# EC3333 Tutorial 2 Suggested Answers

1. Consider a world with two possible states of the world, 1 and 2, which are equally likely.

   Consider two assets A and B whose returns are positively correlated. Their returns have the following probability distribution.

| **State** | **Probability** | $R_A$ | $R_B$ | $R_P$ |
|---|---|---|---|---|
| 1 | 0.5 | 10.0% | 12.0% | ? |
| 2 | 0.5 | -2.0% | -1.0% | ? |

   Consider a parallel universe where the returns on the two assets A and B are negatively correlated instead. Their returns have the following probability distribution.

| **State** | **Probability** | $R_A$ | $R_B$ | $R_P$ |
|---|---|---|---|---|
| 1 | 0.5 | 10.0% | -1.0% | ? |
| 2 | 0.5 | -2.0% | 12.0% | ? |

   Suppose we are interested in a portfolio *P*, where the portfolio weights on asset A and asset B are $x_A = 0.4$ and $x_B = 0.6$, respectively.

   a. Fill in the blank in the above tables (i.e., compute the portfolio returns in different states of the world).
   b. Calculate the expected values $E(R_A)$, $E(R_B)$, and $E(R_P)$ in both cases.
   c. Calculate the volatilities $\sigma_A$, $\sigma_B$, and $\sigma_P$ in both cases.
   d. Calculate the covariances $\sigma_{AB}$ and correlation coefficients $\rho_{AB}$ in both cases.
   e. How are the expected values and volatilities related to the correlation coefficients?

$$\text{Expected Return } = E[R] = \sum_R P_R \times R$$

$$\text{Var }(R) = E[(R - E[R])^2] = \sum_R P_R \times (R - E[R])^2$$

$$Volatility\ (R) = SD(R) = \sigma_R = \sqrt{\text{Var }(R)}$$

$$\text{Cov }(R_i, R_j) = E\left[(R_i - E[R_i])(R_j - E[R_j])\right] = \sum_R P_R \times (R_i - E[R_i])(R_j - E[R_j])$$

$$\rho_{AB} = \frac{\text{Cov}(r_A r_B)}{\sigma_A \sigma_B}$$

| | | Scenario rates of return | | **a. Portfolio return** |
|---|---|---|---|---|
| Scenario | Probability | RA | RB | 0.4*RA+0.6*RB |
| 1 | 0.5 | 0.1 | 0.12 | 0.112 |
| 2 | 0.5 | -0.02 | -0.01 | -0.014 |
| | **b. Expected Return** | **0.04** | **0.055** | **0.049** |
| | | | | |
| | | | | |
| | | Scenario rates of return | | **Portfolio return** |
| Scenario | Probability | RA | RB | 0.4*RA+0.6*RB |
| 1 | 0.5 | 0.1 | -0.01 | 0.034 |
| 2 | 0.5 | -0.02 | 0.12 | 0.064 |
| | **b. Expected Return** | **0.04** | **0.055** | **0.049** |

| | | | | | |
|---|---|---|---|---|---|
| | (R_A-R_Abar)^2 | (R_B-R_Bbar)^2 | (R_P-R_Pbar)^2 | | (R_A-R_Abar)(R_A-R_Abar) |
| | 0.0036 | 0.004225 | 0.003969 | | 0.0039 |
| | 0.0036 | 0.004225 | 0.003969 | | 0.0039 |
| Variance | 0.0036 | 0.004225 | 0.003969 | **d. covariance** | **0.0039** |
| **c. Volatility** | **0.06** | **0.065** | **0.063** | **d. correlation** | **1** |
| | | | | | |
| | | | | | |
| | (R_A-R_Abar)^2 | (R_B-R_Bbar)^2 | (R_P-R_Pbar)^2 | | (R_A-R_Abar)(R_A-R_Abar) |
| | 0.0036 | 0.004225 | 0.000225 | | -0.0039 |
| | 0.0036 | 0.004225 | 0.000225 | | -0.0039 |
| | 0.0036 | 0.004225 | 0.000225 | **d. covariance** | **-0.0039** |
| **c. Volatility** | **0.06** | **0.065** | **0.015** | **d. correlation** | **-1** |

e.
The expected values of $E(R_A)$, $E(R_B)$, and $E(R_P)$, and the volatilities $\sigma_A$ and $\sigma_B$ do not vary with the correlation of returns between asset A and asset B. However, the volatility of the portfolio $\sigma_P$ is lower when the returns of asset A and asset B perfectly negatively correlated.

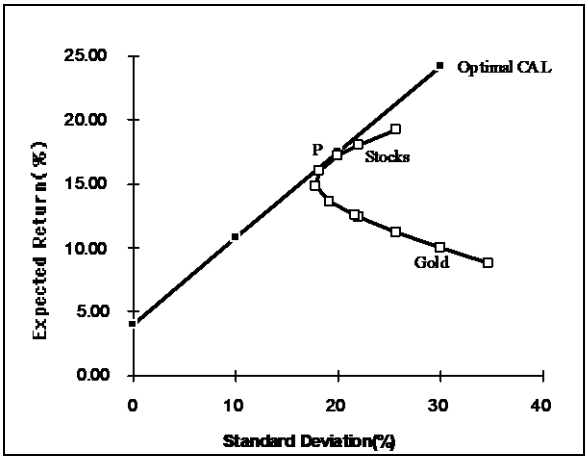
2. Stocks offer an expected rate of return of 18%, with a standard deviation of 22%. Gold offers an expected return of 10% with a standard deviation of 30%.

   a. In light of the apparent inferiority of gold with respect to both mean return and volatility, would anyone hold gold? If so, demonstrate graphically why one would do so.

   b. Given the data above, redo part a with the additional assumption that the correlation coefficient between gold and stocks equals 1. Draw a graph illustrating why one would or would not hold gold in one's portfolio. Could this set of assumptions for expected returns, standard deviations, and correlation represent an equilibrium for the security market?

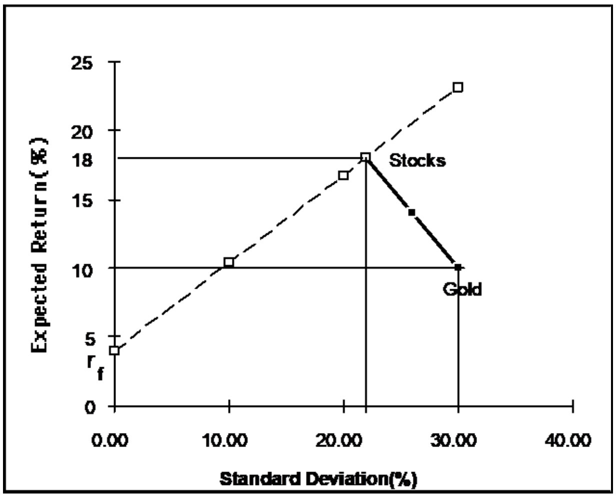
a.

Even though it seems that gold is dominated by stocks, gold might still be an attractive asset to hold as a *part* of a portfolio**. If the correlation between gold and stocks is sufficiently low**, gold will be held as a component in a portfolio, specifically, the optimal tangency portfolio.




b.

**If the correlation between gold and stocks equals +1, then no one would hold gold.** The optimal CAL would be composed of bills and stocks only. Since the set of risk/return combinations of stocks and gold would plot as a straight line with a negative slope, these combinations would be dominated by the stock portfolio. Typically, this situation could not persist. If no one desired gold, its price would fall and its expected rate of return would increase until it became sufficiently attractive to include in a portfolio.



3. Historical data show that the average annual return on the S&P 500 portfolio over the past 90 years has averaged roughly 8% more than the Treasury bill return and that the standard deviation of the return on S&P 500 has been about 20% per year. Assume that these values are representative of the investors' expectations for future performance and that the current T-bill rate is 5%.

a. Calculate the expected return and variance of portfolios invested in T-bills and the S&P 500 index with weights as follows:

| $x_{Bills}$ | $x_{Index}$ |
|---|---|
| -0.2 | 1.2 |
| 0.0 | 1.0 |
| 0.2 | 0.8 |
| 0.4 | 0.6 |
| 0.6 | 0.4 |
| 0.8 | 0.2 |
| 1.0 | 0.0 |

b. Plot the expected return and standard deviation of these portfolios on the E(r) versus σ plane. What do you observe? Do they fall on a straight line? If they do, what are the vertical intercept and the slope? Looking at your plot, when the portfolio weight is non-negative, where is the portfolio located in the E(r) versus σ plane? When the portfolio weight is negative for T-bill, where is the portfolio relative to the portfolio withweight equals zero for T-bill (and weight equals one for S&P500)?

c. Suppose the utility function is given by $U = E(r) - 0.5 \times A\sigma^2$, where $A$ is an index of the investor's risk aversion. Suppose we do not allow short selling.

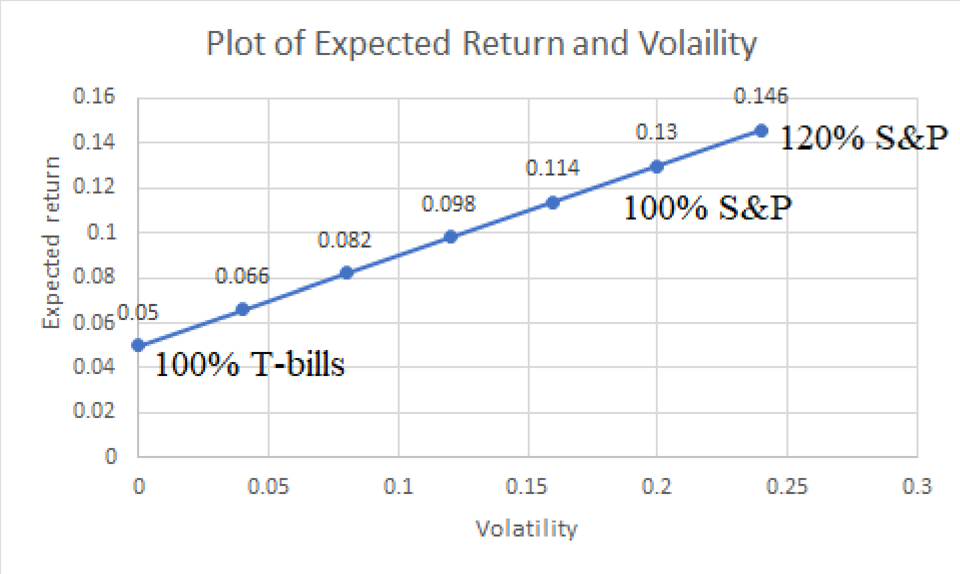
- Calculate the utility levels of each portfolio for an investor with $A$=2. What do you conclude?
- Calculate the utility levels of each portfolio for an investor with $A$=3. What do you conclude?

a.

The portfolio expected return and variance are computed as follows:

| (1) $x_{Bills}$ | (2) $r_{Bills}$ % | (3) $x_{Index}$ | (4) $r_{Index}$ % | $r_{Portfolio}$ (1)×(2)+(3)×(4) | $\sigma_{Portfolio}$ (3) × 20% | $\sigma^2_{Portfolio}$ |
|---|---|---|---|---|---|---|
| -0.2 | 5 | 1.2 | 13.0 | 14.6% = 0.146 | 24% = 0.24 | 0.0576 |
| 0.0 | 5 | 1.0 | 13.0 | 13.0% = 0.130 | 20% = 0.20 | 0.0400 |
| 0.2 | 5 | 0.8 | 13.0 | 11.4% = 0.114 | 16% = 0.16 | 0.0256 |
| 0.4 | 5 | 0.6 | 13.0 | 9.8% = 0.098 | 12% = 0.12 | 0.0144 |
| 0.6 | 5 | 0.4 | 13.0 | 8.2% = 0.082 | 8% = 0.08 | 0.0064 |
| 0.8 | 5 | 0.2 | 13.0 | 6.6% = 0.066 | 4% = 0.04 | 0.0016 |
| 1.0 | 5 | 0.0 | 13.0 | 5.0% = 0.050 | 0% = 0.00 | 0.0000 |

b.




E(r) versus σ of the portfolio consisting of the risk-free T-bills and risky S&P falls on a straight line.
The vertical intercept is the risk-free return of 5%.

The slope is the Sharpe ratio, $S_p = \frac{E(r_p)-r_f}{\sigma_p}$, and it is equals to 0.4.

When the portfolio weights are non-negative, the portfolio is located in the positive quadrant, to the left of 100% S&P (inclusive) on the CAL in the figure above.

When the portfolio weight is negative for T-bills, the portfolio located on the right of the portfolio with 100% S&P on the CAL.

c.
Since short selling is not allowed, computing utility from $U = E(r) - 0.5 \times A\sigma^2$, we get:

| $x_{Bills}$ | $x_{Index}$ | $r_{Portfolio}$ | $\sigma_{Portfolio}$ | $\sigma^2_{Portfolio}$ | *(i)* $U(A = 2)$ | *(ii)* $U(A = 3)$ |
|---|---|---|---|---|---|---|
| 0.0 | 1.0 | 0.130 | 0.20 | 0.0400 | **0.0900** | .0700 |
| 0.2 | 0.8 | 0.114 | 0.16 | 0.0256 | 0.0884 | .0756 |
| 0.4 | 0.6 | 0.098 | 0.12 | 0.0144 | 0.0836 | **.0764** |
| 0.6 | 0.4 | 0.082 | 0.08 | 0.0064 | 0.0756 | .0724 |
| 0.8 | 0.2 | 0.066 | 0.04 | 0.0016 | 0.0644 | .0636 |
| 1.0 | 0.0 | 0.050 | 0.00 | 0.0000 | 0.0500 | .0500 |
| | | | | | | |

Investors, with risk aversion parameter $A = 2$, prefer a portfolio that is invested 100% in the market index to any of the other portfolios in the table.

The more risk averse investors, with risk aversion parameter $A = 3$, prefer the portfolio that is invested 60% in the market, rather than the 100% market weight preferred by investors with $A = 2$.