EC2101: Microeconomic Analysis I

Lecture 3

Theory of the Consumer

- Optimal Choice: Mathematical Analysis
  - BLTC Method
  - Lagrange Multiplier Method
- Voucher vs. Cash
- Revealed Preference

Optimal Choice: Mathematical Analysis

Optimal Choice:

Example 1

Finding the Optimal Choice: Example 1

- Suppose Serena's utility function is:

$$U(x, y) = xy$$

- Suppose gelato (x) costs $\$10$ a pint, coffee (y) costs $\$5$ a cup, and Serena's income is $\$100$.

- What is Serena's optimal basket?

Optimal Choice:

BLTC Method

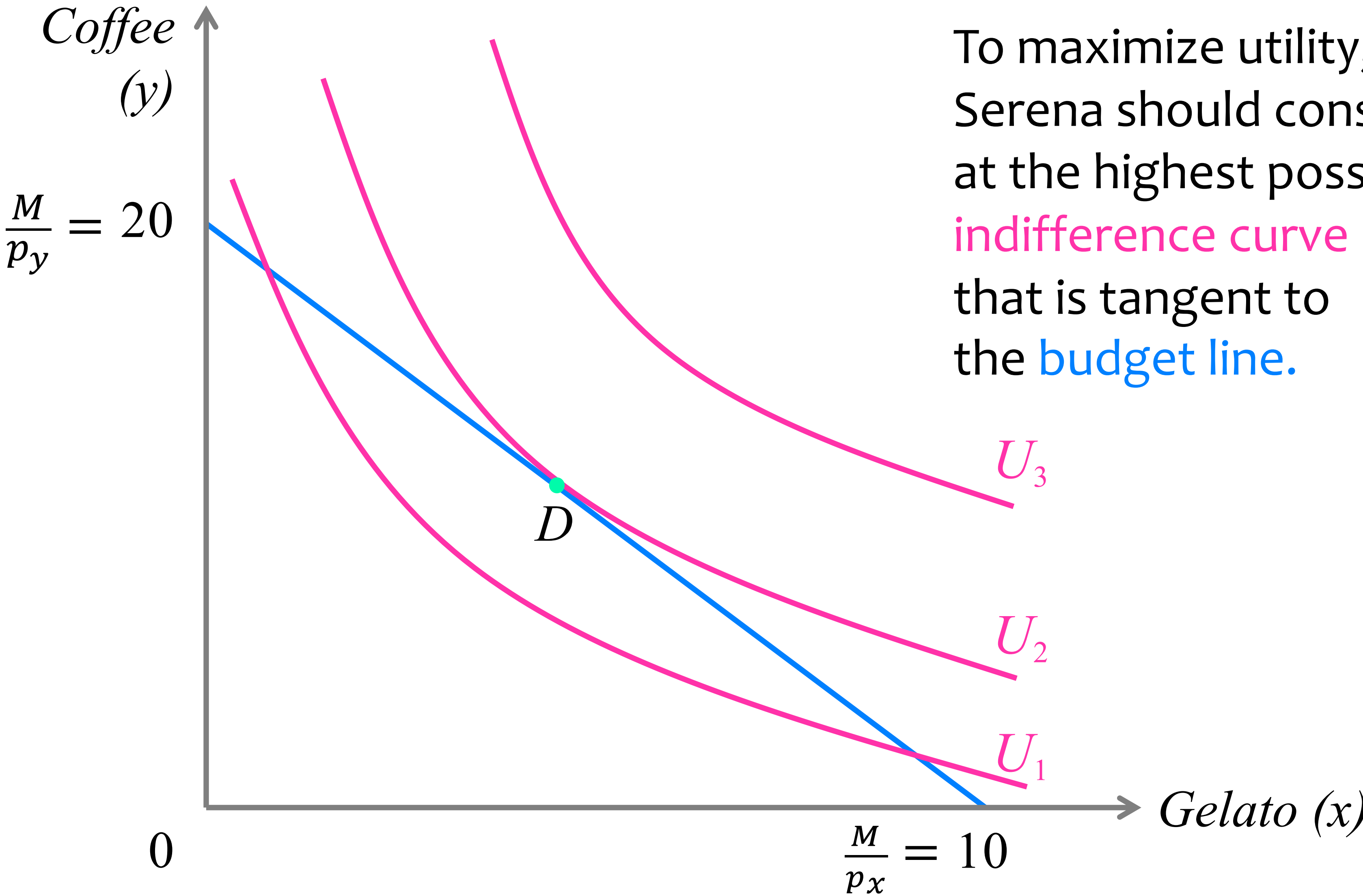
Optimal Choice

- Which consumption basket is optimal?
- Serena chooses the consumption basket that
  gives her the highest utility
  given her budget constraint.
- The constrained optimization problem is:

$$\max_{x,y} U(x,y)$$

$$subject\ to\ \ p_x x + p_y y \leq M$$

Optimal Choice: Graphical Analysis



To maximize utility, Serena should consume at the highest possible indifference curve that is tangent to the budget line.

Optimal Choice

- The optimal basket is:
  - On the budget line: $p_x x + p_y y = M$
  - On the highest indifference curve that is tangent to the budget line:

$$-MRS_{x,y} = -\frac{p_x}{p_y} \quad \Leftrightarrow \quad MRS_{x,y} = \frac{p_x}{p_y}$$

Tangency Condition

- What does this equation mean?

$$MRS_{x,y} = \frac{p_x}{p_y}$$

- To maximize utility,
  the amount of x and y should be such that
  the rate at which the consumer is willing to
  substitute between two goods, holding utility constant
  is equal to
  the rate at which the two goods are exchanged in the market.

Tangency Condition

- Since $$MRS_{x,y} = \frac{MU_x}{MU_y}$$
- At the optimal basket, $$MRS_{x,y} = \frac{p_x}{p_y}$$
- Equivalently, $$\frac{MU_x}{MU_y} = \frac{p_x}{p_y}$$
$$\frac{MU_x}{p_x} = \frac{MU_y}{p_y}$$

Equal Marginal Principle

- What does this equation mean?

$$\frac{MU_x}{p_x} = \frac{MU_y}{p_y}$$

- The marginal utility per dollar spent on gelato
  is exactly the same as
  the marginal utility per dollar spent on coffee.

- To maximize utility,
  Serena sets the marginal utility per dollar of expenditure equal for all goods.

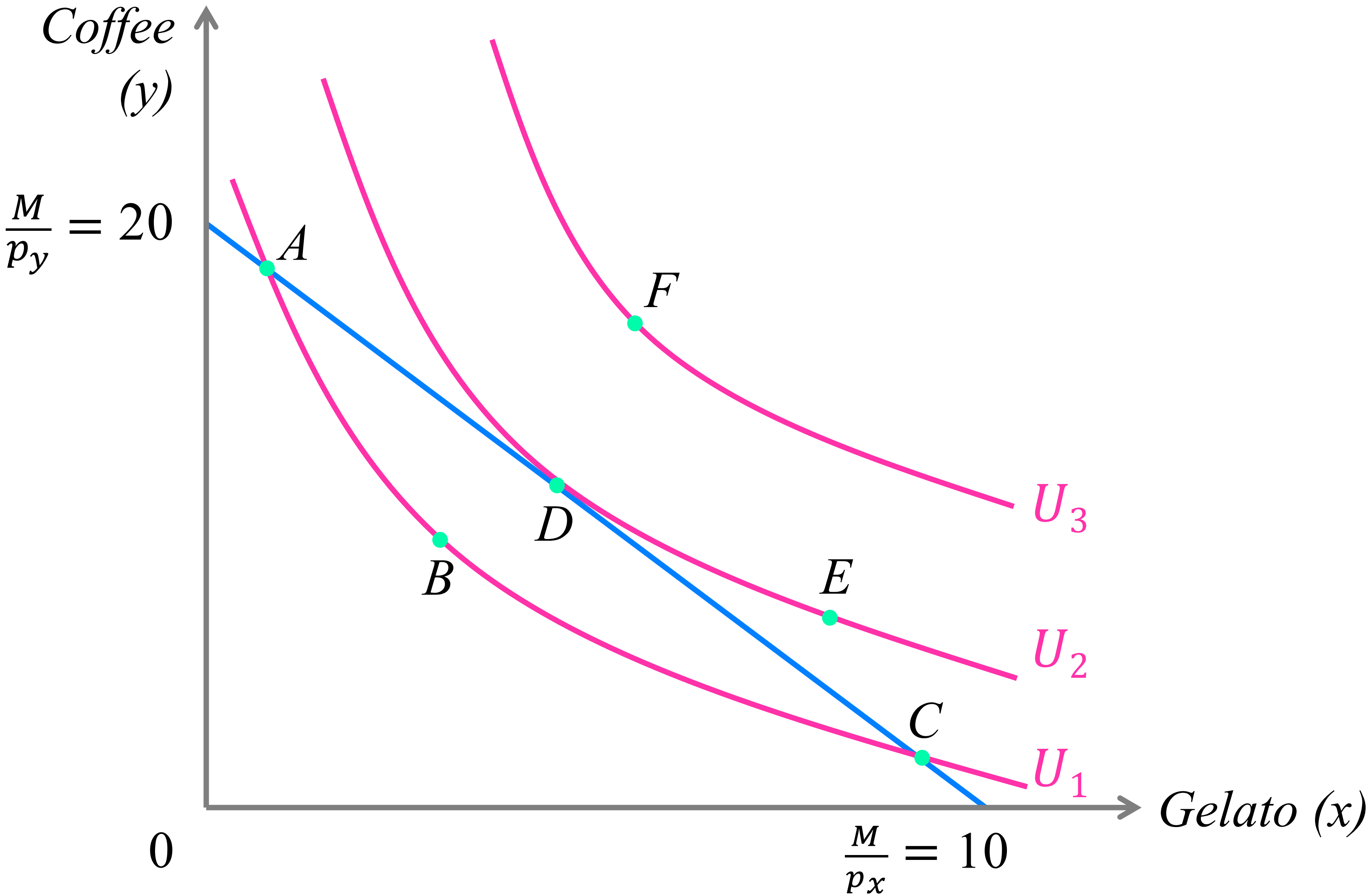
What if *MU* per dollar are not the same?

- Suppose

$$\frac{MU_x}{p_x} < \frac{MU_y}{p_y}$$

- Should Serena buy more x or more y?
  - Serena should buy more y.
  - A dollar spent on y
    brings higher additional utility than
    a dollar spent on x.

Optimal Choice



Why is Basket *A* not optimal?

- At basket *A* on the graph, $MRS_{x,y} > \frac{p_x}{p_y}$

$$\frac{MU_x}{MU_y} > \frac{p_x}{p_y}$$

$$\frac{MU_x}{p_x} > \frac{MU_y}{p_y}$$

- At basket *A*, the per dollar marginal utility of x
  is higher than
  the per dollar marginal utility of y.
- Serena could increase her utility by spending more on x and spending less on y.

Optimal Choice:

Example 1 –
BLTC Method

Finding the Optimal Choice: Example 1

- Suppose Serena's utility function is:

$$U(x, y) = xy$$

- Suppose gelato (x) costs $\$10$ a pint, coffee (y) costs $\$5$ a cup, and Serena's income is $\$100$.

- What is Serena's optimal basket?

Finding the Optimal Choice: Example 1 – BLTC Method

- We need two conditions:
  - Budget line: $10x + 5y = 100$ (i)
  - Tangency condition: ? (ii)
    - At the tangency,
      the slope of the indifference curve equals
      the slope of the budget line.

Finding the Optimal Choice: Example 1 – BLTC Method

- The slope of the indifference curve is: $MRS_{x,y} = \dfrac{MU_x}{MU_y}$

- The slope of the budget line is: $\dfrac{p_x}{p_y}$

- The tangency condition requires: $\dfrac{MU_x}{MU_y} = \dfrac{p_x}{p_y}$

Finding the Optimal Choice: Example 1 – BLTC Method

- Given $U(x, y) = xy$, the marginal utilities are:

$$MU_x = \frac{\partial U}{\partial x} = y$$

$$MU_y = \frac{\partial U}{\partial y} = x$$

- Tangency condition: $$\frac{MU_x}{MU_y} = \frac{p_x}{p_y}$$

$$\frac{y}{x} = \frac{10}{5}$$

$$y = 2x$$

Finding the Optimal Choice: Example 1 – BLTC Method

- We need two conditions:
  - Budget line: $10x + 5y = 100$ (i)
  - Tangency condition: $y = 2x$ (ii)

- Solve the two equations:

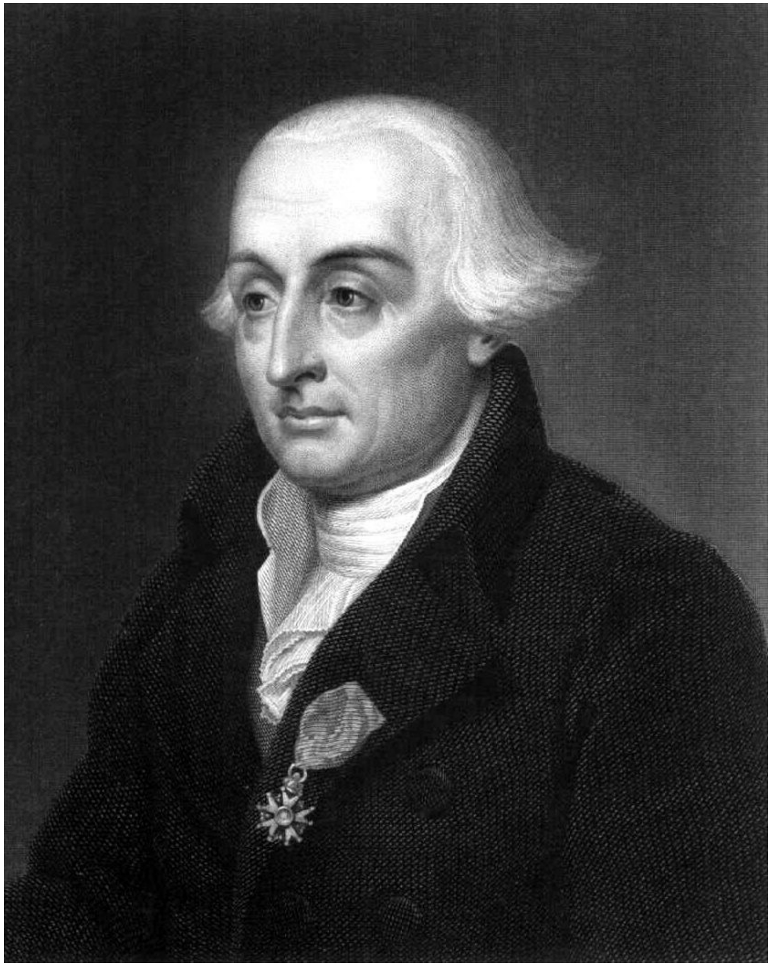
$$10x + 5(2x) = 100 \quad \Rightarrow \quad x = 5$$

$$y = 2(5) \quad \Rightarrow \quad y = 10$$

Optimal Choice:

Lagrange Multiplier Method

Joseph-Louis Lagrange



Joseph-Louis Lagrange
1736–1813

- Baptized Giuseppe Lodovico Lagrangia.
- Italian-French mathematician and astronomer.
- Made great contributions to analysis, number theory, and classical and celestial mechanics.

Lagrange Multiplier Method

- The general form of the constrained maximization problem is:

$$\max_{x,y} f(x, y)$$

$$subject\ to\ g(x, y) = 0$$

- In consumer theory, the constrained maximization problem is:

$$\max_{x,y} U(x, y)$$

$$subject\ to\ M - p_x x - p_y y = 0$$

Lagrange Multiplier Method

- The Lagrangian function is:

$$\Lambda(x, y, \lambda) = U(x, y) + \lambda(M - p_x x - p_y y)$$

- First-order conditions:

$$\frac{\partial \Lambda}{\partial x} = MU_x - \lambda p_x = 0$$

$$\frac{\partial \Lambda}{\partial y} = MU_y - \lambda p_y = 0$$

$$\frac{\partial \Lambda}{\partial \lambda} = M - p_x x - p_y y = 0$$

- Budget line

Lagrange Multiplier Method

- Rearrange the first two equations:

$$\frac{\partial \Lambda}{\partial x} = MU_x - \lambda p_x = 0 \qquad \Rightarrow \qquad \lambda = \frac{MU_x}{p_x}$$

$$\frac{\partial \Lambda}{\partial y} = MU_y - \lambda p_y = 0 \qquad \Rightarrow \qquad \lambda = \frac{MU_y}{p_y}$$

- Therefore:

$$\frac{MU_x}{p_x} = \frac{MU_y}{p_y}$$

$$\frac{MU_x}{MU_y} = \frac{p_x}{p_y}$$

Lagrange Multiplier Method

- The Lagrangian function is:

$$\Lambda(x, y, \lambda) = U(x, y) + \lambda(M - p_x x - p_y y)$$

- First-order conditions:

$$\frac{\partial \Lambda}{\partial x} = MU_x - \lambda p_x = 0$$

➢ Tangency condition

$$\frac{\partial \Lambda}{\partial y} = MU_y - \lambda p_y = 0$$

$$\frac{\partial \Lambda}{\partial \lambda} = M - p_x x - p_y y = 0$$

➢ Budget line

The Meaning of the Lagrange Multiplier

- What is the meaning of the Lagrange multiplier?

$$\lambda = \frac{MU_x}{p_x} = \frac{MU_y}{p_y}$$

- The Lagrange multiplier is
  the additional utility
  from an additional dollar of consumption.

Optimal Choice:

Example 1 – Lagrange Multiplier Method

Finding the Optimal Choice: Example 1

- Suppose Serena's utility function is:

$$U(x, y) = xy$$

- Suppose gelato (x) costs $\$10$ a pint, coffee (y) costs $\$5$ a cup, and Serena's income is $\$100$.

- What is Serena's optimal basket?

Finding the Optimal Choice: Example 1 – Lagrange Multiplier Method

- This is a constrained maximization problem:

$$\max_{x,y} U(x, y) = xy$$

$$subject\ to\ \ 10x + 5y \leq 100$$

- To maximize utility, Serena consumes on the budget line:

$$\max_{x,y} U(x, y) = xy$$

$$subject\ to\ \ 10x + 5y = 100$$

Finding the Optimal Choice: Example 1 – Lagrange Multiplier Method

- Rewrite the budget constraint:

$$\max_{x,y} U(x,y) = xy$$

$$subject\ to \quad 100 - 10x - 5y = 0$$

- The Lagrangian function is:

$$\Lambda(x,y,\lambda) = xy + \lambda(100 - 10x - 5y)$$

Finding the Optimal Choice: Example 1 – Lagrange Multiplier Method

- The Lagrangian function is:

$$\Lambda(x, y, \lambda) = xy + \lambda(100 - 10x - 5y)$$

- First-order conditions:

$$\frac{\partial \Lambda}{\partial x} = y - 10\lambda = 0$$

$$\frac{\partial \Lambda}{\partial y} = x - 5\lambda = 0$$

$$\frac{\partial \Lambda}{\partial \lambda} = 100 - 10x - 5y = 0$$

- Solving the three equations, $x = 5$, $y = 10$, and $\lambda = 1$.

Exercise 3.1

Finding the Optimal Choice

- Suppose Venus's utility function is:

$$U(x, y) = xy^2$$

- Suppose gelato (x) costs $\$10$ a pint, coffee (y) costs $\$5$ a cup, and Venus's income is $\$90$.

- Find Venus's optimal choice using:

  (a) the BLTC method

  (b) the Lagrange multiplier method

Exercise 3.1(a)

BLTC Method

Exercise 3.1(b)

Lagrange Multiplier Method

Optimal Choice:

Example 2

Finding the Optimal Choice: Example 2

- Suppose Billie's utility function is:

$$U(x, y) = xy + 20x$$

- Suppose gelato (x) costs $\$10$ a pint, coffee (y) costs $\$5$ a cup, and Billie's income is $\$80.$

- What is Billie's optimal basket?

- The utility maximization problem is:

$$\max_{x,y} U(x, y) = xy + 20x$$

$$subject\ to \quad 10x + 5y = 80$$

Finding the Optimal Choice: Example 2

- Budget line: $10x + 5y = 80$ (i)
- Tangency condition: $MRS_{x,y} = \frac{p_x}{p_y}$

$$\frac{MU_x}{MU_y} = \frac{p_x}{p_y}$$

$$\frac{y+20}{x} = \frac{10}{5}$$

$$y + 20 = 2x \qquad \text{(ii)}$$

Finding the Optimal Choice: Example 2

- The solution is $x = 9, y = -2.$

- Is this the optimal basket?

- The utility maximization problem is:

$$\max_{x,y} U(x,y) = xy + 20x$$

$$subject\ to\quad 10x + 5y = 80$$

Rewriting the Utility Maximization Problem

- The consumption of each good cannot be negative.
- The utility maximization problem should be:

$$\max_{x,y} U(x, y) = xy + 20x$$

$$subject\ to\quad 10x + 5y = 80$$

$$x \geq 0$$

$$y \geq 0$$

Solving the Utility Maximization Problem

- How should we solve the problem?
- Assuming the non-negative constraints are satisfied, we just need to solve:

$$\max_{x,y} U(x, y) = xy + 20x$$

$$subject\ to\ \ 10x + 5y = 80$$

- Check if the solution satisfies $x \geq 0$ and $y \geq 0.$
  - If the answer is yes, we are done.
  - If the answer is no, our assumption (that the non-negative constraints are satisfied) is incorrect.

Solving the Utility Maximization Problem

- We found that $x = 9, y = -2.$
- As $y = -2$ is not possible, $y = 0$ is the best we can get.
- Plug $y = 0$ into the budget constraint $10x + 5y = 80$:

$$10x + 5(0) = 80$$
$$x = 8$$

- The correct solution is $x = 8, y = 0.$

Solving the Utility Maximization Problem

- Here, the constraint $y \geq 0$ is binding.
  - I.e., the constraint holds with equality, $y = 0$.

- When there are inequality constraints,
  the constraints may or may not be binding.
  - In this example, the constraint $y \geq 0$ is binding
    while the constraint $x \geq 0$ is not binding.

Is (8,0) the Optimal Basket?

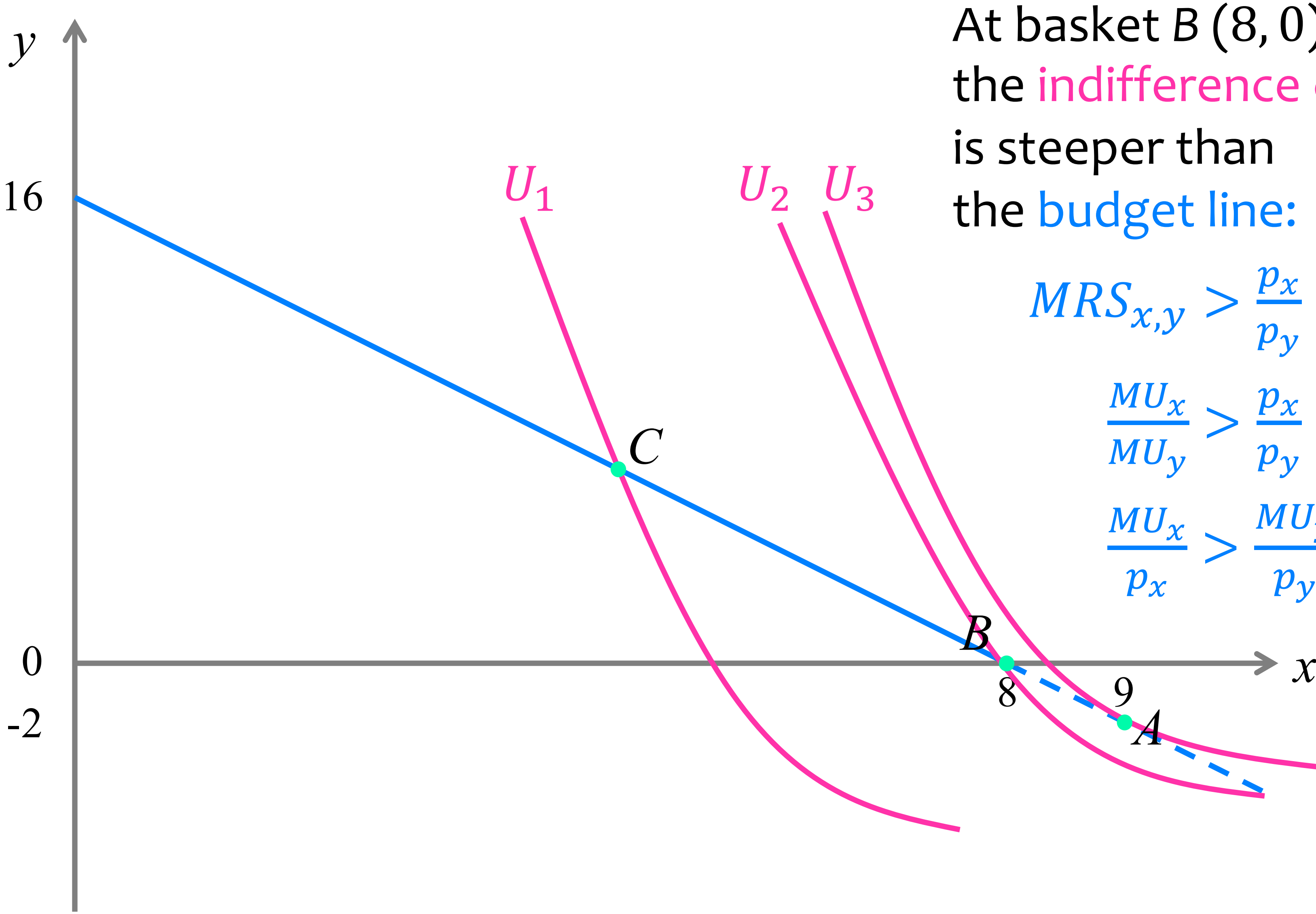
- Compare Billie's per dollar marginal utility of x and of y :

$$\frac{MU_x}{p_x} = \frac{y + 20}{10} = \frac{0 + 20}{10} = 2$$

$$\frac{MU_y}{p_y} = \frac{x}{5} = \frac{8}{5}$$

- Since $\frac{MU_x}{p_x} = 2 > \frac{8}{5} = \frac{MU_y}{p_y}$,
  Billie would like to increase her utility by consuming more x and less y.
  - But her consumption of y is already zero.

Binding and Non-binding Constraints



At basket B $(8, 0)$,
the indifference curve
is steeper than
the budget line:

$$MRS_{x,y} > \frac{p_x}{p_y}$$

$$\frac{MU_x}{MU_y} > \frac{p_x}{p_y}$$

$$\frac{MU_x}{p_x} > \frac{MU_y}{p_y}$$

Interior Solution vs. Corner Solution

- Interior solution: an optimal basket
  where strictly positive amounts of both goods are consumed.

- Corner solution: an optimal basket
  where the consumption of at least one good is zero.
  - The optimal basket is either on the horizontal axis or
    on the vertical axis.
  - The indifference curve may not be tangent
    to the budget line.

Exercise 3.2

Perfect Substitutes

- Naomi's utility function is $U(x, y) = x + y$, and she has an income of $\$3$.

(a) Suppose $p_x = 1$ and $p_y = 2$.

- Compare $\frac{MU_x}{p_x}$ and $\frac{MU_y}{p_y}$.
- Find Naomi's optimal choice.

(b) Suppose $p_x = 2$ and $p_y = 1$.

- Compare $\frac{MU_x}{p_x}$ and $\frac{MU_y}{p_y}$.
- Find Naomi's optimal choice.

Exercise 3.2(a)

Perfect Substitutes $(p_x = 1, p_y = 2)$

Exercise 3.2(b)

Perfect Substitutes $(p_x = 2, p_y = 1)$

Voucher vs. Cash

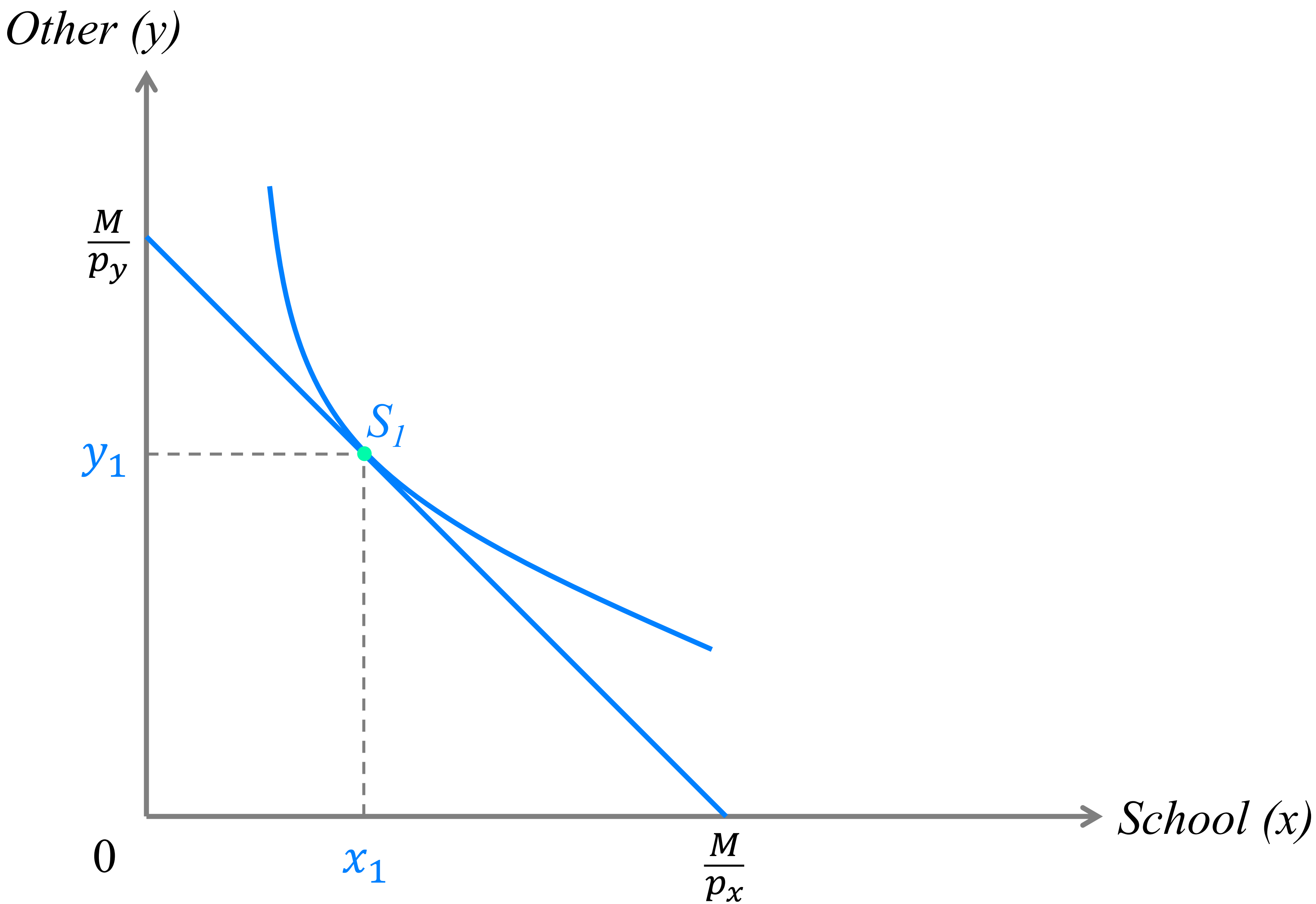
Voucher

- Examples of voucher programs:
  - Supplemental Nutrition Assistance Program (SNAP) in the U.S.
  - School vouchers for private schools in the U.S.
  - In April 2020 in China, digital vouchers for supermarkets, food catering, restaurants, shopping malls, sports, entertainment, tourism.
  - In December 2020 in Singapore, $100 SingapoRediscovers Vouchers for hotels, attractions, and tours.

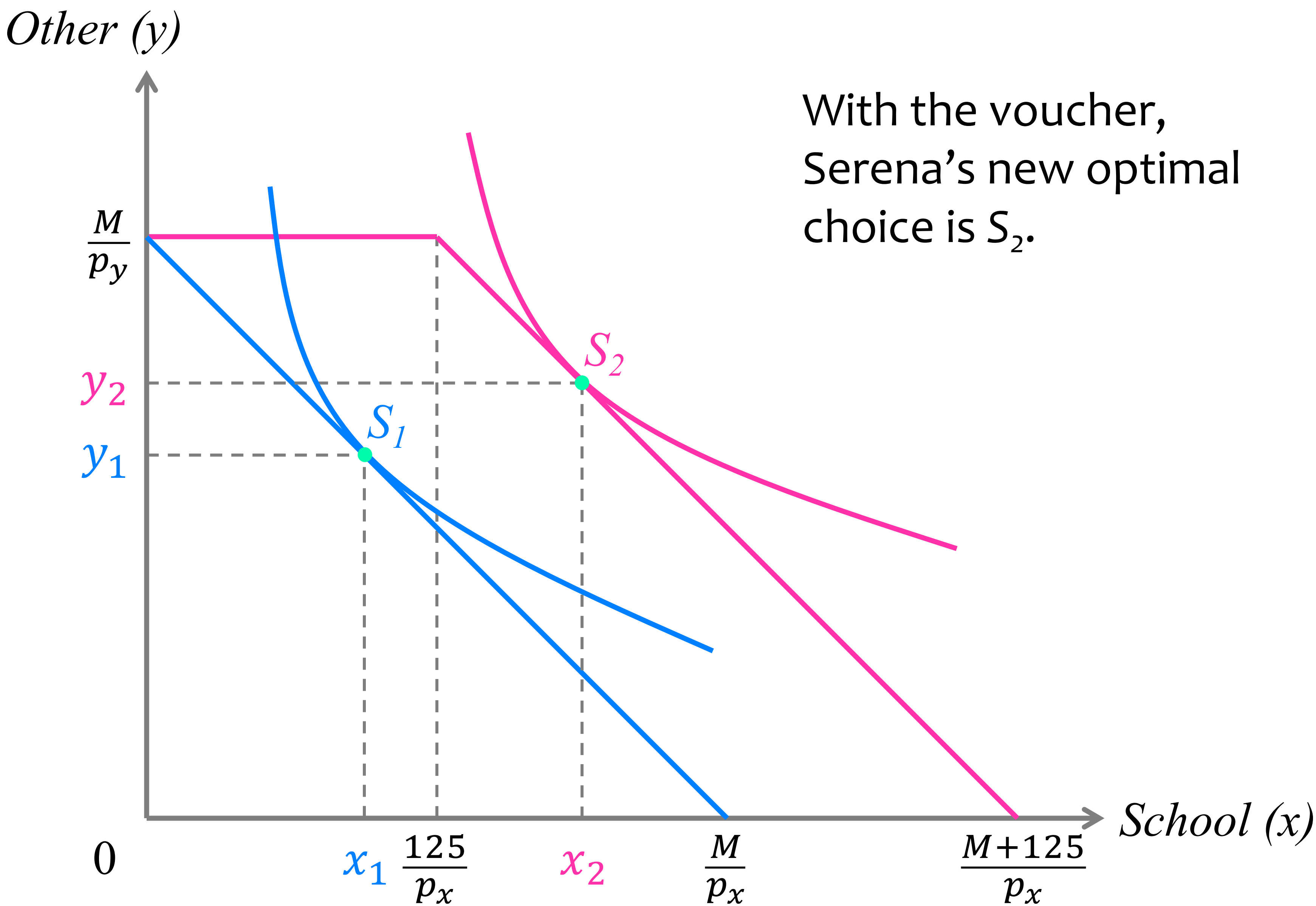
Voucher: Example

- NTUC used to offer back-to-school vouchers to low-income families.
  - $125 voucher per school-going child for school-related merchandise (e.g., school bags and shoes, assessment books, stationery).
- What is the impact of the $125 back-to-school voucher on:
  - Consumer’s choice.
  - Consumer’s utility.

Serena's Indifference Curve and Optimal Choice

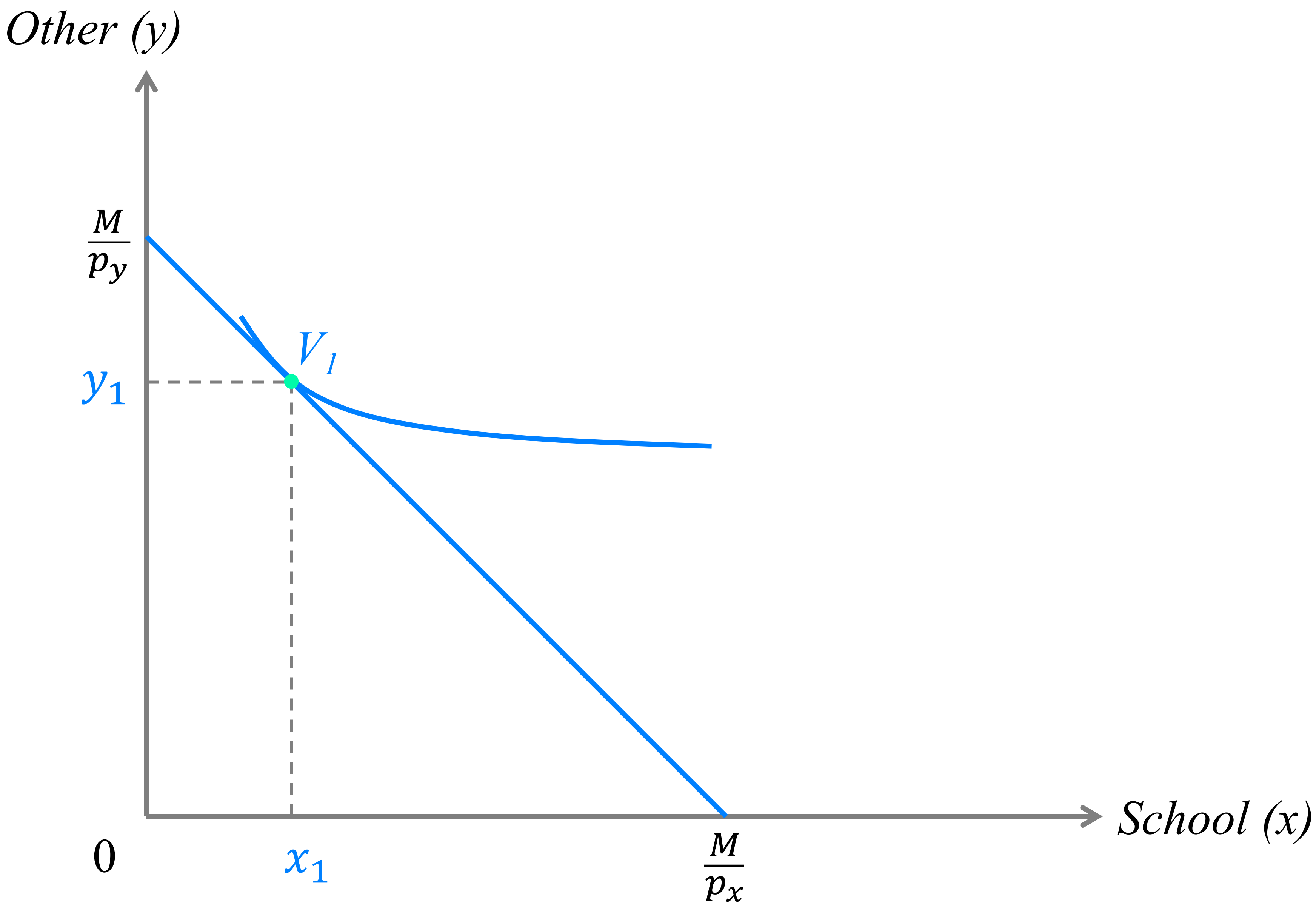


Impact of Voucher on Serena

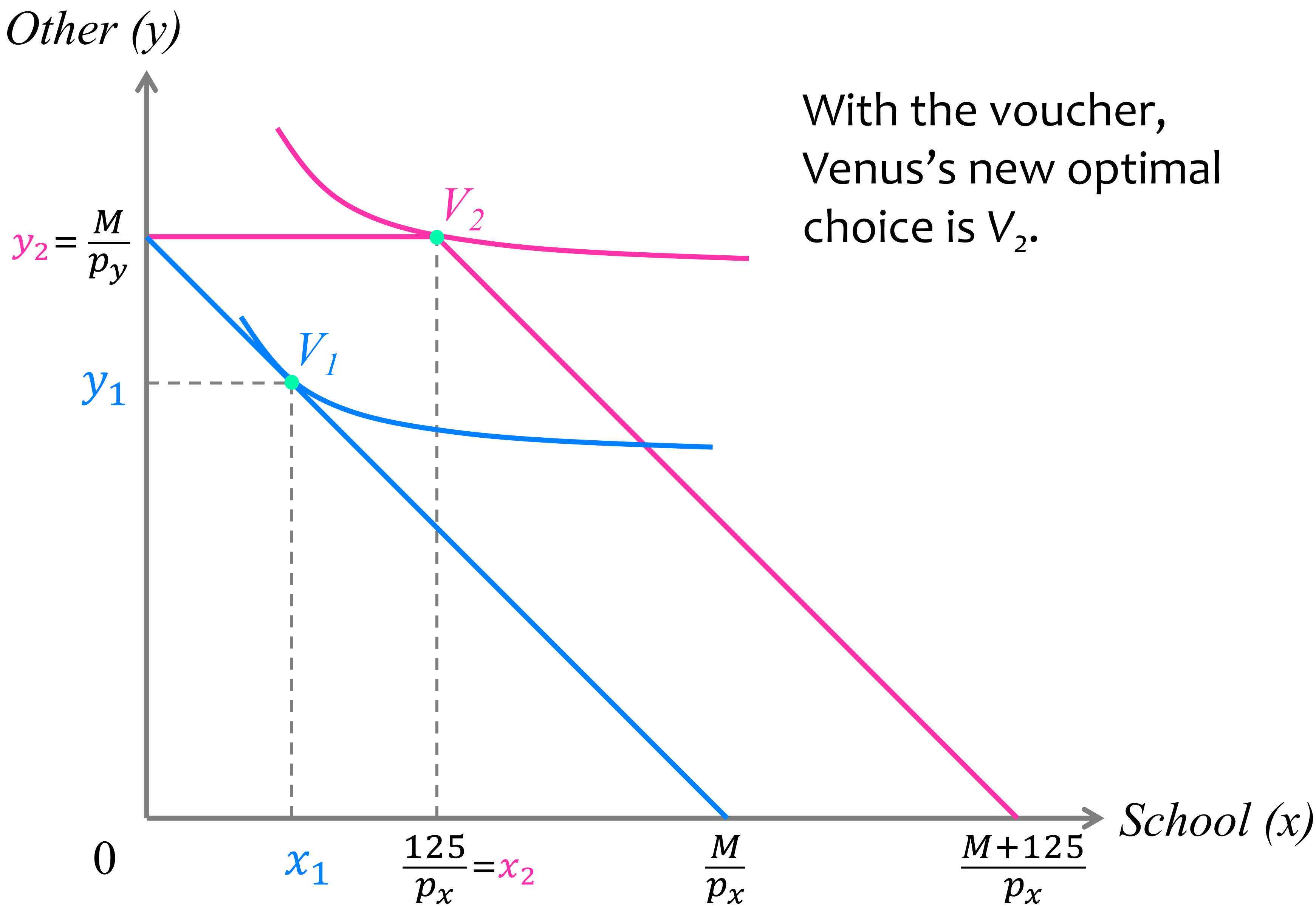


With the voucher, Serena's new optimal choice is S_2.

Venus's Indifference Curve and Optimal Choice



Impact of Voucher on Venus

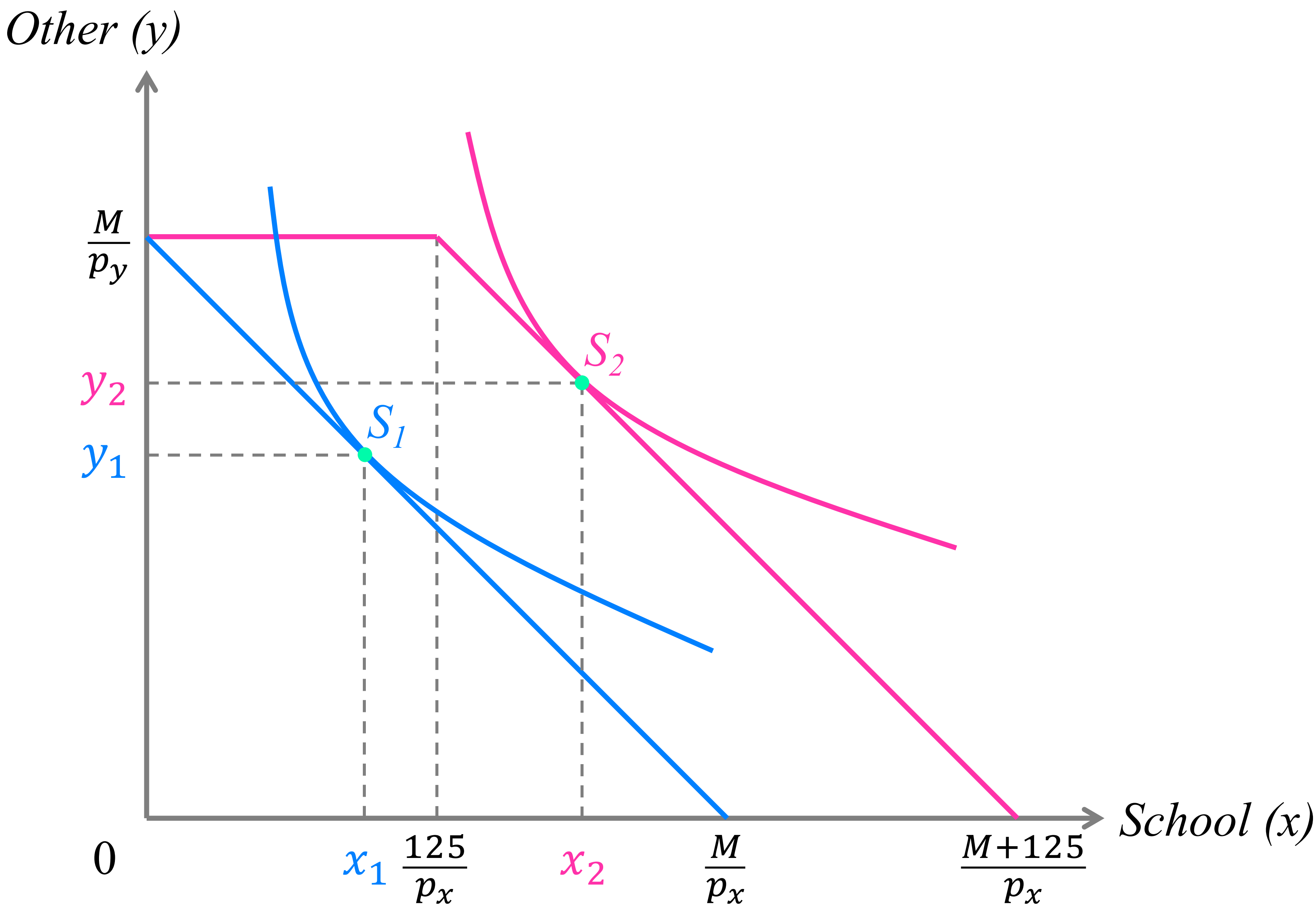


With the voucher, Venus's new optimal choice is V_2.

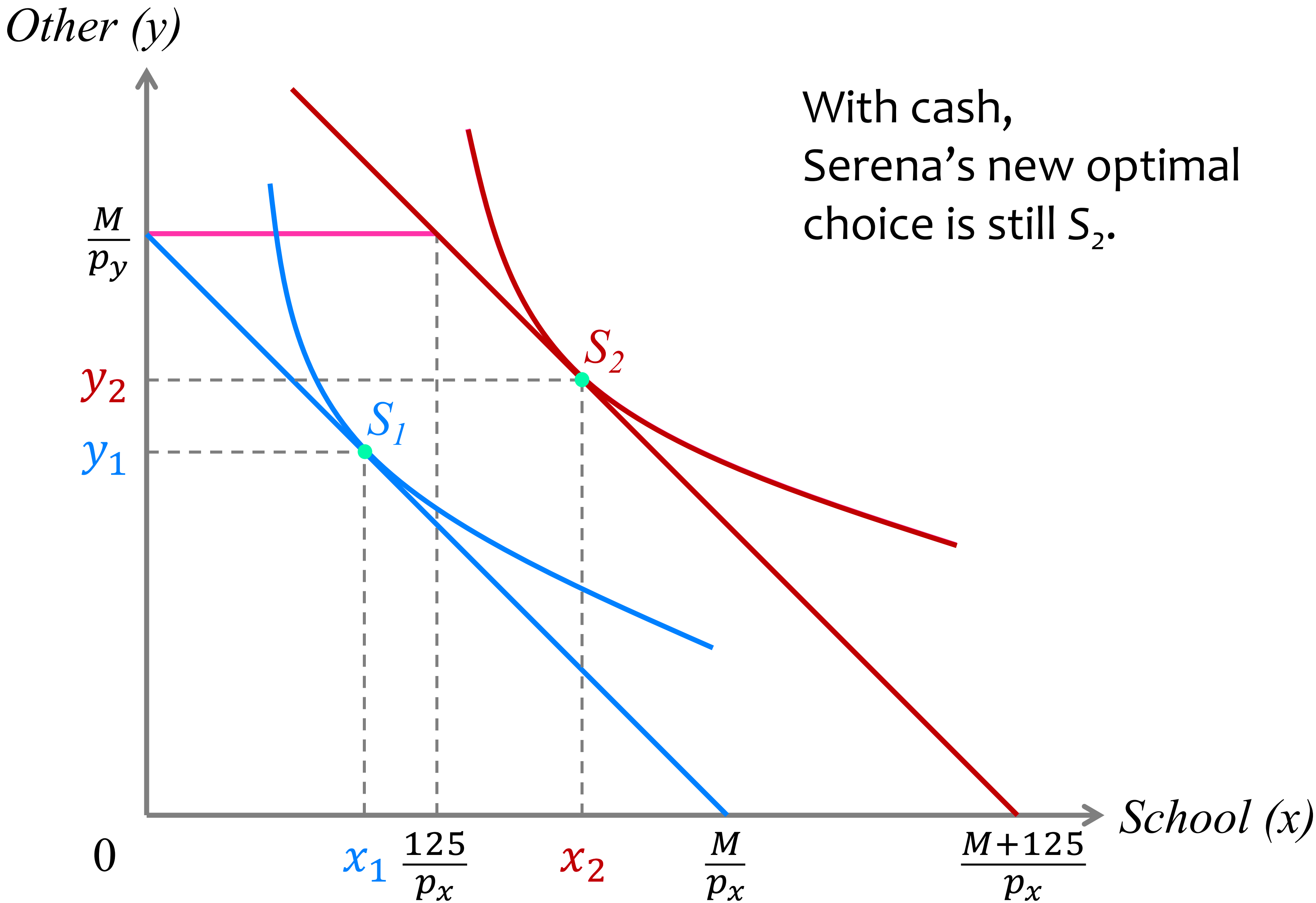
Voucher vs. Cash

- We saw the impact of the $125 back-to-school voucher on:
  - Consumer’s choice.
  - Consumer’s utility.
- What is the impact of $125 in cash on:
  - Consumer’s choice.
  - Consumer’s utility.

Impact of Cash on Serena

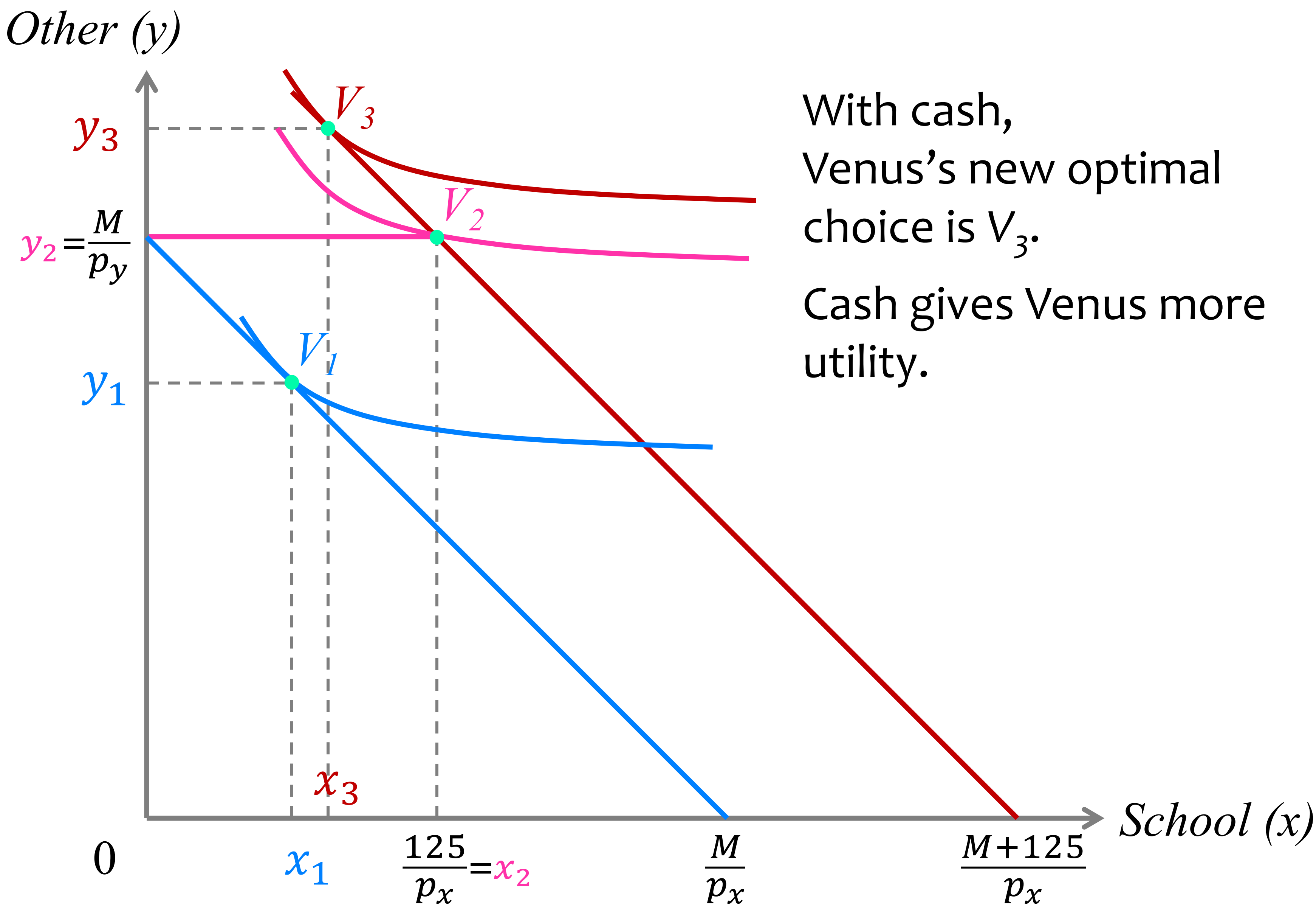


Impact of Cash on Serena



With cash,
Serena's new optimal choice is still S_2.

Impact of Cash on Venus



With cash,
Venus's new optimal choice is V_3.

Cash gives Venus more utility.

Application

Voucher vs. Cash

- Between $\$V$ in vouchers and $\$V$ in cash,
  - Some consumers are indifferent between the two.
  - Some consumers prefer cash to vouchers.
- Cash is never worse than vouchers.
- So why use vouchers?

Revealed Preference

Preference and Optimal Choice

- What have we been doing so far?
  - Given preference (indifference curves/utility functions)

    and the budget constraint …
  - We can find the consumer’s optimal choice.

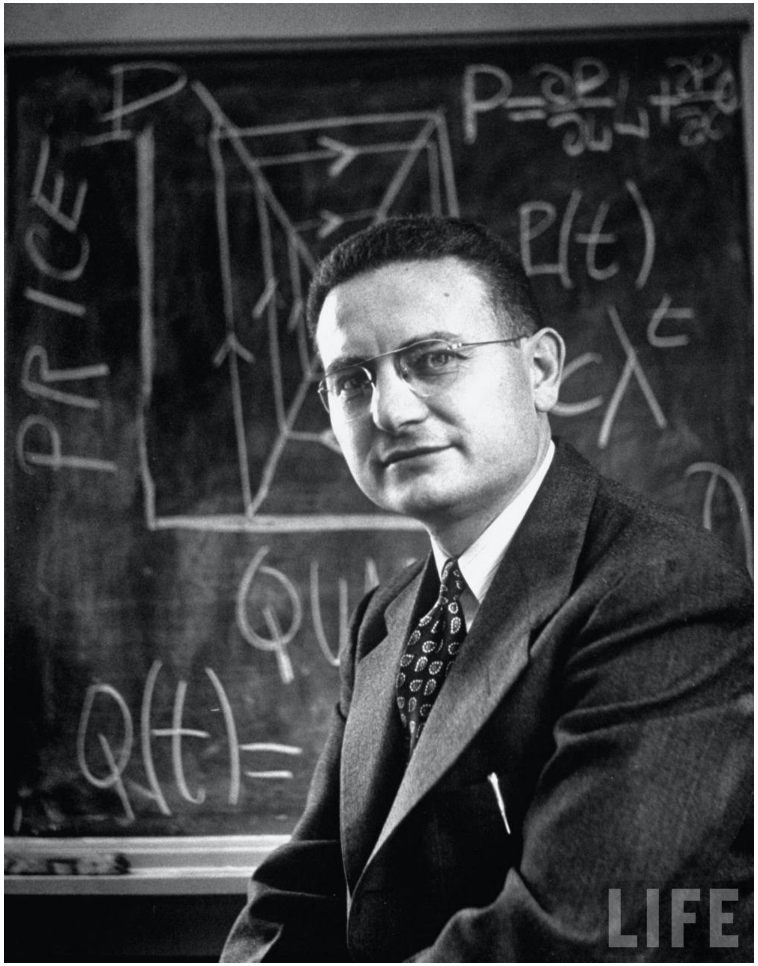
- Can we go the other way round?
  - Given the budget constraint

    and the consumer’s optimal choice …
  - Can we get any information on preference?

Revealed Preference

- Revealed preference:
  - The analysis that enables us to infer preference based on observed prices and choices.

Paul Samuelson



Paul Samuelson
1915–2009

- Awarded the
  1970 Nobel in Economics.
- Contributions include: revealed preference, Samuelson rule, Bergson–Samuelson social welfare function, efficient markets hypothesis, Turnpike theory, Balassa–Samuelson effect, Stolper–Samuelson theorem, overlapping generations model

Strictly Preferred vs. Weakly Preferred

- A is strictly preferred to B : $A \succ B$
- A is weakly preferred to B : $A \succcurlyeq B$

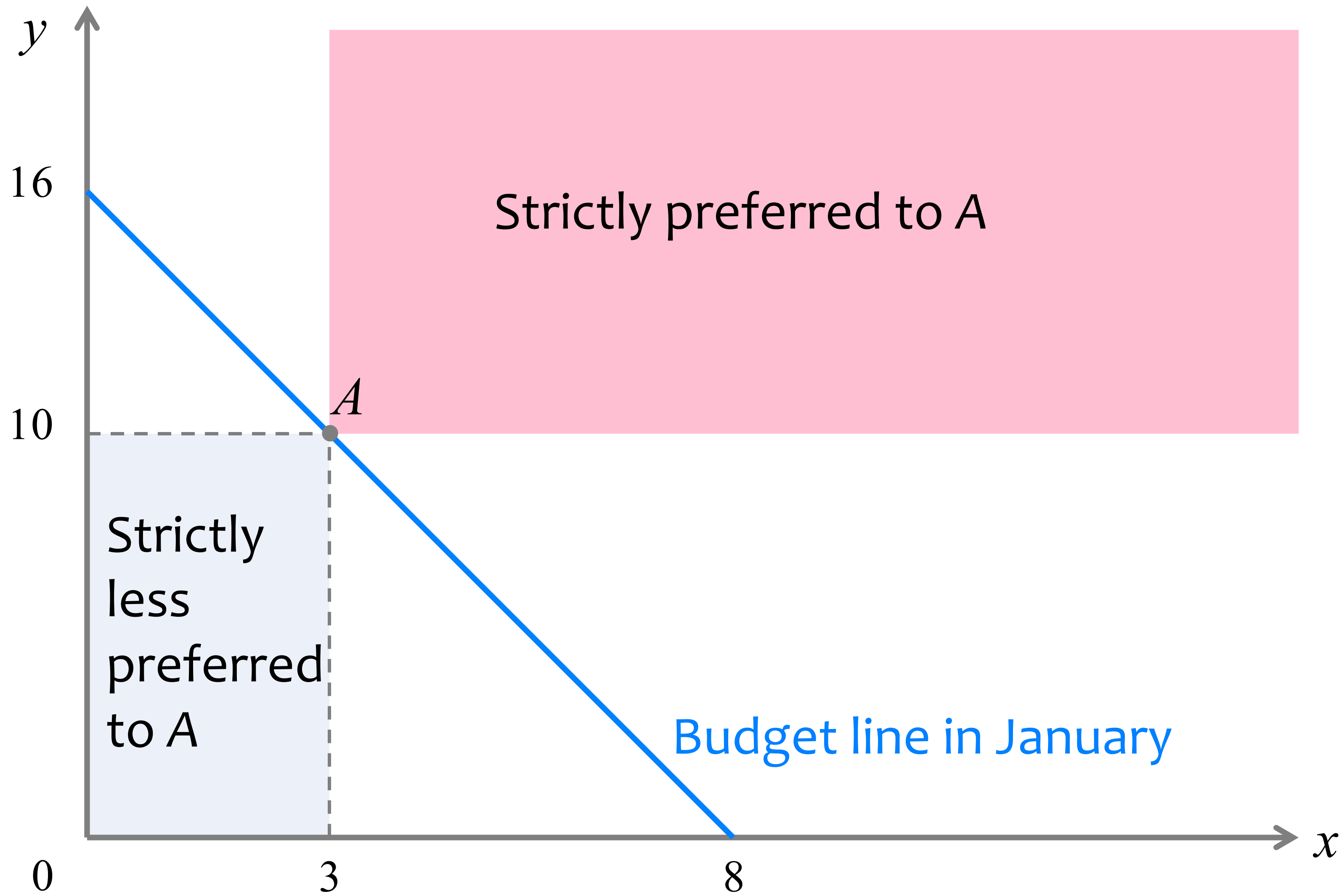
$$\Longleftrightarrow \quad A \succ B \quad \text{or}$$

$$A \sim B$$

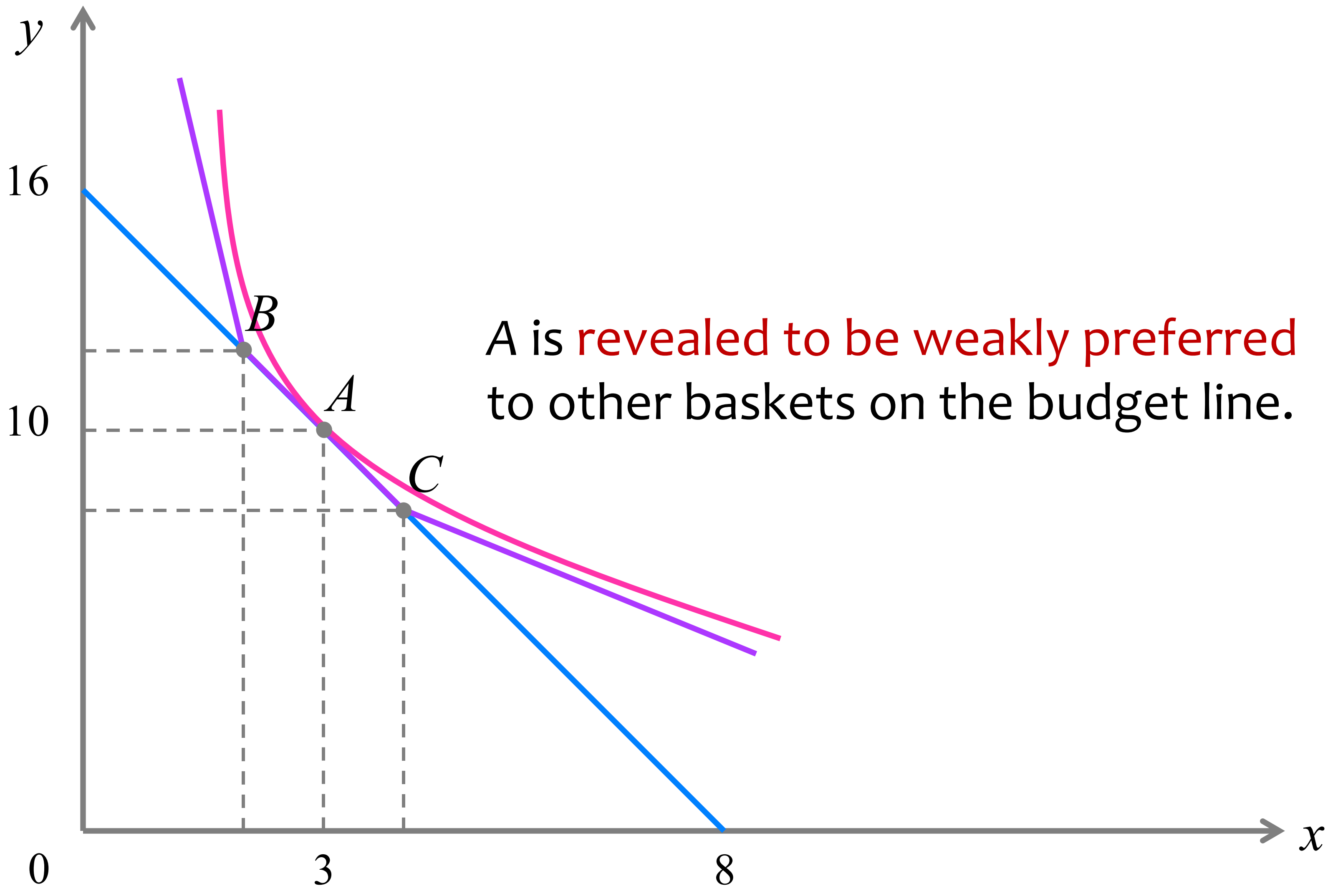
From Choice to Preference

- Suppose we observe Steffi's budget constraint.
- We also know the optimal basket that she has chosen given the budget constraint.
- But we do not know Steffi's preference.
  - We know her preference satisfies the three assumptions.
  - We also know her preference does not change with prices or with income.
- Our goal is to infer Steffi's preference — how she ranks different consumption baskets.

Basket *A* is the Optimal Choice in January

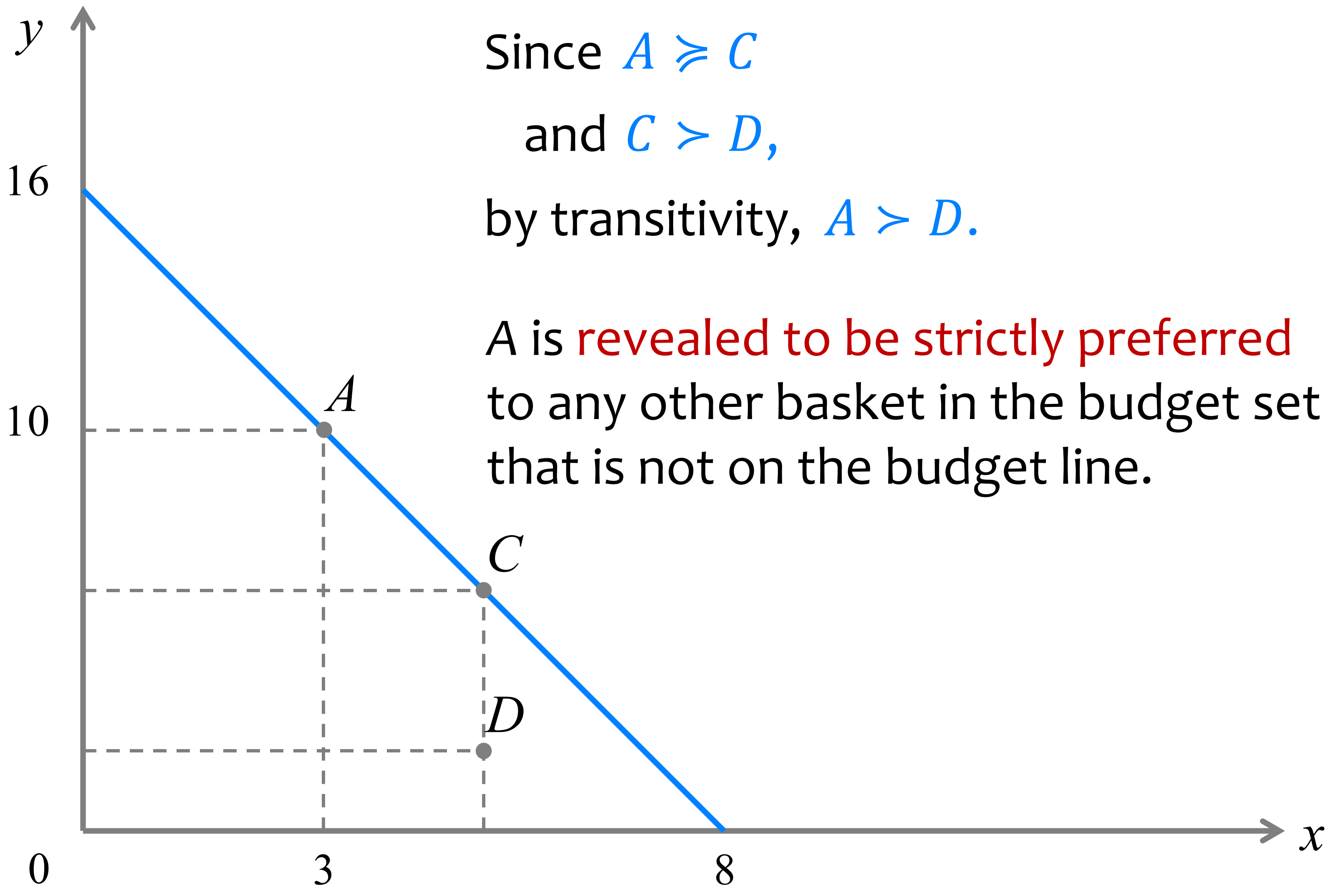


Basket *A* vs. Other Baskets on the Budget Line



A is revealed to be weakly preferred to other baskets on the budget line.

Basket *A* vs. Other Baskets below the Budget Line



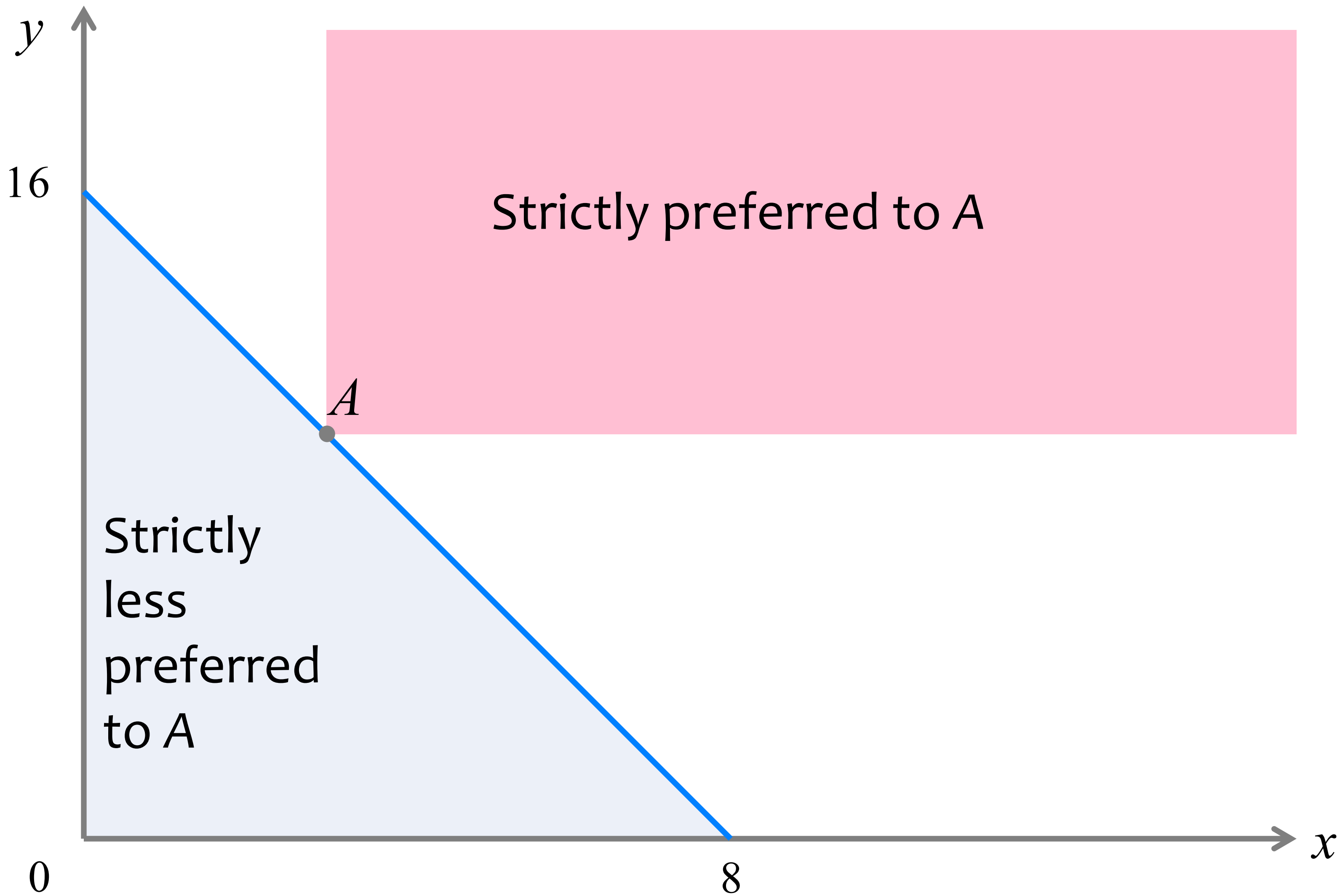
Since $A \succcurlyeq C$

and $C \succ D$,

by transitivity, $A \succ D$.

A is revealed to be strictly preferred to any other basket in the budget set that is not on the budget line.

How Optimal Choice "Reveals" Preference



Another Way to Understand Revealed Preference

- Suppose basket $A = (x_A, y_A)$ is the optimal basket given prices p_x, p_y, and income M.
  - Basket A must be on the budget line.

$$p_x x_A + p_y y_A = M$$

Another Way to Understand Revealed Preference

- No other affordable basket is strictly preferred to basket *A*.
- Suppose basket $B = (x_B, y_B)$ is strictly preferred to basket *A*.
  - Basket *B* cannot be affordable,
    i.e., basket *B* must lie above the budget line.

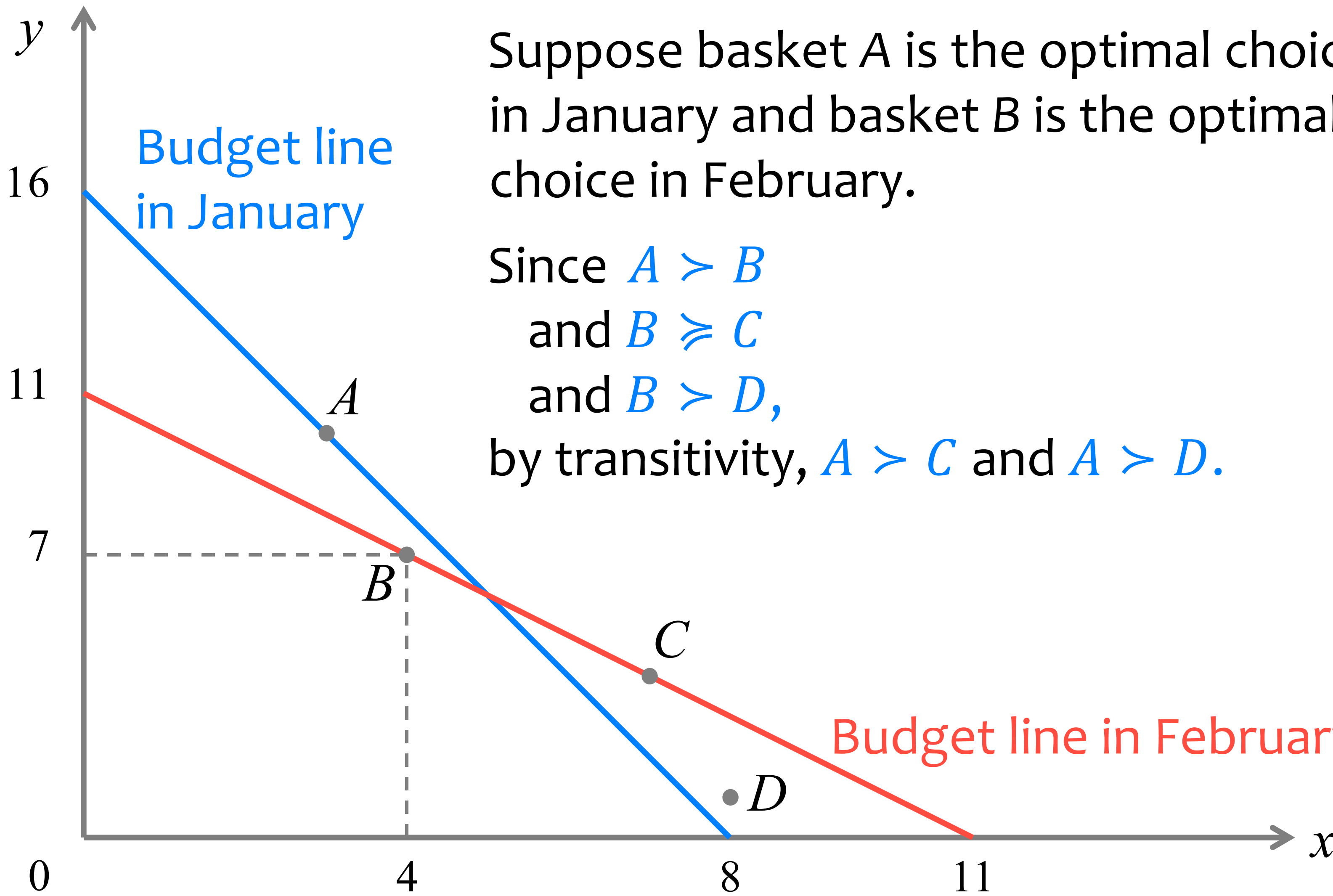
$$p_x x_B + p_y y_B > p_x x_A + p_y y_A = M$$

Another Way to Understand Revealed Preference

- Suppose Steffi is indifferent between basket A and basket C $= (x_C, y_C)$.
  - Basket C cannot cost less than basket A, i.e., basket C cannot lie below the budget line.

$$p_x x_C + p_y y_C \geq p_x x_A + p_y y_A = M$$

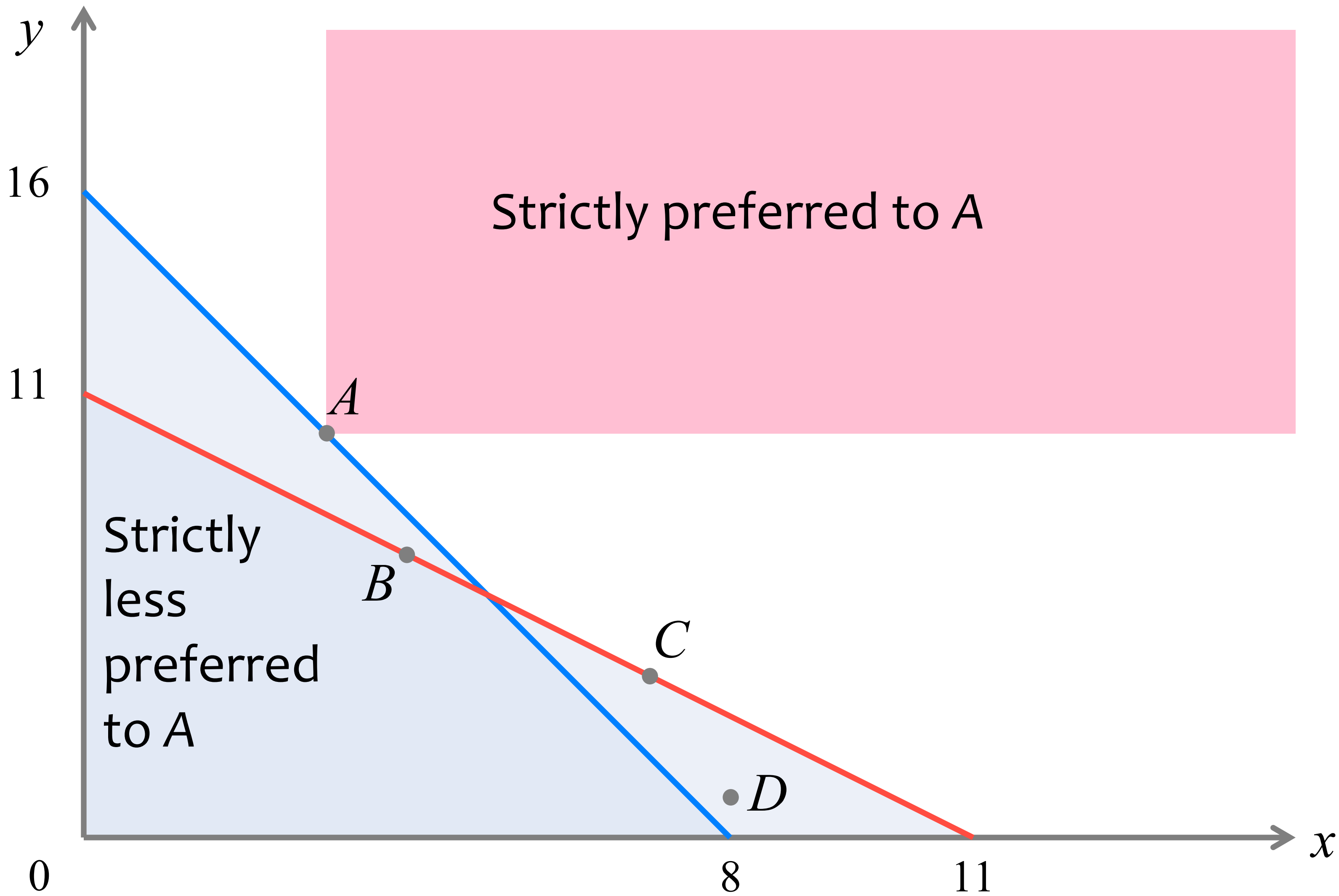
Basket B is the Optimal Choice in February



Suppose basket A is the optimal choice in January and basket B is the optimal choice in February.

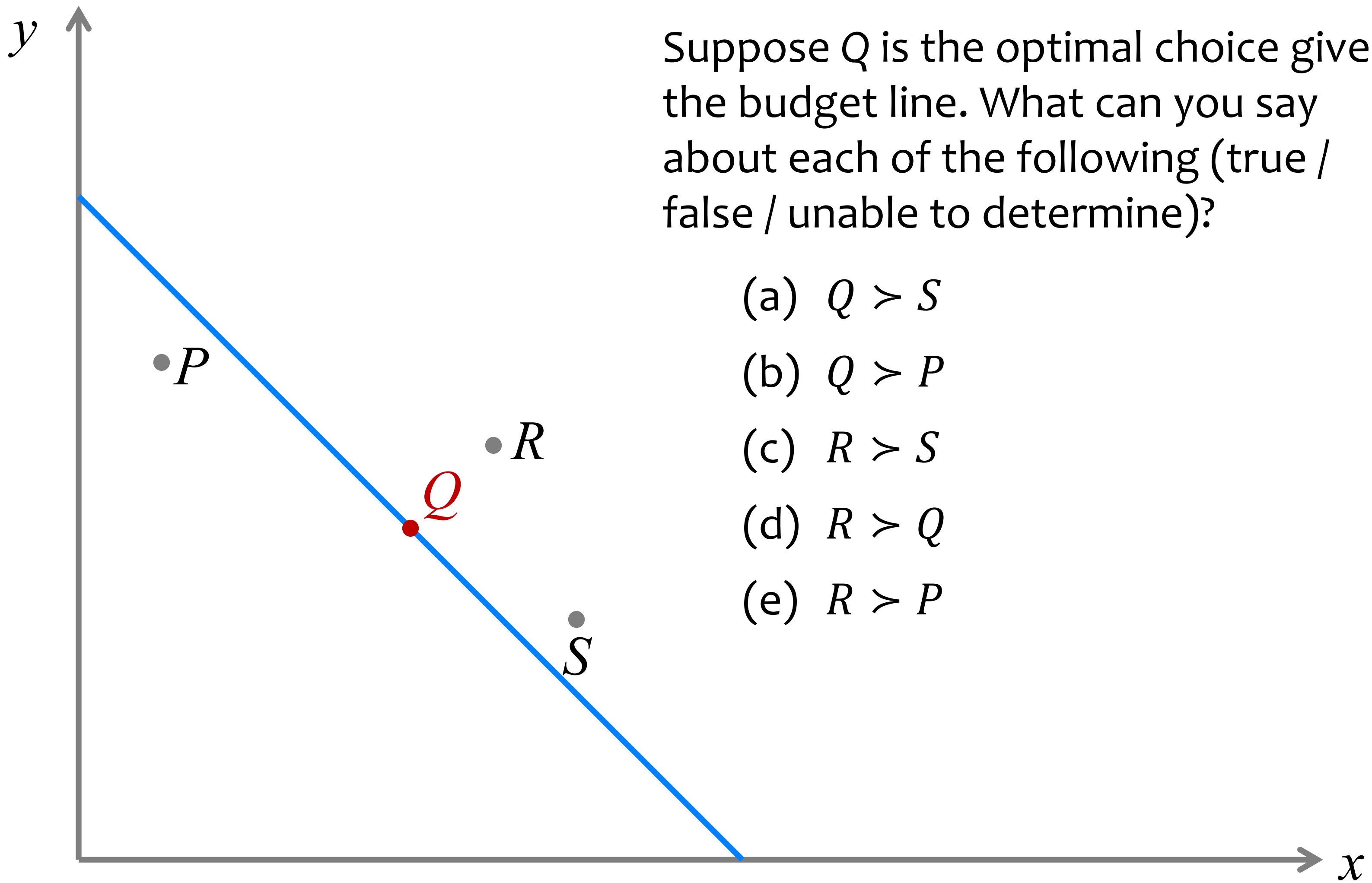
Since $A \succ B$
and $B \succcurlyeq C$
and $B \succ D$,
by transitivity, $A \succ C$ and $A \succ D$.

More Choices Observed, More Information on Preference Revealed



Exercise 3.3

Revealed Preference



Suppose Q is the optimal choice given the budget line. What can you say about each of the following (true / false / unable to determine)?

(a) $Q \succ S$

(b) $Q \succ P$

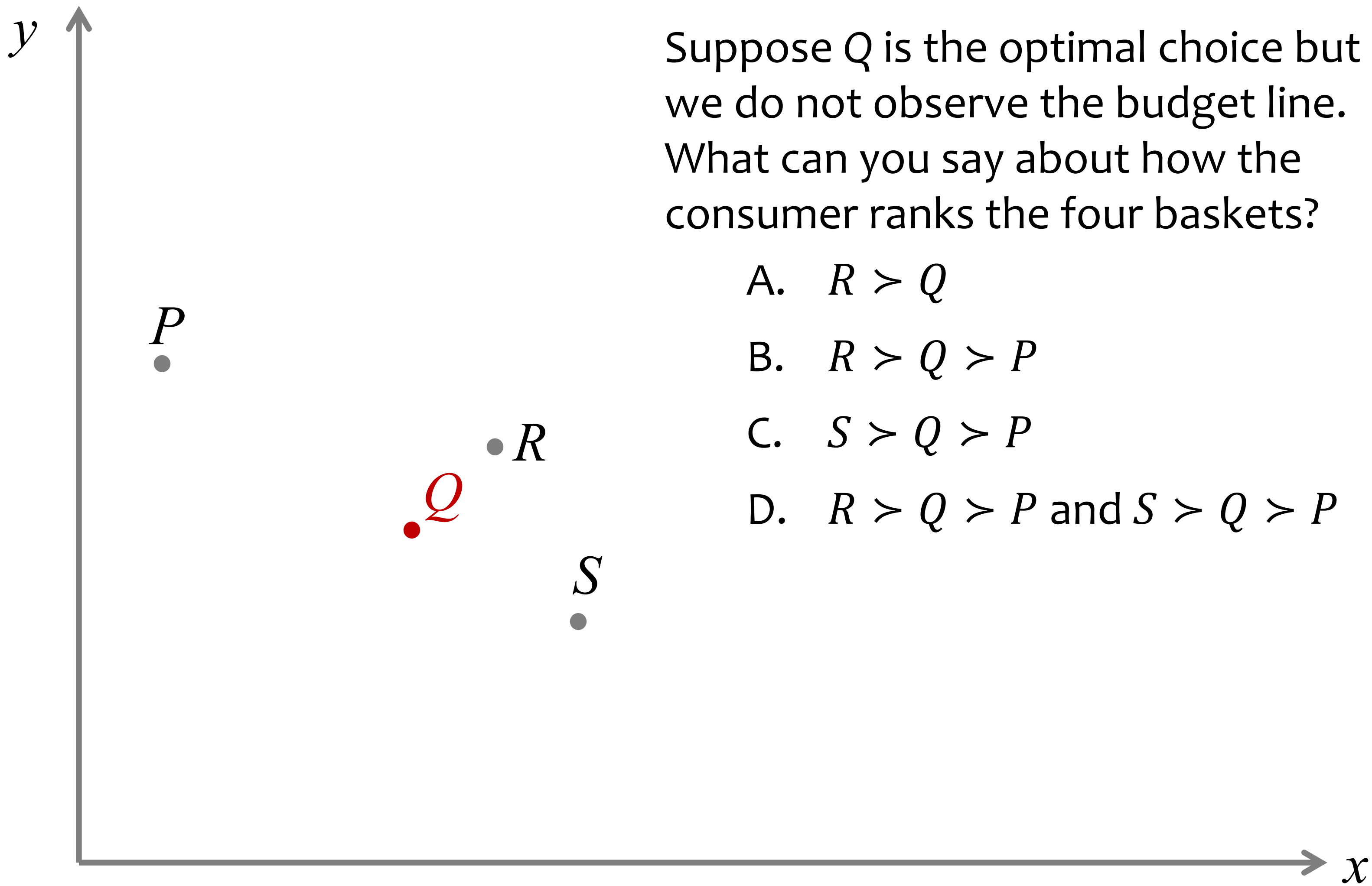
(c) $R \succ S$

(d) $R \succ Q$

(e) $R \succ P$

Exercise 3.4

Revealed Preference



Suppose *Q* is the optimal choice but we do not observe the budget line. What can you say about how the consumer ranks the four baskets?

A. $R \succ Q$

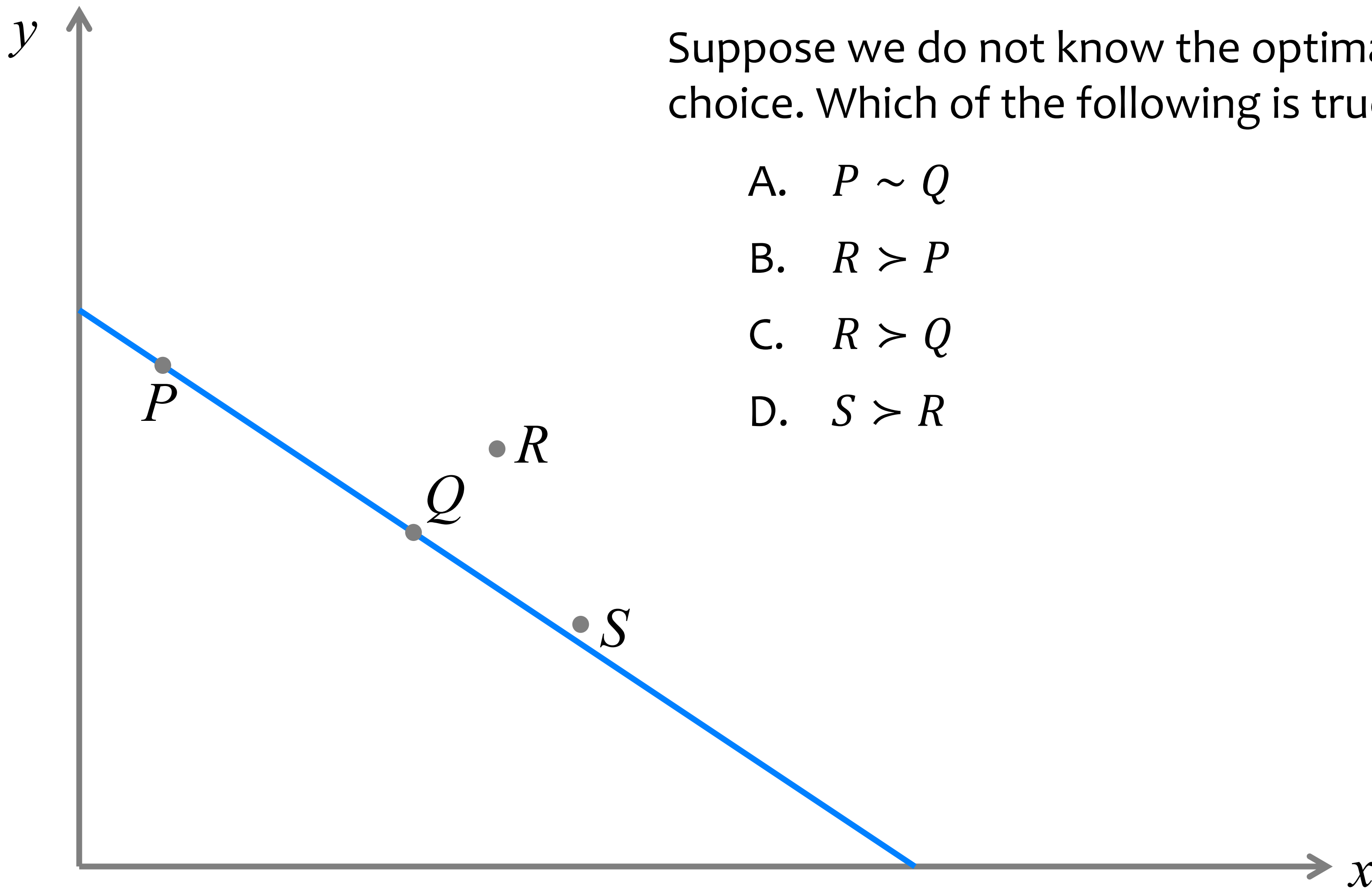
B. $R \succ Q \succ P$

C. $S \succ Q \succ P$

D. $R \succ Q \succ P$ and $S \succ Q \succ P$

Exercise 3.5

Revealed Preference



Suppose we do not know the optimal choice. Which of the following is true?

A. $P \sim Q$

B. $R \succ P$

C. $R \succ Q$

D. $S \succ R$