Class overview today - April 11, 2016

- **Part I - Advection of the Earth's surface**
  - Course feedback summary
  - Mathematics/Python concepts review
  - The advection equation
    - Applications: Bedrock river incision, heat transfer
- **Part II - Solving the advection-diffusion equation** (Jorina)
  - Components of the advection-diffusion equation
  - Solution of the advection-diffusion equation
  - Other boundary conditions, useful values

Introduction to Quantitative Geology
Lecture 7
Advection of the Earth's surface: Fluvial incision and rock uplift

Lecturer: David Whipp
david.whipp@helsinki.fi

11.4.2016

Course feedback summary (based on 12 forms)

- **What do you like best about the course?**
  - Learning Python
  - Step-by-step process for learning
  - Course structure
  - Opportunity to do practical things
  - Learning to program

Course feedback summary (based on 12 forms)

- **What would you change about this course?**
    - Add an introduction to Github
    - More information on finding help
    - More exercise help
    - More time working together on exercises
    - Feedback on the exercises
    - More time using Python

Course feedback summary (based on 12 forms)

- **What suggestions do you have to improve the instructor's teaching?**
  - Don’t skip over things that are obvious to you, but not for complete beginners
  - Make the course materials available as a file that could be printed

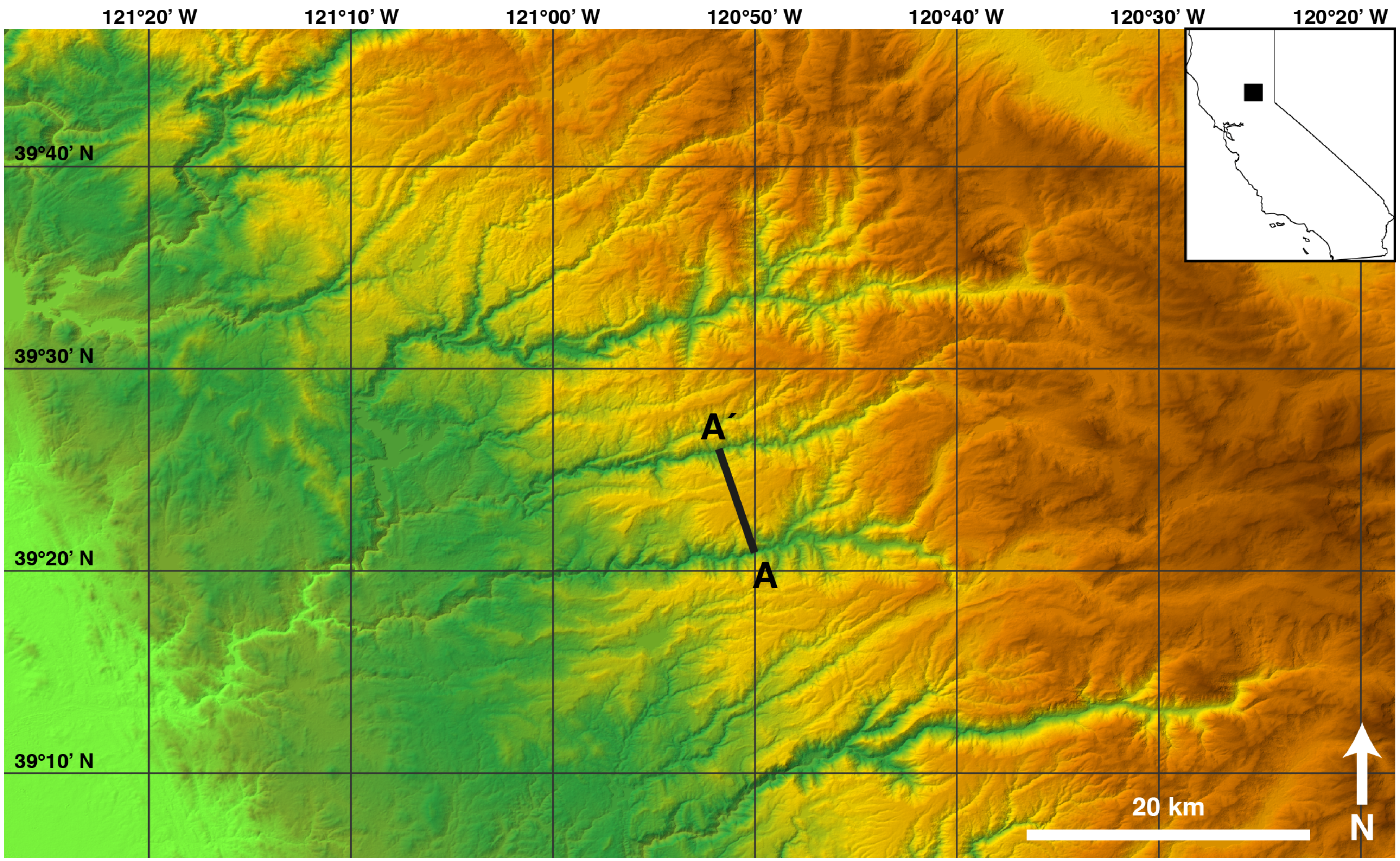
Course feedback summary (based on 12 forms)

- Python
  - (12/12 !) Learning Python is useful for me
  - (7/12) I plan to continue to use Python after finishing this course
  - (0/12) Learning Python is not useful for me
  - (6/12) I am frequently confused when trying to use Python
  - (2/12) There is too little time to learn Python in this course
- Comments
  - Confusing to remember all the new commands
  - I'm often confused

Course feedback summary (based on 12 forms)

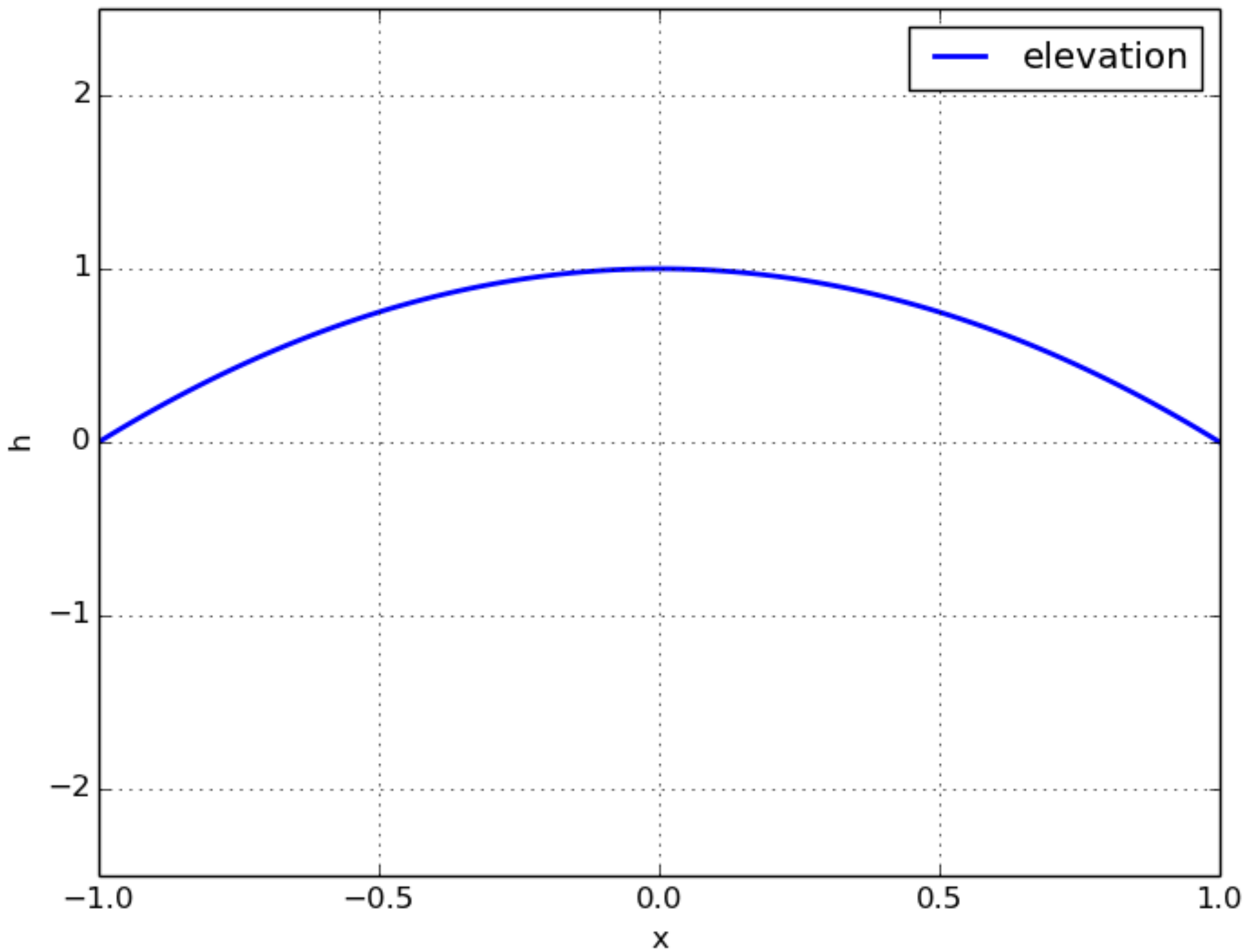
- **Github**
  - (8/12) Learning to use Github is useful for me
  - (2/12) I plan to continue to use Github after finishing this course
  - (2/12) Learning to use Github is not useful for me
  - (4/12) I am frequently confused when trying to use Github
  - (1/12) There is too little time to learn to use Github in this course
- **Comments**
  - Not very intuitive, but useful for uploading scripts
  - Hasn't been used for anything yet that Moodle can't do

Exercise 3



- Any questions or problems?

Mathematics concept review: Derivatives



As we saw with diffusive hillslopes, the equation for the elevation of the hillslopes has the general form of an **inverted parabola**

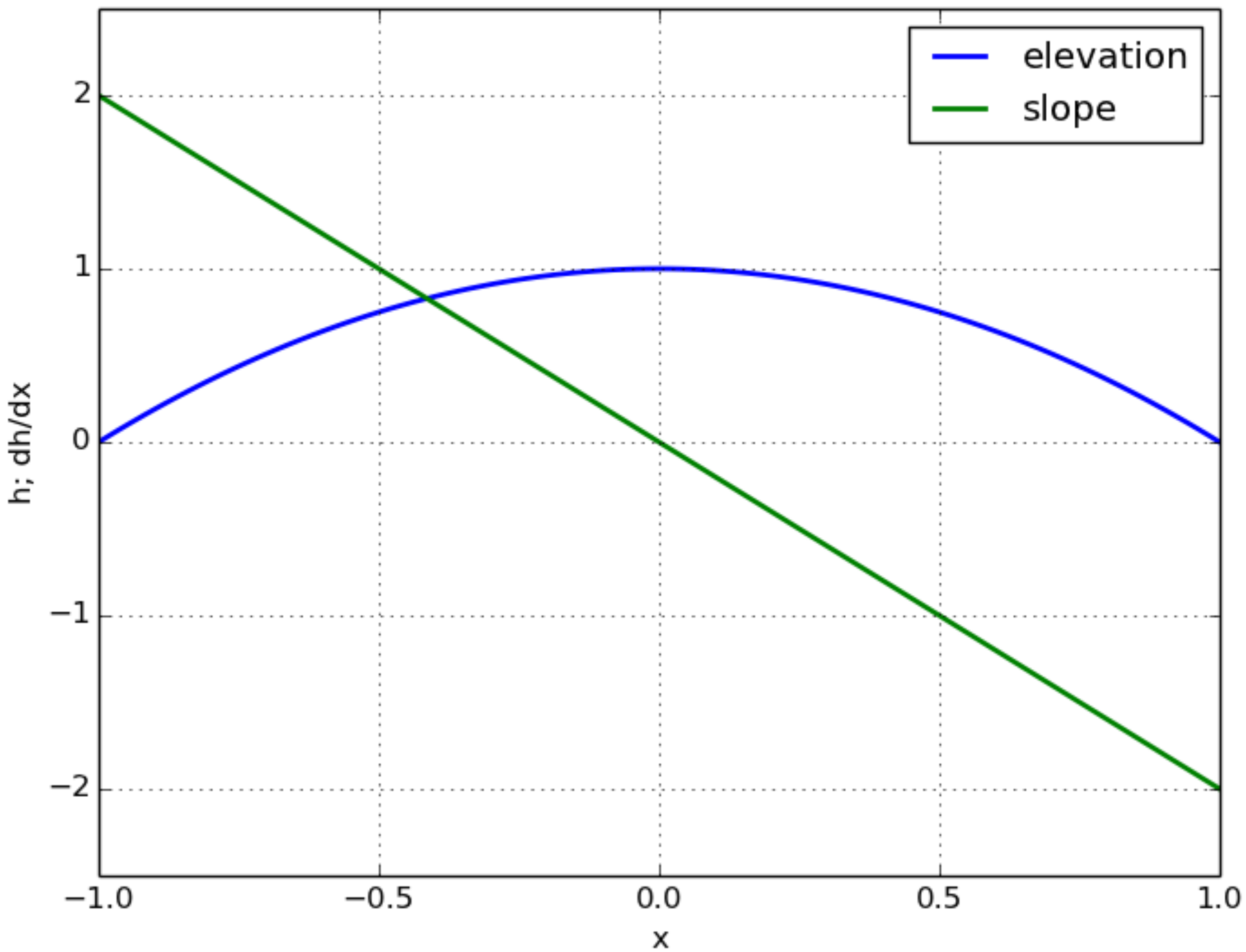
For a hillslope of this form, the following values can be calculated

Elevation: $h(x) = -x^2 + 1$

Gradient (slope):

Curvature:

Mathematics concept review: Derivatives



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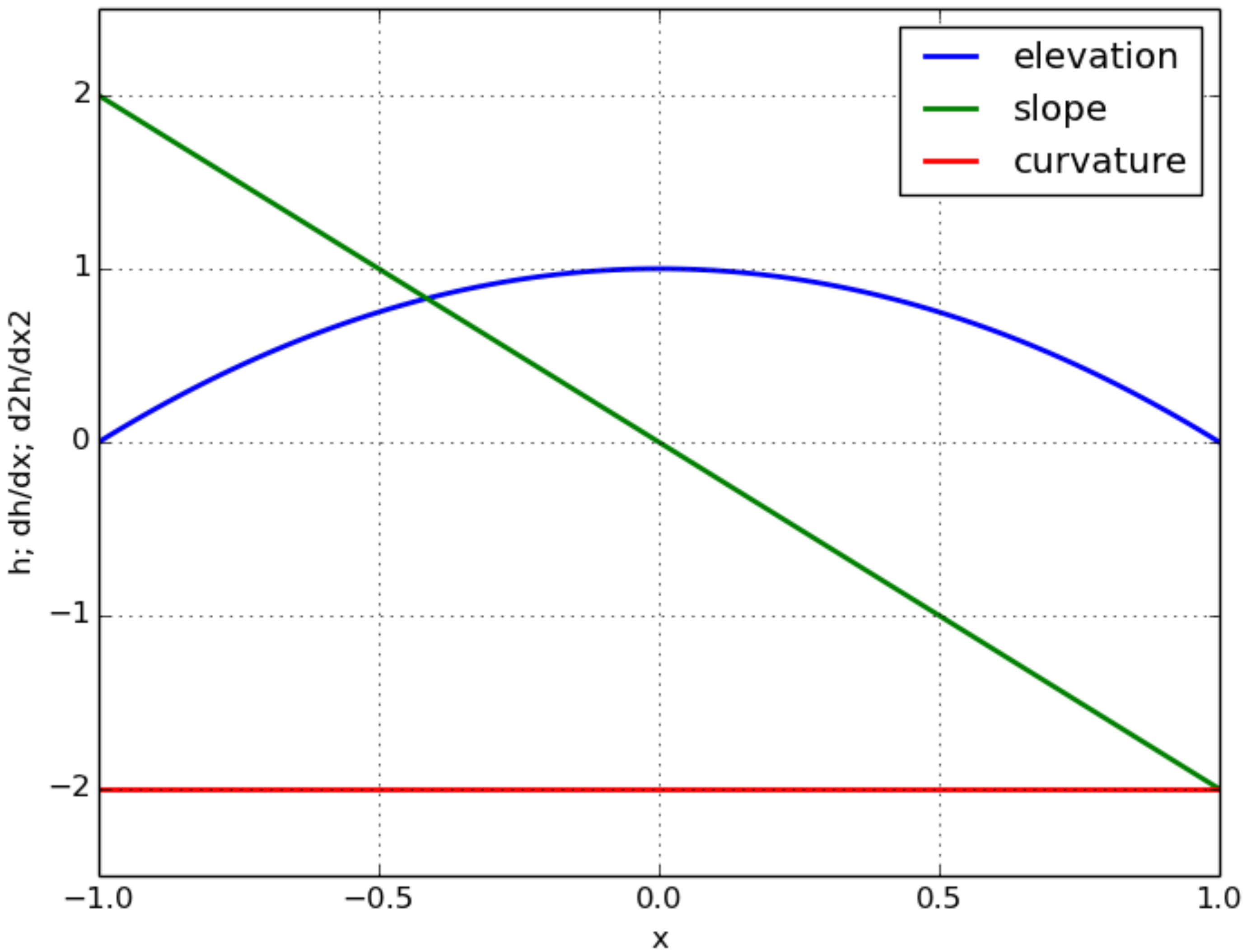
Curvature:

Mathematics concept review: Derivatives



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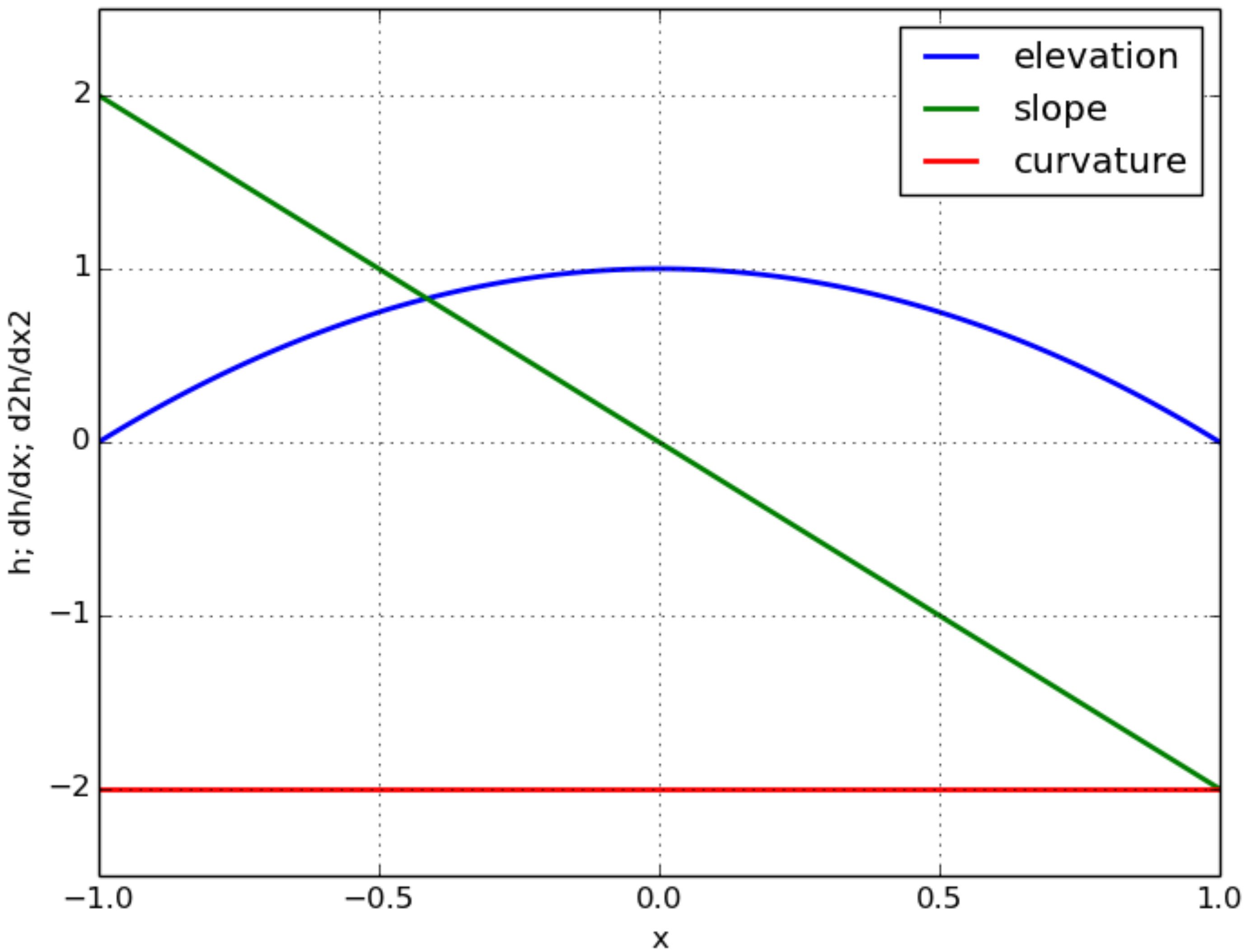
Mathematics concept review: Derivatives



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Gradient (slope): $h'(x) = \frac{\partial h}{\partial x} = -2x$

Curvature: $h''(x) = \frac{\partial^2 h}{dx^2} = -2$

Helpful links

Derivatives: https://www.mathsisfun.com/calculus/derivatives-introduction.html

Integrals: https://www.mathsisfun.com/calculus/integration-introduction.html

Python concept review: Arrays

```
# Define an array
>>> x = np.linspace(0.0,20.0,11)
```

- An **array** is a data structure with a <u>specified number of 'slots' or places that can contain data</u>

Python concept review: Arrays

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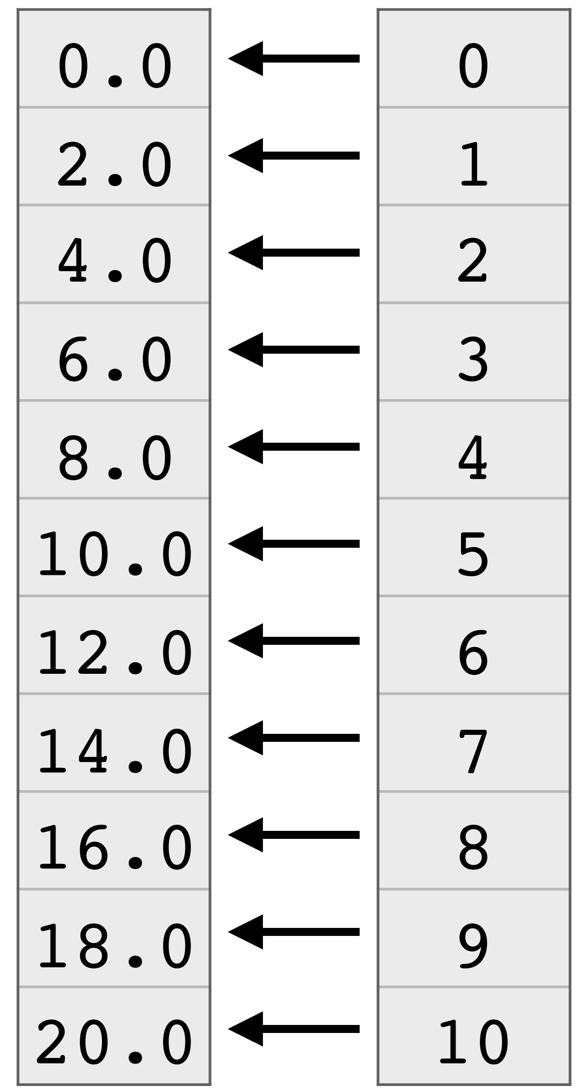
# Define an array
>>> x = np.linspace(0.0,20.0,11)
```

Array x   Array indicies



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Python concept review: Arrays

```
# Define an array
>>> x = np.linspace(0.0,20.0,11)

# Define another array
>>> y = np.arange(0.0, 20.0, 2.0)
```

Array x		Array indicies
0.0	←	0
2.0	←	1
4.0	←	2
6.0	←	3
8.0	←	4
10.0	←	5
12.0	←	6
14.0	←	7
16.0	←	8
18.0	←	9
20.0	←	10

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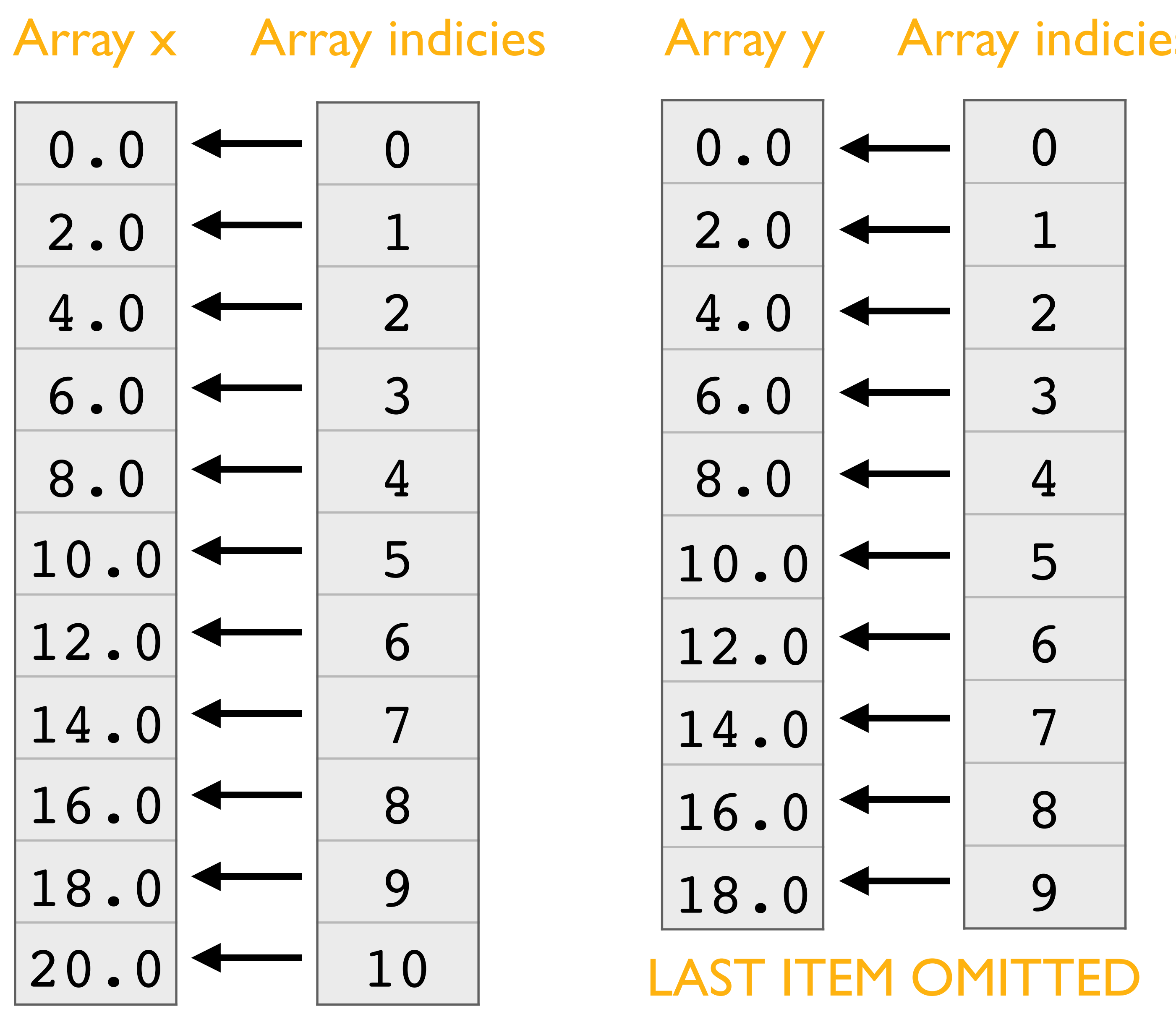

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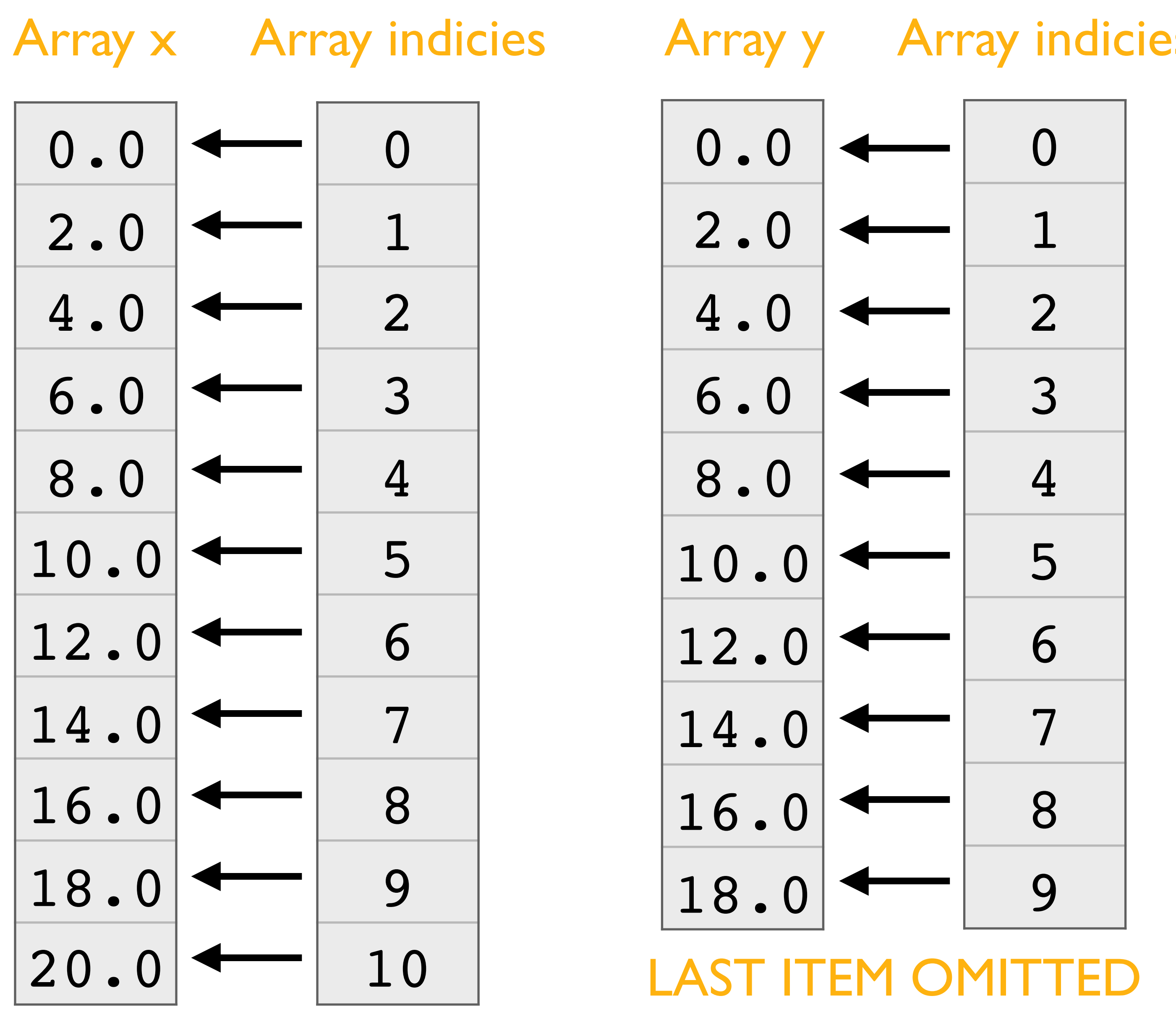
Python concept review: Arrays

```
# Define an array
>>> x = np.linspace(0.0,20.0,11)

# Define another array
>>> y = np.arange(0.0, 20.0, 2.0)

# Define a range
>>> range(5)
[0, 1, 2, 3, 4]  # 5 items in list

# Define the same range
>>> range(0,5,1)
[0, 1, 2, 3, 4]  # 5 items in list!
```



- An **array** is a data structure with a <u>specified number of 'slots' or places that can contain data</u>

Python concept review: `for` loops

```
# Define a for loop
>>> for increment in range(5):
...     print(increment)
...
0
1
2
3
4
```

- A **`for`** loop will repeat a section of code for each value of the given range

Python concept review: **for** loops

```
# Define a for loop
>>> for increment in range(5):
...     print(increment)
...
0
1
2
3
4
```

Code description

```
start for loop

increment = 0
print increment
0 written to screen

increment = 1
print increment
1 written to screen

...

increment = 4
print increment
4 written to screen

hit end of range(5)
exit for loop
```

- A **for** loop will repeat a section of code for each value of the given range

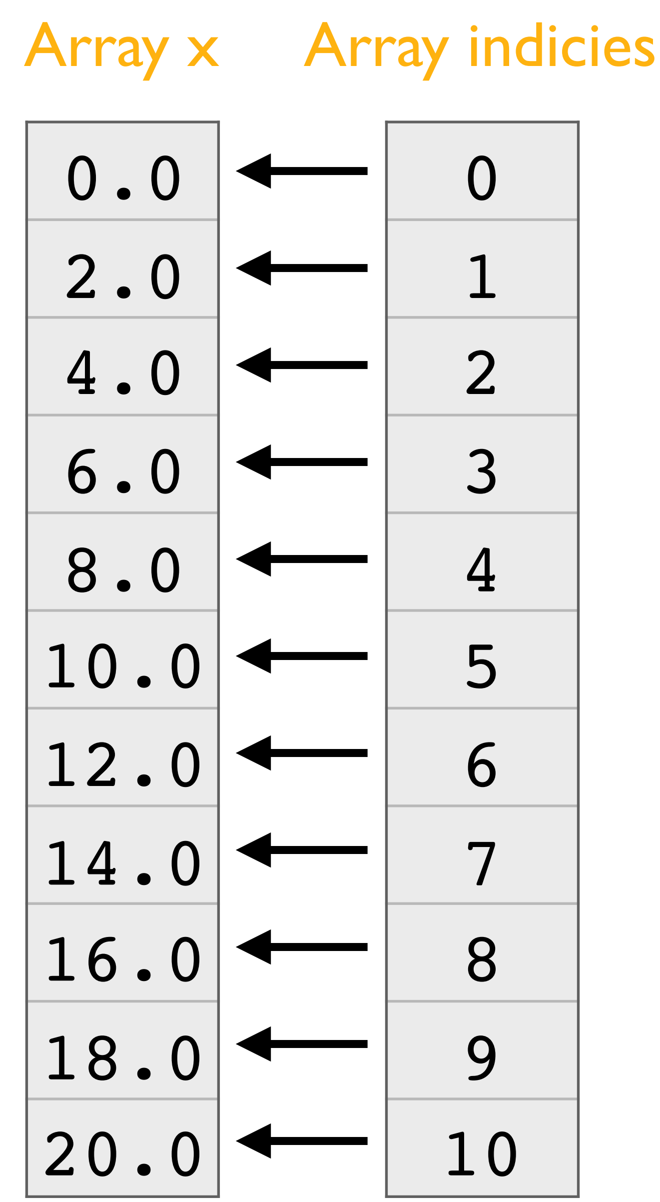
Python concept review: **for** loops

Array x	Array indicies
0.0	0
2.0	1
4.0	2
6.0	3
8.0	4
10.0	5
12.0	6
14.0	7
16.0	8
18.0	9
20.0	10

```
# Define another loop
>>> for increment in range(len(x)):
...     print(x[increment], increment)
...
```

- A **for** loop will repeat a section of code for each value of the given range

Python concept review: **for** loops



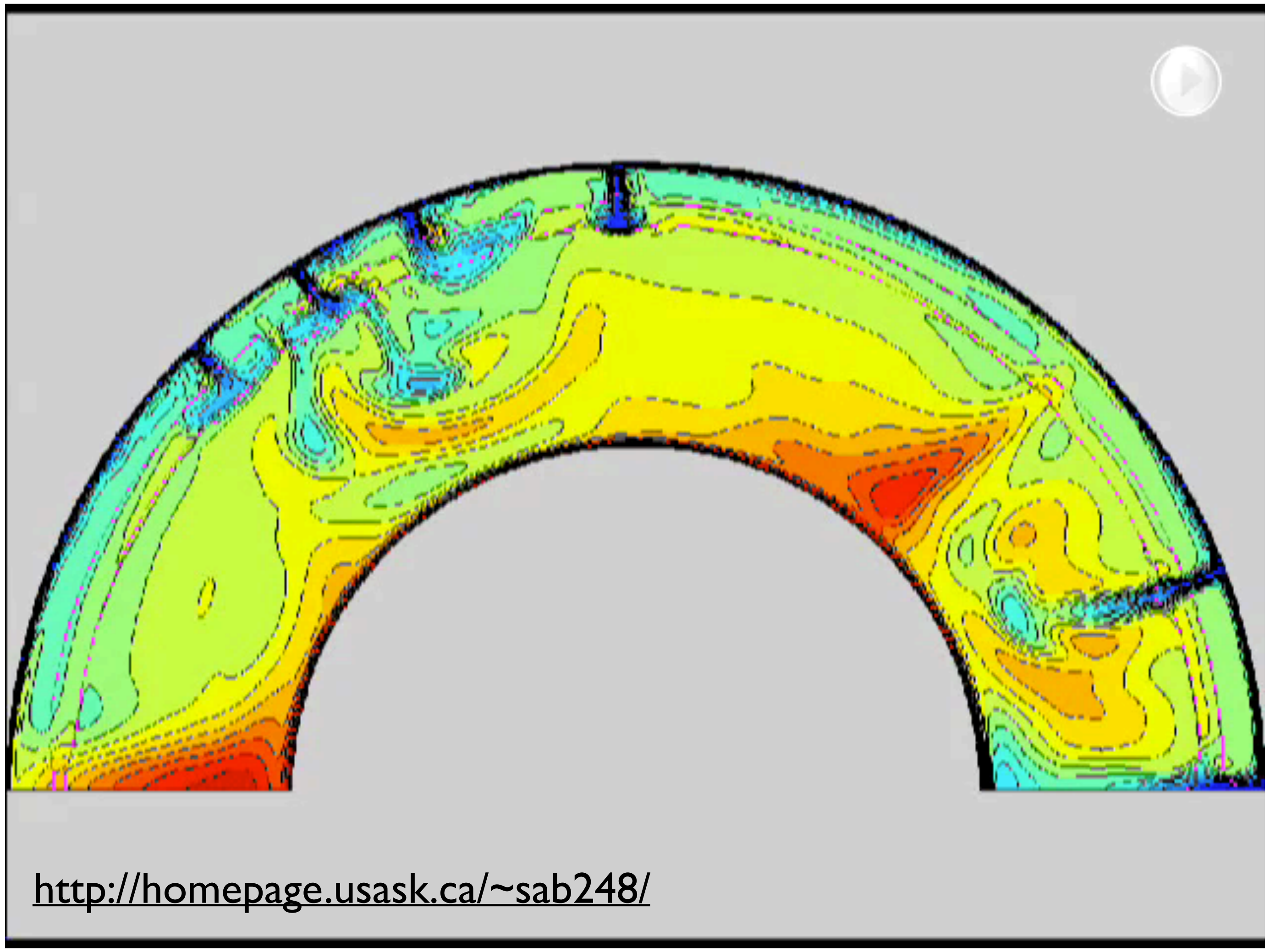

```
# Define another loop
>>> for increment in range(len(x)):
...     print(x[increment], increment)
...
(0.0, 0)
(2.0, 1)
(4.0, 2)
...
(20.0, 10)
```

- A **for** loop will repeat a section of code for each value of the given range

Goals of this lecture

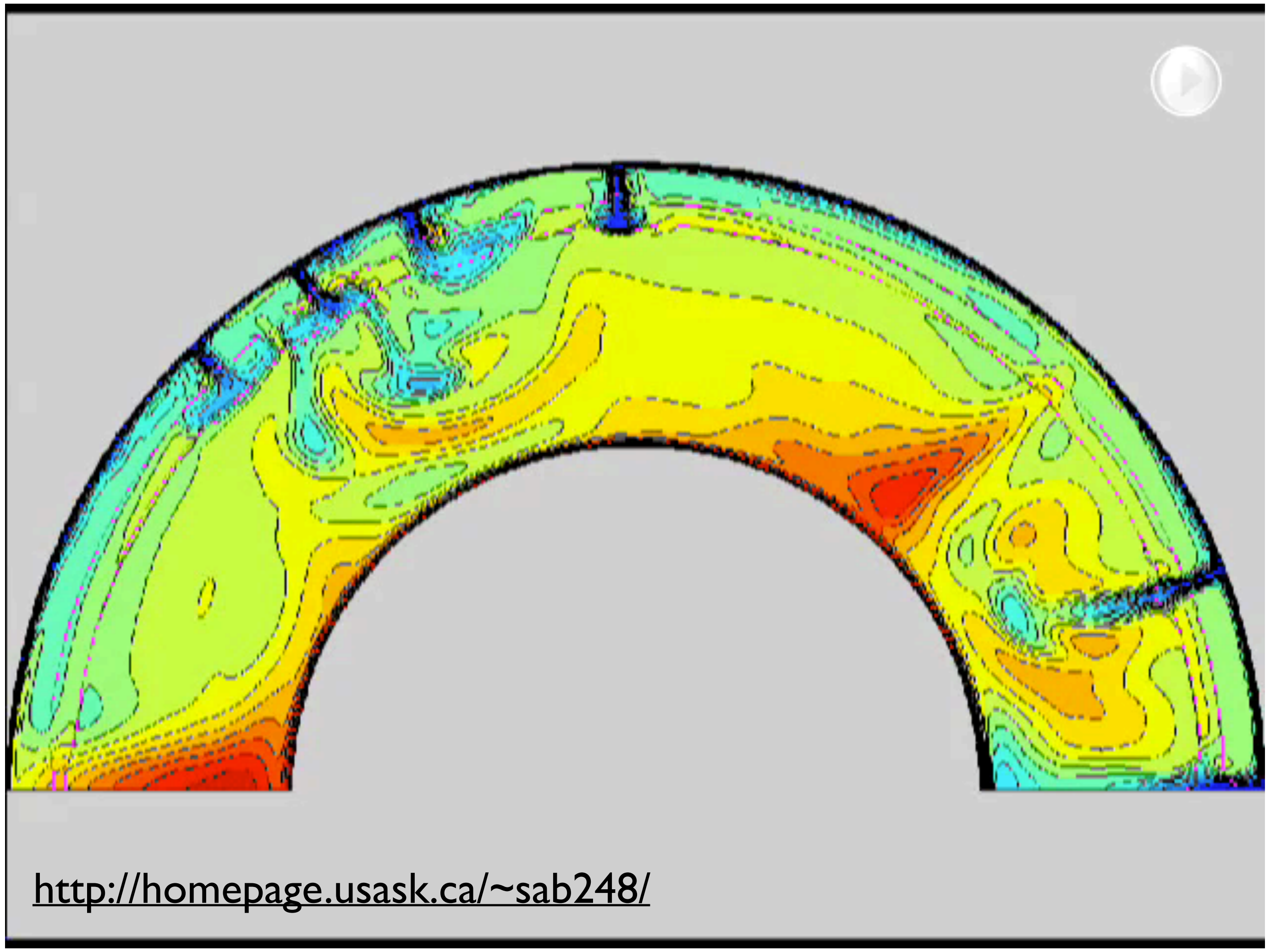
- Introduce the **advection equation**
- Discuss application of the advection equation to **bedrock river erosion**
- Introduce **advective heat transfer**

What is advection?



- Advection involves a lateral translation of some quantity
    - For example, the transfer of heat by physical movement of molecules or atoms within a material. A type of convection, mostly applied to heat transfer in solid materials.

What is advection?



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Diffusion equation

$$q = -\rho\kappa\frac{\partial h}{\partial x}$$

$$\frac{\partial h}{\partial t} = -\frac{1}{\rho}\frac{\partial q}{\partial x}$$

- Last week we were introduced to the **diffusion equation**
  - Flux (transport of mass or transfer of energy) proportional to a gradient
  - Conservation of mass: Any change in flux results in a change in mass/energy

Diffusion equation

Diffusion

$$\frac{\partial h}{\partial t} = -\kappa \frac{\partial^2 h}{\partial x^2}$$

$$q = -\rho\kappa \frac{\partial h}{\partial x}$$

$$\frac{\partial h}{\partial t} = -\frac{1}{\rho}\frac{\partial q}{\partial x}$$

- Substitute the upper equation on the left into the lower to get the classic **diffusion equation**
  - q = sediment flux per unit length
    ρ = bulk sediment density
    κ = sediment diffusivity
    h = elevation
    x = distance from divide
    t = time

Advection and diffusion equations

Diffusion

$$\frac{\partial h}{\partial t} = -\kappa \frac{\partial^2 h}{\partial x^2}$$

Advection

$$\frac{\partial h}{\partial t} = c \frac{\partial h}{\partial x}$$

- This week we meet the **advection equation**

Advection and diffusion equations

Diffusion

$$\frac{\partial h}{\partial t} = -\kappa \frac{\partial^2 h}{\partial x^2}$$

Advection

$$\frac{\partial h}{\partial t} = c \frac{\partial h}{\partial x}$$

- This week we meet the **advection equation**
- Two key differences:
  - Change in mass/energy with time <u>proportional to gradient, rather than curvature</u> (or *change* in gradient)
  - **Advection coefficient** c has units of $[L/T]$, rather than $[L^2/T]$

Advection and diffusion equations

River channel profiles

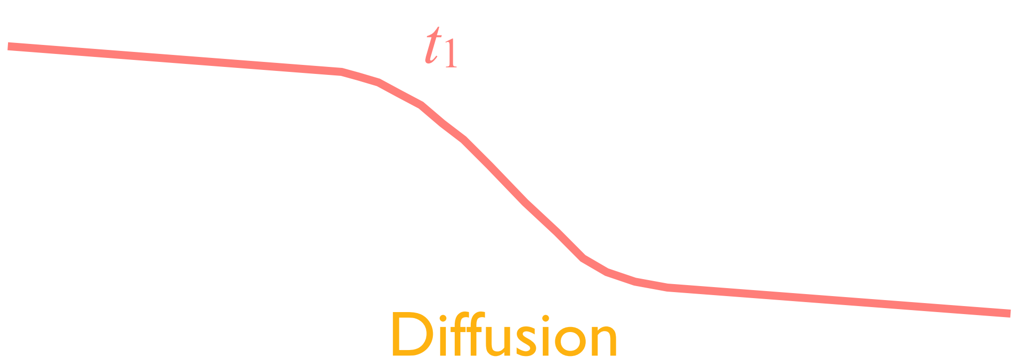

Fig. 1.7, Pelletier, 2008

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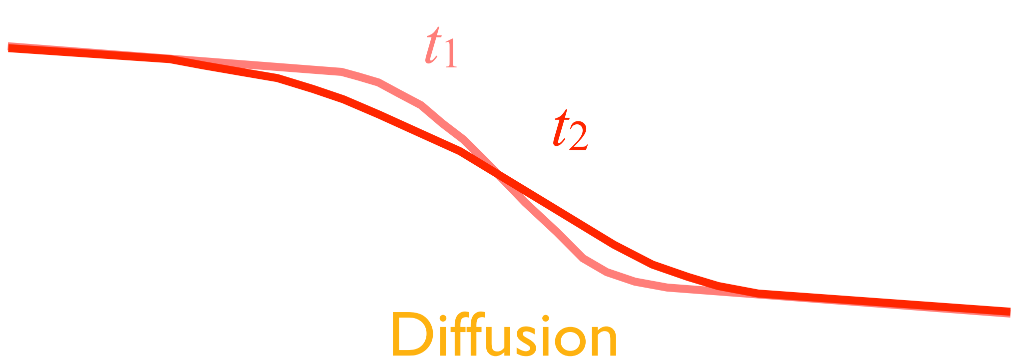

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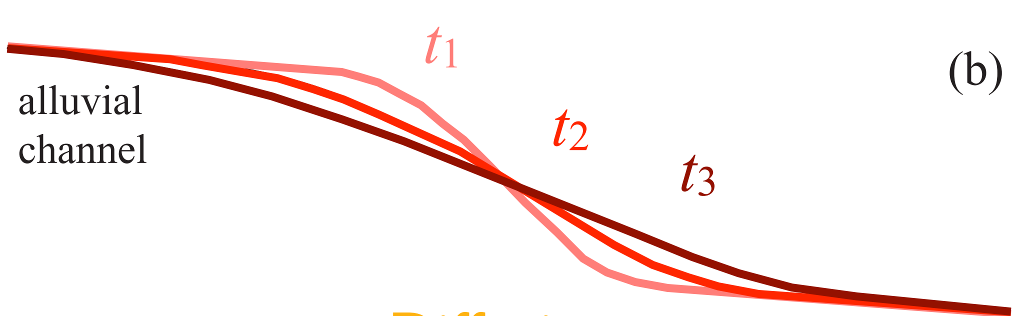

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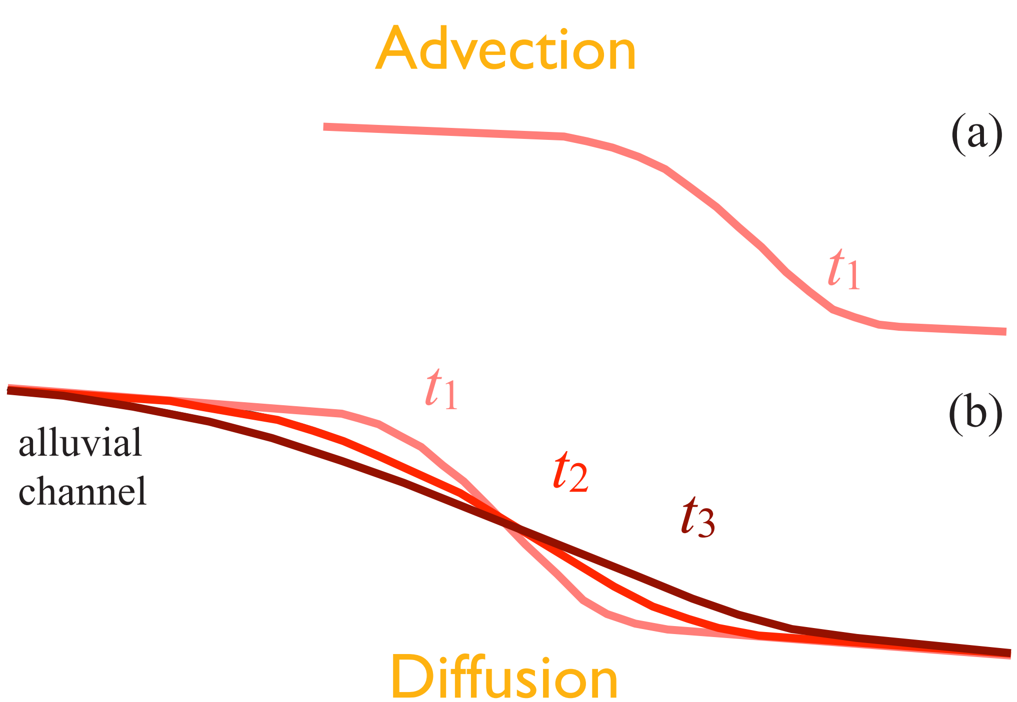

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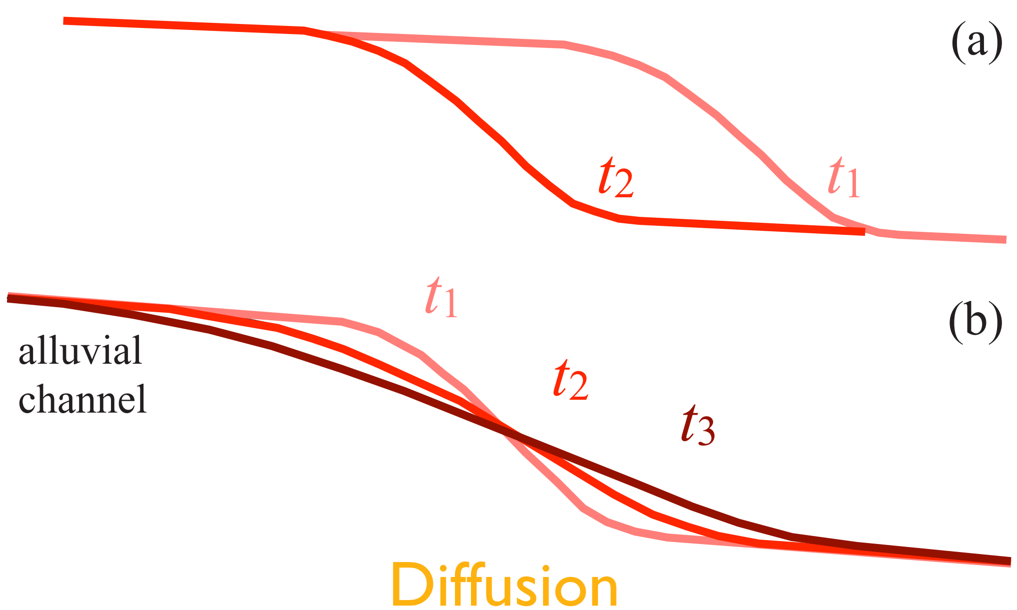

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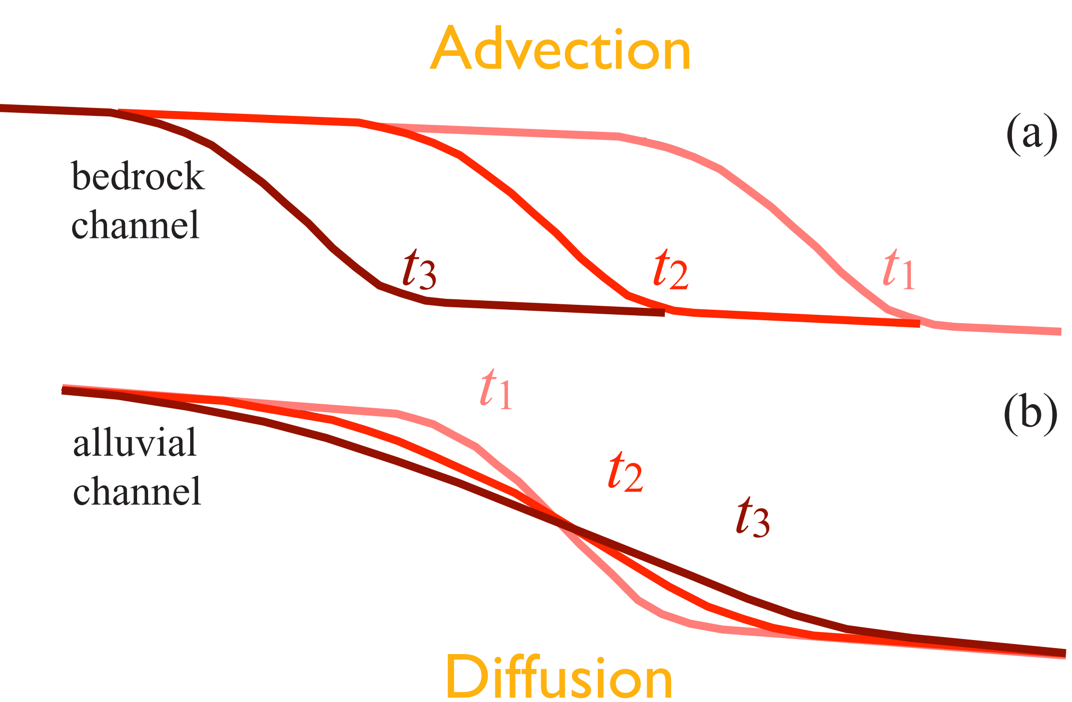

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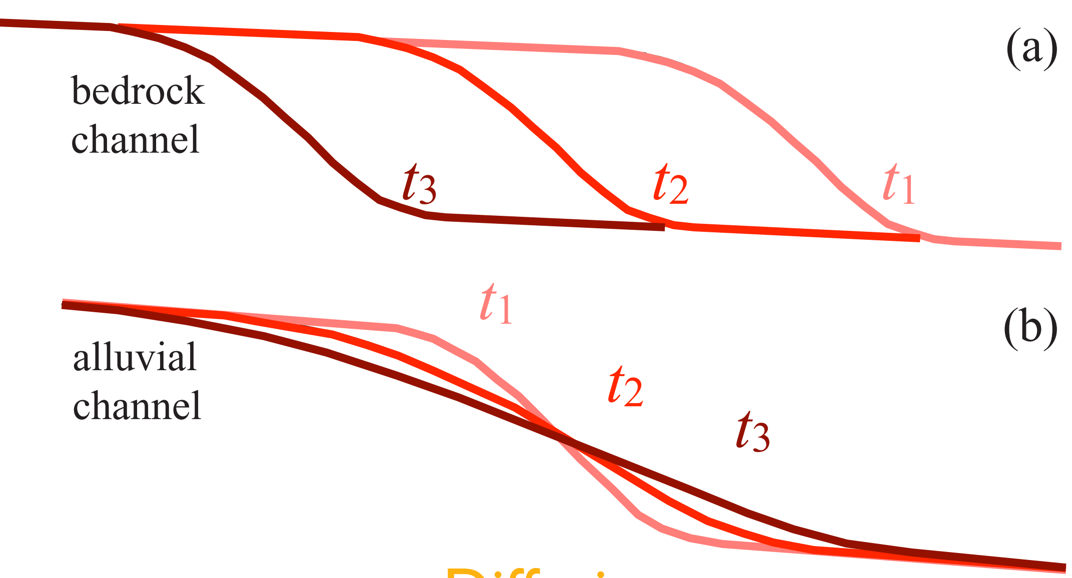
Advection



Diffusion

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Diffusion

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Advection

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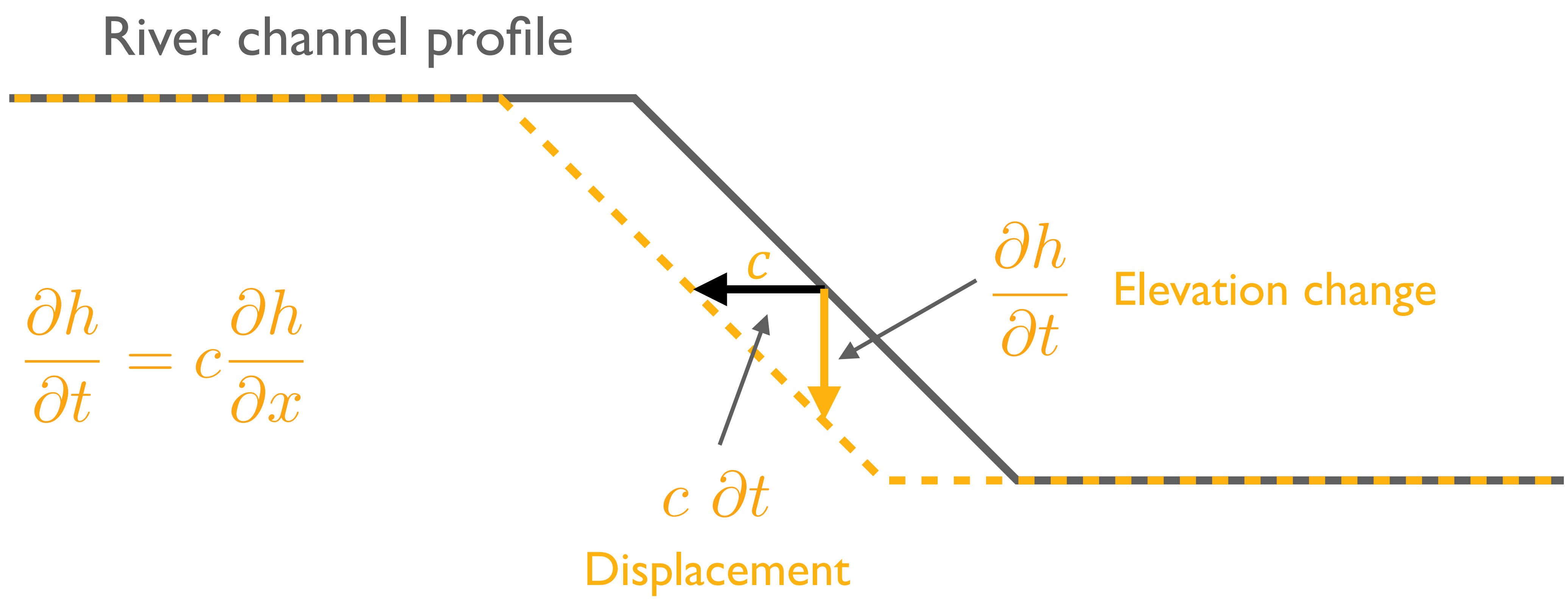
- **Diffusion**: Rate of erosion depends on change in hillslope gradient (curvature)
- **Advection**: Rate of erosion is directly proportional to hillslope gradient
  - Also, no conservation of mass (deposition)

Advection at a constant rate c



$$\frac{\partial h}{\partial t} = c\frac{\partial h}{\partial x}$$

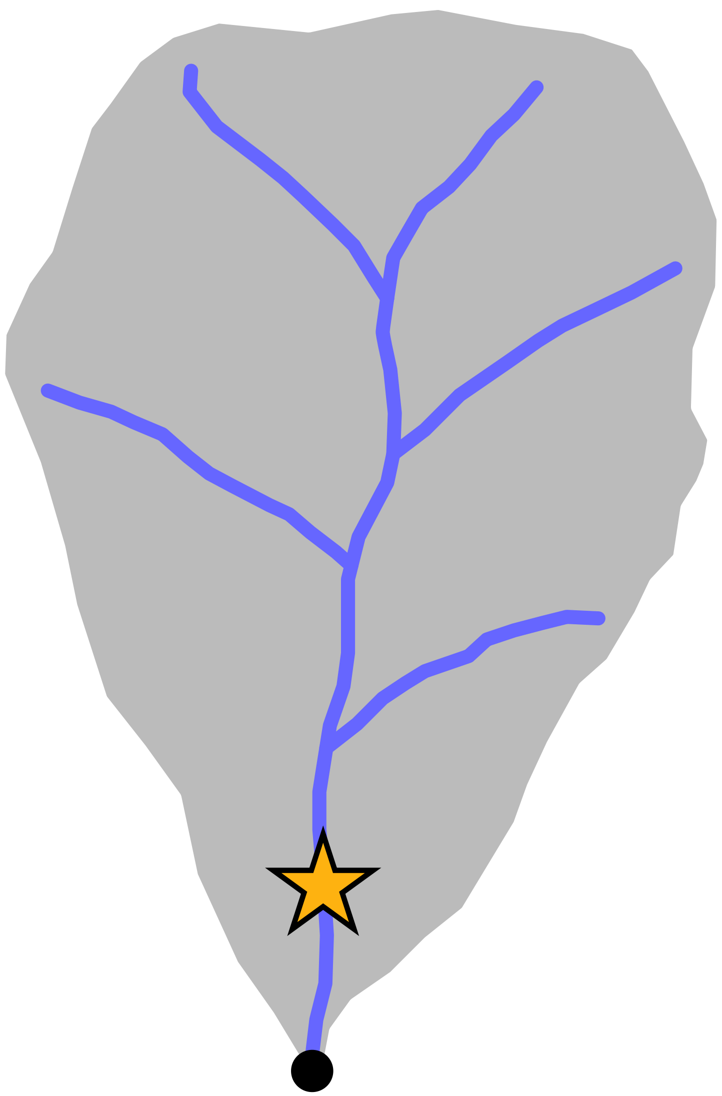
- Surface elevation changes in <u>direct proportion to surface slope</u>
- Result is lateral propagation of the topography or river channel profile
- Although this is interesting, it is <u>not that common in nature</u>

Advection of the Earth's surface: Two cases

- **Bedrock river erosion**
  - Purely an advection problem with a spatially variable advection coefficient
- **Advective heat transport**
  - Combination of advection and diffusion

~~Bedrock~~ river erosion

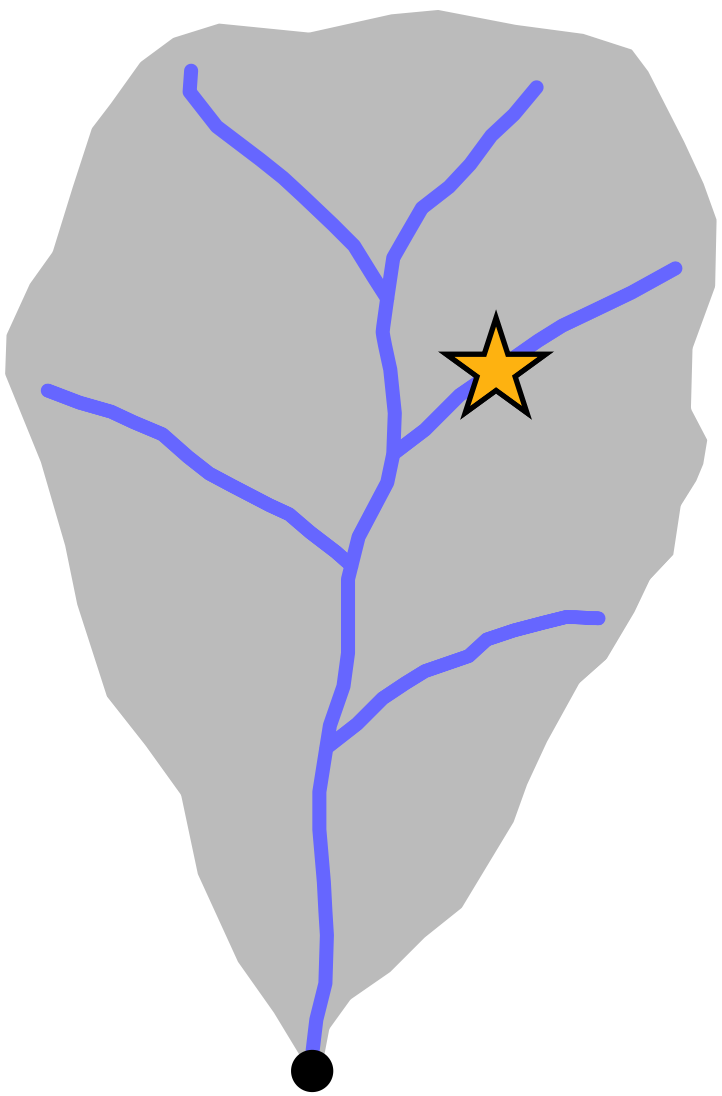
Drainage basin



- Not much bedrock being eroded here…

Bedrock river erosion

Drainage basin



Kali Gandaki river gorge, central Nepal
http://en.wikipedia.org/

- Rapid bedrock incision has formed a steep gorge in this case

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HELSINGFORS UNIVERSITET
UNIVERSITY OF HELSINKI

River erosion as an advection process

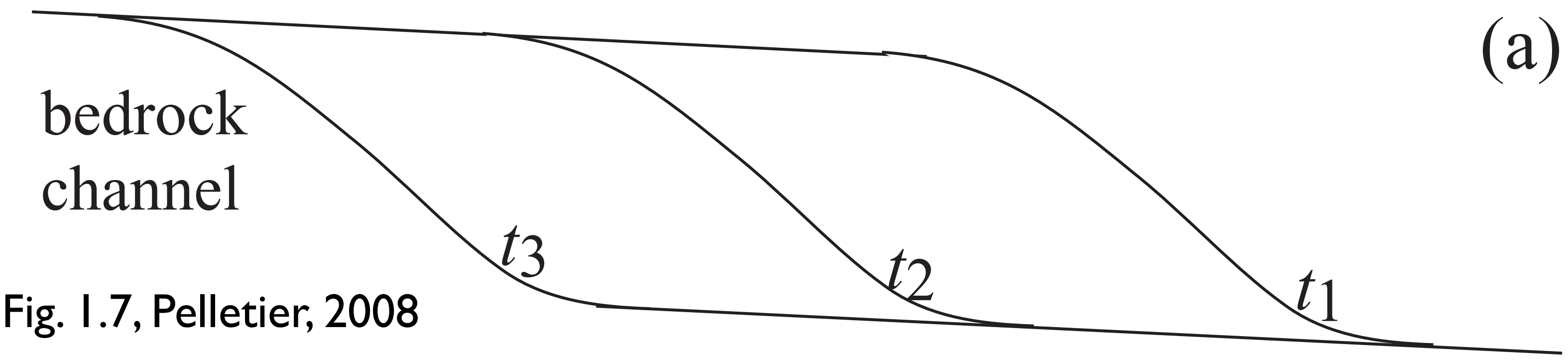

Fig. 1.7, Pelletier, 2008

- With a constant advection coefficient c, we predict lateral migration of the river profile at a constant rate (c)

River erosion as an advection process

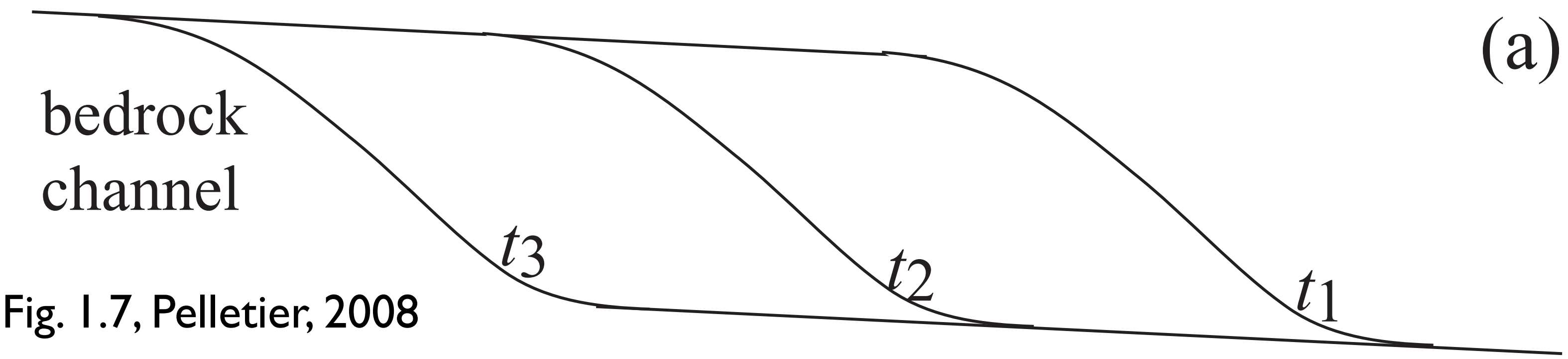

Fig. 1.7, Pelletier, 2008

- With a constant advection coefficient c, we predict <u>lateral migration of the river profile at a constant rate</u> (c)
  - **Do you think this works in real (bedrock) rivers?**

River erosion as an advection process

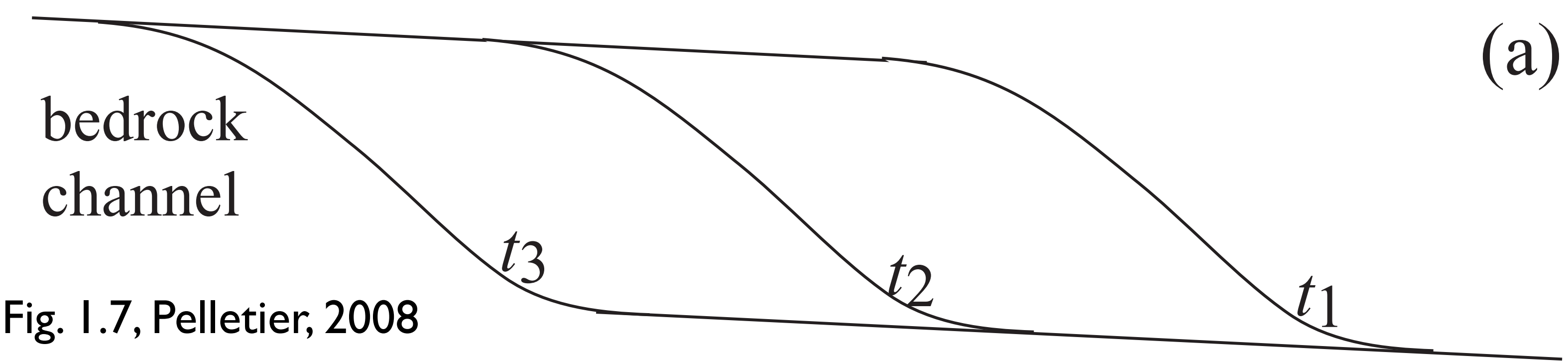

Fig. 1.7, Pelletier, 2008

- With a constant advection coefficient c, we predict lateral migration of the river profile at a constant rate (c)
  - **Do you think this works in real (bedrock) rivers?**
  - **What might affect the rate of lateral migration?**

What affects the efficiency of river erosion?

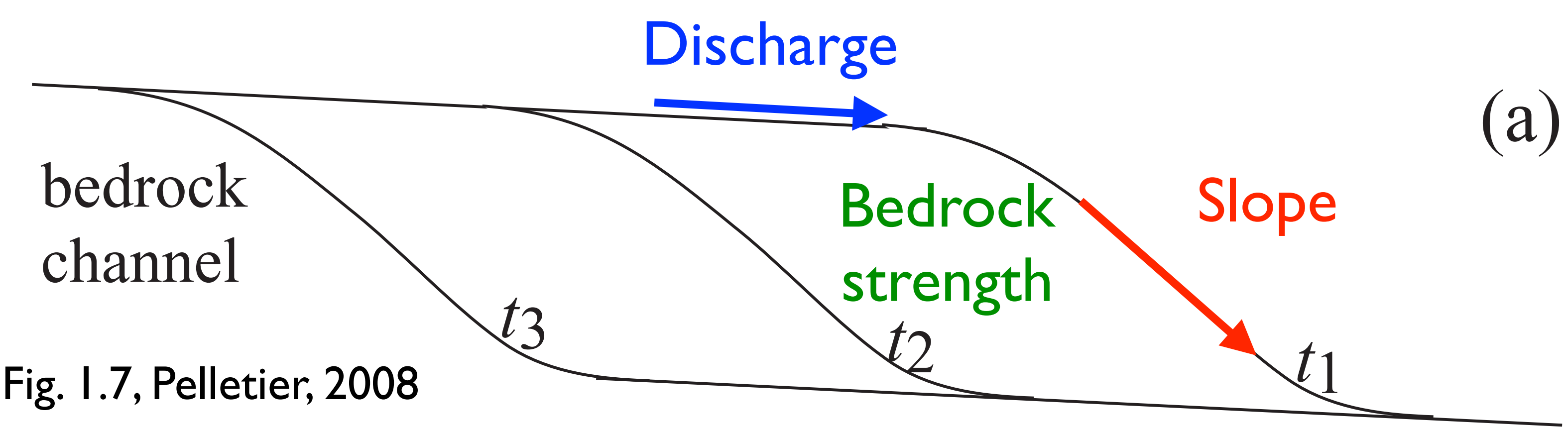

Fig. 1.7, Pelletier, 2008

- The amount of water flowing in the river (discharge) and sediment
- The slope of the river channel
- The strength of the underlying bedrock

What affects the efficiency of river erosion?

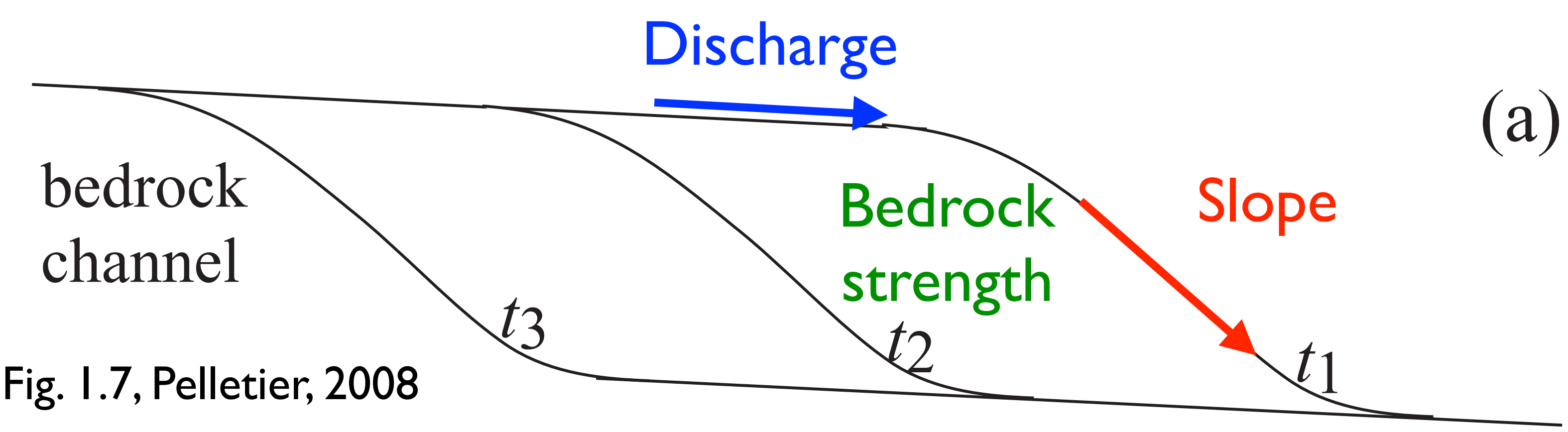

Fig. 1.7, Pelletier, 2008

- The amount of water flowing in the river (discharge) and sediment
- The slope of the river channel
- The strength of the underlying bedrock

- Are these constant?

Stream-power model of river incision

$$\cancel{\frac{\partial h}{\partial t} = c\frac{\partial h}{\partial x}}$$

- Rather than being constant, the rate of lateral advection in river systems is spatially variable

$$\frac{\partial h}{\partial t} = \frac{k_f}{w} Q \frac{\partial h}{\partial x}$$

where k_f is a material property of the bedrock (erodibility), w is the channel width, and Q is discharge

Stream-power model of river incision

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- Rather than being constant, the rate of lateral advection in river systems is <u>spatially variable</u>

$$\frac{\partial h}{\partial t} = \frac{k_f}{w} Q \frac{\partial h}{\partial x}$$

where k_f is a material property of the bedrock (erodibility), w is the channel width, and Q is discharge

  - This is known as the **stream-power erosion model**

Stream-power model of river incision

- If we assume precipitation is uniform in the drainage basin, discharge Q will scale with drainage basin area, so we can modify our equation to read

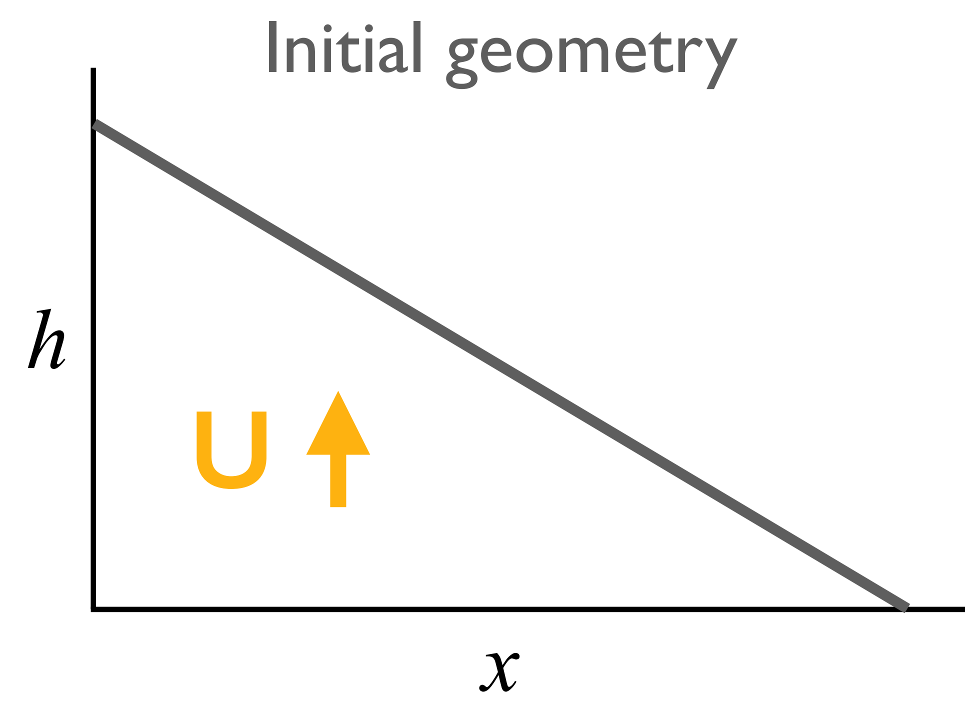
$$\frac{\partial h}{\partial t} = \frac{k_f}{w} Q \frac{\partial h}{\partial x} \longrightarrow \frac{\partial h}{\partial t} = K A^m S^n$$

  where K is an erosional efficiency factor (accounts for lithology, climate, channel geometry, sediment supply, etc. (!)), A is upstream drainage area, S is channel slope, and m and n are area and slope exponents

- If we assume the drainage basin area increases with distance from the drainage divide x, we can replace the area with an estimate $A = x^{5/3}$

Test your might

$$\frac{\partial h}{\partial t} = U - KA^m S^n$$



- Based on our **stream-power erosion** equation, what general form would a channel profile take?
  - If we assume we have reached a steady state ($\partial h/\partial t = 0$) and $n = 1$, erosion must balance uplift U everywhere
  - If we further assume precipitation is constant, bedrock erodibility is constant and $A = x^{5/3}$, how would the channel steepness vary as you move downstream from the divide?
  - Think about how S would change as x increases

Evolution of a channel profile

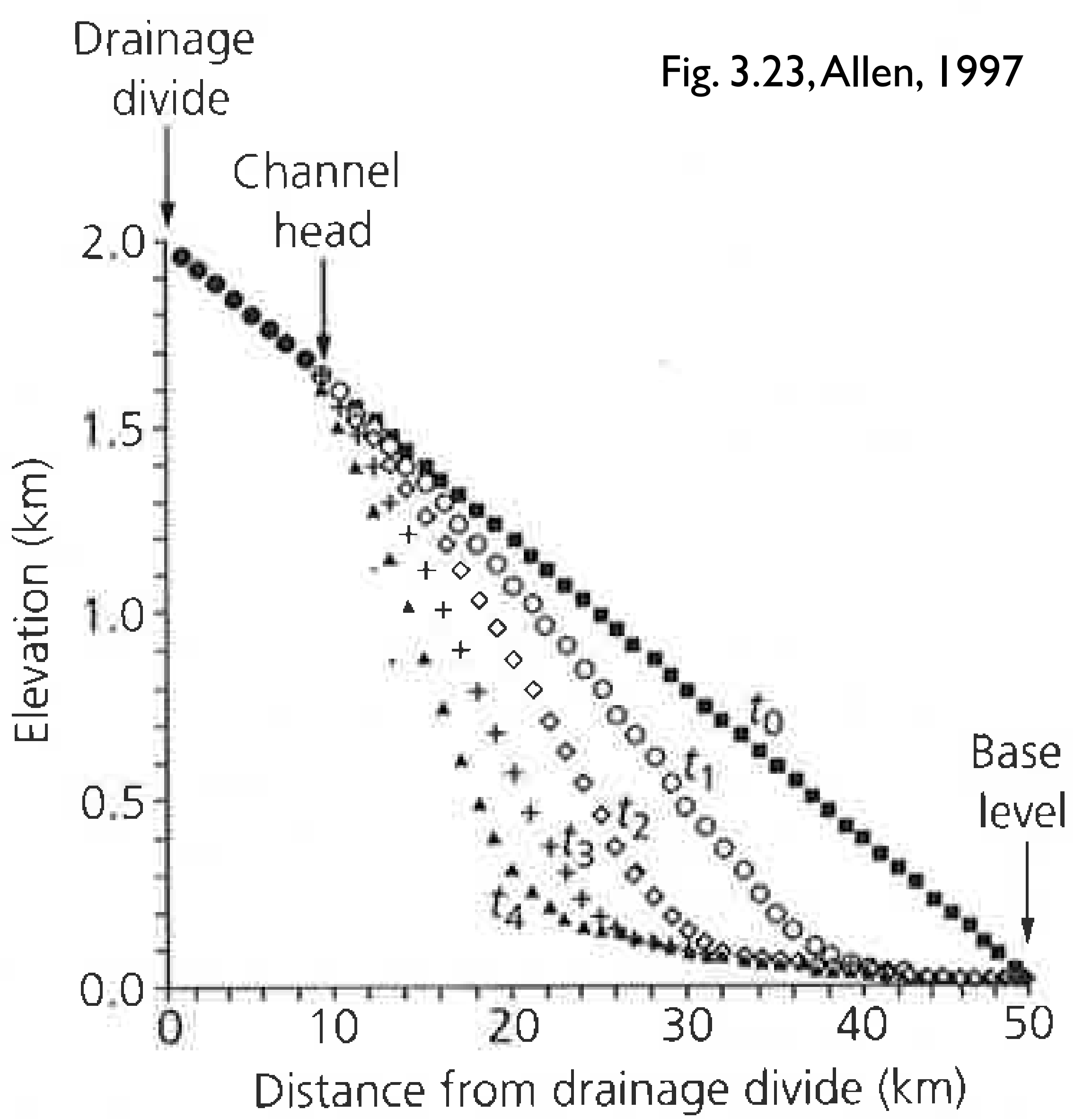
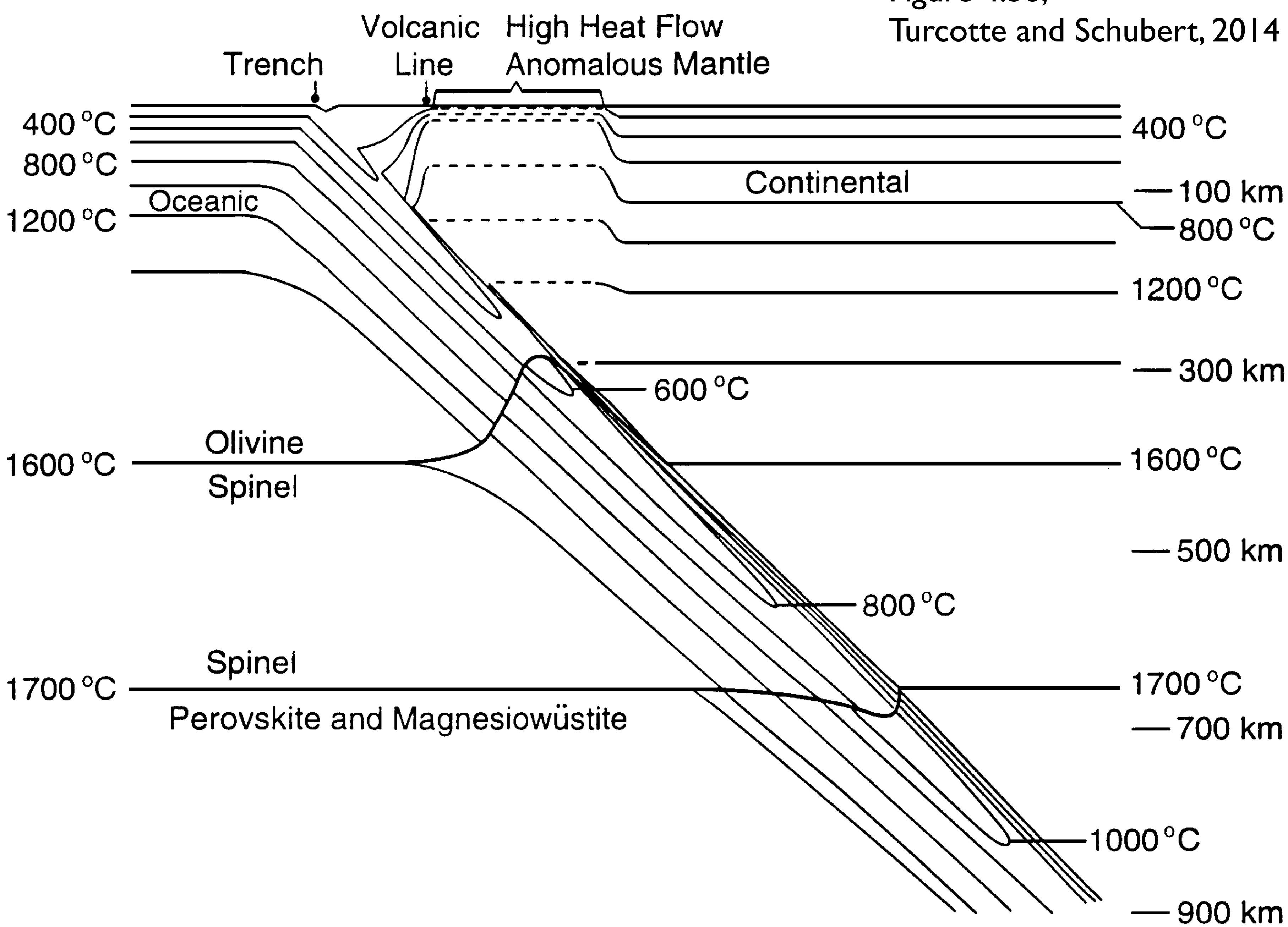


Fig. 3.23, Allen, 1997

- A few stream-power erosion observations:
  - Stream power increases downstream as the discharge grows
  - Steeper slopes occur upstream where the discharge is low
  - Incision migrates upstream until a balance is attained between erosion and uplift

Advective heat transfer

Figure 4.58, Turcotte and Schubert, 2014



- Advection is important in tectonically active settings

Advective heat transfer

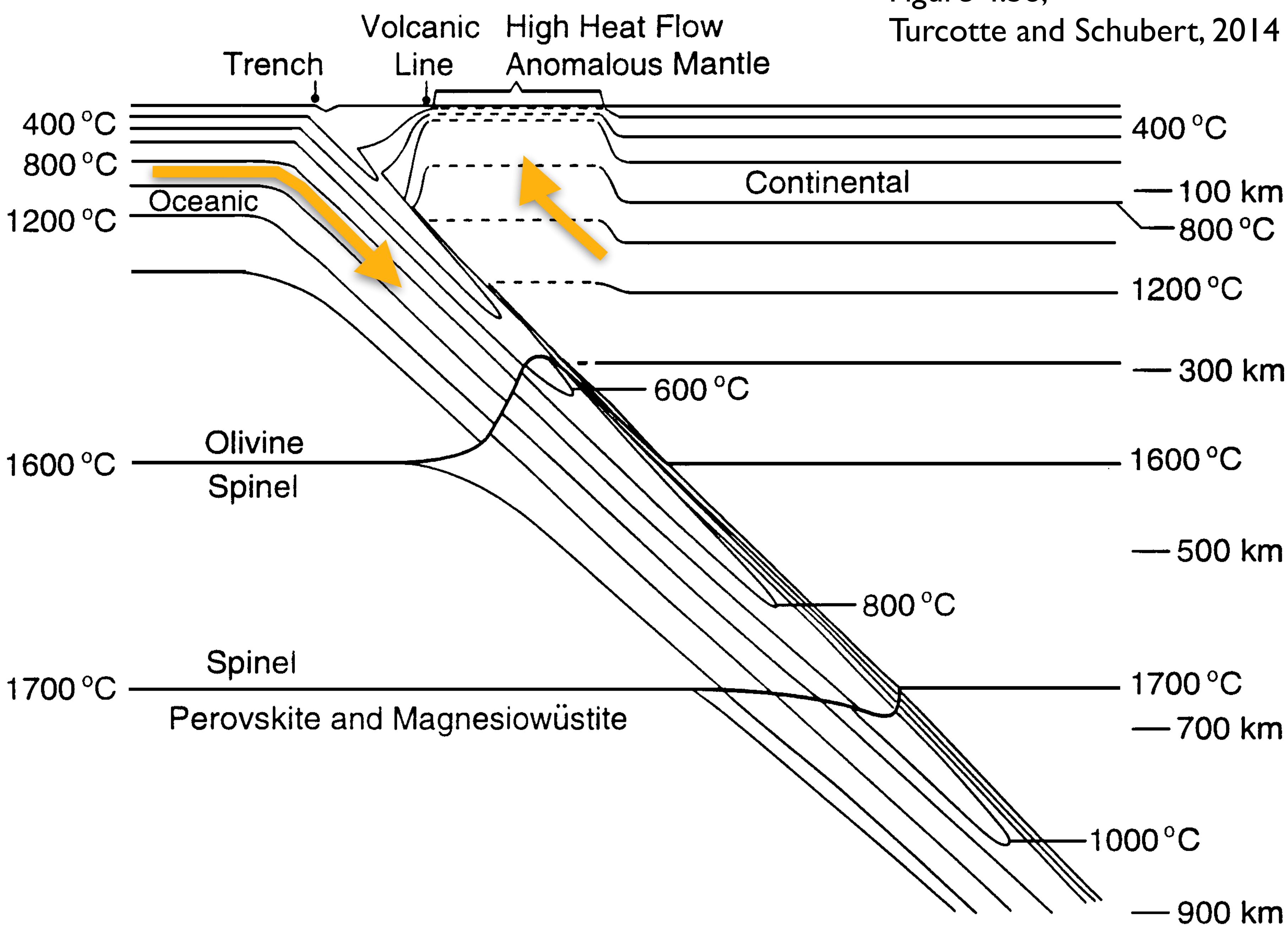


Figure 4.58, Turcotte and Schubert, 2014

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Advective heat transfer

Time-dependent advection

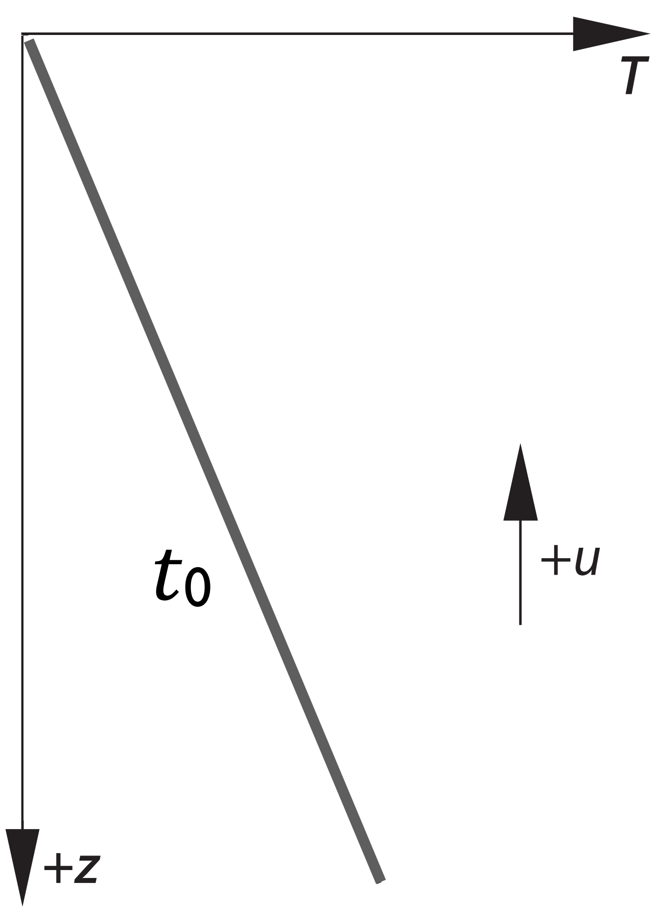

Fig. 3.13, Stüwe, 2007

- **What happens when a parcel of rock is advected?**

Advective heat transfer

Time-dependent advection

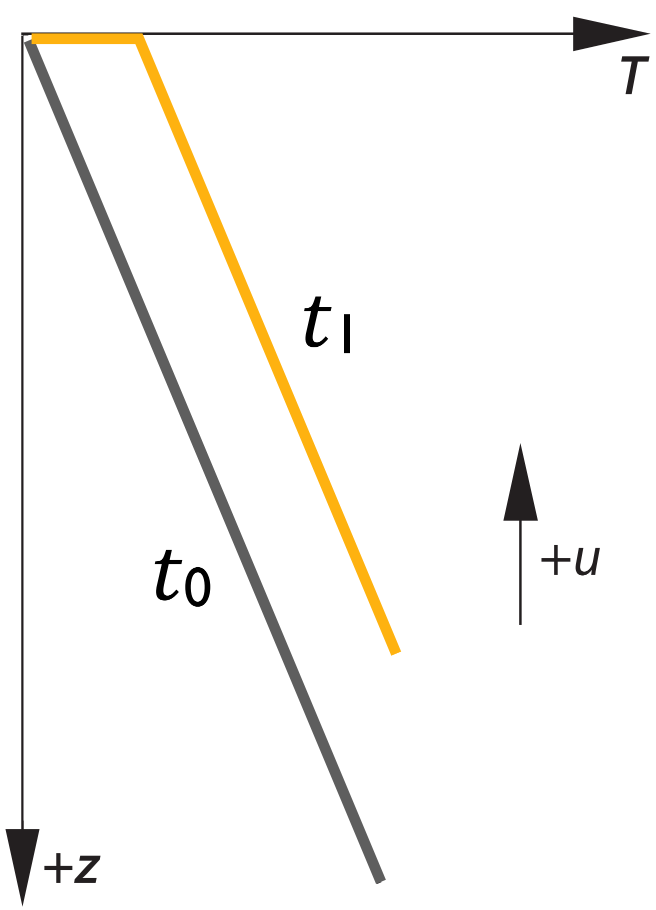

Fig. 3.13, Stüwe, 2007

- **What happens when a parcel of rock is advected?**
  - The rock carries with it **thermal energy** (**heat**)

Advective heat transfer

Time-dependent advection

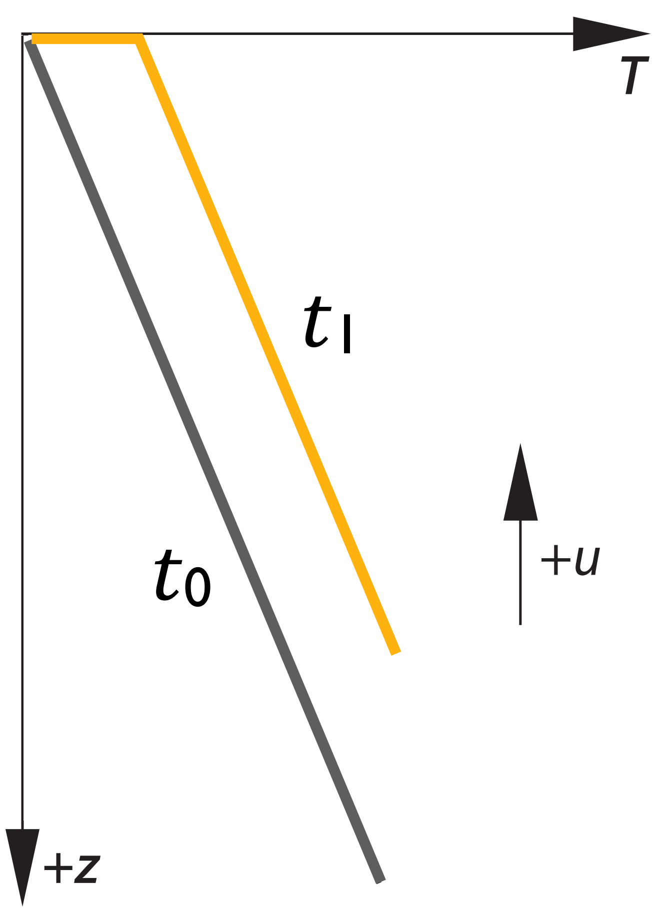

Fig. 3.13, Stüwe, 2007

- **What happens when a parcel of rock is advected?**
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  - The effect of advection (change in temperature) depends on the relative motion

Advective heat transfer

Time-dependent advection and diffusion

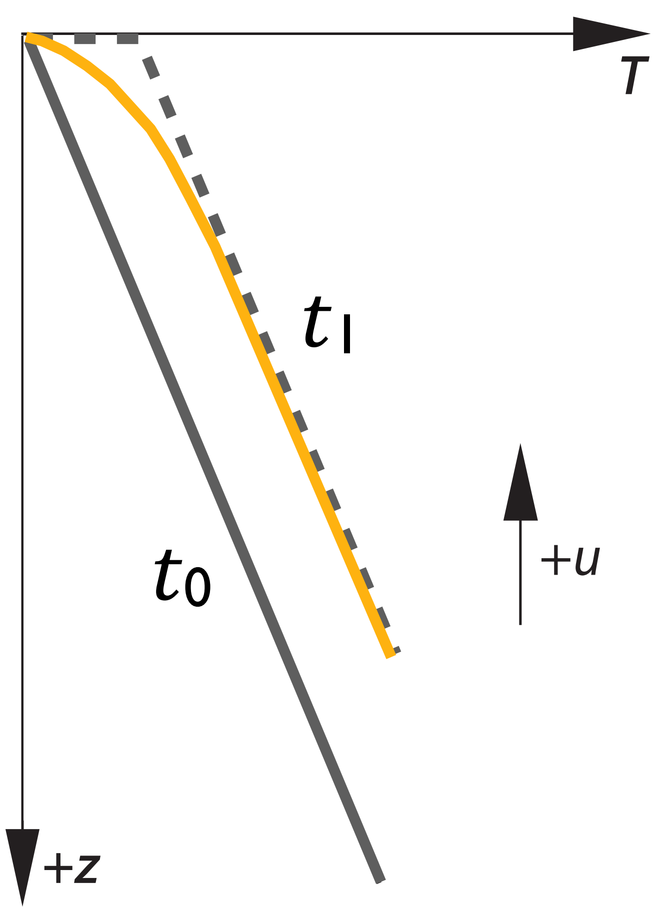
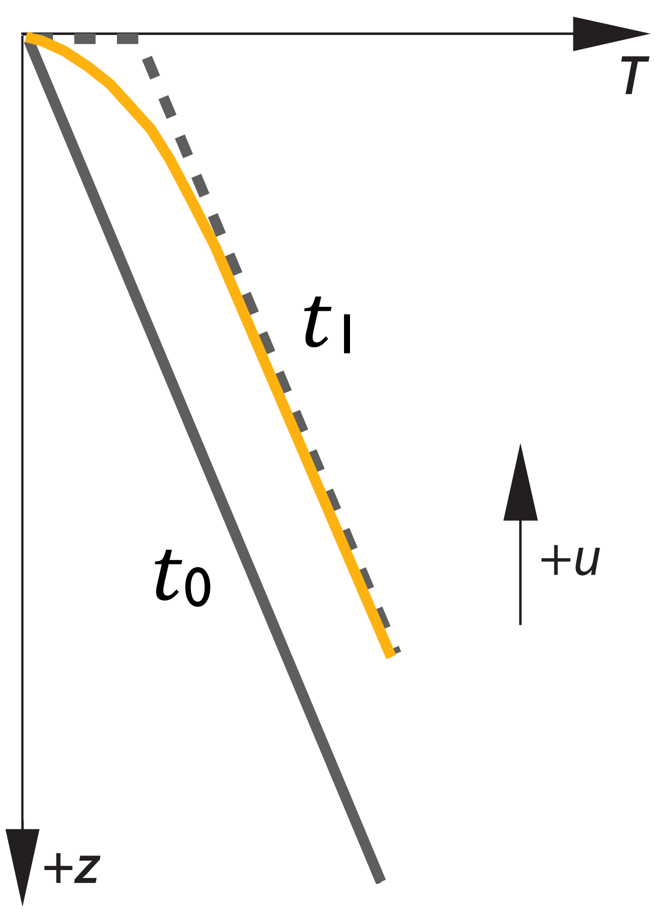


Fig. 3.13, Stüwe, 2007

- **What happens when a parcel of rock is advected?**
  - The rock carries with it **thermal energy** (**heat**)
  - The effect of advection (change in temperature) depends on the relative motion
  - The effect of advection also depends on the rate of motion

Advective heat transfer

Time-dependent advection and diffusion



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  - The rock carries with it **thermal energy** (**heat**)
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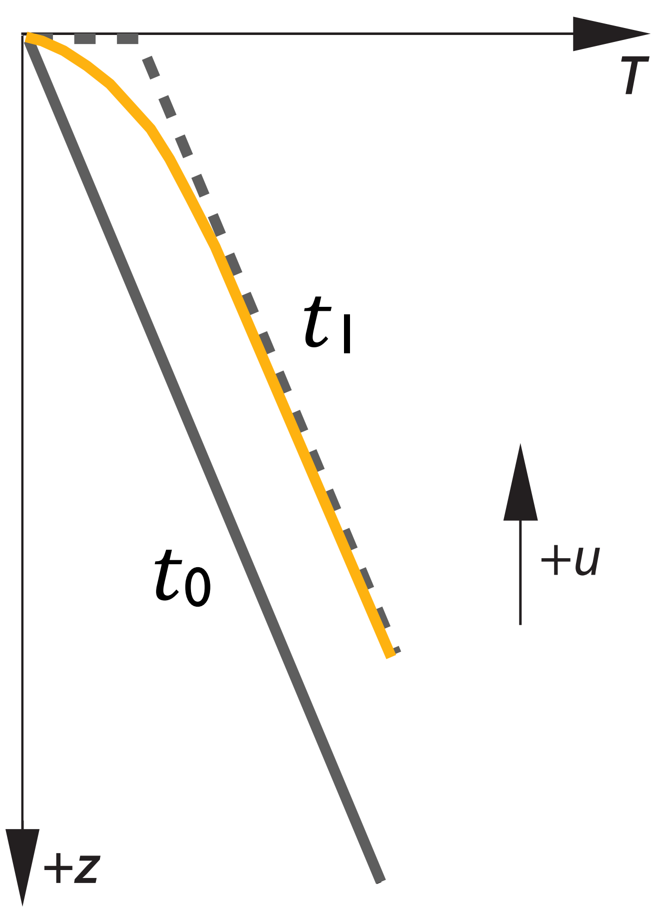
    - **Why?**

Advective heat transfer

Time-dependent advection and diffusion



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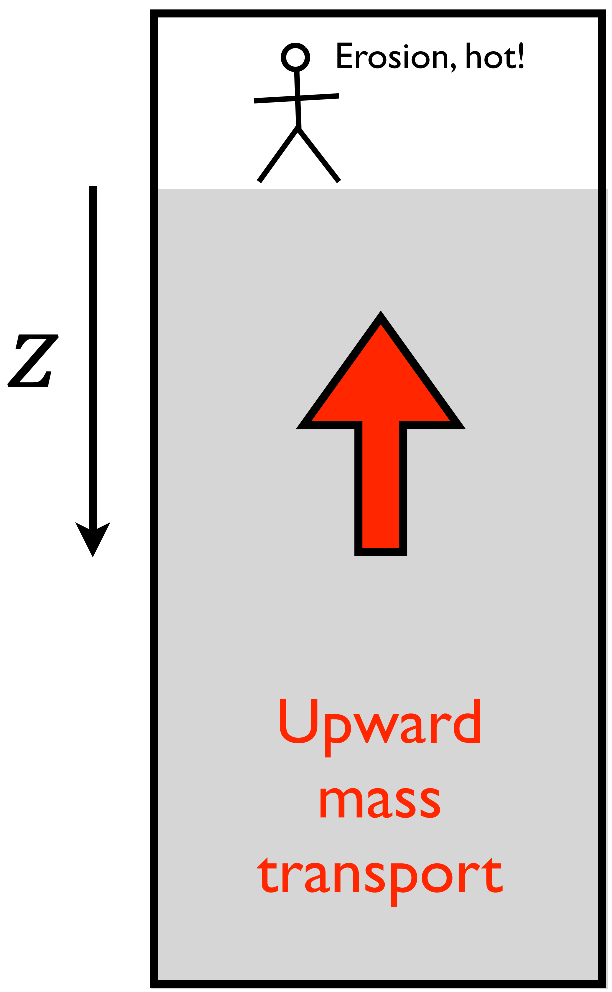
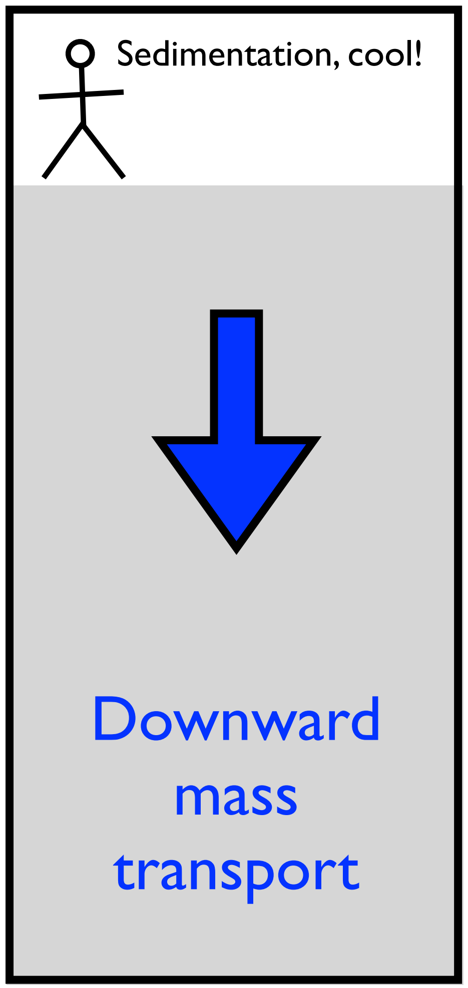
- **What happens when a parcel of rock is advected?**
  - The rock carries with it **thermal energy** (**heat**)
  - The effect of advection (change in temperature) depends on the relative motion
  - The effect of advection also depends on the rate of motion
    - **Why?**
    - If heat is lost to diffusion more rapidly than it is advected, advection will have little effect

Erosion



- **Erosion** results in upward advection of rock as surface rock is eroded and transported away
- Upward motion brings relatively hot rock up from depth toward the surface, increasing the geothermal gradient
- Erosion typically becomes important at advection velocities of >0.1 mm/a

Sedimentation



- **Sedimentation** is essentially the opposite of erosion
  - Sediment deposition results in downward advection of rock as the surface subsides and a basin is filled
- Downward motion pushes relatively cold rock downward, decreasing the geothermal gradient
- Sedimentation typically becomes important at advection velocities of >0.1 mm/a

Recap

- What is the main difference between the advection and diffusion equations?
- What is special about the stream power erosion model compared to the general advection equation?
- Why does the rate of advection matter in heat transfer problems?

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References

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