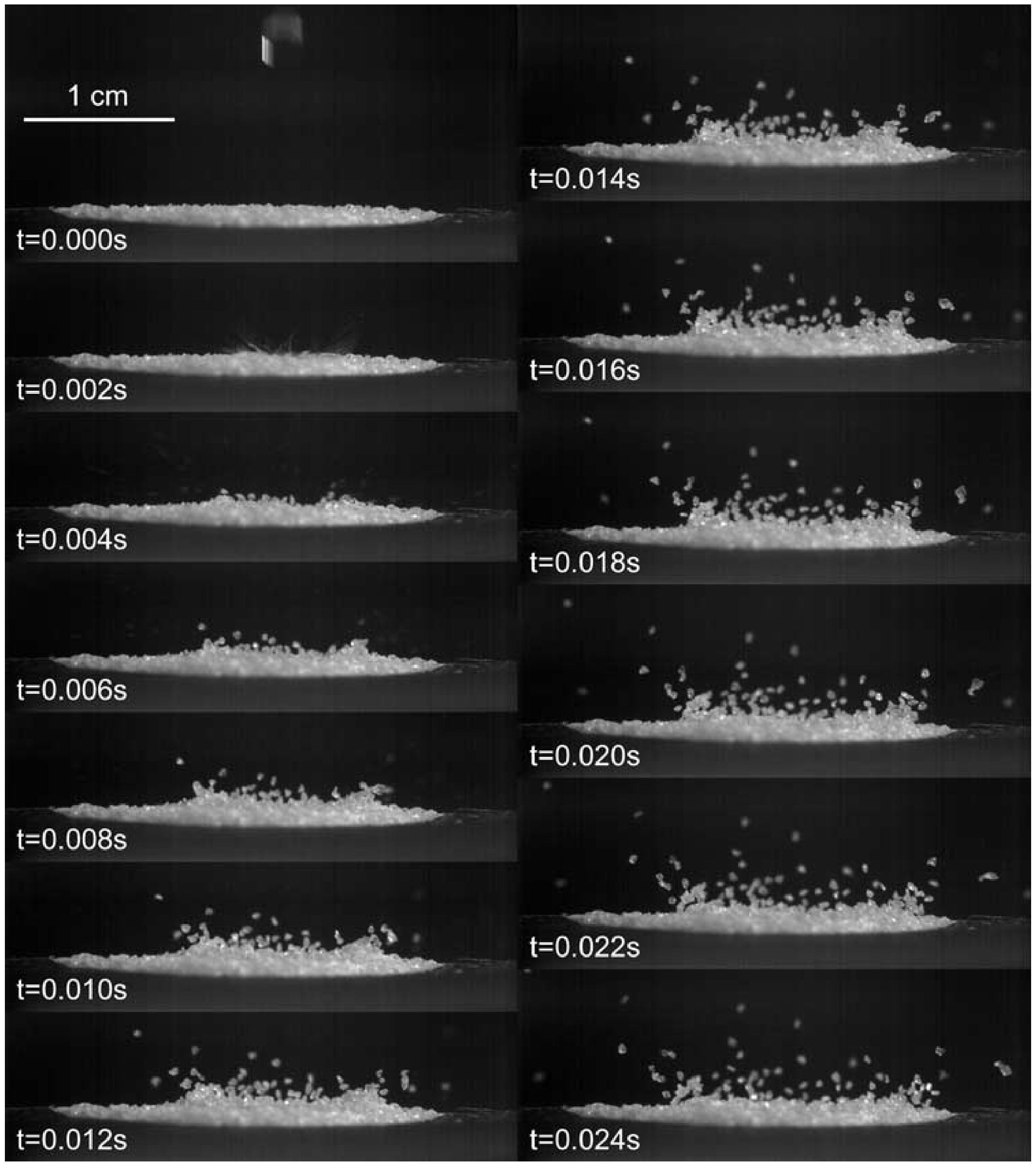
Studying rain splash



Furbish et al., 2007

- Experimental setup:
  - "Rain drops" released from a syringe ~5 m above a dry sand target
  - Drops travel down a pipe to avoid interference by wind
  - Various drop sizes (2-4 mm), sand grain sizes (0.18 - 0.84 mm) and hillslope angles
  - High-speed camera used to capture raindrop impact and sand grain motion

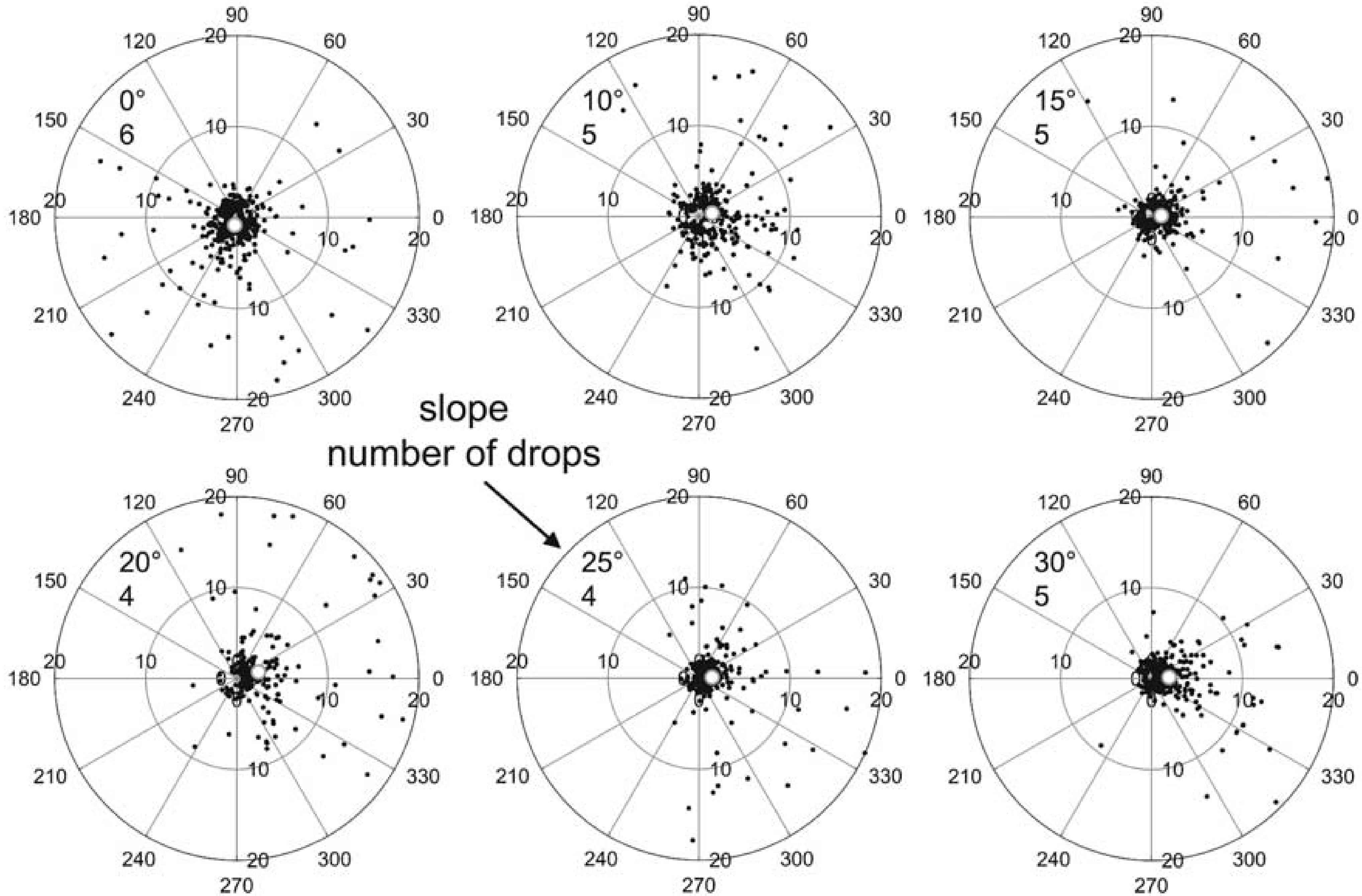
Studying rain splash



Furbish et al., 2007

- Dry sand grains are displaced following raindrop impact
- Miniature bolide impacts (?)

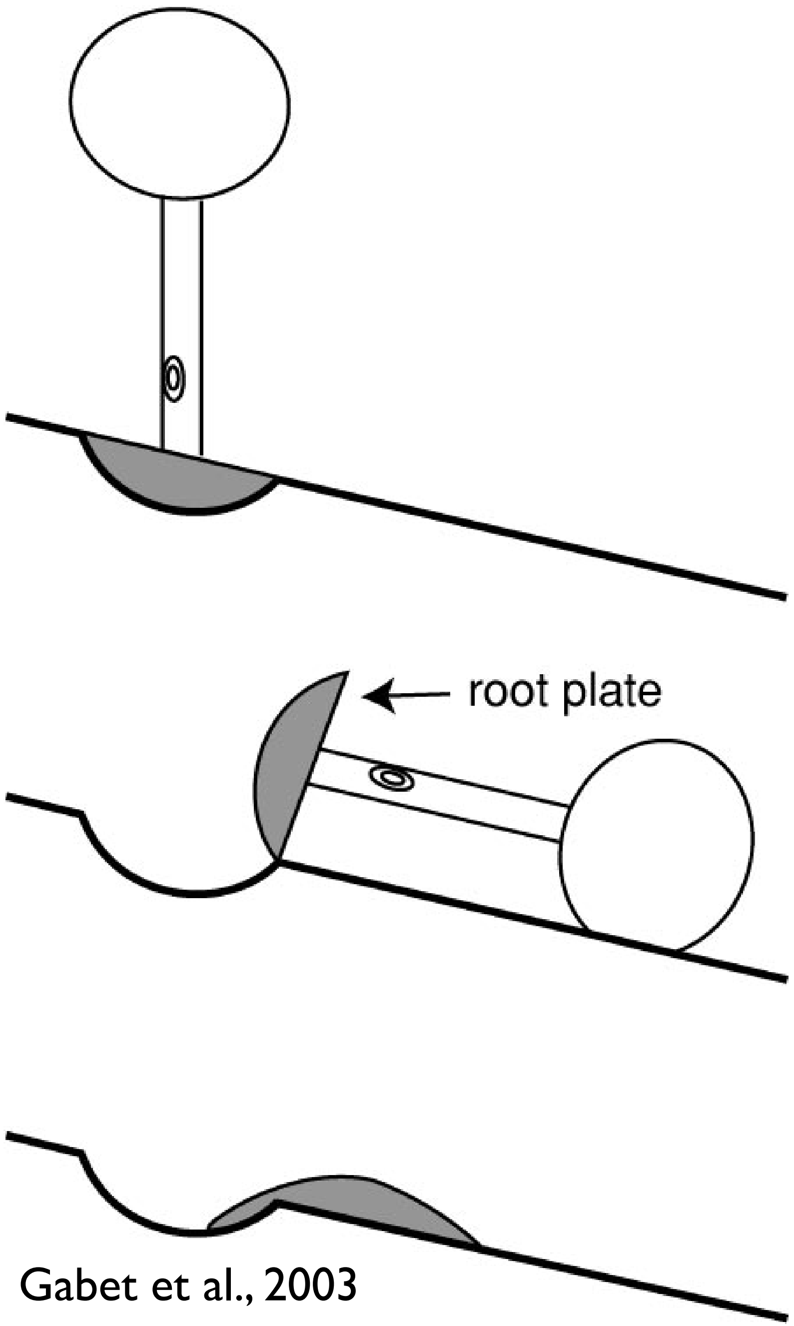
Studying rain splash



More particles drift downslope as slope angle increase

Furbish et al., 2007

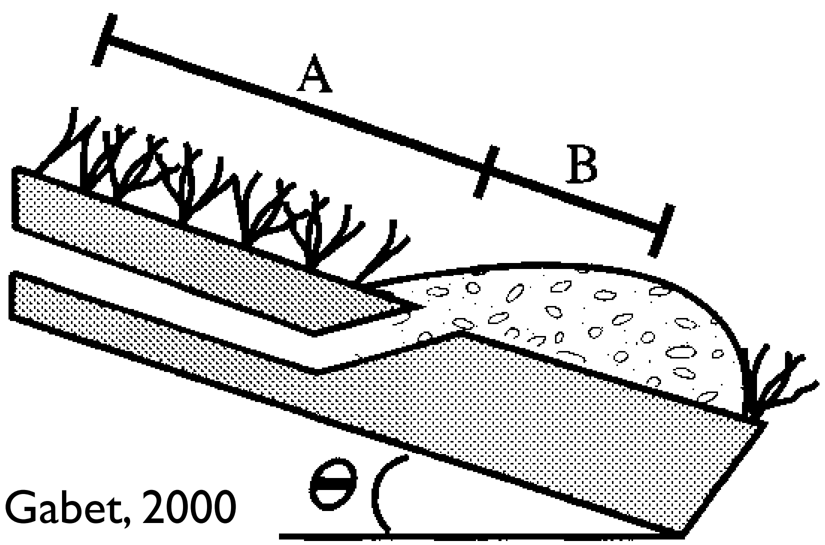
Biogenic transport: Tree throw



Gabet et al., 2003

- Falling trees also displace sediment/soil and can produce <u>downslope motion</u>
  - When trees fall, its root mass rotates soil and rock upward
  - Gradually, this soil/rock falls down beneath the root mass as it decays

Biogenic transport: Gopher holes



Gabet, 2000

- Gophers dig underground tunnels parallel to the surface and displace sediment both under and above ground
- On slopes, this sediment is displaced downslope, resulting in mass movement
- Locally, this process can be the dominant mechanism for sediment transport

Common features

- **What do all of these sediment transport processes have in common?**

- In each case, the rate of transport is strongly dependent on the hillslope angle
  - Steeper slopes result in faster downslope transport
  - In other words, the flux of mass is proportional to the topographic gradient
- This suggests these erosional processes can be modelled as **diffusive**

Recap

- What are the two components of diffusion processes?
- How does soil creep result in diffusion of soil or regolith?
- What are the main factors controlling the rate of hillslope diffusion?

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References

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