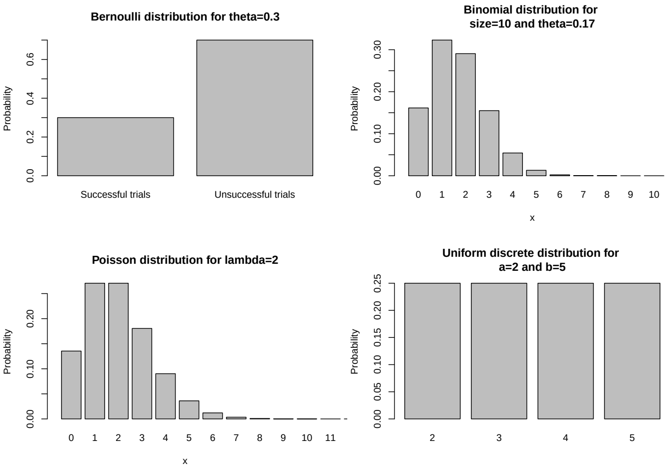
Discrete distributions



Outline

Learning Objectives

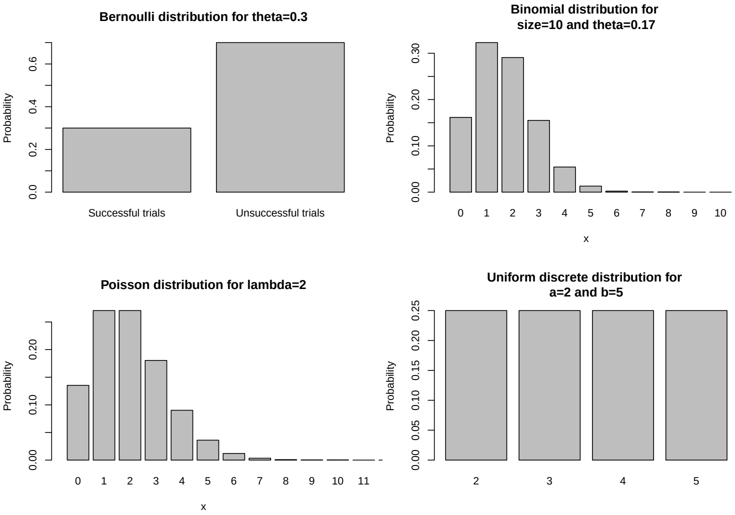
1. Learn what defines the some commonly used discrete distributions.
2. Build a vocabulary of some of the more common discrete distributions

Introduction

A brief review of discrete random variables

- Recall that a discrete random variable can be defined by a probability mass function (pmf).
- A discrete random variable have values that are restricted to finitely separated values.
- A discrete random variable can belong to some pmf, i.e., $X \sim p(x|\theta)$.
  - x denotes the value that the random variable takes on.
  - θ denotes the parameter(s) of the pmf.

Histograms of discrete distributions

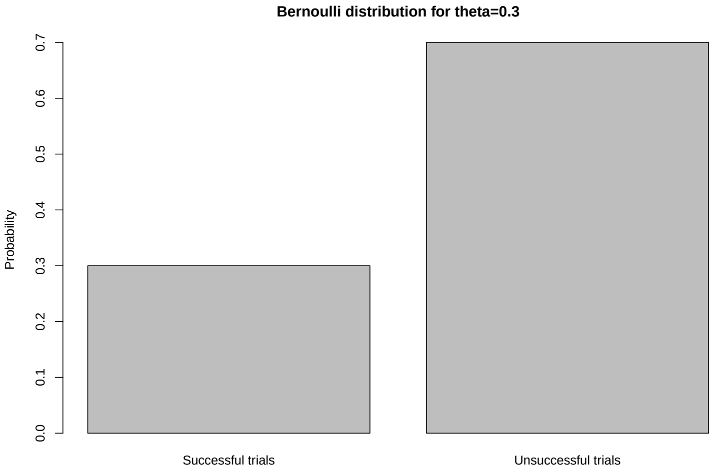


Examples of discrete distributions

Bernoulli distribution

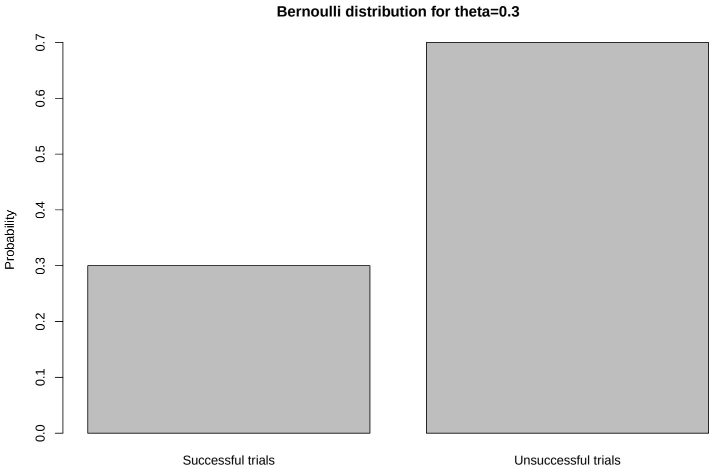
Bernoulli distribution

- Simplest probability distribution.
- $X \sim p(x|\theta) = Bernoulli(x|\theta)$.
- The support is $x = 0, 1$.
  - 0: "Unsuccessful"
  - 1: "Successful"
- One parameter called θ (pronounced theta) which describes the probability of a successful trial.



An example: Will it rain?

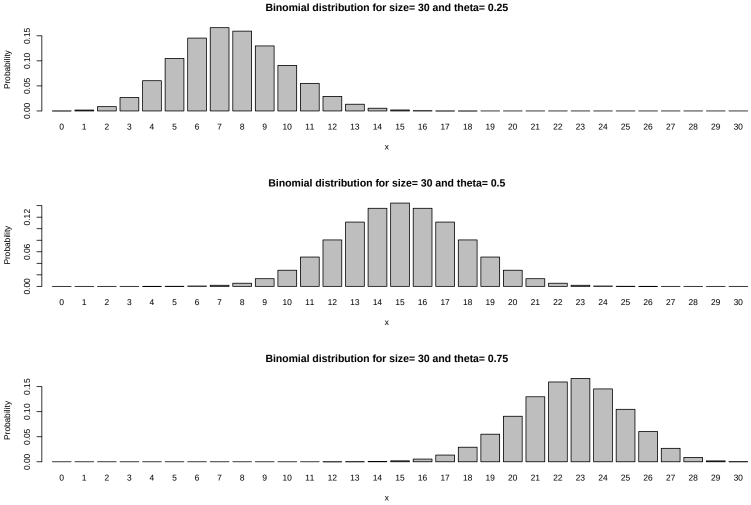
- Will it rain today?
- This is a random event with two possible outcomes.
- We can use a Bernoulli distribution.
  - 1: Rain ("successful")
  - 0: No rain ("unsuccessful")
  - $\theta = 0.3$
- In other words, the probability of rain is just 0.3.
- Likewise, the probability of no rain is 0.7.



Binomial distribution

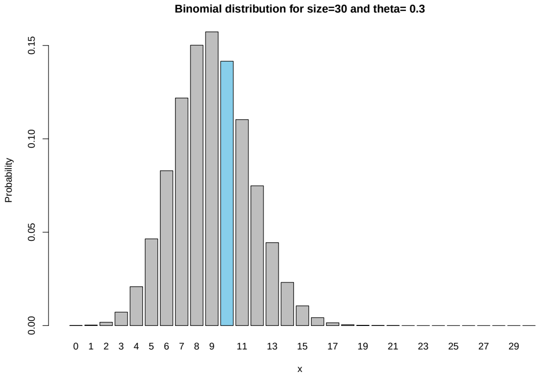
Binomial distribution

- Made up of n identical and independent Bernoulli trials.
- $X \sim p(x|\theta) = Binomial(x|\theta)$.
- The support is $x = 0, 1, \cdots, n$.
    - The value of x denotes the number of successful trials.
- One parameter called θ which describes the probability of a successful trial.
- As an example, consider $n = 30$.



An example: How many rainy days should we expect?

- Out of 30 days, what is the probability of rain on *any* 10 days?
- Simplifying assumption: probability of rain each day is identically 0.3.
- We can use a Binomial distribution.
  - $\theta = 0.3$
- We can use the `dbinom()` function to compute this probability:

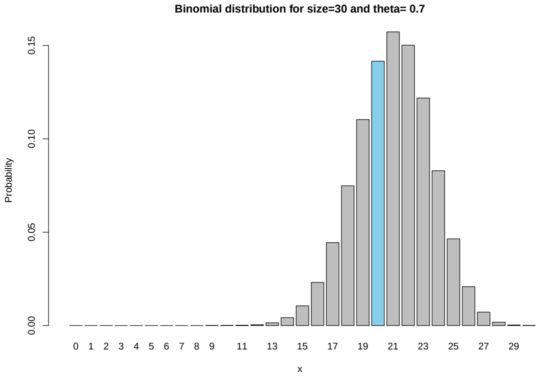



```
dbinom(x=10, # x=10 days of rain
       size=30, # A total of n=30 days
       prob=0.3) # Probability of rain each day is theta=0.3
```

```
## [1] 0.1415617
```

An example: How many rainy days should we expect? (cont'd)

- Out of 30 days, what is the probability of *no* rain on *any* 20 days?
- Probability of no rain each day is identically 0.7.
- In other words, $\theta = 0.7$.
- We can use the `dbinom()` function to compute this probability:

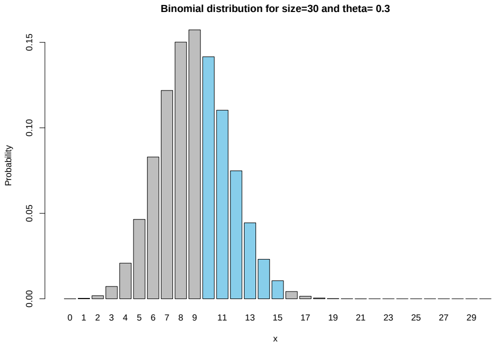



```r
dbinom(x=20, # x=20 days of no rain
       size=30, # A total of n=30 days
       prob=0.7) # Probability of no rain each day is theta=0.7
```

```
## [1] 0.1415617
```

An example: How many rainy days should we expect? (cont'd)

- Out of 30 days, what is the probability of rain on *any* 10 to 15 days?
- Probability of rain each day is 0.3.
- We can use a combination of the `dbinom()` and `sum()` functions to compute this probability:



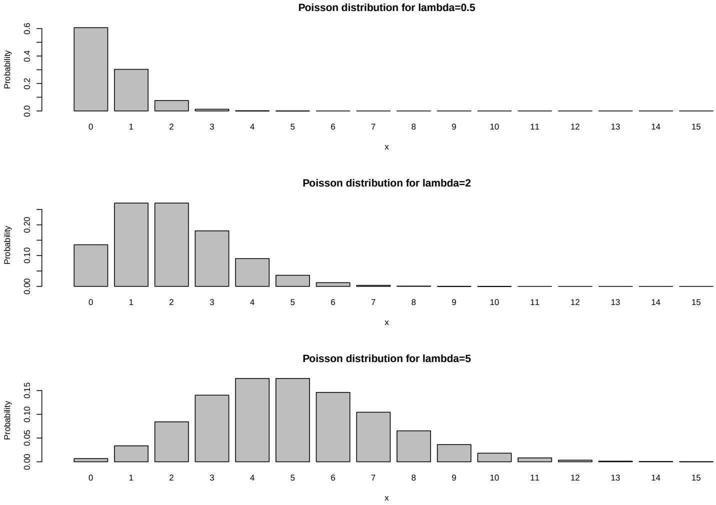

```
p_10_15 <- dbinom(x=10:15, # Between x=10 and x=15 days of rain
                  size=30, # A total of n=30 days
                  prob=0.3) # Probability of rain each day is theta=0.3
sum(p_10_15) # Sum up probabilities between x=10 and x=15
```

```
## [1] 0.404821
```

Poisson distribution

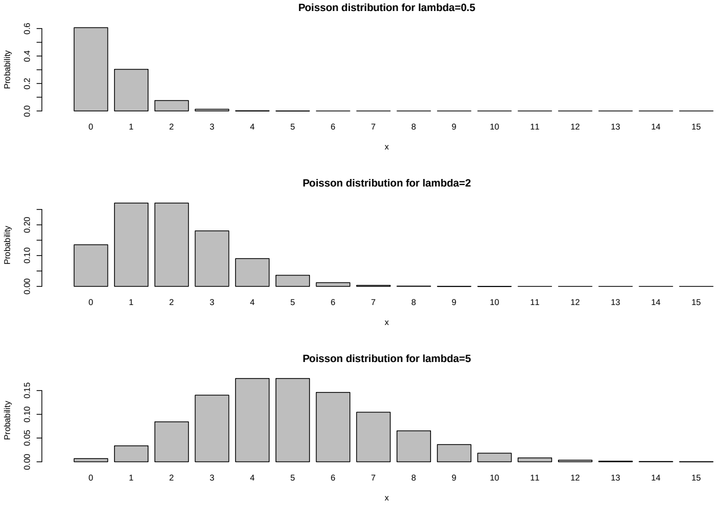
Poisson distribution

- Within one unit of time or space, what is the number of “events” that we observe?
- Some examples of these events:
  - Number of phone calls that a call centre receives in an hour.
  - Number of cars passing by a junction in 10 minutes.
  - Number of typographical errors in a chapter of a book.



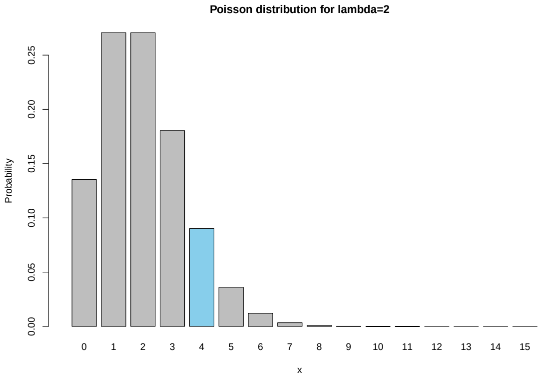
Poisson distribution (cont'd)

- $X \sim p(x|\theta) = Poisson(x|\lambda)$.
- The support is $x = 0, 1, \cdots$.
  - The value of x denotes the number of events.
- One parameter called λ (pronounced lambda) which describes the average number of events in one unit of space or time.
- As an example, consider stalls A, B, C with $\lambda = 0.5, 2, 5$, respectively.



An example: How many customers should we expect at stall B?

- Stall B: If the average number of customers every minute is 2, what is the probability of observing 4 customers in the next minute?
- $\lambda = 2$
- We can use the `dpois()` function to compute this probability:

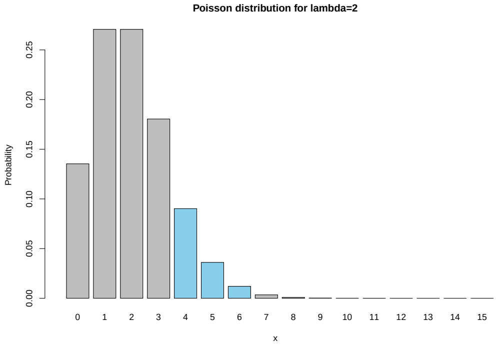



```
dpois(x=4, # x=4 customers within each minute
      lambda = 2) # Average no. of customers per minute is lambda=2
```

```
## [1] 0.09022352
```

An example: How many customers should we expect at stall B? (cont'd)

- What is the probability of observing 4 to 6 customers in the next minute?
- We can use a combination of the `dpois()` and `sum()` functions to compute this probability:



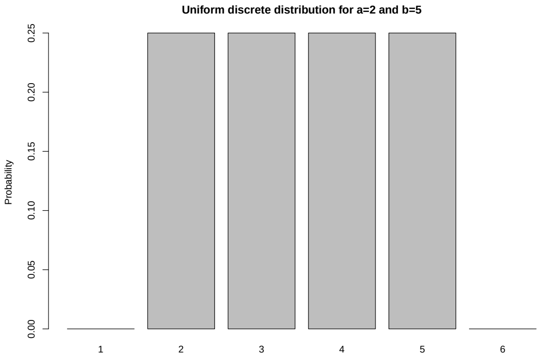

```
p_4_6 <- dpois(x=4:6, # x=4 to x=6 customers within each minute
      lambda = 2) # Average no. of customers per minute is lambda=2
sum(p_4_6) # Sum up probabilities between x=4 and x=6
```

```
## [1] 0.1383427
```

Uniform discrete distribution

Uniform discrete distribution

- Suppose there are k possible outcomes, labelled by $a, a+1, \cdots, b-1, b$
  - a and b are integers.
  - $a < b$
- $X \sim p(x|\theta) = Uniform_{discrete}(x|a, b)$.
- The support is $a, a+1, \cdots, b-1, b$.
- Two parameters, a and b.
- For example, $a = 2$, $b = 5$.
  - 4 possible outcomes.
  - Each outcome has a 0.25 probability.
- E.g., an unbiased lucky draw.



More examples

More examples

- There are many more examples of discrete distributions.
- Geometric distribution
  - Number of successful outcomes until the first unsuccessful one.
  - Can be used to detect defective products in a manufacturing facility.
- Negative binomial distribution
  - Number of successful outcomes until some other number of unsuccessful trials.
- Hypergeometric distribution
- Delaporte distribution
- Different distributions differ in terms of
  - Support
  - Parameters
  - Shape of histogram
- `d<distribution>()` can be used to compute probabilities for *discrete distributions*.
  - e.g., `dbinom()`, `dpois()`, `dgeom()`, etc.

Summary

Summary

In this video, we have:

- Defined some commonly used discrete distributions.
- Built a vocabulary of some of the more common discrete distributions.

References

Diez, D. M., Barr, C. D., and Cetinkaya-Rundel, M. (2012).
OpenIntro statistics.
OpenIntro Boston, MA, USA:.

Kerns, G. J. (2010).
Introduction to probability and statistics using r.
Lulu. com.

Ross, S. (2010).
A first course in probability.
Pearson.