Chapter 3

Probability[1]

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[1] These notes use content from OpenIntro Statistics Slides by Mine Cetinkaya-Rundel.

Sampling from a small population

Sampling with replacement

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- Suppose you did indeed pull a blue chip in the first draw. If drawing with replacement, what is the probability of drawing a blue chip in the second draw?

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1^{st} draw: 5 ●, 3 ●, 2 ●

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1^{st} draw: 5 ●, 3 ●, 2 ●

2^{nd} draw: 5 ●, 3 ●, 2 ●

$$Prob(2^{nd}\ chip\ B | 1^{st}\ chip\ B) = \frac{3}{10} = 0.3$$

Sampling with replacement

- Suppose you actually pulled an orange chip in the first draw. If drawing with replacement, what is the probability of drawing a blue chip in the second draw?

1^{st} draw: 5 ●, 3 ●, 2 ●

2^{nd} draw: 5 ●, 3 ●, 2 ●

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2^{nd} draw: 5 ●, 3 ●, 2 ●

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- If drawing with replacement, what is the probability of drawing two blue chips in a row?

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$$Prob(1^{st}\ chip\ B) \cdot Prob(2^{nd}\ chip\ B|1^{st}\ chip\ B) = 0.3 \times 0.3 = 0.3^2 = 0.09$$

Sampling with replacement

- When drawing with replacement, probability of the second chip being blue does not depend on the color of the first chip since whatever we draw in the first draw gets put back in the bag.

$$Prob(B|B) = Prob(B|O)$$

- In addition, this probability is equal to the probability of drawing a blue chip in the first draw, since the composition of the bag never changes when sampling with replacement.

$$Prob(B|B) = Prob(B)$$

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2^{nd} draw: 5 ●, 2 ●, 2 ●

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2^{nd} draw: 5 ●, 2 ●, 2 ●

$$Prob(2^{nd}\ chip\ B \mid 1^{st}\ chip\ B = \tfrac{2}{9} = 0.22$$

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$$Prob(2^{nd}\ chip\ B \mid 1^{st}\ chip\ B = \tfrac{2}{9} = 0.22$$

- If drawing without replacement what is the probability of drawing two blue chips in a row?

$$Prob(1^{st}\ chipB) \cdot Prob(2^{nd}\ chip\ B \mid 1^{st}\ chip\ B) = 0.3 \times 0.22 = 0.066$$

Sampling without replacement

- When drawing without replacement, the probability of the second chip being blue given the first was blue is not equal to the probability of drawing a blue chip in the first draw since the composition of the bag changes with the outcome of the first draw.

$$Prob(B|B) \neq Prob(B)$$

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- **When drawing without replacement, draws are not independent.**

- This is especially important to take note of when the sample sizes are small. If we were dealing with, say, 10,000 chips in a (giant) bag, taking out one chip of any color would not have as big an impact on the probabilities in the second draw.

Practice

In most card games, cards are drawn without replacement. What is the probability of drawing an ace and then a 3? Choose the closet answer.

A) 0.0045
B) 0.0059
C) 0.0060
D) 0.1553

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$$P(ace\ then\ 3) = \frac{4}{52} \times \frac{4}{51} \approx 0.0060$$