

Chapter 7

Inference for numerical data¹

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¹These notes use content from OpenIntro Statistics Slides by Mine Cetinkaya-Rundel.

Difference of two means

Experiment

- ▶ A 2003 American Journal of Health Education Study investigated the effects of cell phone use on reaction time while driving.
- ▶ In the study, 60 participants were randomly selected and placed into one of two groups:
 - ▶ Treatment Group - Access to text documents on a cell phone.
 - ▶ Control Group - No distractions
- ▶ Participants in each group were then asked to take a computerized reaction time test.
- ▶ Researchers then recorded each subject's reaction time in seconds.

Data Summary

	Treatment	Control
	Phone	No Phone
\bar{x}	0.546	0.356
s	0.213	0.245
n	30	30

Parameter and point estimate

- ▶ **Parameter of interest:** Average difference between the reaction time of **all** drivers using a phone or not.

$$\mu_C - \mu_T$$

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- ▶ **Point estimate:** Average difference between the reaction time of **participants** in the treatment and control group.

$$\bar{x}_C - \bar{x}_T$$

Hypotheses

Which of the following is the correct set of hypotheses for testing if the average reaction time of the drivers using a phone (μ_T) is higher than the average reaction time of the drivers not using a phone (μ_C)?

A) $H_0 : \mu_C = \mu_T$
 $H_A : \mu_C \neq \mu_T$

B) $H_0 : \mu_C = \mu_T$
 $H_A : \mu_C > \mu_T$

C) $H_0 : \mu_C = \mu_T$
 $H_A : \mu_C < \mu_T$

D) $H_0 : \bar{x}_C = \bar{x}_T$
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Conditions

Which of the following does not need to be satisfied in order to conduct this hypothesis test using theoretical methods?

- A) The reaction time of drivers not using a phone in the sample should be independent of another, and the reaction time of drivers using a phone should independent of another as well.
- B) The reaction times of drivers using and not using a phone in the sample should be independent.
- C) Distributions of reaction times of drivers in both groups should not be extremely skewed.
- D) Both sample sizes should be the same.

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Test statistics

The test statistic for inference on the difference of two means where σ_1 and σ_2 are unknown is the T statistic.

$$T_{df} = \frac{\text{point estimate} - \text{null value}}{SE}$$

where

$$SE = \sqrt{\frac{s_1^2}{n_1} + \frac{s_2^2}{n_2}} \quad \text{and} \quad df = \min(n_1 - 1, n_2 - 1)$$

Note: The calculation of the df is actually much more complicated. For simplicity we'll use the above formula to estimate the true df when conducting the analysis by hand.

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$$\begin{aligned} T &= \frac{\text{point estimate} - \text{null value}}{SE} \\ &= \frac{(0.356 - 0.546) - 0}{\sqrt{\frac{0.213^2}{30} + \frac{0.245^2}{30}}} \end{aligned}$$

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Test statistic

- ▶ Coincidentally, in the experiment the number of participants in both groups is the same.
- ▶ Thus, $df = \min(30 - 1, 30 - 1) = \min(29, 29) = 29$.

p-value

Which of the following is the correct p-value for this hypothesis test?

$$T = -3.20 \quad df = 29$$

- A) Between 0.0005 and 0.001
- B) Between 0.001 and 0.0025
- C) Between 0.002 and 0.005
- D) Between 0.01 and 0.02

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- ▶ p-value is small so reject H_0 . The data provide convincing evidence to suggest that the average reaction time of drivers not using a phone is faster than the drivers using a phone while driving.
- ▶ Try not to use your phone while driving because you may never know when you would require the fast reaction time to avoid an accident.

Equivalent confidence level

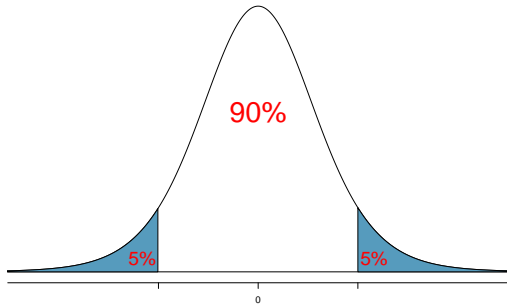
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- A) 90
- B) 92.5
- C) 95
- D) 97.5

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What is the appropriate t^* for a confidence interval for the average difference between using a phone and not using it while driving?

- A) 1.32
- B) 1.70
- C) 2.07
- D) 2.82

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$$(\bar{x}_C - \bar{x}_T) \pm t_{df}^* \times SE = (0.356 - 0.546) \pm 1.70 \times 0.0593$$

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Calculate the interval, and interpret it in context

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$$\begin{aligned}(\bar{x}_C - \bar{x}_T) \pm t_{df}^* \times SE &= (0.356 - 0.546) \pm 1.70 \times 0.0593 \\ &= -0.19 \pm 0.1008\end{aligned}$$

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We are 90% confident that the average reaction time of a driver not using a phone is 0.0892 to 0.2908 seconds faster than the average reaction time of a driver using a phone.

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- ▶ If σ_1 or σ_2 is unknown, difference between the sample means follow a t -distribution with $SE = \sqrt{\frac{s_1^2}{n_1} + \frac{s_2^2}{n_1}}$.

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- ▶ Confidence interval:

$$\text{point estimate} \pm t_{df}^* \times SE$$