Chapter 7

Inference for numerical data[1]

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[1] These notes use content from OpenIntro Statistics Slides by Mine Cetinkaya-Rundel.

Difference of two means

Experiment

- A 2003 American Journal of Health Education Study investigated the effects of cell phone use on reaction time while driving.
- In the study, 60 participants were randomly selected and placed into one of two groups:
  - Treatment Group - Access to text documents on a cell phone.
  - Control Group - No distractions
- Participants in each group were then asked to take a computerized reaction time test.
- Researchers then recorded each subject's reaction time in seconds.

Data Summary

	Treatment Phone	Control No Phone
$\bar{x}$	0.546	0.356
s	0.213	0.245
n	30	30

Parameter and point estimate

- **Parameter of interest:** Average difference between the reaction time of **all** drivers using a phone or not.

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- **Point estimate:** Average difference between the reaction time of **participants** in the treatment and control group.

$$\bar{x}_C - \bar{x}_T$$

Hypotheses

Which of the following is the correct set of hypotheses for testing if the average reaction time of the drivers using a phone (μ_T) is higher than the average reaction time of the drivers not using a phone (μ_C)?

A) $H_0 : \mu_C = \mu_T$
$H_A : \mu_C \neq \mu_{pT}$

B) $H_0 : \mu_C = \mu_T$
$H_A : \mu_C > \mu_T$

C) $H_0 : \mu_C = \mu_T$
$H_A : \mu_C < \mu_T$

D) $H_0 : \bar{x}_C = \bar{x}_T$
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D) $H_0 : \bar{x}_C = \bar{x}_T$
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Conditions

Which of the following does <u>not</u> need to be satisfied in order to conduct this hypothesis test using theoretical methods?

A) The reaction time of drivers not using a phone in the sample should be independent of another, and the reaction time of drivers using a phone should independent of another as well.

B) The reaction times of drivers using and not using a phone in the sample should be independent.

C) Distributions of reaction times of drivers in both groups should not be extremely skewed.

D) Both sample sizes should be the same.

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Test statistics

The test statistic for inference on the difference of two means where σ_1 and σ_2 are unknown is the T statistic.

$$T_{df} = \frac{\text{point estimate} - \text{null value}}{SE}$$

where

$$SE = \sqrt{\frac{s_1^2}{n_1} + \frac{s_2^2}{n_2}} \qquad \text{and} \qquad df = min(n_1 - 1, n_2 - 1)$$

Note: The calculation of the df is actually much more complicated. For simplicity we'll use the above formula to estimate the true df when conducting the analysis by hand.

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\begin{aligned}
T &= \frac{\text{point estimate} - \text{null value}}{SE} \\
&= \frac{(0.356 - 0.546) - 0}{\sqrt{\frac{0.213^2}{30} + \frac{0.245^2}{30}}}
\end{aligned}
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Test statistic

- Coincidentally, in the experiment the number of participants in both groups is the same.
- Thus, $df = min(30 - 1, 30 - 1) = min(29, 29) = 29$.

p-value

Which of the following is the correct p-value for this hypothesis test?

$$T = -3.20 \qquad df = 29$$

A) Between 0.0005 and 0.001

B) Between 0.001 and 0.0025

C) Between 0.002 and 0.005

D) Between 0.01 and 0.02

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```r
pt(q = -3.20, df = 29)
```

```
## [1] 0.001659221
```

Synthesis

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What is the conclusion of the hypothesis test? How (if at all) would this conclusion change your behavior if you went diamond shopping?

- p-value is small so reject H_0. The data provide convincing evidence to suggest that the average reaction time of drivers not using a phone is faster than the drivers using a phone while driving.
- Try not to use your phone while driving because you may never know when you would require the fast reaction time to avoid an accident.

Equivalent confidence level

What is the equivalent confidence level for a one-sided hypothesis test at $\alpha = 0.05$?

A) 90
B) 92.5
C) 95
D) 97.5

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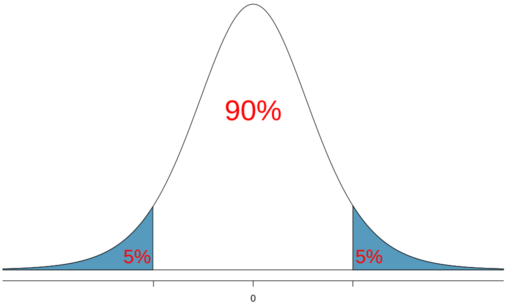

Critical value

What is the appropriate t^* for a confidence interval for the average difference between using a phone and not using it while driving?

A) 1.32
B) 1.70
C) 2.07
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```
qt(p = 0.95, df = 29)
```

```
## [1] 1.699127
```

Confidence interval

Calculate the interval, and interpret it in context

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$$(\bar{x}_C - \bar{x}_T) \pm t^\star_{df} \times SE \quad = \quad (0.356 - 0.546) \pm 1.70 \times 0.0593$$

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$$\begin{aligned}(\bar{x}_C - \bar{x}_T) \pm t^{\star}_{df} \times SE &= (0.356 - 0.546) \pm 1.70 \times 0.0593 \\ &= -0.19 \pm 0.1008\end{aligned}$$

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&= -0.19 \pm 0.1008 \\
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\end{aligned}$$

We are 90% confident that they average reaction time of a driver not using a phone is 0.0892 to 0.2908 seconds faster than the average reaction time of a driver using a phone.

Recap: Inference using difference of two small sample means

- If σ_1 or σ_2 is unknown, difference between the sample means follow a t-distribution with $SE = \sqrt{\frac{s_1^2}{n_1} + \frac{s_2^2}{n_1}}$.

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- Confidence interval:

$$\text{point estimate} \pm t^*_{df} \times SE$$