EC3333 Tutorial 1 Suggested Answers

1.   The last four years of returns for a stock are as follows:

1	2	3	4
-8%	+20%	+10%	+4%

      a. Estimate the average annual return of this stock.
      b. Estimate the variance of the stock's returns.
      c. Estimate the standard deviation of the stock's returns.

a.

To derive the mean estimate from realized returns:

$$\bar{R} = \frac{1}{T}(R_1 + R_2 + \cdots + R_T) = \frac{1}{T}\sum_{t=1}^{T} R_t$$

$$\therefore \text{Average annual return } = \frac{-8\% + 20\% + 10\% + 4\%}{4} = 6.5\%$$

b.

To derive the variance estimate from realized returns:

$$\text{Var }(R) = \frac{1}{T-1}\sum_{t=1}^{T}(R_t - \bar{R})^2$$

$$\therefore \text{Variance of returns } = \frac{(-8\% - 6.5\%)^2 + (20\% - 6.5\%)^2 + (10\% - 6.5\%)^2 + (4\% - 6.5\%)^2}{3}$$

$$= 0.0137$$

c.

$$\text{Standard deviation of returns } = \sqrt{\text{variance}} = \sqrt{0.0137} = 11.7047\%$$

2. Consider the following probability distribution for stocks A and B:

State	Probability	R$_A$	R$_B$
1	0.1	10%	8%
2	0.2	13%	7%
3	0.2	12%	6%
4	0.3	14%	9%
5	0.2	15%	8%

    a. What are the expected returns of stocks A and B?
    b. What are the standard deviations of returns of stocks A and B?
    c. What is the correlation coefficient between returns of A and B?

a.

$$\text{Expected Return} = E[R] = \sum_R P_R \times R$$

$$E[R_A] = 0.1(10\%) + 0.2(13\%) + 0.2(12\%) + 0.3(14\%) + 0.2(15\%) = 13.2\%$$

$$E[R_B] = 0.1(8\%) + 0.2(7\%) + 0.2(6\%) + 0.3(9\%) + 0.2(8\%) = 7.7\%$$

b.

$$\text{Var}(R) = E[(R - E[R])^2] = \sum_R P_R \times (R - E[R])^2$$

$$\text{Var}[R_A] = 0.1 \times (0.1 - 0.132)^2 + 0.2 \times (0.13 - 0.132)^2$$
$$+0.2 \times (0.12 - 0.132)^2 + 0.3 \times (0.14 - 0.132)^2 + 0.2 \times (0.15 - 0.132)^2$$
$$= 0.000216$$

$$\text{Var}[R_B] = 0.1 \times (0.08 - 0.077)^2 + 0.2 \times (0.07 - 0.077)^2$$
$$+0.2 \times (0.06 - 0.077)^2 + 0.3 \times (0.09 - 0.077)^2 + 0.2 \times (0.08 - 0.077)^2$$
$$= 0.000121$$

$$SD(R_A) = \sqrt{\text{Var}(R_A)} = 0.01470$$
$$SD(R_B) = \sqrt{\text{Var}(R_B)} = 0.01100$$

c.

$$SD(R) = \sigma_R = \sqrt{\text{Var}(R)}$$

$$\text{Cov}(R_i, R_j) = E[(R_i - E[R_i])(R_j - E[R_j])] = \sum_R P_R \times (R_i - E[R_i])(R_j - E[R_j])$$

$$\rho_{AB} = \frac{\text{Cov}(r_A r_B)}{\sigma_A \sigma_B}$$

$$\text{Cov}(R_A, R_B) = 0.1 \times (0.1 - 0.132)(0.08 - 0.077)$$
$$+0.2 \times (0.13 - 0.132)(0.07 - 0.077) + 0.2 \times (0.12 - 0.132)(0.06 - 0.077)$$
$$+0.3 \times (0.14 - 0.132)(0.09 - 0.077) + 0.2 \times (0.15 - 0.132)(0.08 - 0.077)$$
$$= 0.092195$$

$$\rho_{AB} = \frac{\text{Cov}(r_A r_B)}{\sigma_A \sigma_B} = \frac{0.092195}{\sqrt{0.000216}\sqrt{0.000121}} = 0.470104092$$

3. Using the data in the following table, calculate the return for investing in Boeing stock (BA) from January 2, 2008, to January 2, 2009, and also from January 3, 2011, to January 3, 2012, assuming all dividends are reinvested in the stock immediately.

Historical Stock and Dividend Data for Boeing

Date	Price	Dividend	Date	Price	Dividend
1/2/2008	86.62		1/3/2011	66.40	
2/6/2008	79.91	0.40	2/9/2011	72.63	0.42
5/7/2008	84.55	0.40	5/11/2011	79.08	0.42
8/6/2008	65.40	0.40	8/10/2011	57.41	0.42
11/5/2008	49.55	0.40	11/8/2011	66.65	0.42
1/2/2009	45.25		1/3/2012	74.22	

For each period, the realized return is:

$$R_{t+1} = \frac{Div_{t+1} + P_{t+1}}{P_t} - 1 = \frac{Div_{t+1}}{P_t} + \frac{P_{t+1} - P_t}{P_t}$$
$$= \text{Dividend Yield } + \text{ Capital Gain Rate}$$

Assume that all dividends are immediately reinvested in the same asset, the total realized return for the holding period, R$_{realized}$, is computed as:

$$R_{\text{realized}} = (1 + R_1)(1 + R_2)(1 + R_3)(1 + R_4)(1 + R_5) - 1$$

Date	Holding Period Return	1+ Holding Period Return	Date	Holding Period Return	1+ Holding Period Return
2/6/2008	-0.072847	0.927153	2/9/2011	0.100151	1.100151
5/7/2008	0.063071	1.063071	5/11/2011	0.094589	1.094589
8/6/2008	-0.221762	0.778238	8/10/2011	-0.268715	0.731285
11/5/2008	-0.236239	0.763761	11/8/2011	0.168263	1.168263
1/2/2009	-0.086781	0.913219	1/3/2012	0.113578	1.113578
Total Return		-0.464994			0.145648
Total Return		-46.50%			14.56%

4. Using the data in the following table,
   a. What was the realized average dividend yield for the SP500 during this period?
   b. What was the realized volatility of the dividend yield during this period?
   c. What was the realized average capital gain rate of the SP500 during this period?
   d. What was the realized volatility of the capital gain rate of the S&P 500 during this period?
   e. Were dividends or capital gains a more important component of the S&P 500's average returns during this period? Which were the more important source of volatility?

Year End	S&P 500 Index	Dividends Paid
2004	1211.92	
2005	1248.29	23.15
2006	1418.30	27.16
2007	1468.36	27.86
2008	903.25	21.85
2009	1115.10	27.19
2010	1257.64	25.44
2011	1257.61	26.59
2012	1426.19	32.67
2013	1848.36	39.75
2014	2058.90	42.47
2015	2043.94	43.45
2016	2238.83	49.56
2017	2673.61	53.99

Year End		Capital Gain Rate	Dividend Yield	(R_CGR-R_CGRbar)^2	(R_div-R_divbar)^2
2004					
2005		3.00%	1.91%	0.002238783	1.33002E-05
2006		13.62%	2.18%	0.003465477	9.8228E-07
2007		3.53%	1.96%	0.00176654	9.64494E-06
2008		-38.49%	1.49%	0.213613999	6.19104E-05
2009		23.45%	3.01%	0.024716847	5.40746E-05
2010		12.78%	2.28%	0.00255036	4.25526E-09
2011		0.00%	2.11%	0.005983001	2.57952E-06
2012		13.40%	2.60%	0.003217375	1.04263E-05
2013		29.60%	2.79%	0.047823761	2.6241E-05
2014		11.39%	2.30%	0.001338124	5.21029E-08
2015		-0.73%	2.11%	0.007155811	2.70722E-06
2016		9.54%	2.42%	0.00032487	2.24527E-06
2017		19.42%	2.41%	0.013659448	1.86707E-06
		c. Average Capital Gain Rate	**a. Average Dividend Yield**	**d. Volatility of Capital Gain Rate**	**b. Volatility of Dividend Yield**
		7.73%	**2.27%**	$=[1/13*\text{sum of }(R\_CGR-R\_CGRbar)^2]^{(1/2)}$	$=[1/13*\text{sum of }(R\_div-R\_divbar)^2]^{(1/2)}$
				15.88%	**0.38%**

e.

As can be seen from the table above, capital gains were more important for both returns and volatility for SP500 during this period.

5. Do you agree with the following statements? If not, how would you modify the statement(s)?
    a. The return that actually occurs over a particular time period is the expected return.
    b. An asset is considered as riskless if its return never deviates from its mean, the variance is equal to one.
    c. Although the variance and the standard deviation are the most common measures of risk, they do not differentiate between upside and downside risk.
    d. Compared to the standard deviation, as a measure the variability of the returns, the variance is easier to interpret because it is in the same units as the returns themselves.
    e. The variance of a return is also referred to as its volatility in the financial markets.

a.
The return that actually occurs over a particular time period is the *realized return*.

b.
An asset is considered as riskless if its return never deviates from its mean, the variance is equal to *zero*.

c.
This statement is correct.

d.
Compared to the *variance*, as a measure the variability of the returns, the *standard deviation (or volatility)* is easier to interpret because it is in the same units as the returns themselves.

e.
The *standard deviation* of a return is also referred to as its volatility in the financial market.