Chapter 7

Inference for numerical data[1]

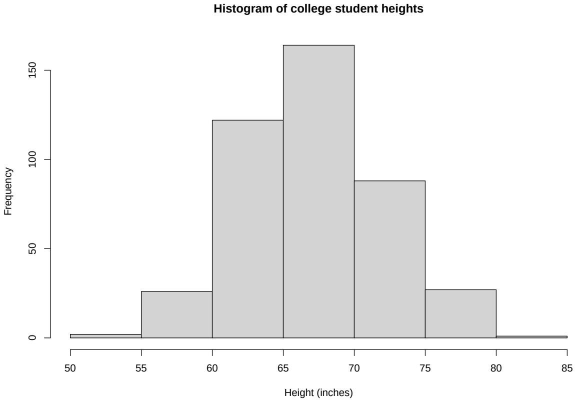
Department of Mathematics & Statistics
North Carolina A&T State University

[1] These notes use content from OpenIntro Statistics Slides by Mine Cetinkaya-Rundel.

One-sample means with the t distribution

Heights

- According to the CDC, the mean height of U.S. adults ages 20 and older is about 66.5 inches (69.3 inches for males, and 63.8 inches for females).
- In our sample data, we have a sample of 430 college students from a single college.



Summary statistics

n	$\bar{x}$	s	minimum	maximum
430	67.09	4.86	53.78	83.21

Objective: We would like to investigate if the mean height of students at this college is significantly different than 66.5 inches.

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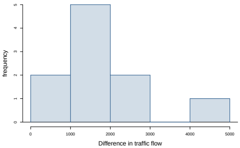

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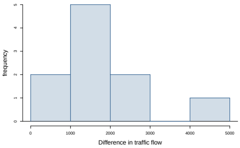

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So what do we do when the sample size is small?

Review: what purpose does a large sample serve?

As long as observations are independent, and the population distribution is not extremely skewed, a large sample would ensure that...

- The sampling distribution of the mean is nearly normal.
- The estimate of the standard error, as $\frac{s}{\sqrt{n}}$, is reliable.

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- While this is helpful special case, it's inherently difficult to verify normality in small data sets.
- We should exercise caution when verifying the normality condition for small samples. It is important to not only examine the data but also think about where the data come from.
  - For example, ask: would I expect this distribution to be symmetric, and am I confident that outliers are rare?

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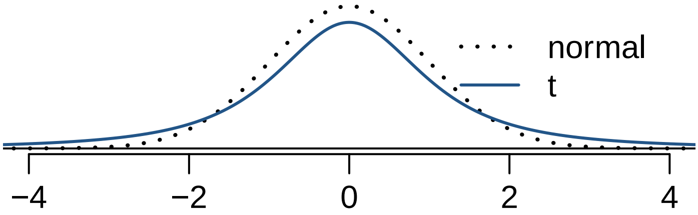
- Therefore observations are more likely to fall beyond two SDs from the mean than under the normal distribution.
- Extra thick tails are helpful for resolving our problem with a less reliable estimate the standard error (since n is small).



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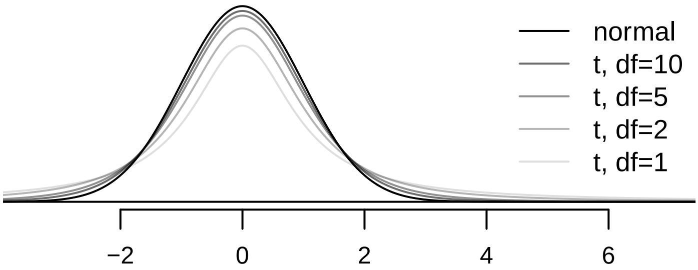

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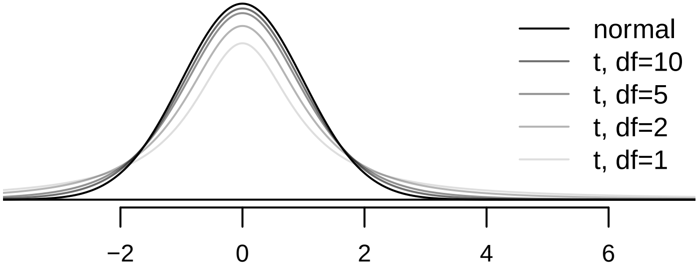

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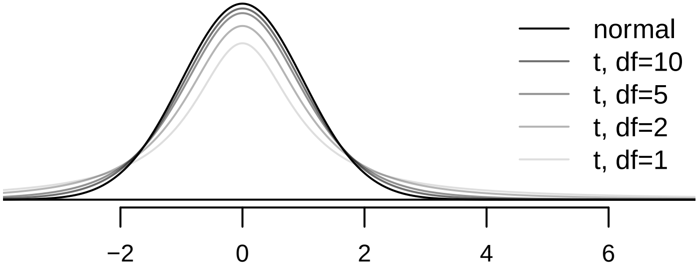

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Approaches normal.

Back to the student heights survey

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Objective: We would like to investigate if the mean height of students at this college is significantly different than 66.5 inches.

Hypotheses

What are the hypotheses for testing for the mean of college student heights being different from 67 inches?

A) $H_0 : \mu = 66.5$
$H_A : \mu \neq 66.5$

B) $H_0 : \mu = 66.5$
$H_A : \mu > 66.5$

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$$df = 430 - 1 = 429$$

Note: Null value is 66.5 because in the null hypothesis we set $\mu = 66.5$.

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  https://gallery.shinyapps.io/dist_calc/
- Or when these aren't available, we can use a t-table.

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We saw that the p-value was extremely low. Thus, we reject the null hypothesis. Based on the p-value, we conclude that the survey provide strong evidence that the mean of the college students height is different from the mean height of U.S. adults over 20.

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- But it would be more interesting to find out what exactly this difference is.
- We can use a confidence interval to estimate this difference.

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- $ME = t^* \times SE$

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Finding the critical $t(t^*)$

- We want to find the 95% confidence interval.

Constructing a CI for a small sample mean

Which of the following is the correct calculation of a 95% confidence interval for the heights of the college students?

$$\bar{x} = 67.09 \quad s = 4.86 \quad n = 430 \quad SE = 0.234$$

A) $66.5 \pm 1.96 \times 0.234$

B) $67.09 \pm 1.97 \times 0.234$

C) $67.09 \pm -2.26 \times 0.234$

D) $66.5 \pm 2.26 \times 4.86$

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B) $67.09 \pm 1.97 \times 0.234 \rightarrow (66.63, 67.55)$

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D) $66.5 \pm 2.26 \times 4.86$

Synthesis

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Yes, the hypothesis test found a significant difference, and the CI does not contain the null value of 66.5.

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- Confidence interval: point estimate $\pm\, t^{*}_{df} \times SE$