Chapter 9

Multiple and logistic regression[1]

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[1] These notes use content from OpenIntro Statistics Slides by Mine Cetinkaya-Rundel.

Introduction to multiple regression

Multuple regression

- Simple linear regression: Bivariate - two variables: y and x.
- Multiple linear regression: Multiple variables: y and $x_1, x_2,$

Poverty vs. Region (east, west)

$$\widehat{poverty} = 11.17 + 0.38 \times west$$

- Explanatory variable: region, **reference level:** east
- **Intercept:** The estimated average poverty percentage in eastern states is 11.17%

Poverty vs. Region (east, west)

$$\widehat{poverty} = 11.17 + 0.38 \times west$$

- Explanatory variable: region, **reference level:** east
- **Intercept:** The estimated average poverty percentage in eastern states is 11.17%
  - This is the value we get if we plug in 0 for the explanatory variable

Poverty vs. Region (east, west)

$$\widehat{poverty} = 11.17 + 0.38 \times west$$

- Explanatory variable: region, **reference level:** east
- **Intercept:** The estimated average poverty percentage in eastern states is 11.17%
  - This is the value we get if we plug in 0 for the explanatory variable
- **Slope:** The estimated average poverty percentage in western states is 0.38% higher than eastern states.

Poverty vs. Region (east, west)

$$\widehat{poverty} = 11.17 + 0.38 \times west$$

- Explanatory variable: region, **reference level:** east
- **Intercept:** The estimated average poverty percentage in eastern states is 11.17%
  - This is the value we get if we plug in 0 for the explanatory variable
- **Slope:** The estimated average poverty percentage in western states is 0.38% higher than eastern states.
  - Then, the estimated average poverty percentage in western states is 11.17 + 0.38 = 11.55%.

Poverty vs. Region (east, west)

$$\widehat{poverty} = 11.17 + 0.38 \times west$$

- Explanatory variable: region, **reference level:** east
- **Intercept:** The estimated average poverty percentage in eastern states is 11.17%
  - This is the value we get if we plug in 0 for the explanatory variable
- **Slope:** The estimated average poverty percentage in western states is 0.38% higher than eastern states.
  - Then, the estimated average poverty percentage in western states is $11.17 + 0.38 = 11.55\%$.
  - This is the value we get if we plug in 1 for the explanatory variable

Poverty vs. Region (northeast, midwest, west, south)

Which region (northeast, midwest, west, or south) is the reference level?

	Estimate	Std. Error	t value	Pr(>|t|)
(Intercept)	9.50	0.87	10.94	0.00
region4midwest	0.03	1.15	0.02	0.98
region4west	1.79	1.13	1.59	0.12
region4south	4.16	1.07	3.87	0.00

A) northeast

B) midwest

C) west

D) south

E) cannot tell

Poverty vs. Region (northeast, midwest, west, south)

Which region (northeast, midwest, west, or south) is the reference level?

	Estimate	Std. Error	t value	Pr(>|t|)
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Poverty vs. Region (northeast, midwest, west, south)

Which region (northeast, midwest, west, or south) has the lowest poverty percentage?

	Estimate	Std. Error	t value	Pr(>|t|)
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A) northeast

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C) west

D) south

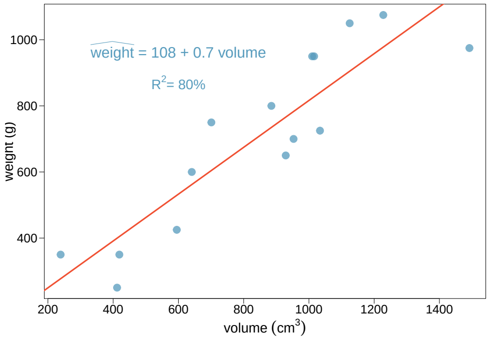
E) cannot tell

Weights of books

	weight (g)	volume (cm^3)	cover
1	800	885	hc
2	950	1016	hc
3	1050	1125	hc
4	350	239	hc
5	750	701	hc
6	600	641	hc
7	1075	1228	hc
8	250	412	pb
9	700	953	pb
10	650	929	pb
11	975	1492	pb
12	350	419	pb
13	950	1010	pb
14	425	595	pb
15	725	1034	pb

Weights of books

The scatterplot shows the relationship between weights and volumes of books as well as the regression output. Which of the below is correct?



A) Weights of 80% of the books can be predicted accurately using this model.

B) Books that are 10 cm^3 over average are expected to weigh 7 g over average.

C) The correlation between weight and volume is $R = 0.80^2 = 0.64$.

D) The model underestimates the weight of the book with the highest volume.

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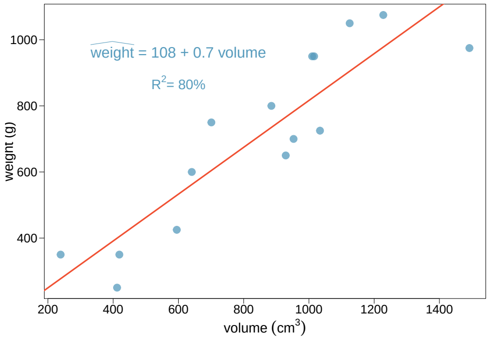

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C) The correlation between weight and volume is $R = 0.80^2 = 0.64$.

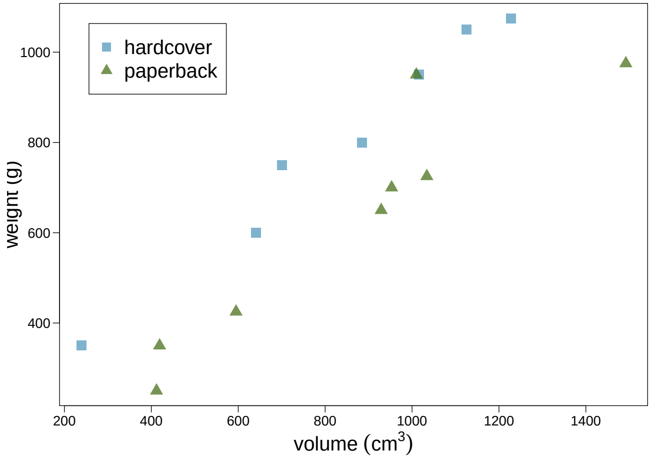
D) The model underestimates the weight of the book with the highest volume.

Modeling weights of books using volume

```
##
## Call:
## lm(formula = weight ~ volume, data = allbacks)
##
## Residuals:
##     Min      1Q  Median      3Q     Max
## -189.97 -109.86   38.08  109.73  145.57
##
## Coefficients:
##              Estimate Std. Error t value Pr(>|t|)
## (Intercept) 107.67931   88.37758   1.218    0.245
## volume        0.70864    0.09746   7.271 6.26e-06 ***
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' '
##
## Residual standard error: 123.9 on 13 degrees of freedom
## Multiple R-squared:  0.8026, Adjusted R-squared:  0.7875
## F-statistic: 52.87 on 1 and 13 DF,  p-value: 6.262e-06
```

Weights of hardcover and paperback books

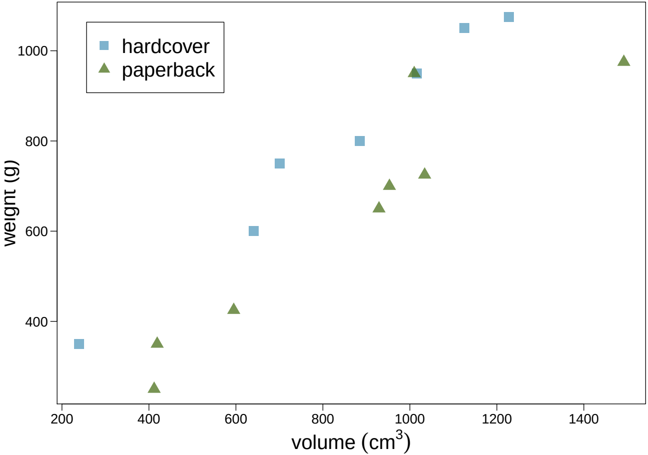
Can you identify a trend in the relationship between volume and weight of hardcover and paper books?



Weights of hardcover and paperback books

Can you identify a trend in the relationship between volume and weight of hardcover and paper books?

Paperbacks generally weight less than hardcover books after controlling for the book's volume.



Modeling weights of books using volume <u>and</u> cover type

```
##
## Call:
## lm(formula = weight ~ volume + cover, data = allbacks)
##
## Residuals:
##     Min      1Q  Median      3Q     Max
## -110.10  -32.32  -16.10   28.93  210.95
##
## Coefficients:
##              Estimate Std. Error t value Pr(>|t|)
## (Intercept)  197.96284   59.19274   3.344 0.005841 **
## volume         0.71795    0.06153  11.669  6.6e-08 ***
## coverpb     -184.04727   40.49420  -4.545 0.000672 ***
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' '
##
## Residual standard error: 78.2 on 12 degrees of freedom
## Multiple R-squared:  0.9275, Adjusted R-squared:  0.9154
## F-statistic: 76.73 on 2 and 12 DF,  p-value: 1.455e-07
```

Determining the reference level

Based on the regression output below, which level of `cover` is the reference level? Note that `pb`: paperback.

	Estimate	Std. Error	t value	Pr(>|t|)
(Intercept)	197.9628	59.1927	3.34	0.0058
volume	0.7180	0.0615	11.67	0.0000
cover:pb	-184.0473	40.4942	-4.55	0.0007

A) paperback

B) hardcover

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Determining the reference level

Which of the below correctly describes the roles of variables in this regression model?

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A) response: weight, explanatory: volume, paperback cover.

B) response: weight, explanatory: volume, hardcover cover.

C) response: volume, explanatory: weight, cover type.

D) response: weight, explanatory: volume, cover type.

Determining the reference level

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Linear model

	Estimate	Std. Error	t value	Pr(>|t|)
(Intercept)	197.96	59.19	3.34	0.01
volume	0.72	0.06	11.67	0.00
cover:pb	-184.05	40.49	-4.55	0.00

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$$\widehat{weight} = 197.96 + 0.72\ volume - 184.05\ cover : pb$$

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1. For **hardcover** books: plug in 0 for `cover`

$$\widehat{weight} = 197.96 + 0.72\ volume - 184.05 \times 0$$

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$$\widehat{weight} = 197.96 + 0.72\ volume - 184.05\ cover:pb$$

1. For **hardcover** books: plug in 0 for `cover`

$$\begin{aligned}\widehat{weight} &= 197.96 + 0.72\ volume - 184.05 \times 0 \\ &= 197.96 + 0.72\ volume\end{aligned}$$

Linear model

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volume	0.72	0.06	11.67	0.00
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$$\widehat{weight} = 197.96 + 0.72\ volume - 184.05\ cover:pb$$

1. For **hardcover** books: plug in 0 for `cover`

$$\begin{aligned}\widehat{weight} &= 197.96 + 0.72\ volume - 184.05 \times 0 \\ &= 197.96 + 0.72\ volume\end{aligned}$$

2. For **paperback** books: plug in 1 for `cover`

$$\widehat{weight} = 197.96 + 0.72\ volume - 184.05 \times 1$$

Linear model

	Estimate	Std. Error	t value	Pr(>|t|)
(Intercept)	197.96	59.19	3.34	0.01
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$$\widehat{weight} = 197.96 + 0.72\ volume - 184.05\ cover:pb$$

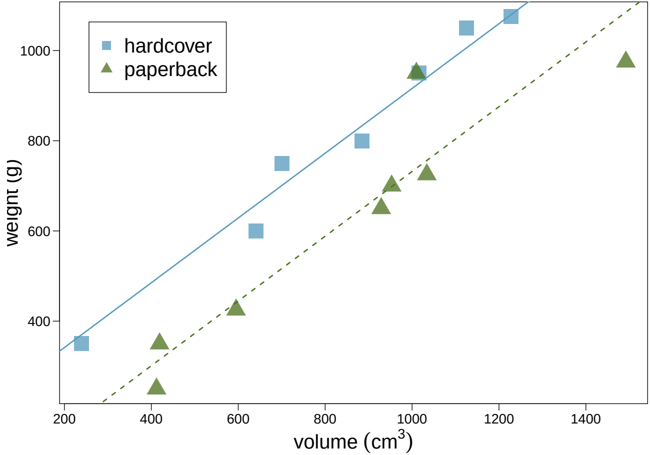
1. For **hardcover** books: plug in 0 for `cover`

$$\begin{aligned}\widehat{weight} &= 197.96 + 0.72\ volume - 184.05 \times 0 \\ &= 197.96 + 0.72\ volume\end{aligned}$$

2. For **paperback** books: plug in 1 for `cover`

$$\begin{aligned}\widehat{weight} &= 197.96 + 0.72\ volume - 184.05 \times 1 \\ &= 13.91 + 0.72\ volume\end{aligned}$$

Visualizing the linear model



Interpretation of the regression coefficients

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- **Slope of volume:** All else held constant, books that are 1 more cubic centimeter in volume tend to weigh about 0.72 grams more.

Interpretation of the regression coefficients

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(Intercept)	197.96	59.19	3.34	0.01
volume	0.72	0.06	11.67	0.00
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- **Slope of volume:** <u>All else held constant</u>, books that are 1 more cubic centimeter in volume tend to weigh about 0.72 grams more.
- **Slope of cover:** <u>All else held constant</u>, the model predicts that paperback books weigh 184 grams lower than hardcover books.

Interpretation of the regression coefficients

	Estimate	Std. Error	t value	Pr(>|t|)
(Intercept)	197.96	59.19	3.34	0.01
volume	0.72	0.06	11.67	0.00
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- **Slope of volume:** All else held constant, books that are 1 more cubic centimeter in volume tend to weigh about 0.72 grams more.
- **Slope of cover:** All else held constant, the model predicts that paperback books weigh 184 grams lower than hardcover books.
- **Intercept:** Hardcover books with no volume are expected on average to weigh 198 grams.

Interpretation of the regression coefficients

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- **Slope of volume:** All else held constant, books that are 1 more cubic centimeter in volume tend to weigh about 0.72 grams more.
- **Slope of cover:** All else held constant, the model predicts that paperback books weigh 184 grams lower than hardcover books.
- **Intercept:** Hardcover books with no volume are expected on average to weigh 198 grams.
  - Obviously, the intercept does not make sense in context. It only serves to adjust the height of the line.

Prediction

Which of the following is the correct calculation for the predicted weight of a paperback book that is $600\ cm^3$?

	Estimate	Std. Error	t value	Pr(>|t|)
(Intercept)	197.96	59.19	3.34	0.01
volume	0.72	0.06	11.67	0.00
cover:pb	-184.05	40.49	-4.55	0.00

A) $197.96 + 0.72 \times 600 - 184.05 \times 1$

B) $184.05 + 0.72 \times 600 - 197.96 \times 1$

C) $197.96 + 0.72 \times 600 - 184.05 \times 0$

D) $197.96 + 0.72 \times 1 - 184.05 \times 600$

Prediction

Which of the following is the correct calculation for the predicted weight of a paperback book that is $600\ cm^3$?

	Estimate	Std. Error	t value	Pr(>|t|)
(Intercept)	197.96	59.19	3.34	0.01
volume	0.72	0.06	11.67	0.00
cover:pb	-184.05	40.49	-4.55	0.00

A) $197.96 + 0.72 \times 600 - 184.05 \times 1 = 445.91\ grams$

B) $184.05 + 0.72 \times 600 - 197.96 \times 1$

C) $197.96 + 0.72 \times 600 - 184.05 \times 0$

D) $197.96 + 0.72 \times 1 - 184.05 \times 600$

Another example: Modeling kid's test scores

Predicting cognitive test scores of three- and four-year-old children using characteristics of their mothers. Data are from a survey of adult American women and their children - a subsample from the National Longitudinal Survey of Youth.

	kid_score	mom_hs	mom_iq	mom_work	mom_age
1	65	yes	121.12	yes	27
⋮	⋮	⋮	⋮	⋮	⋮
5	115	yes	92.75	yes	27
6	98	no	107.90	no	18
⋮	⋮	⋮	⋮	⋮	⋮
434	70	yes	91.25	yes	25

Interpreting the slope

\alert{What is the correct interpretation of the \texttt{mom_work}?}

	Estimate	Std. Error	t value	Pr(>|t|)
(Intercept)	19.59	9.22	2.13	0.03
mom_hs:yes	5.09	2.31	2.20	0.03
mom_iq	0.56	0.06	9.26	0.00
mom_work:yes	2.54	2.35	1.08	0.28
mom_age	0.22	0.33	0.66	0.51

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mom_age	0.22	0.33	0.66	0.51

All else held constant, kids with mothers whose IQs are one point higher tend to score on average 0.56 points higher.

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(Intercept)	19.59	9.22	2.13	0.03
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mom_work:yes	2.54	2.35	1.08	0.28
mom_age	0.22	0.33	0.66	0.51

Kids whose moms haven't gone to HS, did not work during the first three years of the kid's life, have an IQ of 0 and are 0 yrs old are expected on average to score 19.59. Obviously, the intercept does not make any sense in context.

Interpreting the slope

\alert{What is the correct interpretation of the slope for \texttt{mom_work}?}

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(Intercept)	19.59	9.22	2.13	0.03
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mom_age	0.22	0.33	0.66	0.51

All else being equal, kids whose moms worked during the first three years of the kid's life

A) are estimated to score 2.54 points lower.

B) are estimated to score 2.54 points higher.

than those whose moms did not work.

Interpreting the slope

What is the correct interpretation of the slope for \texttt{\alert{mom_work}} ?

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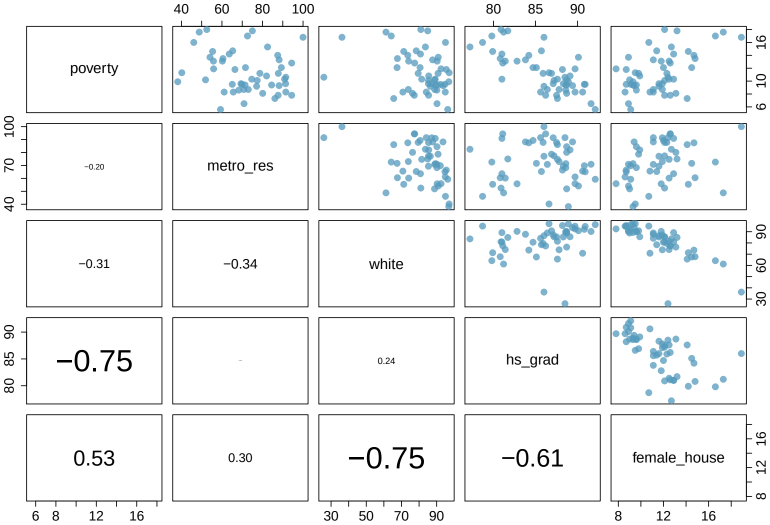
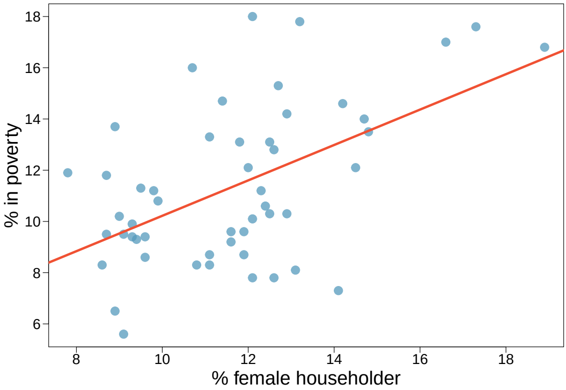
Revisit: Modeling poverty



Predicting poverty using % female householder

	Estimate	Std. Error	t value	Pr(>|t|)
(Intercept)	3.31	1.90	1.74	0.09
female_house	0.69	0.16	4.32	0.00



$$R = 0.53$$

$$R^2 = 0.53^2 = 0.28$$

Another look at R^2

R^2 can be calculated in three ways:

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1. square the correlation coefficient of x and y (how we have been calculating it)

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Another look at R^2

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1. square the correlation coefficient of x and y (how we have been calculating it)
2. square the correlation coefficient of y and $\hat{y}$
3. based on definition:

$$R^2 = \frac{\text{explained variability in } y}{\text{total variability in } y}$$

Another look at R^2

R^2 can be calculated in three ways:

1. square the correlation coefficient of x and y (how we have been calculating it)
2. square the correlation coefficient of y and $\hat{y}$
3. based on definition:
$$R^2 = \frac{\text{explained variability in } y}{\text{total variability in } y}$$

Using **ANOVA** we can calculate the explained variability and total variability in y.

Sum of squares

	Df	Sum Sq	Mean Sq	F value	Pr(>F)
female_house	1	132.57	132.57	18.68	0.00
Residuals	49	347.68	7.10		

Sum of squares

	Df	Sum Sq	Mean Sq	F value	Pr(>F)
female_house	1	132.57	132.57	18.68	0.00
Residuals	49	347.68	7.10		

Sum of squares of y: $SS_{Total} = \sum (y - \bar{y})^2 = 480.25$

$\rightarrow$ *total variability*

Sum of squares

	Df	Sum Sq	Mean Sq	F value	Pr(>F)
female_house	1	132.57	132.57	18.68	0.00
Residuals	49	347.68	7.10		

$$\begin{aligned}
\text{Sum of squares of y: } SS_{Total} &= \sum (y - \bar{y})^2 = 480.25 \\
&\quad \color{red}{\rightarrow \ total\ variability} \\
\text{Sum of squares of residuals: } SS_{Error} &= \sum e_i^2 = 347.68 \\
&\quad \color{red}{\rightarrow \ unexplained\ variability}
\end{aligned}$$

Sum of squares

	Df	Sum Sq	Mean Sq	F value	Pr(>F)
female_house	1	132.57	132.57	18.68	0.00
Residuals	49	347.68	7.10		

$$
\begin{aligned}
\text{Sum of squares of y: } SS_{Total} &= \sum (y - \bar{y})^2 = 480.25 \\
&\quad \rightarrow \textit{ total variability} \\
\text{Sum of squares of residuals: } SS_{Error} &= \sum e_i^2 = 347.68 \\
&\quad \rightarrow \textit{ unexplained variability} \\
\text{Sum of squares of x: } SS_{Model} &= SS_{Total} - SS_{Error} \\
&\quad \rightarrow \textit{ explained variability} \\
&= 480.25 - 347.68 = 132.57
\end{aligned}
$$

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- For single-predictor linear regression, having three ways to calculate the same value may seem like overkill.
- However, in multiple linear regression, we can't calculate R^2 as the square of the correlation between x and y because we have multiple xs.
- And next we'll learn another measure of explained variability, $\mathbf{adjusted\ R^2}$, that requires the use of the third approach, ratio of explained and unexplained variability.

Predicting poverty using % female hh + % white

Linear model:	Estimate	Std. Error	t value	Pr(>|t|)
(Intercept)	-2.58	5.78	-0.45	0.66
female_house	0.89	0.24	3.67	0.00
white	0.04	0.04	1.08	0.29

ANOVA:	Df	Sum Sq	Mean Sq	F value	Pr(>F)
female_house	1	132.57	132.57	18.74	0.00
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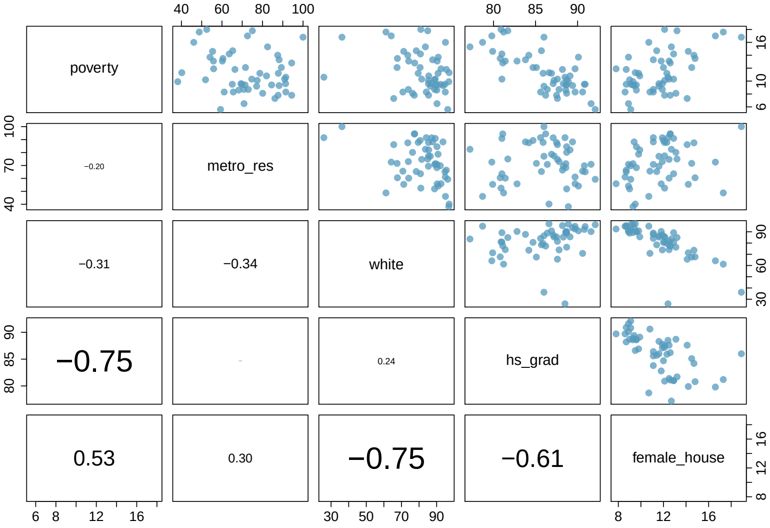
$$R^2 = \frac{\text{explained variability}}{\text{total variability}} = \frac{132.57 + 8.21}{480.25} = 0.29$$

Does adding the variable `white` to the model add valuable information that wasn't provided by `female_house`?



Collinearity between explanatory variables

poverty vs. % female head of household

	Estimate	Std. Error	t value	Pr(>|t|)
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poverty vs. % female head of household and % female hh

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  Remember: Predictors are also called explanatory or independent variables. Ideally, they would be independent of each other.
- We don't like adding predictors that are associated with each other to the model, because often times the addition of such variable brings nothing to the table. Instead, we prefer the simplest best model, i.e. **parsimonious** model.
- While it's impossible to avoid collinearity from arising in observational data, experiments are usually designed to prevent correlation among predictors.

R^2 vs. adjusted R^2

	R^2	Adjusted R^2
Model 1 (Single-predictor)	0.28	0.26
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- When any variable is added to the model R^2 increases.
- But if the added variable doesn't really provide any new information, or is completely unrelated, adjusted R^2 does not increase.

Adjusted R^2

$$R^2_{adj} = 1 - \left(\frac{SS_{Error}}{SS_{Total}} \times \frac{n-1}{n-p-1}\right)$$

where n is the number of cases and p is the number of predictors (explanatory variables) in the model. $\}$

- Because p is never negative, R^2_{adj} will always be smaller than R^2.
- R^2_{adj} applies a penalty for the number of predictors included in the model.
- Therefore, we choose models with higher R^2_{adj} over others.

Calculate adjusted R^2

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