EC2101: Microeconomic Analysis I

Lecture 2

Theory of the Consumer

- Utility Function
- Types of Preferences
- Budget Constraint
- Optimal Choice: Graphical Analysis

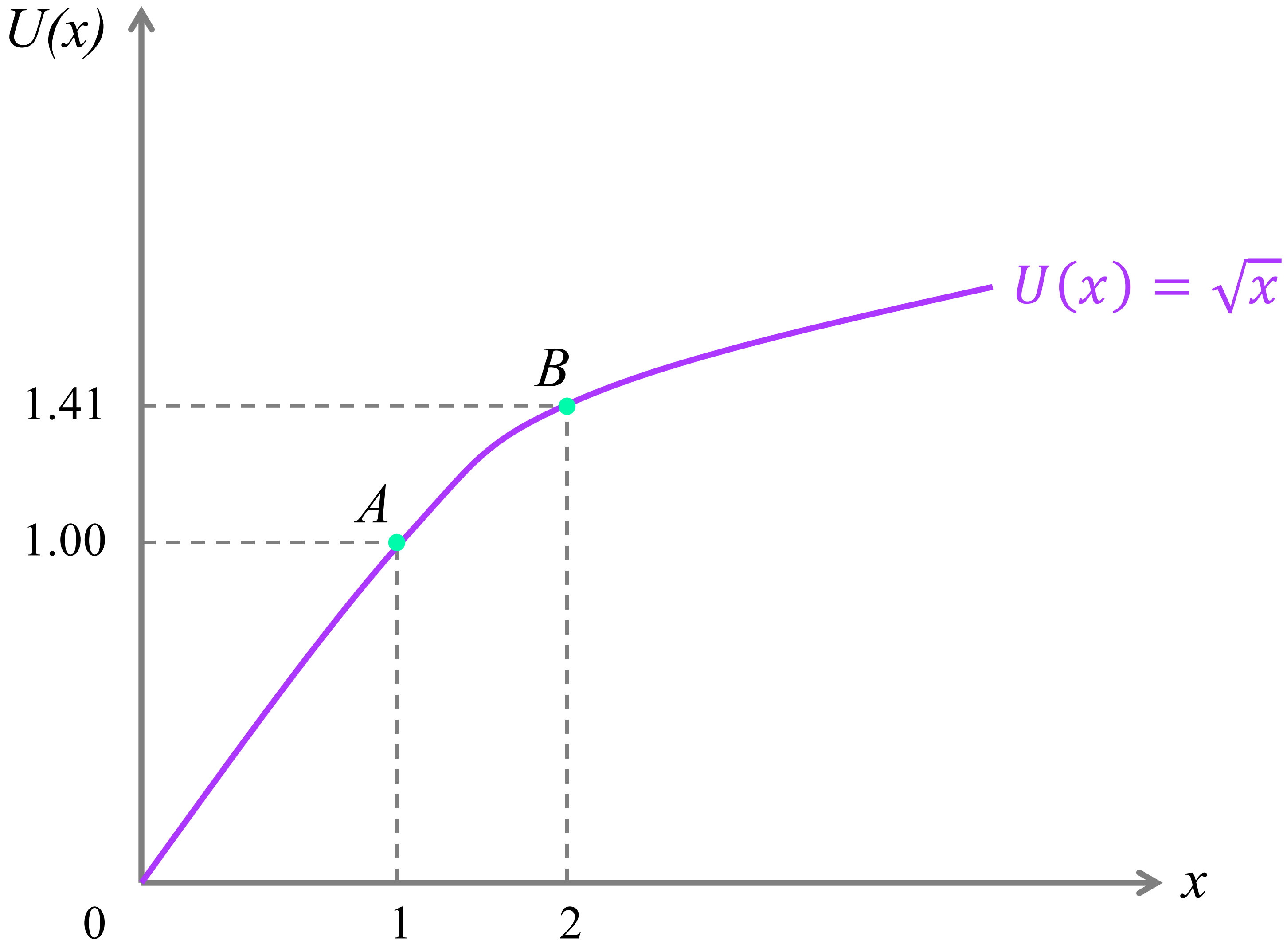
Utility Function

Utility Function

- Preferences can be mathematically represented by a utility function.
- Utility is a numeric value indicating the consumer's level of satisfaction.
- A utility function assigns a level of utility to each consumption basket such that:
  - If $A \sim B$, then $U(A) = U(B)$.
  - If $A \succ B$, then $U(A) > U(B)$.

Utility Function with One Good

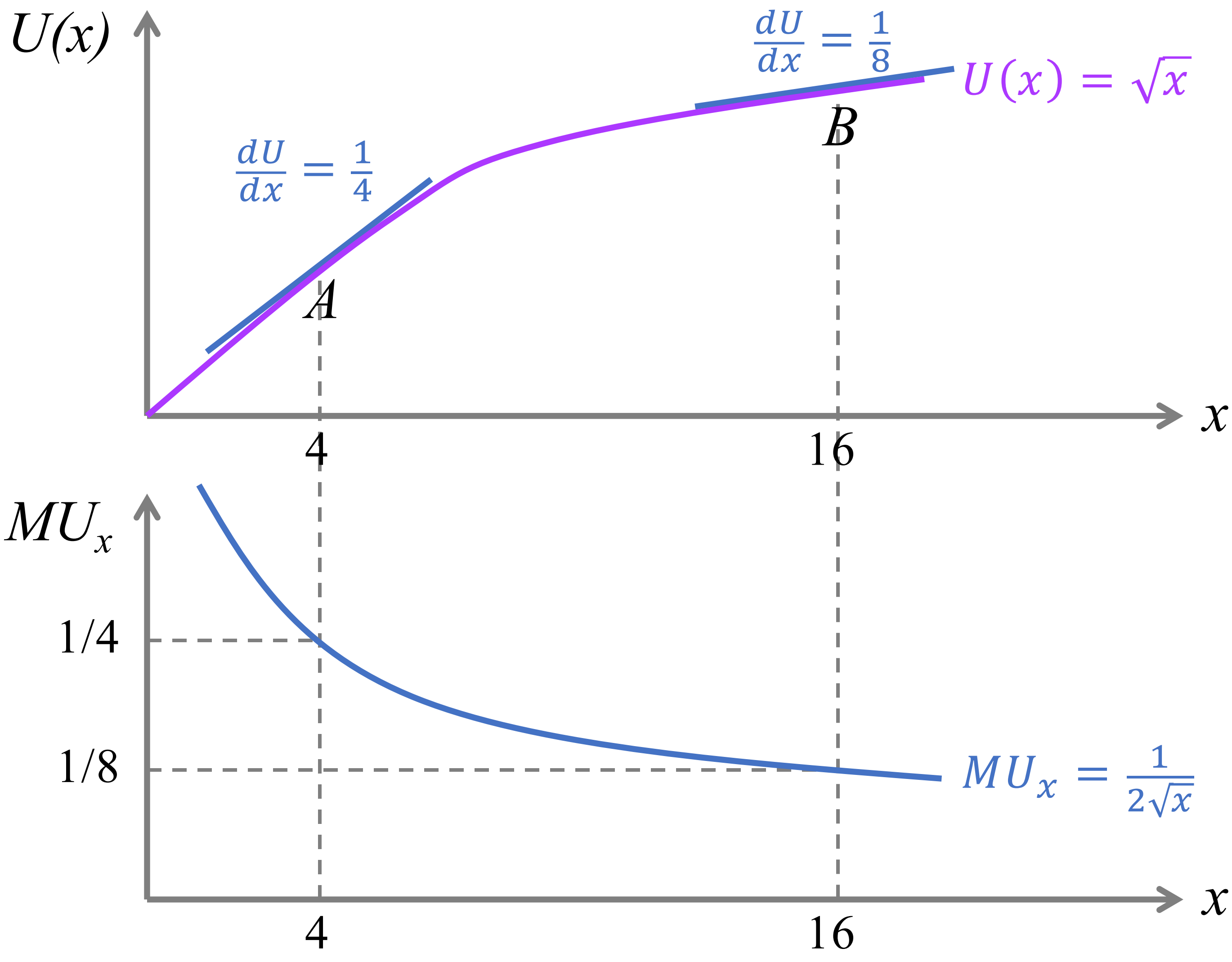
Utility Function with One Good



Marginal Utility with One Good

- Marginal utility:
  - The rate at which utility changes
    as the level of consumption of a good changes.
  - $MU_x = \frac{dU}{dx} = \frac{\Delta U}{\Delta x}$ where Δx is extremely small.
- MU_x is the derivative of the utility function.
- What does the sign of MU_x tell us?
  - Whether monotonicity holds.

Marginal Utility: Graphical Representation



Principle of Diminishing Marginal Utility

- Principle of diminishing marginal utility:
  - As the level of consumption increases, marginal utility decreases.
- Graphically, the slope of the utility function becomes flatter.

Utility Function
with Two Goods

Utility Function with Two Goods

- Suppose there are two goods, x and y.
- Naomi's utility function is:

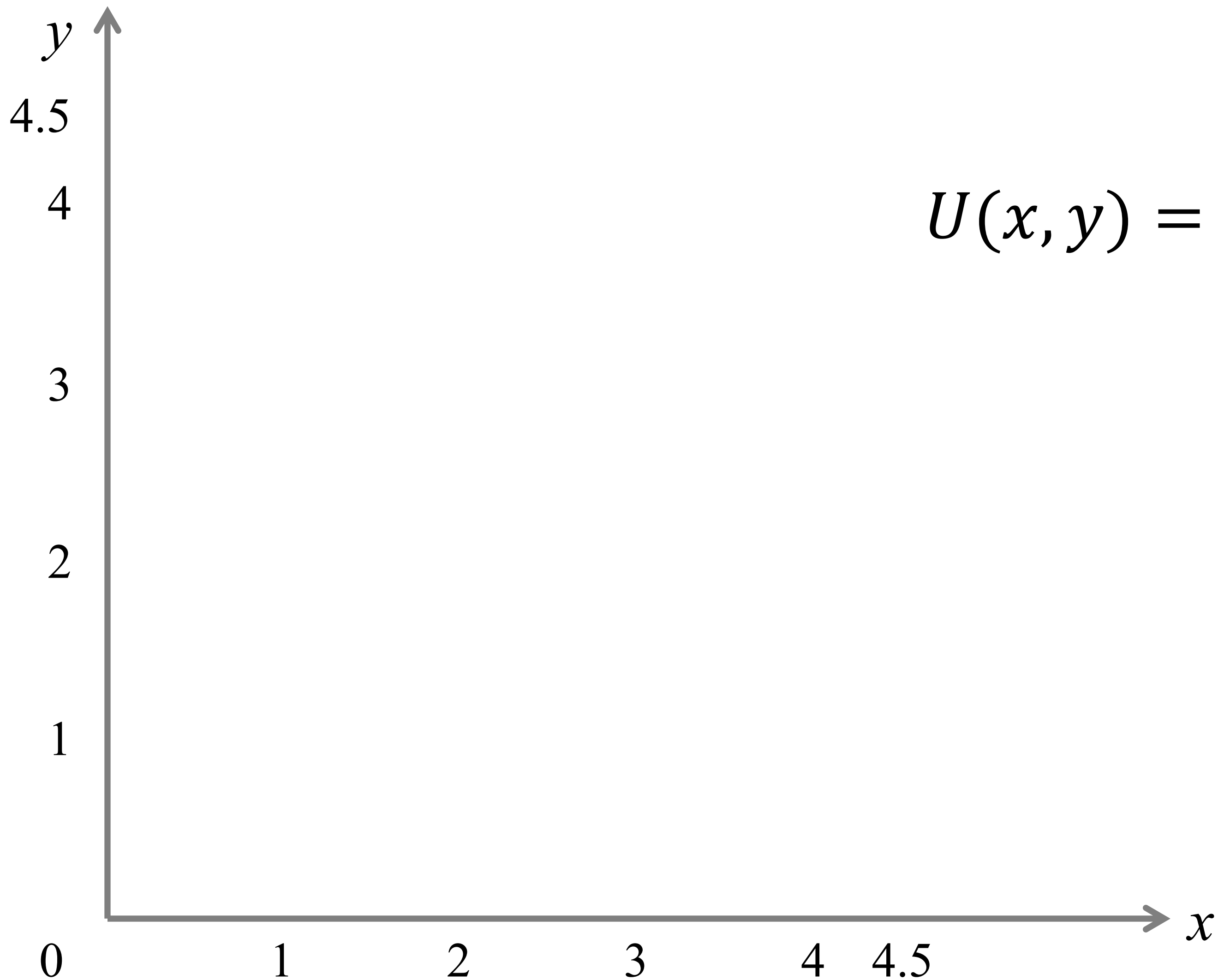
$$U(x, y) = \sqrt{xy}$$

- What does the utility function tell us about her preferences?

Marginal Utility with Two Goods

- Given a utility function, $U(x, y)$:
  - Marginal utility of x : $MU_x = \frac{\partial U}{\partial x}$
  - Marginal utility of y : $MU_y = \frac{\partial U}{\partial y}$
- Principle of diminishing marginal utility:
  - MU_x decreases as x increases, holding y constant.
  - MU_y decreases as y increases, holding x constant.

Utility Function with Two Goods



$$U(x, y) = \sqrt{xy}$$

Utility Function and Indifference Curves

- We can draw indifference curves for Naomi's utility function:

$$U(x,y) = \sqrt{xy}$$

- Naomi is indifferent between (2,2), (1,4), and (4,1):

$$U(2,2) = U(1,4) = U(4,1) = \sqrt{4} = 2$$

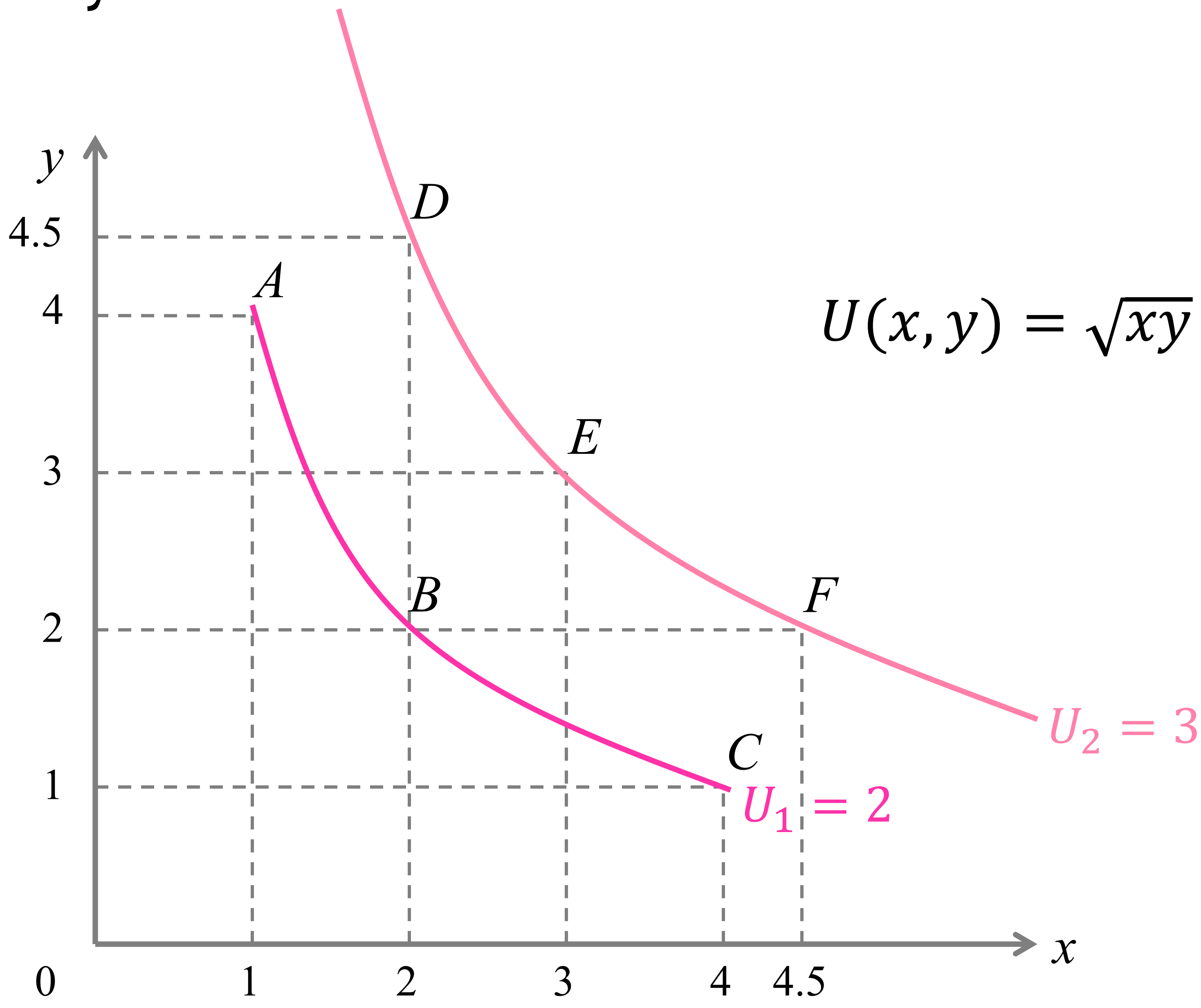
- Naomi is indifferent between (3,3), (2,4.5), and (4.5,2):

$$U(3,3) = U(2,4.5) = U(4.5,2) = \sqrt{9} = 3$$

- Naomi prefers (3,3) to (2,2):

$$U(3,3) = 3 > 2 = U(2,2)$$

Utility Function and Indifference Curves



Exercise 2.1

Utility Function

Suppose Naomi's utility function is:

$$U(x, y) = \sqrt{xy}$$

(a) What does the utility function tell you about her preferences?

Hint: Find $MU_x = \frac{\partial U}{\partial x}$ *and* $MU_y = \frac{\partial U}{\partial y}$.

(b) Does the utility function exhibit diminishing marginal utility in each good?

Hint: Find $\frac{\partial MU_x}{\partial x} = \frac{\partial^2 U}{\partial x^2}$ *and* $\frac{\partial MU_y}{\partial y} = \frac{\partial^2 U}{\partial y^2}$.

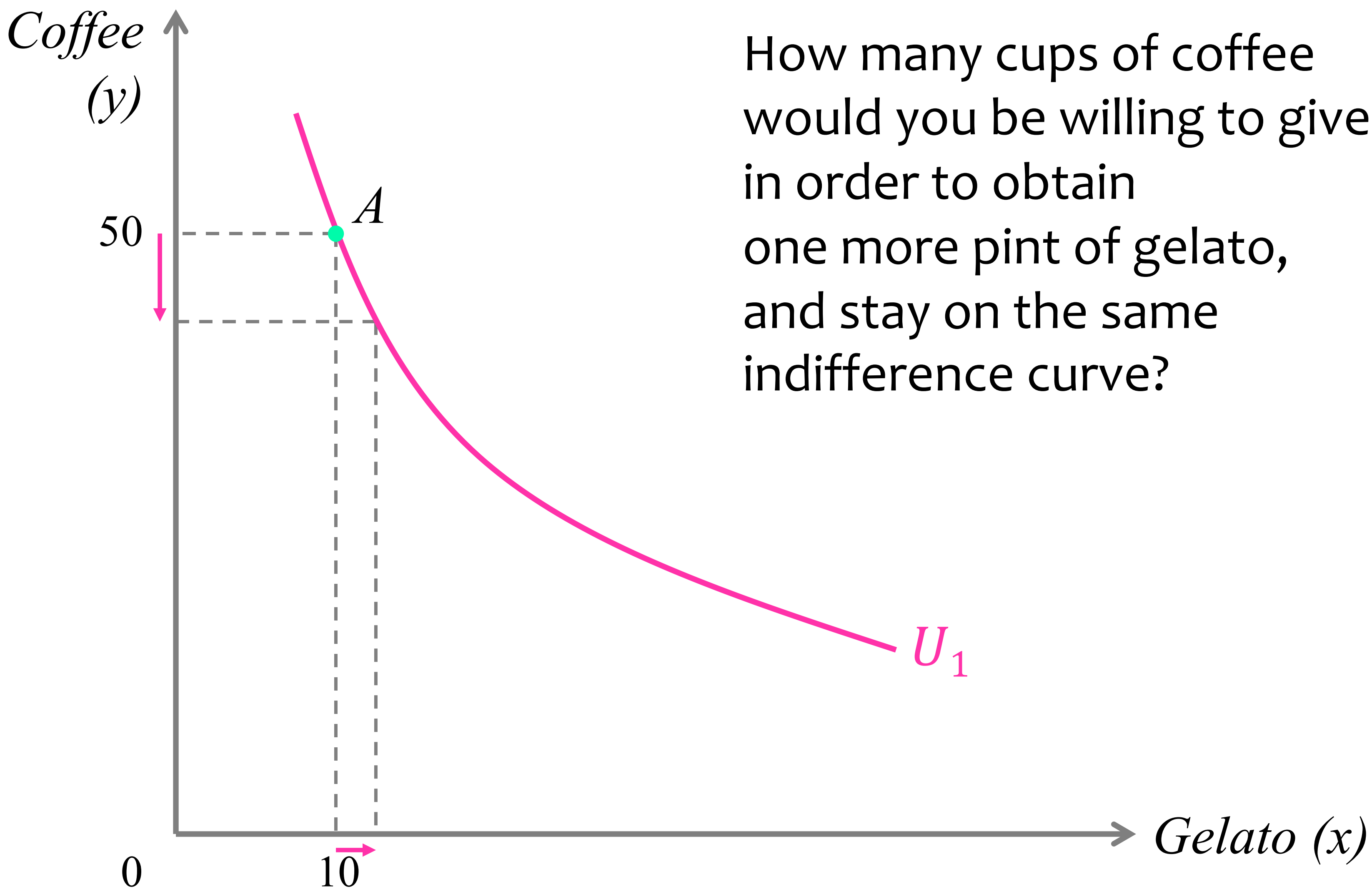
Exercise 2.1(a)

Utility Function: First Derivative

Exercise 2.1(b)

Utility Function: Second Derivative

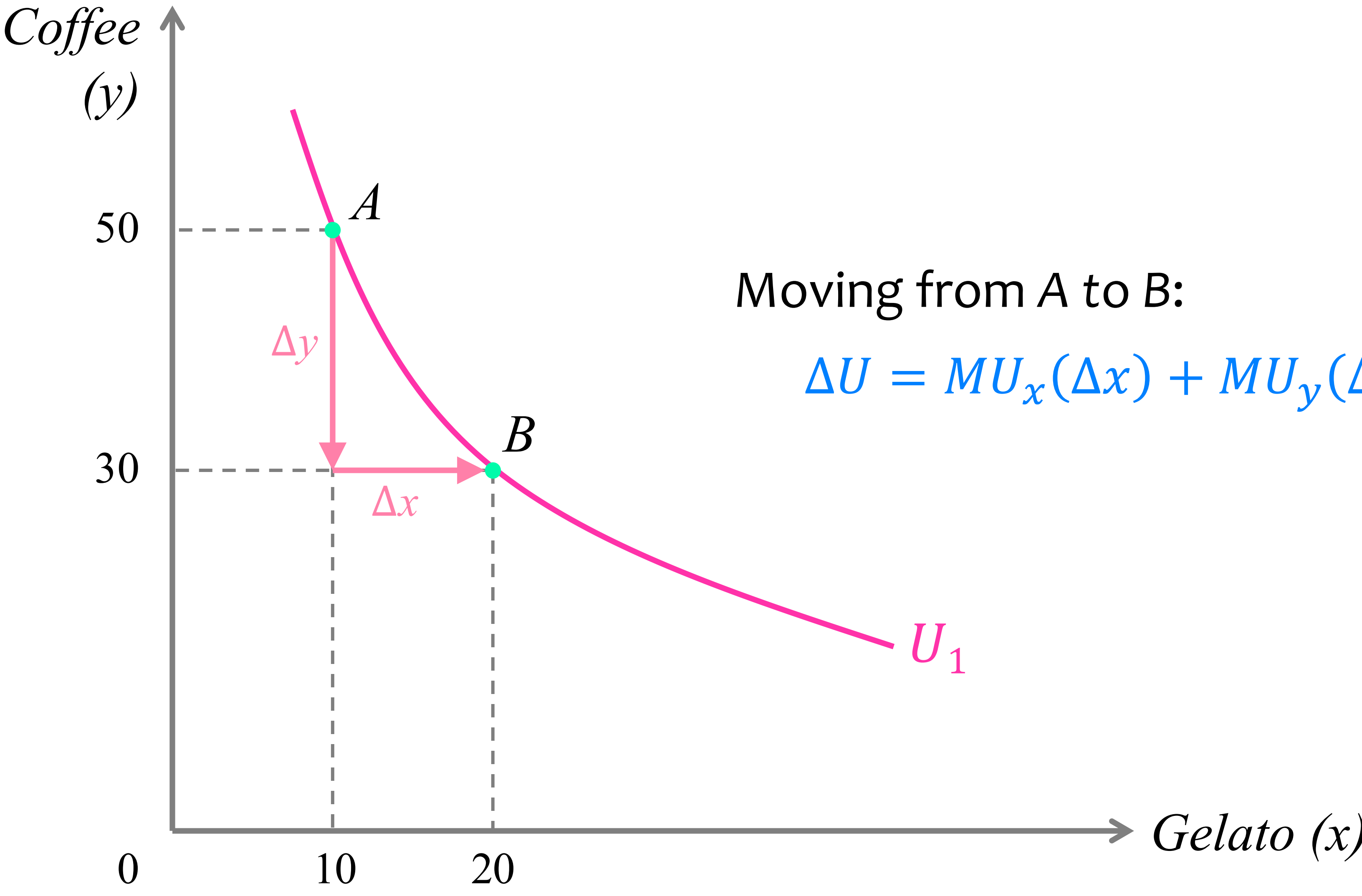
Trade-off between Gelato and Coffee



How many cups of coffee would you be willing to give up in order to obtain one more pint of gelato, and stay on the same indifference curve?

Utility Function and **Marginal Rate of Substitution**

Utility Function and *MRS*



Moving from *A* to *B*:

$$\Delta U = MU_x(\Delta x) + MU_y(\Delta y)$$

Utility Function and *MRS*

- Suppose the consumer moves from one basket to another basket on the same indifference curve.
- The total change in utility is:

$$\Delta U = MU_x(\Delta x) + MU_y(\Delta y)$$

$$0 = MU_x(\Delta x) + MU_y(\Delta y)$$

$$MU_x(\Delta x) = -MU_y(\Delta y)$$

$$\frac{MU_x}{MU_y} = -\frac{\Delta y}{\Delta x}$$

$$\frac{MU_x}{MU_y} = MRS_{x,y}$$

Marginal Rate of Substitution (*MRS*)

- Marginal rate of substitution of x for y:
  - The consumer's valuation of a unit of x,
    measured in terms of units of y.
  - The rate at which the consumer is willing to give up y
    in order to get more of x,
    maintaining the same level of utility.
  - $MRS_{x,y} = -\frac{dy}{dx}\Big|_{same\ U} = -\frac{\Delta y}{\Delta x}\Big|_{same\ U}$
    where Δx is extremely small.
- $MRS_{x,y}$ is the negative of the slope of the indifference curve.

Utility Function and *MRS*

- What does this equation mean?

$$MRS_{x,y} = \frac{MU_x}{MU_y}$$

- The rate at which the consumer is willing to substitute between two goods, holding utility constant is equal to the ratio of the marginal utilities of the two goods.

Exercise 2.2

Utility Function and *MRS*

Suppose Naomi's utility function is:

$$U(x, y) = \sqrt{xy}$$

(a) Find Naomi's marginal rate of substitution, $MRS_{x,y}$.

(b) Does the utility function exhibit diminishing marginal rate of substitution?

Exercise 2.2

Utility Function and *MRS*

Types of Preferences

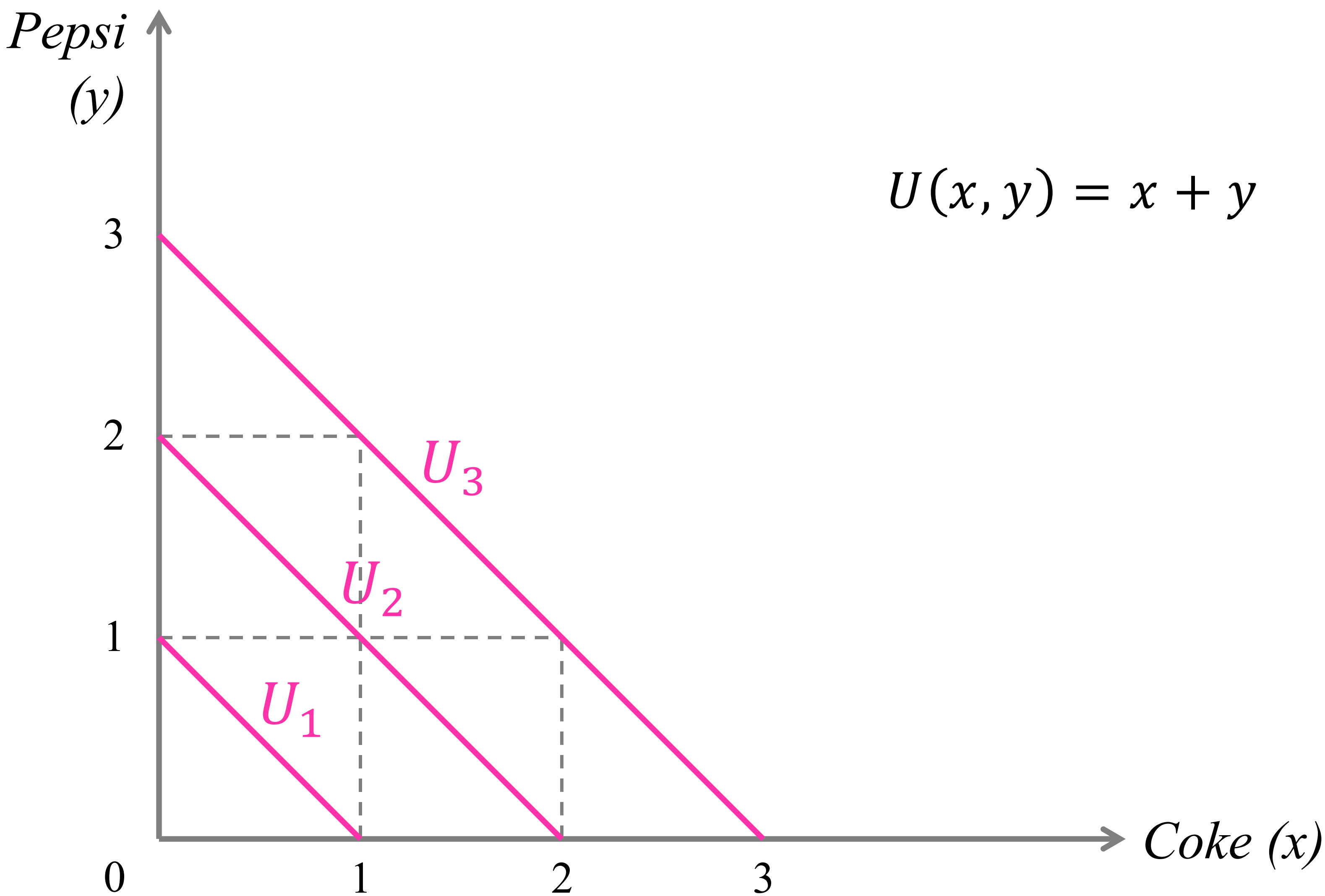
Types of Preferences:

Perfect Substitutes

Perfect Substitutes

- Li Na is equally happy with Coke or Pepsi.
  - The utility she derives from Coke
    is exactly the same as
    the utility she derives from Pepsi.
  - To Li Na, Coke and Pepsi are perfect substitutes.

Perfect Substitutes: Indifference Curves



Perfect Substitutes: Utility Function

- Two goods are perfect substitutes
  when the utility function for the two goods is of the form:

$$U(x, y) = \alpha x + \beta y$$

- Li Na's utility function for Coke (x) and Pepsi (y) is:

$$U(x, y) = x + y$$

  - E.g., if $x = 1$ and $y = 0$, then $U(x, y) = 1 + 0 = 1.$
  - E.g., if $x = 0$ and $y = 1$, then $U(x, y) = 0 + 1 = 1.$

Perfect Substitutes: *MRS*

- Li Na's utility function for Coke (x) and Pepsi (y) is:

$$U(x,y) = x + y$$

- Marginal utility of Coke (x): $MU_x = \frac{\partial U}{\partial x} = \alpha$
- Marginal utility of Pepsi (y): $MU_y = \frac{\partial U}{\partial y} = \beta$
- Marginal rate of substitution: $MRS_{x,y} = \frac{MU_x}{MU_y} = \frac{\alpha}{\beta}$
  - In our example, we can verify from the graph that the slope of the indifference curve is -1.

Perfect Substitutes

- Two goods are perfect substitutes if:
  - The indifference curves are linear.
  - The utility function is linear.
  - The *MRS* is constant,
    i.e., the *MRS* is independent of
    the quantity of good x consumed
    and the quantity of good y consumed.

Exercise 2.3

Perfect Substitutes

Suppose Li Na views two goods as perfect substitutes, i.e.,

$$U(x, y) = x + 2y$$

(a) Draw a graph showing her indifference curves.

(b) Find the marginal utility of x, the marginal utility of y, and the marginal rate of substitution.

Exercise 2.3(a)

Perfect Substitutes

Exercise 2.3(b)

Perfect Substitutes

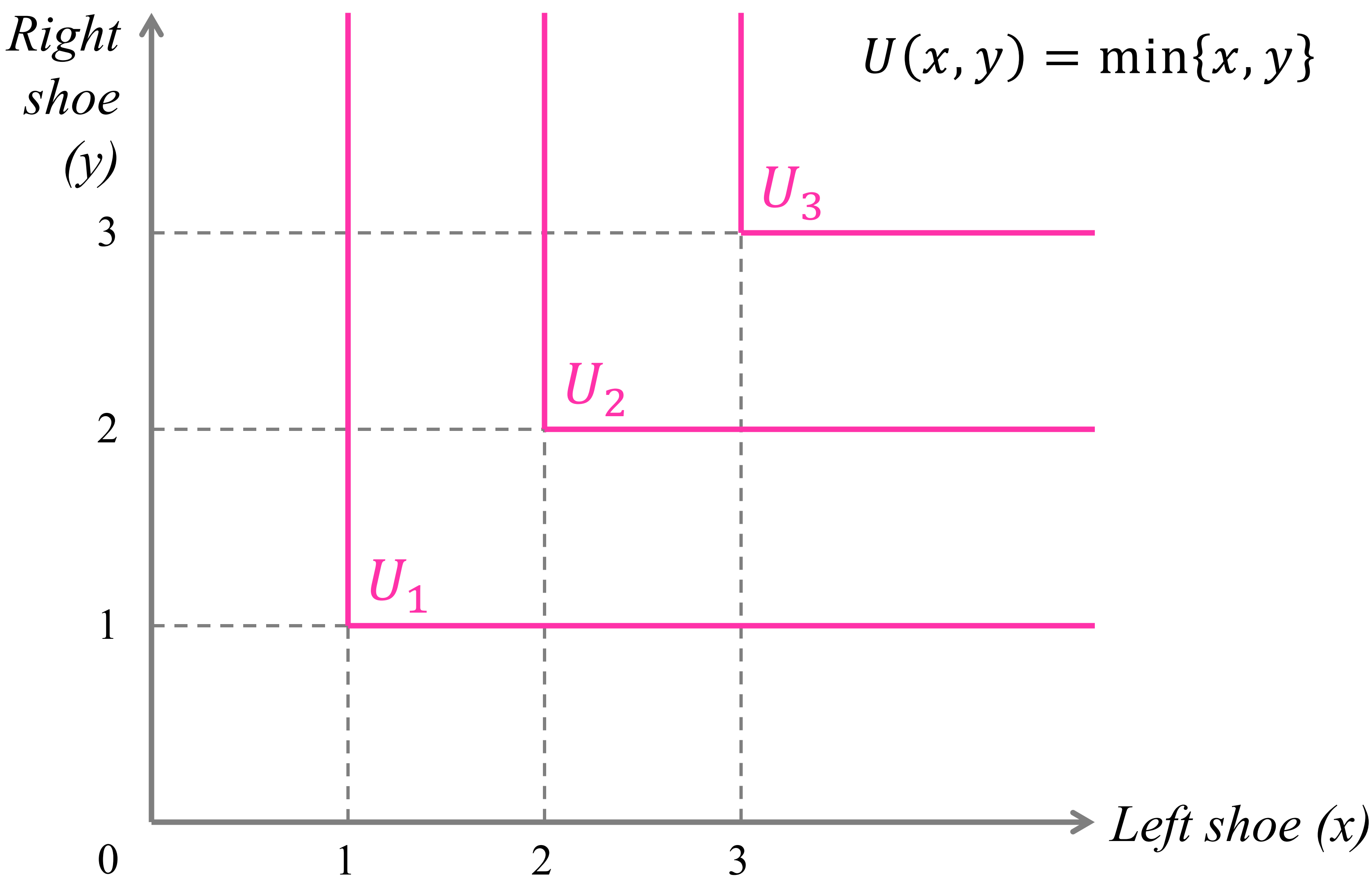
Types of Preferences:

Perfect Complements

Perfect Complements

- Li Na says: “For every left shoe, I need exactly one right shoe.”
  - The utility she derives from 10 left shoes and 1 right shoe
    is exactly the same as
    the utility she derives from 1 left shoe and 1 right shoe.
  - To Li Na, left shoes and right shoes are
    perfect complements.

Perfect Complements: Indifference Curves



Perfect Complements: Utility Function

- Two goods are perfect complements when the utility function for the two goods is of the form:

$$U(x, y) = \min\{\alpha x, \beta y\}$$

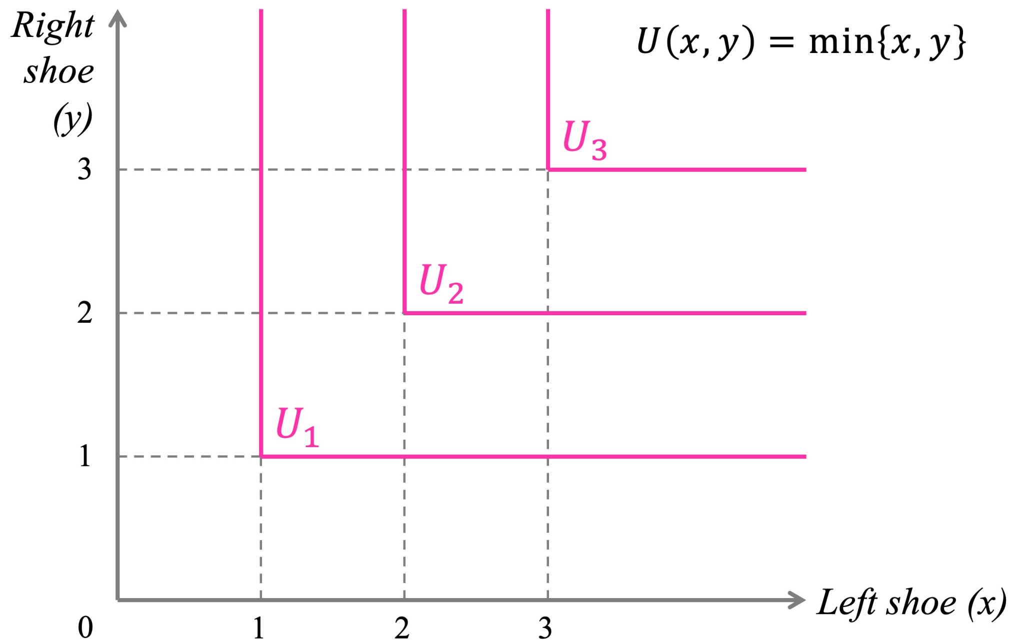
- Li Na's utility function for left shoes (x) and right shoes (y) is:

$$U(x, y) = \min\{x, y\}$$

  - E.g., if $x = 10$ and $y = 1$, then $U(x, y) = \min\{10,1\} = 1$.
  - E.g., if $x = 1$ and $y = 1$, then $U(x, y) = \min\{1,1\} = 1$.

Perfect Complements: *MRS*

- When $\alpha x > \beta y$,
  - $MRS_{x,y} = 0$
- When $\alpha x < \beta y$,
  - $MRS_{x,y} = \infty$
- When $\alpha x = \beta y$,
  - $MRS_{x,y} = \text{Undefined}$



Perfect Complements

- Two goods are perfect complements if:
  - The indifference curves are L-shaped.
  - The utility function is a “minimum” function.
  - The *MRS* is:
    - zero in the horizontal part
    - infinity in the vertical part
    - undefined at the kink

Exercise 2.4

Perfect Complements

Suppose Li Na views two goods as perfect complements, i.e.,

$$U(x, y) = \min\{x, y\}$$

(a) Draw a graph showing her indifference curves.

(b) Does monotonicity hold for x? Does monotonicity hold for y? Explain.

(c) Explain in words why $MRS_{x,y}$ is zero in the horizontal part and infinity in the vertical part.

Exercise 2.4(a)

Perfect Complements

Exercise 2.4(b)

Perfect Complements

Exercise 2.4(c)

Perfect Complements

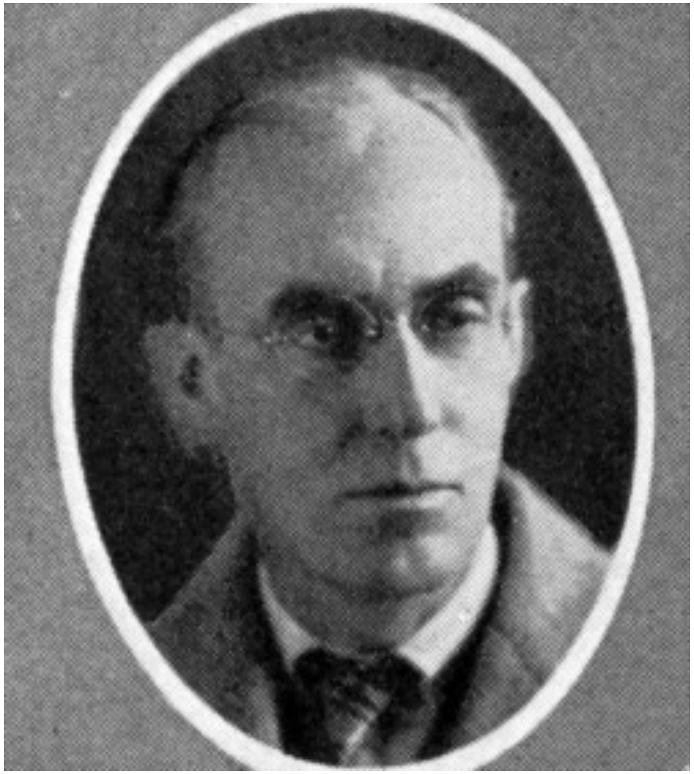
Summary

Perfect Substitutes and Perfect Complements

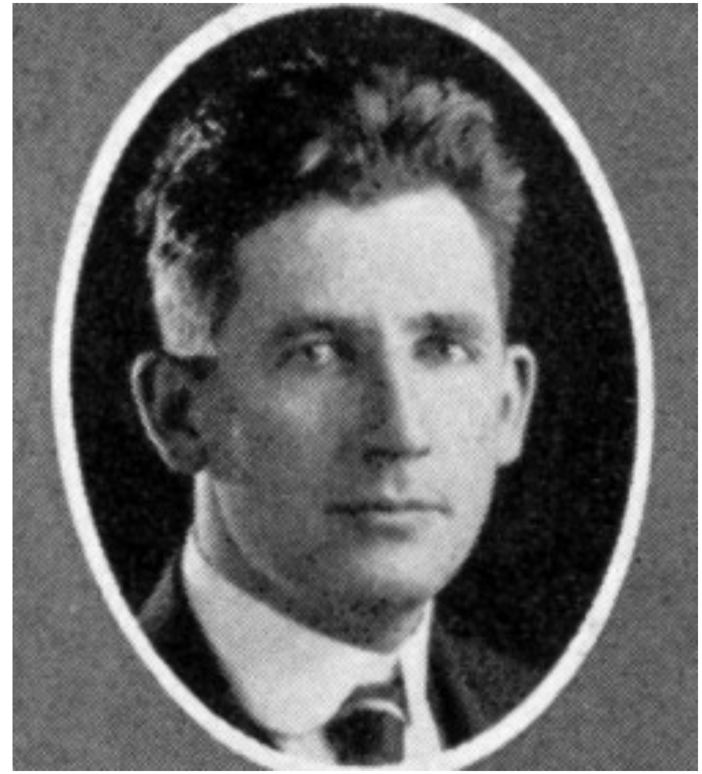
- Perfect Substitutes
  - The indifference curves are ______________________________.
  - The utility function is ______________________________.
  - The *MRS* is ______________________________.
- Perfect Complements
  - The indifference curves are ______________________________.
  - The utility function is ______________________________.
  - The *MRS* is ______________________________.

Types of Preferences:

Cobb-Douglas Preferences



Charles W. Cobb
(1875–1949)



Paul H. Douglas
(1892–1976)

Cobb-Douglas Utility Function

- A Cobb-Douglas utility function has the following form:

$$U(x, y) = Ax^{\alpha}y^{\beta}$$

  where $A > 0$, $\alpha > 0$, and $\beta > 0$.

- Examples of Cobb-Douglas utility function:
  - $U(x, y) = xy$
  - $U(x, y) = \sqrt{xy}$
  - $U(x, y) = 5x^2y^3$
  - $U(x, y) = \frac{1}{2}x^3y^{\frac{1}{3}}$

Marginal Utilities

- Given $U(x, y) = Ax^{\alpha}y^{\beta}$, the marginal utilities are:

$$MU_x = \frac{\partial U}{\partial x} = A\alpha x^{\alpha-1}y^{\beta}$$

$$MU_y = \frac{\partial U}{\partial y} = A\beta x^{\alpha}y^{\beta-1}$$

- Both marginal utilities are always positive.
  - Monotonicity holds for both goods.
  - Indifference curves are downward sloping.

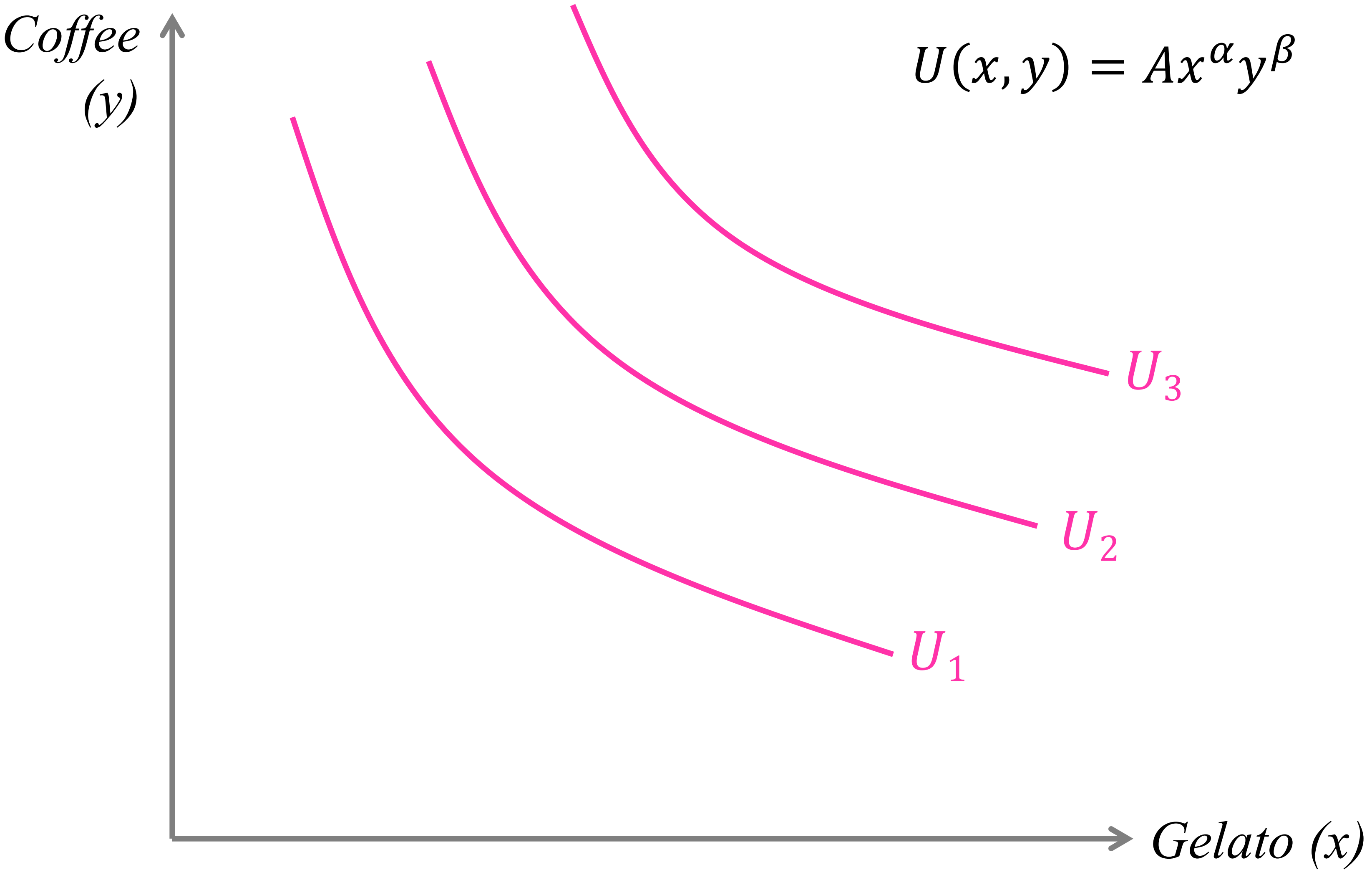
Marginal Rate of Substitution

- The marginal rate of substitution is:

$$MRS_{x,y} = \frac{MU_x}{MU_y} = \frac{A\alpha x^{\alpha-1}y^{\beta}}{A\beta x^{\alpha}y^{\beta-1}} = \frac{\alpha y}{\beta x}$$

- Moving from left to right along an indifference curve, as the consumer consumes more x and less y, $MRS_{x,y}$ is diminishing.
  - Indifference curves are convex.

Indifference Curves



Is Marginal Utility Diminishing?

- Given $U(x, y) = Ax^{\alpha}y^{\beta}$, the marginal utility of x is:

$$MU_x = A\alpha x^{\alpha-1}y^{\beta}$$

- To determine whether marginal utility is diminishing, we need to find out how marginal utility changes as the consumption of x increases.

$$\frac{\partial MU_x}{\partial x} = A\alpha(\alpha - 1)x^{\alpha-2}y^{\beta}$$

- If $\alpha < 1$, then $\frac{\partial MU_x}{\partial x} < 0$.

- For Cobb-Douglas utility functions, marginal utility may or may not be diminishing.

Why Study Cobb-Douglas Utility Functions?

- Cobb-Douglas utility functions have convenient mathematical/economic properties:
  - Simple functional form.
  - Monotonicity is satisfied.
  - Diminishing marginal rate of substitution.
  - Indifference curves do not intersect the axes.

- What kind of preferences can be represented by a Cobb-Douglas utility function?

Exercise 2.5

Cobb-Douglas Preferences

Consider the Cobb-Douglas utility function $U(x, y) = x^2y^2$.

(a) Find MU_x, MU_y, and $MRS_{x,y}$.

(b) Does the utility function exhibit diminishing marginal utility in each good?

(c) Does the utility function exhibit diminishing marginal rate of substitution?

Exercise 2.5

Cobb-Douglas Preferences

Budget Constraint

Budget Constraint

- Suppose Serena chooses x pints of gelato and y cups of coffee.
- The price of a pint of gelato is p_x and the price of a cup of coffee is p_y.
- Serena has income M.
- Her budget constraint is:

$$p_x x + p_y y \leq M$$

Budget Set vs. Budget Line

- Budget set:
  - The set of all baskets that a consumer can afford.
  - $p_x x + p_y y \leq M$
- Budget line:
  - The set of all baskets that a consumer can afford if she spends all her income.
  - $p_x x + p_y y = M$

Budget Constraint: Example

- The price of a pint of gelato is $p_x = \$10$ and the price of a cup of coffee is $p_y = \$5$.

- Serena's income is $M = \$100$.

- Serena's budget constraint (and budget set) is:

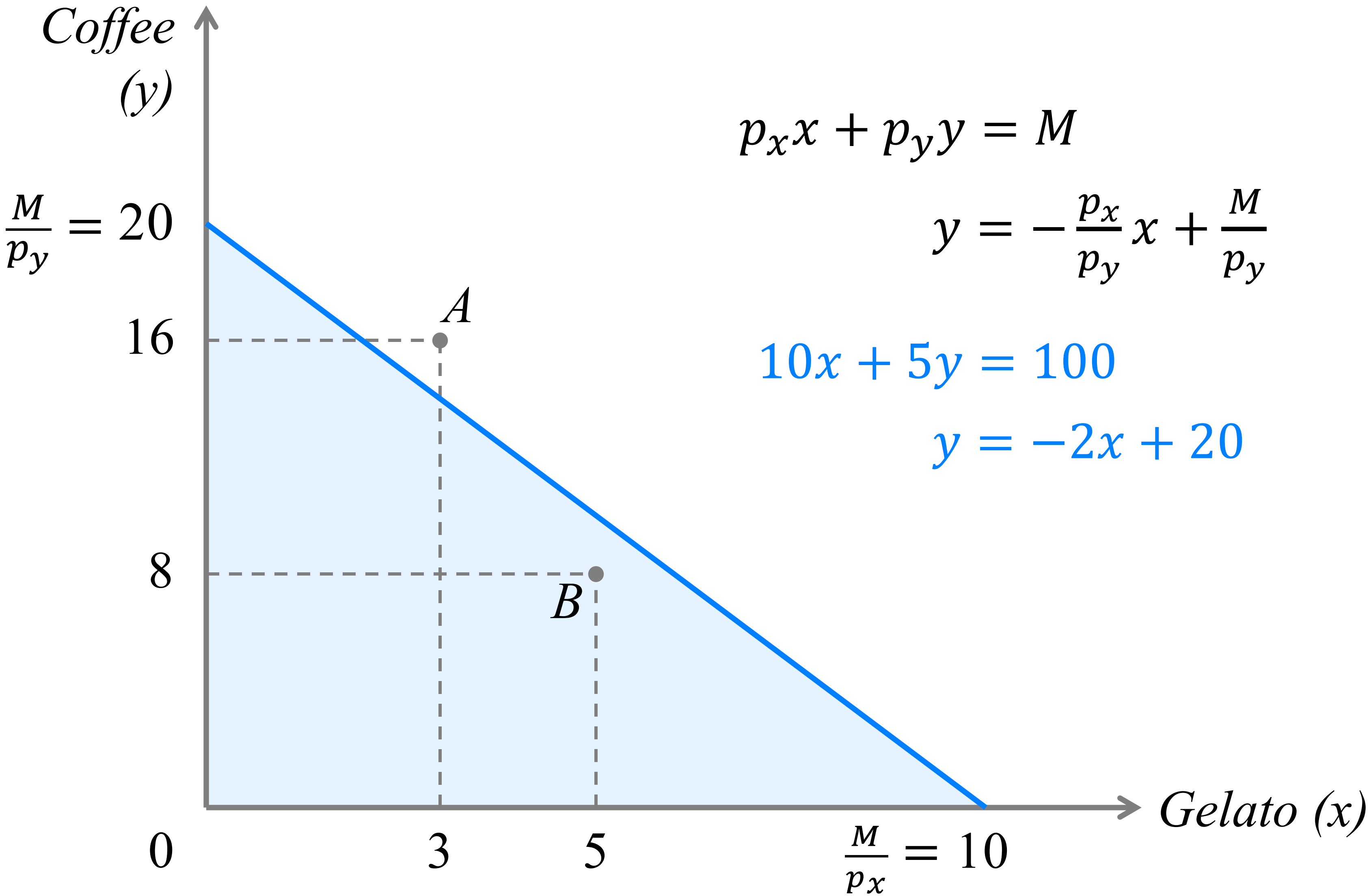
$$p_x x + p_y y \leq M$$

$$10x + 5y \leq 100$$

- Serena's budget line is:

$$10x + 5y = 100$$

Budget Constraint: Graphical Representation



$$p_x x + p_y y = M$$

$$y = -\frac{p_x}{p_y} x + \frac{M}{p_y}$$

$$10x + 5y = 100$$

$$y = -2x + 20$$

Slope of Budget Line

- The slope of the budget line is:

$$-\frac{\left(\frac{M}{p_y}\right)}{\left(\frac{M}{p_x}\right)} = -\frac{p_x}{p_y} = -\frac{10}{5} = -2$$

- The slope of the budget line represents the rate at which the two goods are exchanged in the market.
  - To get an additional pint of gelato, Serena must give up 2 cups of coffee.
- The absolute value of the slope is the relative price of good x.
  - Gelato (x) is twice as expensive as coffee (y).

Exercise 2.6

Budget Line

Serena's budget line is:

$$10x + 5y = 100$$

(a) Suppose Serena's income decreases from \$100 to \$60. How does the budget line change? Draw a graph showing the original budget line and the new budget line.

(b) Suppose Serena's income is \$100 but the price of gelato increases from \$10 a pint to \$20 a pint. How does the budget line change? Draw a graph showing the original budget line and the new budget line.

Exercise 2.6(a)

Budget Line: Income Decreases

Exercise 2.6(b)

Budget Line: Price of Gelato Increases

Optimal Choice: Graphical Analysis

Optimal Choice

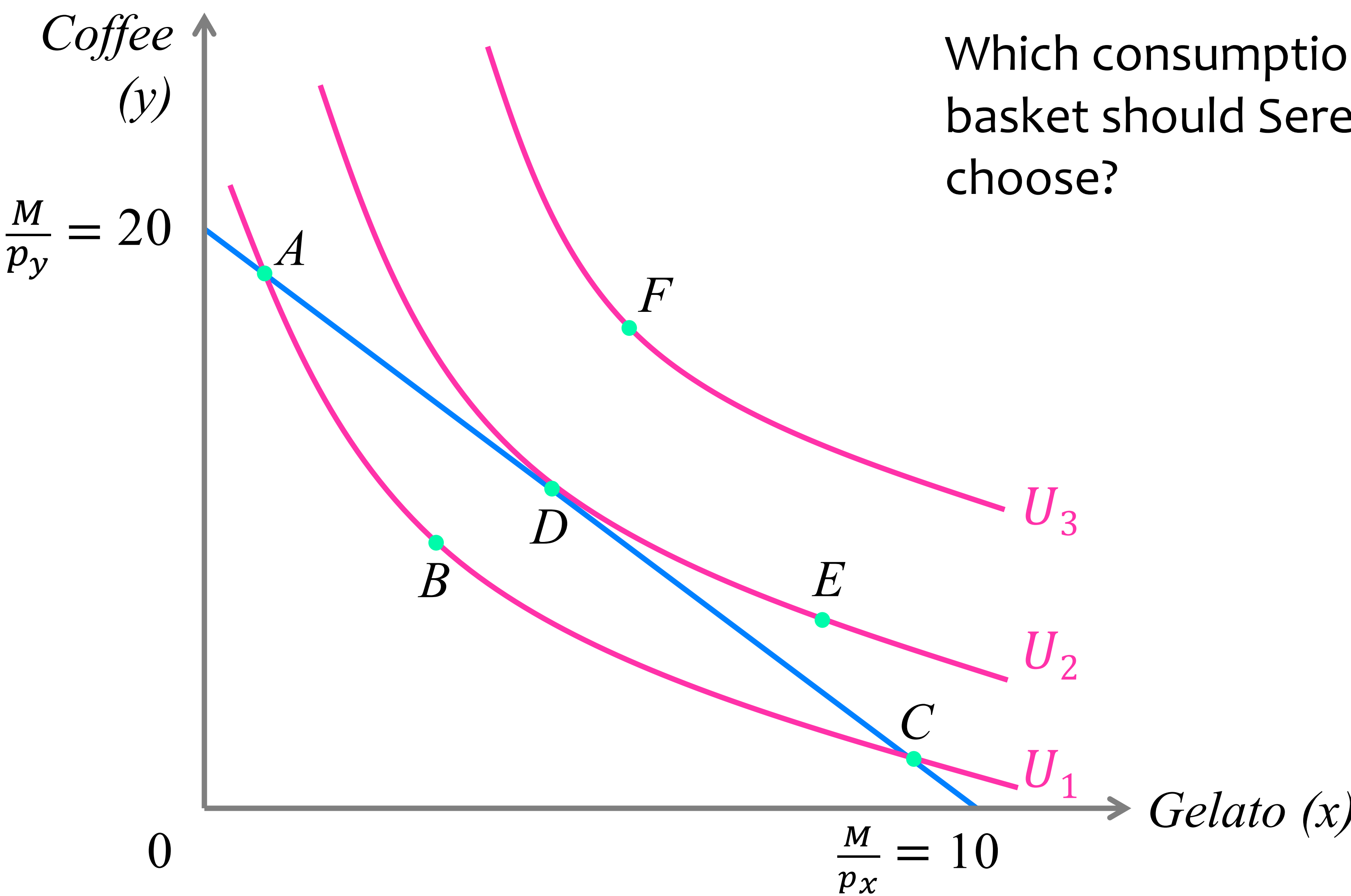
- Which consumption basket is optimal?
- Serena chooses the consumption basket that
  gives her the highest utility
  given her budget constraint.
- The constrained optimization problem is:

$$\max_{x,y} U(x, y)$$

$$subject\ to \quad p_x x + p_y y \leq M$$

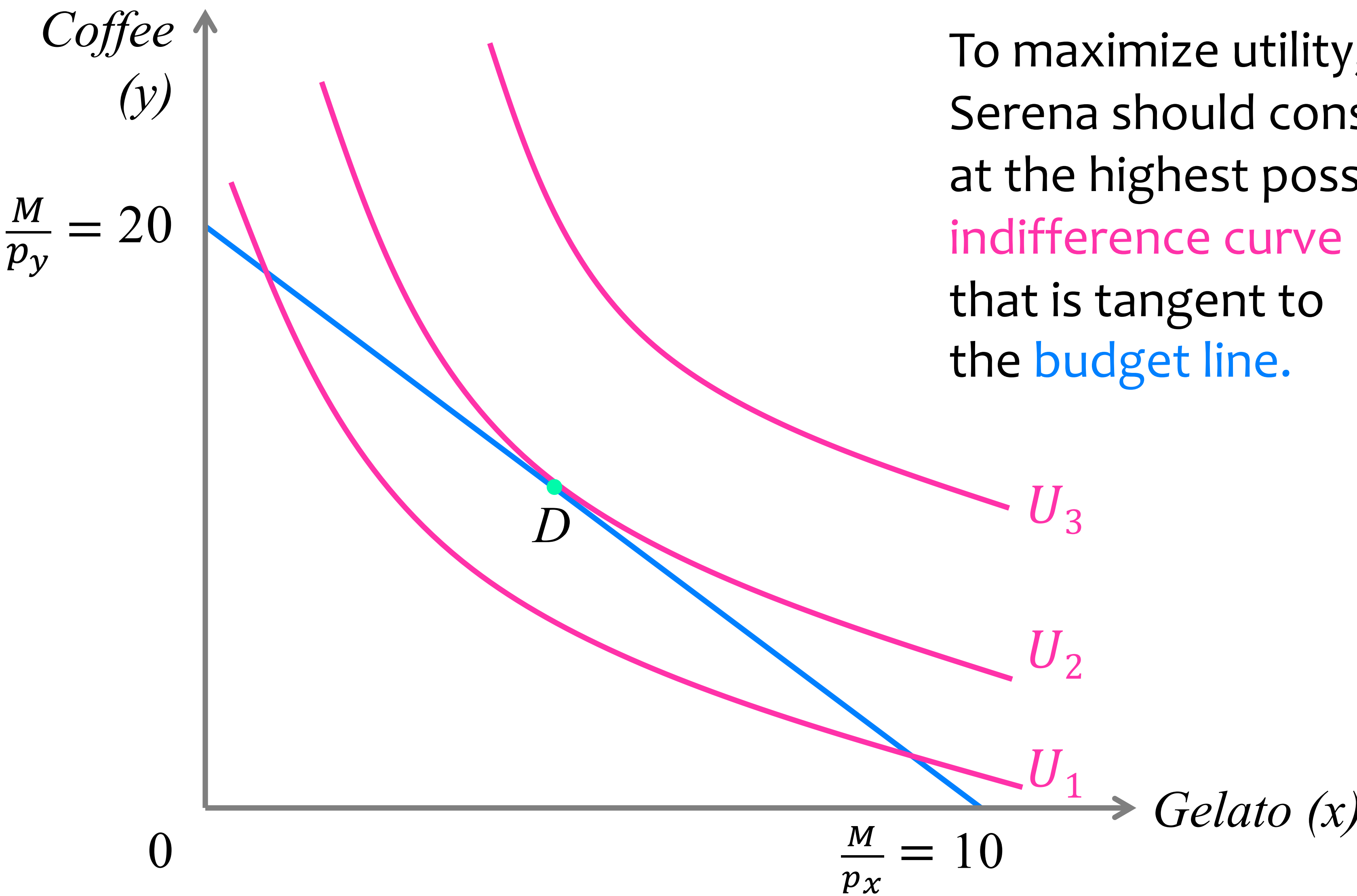
  - x and y are the choice variables.
  - p_x, p_y, and M are the parameters.

Optimal Choice: Graphical Analysis



Which consumption basket should Serena choose?

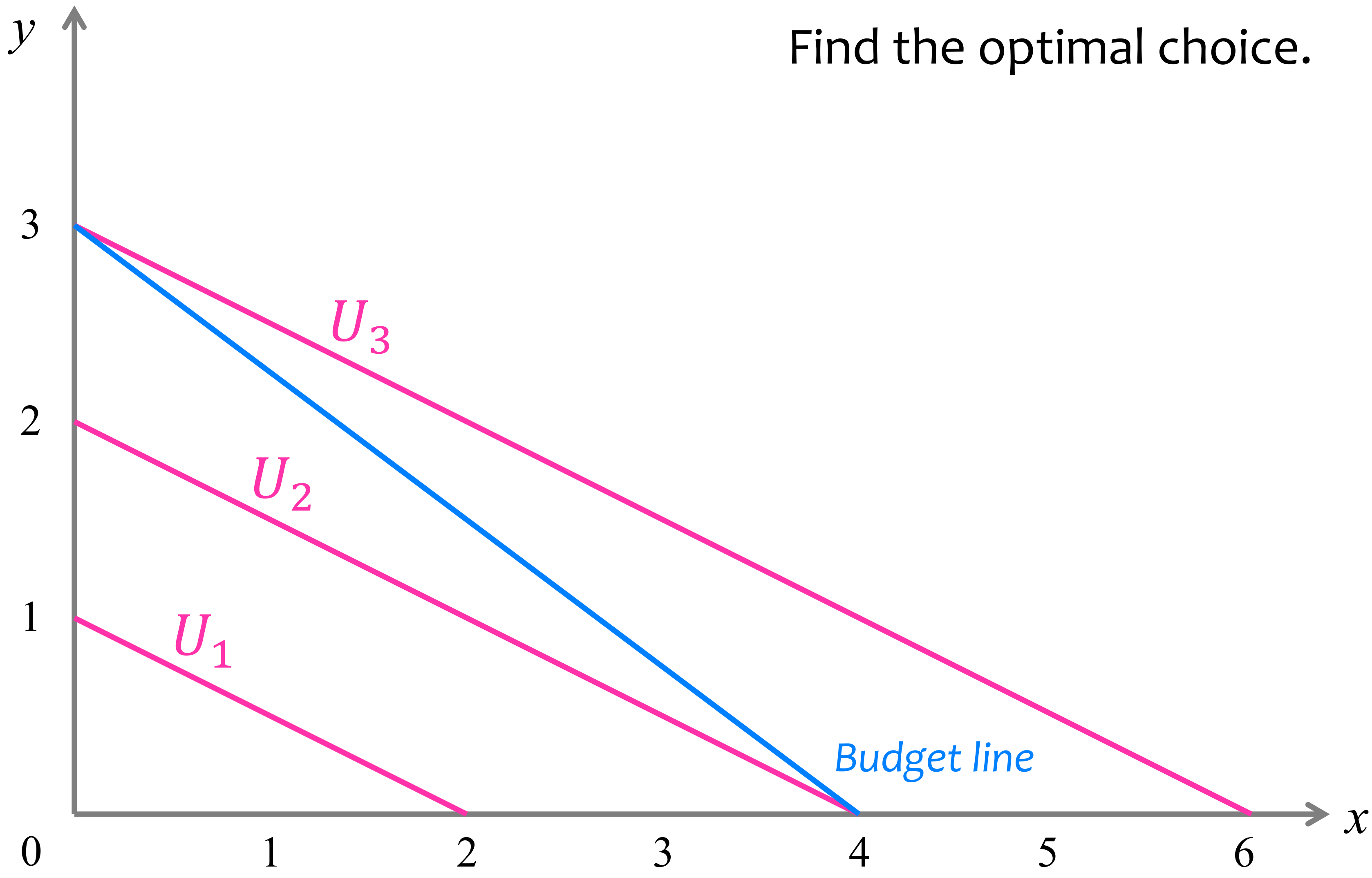
Optimal Choice: Graphical Analysis



To maximize utility, Serena should consume at the highest possible indifference curve that is tangent to the budget line.

Exercise 2.7

Optimal Choice: Perfect Substitutes



Find the optimal choice.

Exercise 2.8

Optimal Choice: Perfect Complements