EC3312: Game Theory & Applications to Economics

Lecture 1: What is game theory?

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NUS, Autumn 2023

A birthday party

It's my birthday! I've brought two cakes for the class – one is double chocolate and one is red velvet. Everyone can pick which cake they want, and then I'll divide the cakes.

You can make your choice here:

What is game theory?

From the 1994 Nobel Prize citation:

> Game theory is a mathematical method for analyzing strategic interaction. Many classical analyses in economics presuppose such a large number of agents that each of them can disregard the others' reactions to their own decision. In many cases, this assumption is a good description of reality, but in other cases it is misleading. When a few firms dominate a market, when countries have to make an agreement on trade policy or environmental policy, when parties on the labor market negotiate about wages, and when a government deregulates a market, privatizes companies or pursues economic policy, each agent in question has to consider other agents' reactions and expectations regarding their own decisions, i.e., strategic interaction.

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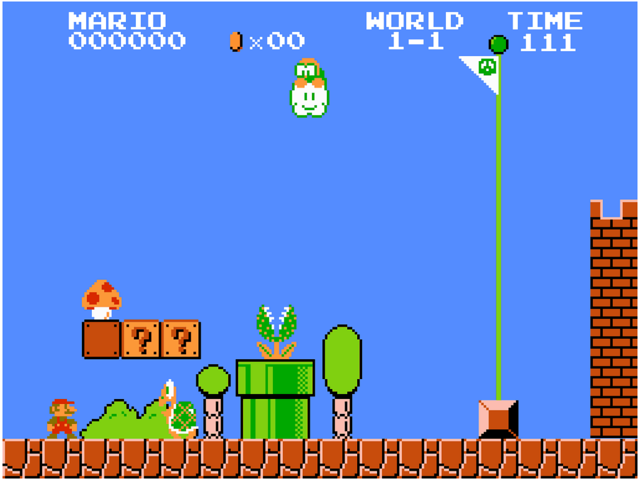
$\Rightarrow$ A game is a situation in which what is best for me to do depends on what others do.

Is this a game?



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Exercise

Think of a game. The person with the most distinctive example wins a slice of (fictitious) cake.

Is this a game?

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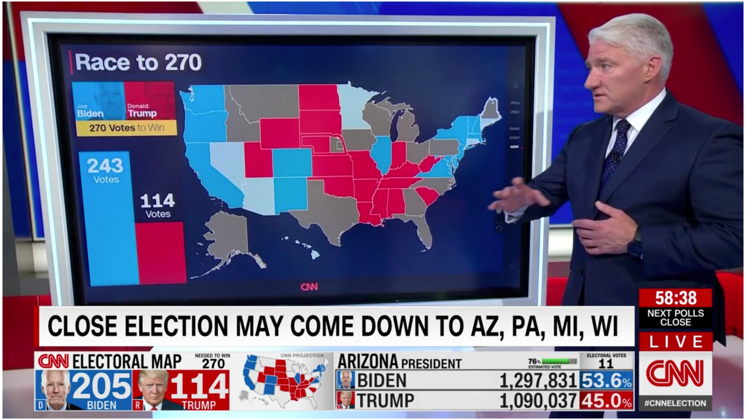

A birthday party (again)

Now that you know what other people chose, you can update your choice:

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If we allow people to update 100 times and only divide the cakes after the final update, what do you think will happen?

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This leads us to the key concept in game theory: equilibrium.

Overview

This course will introduce students to game theory, a theory of strategic interactions. The course studies mathematical models of non-cooperative games. The basic solution concepts are Nash equilibrium and its refinements (subgame-perfect Nash equilibrium, Bayes–Nash equilibrium, and perfect Bayesian equilibrium). This course emphasizes applications of game theory to economics. Students will learn how to:

- Apply appropriate games to model strategic situations (normal-form, extensive-form, perfect-information, and Bayesian games).
- Solve games using appropriate equilibrium concepts (Nash equilibrium, subgame-perfect equilibrium, Bayes–Nash equilibrium, and perfect Bayesian equilibrium).
- Apply game-theoretic methods to different economic contexts, including competition between firms, trade negotiations, environmental policy, wage bargaining, and auctions; derive insights from such applications.

Outline

1. Static games of complete information
2. Dynamic games of complete information
3. Static games of incomplete information
4. Dynamic games of incomplete information

Readings

The key text is

- Gibbons, R. 1992. *A primer in game theory*. Pearson.[1]

The following may also be of interest:

- Schelling, T. C. 1960. *The strategy of conflict*. HUP.
- Schelling, T. C. 1978. *Micromotives and macrobehavior*. HUP.
- Dixit, A. K. and Nalebuff, B. J. 1991. *Thinking strategically*. Norton.

[1] Also published as *Game theory for applied economists* in the US.

Practicalities

Lectures: Mondays, 4–6pm, LT9

Tutorials (25%): 10 tutorials in weeks 3–12; you will solve problems on the board

Midterm (25%): 2 October, 4–6pm, LT9, closed book

Final (50%): 25 November, 9–11am, closed book

Office hours: Tuesdays 2–3pm

Contact: `sam.jindani@nus.edu.sg`; I will typically answer questions in the following class

Prerequisites

- EC2101 and
- EC2104 or any MA course that is not MA1301/MA1301FC/MA1301X, MA1311, MA1312 or MA1421.

Note that EC3312 precludes MA4264.

Mathematical prerequisites: logic and set notation

Notation:

- $\{a, b\}$: the set containing elements a and b
- $\mathbb{R}$: the set of real numbers
- $A \cup B$: union of A and B
- $A \cap B$: intersection of A and B
- '$a \in A$': 'a belongs to the set A' or 'the set A contains a'
- $[a, b]$, where $a, b \in \mathbb{R}$ and $a < b$: the closed interval from a to b
- (a, b), where $a, b \in \mathbb{R}$ and $a < b$: the open interval from a to b

Induction: If P_1 is true and $P_n \implies P_{n+1}$ for all $n \in \mathbb{N}$, then P_n is true for all $n \in \mathbb{N}$

Logic:

- '$\forall$': 'for all'
- '$\exists$': 'there exists'
- $\neg P$: P's negation
- '$P \iff Q$': 'P is equivalent to Q'
- Law of contraposition: $(P \implies Q) \iff (\neg Q \implies \neg P)$

Mathematical prerequisites: calculus and maximisation

- $\max A$: maximal element of A (if one exists)
- $\max_{x \in X} f(x)$: maximum value of $f(x)$ subject to $x \in X$
- $\arg\max_{x \in X} f(x) = \{x \in X : f(x) = \max_{x' \in X} f(x')\}$
- '$f'(x)$': derivative of f at x
- '$f''(x)$': second derivative of f at x

How to solve $\max_{x \in X} f(x)$?

- First-order condition: $f'(x^*) = 0$
- Second-order condition: $f''(x) \leq 0$ for all $x \in X$

Mathematical prerequisites: probability

- $P(A)$: probability of A
- Conditional probability:

$$P(A|B) = \frac{P(A \cap B)}{P(B)}$$

- Bayes's rule:

$$P(A|B) = \frac{P(B|A)}{P(B)} P(A)$$

For a continuous random variable X:

- $F(x)$: cumulative distribution function
- $f(x)$: density function, derivative of the CDF
- Uniform random variable on $[a, b]$:

$$F(x) = \begin{cases} 0 & \text{if } x < a \\ \frac{x}{b-a} & \text{if } x \in [a, b] \\ 1 & \text{otherwise.} \end{cases} \qquad f(x) = \begin{cases} \frac{1}{b-a} & \text{if } x \in [a, b] \\ 0 & \text{otherwise.} \end{cases}$$

Kaya Toast game

Valerie and Ryan agree to meet for kaya toast at ION. Unfortunately they forgot that ION has both a Toast Box and a Ya Kun. Moreover, it's so crowded that they don't have any phone reception! They each have to decide where to go, and hope that the other chooses the same spot. Valerie prefers Toast Box and Ryan prefers Ya Kun.

		Ryan	
		Toast Box	Ya Kun
Valerie	Toast Box	2, 1	0, 0
	Ya Kun	0, 0	1, 2

(The traditional name for this game is 'Battle of the Sexes'.)

Normal-form games

1.1. Definition. A normal-form game consists of:

1. A set of players $N = \{1, 2, \ldots, n\}$
2. A set of strategies S_i for each player $i \in N$.
   - A strategy profile $s = (s_1, s_2, \ldots, s_n)$ consists of a strategy $s_i \in S_i$ for each player $i \in N$.
   - The set of strategy profiles is $S = \times_{i \in N} S_i$
3. A utility function $u_i : S \to \mathbb{R}$ for each player $i \in N$, that assigns a utility to each strategy profile.

Players can choose between different strategies and their payoffs depend on what each player has chosen.

Usually, utilities have only an ordinal interpretation, not cardinal.

The Kaya Toast game in normal form

$N = \{V, R\}$

$S_V = S_R = \{T, Y\}$

$S = \{(T,T), (T,Y), (Y,T), (Y,Y)\}$

$$u_N(s) = \begin{cases} 2 & \text{if } s_V = s_R = T \\ 1 & \text{if } s_V = s_R = Y \\ 0 & \text{otherwise} \end{cases} \qquad u_R(s) = \begin{cases} 1 & \text{if } s_V = s_R = T \\ 2 & \text{if } s_V = s_R = Y \\ 0 & \text{otherwise.} \end{cases}$$

Nash equilibrium

1.2. Definition. A strategy profile s^* is a Nash equilibrium if

$$u_i(s^*) \geq u_i(s_i, s^*_{-i})$$

for all $s_i \in S_i$ and $i \in N$.

Given the others' choices, no player regrets their choice; no player has an incentive to deviate.

Nash equilibria of the Kaya Toast game

		Ryan	
		Toast Box	Ya Kun
Valerie	Toast Box	1 2	0 0
	Ya Kun	0 0	2 1

Nash equilibria of the Kaya Toast game

		Ryan	
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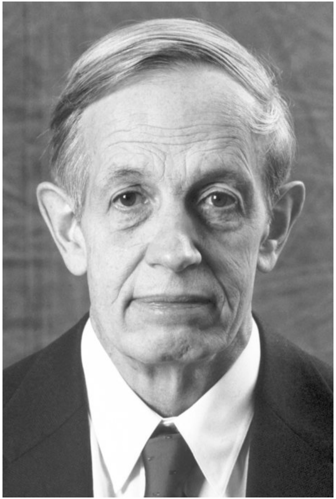
There are two Nash equilibria: (T, T) and (Y, Y).

John Nash



The only person to win both a Nobel Prize and the Abel Prize.

The concept of Nash equilibrium is one of the most important ideas of the twentieth century, with applications not just in economics, but in all of social sciences and in biology.

The Prisoner's Dilemma

Bart and Milhouse have sprayed graffiti on the school facade. Principal Skinner is furious. He is convinced he knows who did it, but he doesn't have proof. He calls each to his office separately, and makes the following offer: if one denounces the other, he will get a reduced amount of detention; if neither says anything, Skinner will give them detention on a trumped-up littering charge.

		Milhouse	
		Cooperate	Defect
Bart	Cooperate	-1 -1	0 -5
	Defect	-5 0	-4 -4

What are the Nash equilibria of this game?

Dominant strategies

Given $i \in N$, s_{-i} denotes a profile of strategies for all players other than i; $S_{-i} = \times_{i \in N \setminus \{i\}} S_i$ is the set of such profiles.

1.3. Definition. Strategy s_i is strictly dominant for player i if

$$u_i(s_i, s_{-i}) > u_i(s_i', s_{-i})$$

for all $s_{-i} \in S_{-i}$ and $s_i' \in S_i \setminus \{s_i\}$.

If s is such that s_i is strictly dominant for each i, then s is the unique Nash equilibrium. Why?

Equilibrium in the Prisoner's Dilemma

		Milhouse	
		Cooperate	Defect
Bart	Cooperate	-1, -1	-5, 0
	Defect	0, -5	-4, -4

Defect is a strictly dominant strategy for each player, so (D, D) is the unique Nash equilibrium.

Is this a realistic prediction?

Conclusion

A game is a situation in which what is best for me to do depends on what others do.

Games arise in many contexts: economics, politics, and social settings in general.

A Nash equilibrium is a strategy profile such that no player has an incentive to deviate.

Next time: mixed strategies, existence of equilibrium.

Image credits