Chapter 7

Inference for numerical data[1]

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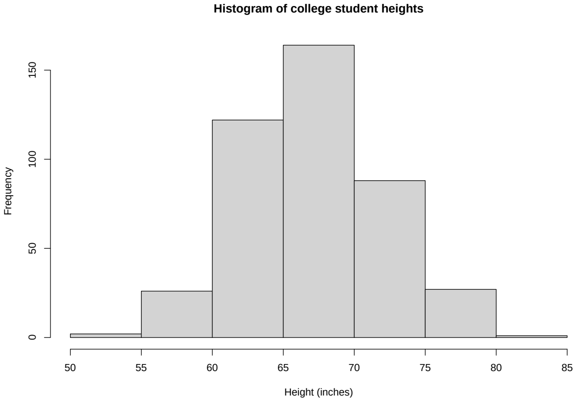
[1] These notes use content from OpenIntro Statistics Slides by Mine Cetinkaya-Rundel.

One-sample means with the t distribution

Heights

- According to the CDC, the mean height of U.S. adults ages 20 and older is about 66.5 inches (69.3 inches for males, and 63.8 inches for females).
- In our sample data, we have a sample of 430 college students from a single college.



Summary statistics

n	$\bar{x}$	s	minimum	maximum
430	67.09	4.86	53.78	83.21

Objective: We would like to investigate if the mean height of students at this college is significantly different than 66.5 inches.

From the Z-Test to the T-Test

Similar to the case of proportions, under certain conditions, we can perform a hypothesis test about the mean μ using the test statistic

$$Z = \frac{\bar{x} - \mu_0}{\frac{\sigma}{\sqrt{n}}}$$

where σ is the population standard deviation and $\mu_0 = 66.5$ is the hypothesized value for μ.

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- Then the test statistic becomes

$$T = \frac{\bar{x} - \mu_0}{\frac{s}{\sqrt{n}}}$$

Conditons

As long as observations are independent, and the population distribution is not extremely skewed, a large sample would ensure that...

- The sampling distribution of the mean is nearly normal by the central limit theorem.
- The estimate of the standard error, as $\frac{s}{\sqrt{n}}$, is reliable.

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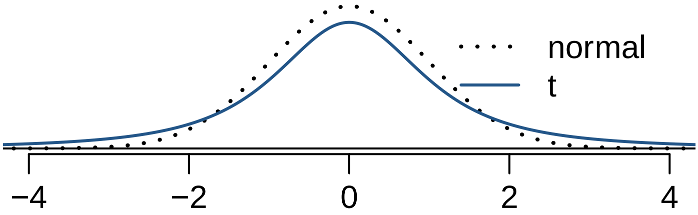
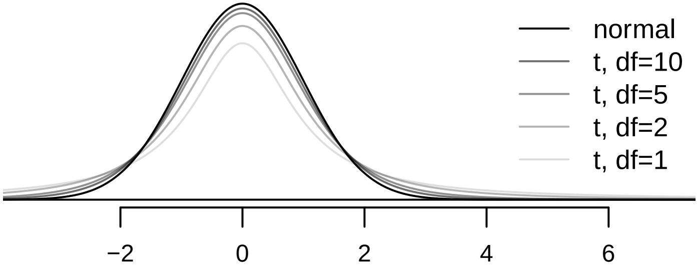
- Therefore observations are more likely to fall beyond two SDs from the mean than under the normal distribution.
- Extra thick tails are helpful for resolving our problem with a less reliable estimate the standard error (since *n* is small).



The t distribution

- Always centered at zero, like the standard normal (z) distribution.
- Has a single parameter: **degrees of freedom** (df).



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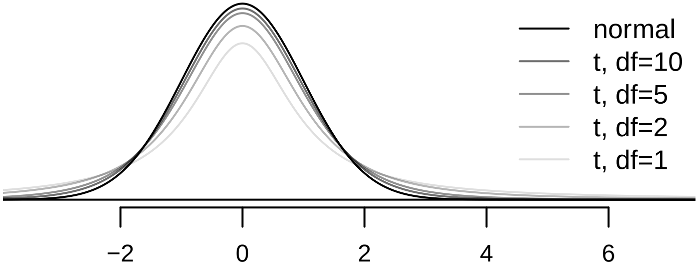

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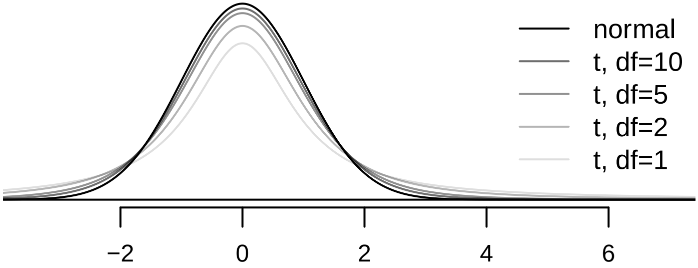

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Approaches normal.

Back to the student heights survey

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Objective: We would like to investigate if the mean height of students at this college is significantly different than 66.5 inches.

Hypotheses

What are the hypotheses for testing for the mean of college student heights being different from 66.5 inches?

A) $H_0 : \mu = 66.5$
   $H_A : \mu \neq 66.5$

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Finding the test statistic

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$$df = 430 - 1 = 429$$

Note: Null value is 66.5 because in the null hypothesis we set $\mu = 66.5$.

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- Or when these aren't available, we can use a t-table.

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We saw that the p-value was extremely low. Thus, we reject the null hypothesis. Based on the p-value, we conclude that the survey provide strong evidence that the mean of the college students height is different from the mean height of U.S. adults over 20.

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- But it would be more interesting to find out what exactly this difference is.
- We can use a confidence interval to estimate this difference.

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- $ME = t^* \times SE$

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Finding the critical $t(t^*)$

- We want to find the 95% confidence interval.
- Using R:

```
qt(p = (1+0.95)/2, df = 429)
```

```
## [1] 1.965509
```

- Or use the t-table.

Constructing a CI for a small sample mean

Which of the following is the correct calculation of a 95% confidence interval for the heights of the college students?

$$\bar{x} = 67.09 \quad s = 4.86 \quad n = 430 \quad SE = 0.234$$

A) $66.5 \pm 1.96 \times 0.234$

B) $67.09 \pm 1.97 \times 0.234$

C) $67.09 \pm -2.26 \times 0.234$

D) $66.5 \pm 2.26 \times 4.86$

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B) $67.09 \pm 1.97 \times 0.234 \rightarrow (66.63, 67.55)$

C) $67.09 \pm -2.26 \times 0.234$

D) $66.5 \pm 2.26 \times 4.86$

Synthesis

Does the conclusion from the hypothesis test agree with the findings of the confidence intereval?

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Yes, the hypothesis test found a significant difference, and the CI does not contain the null value of 66.5.

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- Confidence interval: point estimate $\pm\, t^\star_{df} \times SE$