Chapter 8

Introduction to linear regression[1]

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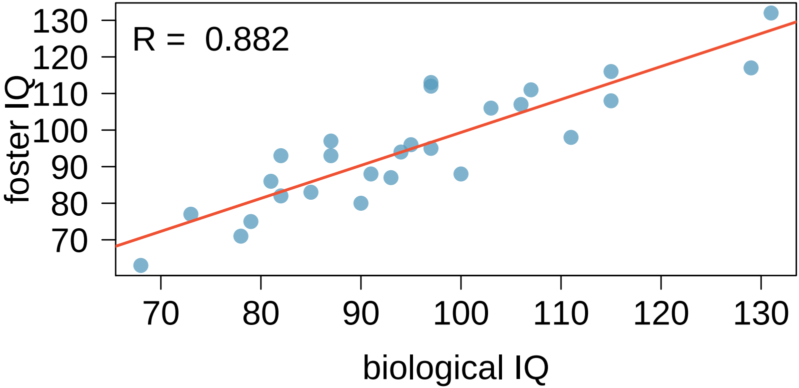
[1] These notes use content from OpenIntro Statistics Slides by Mine Cetinkaya-Rundel.

Inference for linear regression

Nature or nurture?

In 1966 Cyril Burt published a paper called "The genetic determination of differences in intelligence: A study of monozygotic twins reared together and apart". The data consist of IQ scores for [an assumed random sample of] 27 identical twins, one raised by foster parents, the other by the biological parents.



Which of the following is false?

```
##
## Call:
## lm(formula = twins$Foster ~ twins$Biological)
##
## Residuals:
##      Min       1Q   Median       3Q      Max
## -11.3512  -5.7311   0.0574   4.3244  16.3531
##
## Coefficients:
##                  Estimate Std. Error t value Pr(>|t|)
## (Intercept)       9.20760    9.29990   0.990    0.332
## twins$Biological  0.90144    0.09633   9.358  1.2e-09 ***
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
##
## Residual standard error: 7.729 on 25 degrees of freedom
## Multiple R-squared:  0.7779, Adjusted R-squared:  0.769
## F-statistic: 87.56 on 1 and 25 DF,  p-value: 1.204e-09
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A) Additional 10 points in the biological twin's IQ is associated with additional 9 points in the foster twin's IQ, on average.

B) Roughly 78% of the foster twins' IQs can be accurately predicted by the model.

C) The linear model is $\widehat{fosterIQ} = 9.2 + 0.9 \times bioIQ$.

D) Foster twins with IQs higher than average IQs tend to have biological twins with higher than average IQs as well.

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Testing for the slope

Assuming that these 27 twins comprise a representative sample of all twins separated at birth, we would like to test if these data provide convincing evidence that the IQ of the biological twin is a significant predictor of IQ of the foster twin. What are the appropriate hypotheses?

A) $H_0 : b_0 = 0; H_A : b_0 \neq 0$

B) $H_0 : \beta_0 = 0; H_A : \beta_0 \neq 0$

C) $H_0 : b_1 = 0; H_A : b_1 \neq 0$

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  Remember: We lose 1 degree of freedom for each parameter we estimate, and in simple linear regression we estimate 2 parameters, β_0 and β_1.

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p - value &= P(|T| > 9.36) < 0.01
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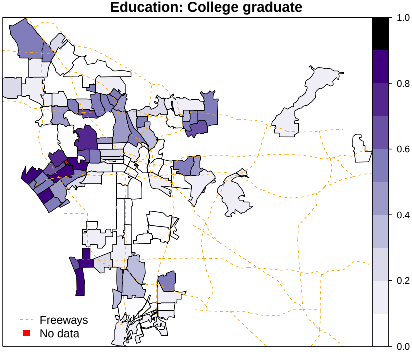
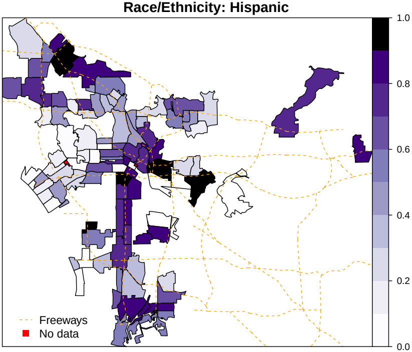
$$

% College graduate vs. % Hispanic in LA

What can you say about the relationship between % college graduate and % Hispanic in a sample of 100 zip code areas in LA?





% College graduate vs. % Hispanic in LA - another look

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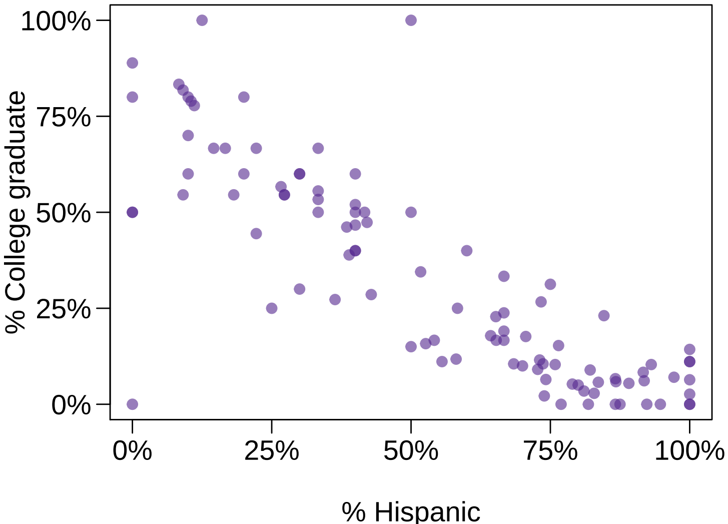

% College graduate vs. % Hispanic in LA - linear model

Which of the below is the best interpretation of the slope?

	Estimate	Std. Error	t value	Pr(>|t|)
(Intercept)	0.7290	0.0308	23.68	0.0000
%Hispanic	-0.7527	0.0501	-15.01	0.0000

A) A 1% increase in Hispanic residents in a zip code area in LA is associated with a 75% decrease in % of college grads.

B) A 1% increase in Hispanic residents in a zip code area in LA is associated with a 0.75% decrease in % of college grads.

C) An additional 1% of Hispanic residents decreases the % of college graduates in a zip code area in LA by 0.75%.

D) In zip code areas with no Hispanic residents, % of college graduates is expected to be 75%.

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Do these data provide convincing evidence that there is a statistically significant relationship between % Hispanic and % college graduates in zip code areas in LA?

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Not very...

Confidence interval for the slope

Remember that a confidence interval is calculated as $point\ estimate \pm ME$ and the degrees of freedom associated with the slope in a simple linear regression is $n - 2$. Which of the below is the correct 95% confidence interval for the slope parameter? Note that the model is based on observations from 27 twins.

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(0.7 & , & 1.1)
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- The regression output gives b_1, SE_{b_1}, and **two-tailed** p-value for the t-test for the slope where the null value is 0.
- We rarely do inference on the intercept, so we'll be focusing on the estimates and inference for the slope.

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- The ultimate goal is to have independent observations.