Chapter 7

Inference for numerical data[1]

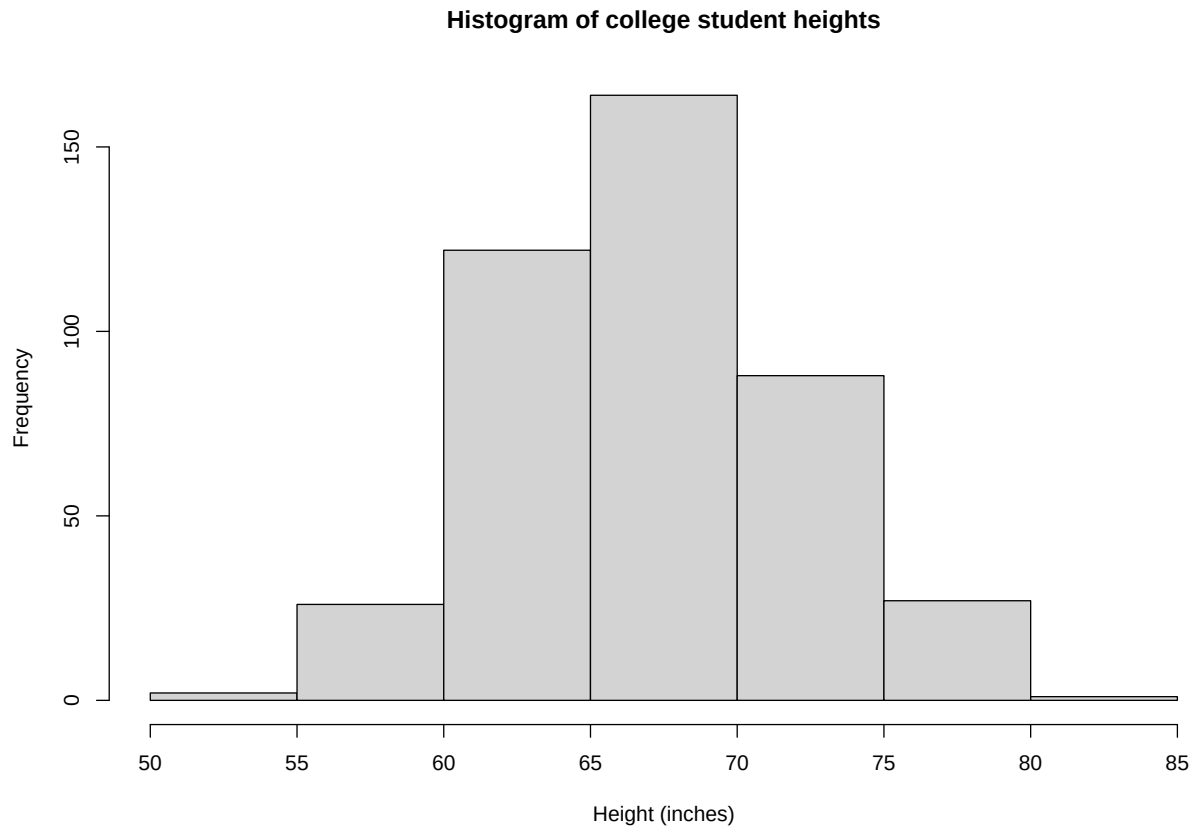
Department of Mathematics & Statistics
North Carolina A&T State University

[1] These notes use content from OpenIntro Statistics Slides by Mine Cetinkaya-Rundel.

One-sample means with the t distribution

Heights

- According to the CDC, the mean height of U.S. adults ages 20 and older is about 66.5 inches (69.3 inches for males, and 63.8 inches for females).
- In our sample data, we have a sample of 430 college students from a single college.



Summary statistics

n	$\bar{x}$	s	minimum	maximum
430	67.09	4.86	53.78	83.21

Objective: We would like to investigate if the mean height of students at this college is significantly different than 66.5 inches.

From the Z-Test to the T-Test

Similar to the case of proportions, under certain conditions, we can perform a hypothesis test about the mean μ using the test statistic

$$Z = \frac{\bar{x} - \mu_0}{\frac{\sigma}{\sqrt{n}}}$$

where σ is the population standard deviation and $\mu_0 = 66.5$ is the hypothesized value for μ.

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- The estimated SE will be SE$=s/\sqrt{n}$
- Then the test statistic becomes

$$T = \frac{\bar{x} - \mu_0}{\frac{s}{\sqrt{n}}}$$

Conditons

As long as observations are independent, and the population distribution is not extremely skewed, a large sample would ensure that...

- The sampling distribution of the mean is nearly normal by the central limit theorem.
- The estimate of the standard error, as $\frac{s}{\sqrt{n}}$, is reliable.

The t distribution

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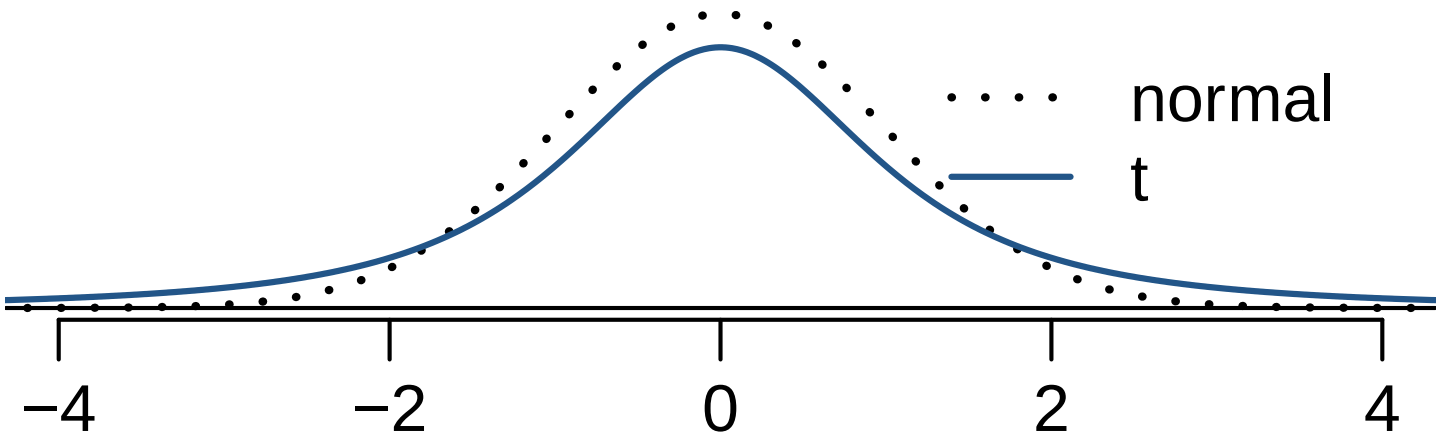
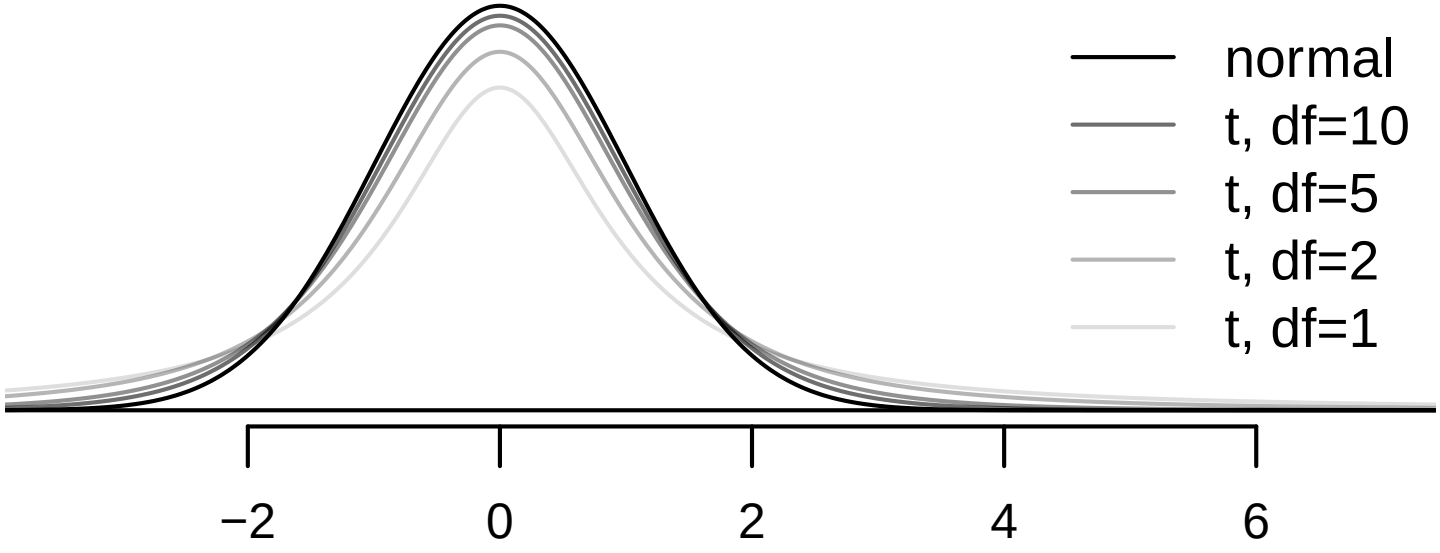
- Therefore observations are more likely to fall beyond two SDs from the mean than under the normal distribution.
- Extra thick tails are helpful for resolving our problem with a less reliable estimate the standard error (since n is small).



The t distribution

- Always centered at zero, like the standard normal (z) distribution.
- Has a single parameter: **degrees of freedom** ($\mathbf{df}$).



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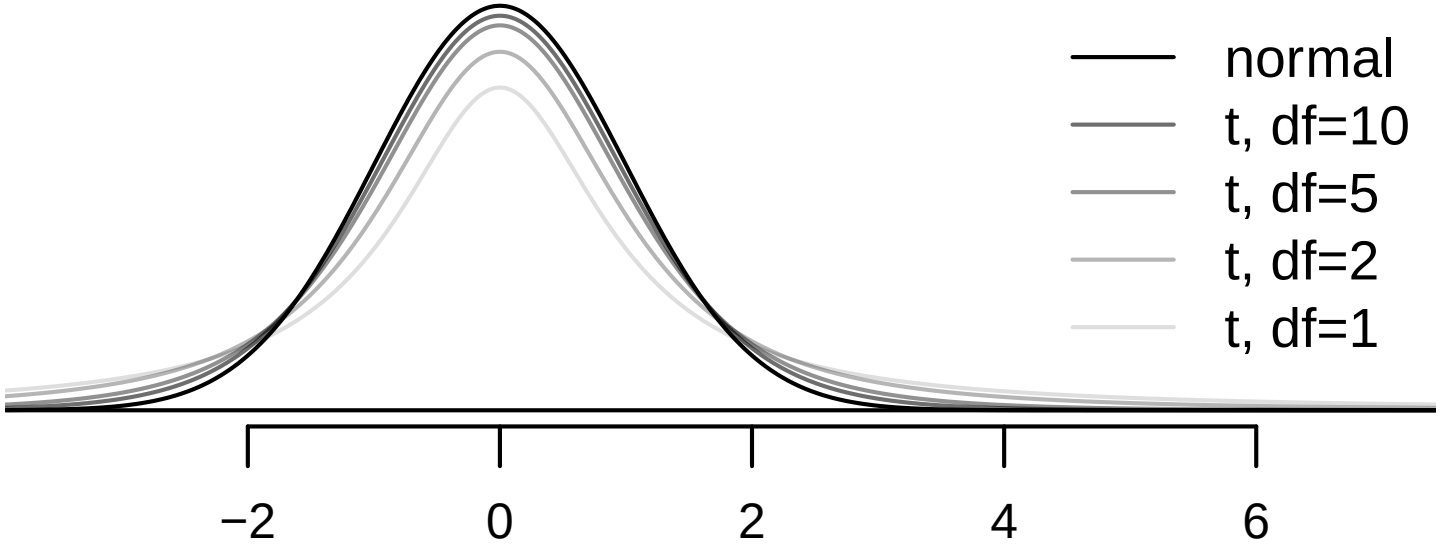

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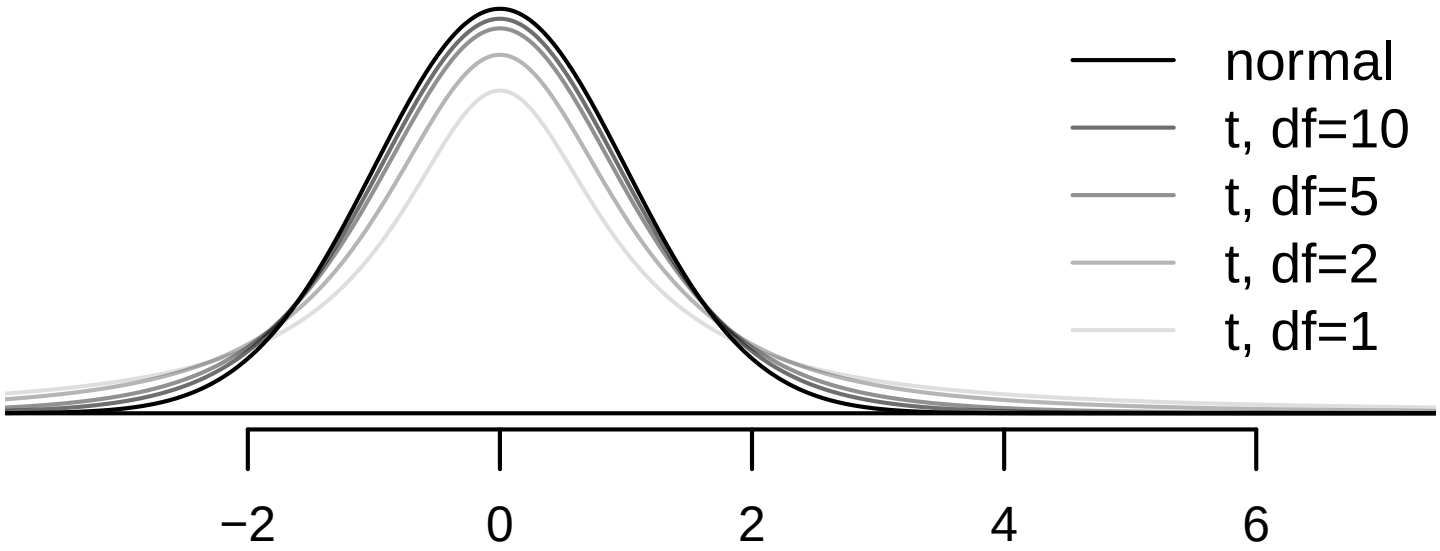

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Approaches normal.

Back to the student heights survey

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Hypotheses

What are the hypotheses for testing for the mean of college student heights being different from 66.5 inches?

A) $H_0 : \mu = 66.5$
$H_A : \mu \neq 66.5$

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$$df = 430 - 1 = 429$$

Note: Null value is 66.5 because in the null hypothesis we set $\mu = 66.5$.

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- Or when these aren't available, we can use a t-table.

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We saw that the p-value was extremely low. Thus, we reject the null hypothesis. Based on the p-value, we conclude that the survey provide strong evidence that the mean of the college students height is different from the mean height of U.S. adults over 20.

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- But it would be more interesting to find out what exactly this difference is.
- We can use a confidence interval to estimate this difference.

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- $ME = t^* \times SE$

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Finding the critical $t(t^*)$

- We want to find the 95% confidence interval.
- Using R:

```
qt(p = (1+0.95)/2, df = 429)
```

```
## [1] 1.965509
```

- Or use the t-table.

Constructing a CI for a small sample mean

Which of the following is the correct calculation of a 95% confidence interval for the heights of the college students?

$$\bar{x} = 67.09 \quad s = 4.86 \quad n = 430 \quad SE = 0.234$$

A) $66.5 \pm 1.96 \times 0.234$

B) $67.09 \pm 1.97 \times 0.234$

C) $67.09 \pm -2.26 \times 0.234$

D) $66.5 \pm 2.26 \times 4.86$

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B) $67.09 \pm 1.97 \times 0.234 \rightarrow (66.63, 67.55)$

C) $67.09 \pm -2.26 \times 0.234$

D) $66.5 \pm 2.26 \times 4.86$

Synthesis

Does the conclusion from the hypothesis test agree with the findings of the confidence intereval?

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Yes, the hypothesis test found a significant difference, and the CI does not contain the null value of 66.5.

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- Confidence interval: point estimate $\pm\, t^*_{df} \times SE$