



# Introduction to geodynamic modelling

## **Key physical processes and concepts**

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# Goals of this lecture

- Present the main **physical processes and concepts** we need to consider to understand **geodynamics (mainly in the lithosphere)**
- Provide **necessary background** for the rest of the course



# Dissecting the course title

- This course is titled “Introduction to geodynamic modelling”
- **What does this title bring to mind for you?**



# Dissecting the course title

- This course is titled “Introduction to (**lithospheric**) geodynamic modelling”
  - Our focus is on the **lithosphere**
    - Outermost layer of the Earth that is rigid over geological timescales
    - Thermal lithosphere: Portion of outer layers below ~1300°C
    - Crust and lithospheric mantle
  - Mantle convection not part of this course



# Dissecting the course title

- This course is titled “Introduction to **geodynamic** modelling”
  - Our focus is on **geodynamics**
    - Plate tectonics and related phenomena
    - Physical processes/topics
      - Stress and strain
      - Heat transfer
      - Deformation: Faulting and folding, rheology



# Dissecting the course title

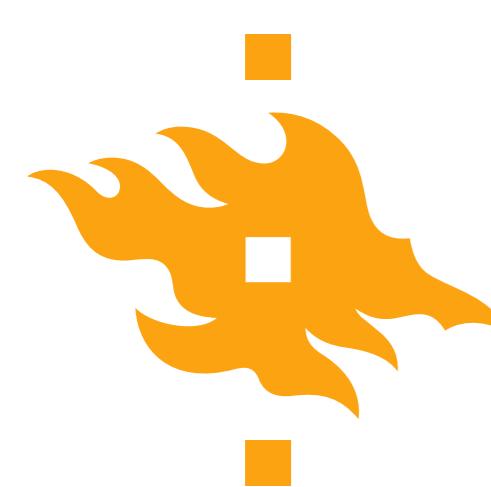
- This course is titled “Introduction to geodynamic **modelling**”
  - Our focus is on **modelling**
    - Using computers to solve equations and simulate geodynamic processes
    - We will learn how to solve equations using numerical methods, and how to implement those numerical solutions in computer code
    - Few geodynamic processes are simple enough to be explored without the use of computers



# The path forward

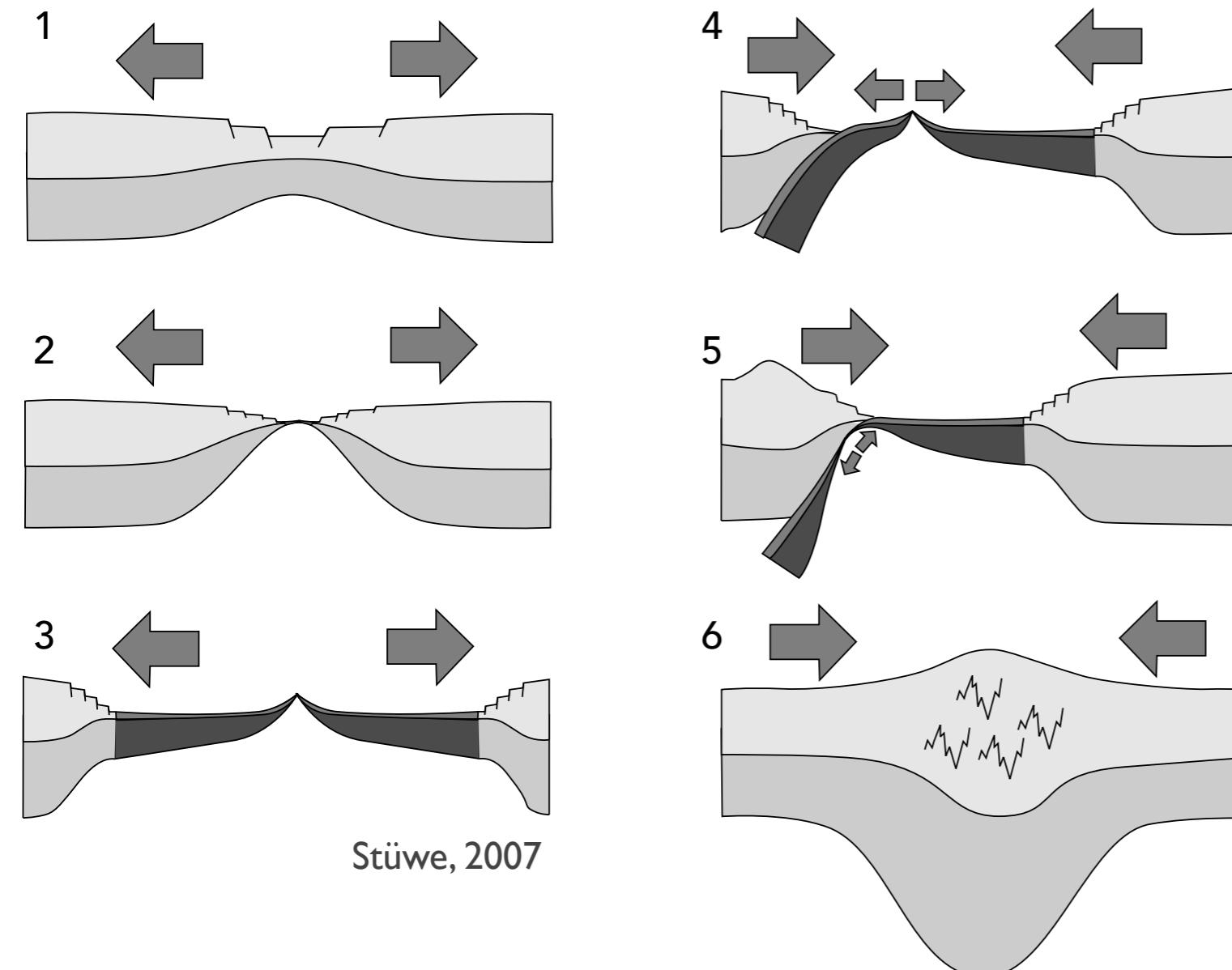
- Now we will briefly review the different **physical processes** and **concepts** related to **geodynamics**
- In the next lecture, we'll take an example equation from this lecture and discuss what is needed to solve it
- In the afternoon you will have time to review basic computing concepts using the Python programming language
- Some of this may be review for you, but it never hurts to revisit these fundamental topics before moving on to more challenging topics

# Plate Tectonics and related phenomena

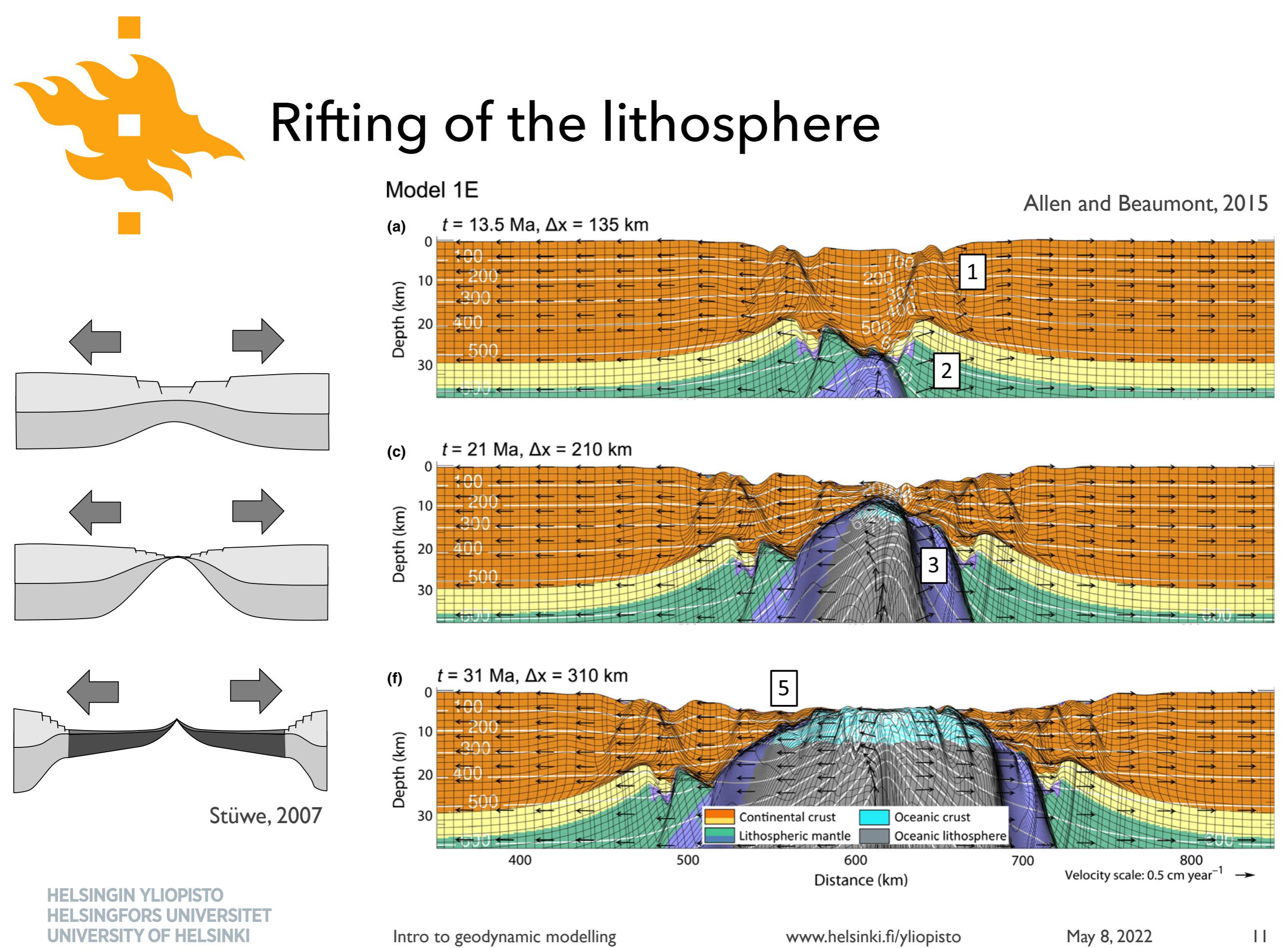


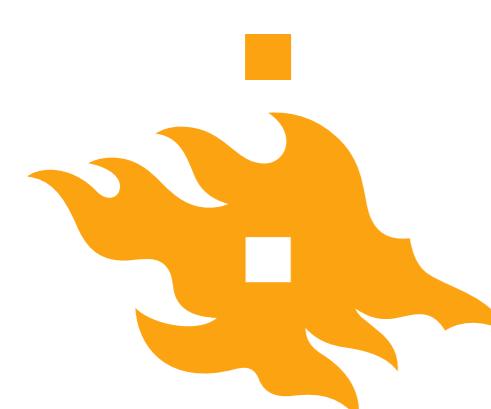
# Lithospheric geodynamic processes

## The Wilson cycle

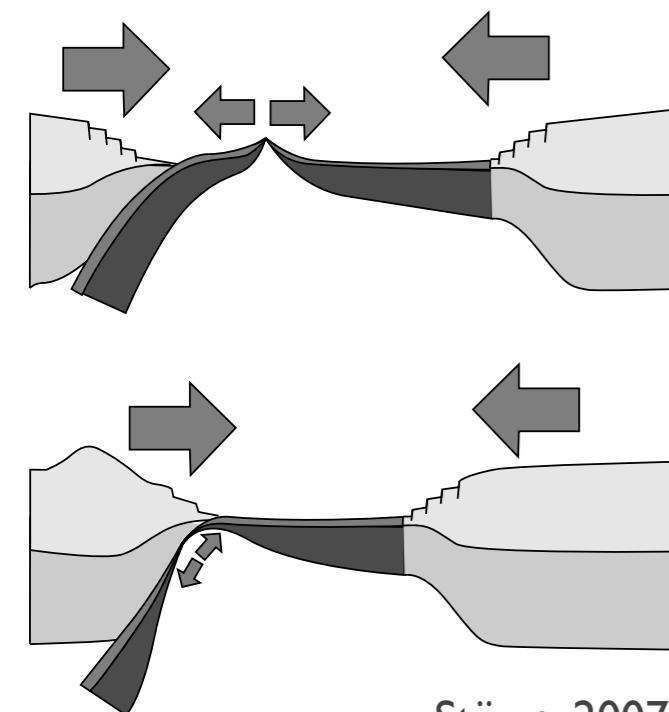


- The focus for this lecture will be on the lithosphere and the dynamic processes involved in its deformation and evolution
- Many of these processes can be directly linked to Plate Tectonics and the Wilson cycle

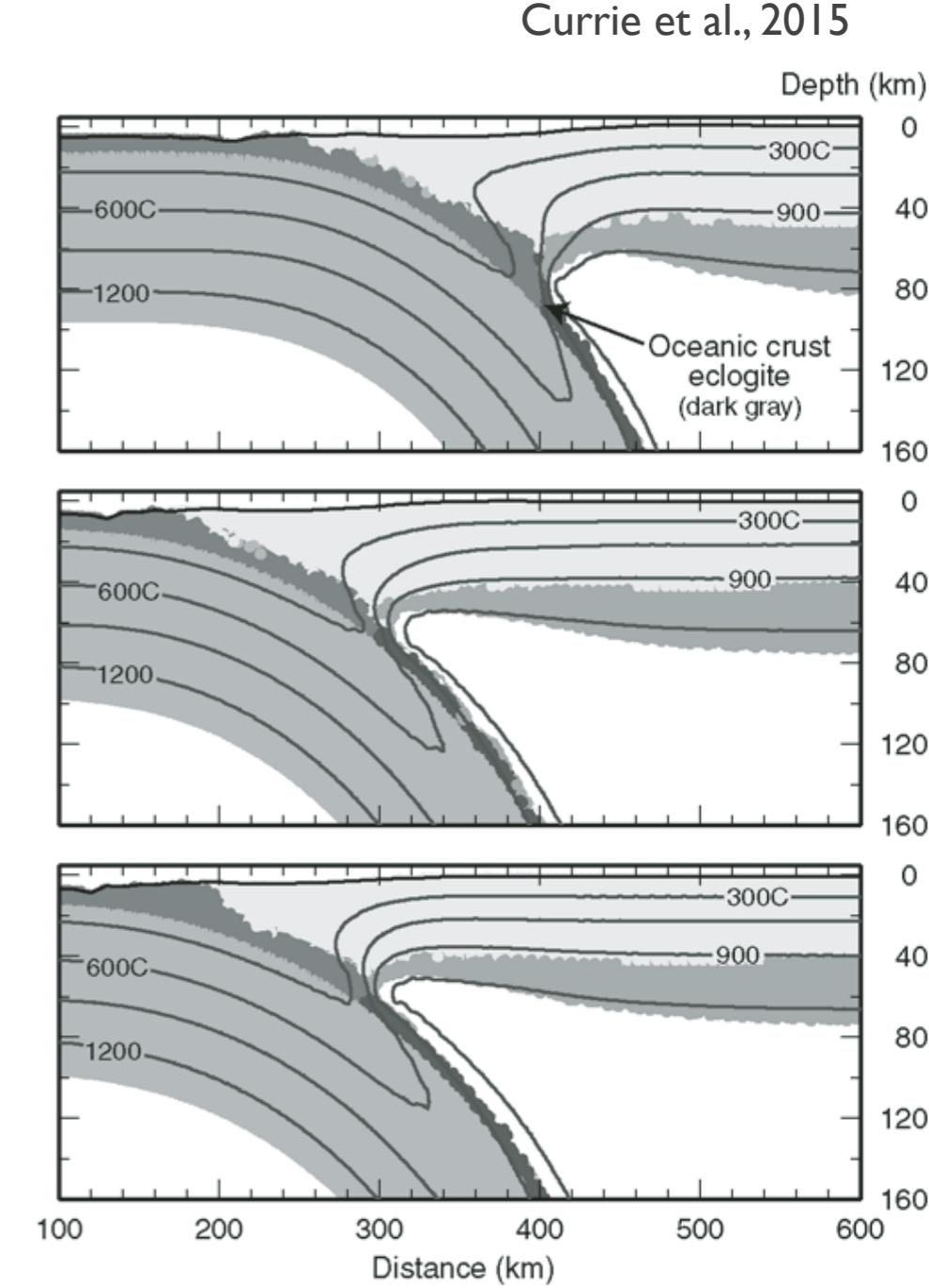
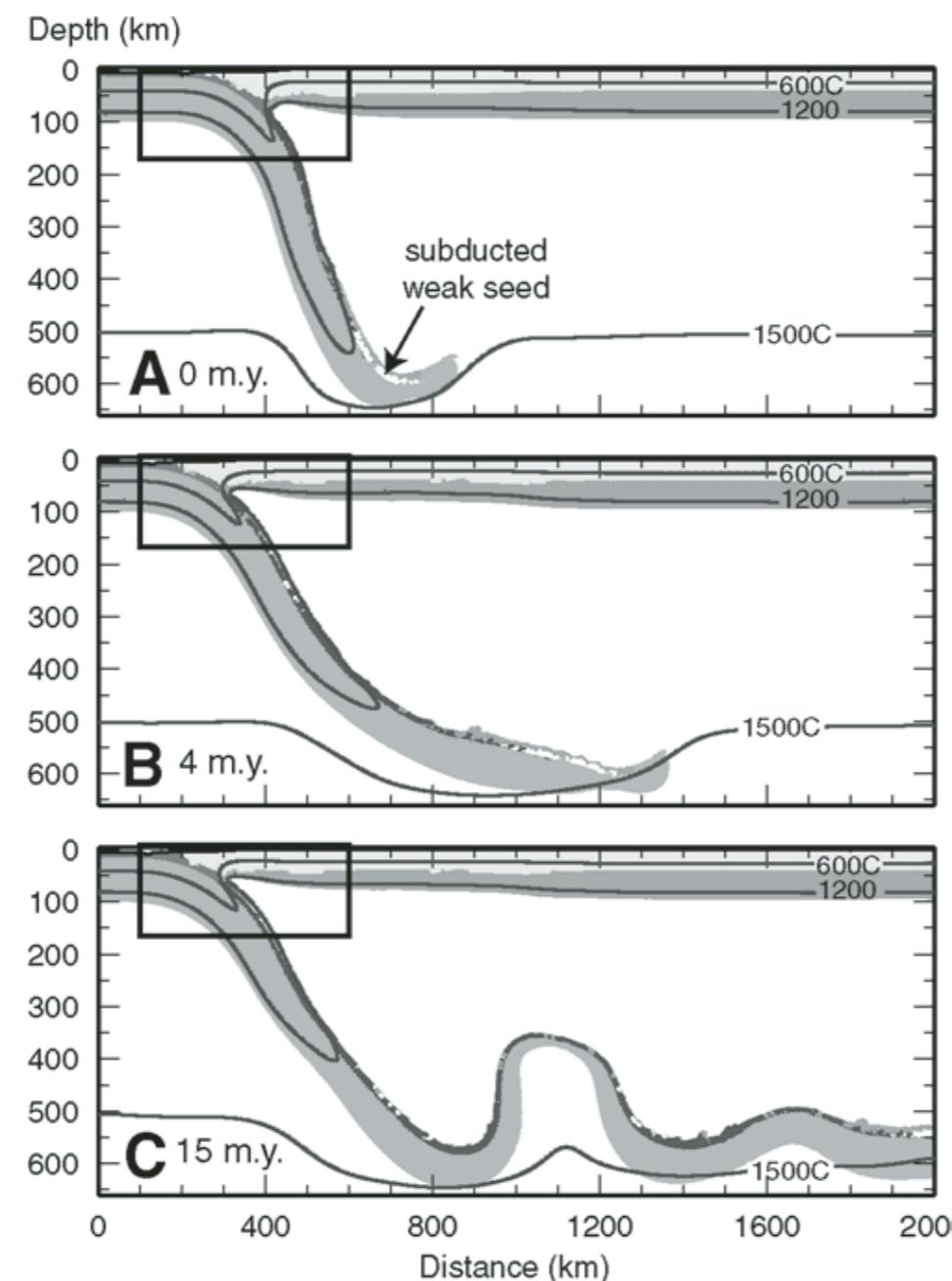




# Oceanic subduction

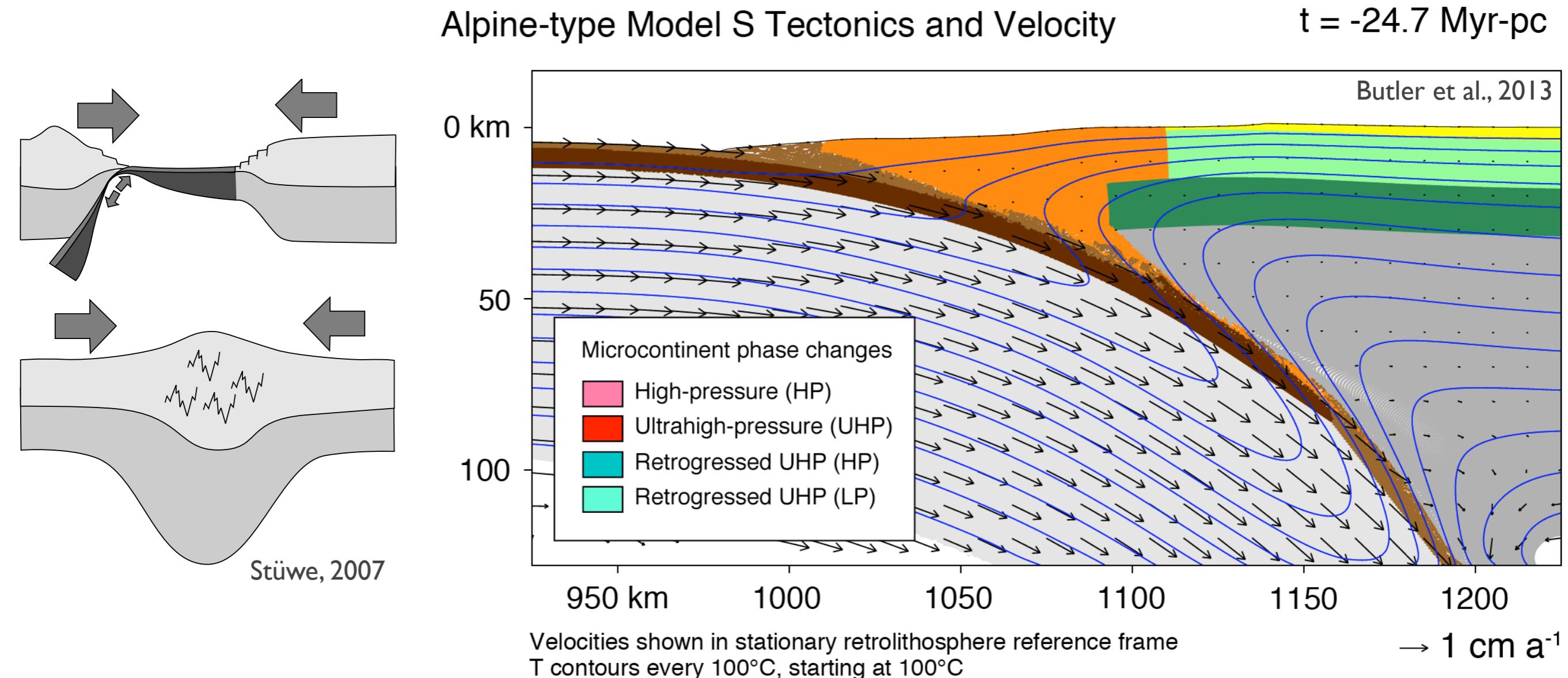


Stüwe, 2007





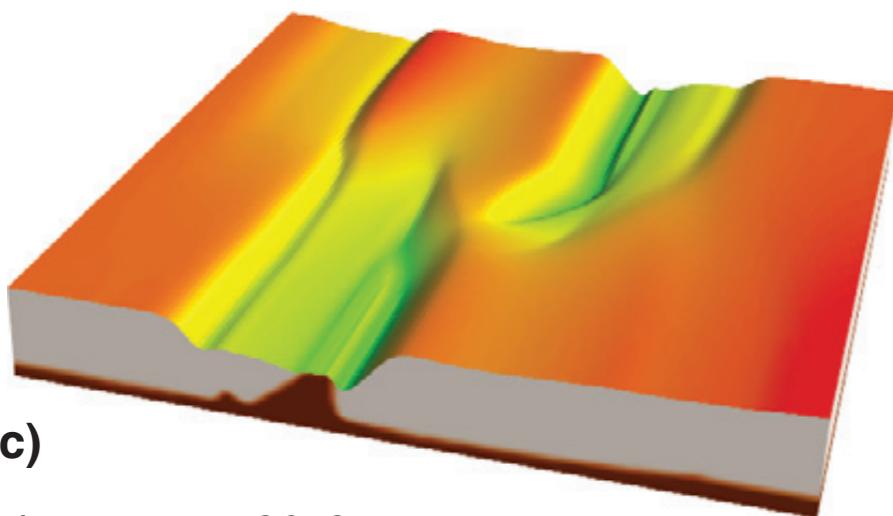
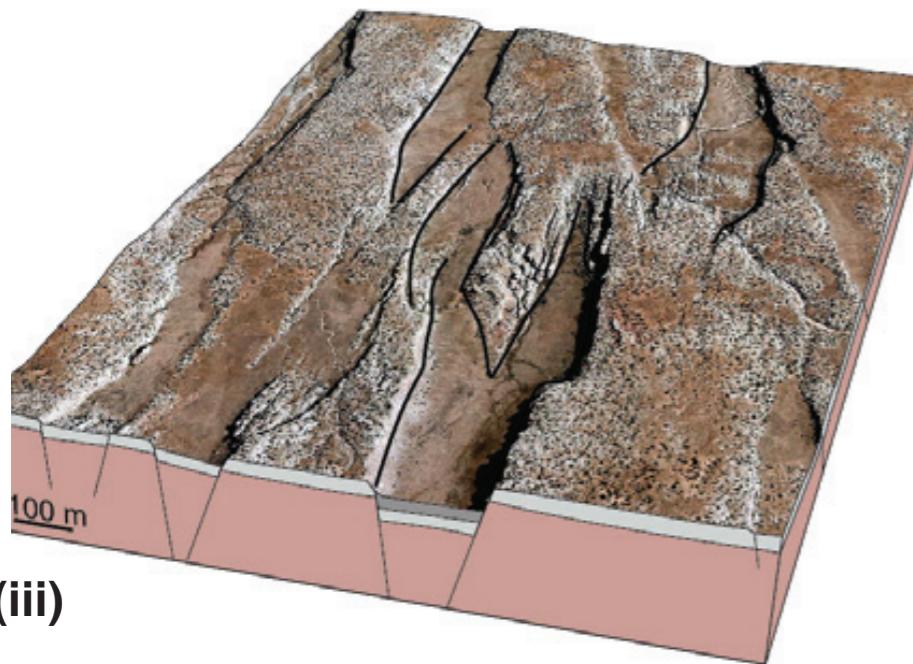
# Continental collision





# Toward three dimensions

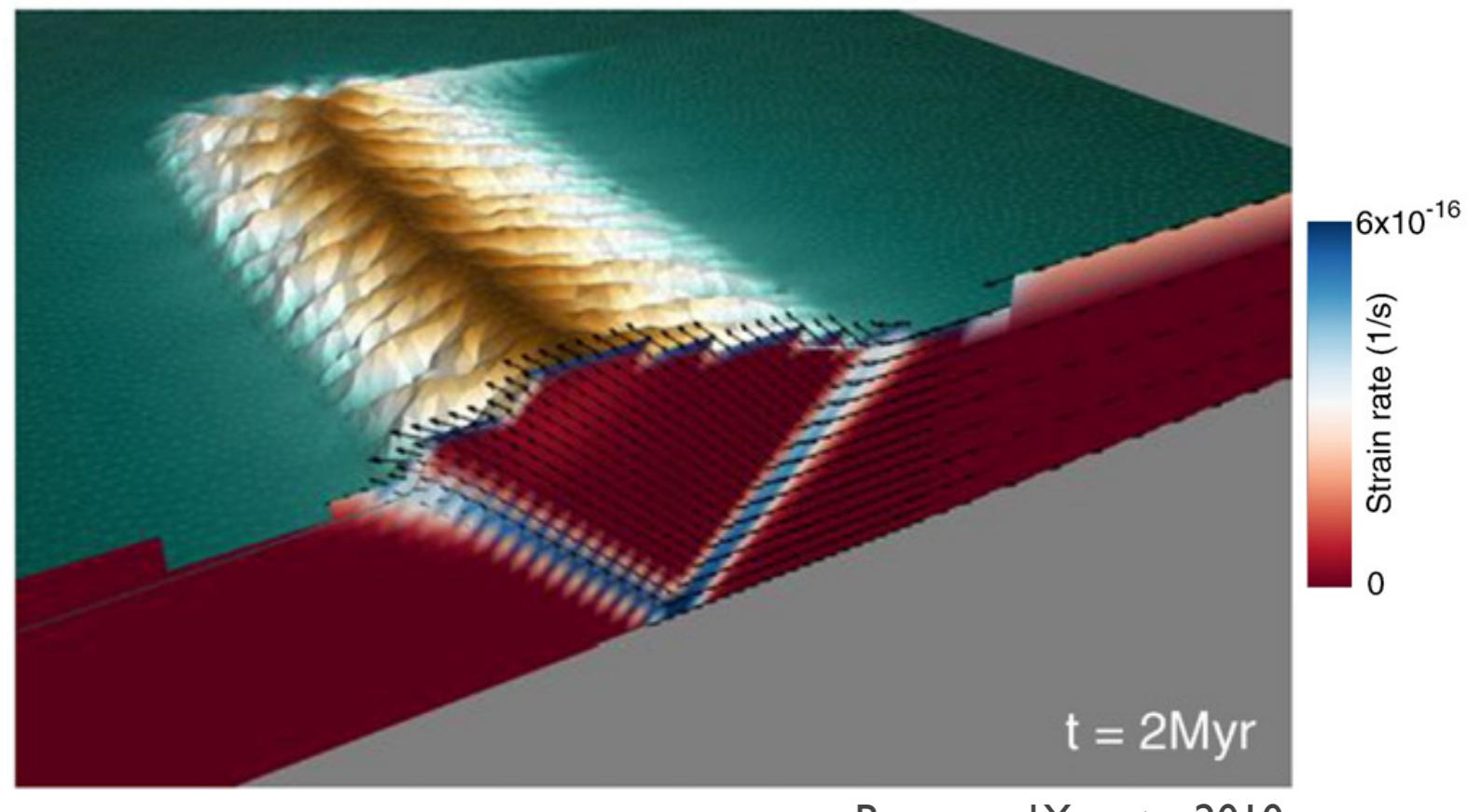
Rifting

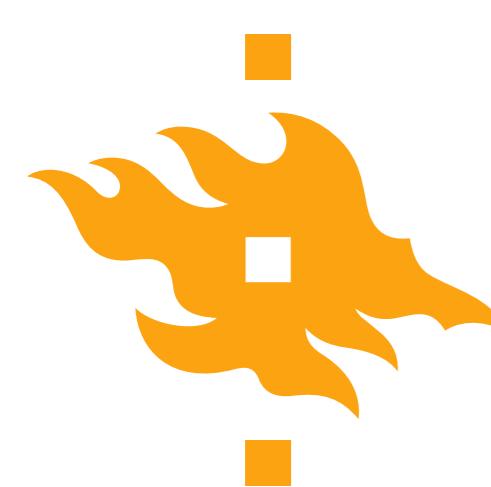


Allken et al., 2012

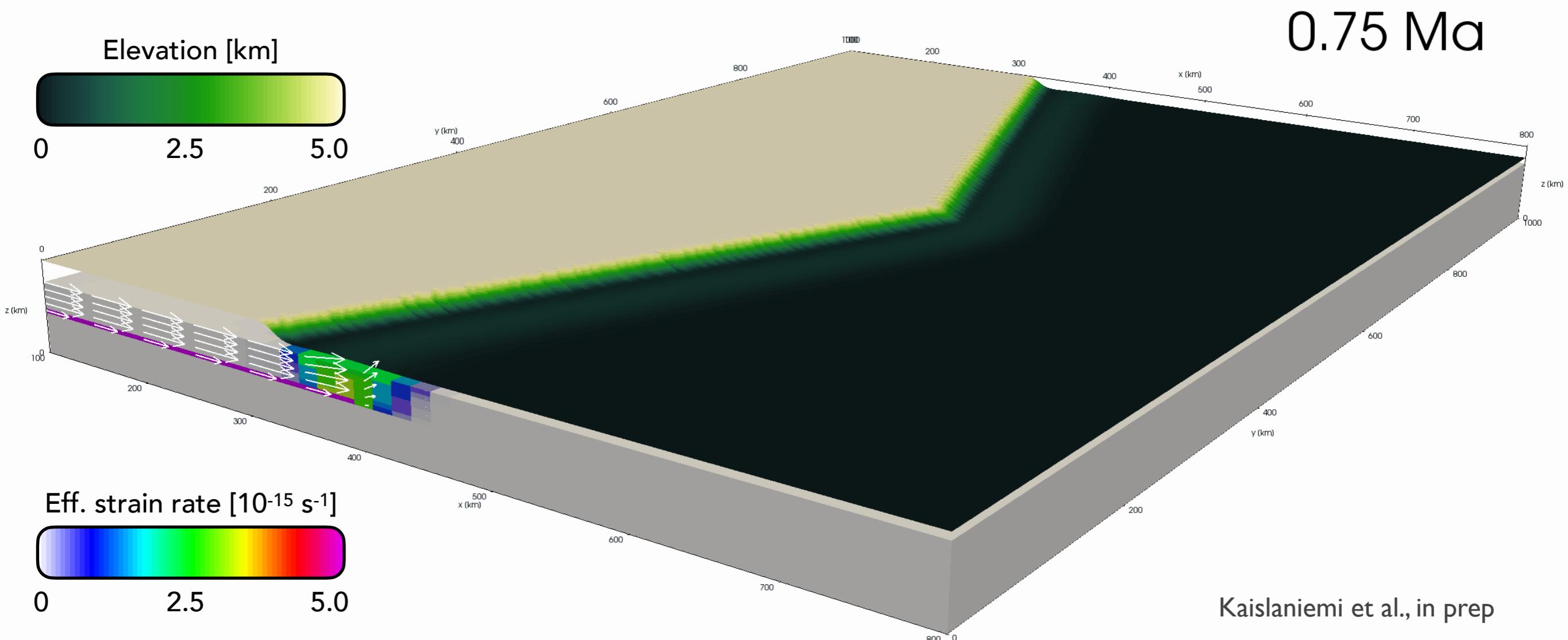
HELSINGIN YLIOPISTO  
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UNIVERSITY OF HELSINKI

Continental collision





# Fold-and-thrust belt growth and erosion



# What is a model?



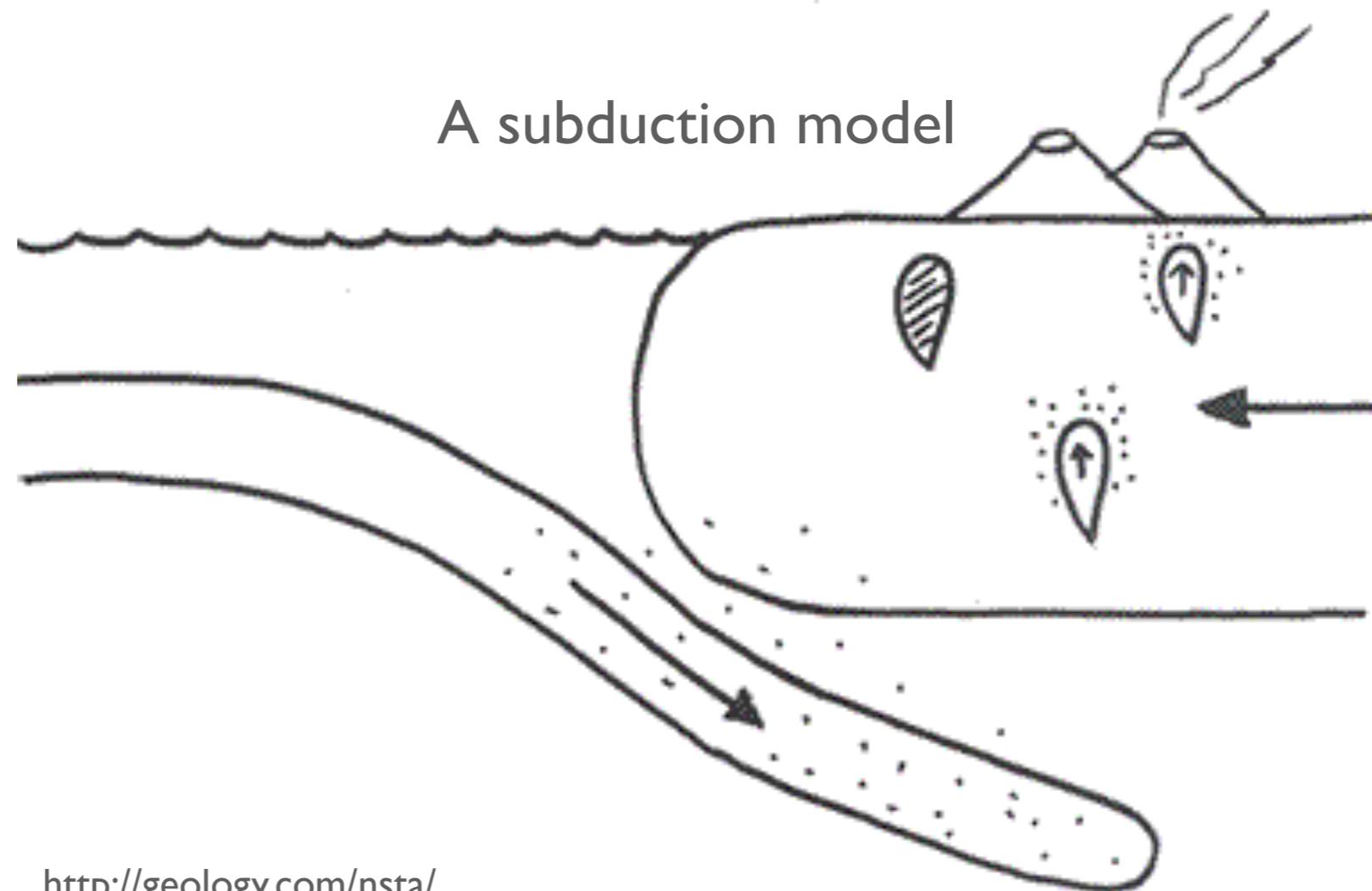
# What is a model?

- “A **model** is tool used to describe the world around us in a *simplified way* so that we can *understand it better*”

Stüwe, 2007



# What is a model?



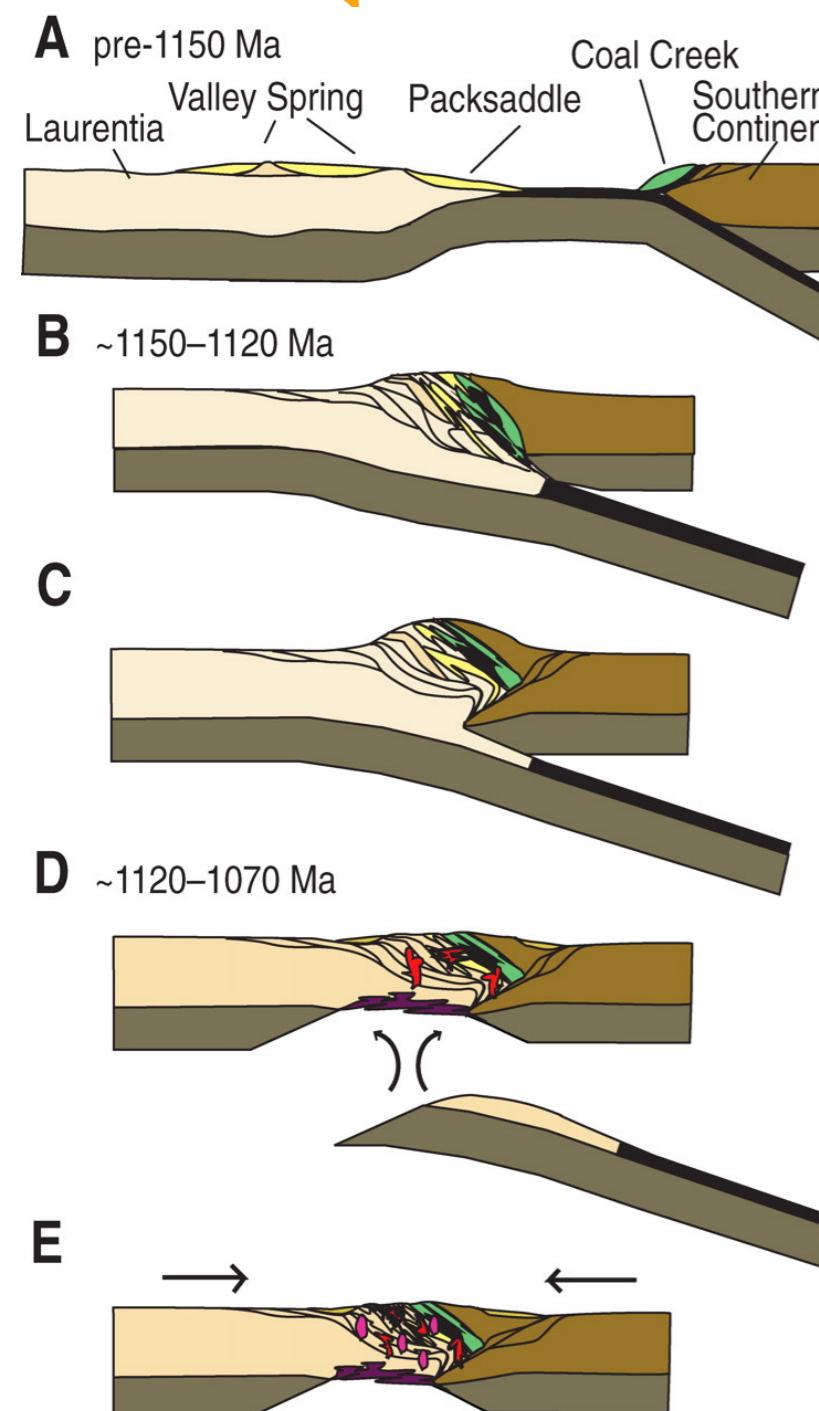
<http://geology.com/nsta/>

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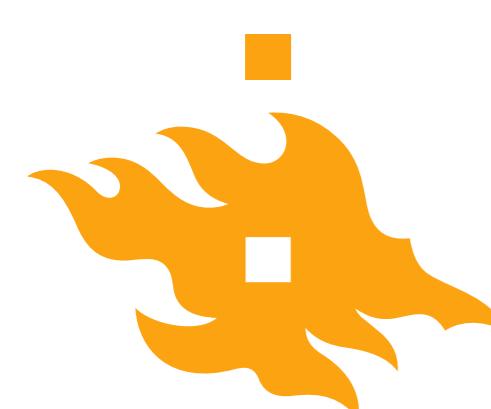
Stüwe, 2007



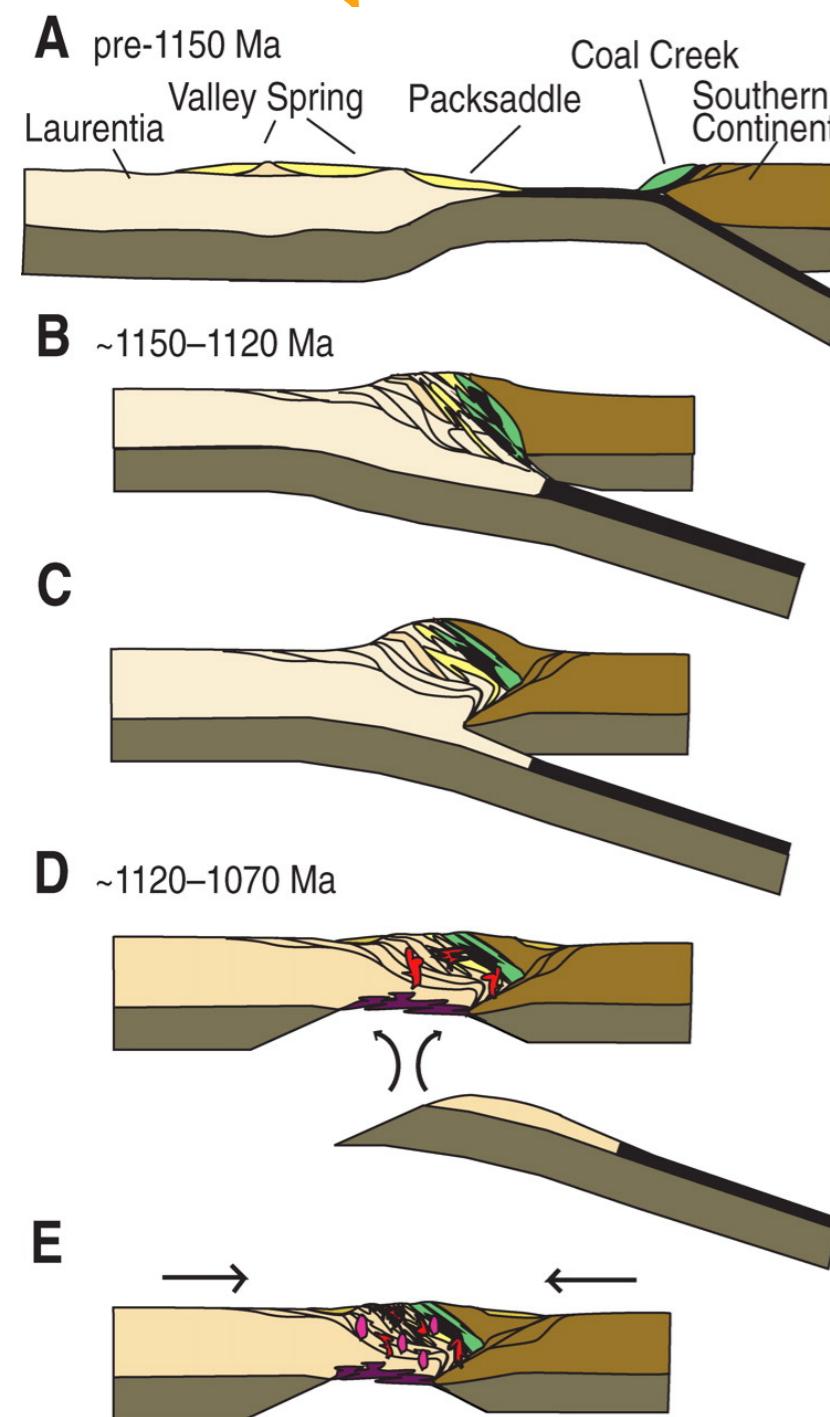
# Types of geological models



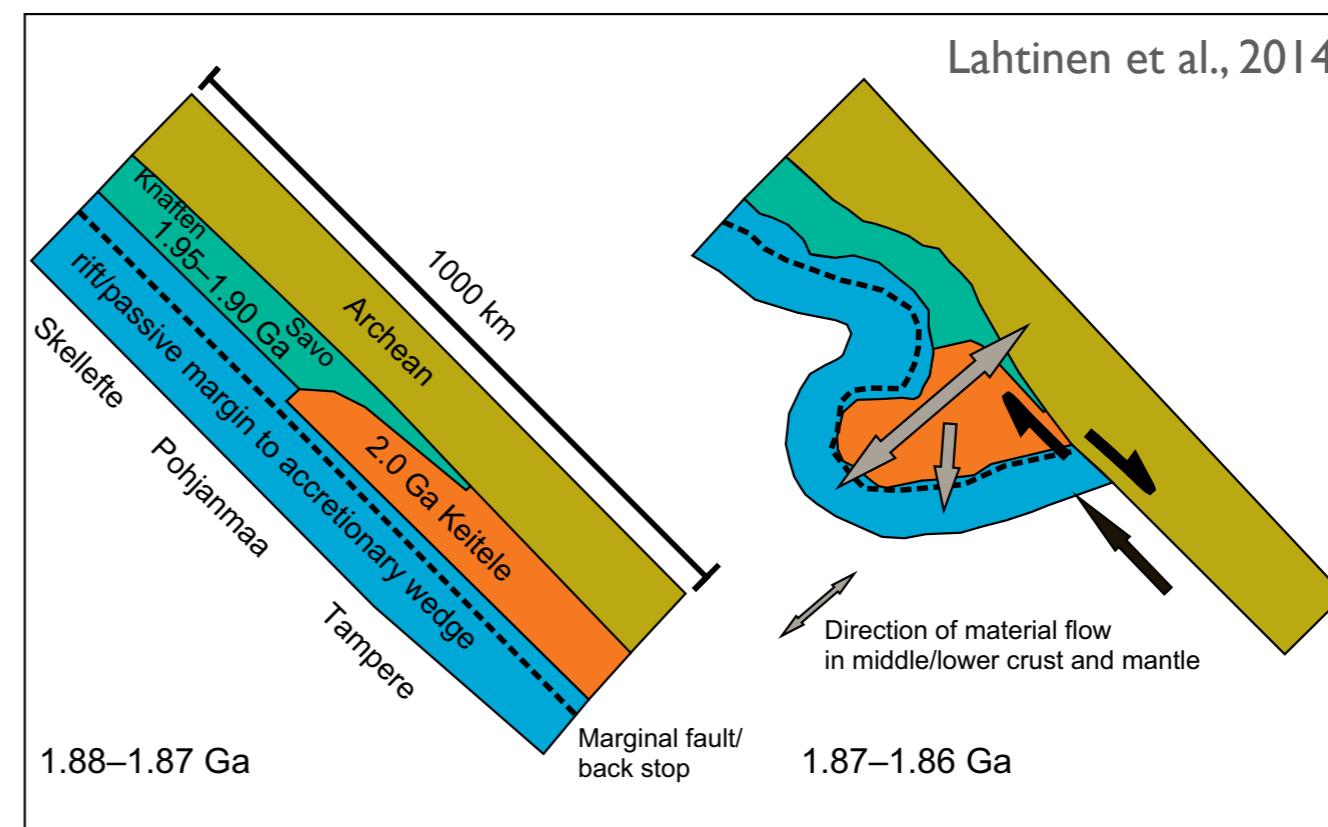
- **Tectonic diagrams** are a familiar form of model to help clarify the time evolution of a study area
  - Typically this kind of model is used to simplify the complex modern geology and restore it to a pre-deformation state
  - These models, though, have no basis in physics

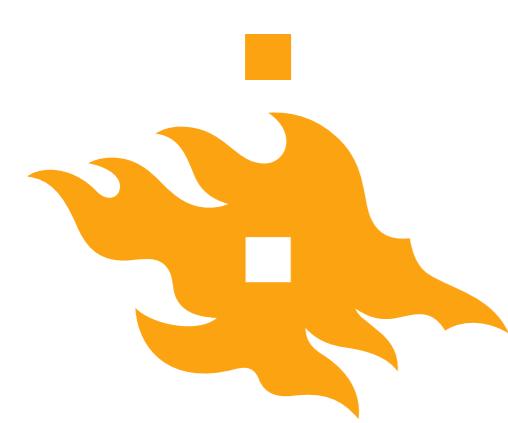


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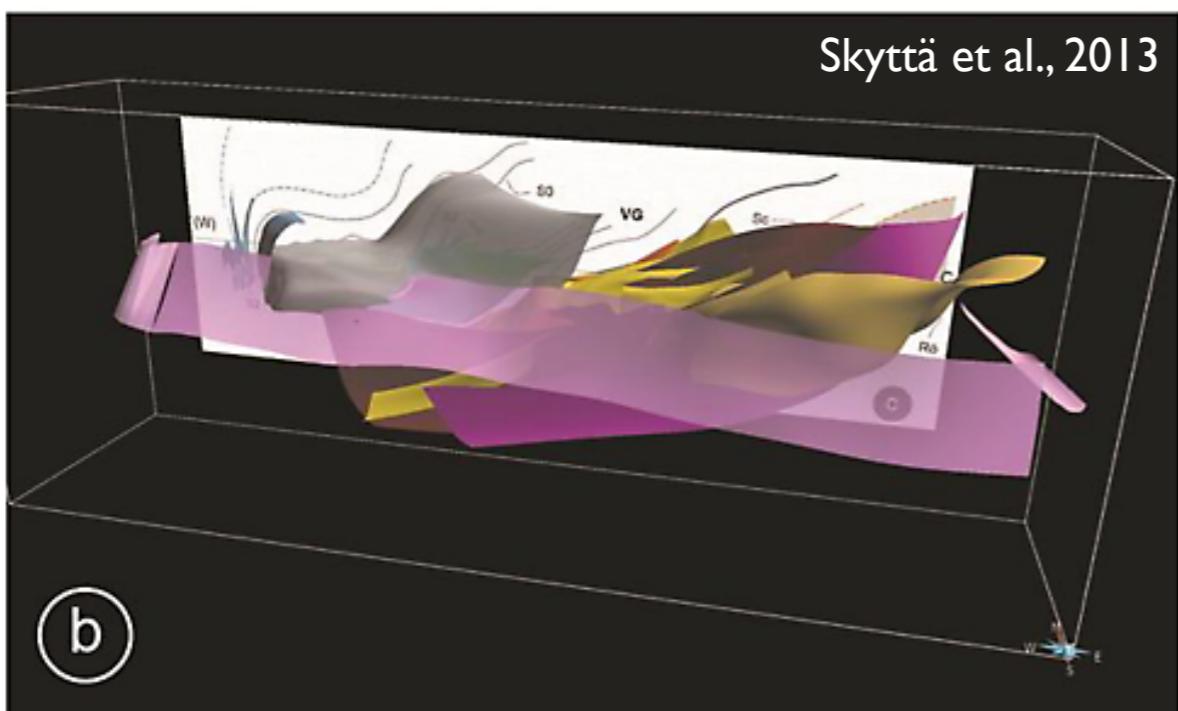
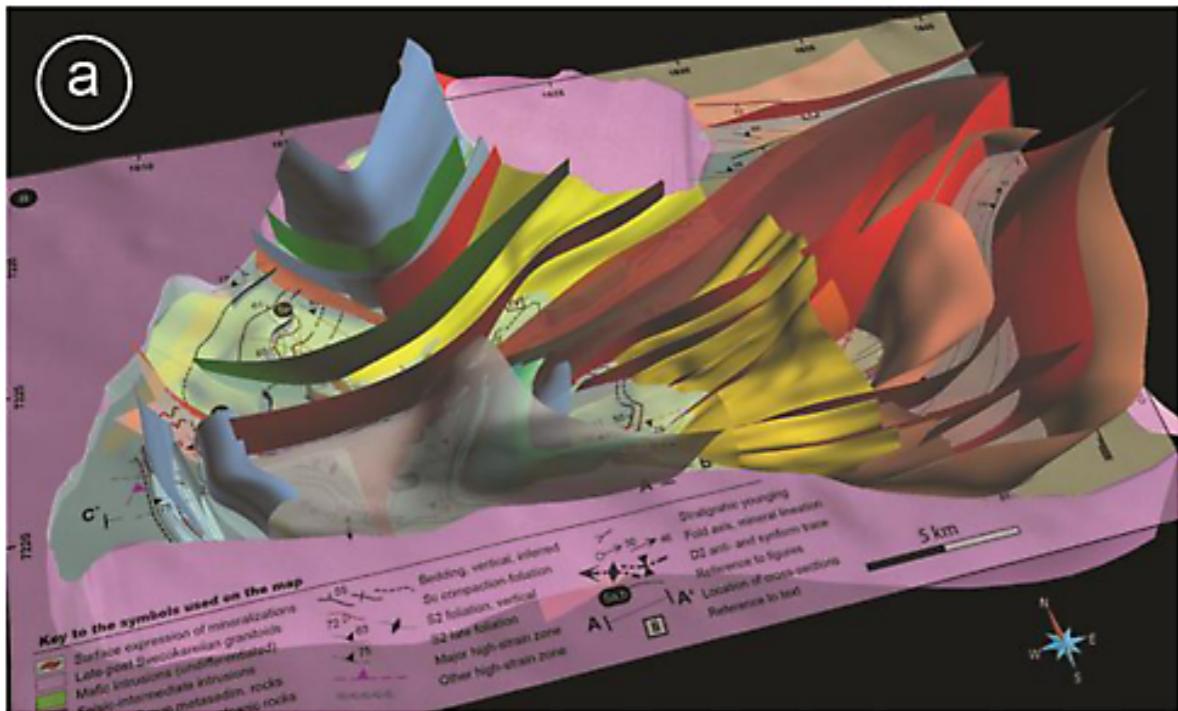


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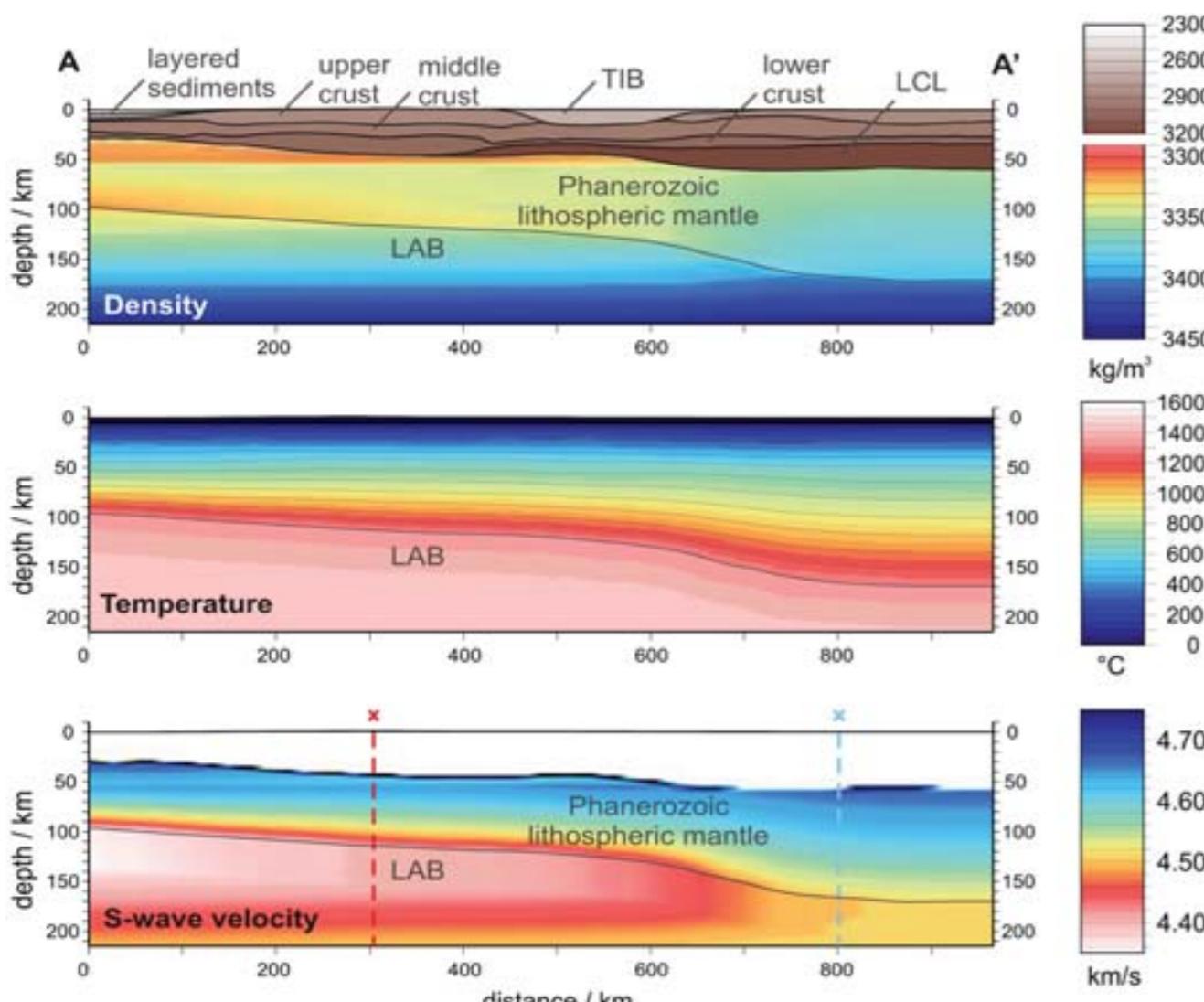
# Types of geological models



- **3D structural or geological models** are closer to reality in that they are based on a combination of surface and subsurface geological and geophysical observations
- The primary goal of these models is data visualisation, again helping us understand complex geometries
- Models of this type typically do not simulate physical processes



# Integrated geophysical modelling

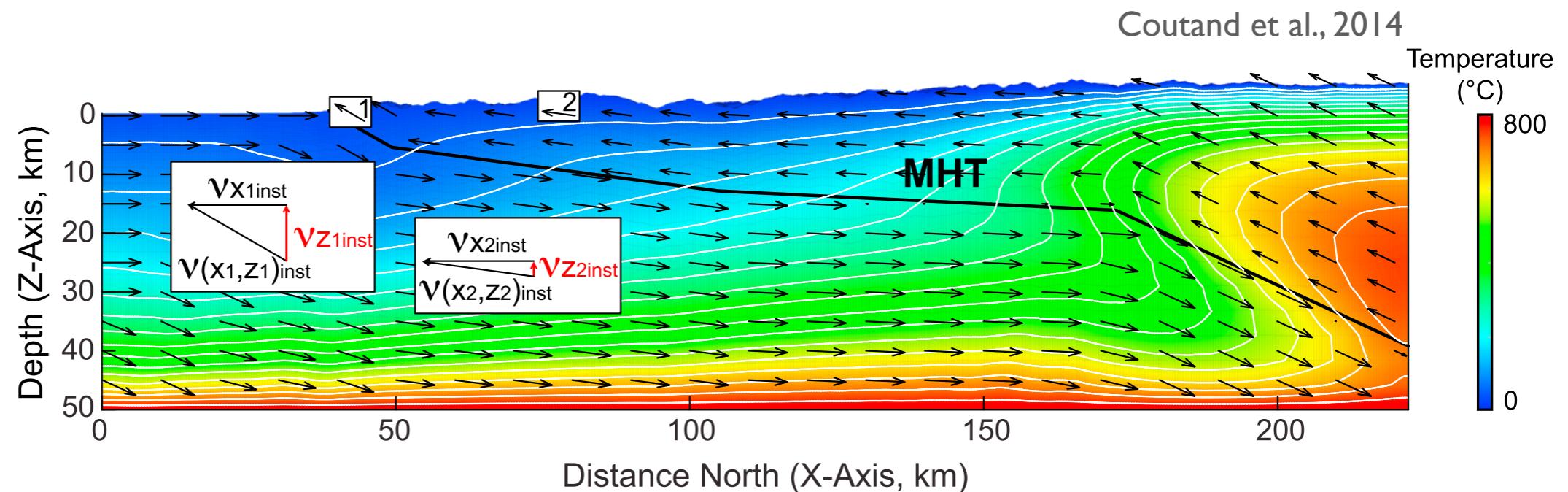


Gradmann et al., 2013

- **Integrated geophysical models** use a combination of an input crustal structure and composition, and rock thermal properties to calculate various properties of the lithosphere (gravity anomalies, seismic velocities, surface heat flow, etc.)
- These models involve a 2D or 3D geometrical model and calculation of heat transfer in the lithosphere and upper mantle



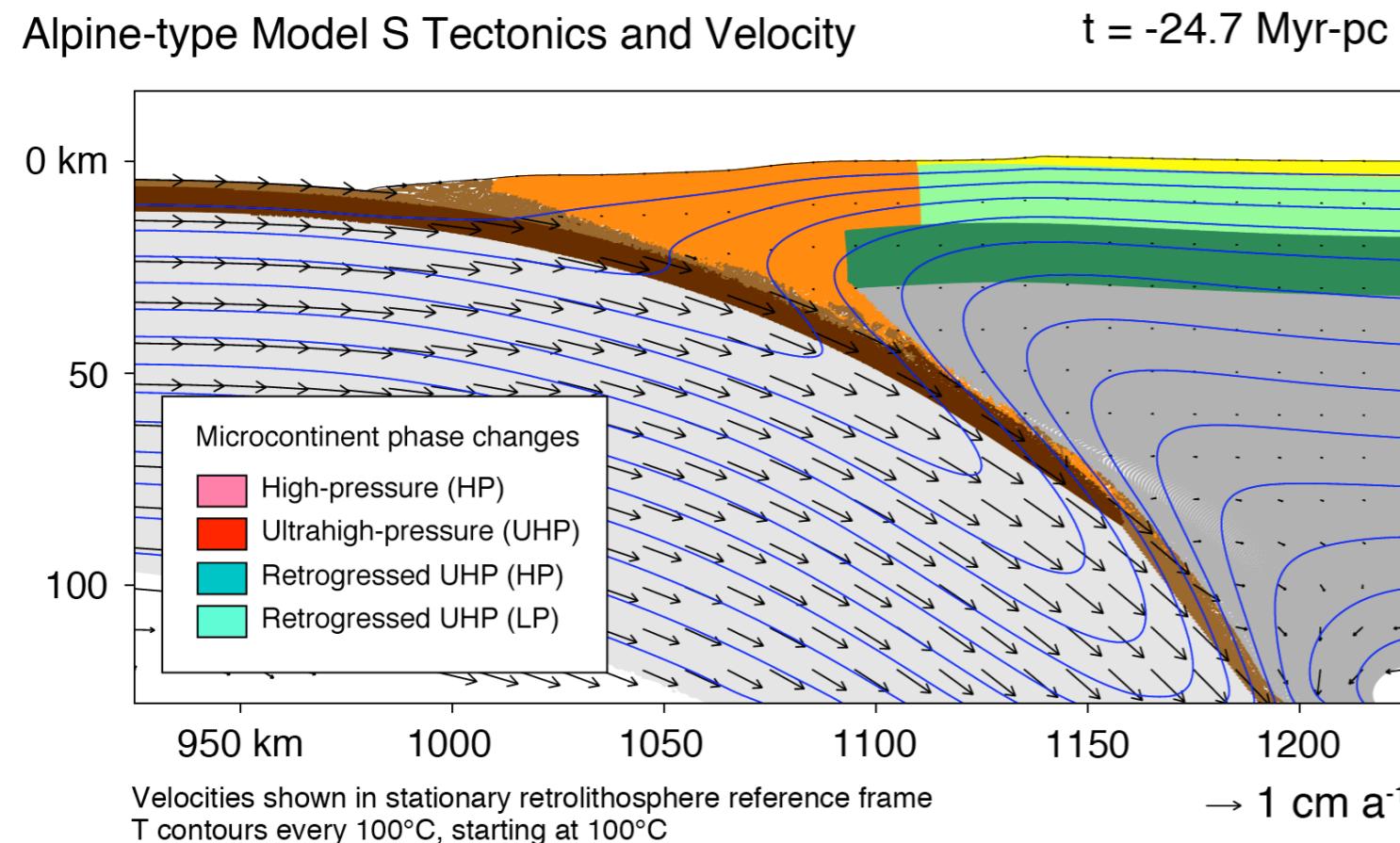
# Types of geological models



- **Thermo-kinematic (or thermokinematic) models** simulate both mass transport and heat transfer using a pre-defined velocity field and input rock thermal/physical properties
- Models of this type can be compared to a number of observables, including surface heat flow and mineral cooling ages, and typically have a geometry based on surface geological observations and geophysical data such as reflection seismics



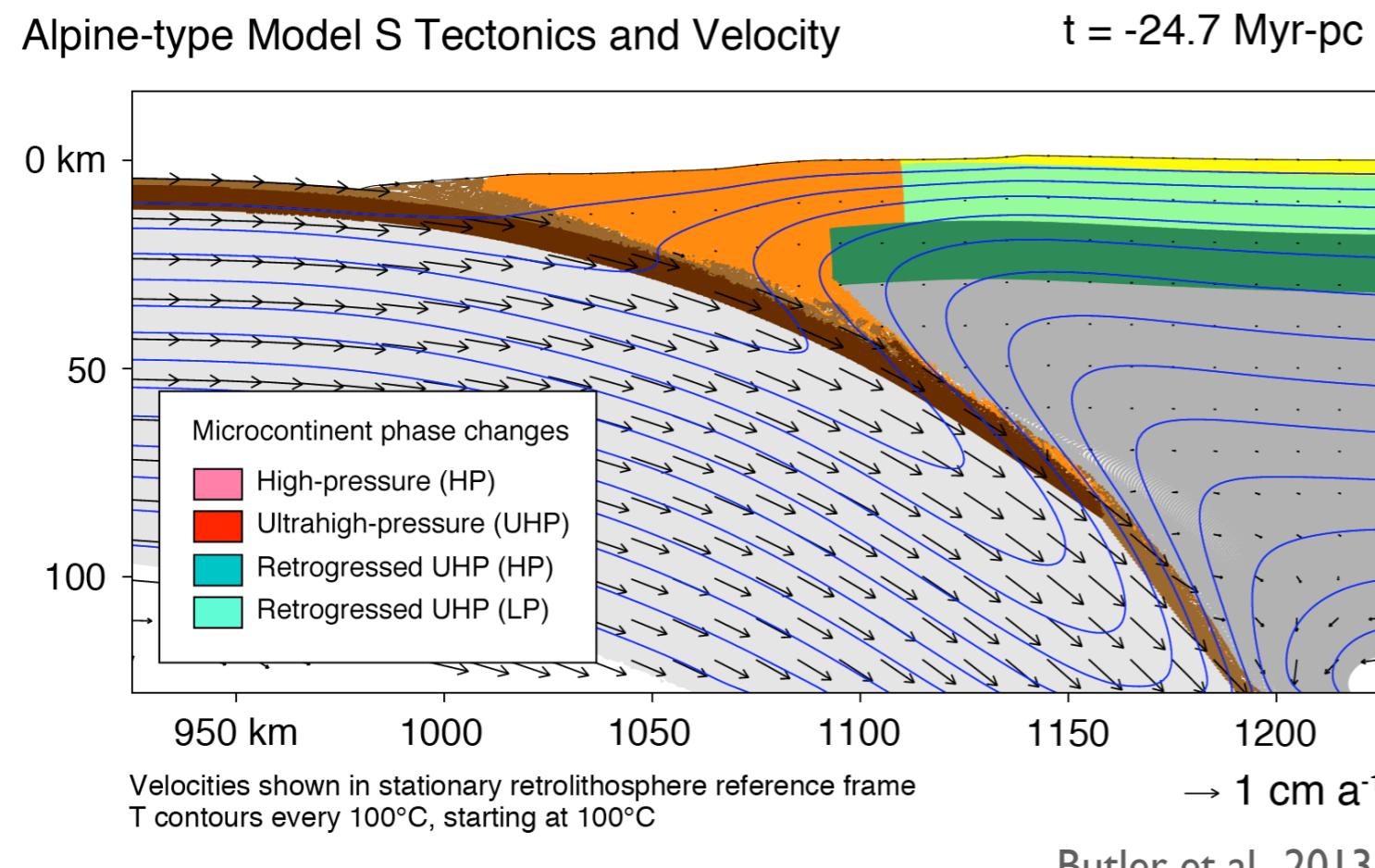
# Types of geological models



- **Thermo-mechanical models** simulate true lithospheric dynamics
  - Internal deformation in the model is determined based on physical forces acting on the model and material properties of rock in the model
  - Heat transfer will vary as a result of model deformation, but also affect the model material properties



# Types of geological models



- **Thermo-mechanical models** simulate true lithospheric dynamics
  - This type of model offers the greatest predictive power, but can be difficult to directly link to geological observations because the model evolution is not known *a priori*
  - This kind of model is the focus for the remainder of this presentation



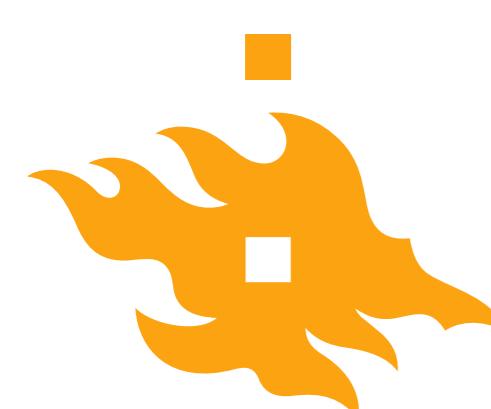
# Analogue versus numerical models



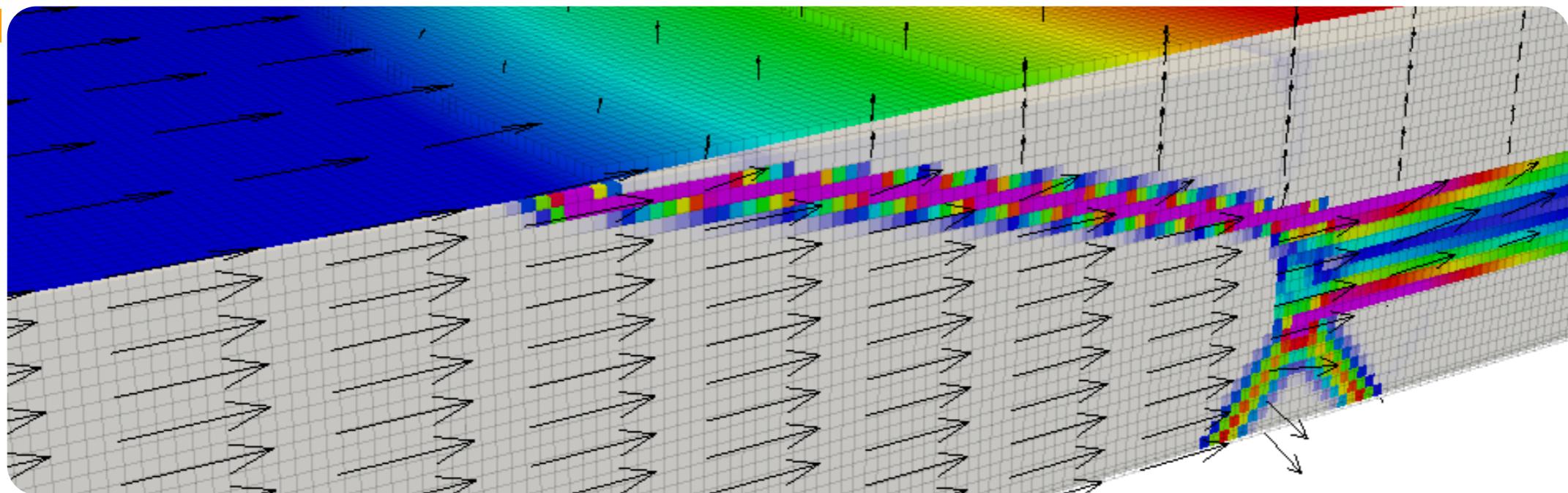
Tapponnier et al., 1982

- **Analogue models** are an alternative to thermomechanical models where materials analogous to Earth materials are used to simulate deformation of the Earth in physical models
- These models do not prescribe any material behavior, but rather allow the material to deform subject to imposed deformation at the boundaries
- Though these are a viable alternative to numerical models, it is difficult to simulate temperature-dependent materials and scaling of the model properties can be a problem

# Physical model concepts: Earth as a continuum



# What does that even mean?



*Velocities and strain rates in a lithospheric geodynamic model*

- Most geodynamic models treat the Earth as a **continuum** such that there are no material gaps or voids at the macroscopic scale
  - Field variables such as pressure, velocity, or stress are thus fully continuous
  - In this context the Earth is a fluid with a very high viscosity (typically  $10^{18}$  -  $10^{23}$  Pa s)



# Earth as a fluid? Even the lithosphere?

- **Fluid:** Any material that flows in response to an applied stress
- Differences between **solids** and **fluids**

Solids	Fluids
Strain from being stressed	<b>Continuous</b> deformation under applied forces
Stresses related to strains	Stresses related to <b>rates of strain</b>
Strain result of displacement gradients	Strain result of <b>velocity gradients</b>

- **Rheological** (or **constitutive**) **law:** An equation relating stress to strain rates in a fluid



# Fluid mechanics

- **Fluid mechanics** is the science of fluid motion



# Fluid mechanics

- **Fluid mechanics** is the science of fluid motion
- Based on conservation of three basic physical property and their corresponding mathematical representations
  - **Conservation of mass** - The continuity equation
  - **Conservation of momentum** - The momentum equation
  - **Conservation of energy** - The heat transfer equation



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  - **Conservation of mass** - The continuity equation
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  - **Conservation of energy** - The heat transfer equation
- Conservation of mass, momentum and energy are combined with rheological laws to describe fluid movement under an applied force

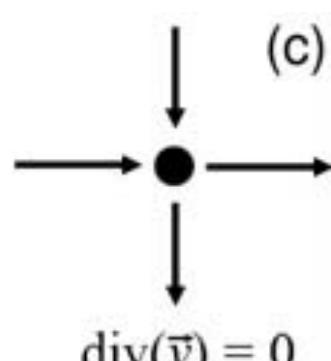
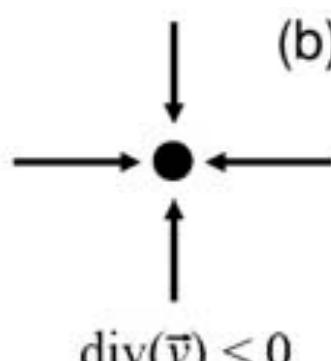
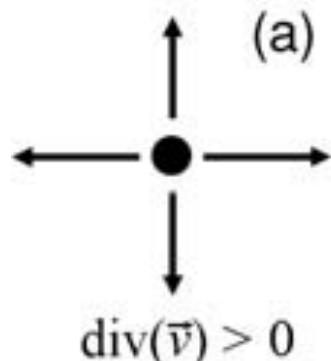


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- Conservation of mass, momentum and energy are combined with rheological laws to describe fluid movement under an applied force



# The continuity equation



Gerya, 2010

- Calculations in the continuum are performed by considering an infinitesimal volume of the material, the local volume
- The general form of **conservation of mass** for a local volume of a continuum in an Eulerian reference frame is

$$\frac{\partial \rho}{\partial t} + \rho (\nabla \cdot \mathbf{V}) = 0$$

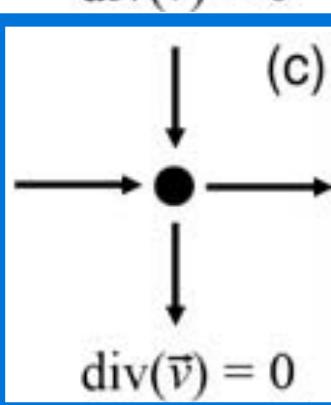
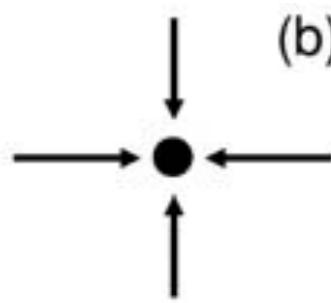
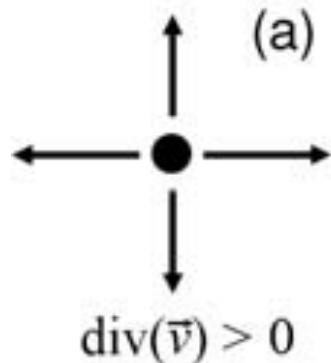
Change in local density      Mass or volume flux  
(divergence of velocity)

where  $\rho$  is the local density,  $t$  is time and  $\mathbf{V}$  is the local velocity

$$\frac{\partial \rho}{\partial t} = -\rho (\nabla \cdot \mathbf{V}) \quad \text{Alternative form}$$



# The continuity equation



- It is common in geodynamic numerical models, particularly in the crust or lithosphere, to assume the material is incompressible
- In this case, the **continuity equation** simplifies to
$$\nabla \cdot V = 0$$
stating simply that there is no divergence in the velocity field of the continuum
- In many numerical models, this condition is not strictly obeyed, allowing a very small amount of compressibility in the materials

Gerya, 2010



# What drives a fluid to flow?



Sir George Stokes

- At this point, we have established that geodynamicists often model the Earth as a continuous, highly viscous fluid. Since we're interested in the dynamics of this fluid, a reasonable question to ask is **what forces might cause a fluid to flow?**



# The momentum equation



Sir George Stokes

- The basic relationship that thus determines the dynamics of material in the continuum is conservation of momentum, the balance of internal and external forces acting on the material
- The **conservation of momentum** for a fluid subject to gravity is the Navier-Stokes equation

$$\nabla \cdot \eta(\nabla V + \nabla V^T) - \nabla P - \rho g = \rho \dot{V}$$

↑                      ↑                      ↑                      ↑  
Fluid velocity   Fluid pressure   Body forces   Acceleration

where  $\eta$  is the fluid shear viscosity,  $P$  is pressure,  $g$  is the acceleration due to gravity, and  $\dot{V}$  is the material time derivative of the fluid velocity (acceleration)



# The momentum equation



Sir George Stokes

- For highly viscous fluids with a very small Reynolds number the acceleration term of the Navier-Stokes equation can be ignored reducing to the equation of **Stokes flow** (and simplifying the solutions)

$$\nabla \cdot \eta(\nabla V + \nabla V^T) - \nabla P = \rho g$$

↑                      ↑                      ↑  
Fluid velocity    Fluid pressure    Body forces

- It is trivial to demonstrate that the Reynolds number of most geodynamic flows is extremely low

$$Re = \frac{\rho V L}{\eta} \quad \frac{\text{Inertial forces}}{\text{Viscous forces}}$$

The Reynolds number



# Pre-break exercise

$$\text{Re} = \frac{\rho V L}{\eta}$$

- The Reynolds number determines whether or not a fluid flow should be expected to be turbulent or laminar
  - The critical value of the Reynolds number is  $\sim 2000$ , above which flow is turbulent

The Reynolds number

- Using your best guess at the equation values, **estimate the Reynolds number for convection of the upper mantle**
- **Do we need to worry about turbulence?**

# Physical model concepts: Stress and strain



# Forces

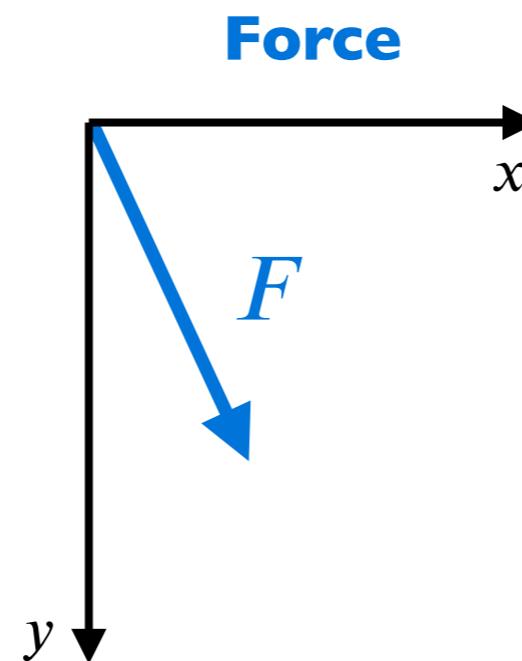
- **Force:** A push or pull applied to a body.  
Force = mass × acceleration (Newton's second law)
- **Units:** Newtons [N];  $1\text{ N} = 1\text{ kg m s}^{-2}$

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# Forces

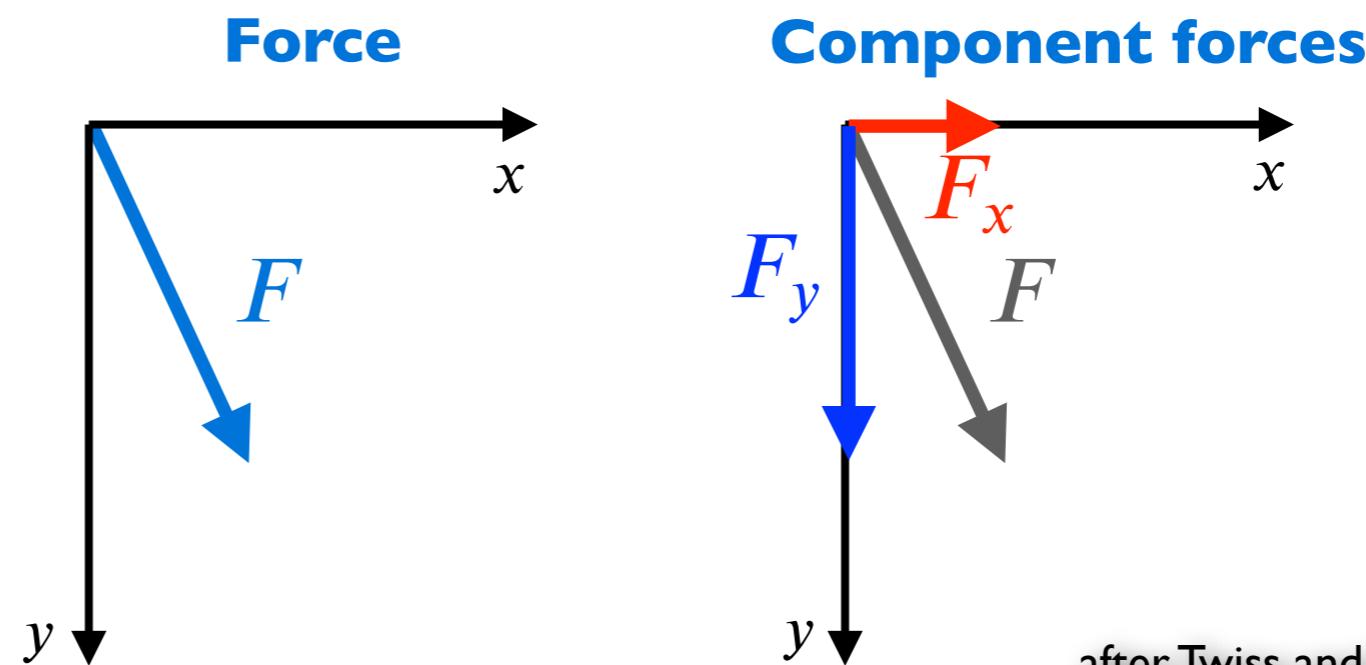
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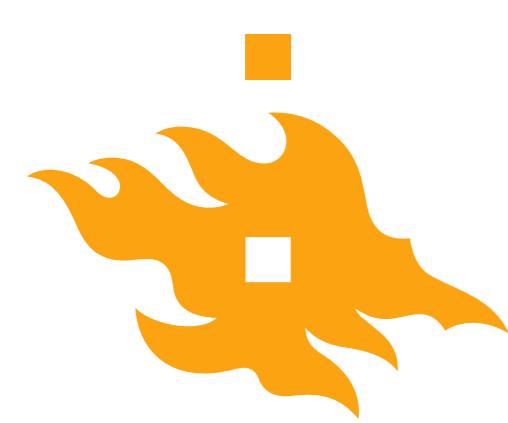
# Body forces versus surface forces

- **Body force:** Forces that act throughout the volume of a solid. Proportional to its volume or mass.
- Example: Slab pull (gravity)
- **Surface force:** Forces that act on the surface area bounding an element or volume. Proportional to the area upon which the force acts.
- Example: Friction along a fault plane



# Surface stresses

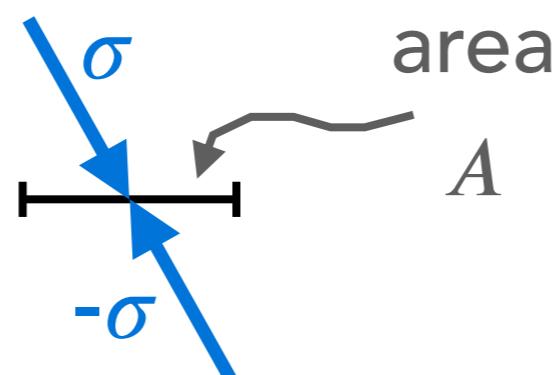
- **Stress:** A force per unit area transmitted through a material by interatomic force fields
- **Surface stress:** A pair of equal and opposite forces acting on the area of a surface in a specific orientation

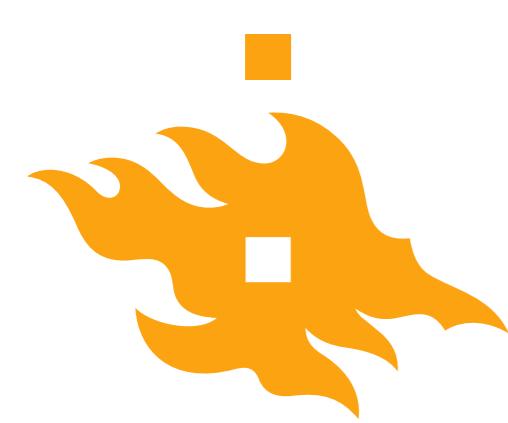


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- **Representation:** Pair of vectors with a specified surface area/orientation
- **Example:** Hand pushing on table, table pushing back

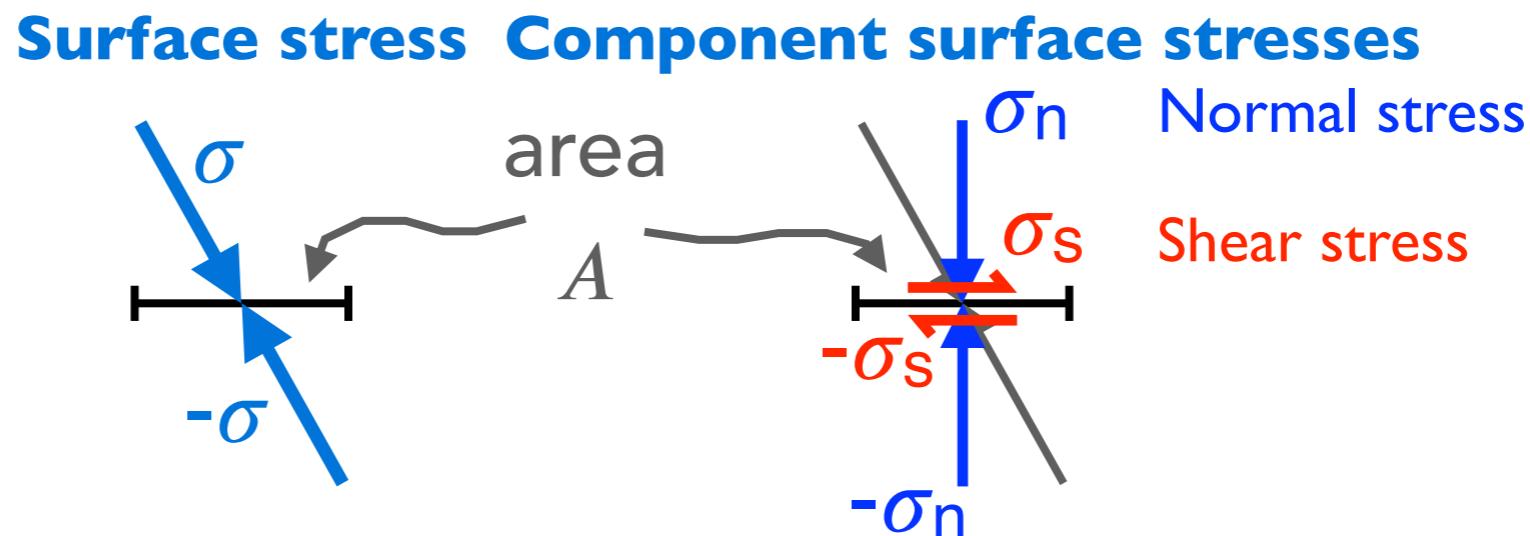
## Surface stress





# Surface stresses

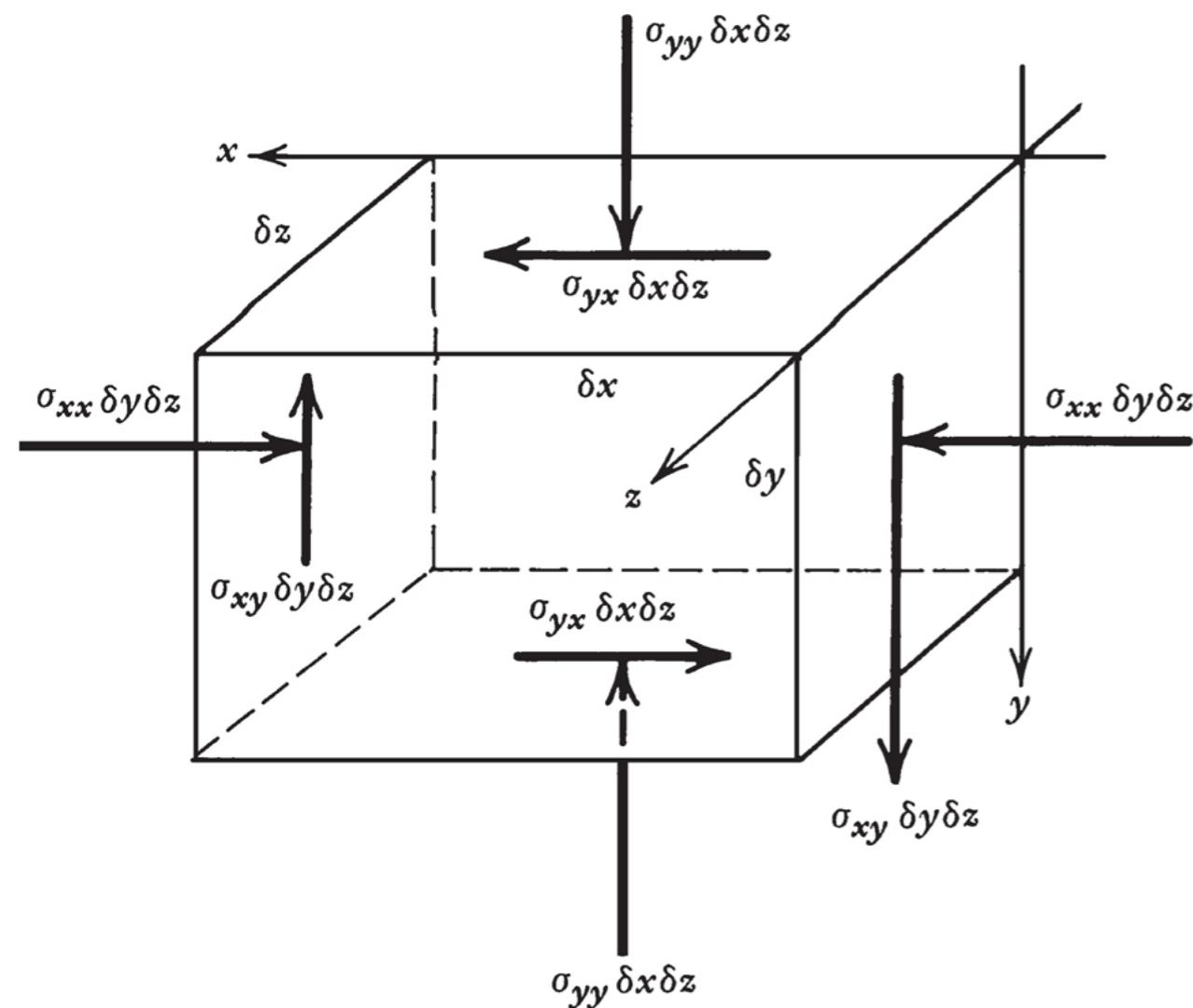
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# Stress in two dimensions

## Surface forces in 2D



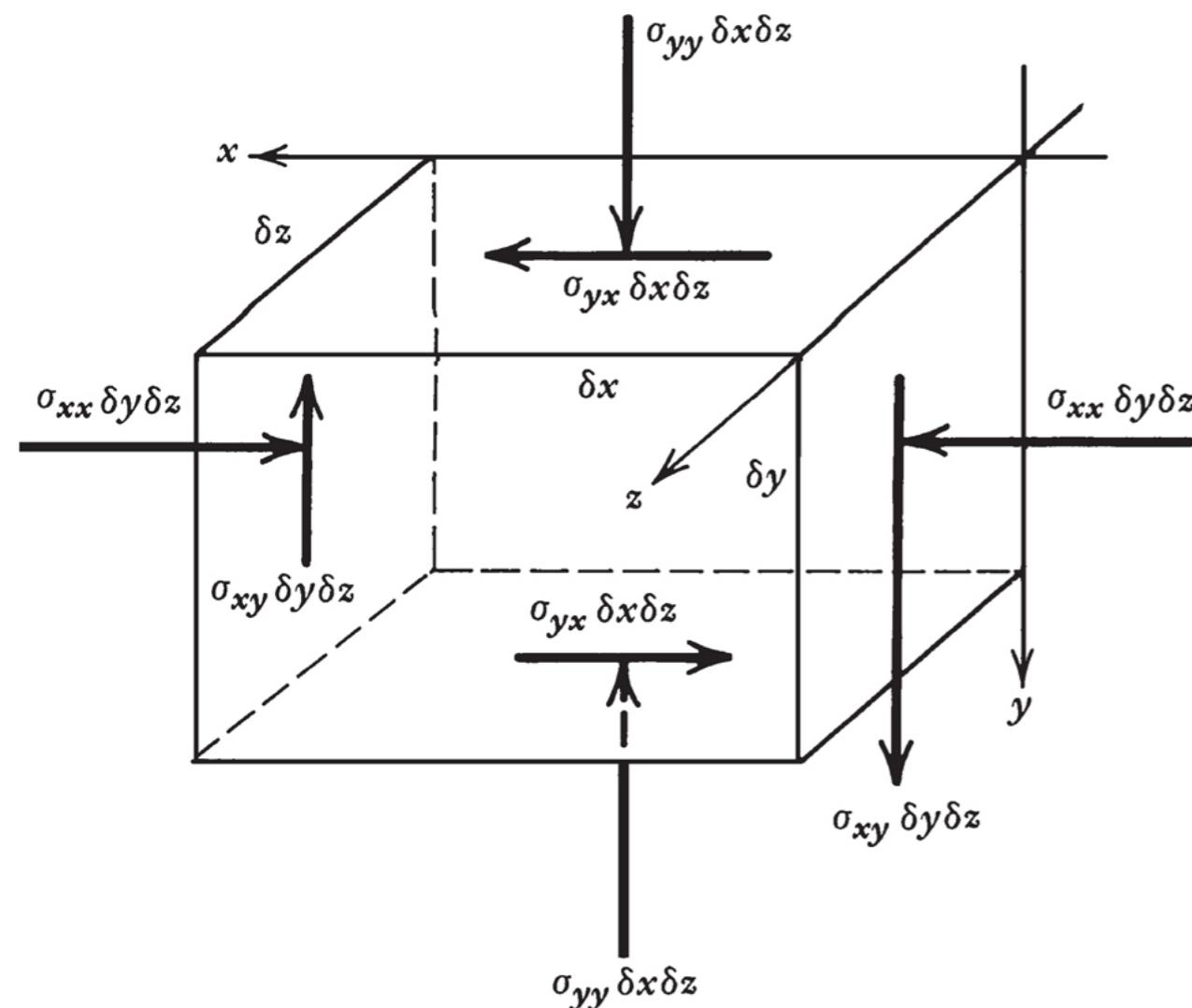
- In two dimensions, we consider forces acting on four faces of an infinitesimal cube of dimension  $\delta x \times \delta y \times \delta z$
- Here we assume no forces act or vary in the  $z$  direction

Fig. 2.13, Turcotte and Schubert, 2014



# Stress in two dimensions

## Surface forces in 2D



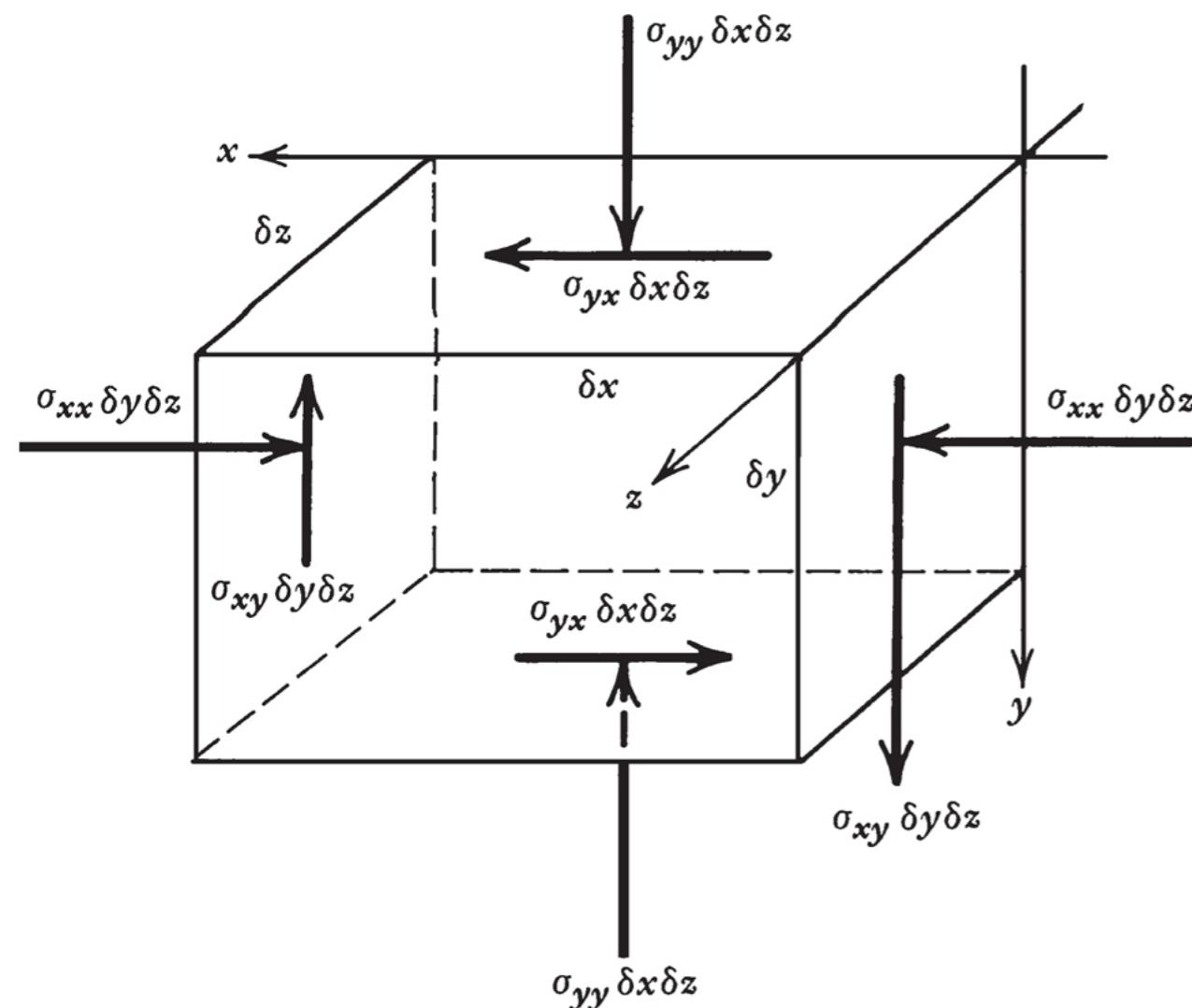
- In two dimensions, we consider forces acting on four faces of an infinitesimal cube of dimension  $\delta x \times \delta y \times \delta z$
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- Normal stresses:  $\sigma_{xx}$ ,  $\sigma_{yy}$
- Shear stresses:  $\sigma_{xy}$ ,  $\sigma_{yx}$
- At equilibrium we can state  $\sigma_{xy} = \sigma_{yx}$

Fig. 2.13, Turcotte and Schubert, 2014



# Stress in two dimensions

## Surface forces in 2D

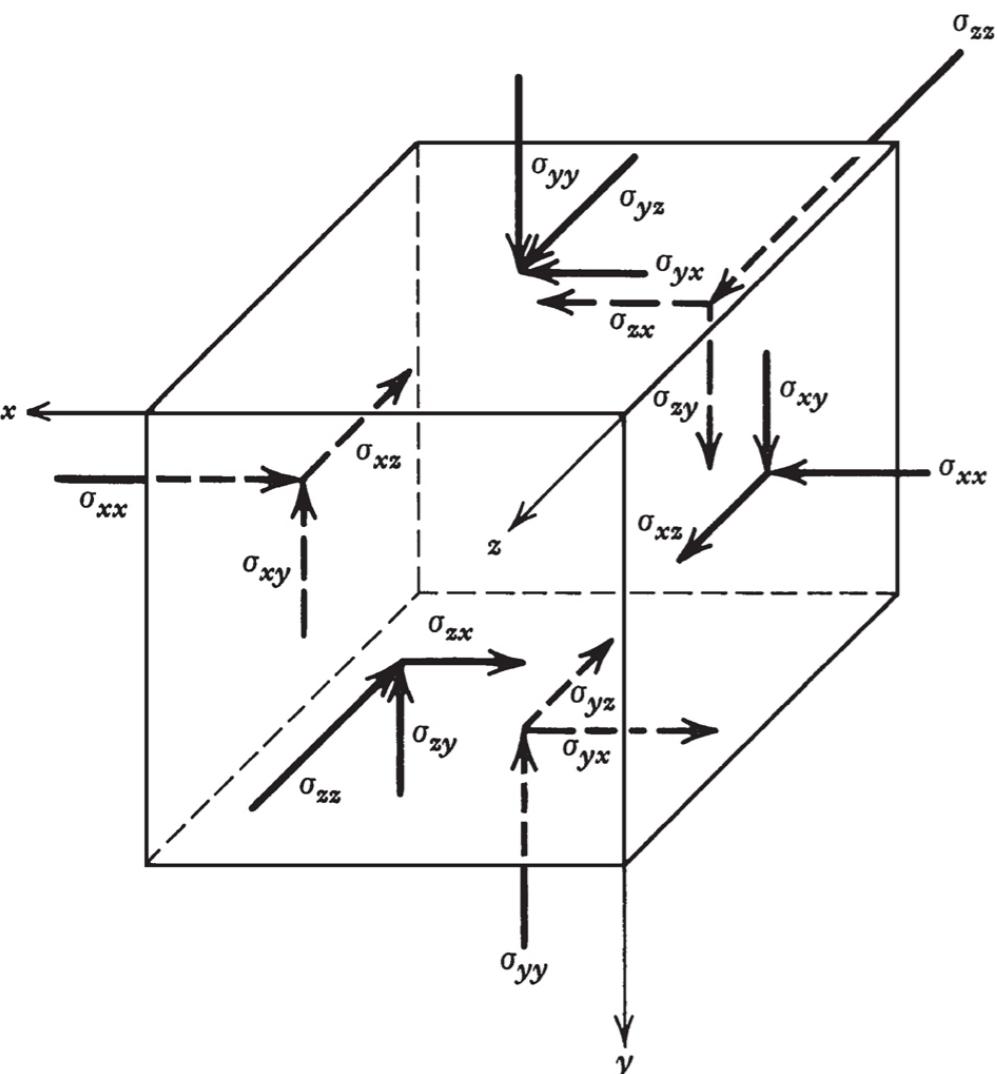


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- Why?

Fig. 2.13, Turcotte and Schubert, 2014



# Stress in three dimensions

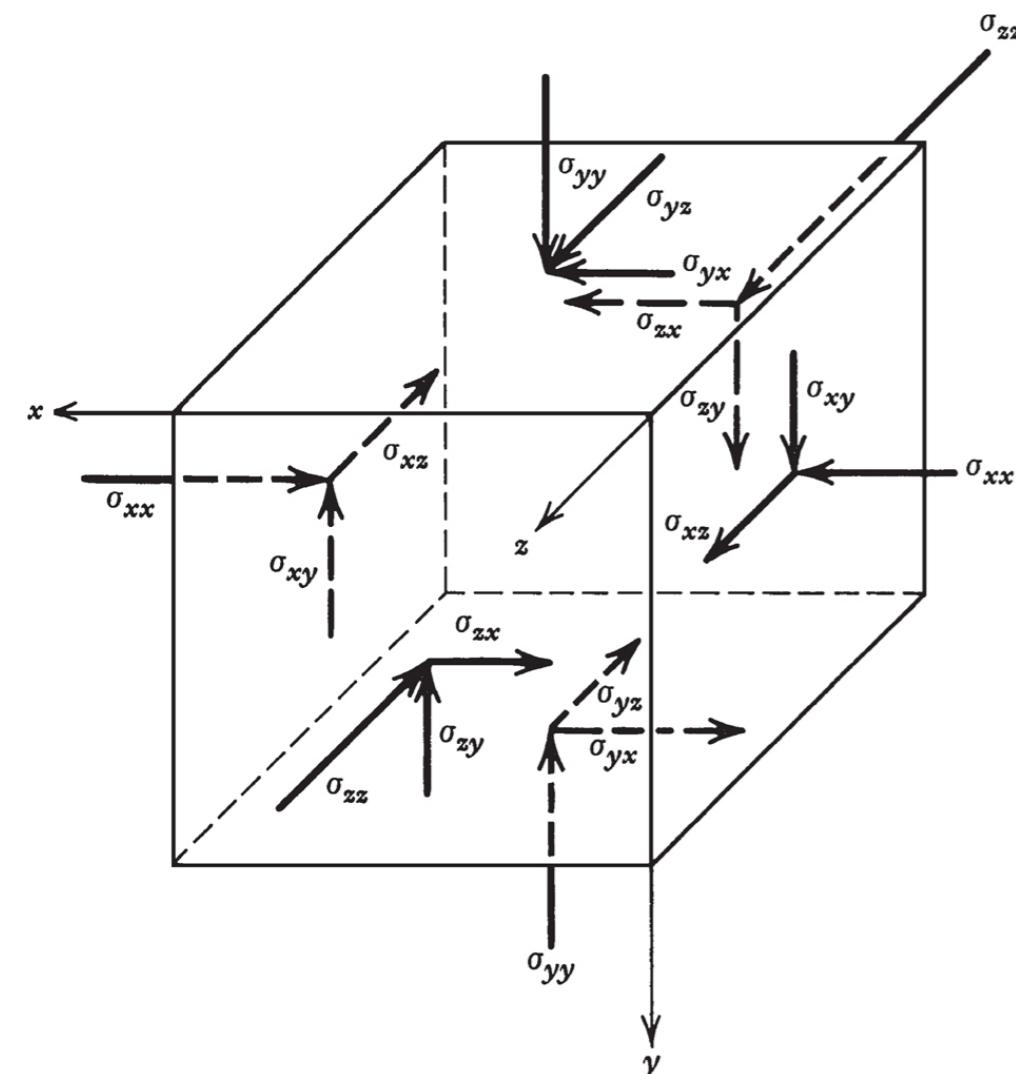


- In three dimensions, we consider forces acting on all six faces of an infinitesimal cube of dimension  $\delta x \times \delta y \times \delta z$
- Normal stresses:  $\sigma_{xx}$ ,  $\sigma_{yy}$ ,  $\sigma_{zz}$
- Shear stresses:  $\sigma_{xy}$ ,  $\sigma_{yx}$ ,  $\sigma_{xz}$ ,  $\sigma_{zx}$ ,  $\sigma_{yz}$ ,  $\sigma_{zy}$
- At equilibrium we can state  $\sigma_{xy} = \sigma_{yx}$ ,  $\sigma_{xz} = \sigma_{zx}$ ,  $\sigma_{yz} = \sigma_{zy}$

Fig. 2.15, Turcotte and Schubert, 2014



# Stress in three dimensions



- A few useful stress values
- **Pressure** (mean stress)

$$p = \frac{1}{3} (\sigma_{xx} + \sigma_{yy} + \sigma_{zz})$$

- **Deviatoric stress** (indicated by primes)

$$\sigma'_{xx} = \sigma_{xx} - p$$

$$\sigma'_{xy} = \sigma_{xy}$$

$$\sigma'_{yy} = \sigma_{yy} - p$$

$$\sigma'_{xz} = \sigma_{xz}$$

$$\sigma'_{zz} = \sigma_{zz} - p$$

$$\sigma'_{yz} = \sigma_{yz}$$

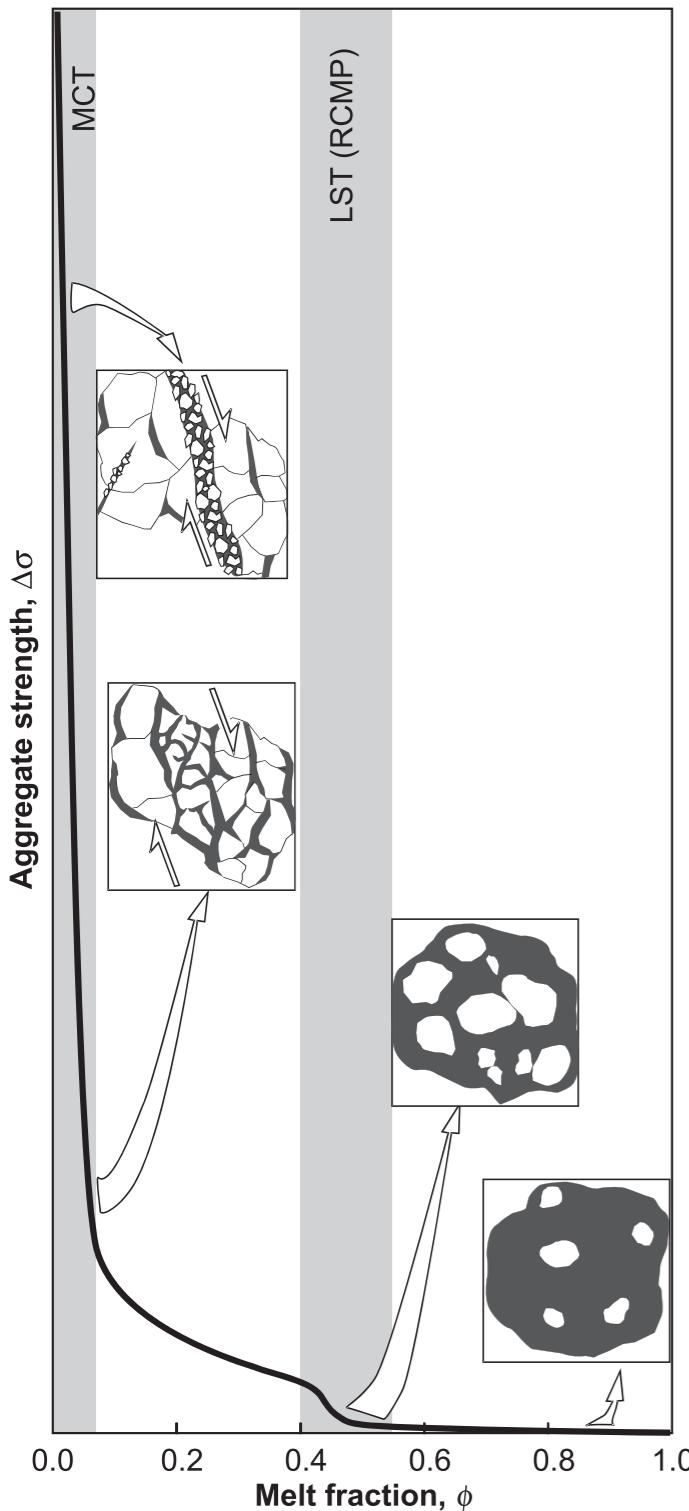
Fig. 2.15, Turcotte and Schubert, 2014

# Physical model concepts: Heat transfer



# Why does temperature matter?

Rosenberg et al., 2007



- Rock deformation strongly depends upon temperature
- Rock strength drops 80-90% for even small amounts of partial melt (5-7%)
- Whether rocks are brittle and fault, or ductile and fold is largely determined by temperature



# Heat transfer processes in the lithosphere

- **Conduction**
- **Production**
- **Advection**



# Heat transfer processes in the lithosphere

- **Conduction:** The diffusive transfer of heat by kinetic atomic or molecular interactions within the material. Also known as thermal diffusion.
- **Production**
- **Advection**



# Fourier's first law of heat conduction

- In 1D, the mathematical translation of “Heat flux  $q$  is *directly* proportional to the thermal gradient in a material” is

$$q_z = -k \frac{dT}{dz}$$

- Here,  $T$  represents **temperature** and  $z$  represents spatial position, **depth** in the Earth for our example
- Thus,  $dT/dz$  is the change in temperature with depth, or the **thermal gradient**
- The proportionality constant  $k$  is known as the **thermal conductivity**



# Fourier's first law of heat conduction

- In 1D, the mathematical translation of “Heat flux  $q$  is *directly* proportional to the thermal gradient in a material” is

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- **Why is there a negative sign?**



# Fourier's first law of heat conduction

- In 1D, the mathematical translation of “Heat flux  $q$  is *directly* proportional to the thermal gradient in a material” is

$$q_z = -k \frac{dT}{dz}$$

- **Why is there a negative sign?**
- In general, we can state Fourier's first law of heat conduction as

$$q = -k \nabla T$$



# Heat transfer processes in the lithosphere

- **Conduction:** The diffusive transfer of heat by kinetic atomic or molecular interactions within the material. Also known as thermal diffusion.
- **Production:** Not really a heat transfer process, but rather a source of heat. Sources in the lithosphere include radioactive decay, friction in deforming rock or chemical reactions such as phase transitions.
- **Advection**



# Radiogenic heat production

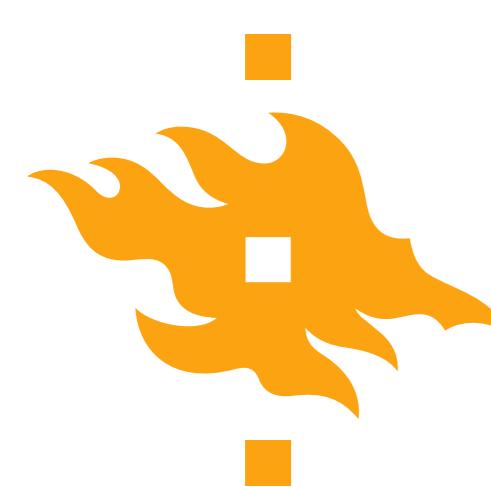
- **Radiogenic heat production**,  $A$  or  $H$ , is one of several heat sources and results from the decay of radioactive isotopes in the Earth, mainly  $^{238}\text{U}$ ,  $^{235}\text{U}$ ,  $^{232}\text{Th}$  and  $^{40}\text{K}$ .  $A$  is often used for volumetric heat production and  $H$  for heat production by mass.
- These elements occur in the mantle, but are concentrated in the crust, where radiogenic heating can be significant
- The surface heat flow in continental regions is  $\sim 65 \text{ mW m}^{-2}$  and  $\sim 37 \text{ mW m}^{-2}$  is from radiogenic heat production (57%)

Rock Type	U (ppm)	Th (ppm)	Concentration K (%)
Reference undepleted (fertile) mantle	0.031	0.124	0.031
“Depleted” peridotites	0.001	0.004	0.003
Tholeiitic basalt	0.07	0.19	0.088
Granite	4.7	20	4.2
Shale	3.7	12	2.7
Average continental crust	1.42	5.6	1.43
Chondritic meteorites	0.008	0.029	0.056



# Heat transfer processes in the lithosphere

- **Conduction:** The diffusive transfer of heat by kinetic atomic or molecular interactions within the material. Also known as thermal diffusion.
- **Production:** Not really a heat transfer process, but rather a source of heat. Sources in the lithosphere include radioactive decay, friction in deforming rock or chemical reactions such as phase transitions.
- **Advection:** The transfer of heat by physical movement of molecules or atoms within a material. A type of convection, mostly applied to heat transfer in solid materials.



# Mathematical description of advection

## Time-dependent advection and diffusion

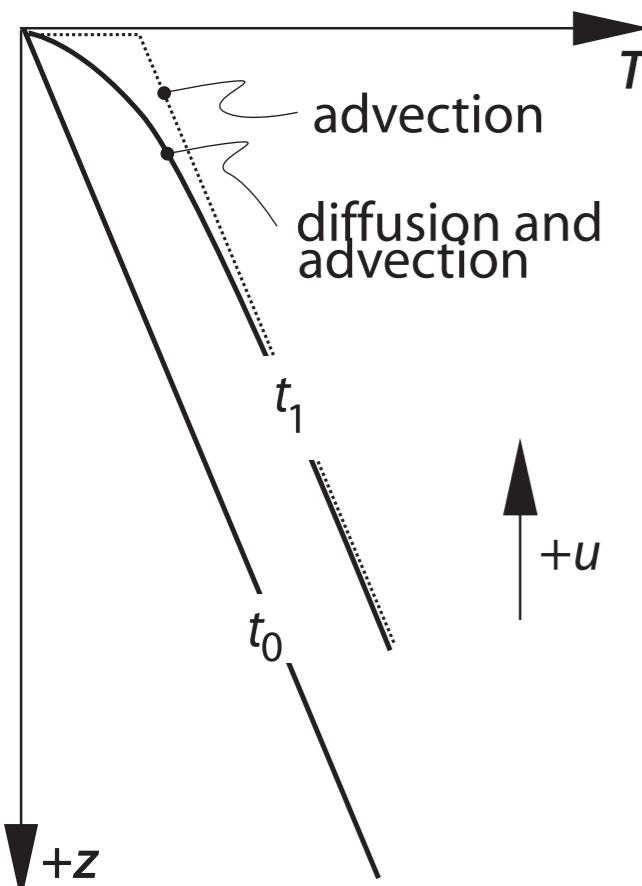


Fig. 3.13, Stüwe, 2007

- Advection in the vertical direction at velocity  $v_z$  at steady state can be represented mathematically as
$$v_z \frac{dT}{dz} = 0$$
- Note that this equation simply describes the vertical translation of temperatures, and that in order for any change in temperature to occur, advection must be combined with other heat transfer processes such as conduction
- In general, we can describe heat advection as
$$\mathbf{V} \cdot \nabla T = 0$$



# The heat conservation equation

- We now combine our three heat transfer components (**conduction**, **production**, **advection**) into the heat conservation equation, which describes heat transfer subject to each of these processes
- In one dimension (vertical), this equation is

$$\rho c_P \left( \frac{\partial T}{\partial t} + v_z \frac{\partial T}{\partial z} \right) = -\frac{\partial q_z}{\partial z} + A$$

Time dependence      Advection      Conduction      Production

- Alternatively, we can state the same equation in substituting in Fourier's first law for the heat flux  $q$

$$\rho c_P \left( \frac{\partial T}{\partial t} + v_z \frac{\partial T}{\partial z} \right) = \frac{\partial}{\partial z} \left( k \frac{\partial T}{\partial z} \right) + A$$



# The heat conservation equation

- We now combine our three heat transfer components (**conduction**, **production**, **advection**) into the heat conservation equation, which describes heat transfer subject to each of these processes
- In general, we can state

$$\rho c_P \left( \frac{\partial T}{\partial t} + \mathbf{V} \cdot \nabla T \right) = \nabla \cdot k \nabla T + A$$

Time dependence      Advection      Conduction      Production

# Physical model concepts: Rheological laws



# Rheology of the lithosphere

- The term **rheology** refers to the flow characteristics of materials
  - For most geoscientists this term describes the deformation behavior of materials regardless of whether deformation occurs by flow, fracture, or other mechanisms
- Rock deformation mainly occurs by three **deformation mechanisms**:
  - Elasticity
  - Plasticity
  - Viscous flow



# Rheology of the lithosphere

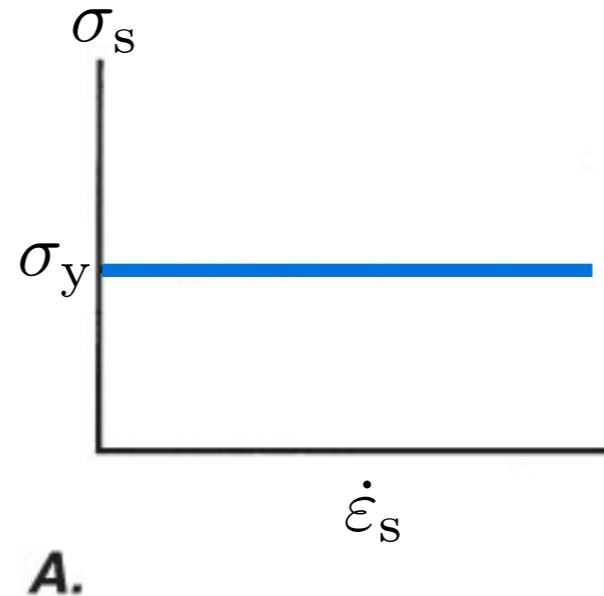
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  - For most geoscientists this term describes the deformation behavior of materials regardless of whether deformation occurs by flow, fracture, or other mechanisms
- Rock deformation mainly occurs by three **deformation mechanisms**:
  - Elasticity - Can ignore this, not relevant for long time scales
  - Plasticity
  - Viscous flow



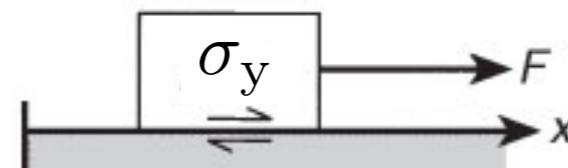
# Perfectly plastic behavior

Twiss and Moores, 2007

- **Constant stress** required for deformation
    - No deformation prior to exceeding yield stress
    - Infinite deformation if applied stress equals (or exceeds) yield stress
  - Nonrecoverable
- $\begin{cases} \sigma < \sigma_y & \text{no deformation} \\ \sigma = \sigma_y & \text{failure; infinite deformation} \end{cases}$



**A.**



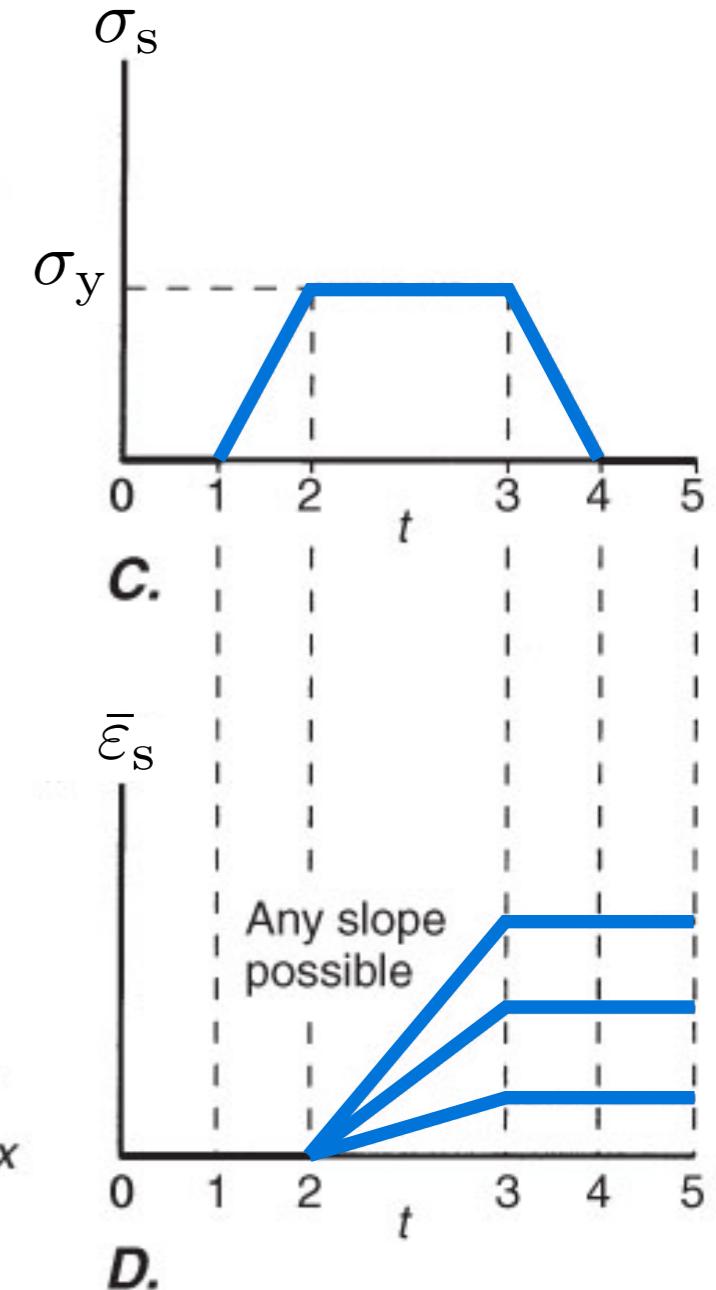
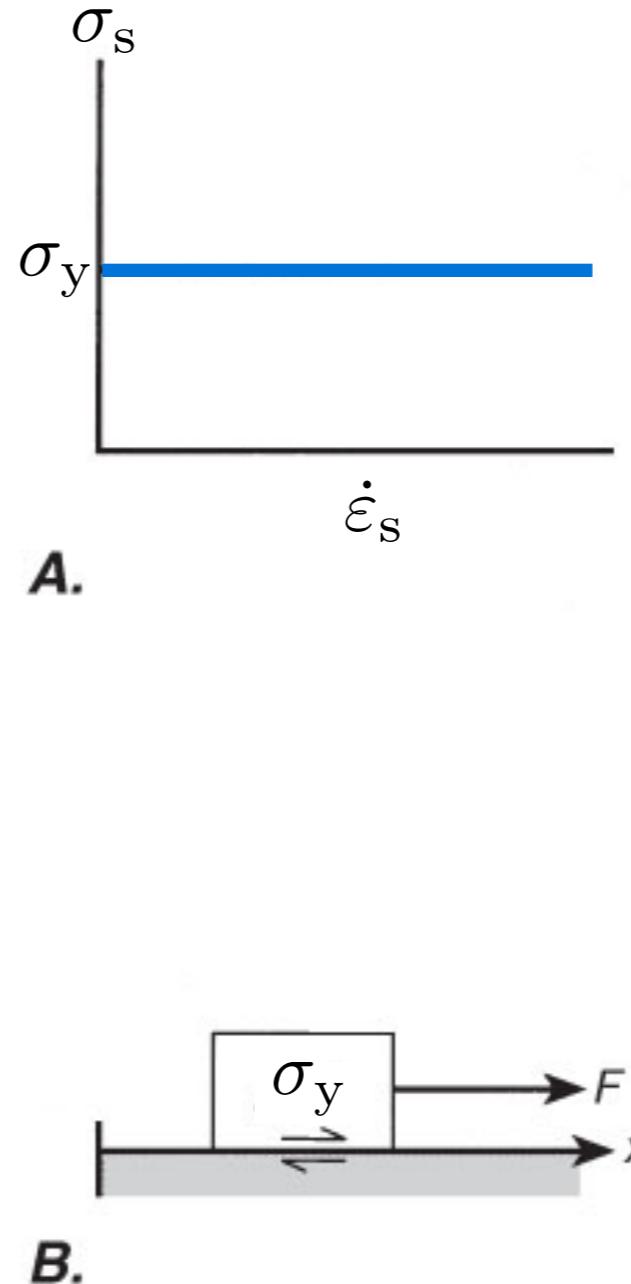
**B.**



# Perfectly plastic behavior

Twiss and Moores, 2007

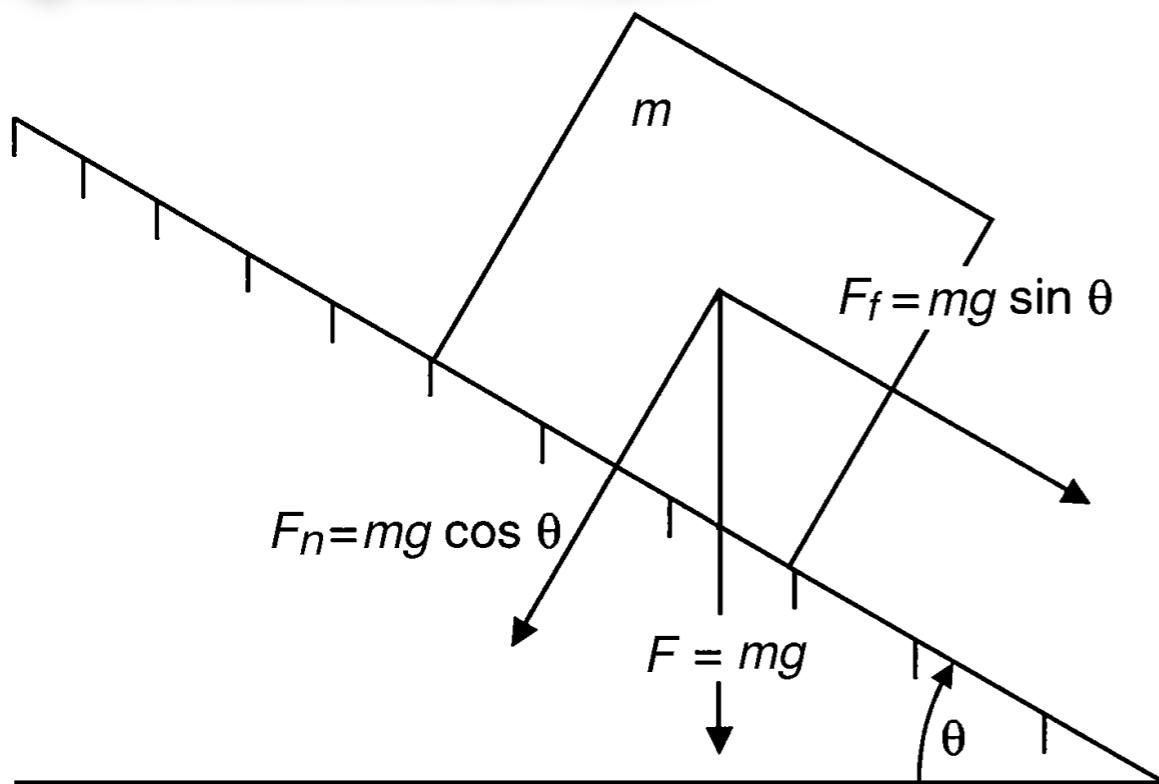
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- Nonrecoverable





# Frictional plasticity

Fig. 8.5, Turcotte and Schubert, 2014



Normal stress

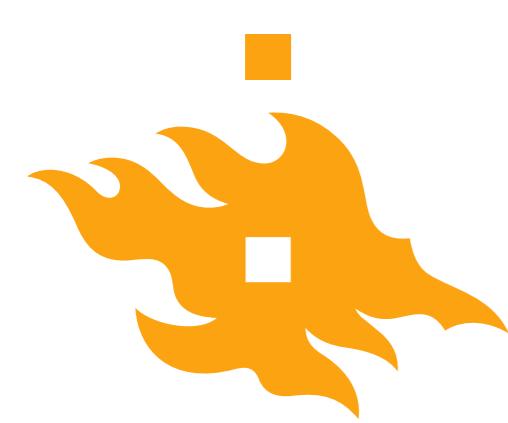
$$\sigma_n = \frac{mg \cos \theta}{A}$$

Shear stress

$$\sigma_s = \frac{mg \sin \theta}{A}$$

- Fault slip accounts for a large portion of deformation of the upper crust
- Friction must be overcome for slip to occur
  - After exceeding the frictional resistance, slip will occur on the fault or shear zone
  - Known as **frictional plasticity**
  - The basic relationship for static friction is  $\tau_{f_s} = f_s \sigma_n$  (Amonton's law)

where  $f_s$  is the **coefficient of static friction**, and  $\tau_{f_s}$  is the **static frictional stress** required for slip



# (Linear) Viscous deformation

- In simple shear,

$$\tau_s = \eta \dot{\gamma} \quad \eta \text{ Dynamic viscosity}$$

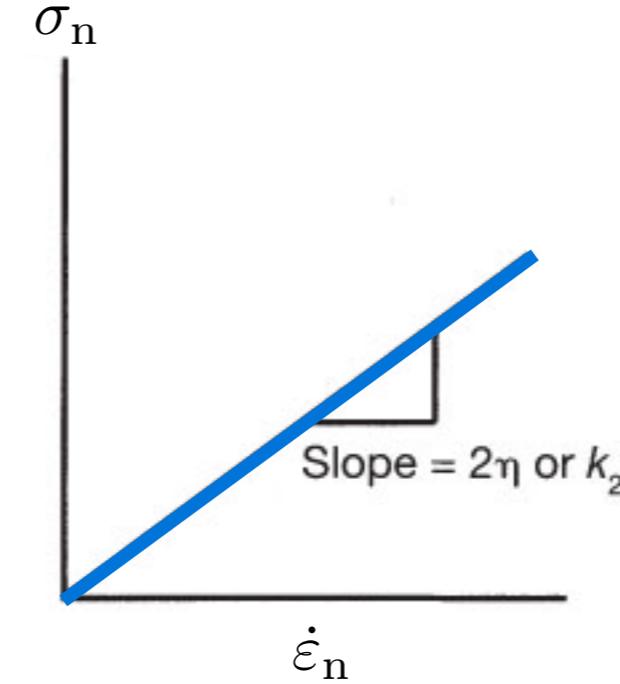
**Shear stress** proportional to  
**shear strain rate**

- In general,

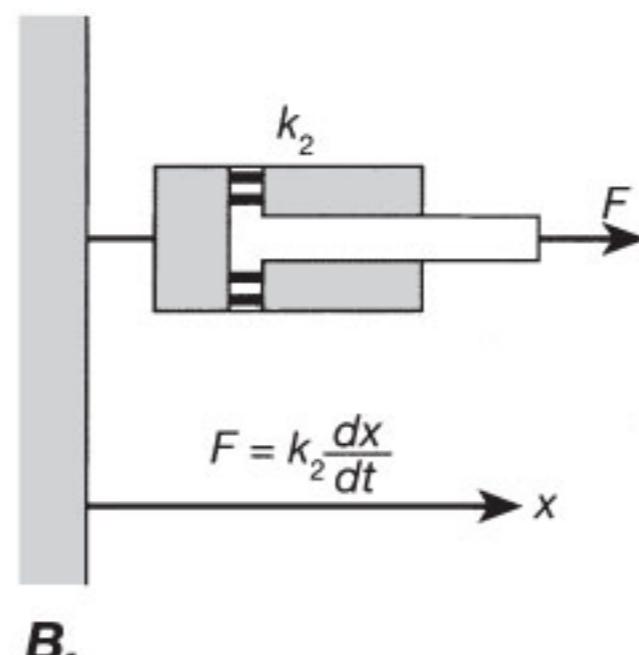
$$\tau = 2\eta \dot{\epsilon}$$

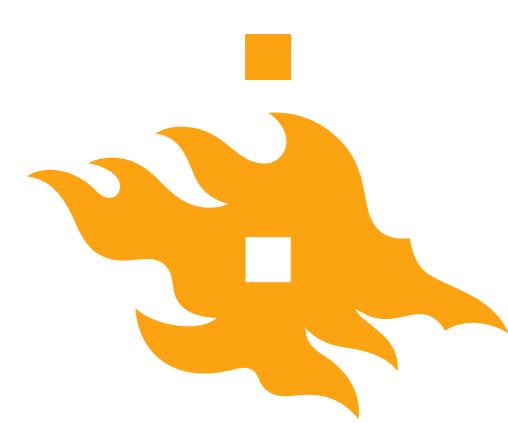
**deviatoric stress** is proportional  
to **strain rate**

- For linear viscous (Newtonian) materials,  $\eta$  is constant
- Nonrecoverable



A.





# (Linear) Viscous deformation

- In simple shear,

$$\tau_s = \eta \dot{\gamma} \quad \eta \text{ Dynamic viscosity}$$

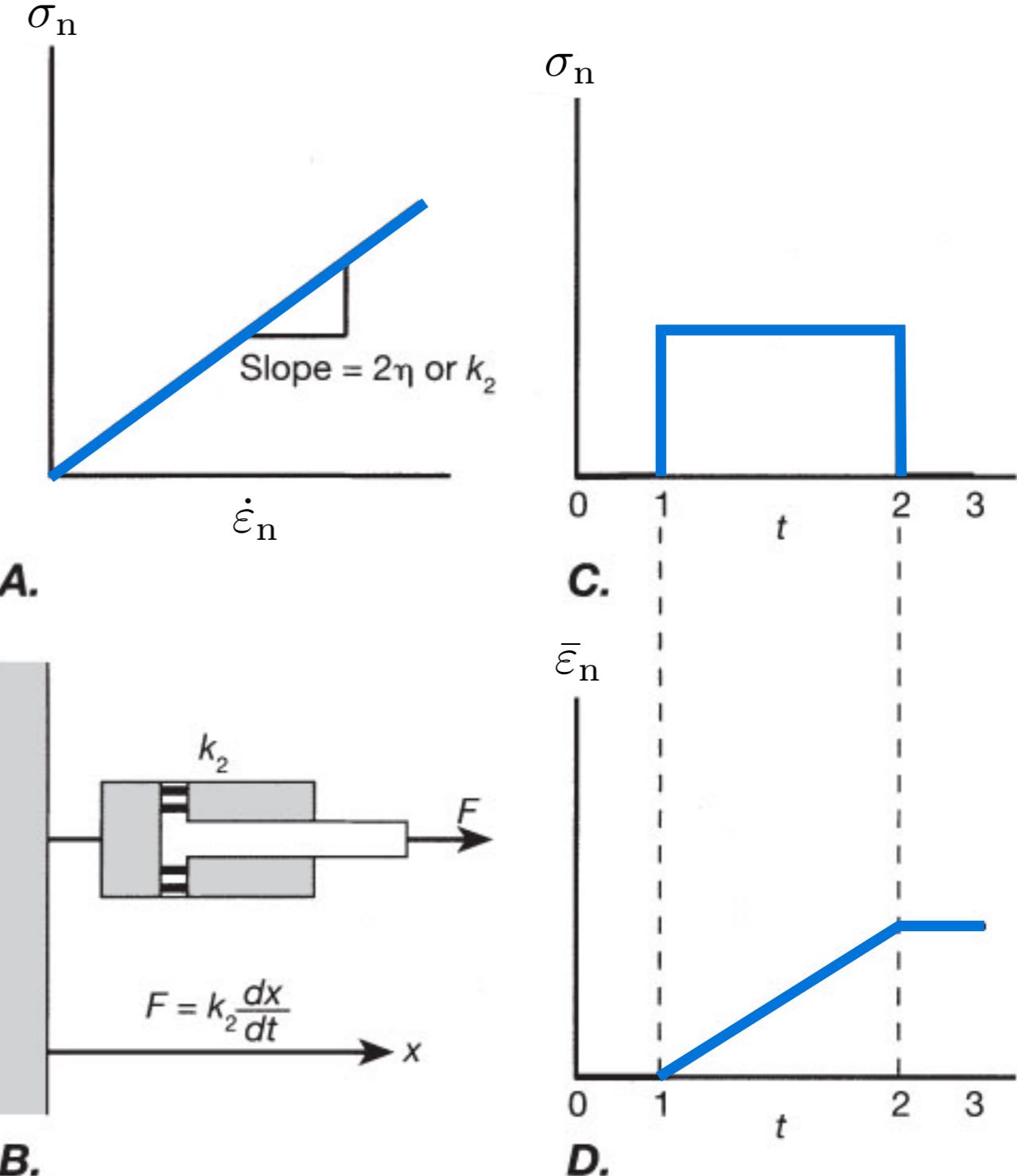
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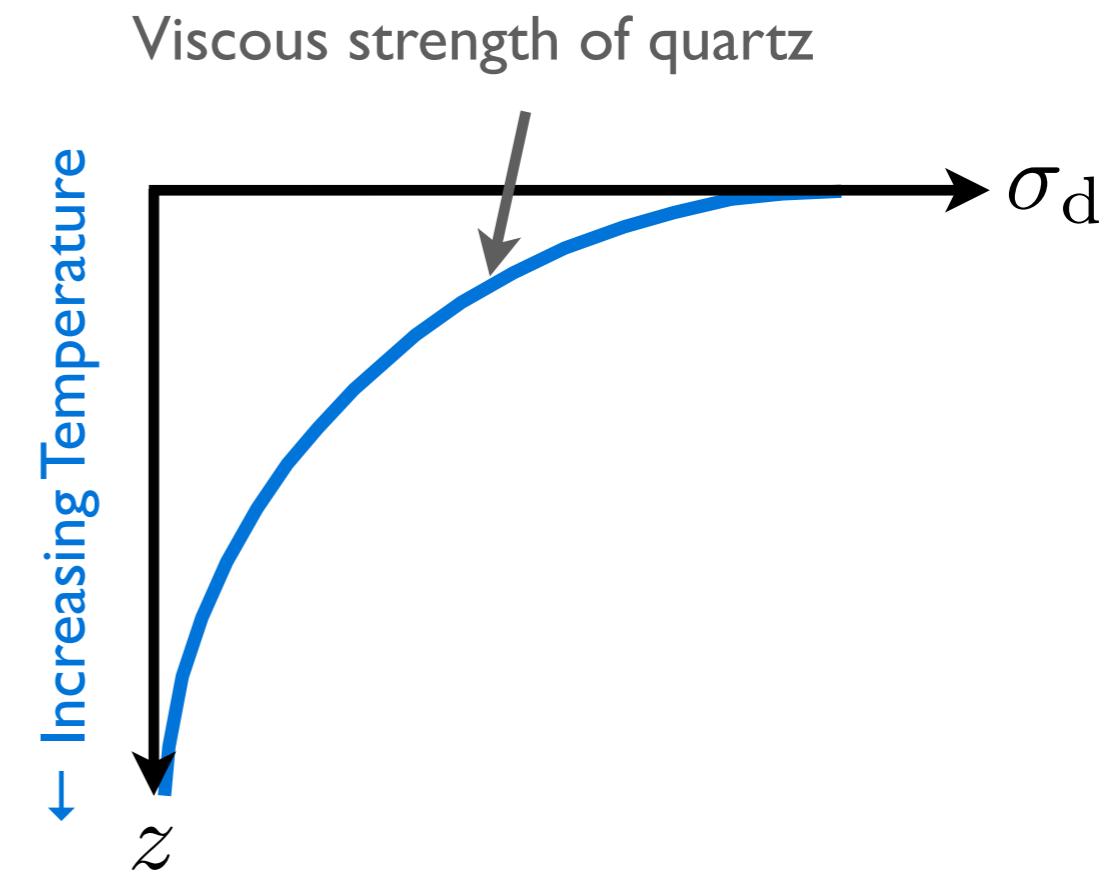


# Nonlinear viscous deformation

- Most rocks do not behave as **Newtonian viscous** materials
- Why not?
- Two main reasons:
  - **Temperature dependence**

$$\eta = A_0 e^{Q/RT_K}$$

$A_0$  is the pre-exponent constant,  $Q$  is the activation energy,  $R$  is the universal gas constant and  $T_K$  is temperature in Kelvins



Stüwe, 2007



# Nonlinear viscous deformation

- Most rocks do not behave as **Newtonian viscous** materials

- Why not?

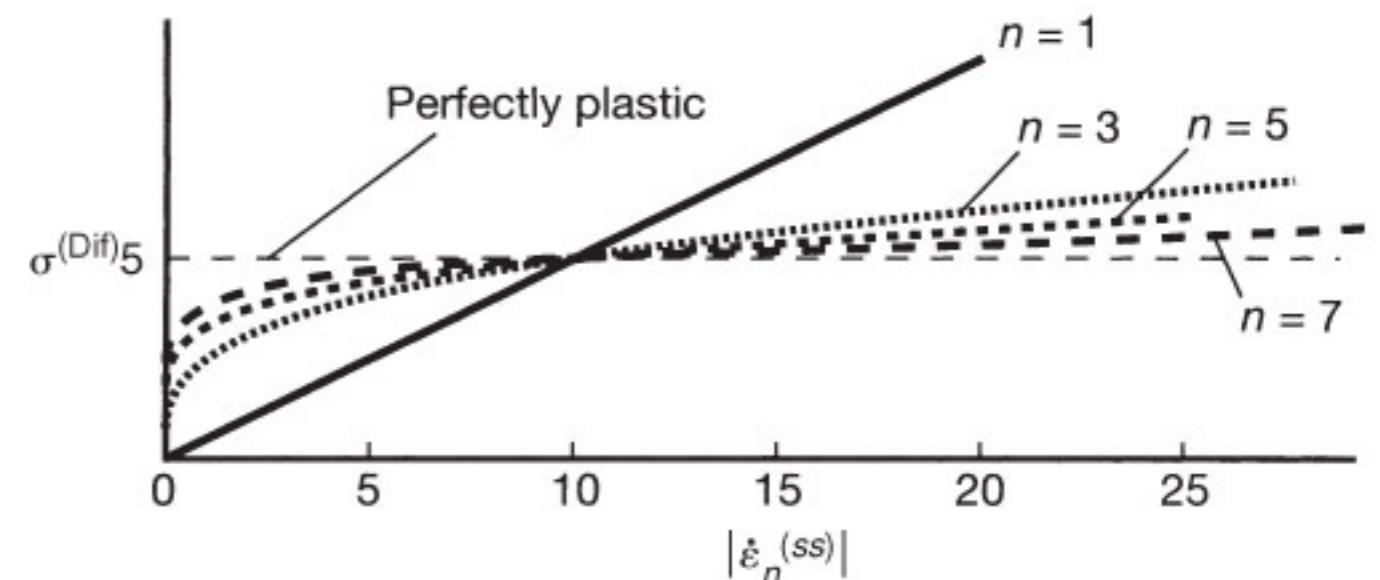
- Two main reasons:

- **Nonlinearity**

$$\tau_s^n = A_{\text{eff}} \dot{\gamma}$$

*n* is the power law exponent and

*A<sub>eff</sub>* is a material constant in Pa<sup>*n*</sup> s



Twiss and Moores, 2007

- Many rocks deform 8 times as fast when stress is doubled

# Lithospheric strength envelopes

- There are many ways in which lithospheric strength can be modelled, here are a few

## ● Jelly sandwich

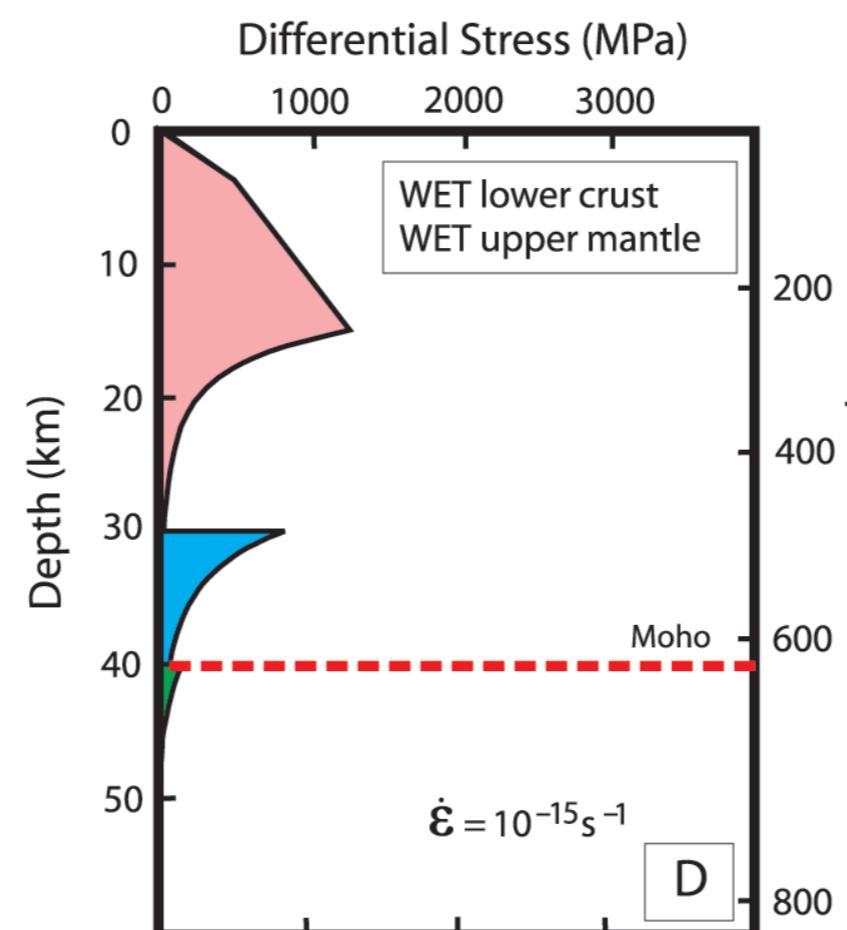
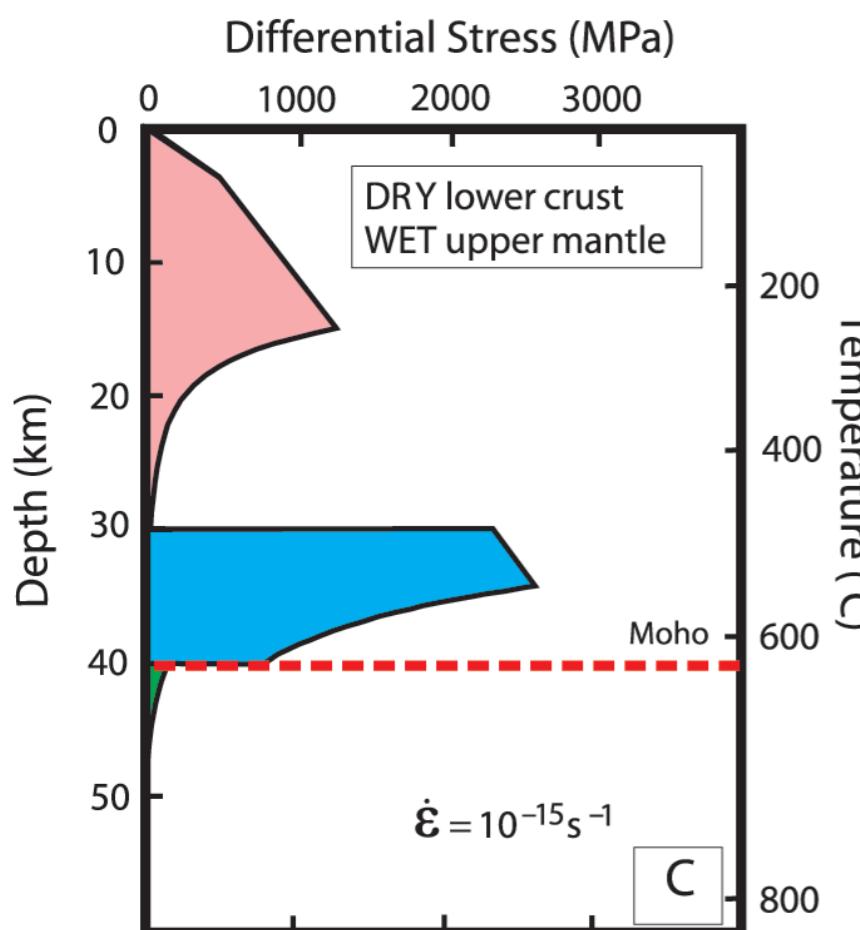
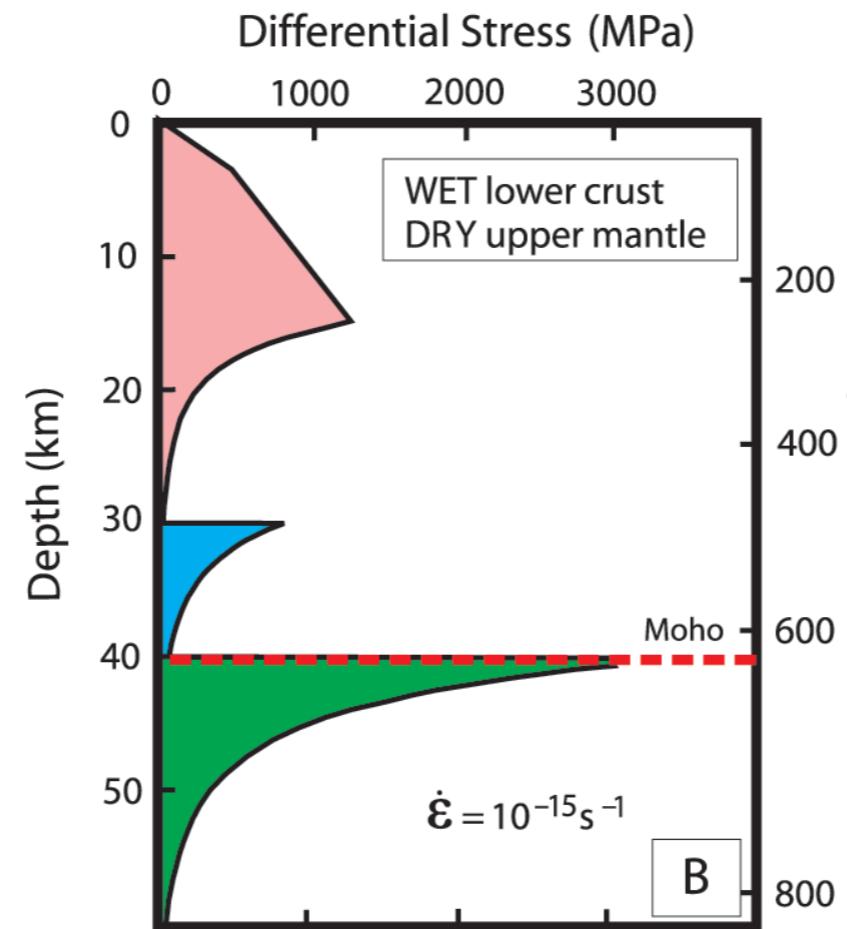
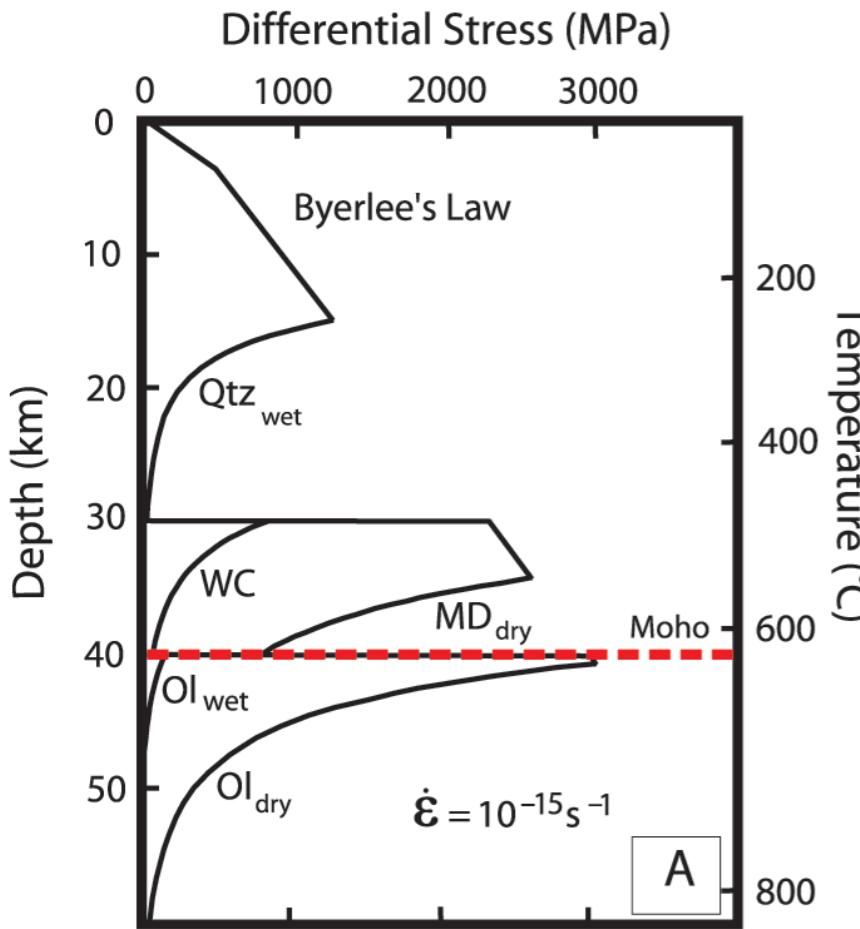
- A - Brace-Goetze

- B - Wet LC

## ● Crème brûlée

- C - Wet UM

- D - Wet LC, UM





# Summary I

- The aim of this course is to help you understand geodynamic models (mainly in the lithosphere)
  - The models are **(thermo-)mechanical**, where the internal and external forces acting on the extremely viscous fluid in the model determine how the model will deform
  - The physics and general concepts of the equations are fairly simple, but as you will see, the numerical solution of the equations and the output can be complex



# Summary II

- Deformation of the Earth in numerical geodynamic models is based on three simple conservation equations
  - **Conservation of mass** - The continuity equation
  - **Conservation of momentum** - The momentum equation
  - **Conservation of energy** - The heat transfer equation
- Conservation of mass, momentum and energy are combined with rheological laws to describe fluid movement under an applied force



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# Extra slides



# Elasticity

Twiss and Moores, 2007

$$\sigma \propto \epsilon$$

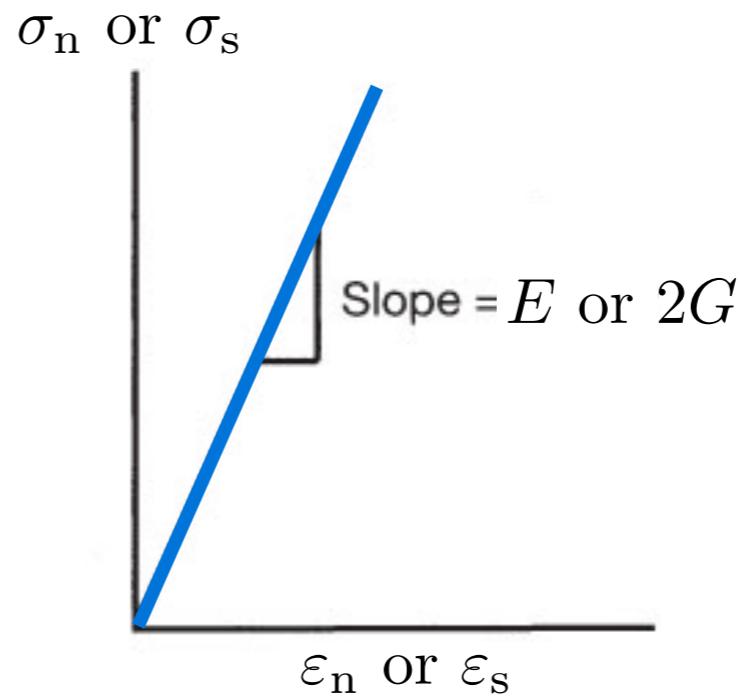
- **Stress** is proportional to **strain**
- For 1-D normal stress

$$\sigma_{xx} = E \epsilon_{xx}$$

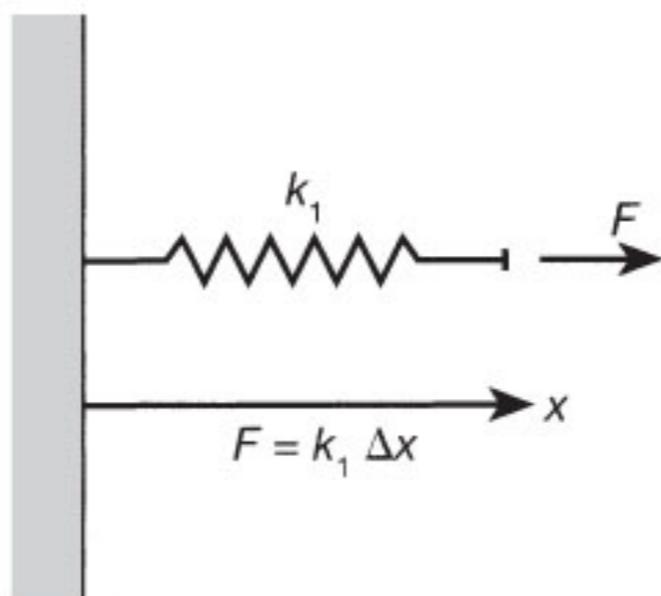
$E$  : Young's modulus (1D)

$G$  : Shear modulus (1D)

- If stress  $\rightarrow 0$ , strain  $\rightarrow 0$  (recoverable)



**A.**



**B.**



# Elasticity

Twiss and Moores, 2007

$$\sigma \propto \epsilon$$

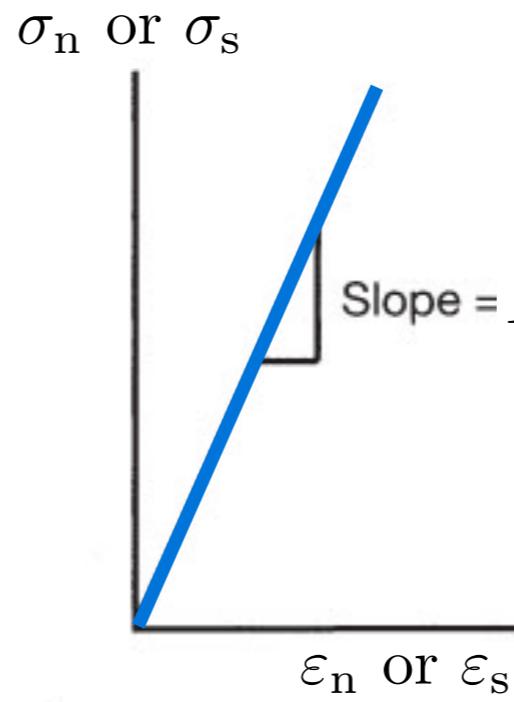
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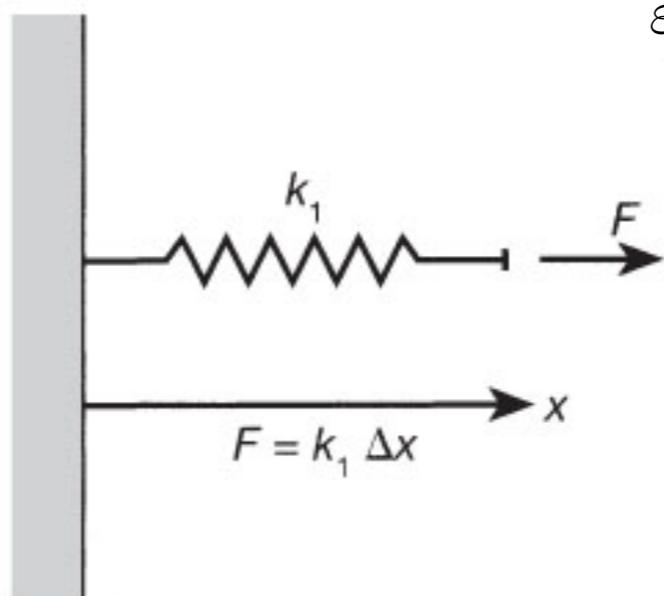
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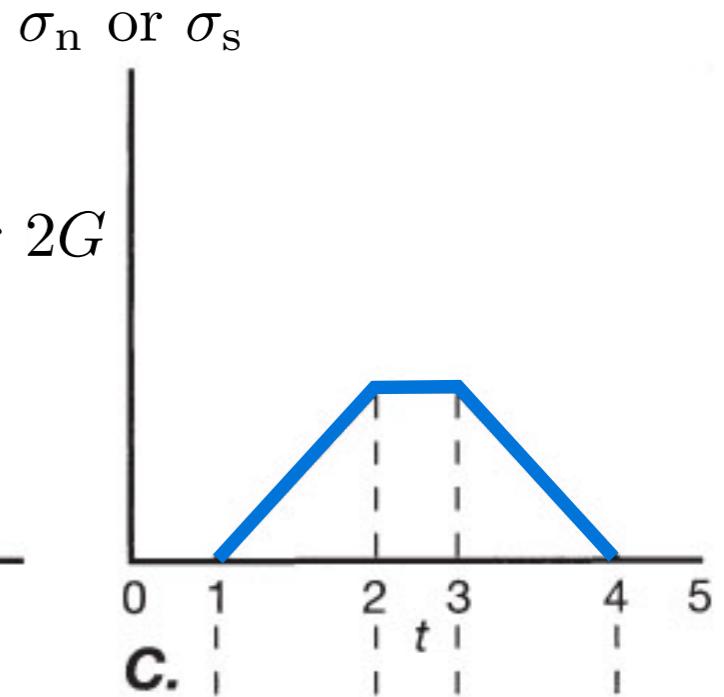
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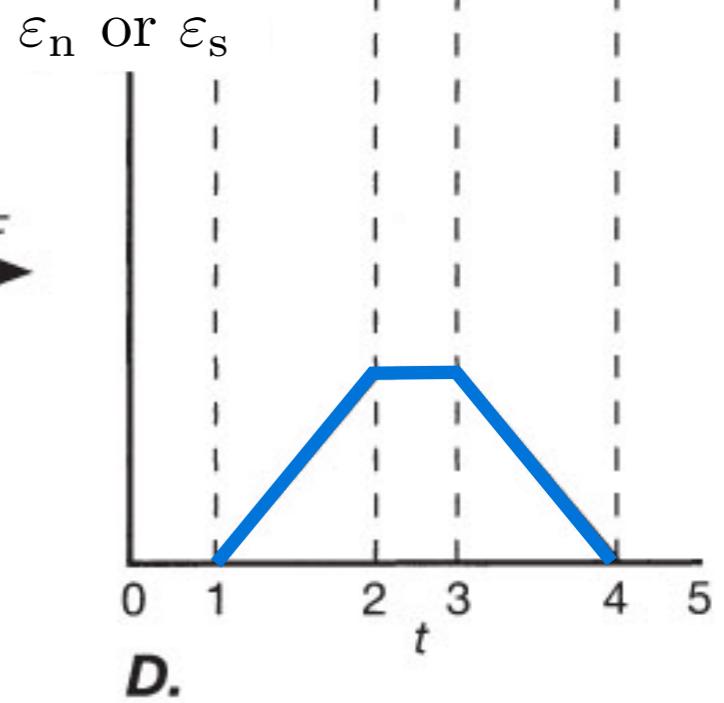
A.



B.



C.



D.