UW CSSS/POLS 512:
Time Series and Panel Data for the Social Sciences

Multiple Imputation for Panel Data

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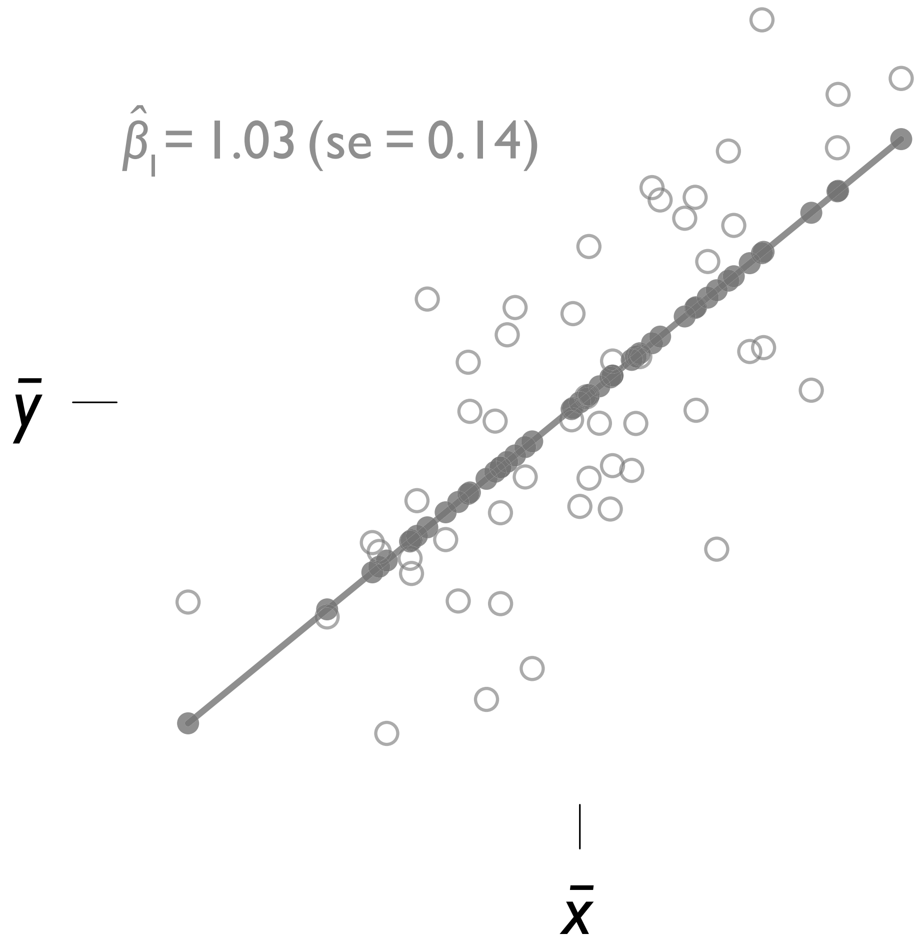
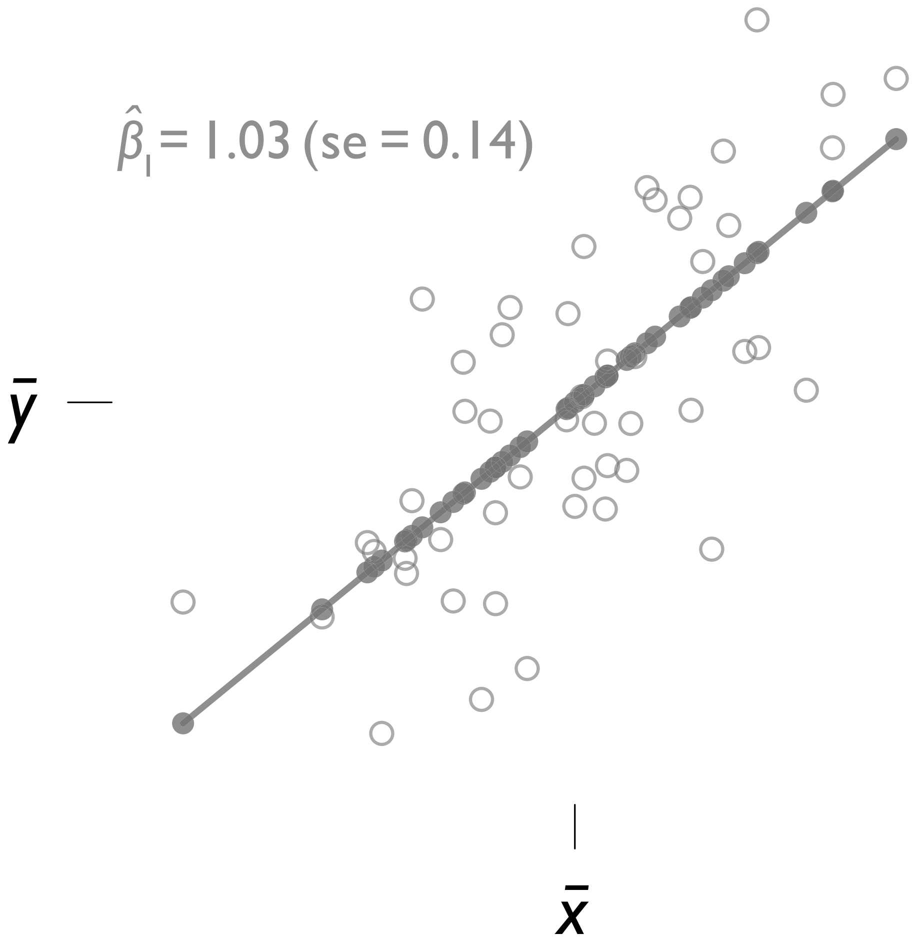
Using a random sample



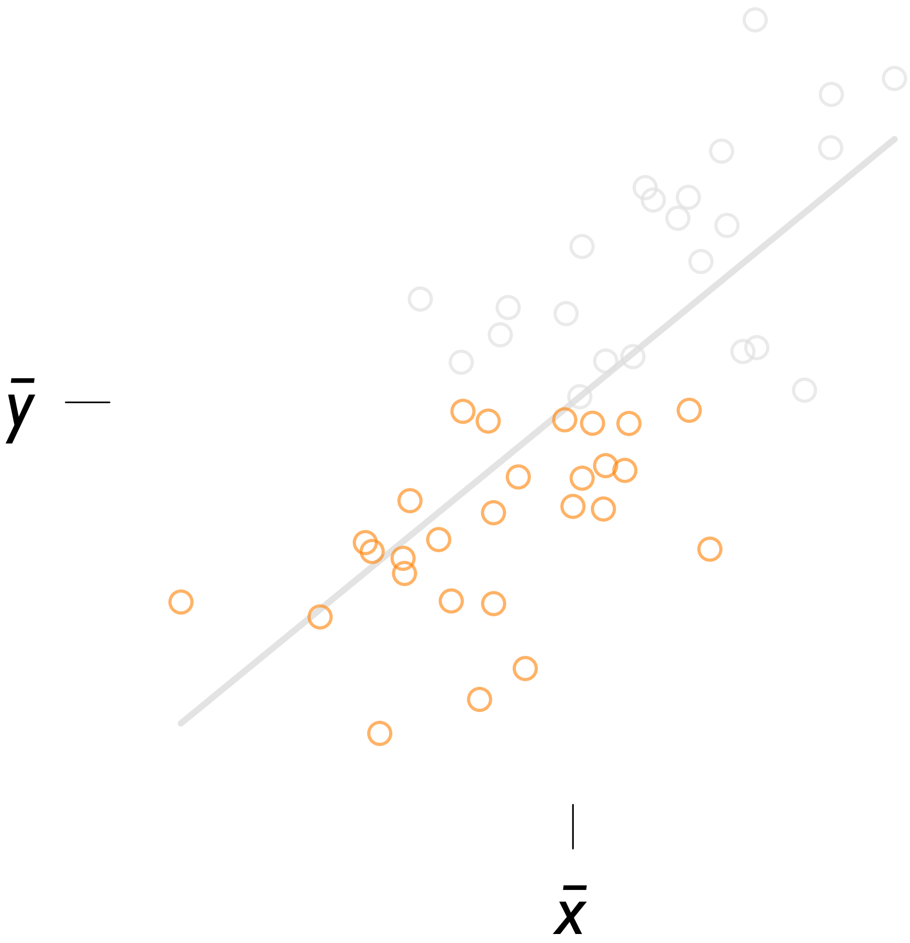
Suppose the population relationship between x and y is $y = x + \varepsilon$, $\varepsilon \sim \mathcal{N}(0, 1)$

If we randomly sample 50 cases, we recover $\hat{\beta}_1$ close to the true value of 1

Using a random sample

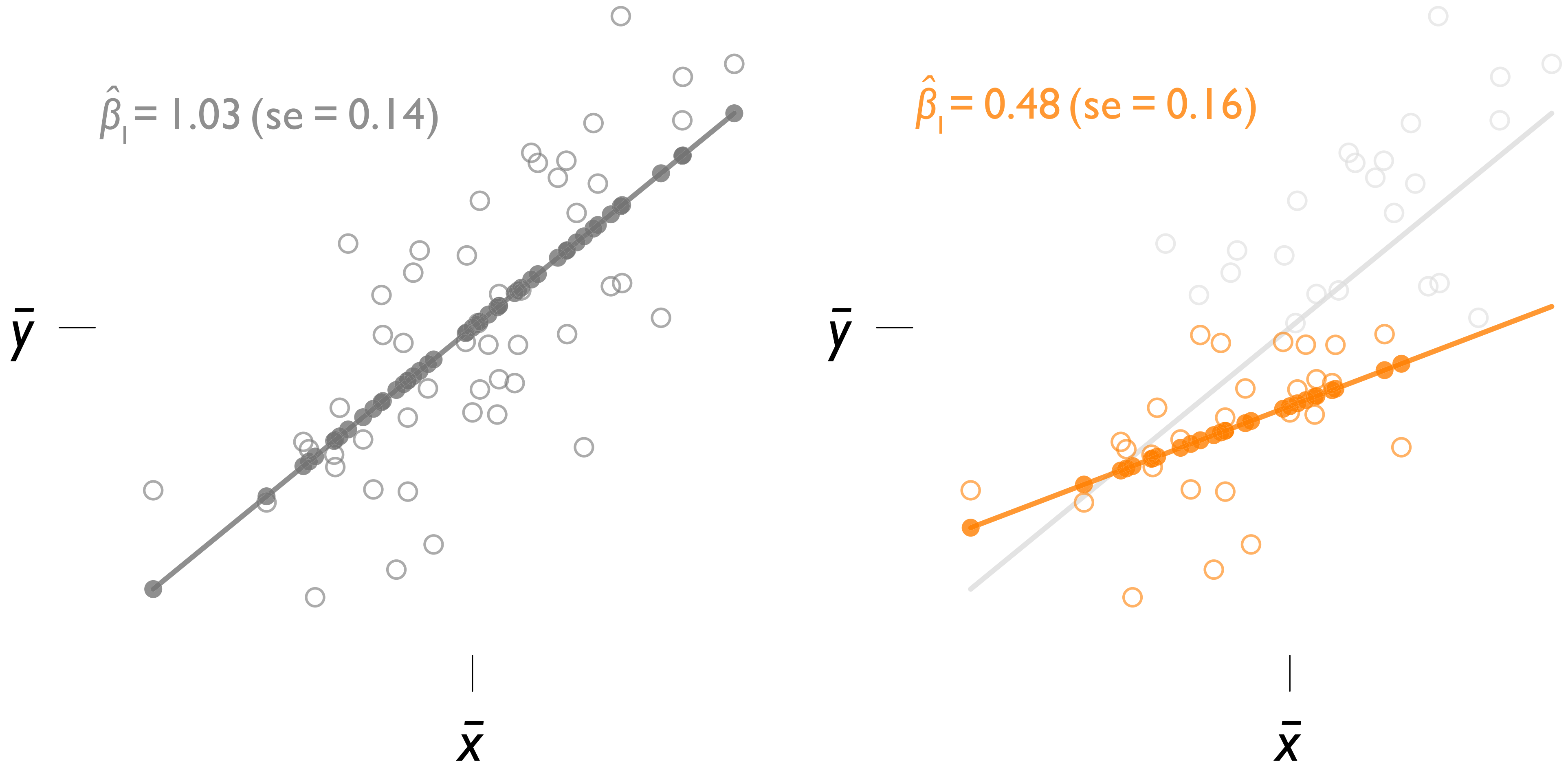


Sampling only $y < \bar{y}$



Suppose we have sample selection bias: we can only collect cases with low y

What happens if we run a regression on the orange dots only?

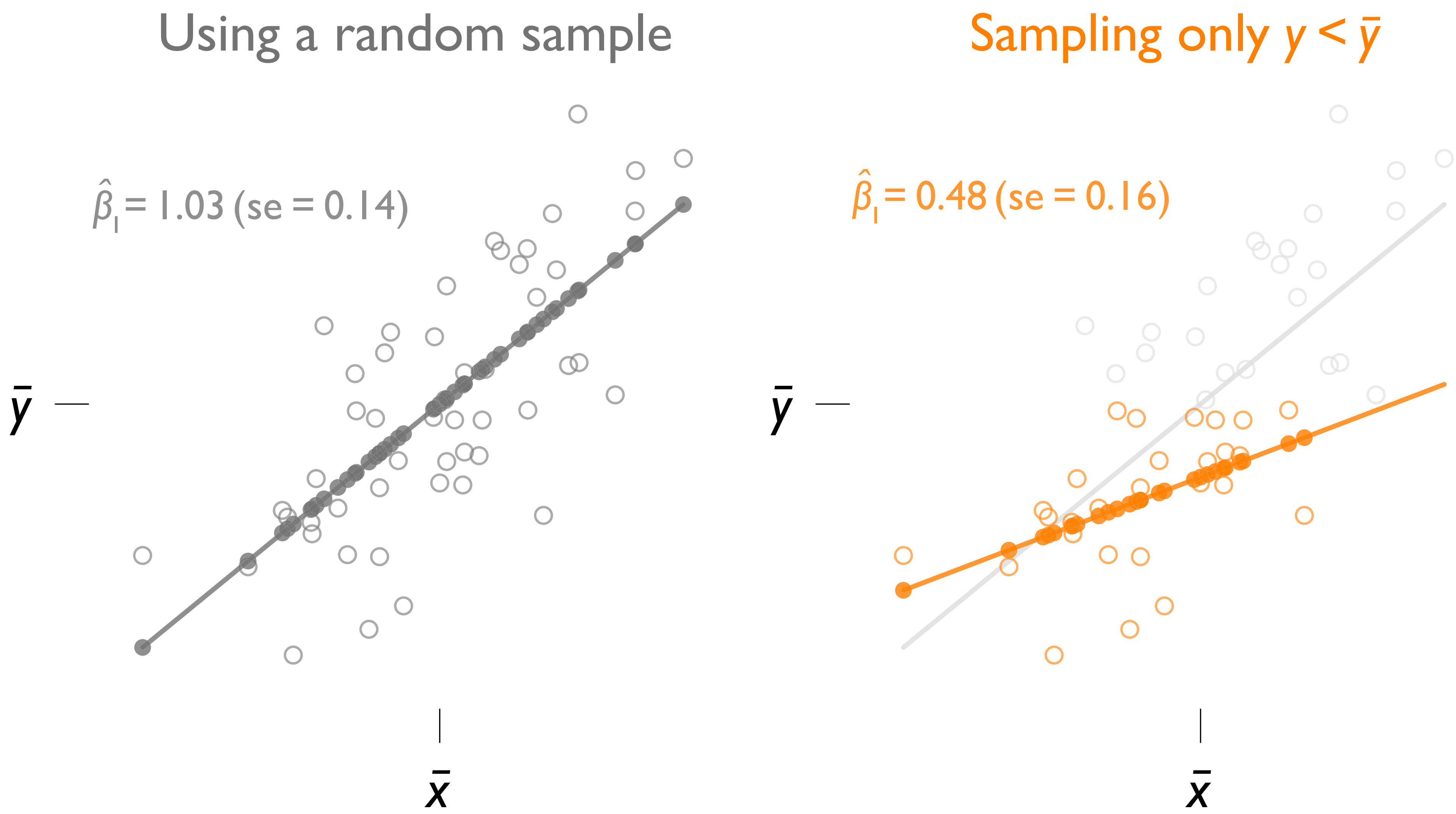


Using a random sample

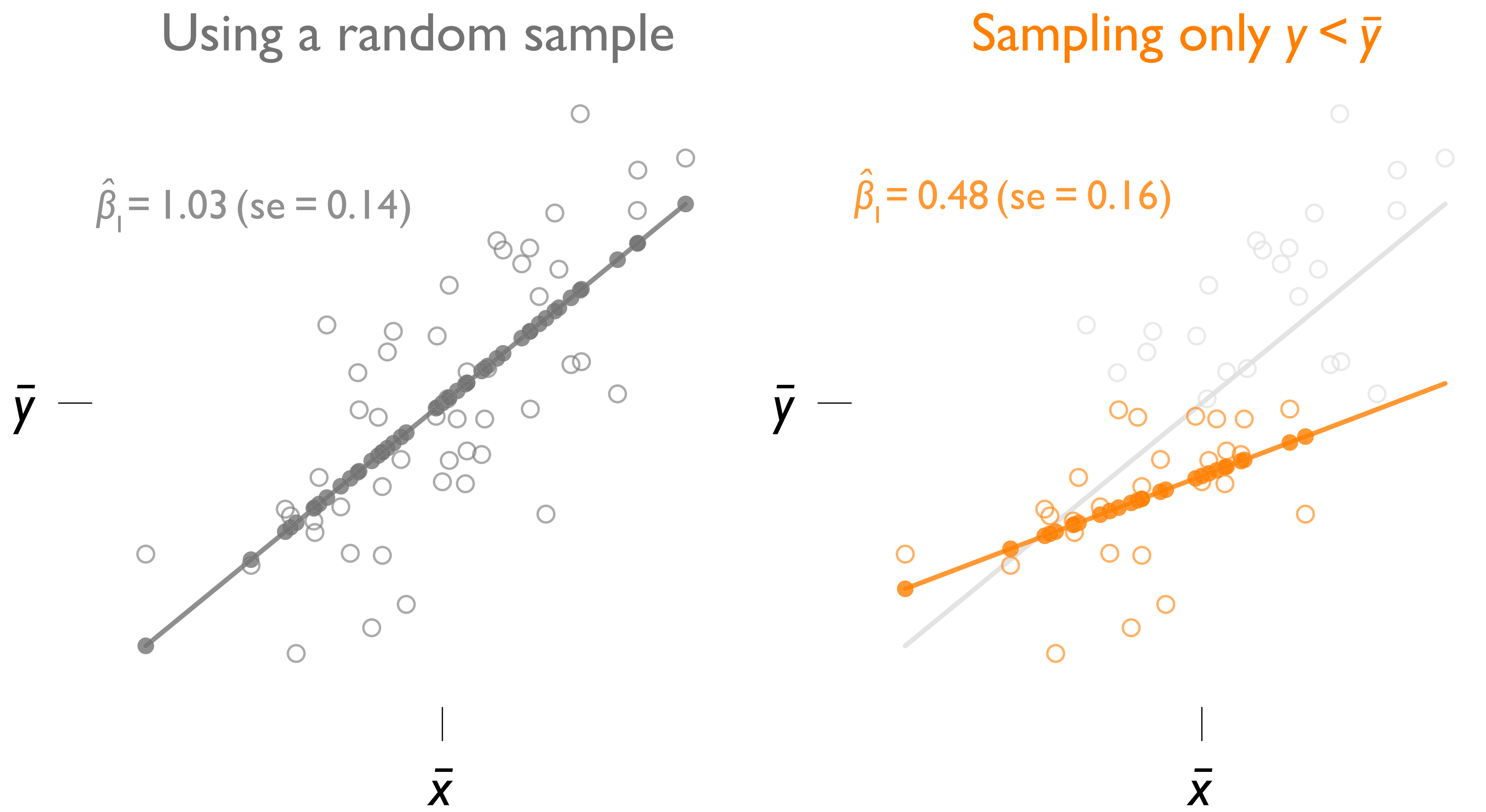
Sampling only $y < \bar{y}$

This pattern of missingness biased our result biased towards 0, whether we selected cases intentionally or had them selected for us by accident

Why? Selecting on y truncates the variation in outcomes, but not in covariates



If I call this *sample selection bias* or *compositional bias*,
all would agree I have a serious problem



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If I say "I had some missing data, so I listwise deleted," would you object as strongly?

Agenda

Why listwise deletion can be harmful

Why crude methods of imputation are no cure

A generic approach to multiple imputation

When multiple imputation is most needed

Multiple imputation for panel data

Sources

The methods and ideas emphasized here come from:

Gary King et al (2001) "Analyzing Incomplete Political Science Data: An Alternative Algorithm for Multiple Imputation", *American Political Science Review*

James Honaker and Gary King (2010) "What to Do about Missing Values in Time-Series Cross-Section Data", *American Journal of Political Science*

Stef van Buuren and Karin Groothuis-Oudshoorn (2011) "`mice`: Multivariate Imputation by Chained Equations in R." *Journal of Statistical Software*

while the classic source on missing data imputation is

Roderick Little and Donald Rubin (2002), *Statistical Analysis with Missing Data,* 2nd Ed., Wiley.

From a certain point of view, all inference problems are missing data problems; we could just treat unknown parameters as "missing data"

For today, we will just consider missingness in the data itself

A Monte Carlo experiment

$$y_i = -1x_i + 1z_i + \varepsilon_i$$

$$\begin{bmatrix} x_i \\ z_i \end{bmatrix} \sim \text{Multivariate Normal}\left(\begin{bmatrix} 0 \\ 0 \end{bmatrix}, \begin{bmatrix} 1 & -0.5 \\ -0.5 & 1 \end{bmatrix} \right)$$

$$\varepsilon \sim \text{Normal}(0, 4)$$

We will create some data using this model, then delete some of it,
and compare the effectiveness of different methods of coping with missing data

In our data, y and z_i are always observed, but x_i is sometimes missing

In our setup, we allow this to happen 3 different ways. . .

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Missing at random given z_i: Probability of missingness a function of quantile of z_i: 60% at $\min z_i$, 30% at 25th percentile of z_i, 0% at median and above

Missing at random given y_i: Probability of missingness a function of quantile of y_i: 60% at $\min y_i$, 30% at 25th percentile of y_i, 0% at median and above

Missing completely at random: In addition to the above conditional missingness, 20% of the time, x_i is missing regardless of the values of z_i and y_i

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Net effect of three patterns of missingness: x_i missing about 60% of the time

In our experiments, we will simulate 200 observations:

about 120 will be missing, and about 80 will be full observed

Exact number of missing cases will vary randomly from dataset to dataset

A Monte Carlo experiment

$$\text{Democracy}_i = -1 \times \text{Inequality}_i + 1 \times \text{GDP}_i + \varepsilon_i$$

$$\begin{bmatrix} \text{Inequality}_i \\ \text{GDP}_i \end{bmatrix} \sim \text{Multivariate Normal}\left(\begin{bmatrix} 0 \\ 0 \end{bmatrix}, \begin{bmatrix} 1 & -0.5 \\ -0.5 & 1 \end{bmatrix} \right)$$

$$\varepsilon \sim \text{Normal}(0, 4)$$

It may help to imagine some context, but remember this example is fictive:

Imagine democracy is hampered by inequality and aided by development,

Inequality tends to be lower in developed countries,

Poorer countries & non-democracies less likely to collect/publish inequality data,

And sometimes even rich democracies fail to collect such complex data

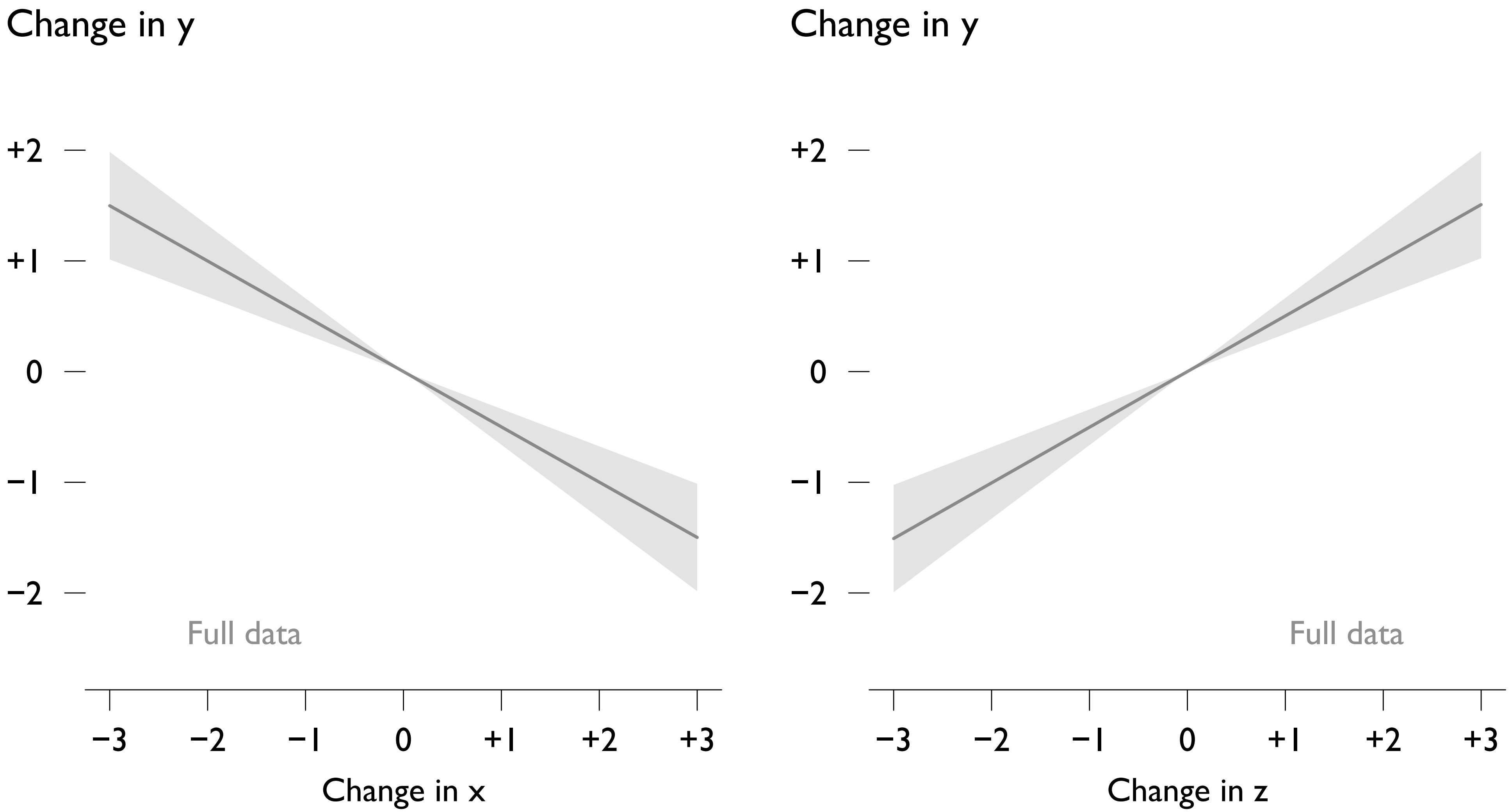
Monte Carlo run 1, fully observed

i	Democracy$_i$	Inequality$_i$	GDP$_i$
[1]	1.94	-0.16	1.28
[2]	0.26	0.27	-0.21
[3]	0.97	-0.05	-0.66
[4]	0.17	-0.46	-0.31
[5]	3.17	-2.94	0.96
[6]	-1.56	-1.16	0.28
⋮	⋮	⋮	⋮

I will generate many datasets from this true model
as part of the Monte Carlo experiement

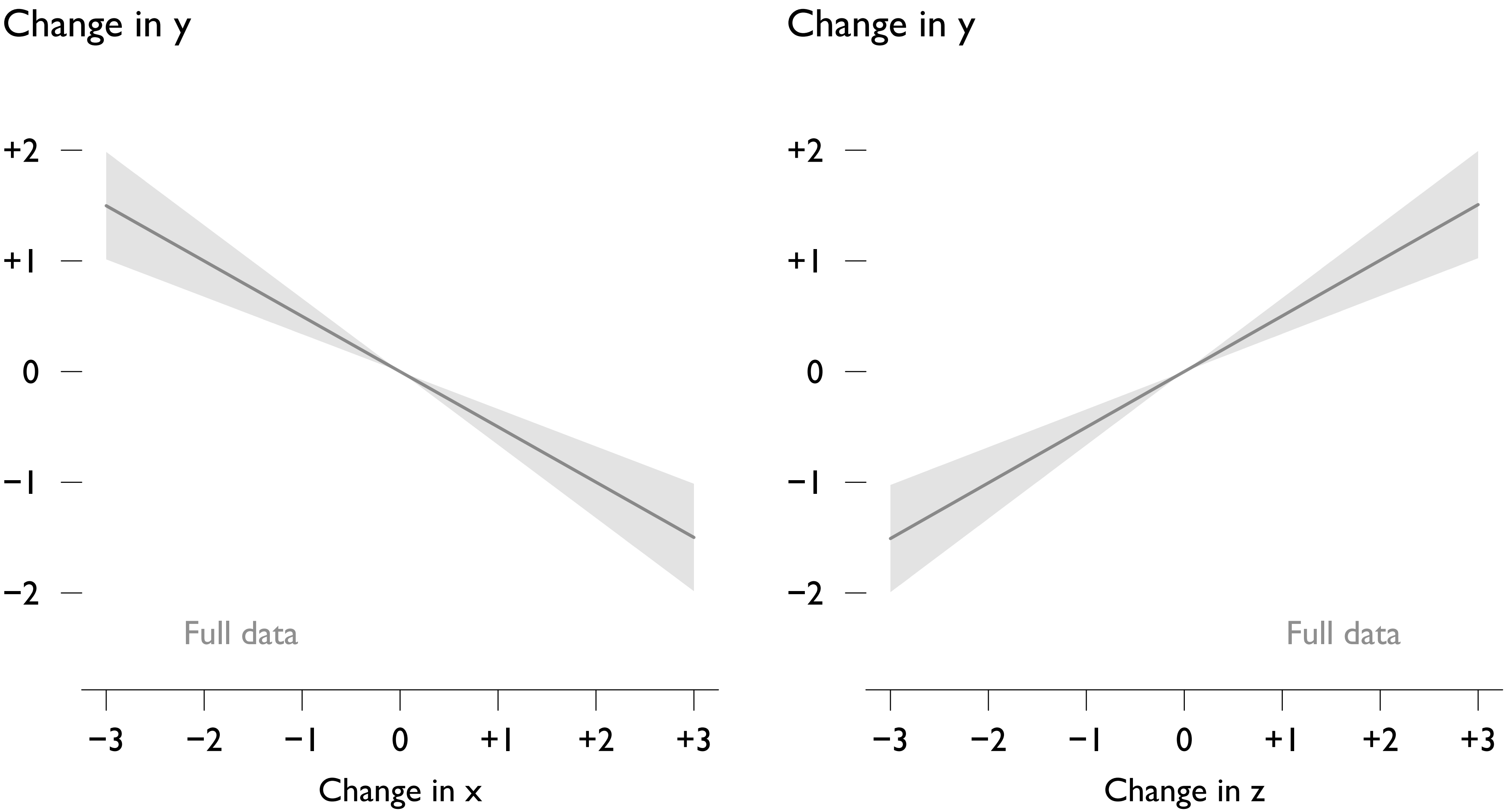
But to illustrate how data goes missing and get imputed,
I'll show what happens to the first 6 cases of the first Monte Carlo dataset

First, let's establish a baseline:
what we would find if we could use the full dataset. . .



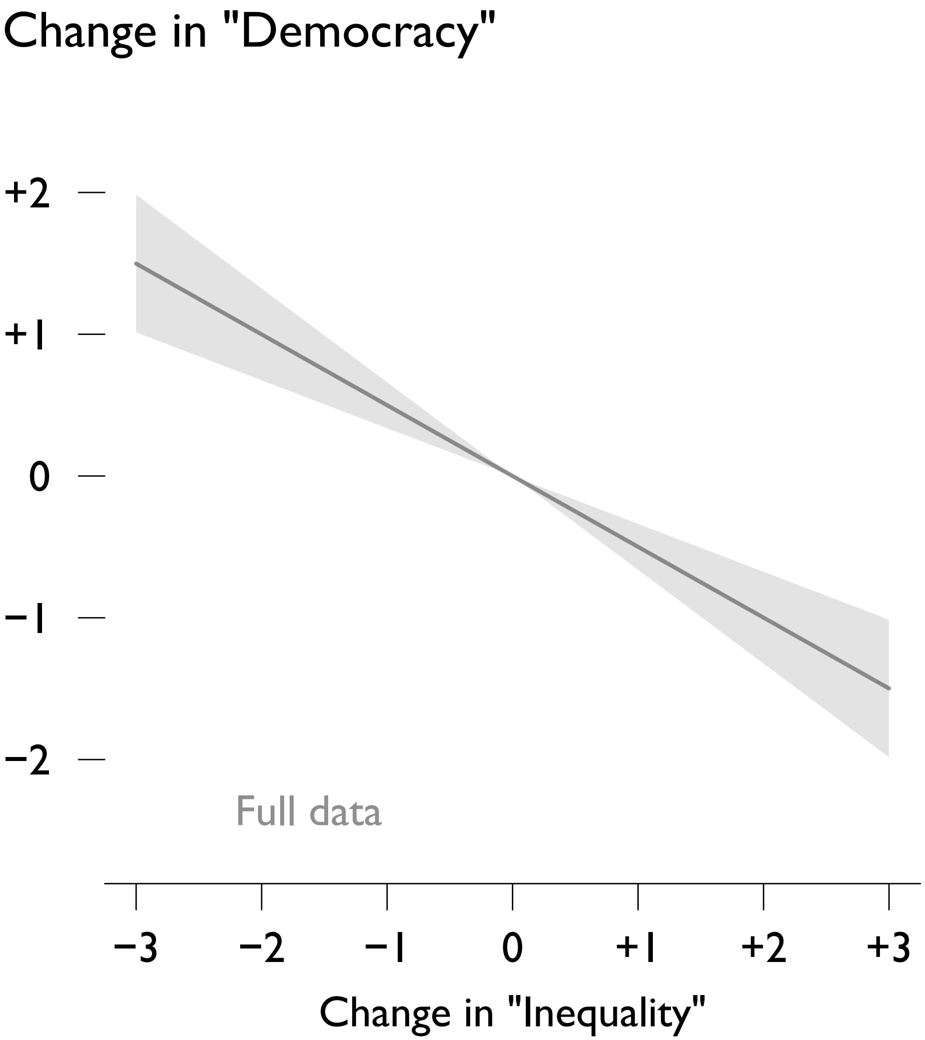
Above shows the first differences we'd get if we fully observed our 200 cases

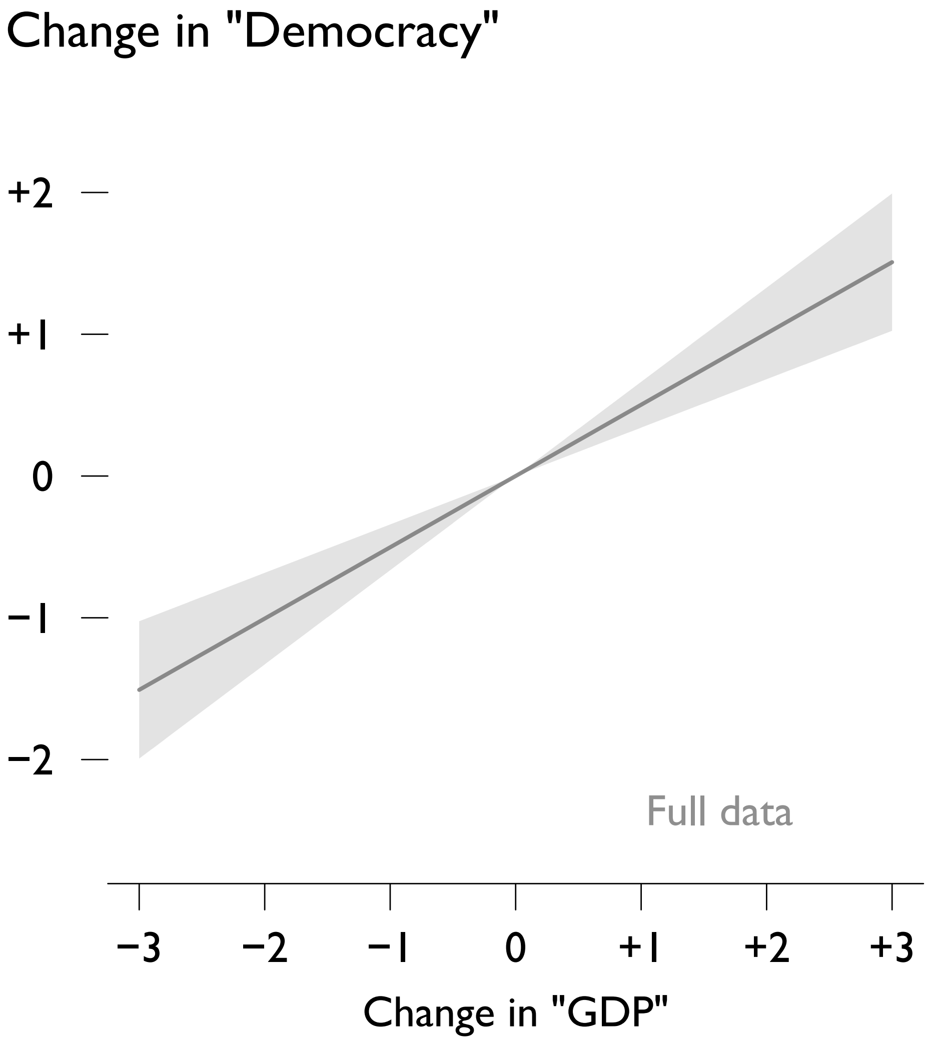
Our goal henceforth is to reproduce these effects & 95% CIs as closely as possible



For all first difference plots, I've actually averaged results after running the whole experiment (creating a dataset, then estimating the model) 1000×

This eliminates Monte Carlo error
to shows us what will happen on average for each missing data strategy





To make the example easier to follow,
I've replaced x, y, and z with our fictive variable names

Of course, we don't have any real evidence on this hypothetical research question;
all the data are made up

Costs of listwise deletion

Our dataset contains 3 variables and 200 cases

But for about 120 of our cases, a single variable has a missing value

This means that only $120/(3 \times 200) = 20\%$ of our cells are missing

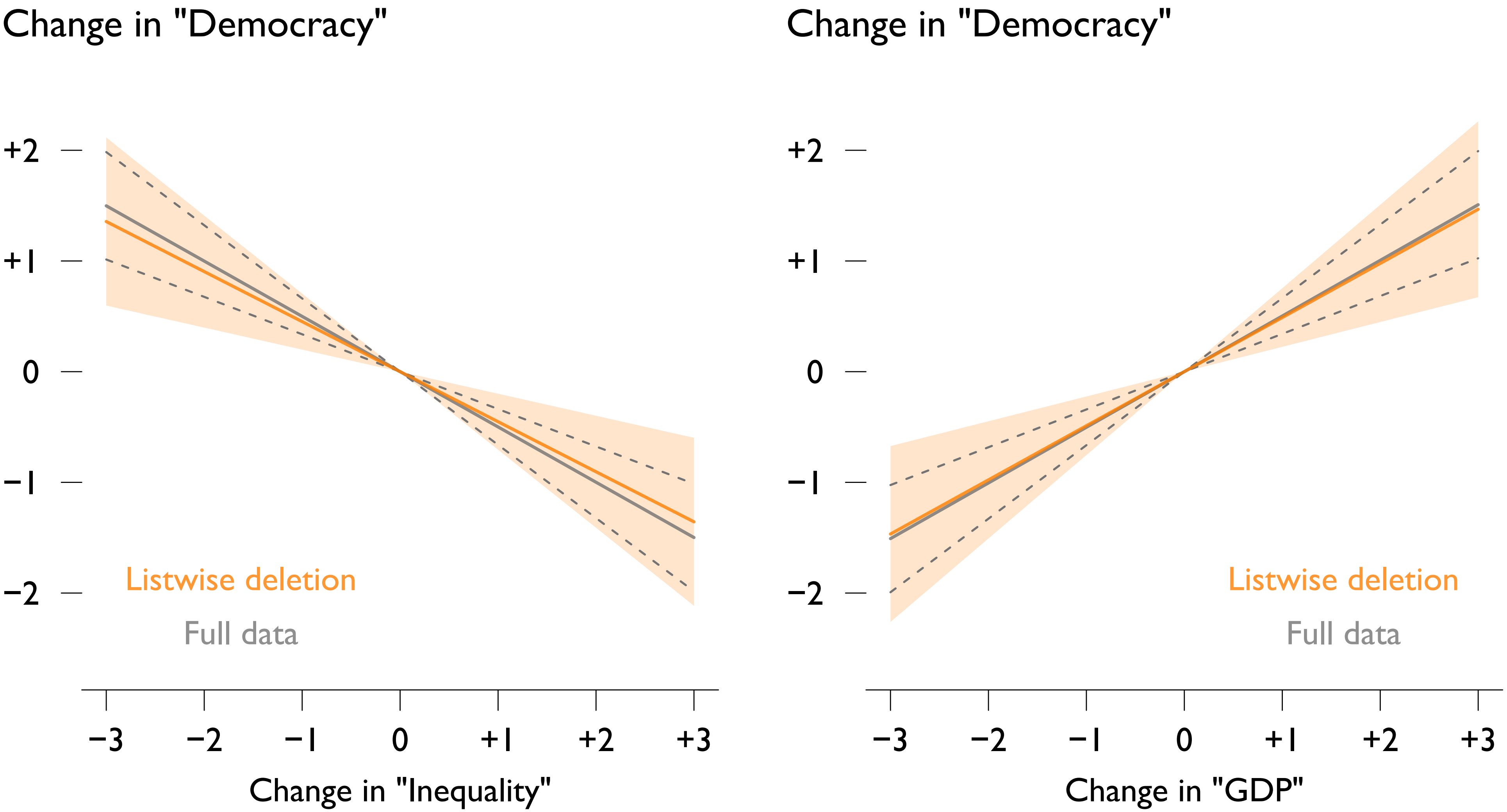
But listwise deletion will remove 60% of our cases,
increasing standard errors considerably

We've thrown away 240 cells containing actual data – *half* the observed cells

Imagine collecting your dataset by hand, then tossing half of it the trash

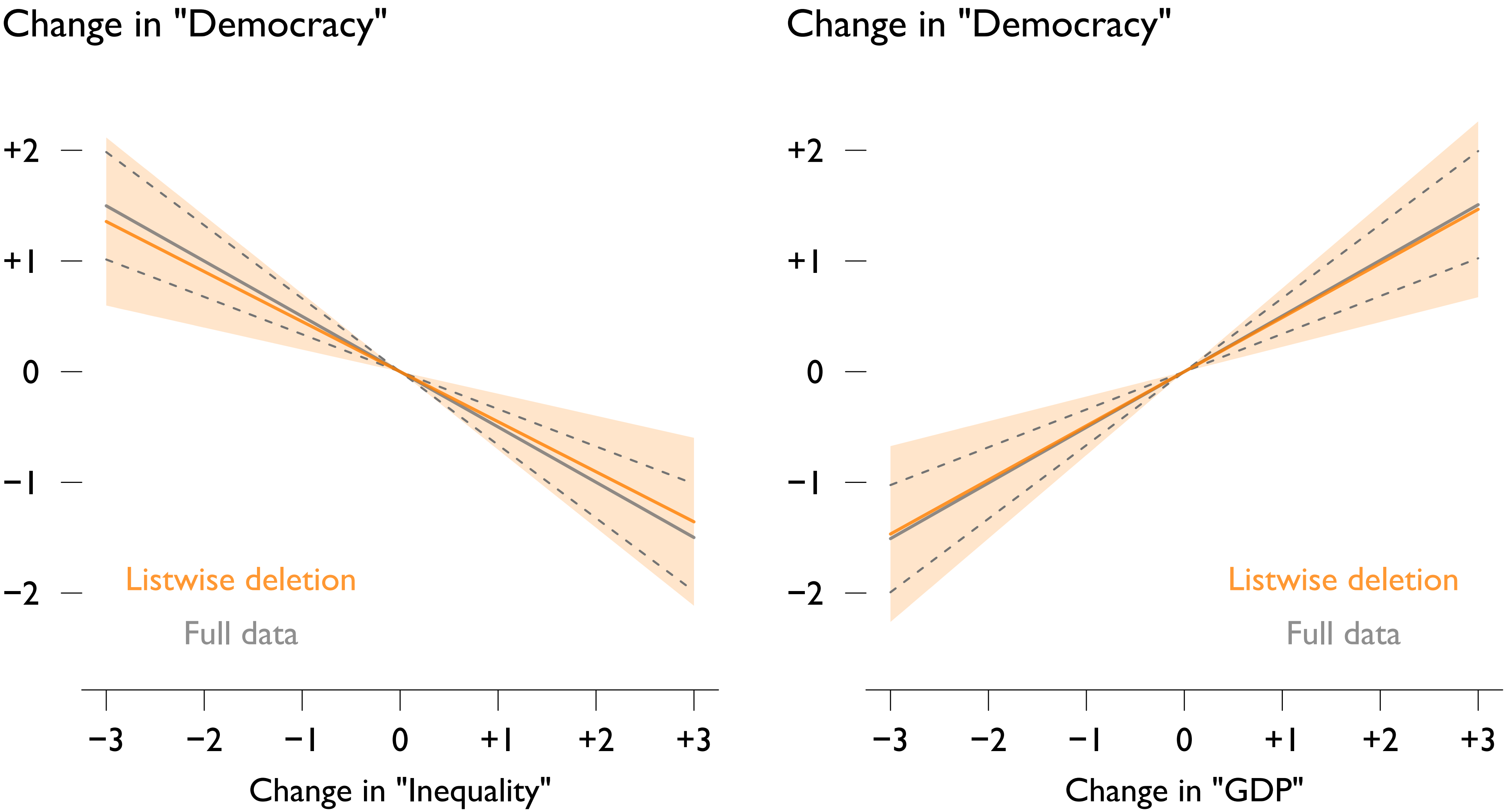
But this isn't just wasted data collection effort:

listwise deletion is statistically inefficient
and often creates statistical bias



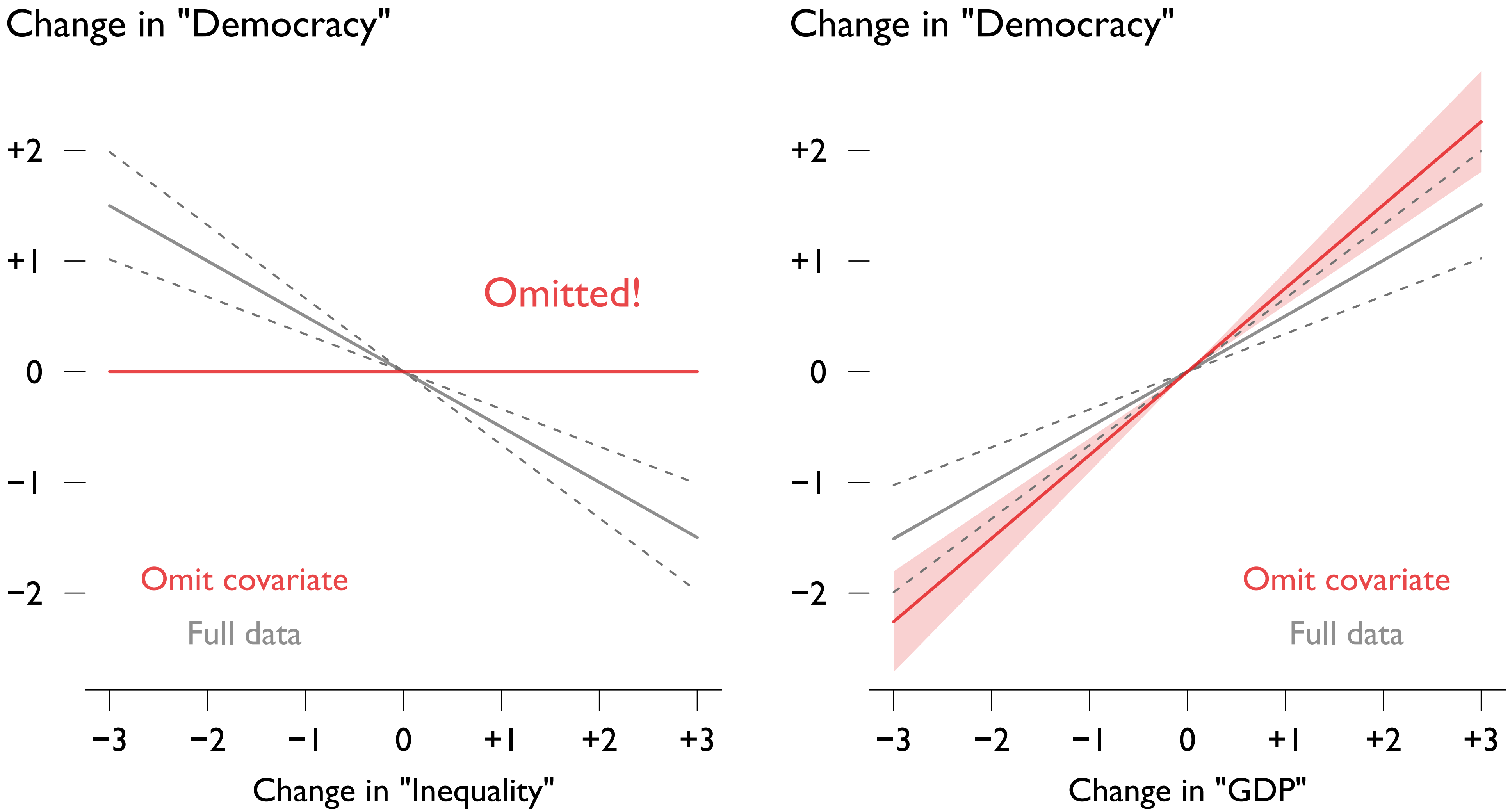
In our hypothetical example, listwise deletion is biased:
the relationship between Democracy & Inequality is attenuated

It's also inefficient: CIs are wider than they should be,
so we might fail to detect significant relationships because of missingness



Why did we listwise delete?

Why not drop Inequality from the model instead?



Even if we didn't care about estimating the relationship between Inequality and GDP, we still need it in the model

Including Inequality is necessary to get unbiased estimates of the effect of GDP, because it is correleted with both Inequality & Democracy

Crude imputation methods don’t help

Listwise deletion just trades one problem – omitted variable bias – for another – inefficiency and possible bias from sample selection

The latter occurs, as in the introductory example, when the missingness of a covariate is correlated with the value of the outcome

If both approaches are statistically flawed, what about filling in the missing data?

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The latter occurs, as in the introductory example, when the missingness of a covariate is correlated with the value of the outcome

If both approaches are statistically flawed, what about filling in the missing data?

This approach called *imputation*, and there are obvious crude methods:

Mean imputation Fill in missing x_i's with unconditional expected values, $\bar{x}_i$

Single imputation Fill in missing x_i's with conditional expected values, $\mathbb{E}(x_i|y_i, z_i)$

Neither crude approach works

Both are *worse* than listwise deletion most of the time

Monte Carlo run 1, with missing values

i	Democracy$_i$	Inequality$_i$	GDP$_i$
[1]	1.94	-0.16	1.28
[2]	0.26	NA	-0.21
[3]	0.97	NA	-0.66
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[5]	3.17	-2.94	0.96
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⋮	⋮	⋮	⋮

Above are the first six observations, now showing the effects of missing data

Mean imputation says to replace each `NA` with the observed mean of that variable

Monte Carlo run 1, mean imputation

i	Democracy$_i$	Inequality$_i$	GDP$_i$
[1]	1.94	-0.16	1.28
[2]	0.26	-0.23	-0.21
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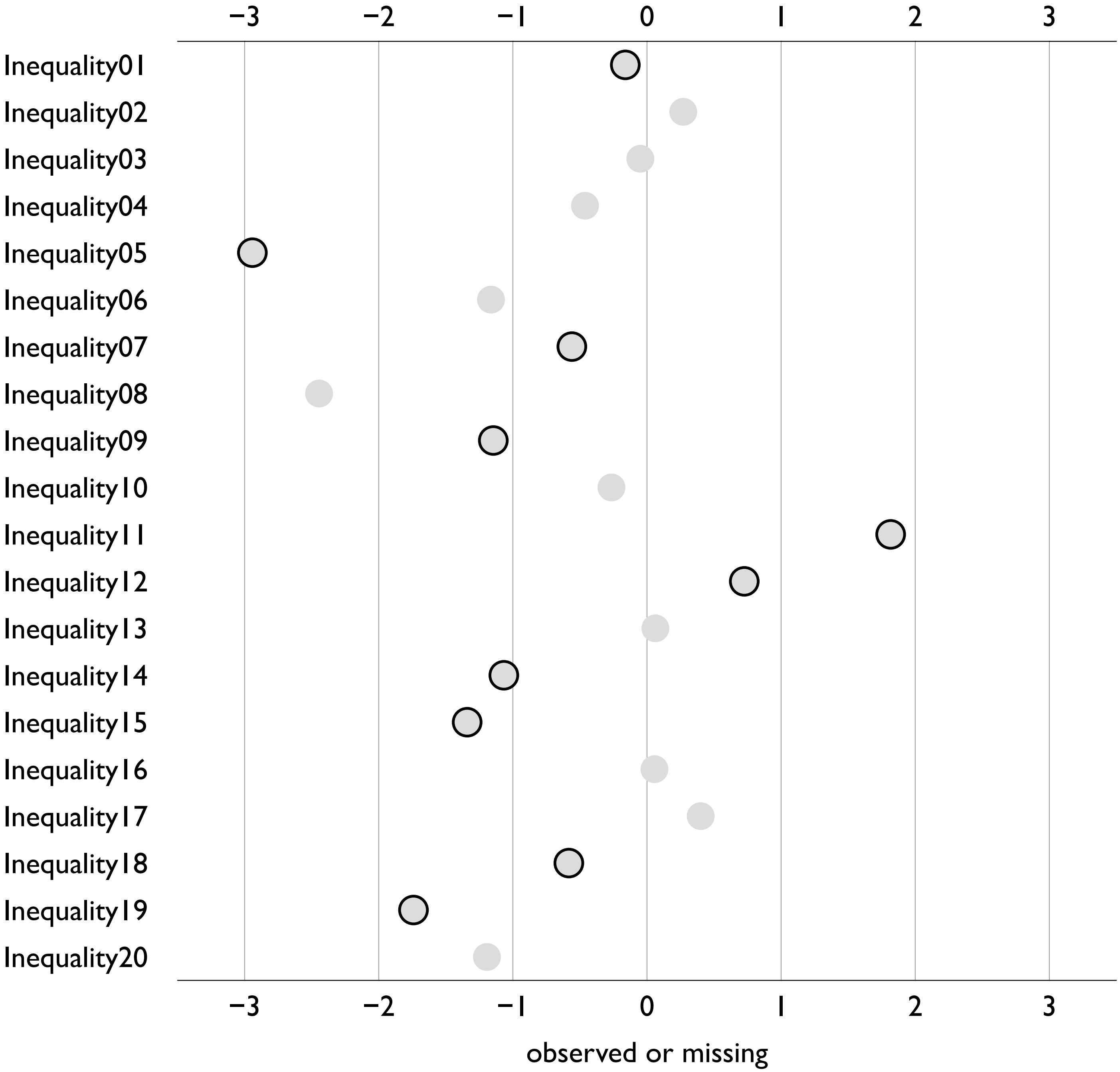
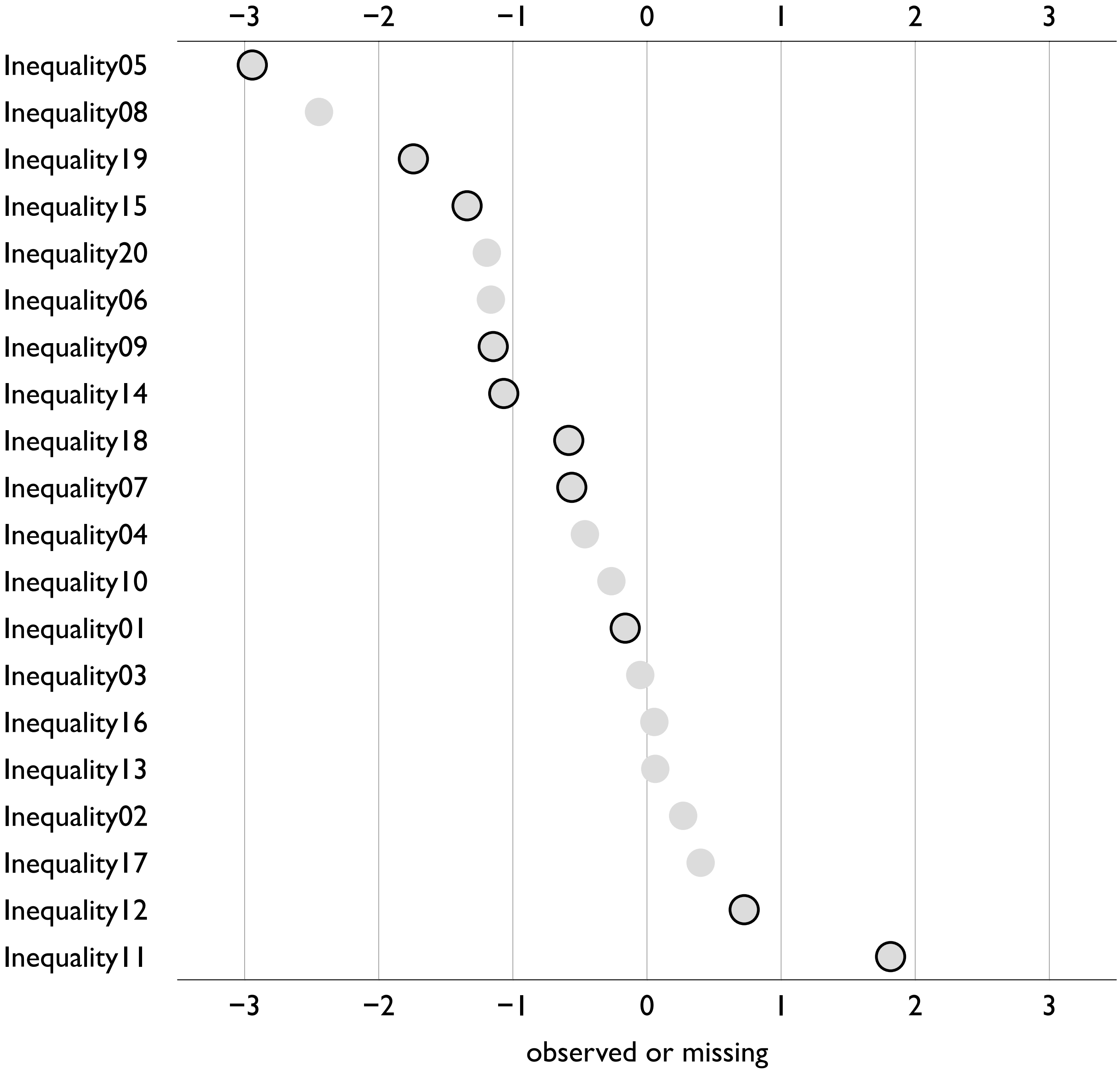
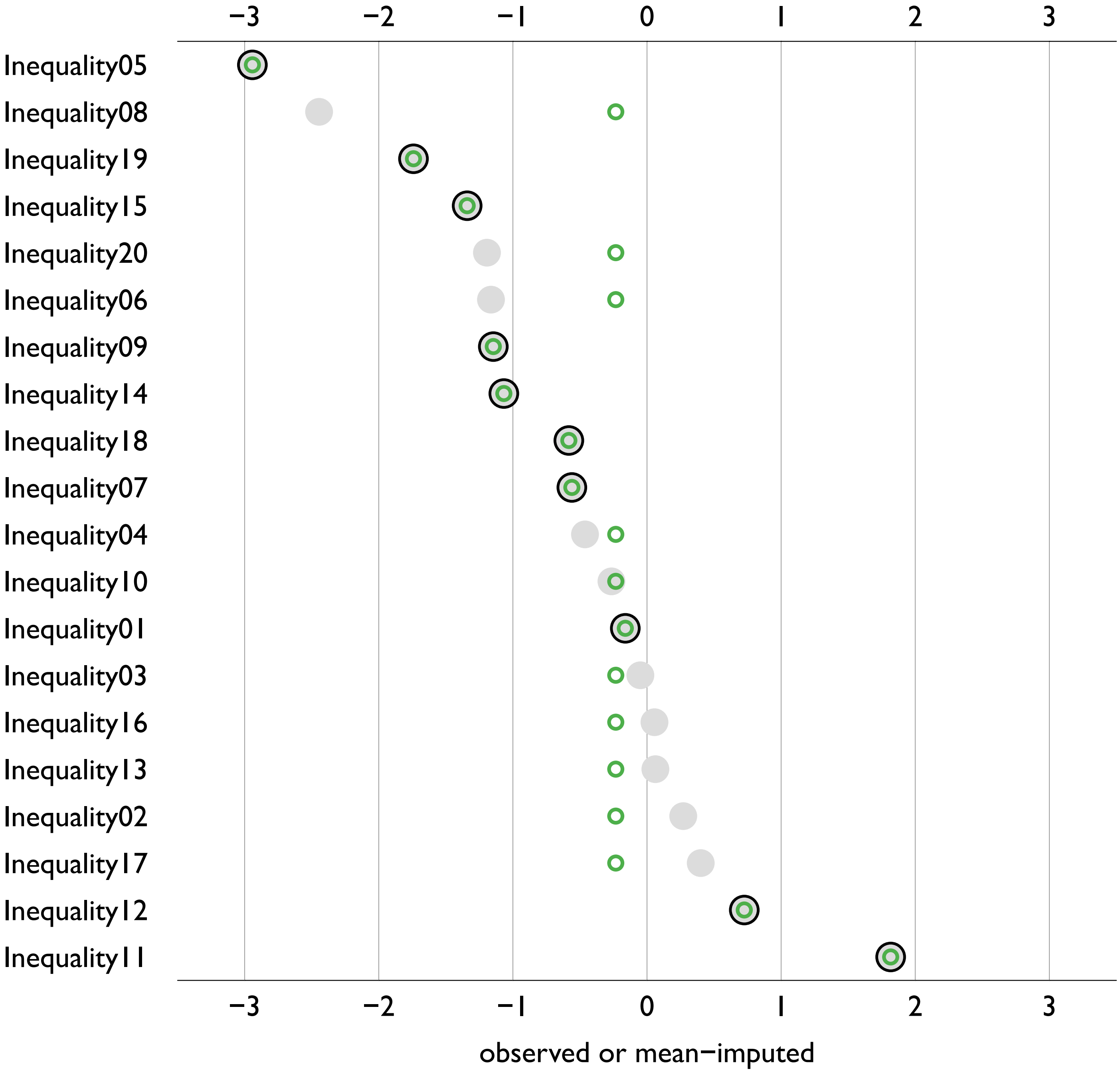
The observed mean of Inequality is -0.23



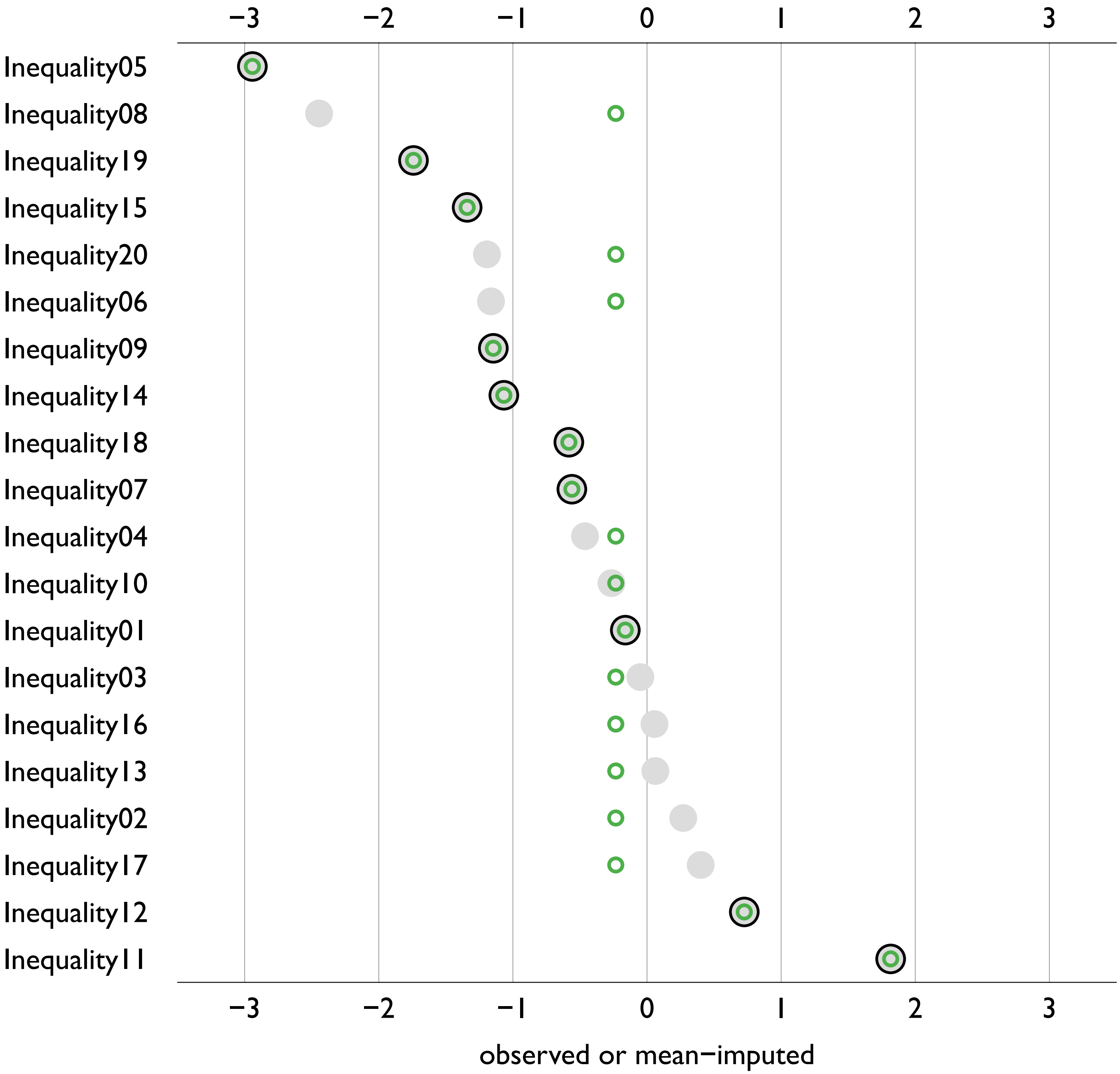
A visual representation of the first 20 cases, with non-missing cases ringed in black



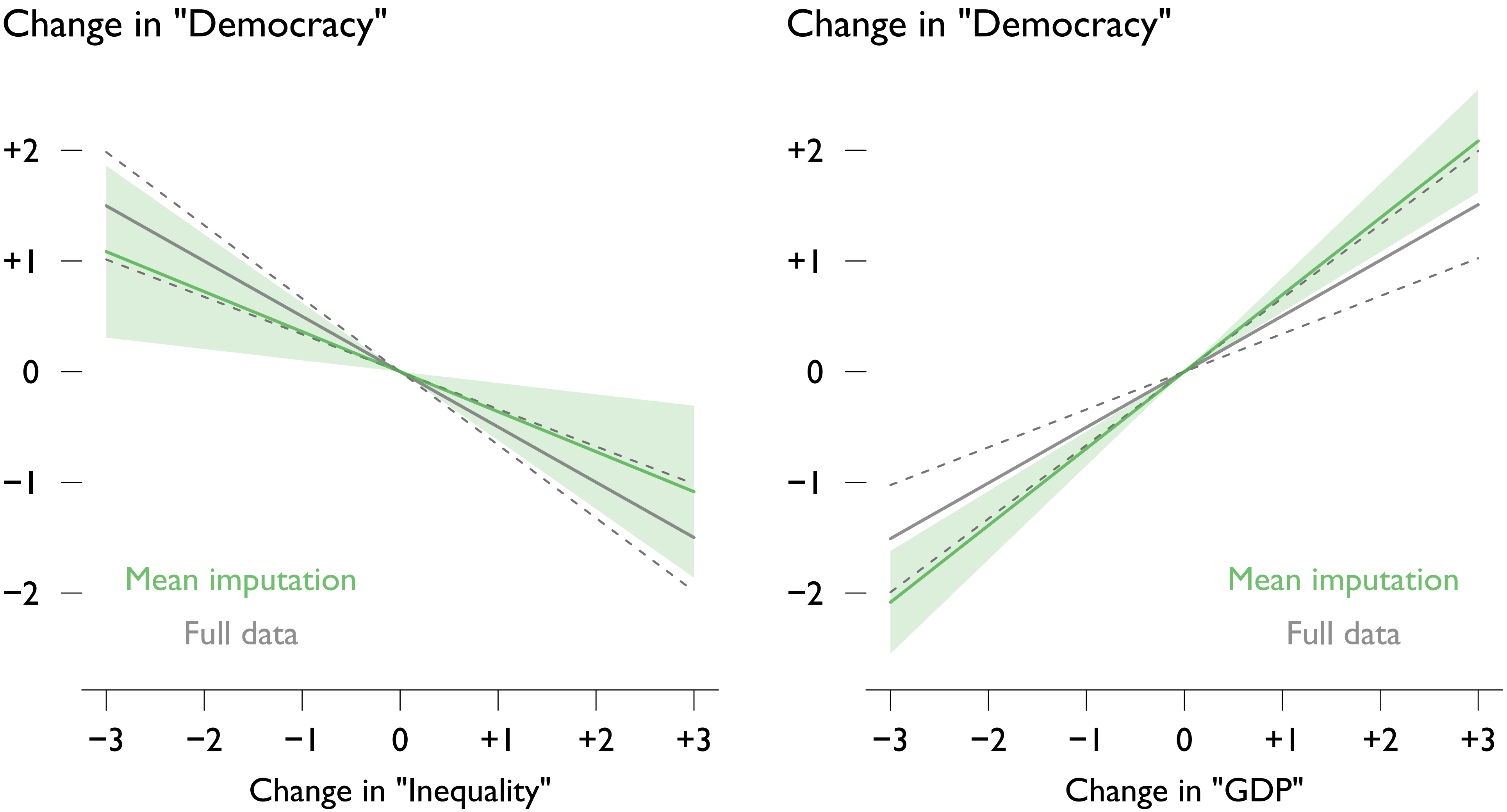
Sorting the cases by level of Inequality will aid comparison across methods



The mean-imputation completed dataset



Half real data, half very different (made-up) outliers!



Mean imputation biases coefficients for missing variables downwards

And biases correlated observed variables upwards

Why did this happen?

Why mean imputation doesn't work

1. *Filling in missings with the mean* assumes
there's no relationship among our variables

But the whole reason for the model is to *measure* the conditional relationship!

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For example, we to fill in the sixth observation, we need $\mathbb{E}(\text{Inequality}_6|\text{Democracy}_6, \text{GDP}_6)$, not the unconditional $\mathbb{E}(\text{Inequality})$

If Democracy is low in case 6,
and if Democracy is inversely correlated with Inequality,
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2. *Missing data has also biased our estimate of the mean, and we've translated that bias into our imputations*

The true sample mean of Inequality in the fully observed data is -0.03, not -0.23

Monte Carlo run 1, with missing values

i	Democracy$_i$	Inequality$_i$	GDP$_i$
[1]	1.94	-0.16	1.28
[2]	0.26	NA	-0.21
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$\vdots$	$\vdots$	$\vdots$	$\vdots$

Mean imputation failed because we didn't take the model into account

If our variables are correlated – and we think they are –
we need to condition on that correlation when imputing

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Suppose that we fit the following model for our fully observed cases:

$$\text{Inequality}_i = \gamma_0 + \gamma_1 \text{GDP}_i + \gamma_2 \text{Democracy}_i + \nu_i$$

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$$\text{Inequality}_i = \gamma_0 + \gamma_1 \text{GDP}_i + \gamma_2 \text{Democracy}_i + \nu_i$$

And then use the fitted values to fill-in missing values of Inequality j:

$$\mathbb{E}(\text{Inequality}_j) = \hat{\gamma}_0 + \hat{\gamma}_0 \text{GDP}_j + \hat{\gamma}_2 \text{Democracy}_j$$

Monte Carlo run 1, single imputation

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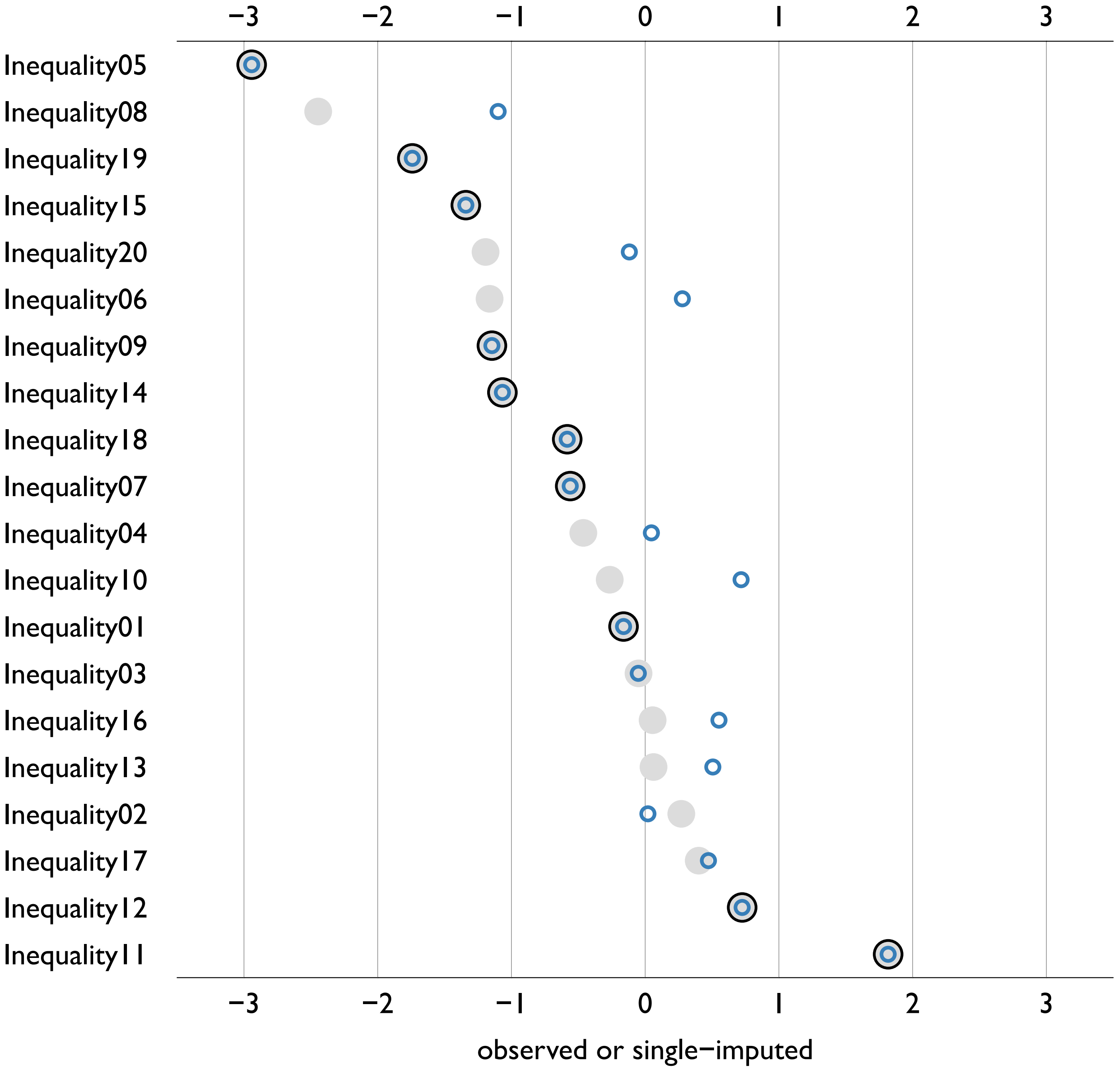
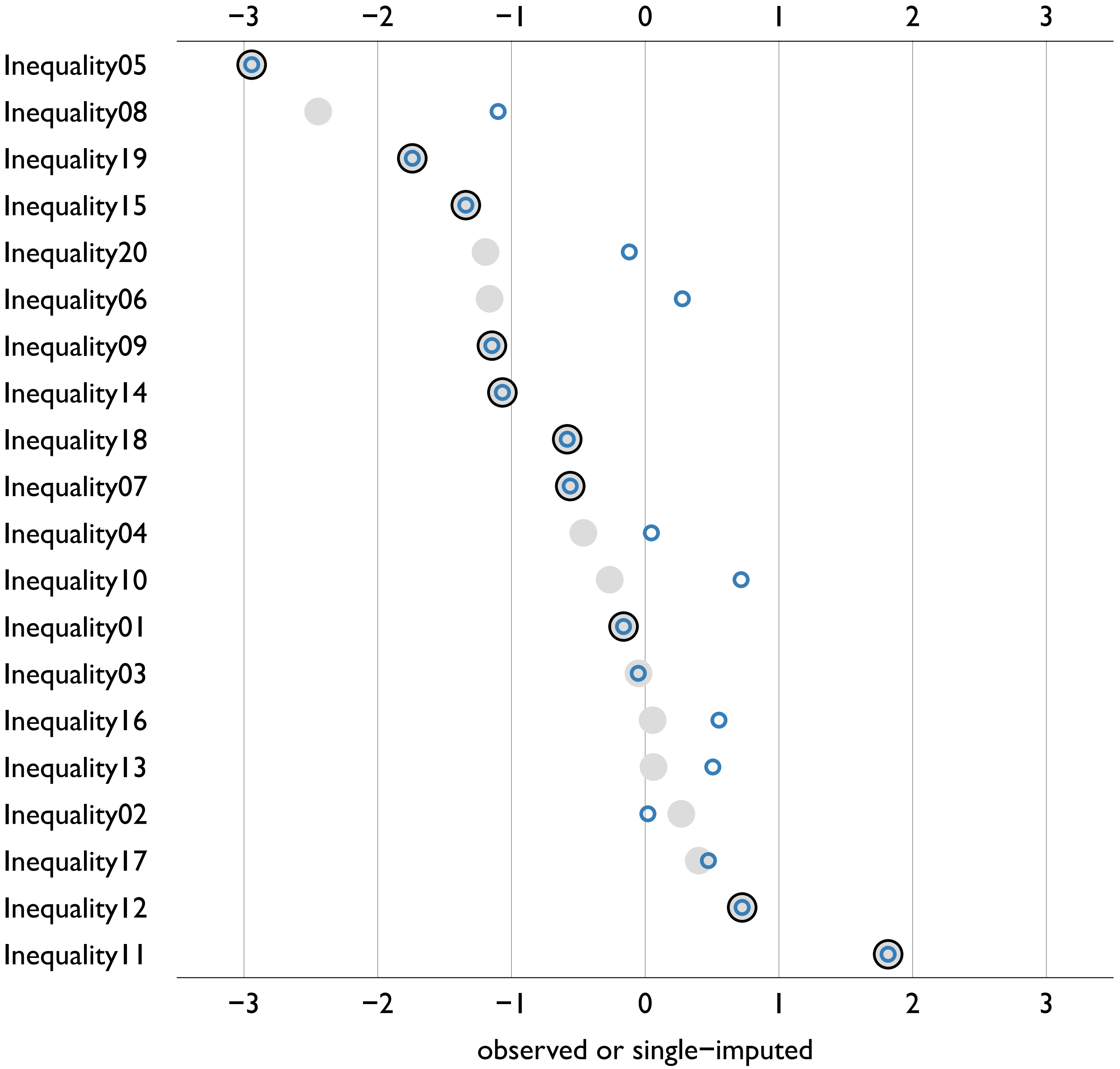
This seems better:
the imputed Inequality values at least seem consistent with the rest of the data

As noted, observation 6 has low democracy and is imputed to have higher inequality

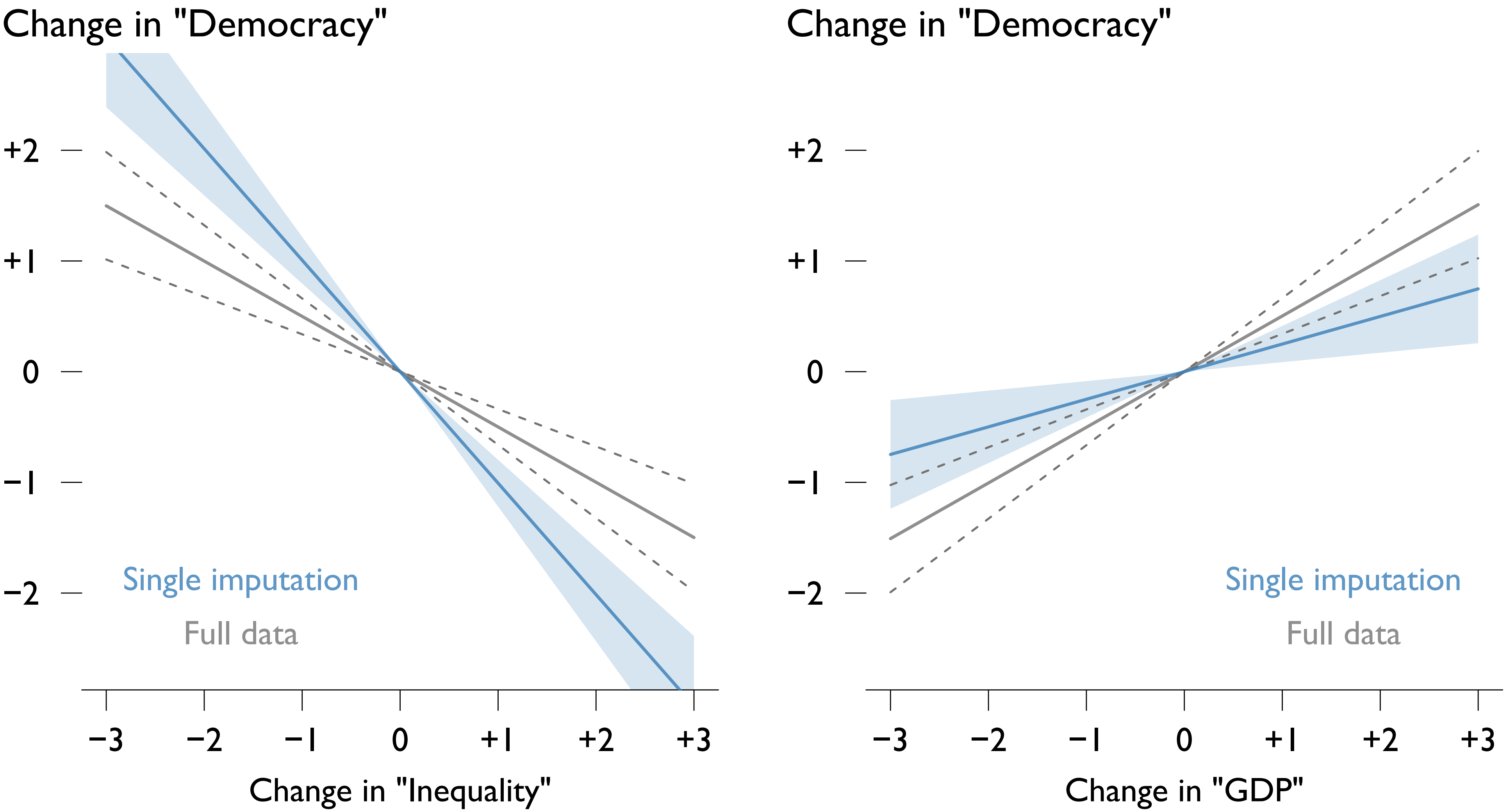
But actually, what we've done is *worse* than before



Our imputations still miss by a lot – yet we treat them as known *data*



For example, case 6 had a large random error – it's much lower than expected



Single imputation biases imputed variables upwards

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Why did this happen?

Why single imputation doesn't work

1. *We assumed any missing values were exactly equal to their conditional expected values, with* no error

But randomness is fundamental to all real world variables –
none of our other variables are deterministic functions of covariates

$\rightarrow$ we've assumed that the cases we didn't see are more consistent with our model than the cases we did see!

This leads to considerable overconfidence, and biases our β's upwards

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2. *How would we implement this approach consistently across cases if different or multiple variables are missing?*

3. *The linear model of Inequality is still estimated using listwise deletion, so the bias from LWD still passes on to our imputations*

This last objection suggests an infinite regress – how do we escape it?

Multiple imputation

Goals: (1) treat all observed values in our original data as known with certainty; (2) summarize the *uncertainty* about missing values implied by the observed data

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3. Includes the randomness of real world variables,
which can't be exactly predicted even by the true model

Multiple imputation is a family of methods that achieve these goals

Unless stringent assumptions are met, MI improves on listwise deletion

We focus on the King, Honaker *et al* method known as Amelia

How Amelia works

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Key assumption of Amelia: all these variables are jointly multivariate normal

$$\mathbf{D} \overset{\text{iid}}{\sim} \text{Multivariate Normal}(\boldsymbol{\mu}, \boldsymbol{\Sigma})$$

To impute missing elements of $\mathbf{D}$, we first need to estimate $\boldsymbol{\mu}$ and $\boldsymbol{\Sigma}$

The iid MVN assumption implies this likelihood for the joint distribution of the data

$$\mathcal{L}(\boldsymbol{\mu}, \boldsymbol{\Sigma}|\mathbf{D}) = \prod_{i=1}^{N} f_{\mathcal{MVN}}(\mathbf{d}_i|\boldsymbol{\mu}, \boldsymbol{\Sigma})$$

where $\mathbf{d}_i$ refers to the ith observation in the dataset $\mathbf{D}$

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Variation across $\tilde{\mathbf{D}}_{\text{miss}}$ would summarize uncertainty about these imputations,

while the mean value of $\tilde{\mathbf{D}}_{\text{miss}}$ would capture the expected value the missing data

Often even a small number of imputed datasets is enough to summarize uncertainty

How Amelia works

$$\mathcal{L}(\boldsymbol{\mu}, \boldsymbol{\Sigma} | \mathbf{D}) = \prod_{i=1}^{N} f_{\mathcal{MVN}}(\mathbf{d}_i | \boldsymbol{\mu}, \boldsymbol{\Sigma})$$

But we *don't* know the true $\boldsymbol{\mu}$ and $\boldsymbol{\Sigma}$

If we try to estimate them from $\mathbf{D}_{\mathrm{obs}}$ only using listwise deletion, we will have biased estimates, as in single imputation

How Amelia works

$$\mathcal{L}(\boldsymbol{\mu}, \boldsymbol{\Sigma}|\mathbf{D}) = \prod_{i=1}^{N} f_{\mathcal{MVN}}(\mathbf{d}_i|\boldsymbol{\mu}, \boldsymbol{\Sigma})$$

Instead, we use a method called *Expectation Maximization* (EM)
which iterates back and forth between two steps:

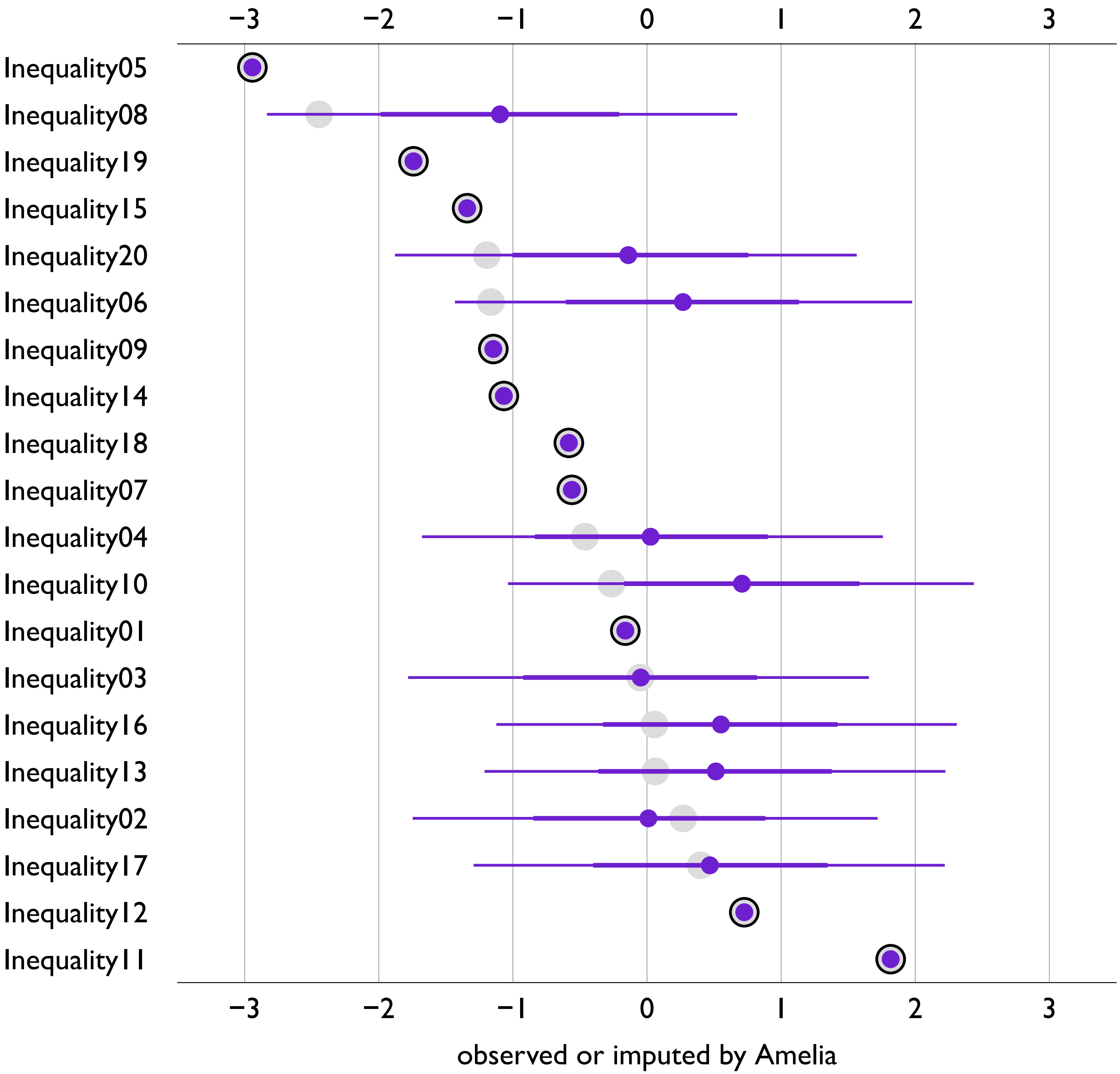
Expectation step Use the estimates $\hat{\boldsymbol{\mu}}$ and $\hat{\boldsymbol{\Sigma}}$ to fill in missing data $\mathbf{D}_{\text{miss}}$

Maximization step Use the filled-in matrix $\mathbf{D}$ to estimate $\hat{\boldsymbol{\mu}}$ and $\hat{\boldsymbol{\Sigma}}$

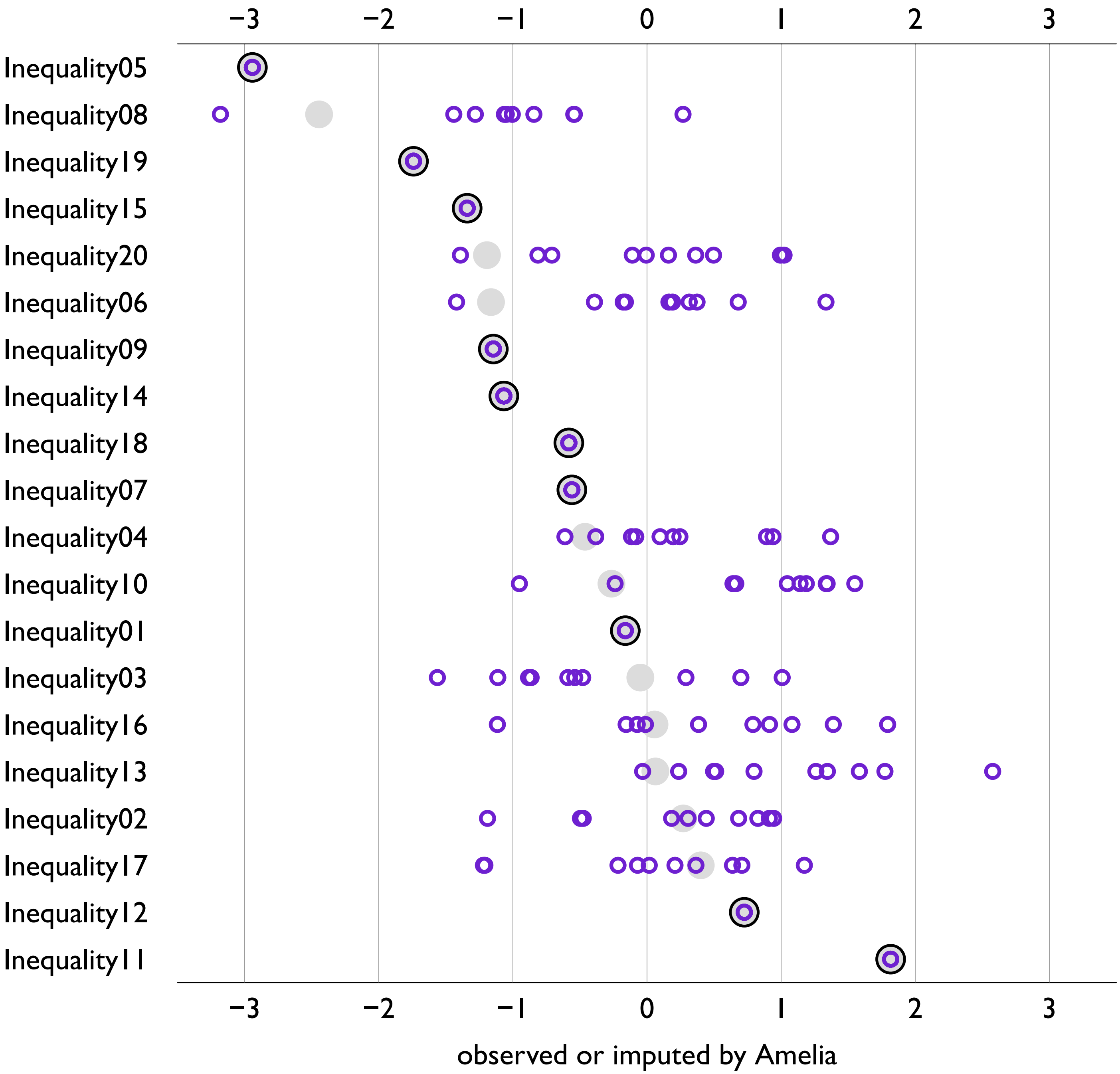
To get this "chicken-and-egg" process rolling, we supply starting values for $\hat{\boldsymbol{\mu}}$ and $\hat{\boldsymbol{\Sigma}}$

Then we iterate back-and-forth until convergence
and never need to delete any rows with missing data

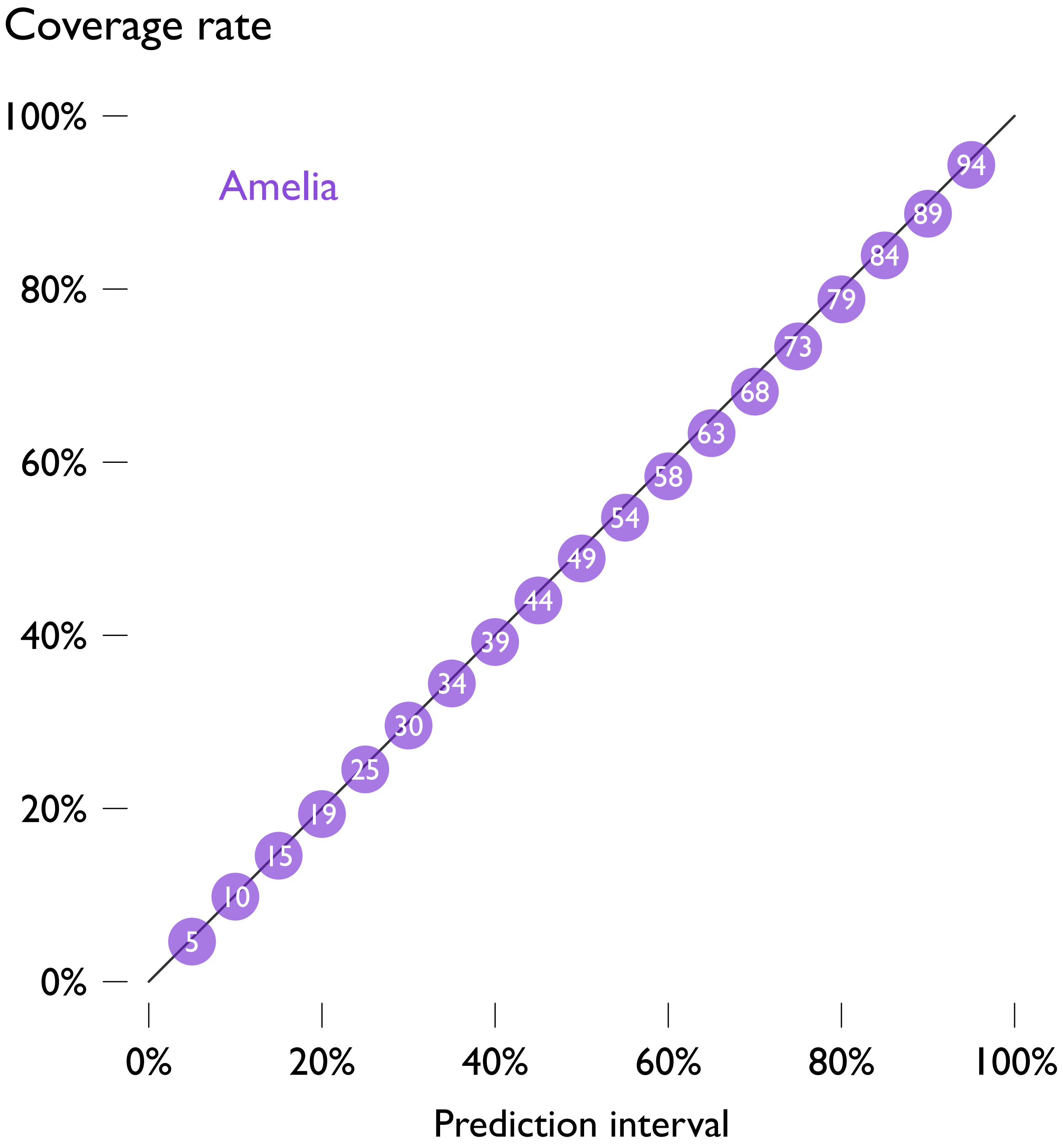
Naturally, there are a few extra pieces to the model
Bayesian priors, empirical priors, etc.



$\hat{\mu}$ and $\hat{\Sigma}$ allow us to compute posterior distributions over each missing datum



We summarize uncertainty with 5 (or 10, or more) draws from these posteriors



Across MC runs, Amelia’s posteriors over missing values have correct *coverage*

Monte Carlo run 1, multiple imputation I

i	Democracy$_i$	Inequality$_i$	GDP$_i$
[1]	1.94	-0.16	1.28
[2]	0.26	0.91	-0.21
[3]	0.97	-0.54	-0.66
[4]	0.17	0.10	-0.31
[5]	3.17	-2.94	0.96
[6]	-1.56	-0.18	0.28
$\vdots$	$\vdots$	$\vdots$	$\vdots$

Imputed dataset 1

We end up with not one but five or more imputed datasets

Collectively, these datasets provide the central tendency *and* uncertainty of the missing cases

Monte Carlo run 1, multiple imputation 2

i	Democracy$_i$	Inequality$_i$	GDP$_i$
[1]	1.94	-0.16	1.28
[2]	0.26	0.68	-0.21
[3]	0.97	-1.56	-0.66
[4]	0.17	0.89	-0.31
[5]	3.17	-2.94	0.96
[6]	-1.56	-0.39	0.28
⋮	⋮	⋮	⋮

Imputed dataset 2

We need to run all our analyses in parallel on the five datasets, then combine the results using simulation

Monte Carlo run 1, multiple imputation 3

i	Democracy$_i$	Inequality$_i$	GDP$_i$
[1]	1.94	-0.16	1.28
[2]	0.26	0.44	-0.21
[3]	0.97	0.29	-0.66
[4]	0.17	-0.61	-0.31
[5]	3.17	-2.94	0.96
[6]	-1.56	1.33	0.28
⋮	⋮	⋮	⋮

Imputed dataset 3

Specifically, take one-fifth of your simulated $\hat{\beta}$'s from each of your five analyses, then `rbind()` them together before computing counterfactual scenarios

Monte Carlo run 1, multiple imputation 4

i	Democracy$_i$	Inequality$_i$	GDP$_i$
[1]	1.94	-0.16	1.28
[2]	0.26	0.94	-0.21
[3]	0.97	-0.88	-0.66
[4]	0.17	0.25	-0.31
[5]	3.17	-2.94	0.96
[6]	-1.56	-0.16	0.28
⋮	⋮	⋮	⋮

Imputed dataset 4

`zelig()` in the `Zelig` package can automate this for you, but it only works for certain statistical models

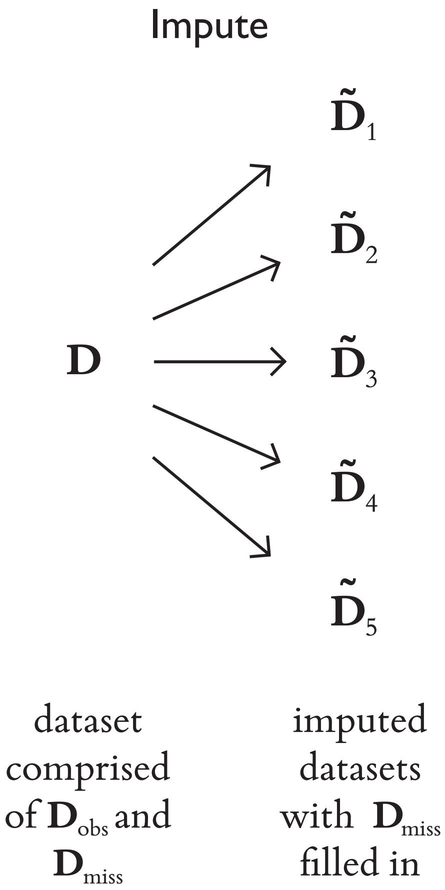
Monte Carlo run 1, multiple imputation 5

i	Democracy_i	Inequality_i	GDP_i
[1]	1.94	-0.16	1.28
[2]	0.26	0.18	-0.21
[3]	0.97	1.01	-0.66
[4]	0.17	0.94	-0.31
[5]	3.17	-2.94	0.96
[6]	-1.56	0.19	0.28
⋮	⋮	⋮	⋮

Imputed dataset 5

Instead, I recommend you write your own code, which is more flexible

Here's the multiple imputation workflow. . .



Impute

$\mathbf{D}$

$\tilde{\mathbf{D}}_1$

$\tilde{\mathbf{D}}_2$

$\tilde{\mathbf{D}}_3$

$\tilde{\mathbf{D}}_4$

$\tilde{\mathbf{D}}_5$

dataset comprised of $\mathbf{D}_{\text{obs}}$ and $\mathbf{D}_{\text{miss}}$

imputed datasets with $\mathbf{D}_{\text{miss}}$ filled in

Step 1: Perform multiple imputation to create $m = 5$ or more imputation datasets

(Very time consuming, especially if run multiple times under different assumptions)

Imputing splits the analysis into M streams,
so it helps to loop over the imputed datasets for each subsequent step

$$\begin{tikzcd}
\text{Impute} & & \text{Process} & \\
 & \tilde{\mathbf{D}}_1 \arrow[rr] & & \tilde{\mathbf{D}}'_1 \\
 & \tilde{\mathbf{D}}_2 \arrow[rr] & & \tilde{\mathbf{D}}'_2 \\
\mathbf{D} \arrow[ruu] \arrow[ru] \arrow[r] \arrow[rd] \arrow[rdd] & \tilde{\mathbf{D}}_3 \arrow[rr] & & \tilde{\mathbf{D}}'_3 \\
 & \tilde{\mathbf{D}}_4 \arrow[rr] & & \tilde{\mathbf{D}}'_4 \\
 & \tilde{\mathbf{D}}_5 \arrow[rr] & & \tilde{\mathbf{D}}'_5
\end{tikzcd}$$

dataset comprised of $\mathbf{D}_{\text{obs}}$ and $\mathbf{D}_{\text{miss}}$

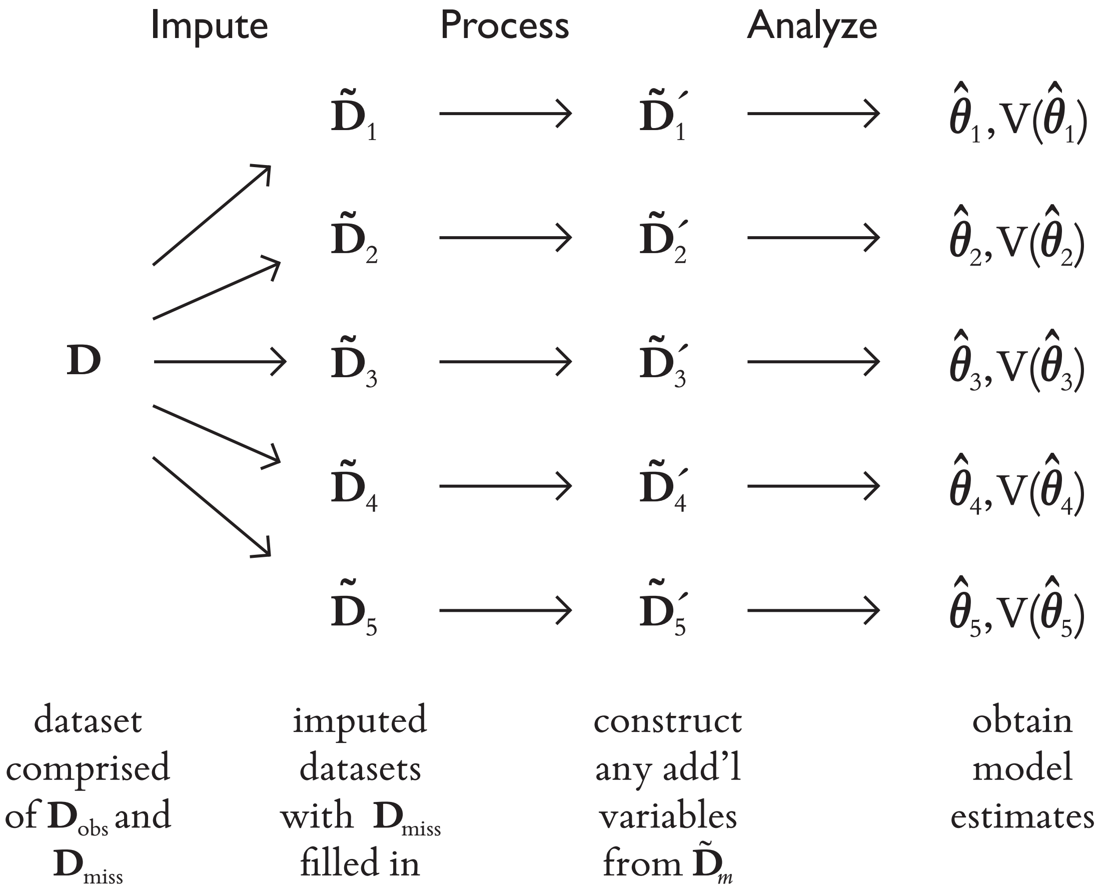
imputed datasets with $\mathbf{D}_{\text{miss}}$ filled in

construct any add’l variables from $\tilde{\mathbf{D}}_m$

Step 2: Construct additional variables from the imputed datasets

E.g., interaction terms, sums of components, or other products and sums

(e.g., if you impute GDP and popuation,
construct GDP per capita *after* all missings in either are imputed)



Impute

Process

Analyze

$\mathbf{D}$

$\tilde{\mathbf{D}}_1$ $\longrightarrow$ $\tilde{\mathbf{D}}'_1$ $\longrightarrow$ $\hat{\theta}_1, \mathrm{V}(\hat{\theta}_1)$

$\tilde{\mathbf{D}}_2$ $\longrightarrow$ $\tilde{\mathbf{D}}'_2$ $\longrightarrow$ $\hat{\theta}_2, \mathrm{V}(\hat{\theta}_2)$

$\tilde{\mathbf{D}}_3$ $\longrightarrow$ $\tilde{\mathbf{D}}'_3$ $\longrightarrow$ $\hat{\theta}_3, \mathrm{V}(\hat{\theta}_3)$

$\tilde{\mathbf{D}}_4$ $\longrightarrow$ $\tilde{\mathbf{D}}'_4$ $\longrightarrow$ $\hat{\theta}_4, \mathrm{V}(\hat{\theta}_4)$

$\tilde{\mathbf{D}}_5$ $\longrightarrow$ $\tilde{\mathbf{D}}'_5$ $\longrightarrow$ $\hat{\theta}_5, \mathrm{V}(\hat{\theta}_5)$

dataset comprised of $\mathbf{D}_{\text{obs}}$ and $\mathbf{D}_{\text{miss}}$

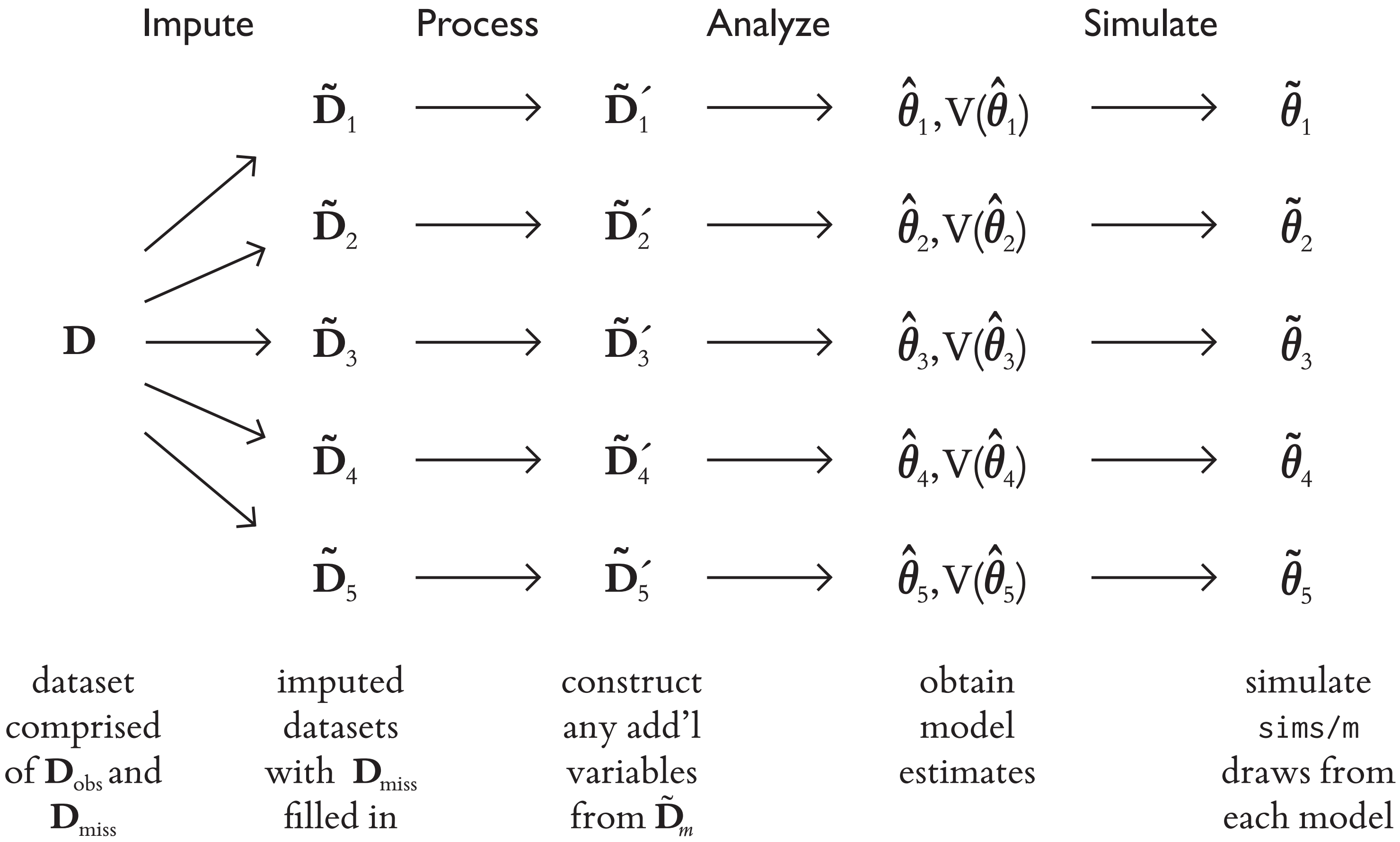
imputed datasets with $\mathbf{D}_{\text{miss}}$ filled in

construct any add'l variables from $\tilde{\mathbf{D}}_m$

obtain model estimates

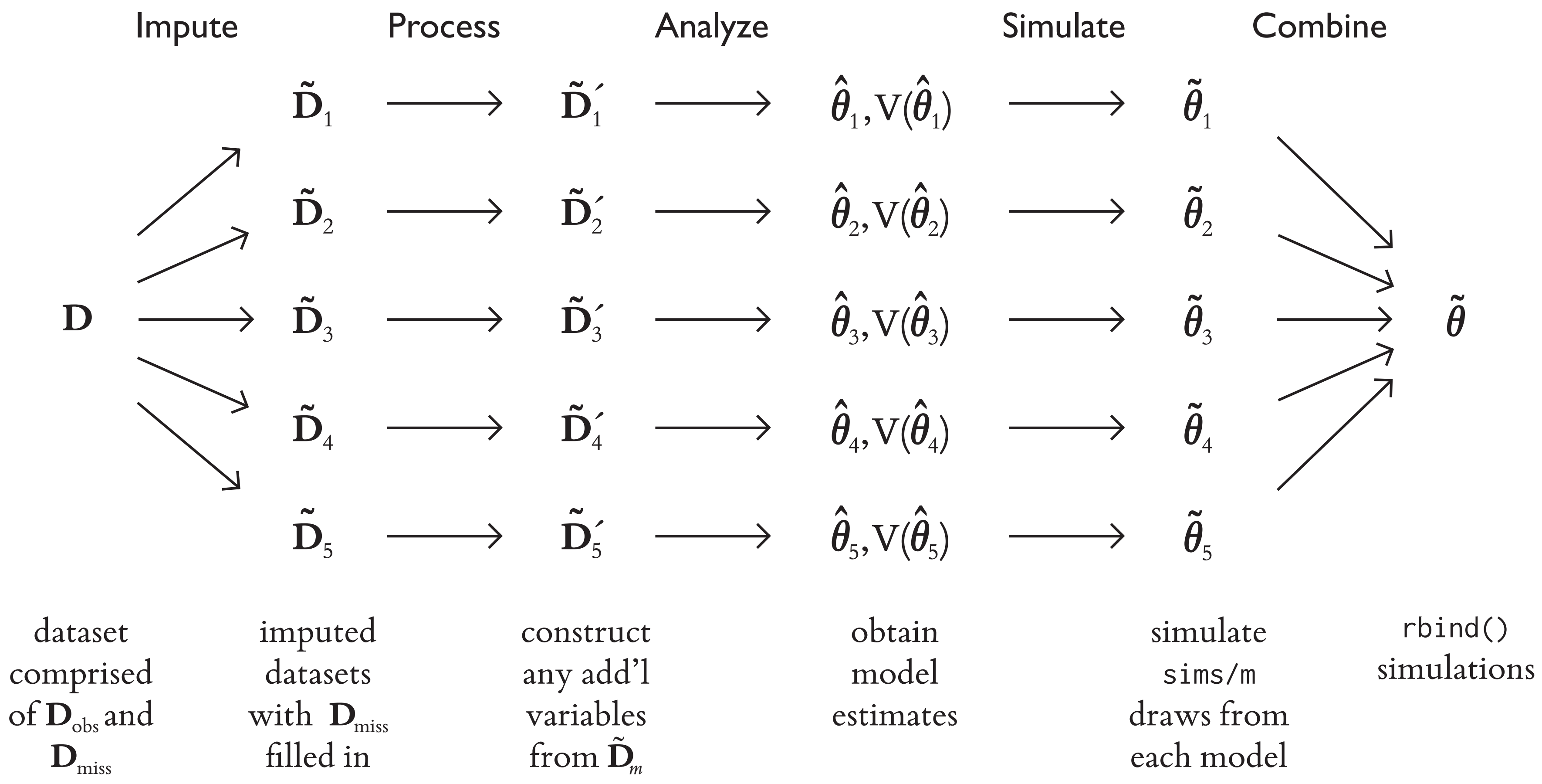
Step 3: Estimate the analysis model separately on each dataset m,
and save each set of estimates $\boldsymbol{\theta}_m$ and variance-covariance matrix $\mathrm{V}(\hat{\boldsymbol{\theta}}_m)$

Each model should be the *same*, so use a loop or `lapply()`


Impute
Process
Analyze
Simulate
$\mathbf{D}$
$\tilde{\mathbf{D}}_1$ $\tilde{\mathbf{D}}_1'$ $\hat{\theta}_1, \mathrm{V}(\hat{\theta}_1)$ $\tilde{\theta}_1$
$\tilde{\mathbf{D}}_2$ $\tilde{\mathbf{D}}_2'$ $\hat{\theta}_2, \mathrm{V}(\hat{\theta}_2)$ $\tilde{\theta}_2$
$\tilde{\mathbf{D}}_3$ $\tilde{\mathbf{D}}_3'$ $\hat{\theta}_3, \mathrm{V}(\hat{\theta}_3)$ $\tilde{\theta}_3$
$\tilde{\mathbf{D}}_4$ $\tilde{\mathbf{D}}_4'$ $\hat{\theta}_4, \mathrm{V}(\hat{\theta}_4)$ $\tilde{\theta}_4$
$\tilde{\mathbf{D}}_5$ $\tilde{\mathbf{D}}_5'$ $\hat{\theta}_5, \mathrm{V}(\hat{\theta}_5)$ $\tilde{\theta}_5$
dataset comprised of $\mathbf{D}_{\text{obs}}$ and $\mathbf{D}_{\text{miss}}$
imputed datasets with $\mathbf{D}_{\text{miss}}$ filled in
construct any add'l variables from $\tilde{\mathbf{D}}_m$
obtain model estimates
simulate `sims/m` draws from each model

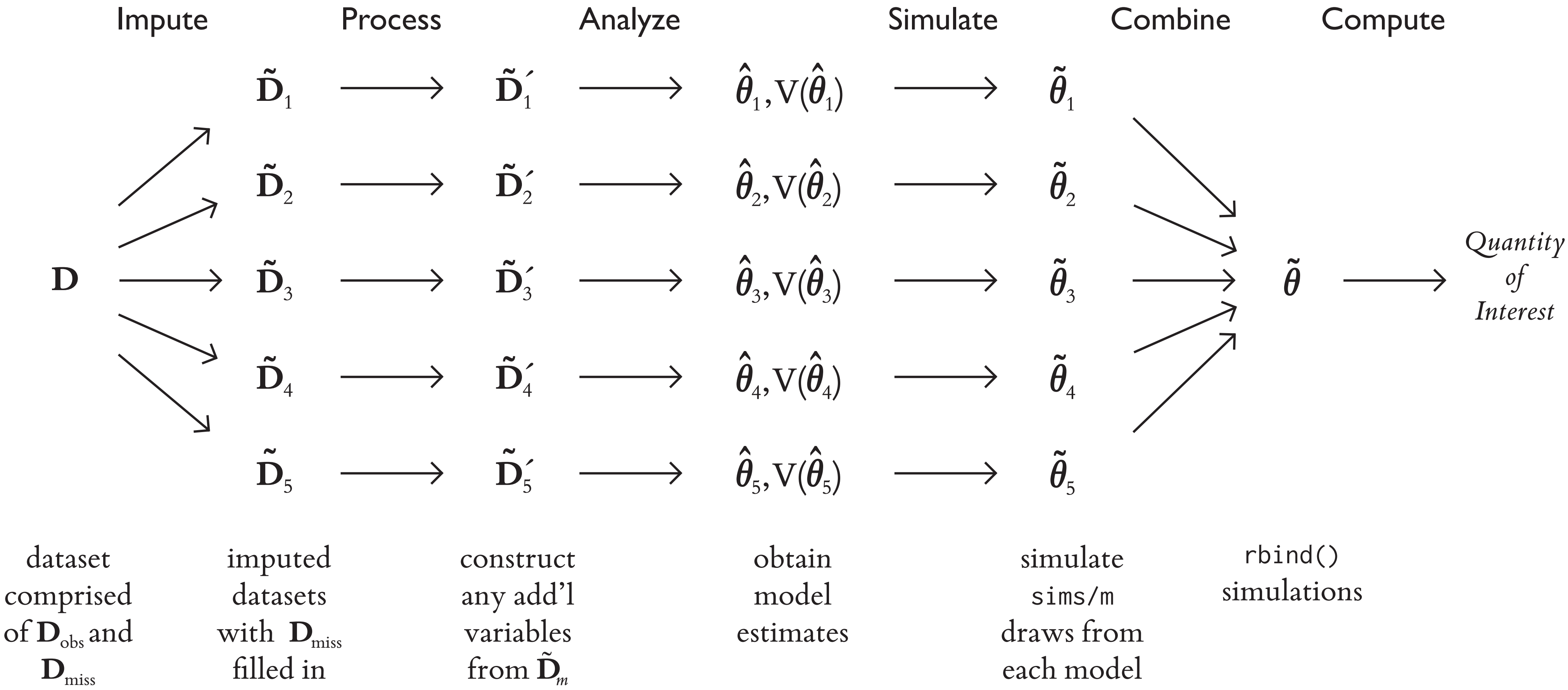
Step 4: Draw `sims/M` sets of simulated parameters from each of the M analyses

Use `mvrnorm()` as usual for this step, but in a loop over the M analysis runs


Impute
Process
Analyze
Simulate
Combine
$\mathbf{D}$
$\tilde{\mathbf{D}}_1$ ⟶ $\tilde{\mathbf{D}}_1'$ ⟶ $\hat{\theta}_1, \mathrm{V}(\hat{\theta}_1)$ ⟶ $\tilde{\theta}_1$
$\tilde{\mathbf{D}}_2$ ⟶ $\tilde{\mathbf{D}}_2'$ ⟶ $\hat{\theta}_2, \mathrm{V}(\hat{\theta}_2)$ ⟶ $\tilde{\theta}_2$
$\tilde{\mathbf{D}}_3$ ⟶ $\tilde{\mathbf{D}}_3'$ ⟶ $\hat{\theta}_3, \mathrm{V}(\hat{\theta}_3)$ ⟶ $\tilde{\theta}_3$
$\tilde{\mathbf{D}}_4$ ⟶ $\tilde{\mathbf{D}}_4'$ ⟶ $\hat{\theta}_4, \mathrm{V}(\hat{\theta}_4)$ ⟶ $\tilde{\theta}_4$
$\tilde{\mathbf{D}}_5$ ⟶ $\tilde{\mathbf{D}}_5'$ ⟶ $\hat{\theta}_5, \mathrm{V}(\hat{\theta}_5)$ ⟶ $\tilde{\theta}_5$
$\tilde{\theta}$
dataset comprised of $\mathbf{D}_{\text{obs}}$ and $\mathbf{D}_{\text{miss}}$
imputed datasets with $\mathbf{D}_{\text{miss}}$ filled in
construct any add'l variables from $\tilde{\mathbf{D}}_m$
obtain model estimates
simulate `sims/m` draws from each model
`rbind()` simulations

Step 5: Combine the M sets of simulated parameters into a single matrix using `rbind()`

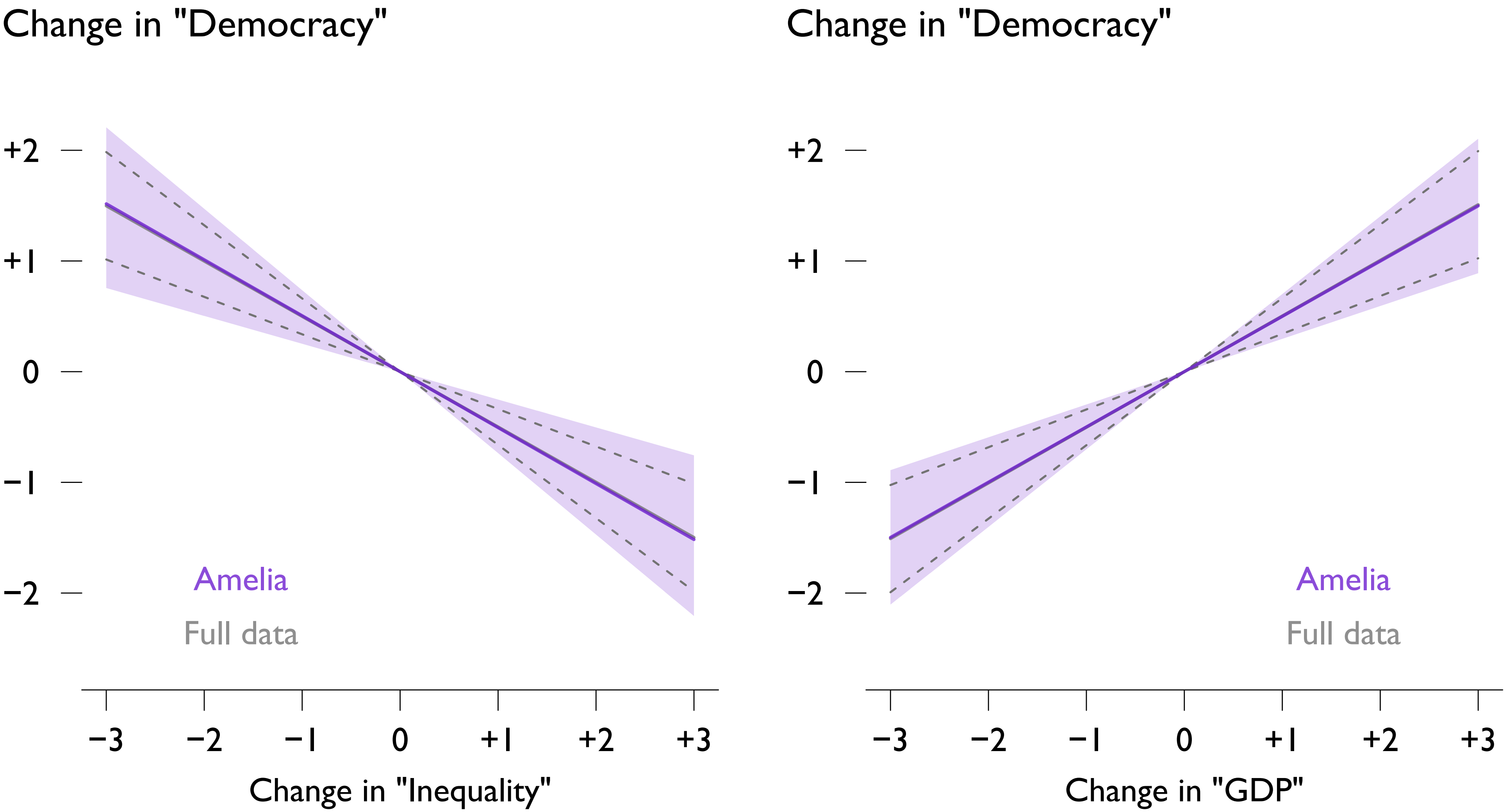
This brings the $M = 5$ streams of the analysis back together;
after this step, we only need to do things *once* for the whole analysis


Impute
Process
Analyze
Simulate
Combine
Compute
$\mathbf{D}$
$\tilde{\mathbf{D}}_1$
$\tilde{\mathbf{D}}_2$
$\tilde{\mathbf{D}}_3$
$\tilde{\mathbf{D}}_4$
$\tilde{\mathbf{D}}_5$
$\tilde{\mathbf{D}}'_1$
$\tilde{\mathbf{D}}'_2$
$\tilde{\mathbf{D}}'_3$
$\tilde{\mathbf{D}}'_4$
$\tilde{\mathbf{D}}'_5$
$\hat{\theta}_1, \mathrm{V}(\hat{\theta}_1)$
$\hat{\theta}_2, \mathrm{V}(\hat{\theta}_2)$
$\hat{\theta}_3, \mathrm{V}(\hat{\theta}_3)$
$\hat{\theta}_4, \mathrm{V}(\hat{\theta}_4)$
$\hat{\theta}_5, \mathrm{V}(\hat{\theta}_5)$
$\tilde{\theta}_1$
$\tilde{\theta}_2$
$\tilde{\theta}_3$
$\tilde{\theta}_4$
$\tilde{\theta}_5$
$\tilde{\theta}$
Quantity of Interest
dataset comprised of $\mathbf{D}_{\text{obs}}$ and $\mathbf{D}_{\text{miss}}$
imputed datasets with $\mathbf{D}_{\text{miss}}$ filled in
construct any add'l variables from $\tilde{\mathbf{D}}_m$
obtain model estimates
simulate `sims/m` draws from each model
`rbind()` simulations

Step 6: Produce counterfactual scenarios and graphics as usual

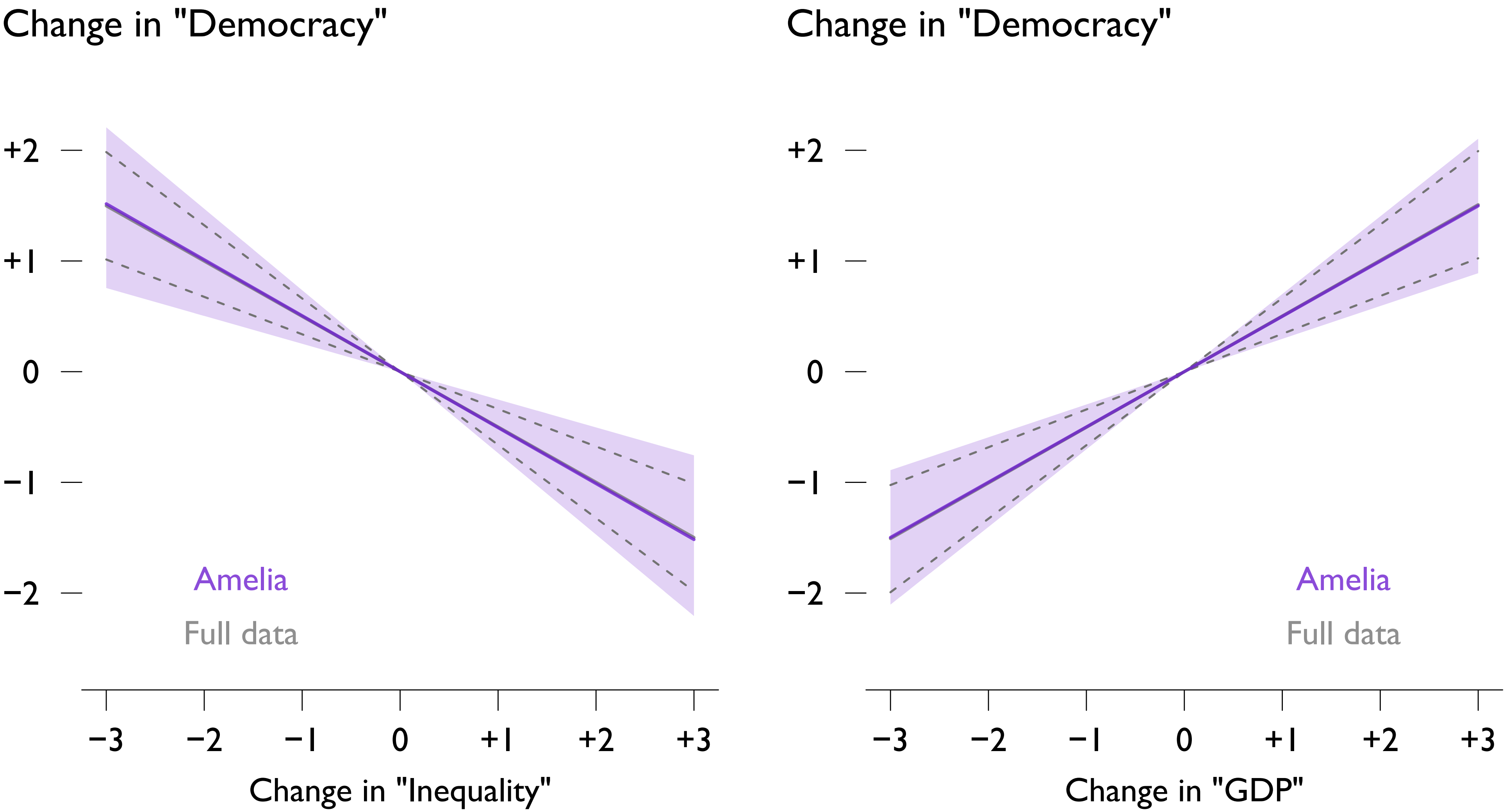
The code for this step can be exactly the same as for a non-imputation analysis

You may wish to average the $M = 5$ datasets at this stage for computing factual and counterfactual values of the covariates



Success! We have closely matched the original full data results

We've gotten more information & precision out of our data than with LWD, and not added any bias despite imputing



Will multiple imputation always work this well?

Should we ever listwise delete instead?

Outcome y is missing as a function of. . .

	Itself	*Covariate x*	*Covariate z*	*Auxilliaries*	*None of these*
	NI	MAR	MAR	MAR	MCAR
LWD	Biased[★]			Inefficient	
MI	Biased				

Covariate x is missing as a function of. . .

	Outcome y	*Itself*	*Covariate z*	*Auxilliaries*	*None of these*
	MAR	NI	MAR	MAR	MCAR
LWD	Biased	Inefficient[†]	Inefficient	Inefficient	Inefficient[‡]
MI		Biased			

Choose the row with your method for dealing with missing data:
either listwise deletion or multiple imputation

Each column describes a potential mechanism by which missingness occurs

Your method has all the problems listed in the relevant cells

If you have all blank cells, your method is unbiased and efficient

Outcome y is missing as a function of…

	Itself	*Covariate x*	*Covariate z*	*Auxilliaries*	*None of these*
	NI	MAR	MAR	MAR	MCAR
LWD	Biased[*]			Inefficient	
MI	Biased				

Covariate x is missing as a function of…

	Outcome y	*Itself*	*Covariate z*	*Auxilliaries*	*None of these*
	MAR	NI	MAR	MAR	MCAR
LWD	Biased	Inefficient[†]	Inefficient	Inefficient	Inefficient[‡]
MI		Biased			

Non-ignorable (NI) missingness. After controlling for observables, whether a datum is missing depends on the missing datum. Unbiased imputation impossible

Missing at random (MAR). Pattern of missingness is related to observed values in dataset, and seemingly purely random once that pattern is controlled for

Missing completely at random (MCAR). Pattern of missingness is uncorrelated with all variables in the model, and thus seemingly purely random

Outcome y is missing as a function of…

	Itself	*Covariate x*	*Covariate z*	*Auxilliaries*	*None of these*
	NI	MAR	MAR	MAR	MCAR
LWD	Biased[*]			Inefficient	
MI	Biased				

Covariate x is missing as a function of…

	Outcome y	*Itself*	*Covariate z*	*Auxilliaries*	*None of these*
	MAR	NI	MAR	MAR	MCAR
LWD	Biased	Inefficient[†]	Inefficient	Inefficient	Inefficient[‡]
MI		Biased			

[*] Logit unbiased in this case if missingness does not depend on covariates

[†] It's complicated: unbiased if missingness of x only depends on x (!) or other covariates; biased if also depends on y

[‡] Assumes you have multiple covariates, ≥ 1 of which is observed when x is missing

Can you identify cases/assumptions where LWD is superior to MI?

Outcome y is missing as a function of. . .

	Itself	*Covariate x*	*Covariate z*	*Auxilliaries*	*None of these*
	NI	MAR	MAR	MAR	MCAR
LWD	Biased[⋆]			Inefficient	
MI	Biased				

Covariate x is missing as a function of. . .

	Outcome y	*Itself*	*Covariate z*	*Auxilliaries*	*None of these*
	MAR	NI	MAR	MAR	MCAR
LWD	Biased	Inefficient[†]	Inefficient	Inefficient	Inefficient[‡]
MI		Biased			

Most applications of LWD have efficiency costs: MI can produce more efficient results

If pattern of missingness in y depends on x, or vice versa, then LWD will be biased. MI can repair the bias – provided missingness can be predicted using observed data

If the pattern of missingness in y (or x) depends on the values of y (or x) that are missing, no method can eliminate bias, but careful use of MI may help sometimes

Outcome y is missing as a function of…

	Itself	*Covariate x*	*Covariate z*	*Auxilliaries*	*None of these*
	NI	MAR	MAR	MAR	MCAR
LWD	Biased*			Inefficient	
MI	Biased				

Covariate x is missing as a function of…

	Outcome y	*Itself*	*Covariate z*	*Auxilliaries*	*None of these*
	MAR	NI	MAR	MAR	MCAR
LWD	Biased	Inefficient†	Inefficient	Inefficient	Inefficient‡
MI		Biased			

Common misconception: *"you can't impute missing values of an outcome variable"*

1. No benefit to MI if only y has missings & no auxiliary variables present
2. Shouldn't impute if only y has missings in a logistic regression & no aux help
3. *Should* impute y as needed for imputation models of missing covariates, or any time helpful auxillary variables correlated with y are available

Outcome y is missing as a function of…

	Itself	*Covariate x*	*Covariate z*	*Auxilliaries*	*None of these*
	NI	MAR	MAR	MAR	MCAR
LWD	Biased[⋆]			Inefficient	
MI	Biased				

Covariate x is missing as a function of…

	Outcome y	*Itself*	*Covariate z*	*Auxilliaries*	*None of these*
	MAR	NI	MAR	MAR	MCAR
LWD	Biased	Inefficient[†]	Inefficient	Inefficient	Inefficient[‡]
MI		Biased			

Finally, multiple imputation is not magical

1. MI can't help if all of your covariates and auxilliaries are missing for a case

2. May fail if you try to impute a dataset that has a very high percentage of missing values, or some variables which are almost never observed

You may need to give up on some variables in this case (exclude from your study)

Special considerations for effective use of Amelia

Key issue: maintaining the assumption that data are jointly Multivariate Normal

- transform continous variables to be as close to Normal as possible,
e.g., through log, logit, or quadratic transformations

- tell your imputation model which variables are ordered or categorical –
note King et al recommend treating *binary* variables as MVN

- check available diagnostics to make sure imputation worked

Special considerations for effective use of Amelia

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e.g., through log, logit, or quadratic transformations

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note King et al recommend treating *binary* variables as MVN

- check available diagnostics to make sure imputation worked

Two additional best practices for all multiple imputation methods:

- include in the imputation as many well-observed variables related to your partially observed variables as you can find

These auxilliary variables don't need to be in the analysis model later

- *every* variable in the analysis model *must* also be in the imputation model

Implementing Amelia for cross-sectional data

In `R`, the `amelia()` function in the `Amelia` package does multiple imputation for cross-sectional, time series, and TSCS data

For cross-sectional data, it's usually very easy to make your imputed datasets:

```
library(Amelia)

# Run Amelia and save imputed data, and number of imputed datasets
nimp <- 5   # Use nimp=5 at minimum; 10 often a good idea
amelia.res <- amelia(observedData, m=nimp)
miData <- amelia.res$imputations
```

`MiData` is a list object with `nimp` elements, each of which is a complete dataset

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nimp <- 5   # Use nimp=5 at minimum; 10 often a good idea
amelia.res <- amelia(observedData, m=nimp)
miData <- amelia.res$imputations
```

MiData is a list object with nimp elements, each of which is a complete dataset

Then run your analysis nimp times in a loop, saving each result in a list object:

```
# Run least squares on each imputed dataset,
# and save results in a list vector
mi <- vector("list", nimp)
for (i in 1:nimp) {
  mi[[i]] <- lm(y ~ x + z, data=miData[[i]])
}
```

Multiple imputation for cross-sectional data

Next, combine the results by drawing one-`nimp`th of your simulated β's from each model, like so:

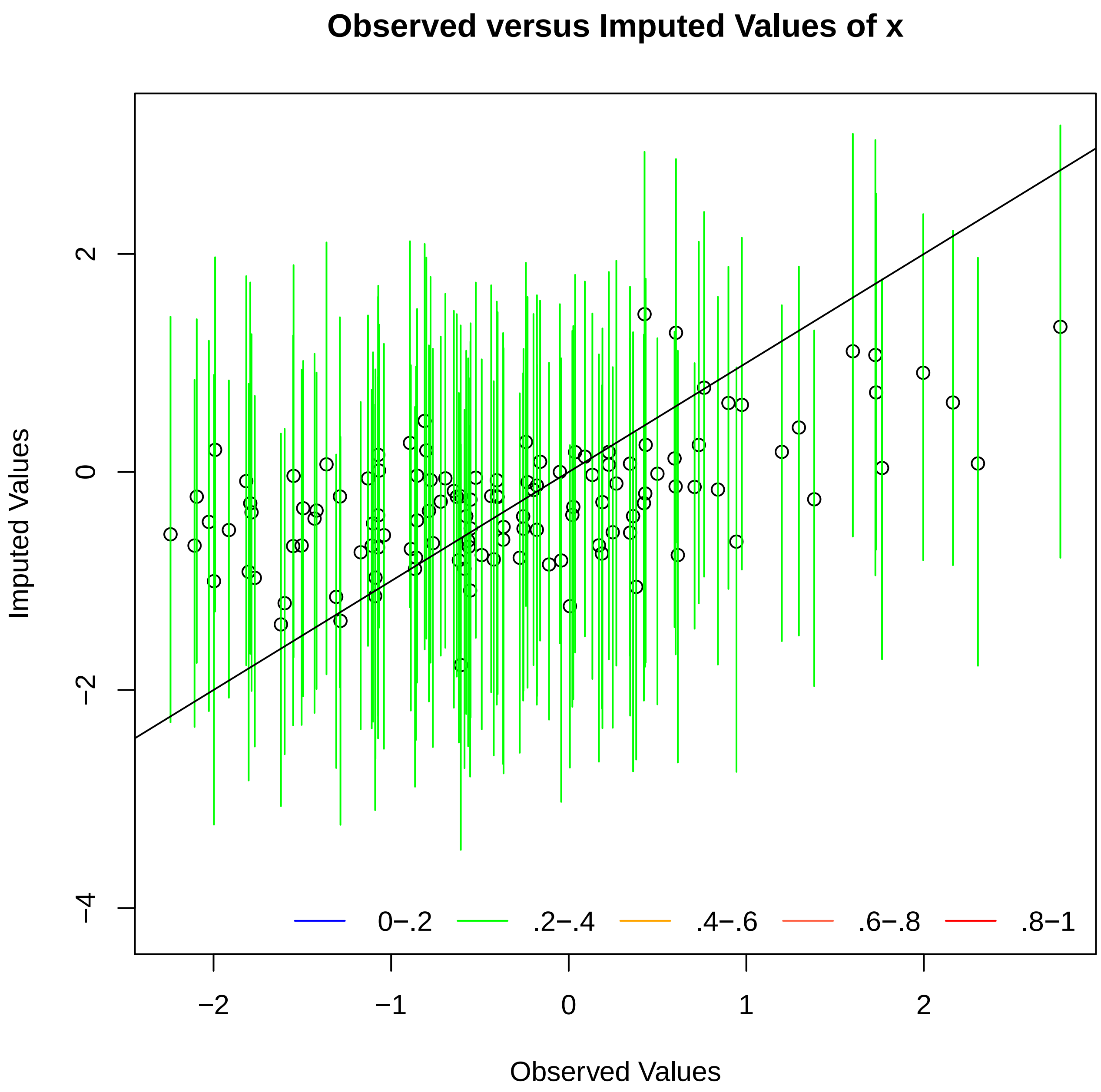
```
# Draw 1/nimp of the beta simulations from each run of least squares
sims <- 10000
simbetas <- NULL
for (i in 1:nimp) {
  simbetas <- rbind(simbetas,
                    mvrnorm(sims/nimp, coef(mi[[i]]), vcov(mi[[i]]))
                    )
}
```

From this point, you can simulate counterfactuals as normal using `simcf`

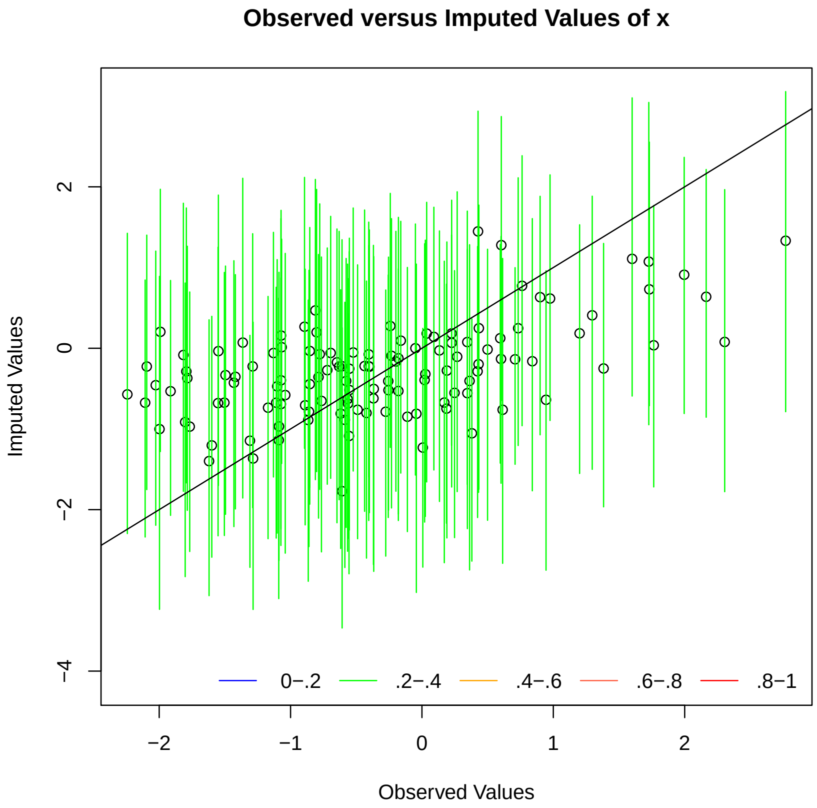
NB: you will need to either select an imputed dataset for computing means of variables, or average them all

Alternatively, you could have `zelig()` automate all of this, as Zelig knows what to do with Amelia objects

But it's usually best to write your own code for flexibility



Overimputation diagnostic: 90% of colored lines should cross the black line



```
pdf("overimputeX.pdf")
overimpute(amelia.res, var="x")
dev.off()
```

We did something similar earlier using MC data;
you could cook up your own version if you like

Multiple imputation for TSCS data

Until recently, off-the-shelf multiple imputation failed for TSCS data

For example, TSCS data are not iid, so we can't "just" treat them as MVN

King and Honaker (2010) propose a straightforward solution

Add to the EM imputation model several tricks to flexibly model time series and panel structures:

1. Allow observations from the same unit to be a smooth function of time.
2. Allow contemporaneous observations to be more or less correlated with other cross-sectional units

Multiple imputation for TSCS data

To use Amelia with panel data, you need to set a few more options, like this:

```
data.mi <- amelia(data, idvars=2, ts=3, cs=1,
                  polytime=3, intercs=FALSE, nom=56,
                  bounds=bounds,
                  log=c(4:38,40:55))
```

In this example, taken from my work, I identify for Amelia the column containing the id of each observation (2), the column with the period (3), and the country name (1)

Multiple imputation for TSCS data

To use Amelia with panel data, you need to set a few more options, like this:

```
data.mi <- amelia(data, idvars=2, ts=3, cs=1,
                  polytime=3, intercs=FALSE, nom=56,
                  bounds=bounds,
                  log=c(4:38,40:55))
```

I request time to be smoothed using a cubic polynomial (`polytime=3`)

An alternative would be to ask for a spline smoother with k knots by setting `splinetime=k`

Splines are faster, especially if k is high

Multiple imputation for TSCS data

To use Amelia with panel data, you need to set a few more options, like this:

```
data.mi <- amelia(data, idvars=2, ts=3, cs=1,
                  polytime=3, intercs=FALSE, nom=56,
                  bounds=bounds,
                  log=c(4:38,40:55))
```

Setting `intercs=TRUE` would allow each country to have a different smoother over time; I turned this off to speed up estimation

Multiple imputation for TSCS data

To use Amelia with panel data, you need to set a few more options, like this:

```
data.mi <- amelia(data, idvars=2, ts=3, cs=1,
                  polytime=3, intercs=FALSE, nom=56,
                  bounds=bounds,
                  log=c(4:38,40:55))
```

I had a nominal variable in column 56, and needed to log the variables
in columns 4 to 38 and 40 to 55 to make them more approximately Normal

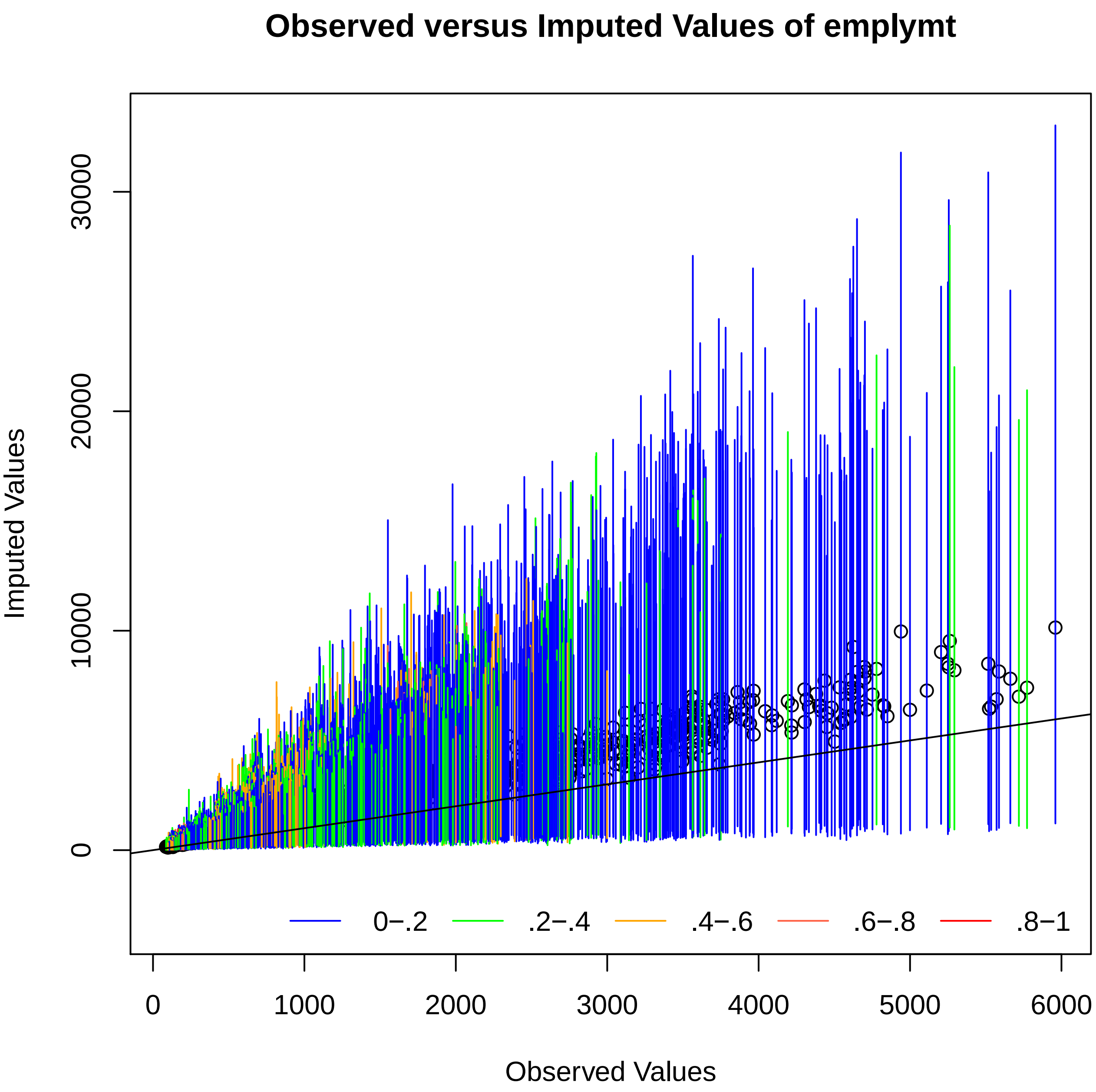
Finally, a number of my variables were bounded (e.g., from 0 to 1);
setting the bounds argument to identify these bounds by variable
allows Amelia handle them appropriately

Multiple imputation for TSCS data

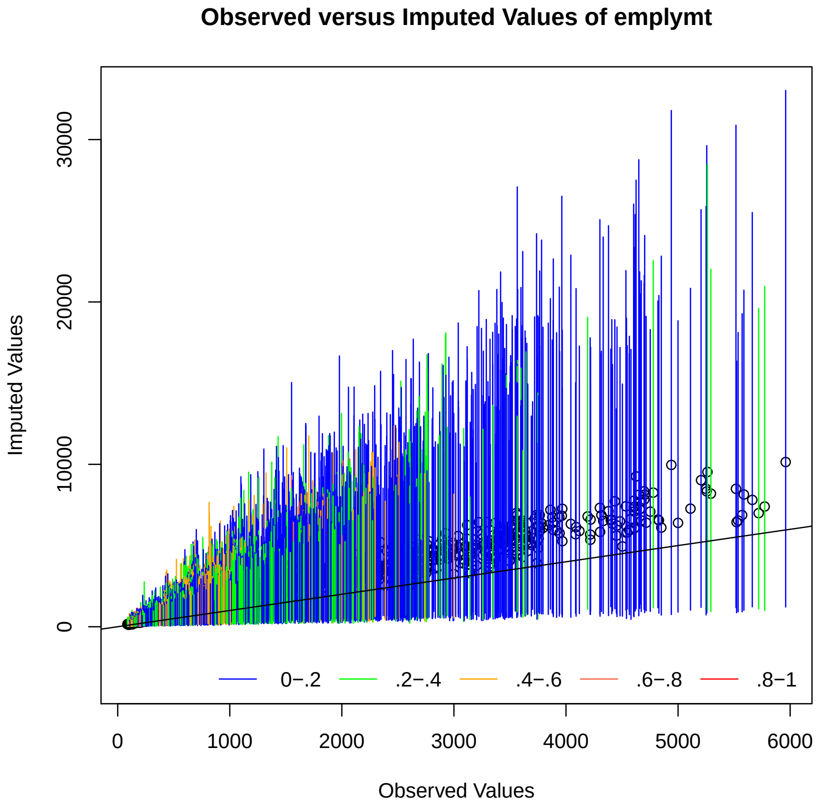
To use Amelia with panel data, you need to set a few more options, like this:

```
data.mi <- amelia(data, idvars=2, ts=3, cs=1,
                  polytime=3, intercs=FALSE, nom=56,
                  bounds=bounds,
                  log=c(4:38,40:55))
```

Other inputs I didn't use (but you might) include:
including `lags` or `leads` of variables in the imputation model,
square root or logit transformations,
adjusting the priors to help estimation in difficult cases

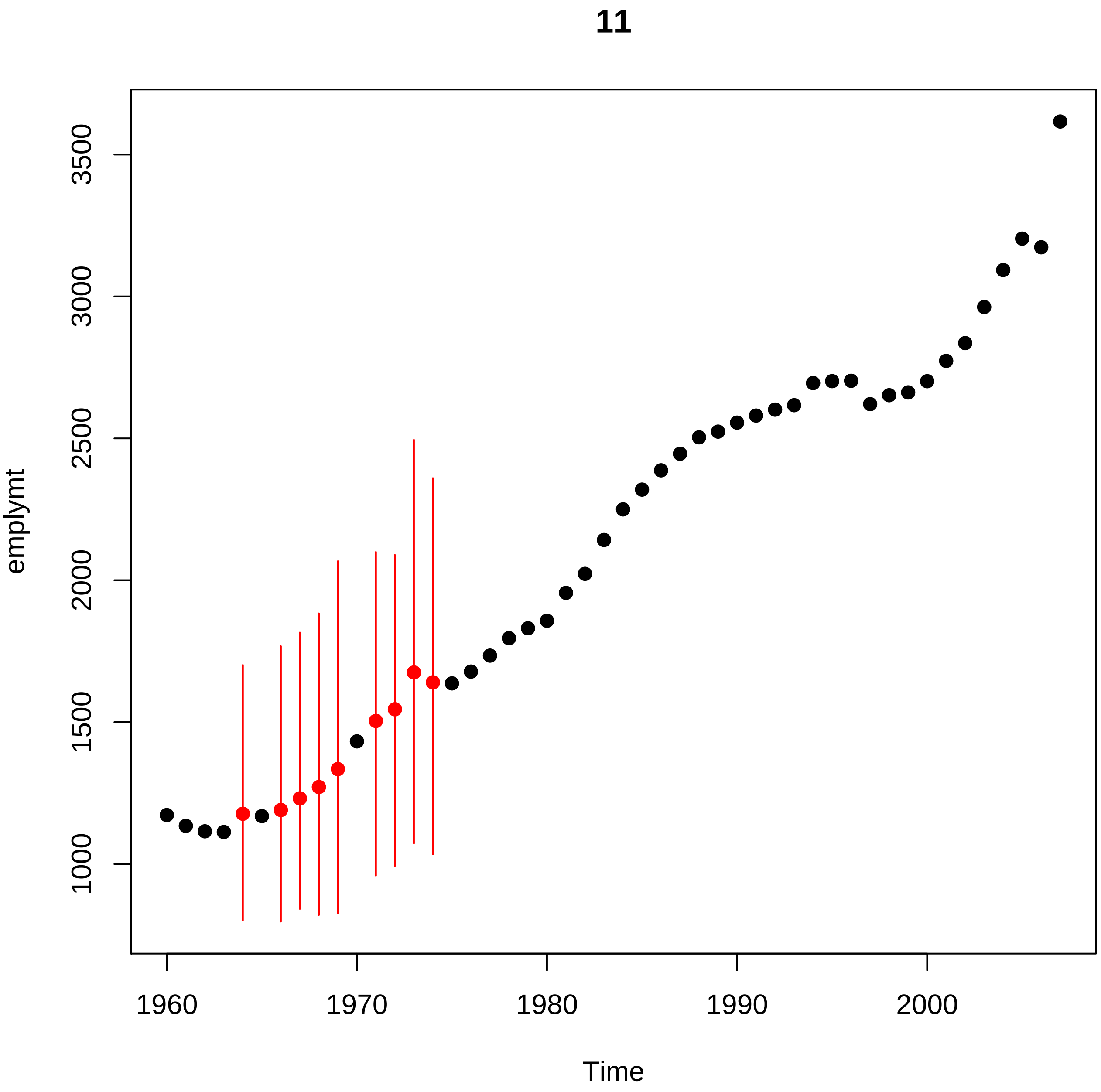


Overimputation diagnostic: multiple coverage levels



```
# Loop over all data variables
for (i in 4:ncol(data)) {

  # Make the overimputation plot for current variable
  pdf(paste("overimp_",names(data[i]),".pdf",sep=""))
  overimpute(data.mi, var=i)
  dev.off()
}
```



Time series diagnostic: Check for a plausible, usually smooth pattern

```
# Get the province codes & number of provinces
allprov <- unique(data$provcode)
nprov <- length(allprov)

# Loop over all data columns
for (i in 4:ncol(data)) {

  # Loop over all provinces
  for (j in 1:nprov) {

    # Make each province’s time series plot, with imputations added
    pdf(paste("tscs_",names(data[i]),"_",allprov[j],".pdf",sep=""))
    tscsPlot(data.mi, var=i, cs=allprov[j])
    dev.off()
  }
}
```