UNCERTAINTY

Week 2

(Chapter 12, except Appendix)

Uncertainty is Pervasive

- What is uncertain in one’s economic life?
  - Tomorrow’s house prices
  - Future health
  - Present and future actions of other people

- What are rational responses to uncertainty?
  - Buying insurance (health, life, auto)
  - Diversification

Preferences Under Uncertainty

Example

- Suppose a lottery costs \$45
- Win \$90 with probability 0.5 and win \$0 with probability 0.5
- Suppose individual's utility: U(\$90) = 12, U(\$0) = 2

General Structure

- *n* possible <u>outcomes</u> (states of nature) $(n = 2)$
- Outcome *i*'s probability is π_i $(\pi_{win} = 0.5, \pi_{nowin} = 0.5)$
- Outcome *i* is X_i $(X_{win} = 90, X_{nowin} = 0)$
- Individual's utility function

Expected Value

Example

- Suppose a lottery costs \$45
- Win \$90 with probability 0.5 and win \$0 with probability 0.5
- Suppose individual's utility: U(\$90) = 12, U(\$0) = 2

Expected value: probability-weighted average of all outcomes

$$EV = \sum_{i=1}^{n} \pi_i \cdot X_i = \pi_1 \cdot X_1 + \pi_2 \cdot X_2 + \cdots + \pi_n \cdot X_n$$

$$EV = 0.5 \times \$90 + 0.5 \times \$0 = \$45$$

Expected Utility

Example

- Suppose a lottery costs \$45
- Win \$90 with probability 0.5 and win \$0 with probability 0.5
- Suppose individual's utility: U(\$90) = 12, U(\$0) = 2

Expected utility: probability-weighted sum of all utilities associated with all possible outcomes:

$$EU = \sum_{i=1}^{n} \pi_i \cdot U(X_i) = \pi_1 \cdot U(X_1) + \pi_2 \cdot U(X_2) + \cdots + \pi_n \cdot U(X_n)$$

$$EU = 0.5 \times 12 + 0.5 \times 2 = 7$$

Expected Value ≠ Expected Utility

Risk Attitude

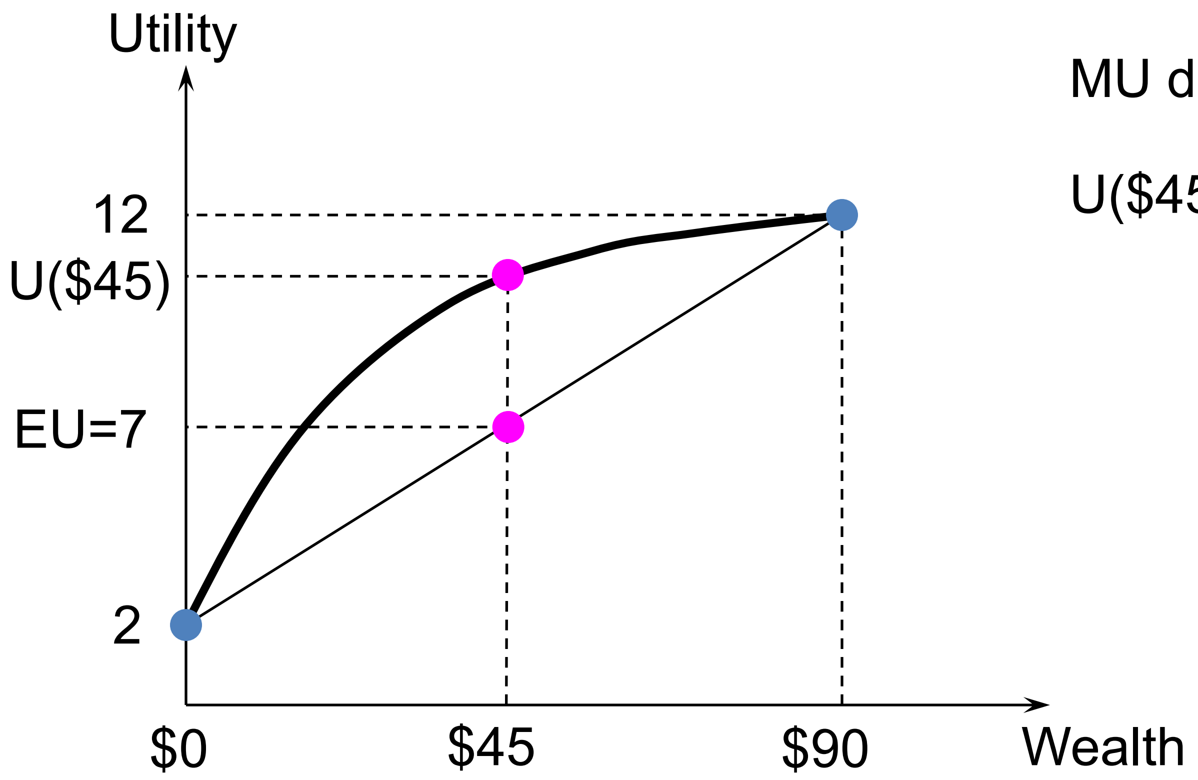
- EV = \$45 and EU=7

- If U(\$45) > 7
  $\Rightarrow$ Individual prefers \$45 for sure to lottery $\Rightarrow$ risk-aversion

- If U(\$45) < 7
  $\Rightarrow$ Individual prefers lottery to \$45 for sure $\Rightarrow$ risk-loving

- If U(\$45) = 7
  $\Rightarrow$ Individual indifferent between lottery and \$45 for sure $\Rightarrow$ risk-neutral

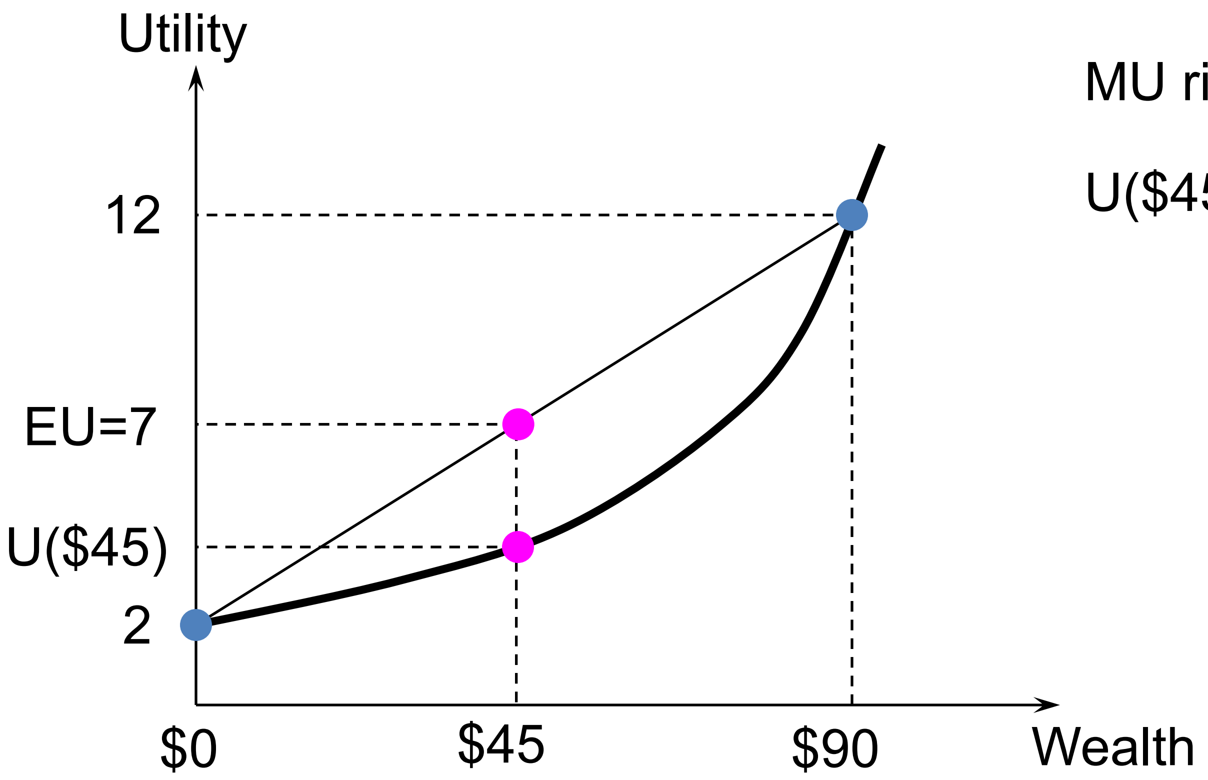
Risk-Averse Individual



MU declines as wealth rises

U($45) > EU $\Rightarrow$ risk averse

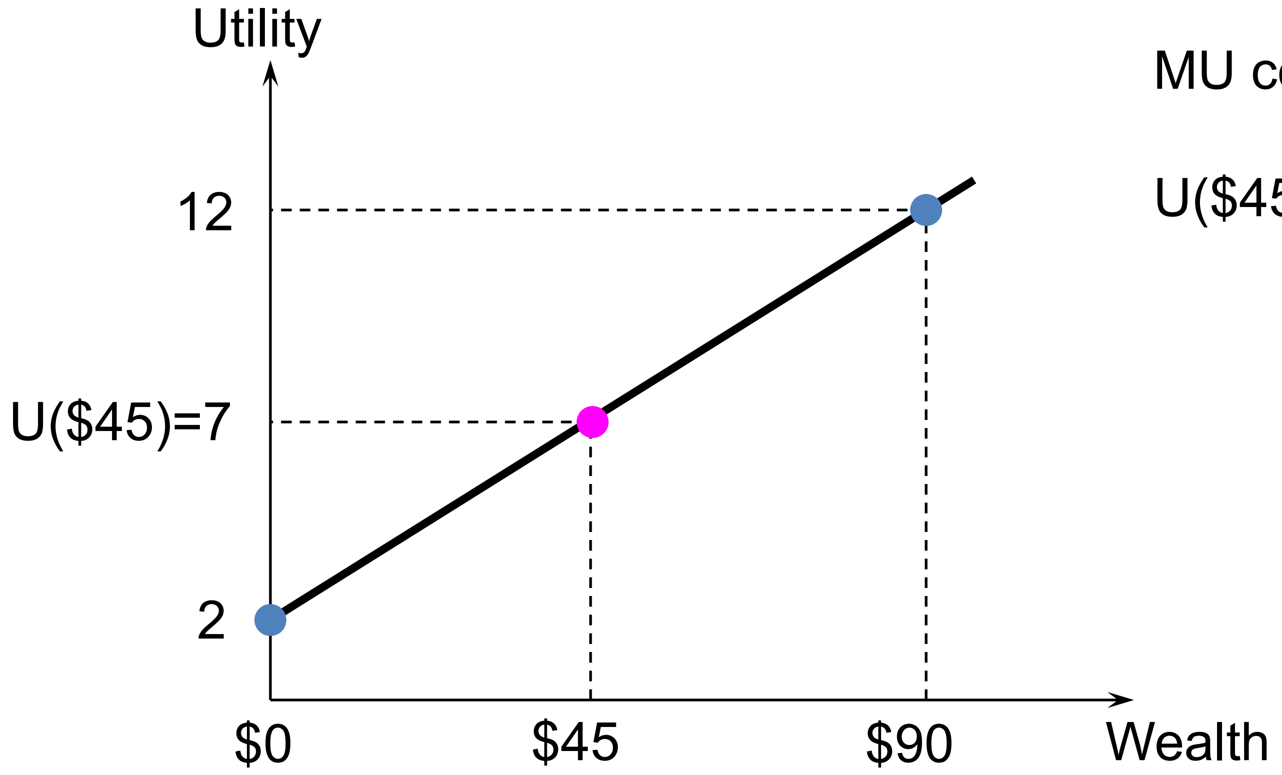
Risk-Loving Individual



MU rises as wealth rises

U($45) < EU $\Rightarrow$ risk loving

Risk-Neutral Individual



MU constant as wealth rises

U($45) = EU $\Rightarrow$ risk neutral

Risk Attitude and Marginal Utility

- Risk averse
  - Diminishing marginal utility of income
- Risk neutral
  - Constant marginal utility of income
- Risk loving
  - Increasing marginal utility of income

Measuring Risk Aversion

- Arrow-Pratt measure of absolute risk aversion at income level *x*:

$$A(x) = -\frac{u''(x)}{u'(x)}$$

- Person 1 is less risk averse than person 2 at *x* iff:

$$A_1(x) = -\frac{u_1''(x)}{u_1'(x)} < -\frac{u_2''(x)}{u_2'(x)} = A_2(x)$$

Risk Premium

- Most individuals are risk averse

- Risk-averse individual may still prefer a risky income
  - Willing to bear risk if there is enough reward to compensate for the risk
  - Will not buy risky asset if its price is equal to its expected value
  - But will buy risky asset if its price is sufficiently low

- *Risk premium* is the difference between the expected value of a risky asset and the risk-free income that makes the individual indifferent (between the risky asset and the risk-free income)
  - the value in excess of the risk-free value that a risky asset is expected to yield
  - (risk-free income = certainty equivalent)

Calculating Risk Premium

- Suppose the utility function of a consumer is

$$U(I) = \sqrt{I}$$

- Consumer can buy the following risky asset
  - $900 with probability 60%
  - $400 with probability 40%

- What is the risk premium associated with this asset?

Calculating Risk Premium

- Expected value of asset: $0.6 \times 900 + 0.4 \times 400 = 700$
- Expected utility of asset: $0.6 \times \sqrt{900} + 0.4 \times \sqrt{400} = 26$
- If individual can opt for an income of $\$C$ with certainty, prefer $\$C$ if

$$\sqrt{C} > 26 \implies C > 26^2 = 676$$

- To convince consumer to purchase, price of asset has to be <$676
- Risk premium is $700 - 676 = \$24$

States of Nature (Outcomes)

- Suppose there are two possible States of Nature:
  - "car accident" (a)
  - "no car accident" (na)

- Accident occurs with probability π_a, does not with probability π_{na}; where

$$\pi_a + \pi_{na} = 1$$

- Accident causes a loss of $\$L$

Contingencies

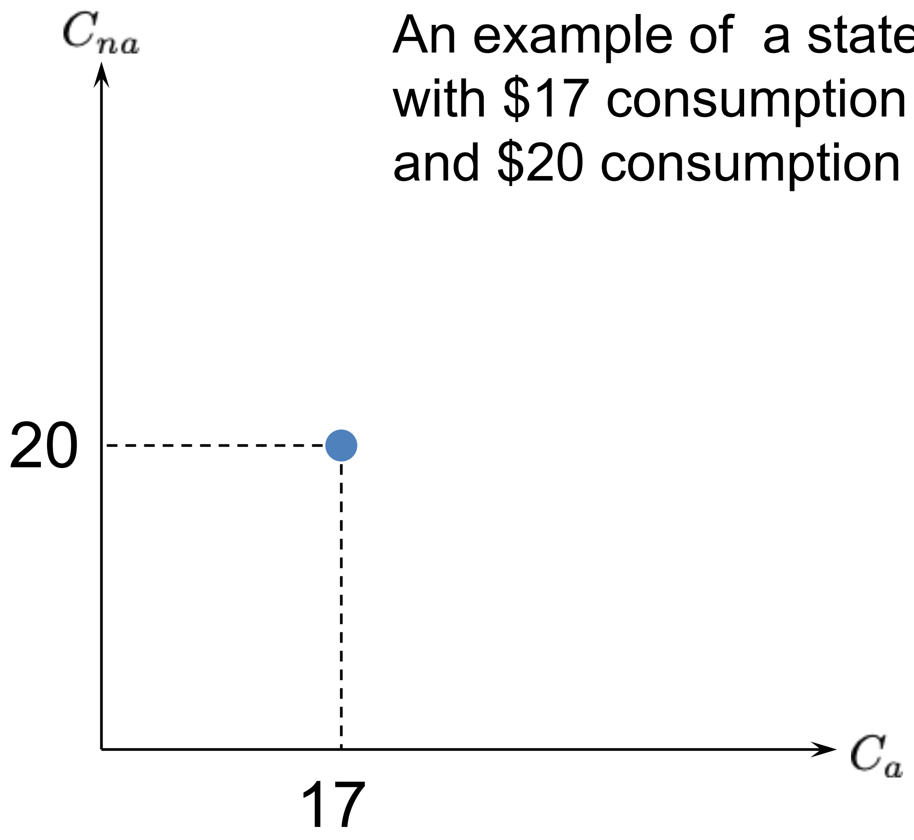
- A contract implemented only when a particular State of Nature occurs is state-contingent
  - e.g., the insurer pays only if there is an accident

- Likewise, a state-contingent consumption plan is implemented only when a particular State of Nature occurs
  - e.g., take a vacation only if there is no accident

State-Contingent Budget Constraint

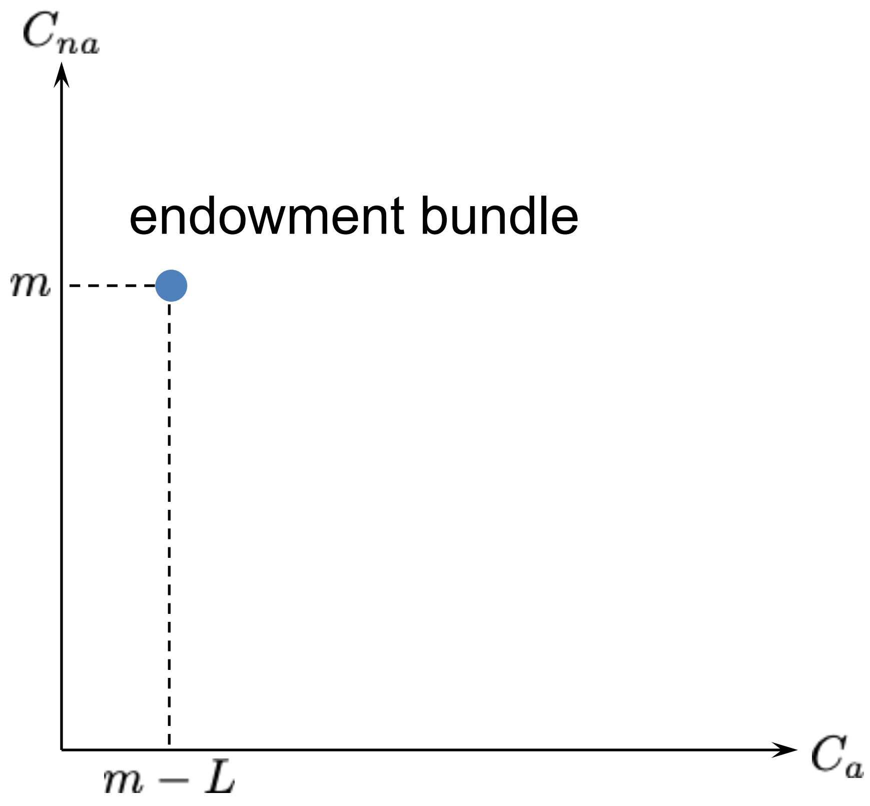
- Suppose each \$1 of insurance costs $0 < \$\gamma < 1$
- Consumer has \$$m$ of wealth
- c_{na} is consumption value in the no-accident state
- c_a is consumption value in the accident state

State-Contingent Budget Constraint



An example of a state-contingent consumption plan with \$17 consumption value in the accident state and \$20 consumption value in the no-accident state

State-Contingent Budget Constraint



More generally,

(With no insurance)

- $C_a = m - L$
- $C_{na} = m$

State-Contingent Budget Constraint

- If individual buys $*K* of accident insurance,

$$C_{na} = m - \gamma K \tag{1}$$
$$C_a = m - L - \gamma K + K = m - L + (1-\gamma)K \tag{2}$$

- Rearranging (2),

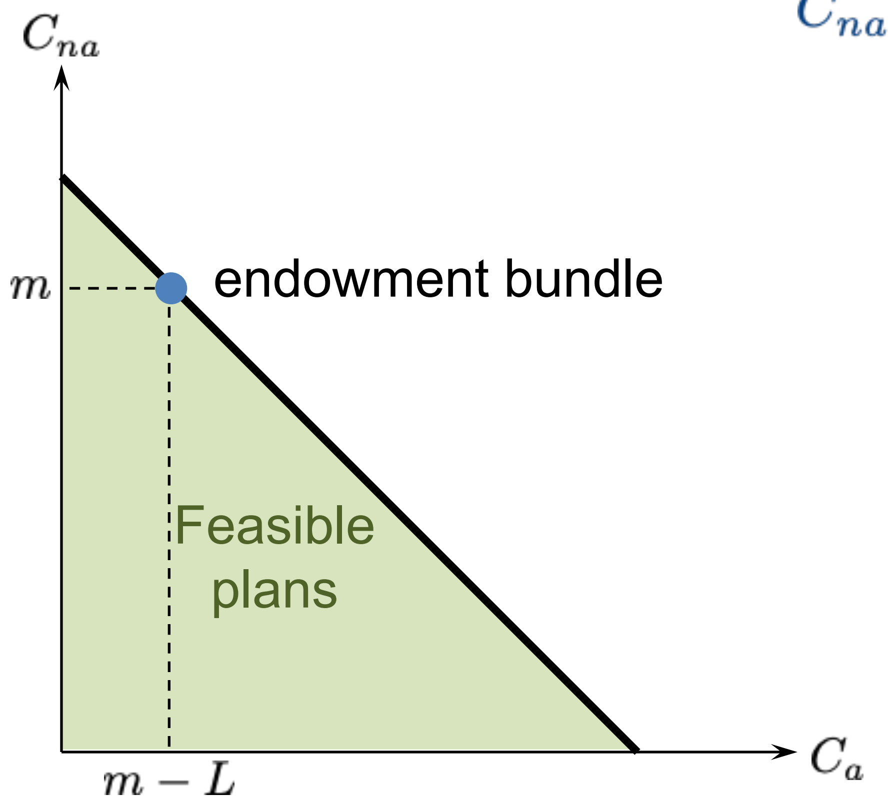
$$K = \frac{C_a - m + L}{1-\gamma} \tag{3}$$

- Substituting (3) into (1),

$$C_{na} = m - \gamma \cdot \frac{C_a - m + L}{1-\gamma} \tag{4}$$

$$\implies C_{na} = -\frac{\gamma}{1-\gamma} \cdot C_a + \frac{m - \gamma L}{1-\gamma}$$

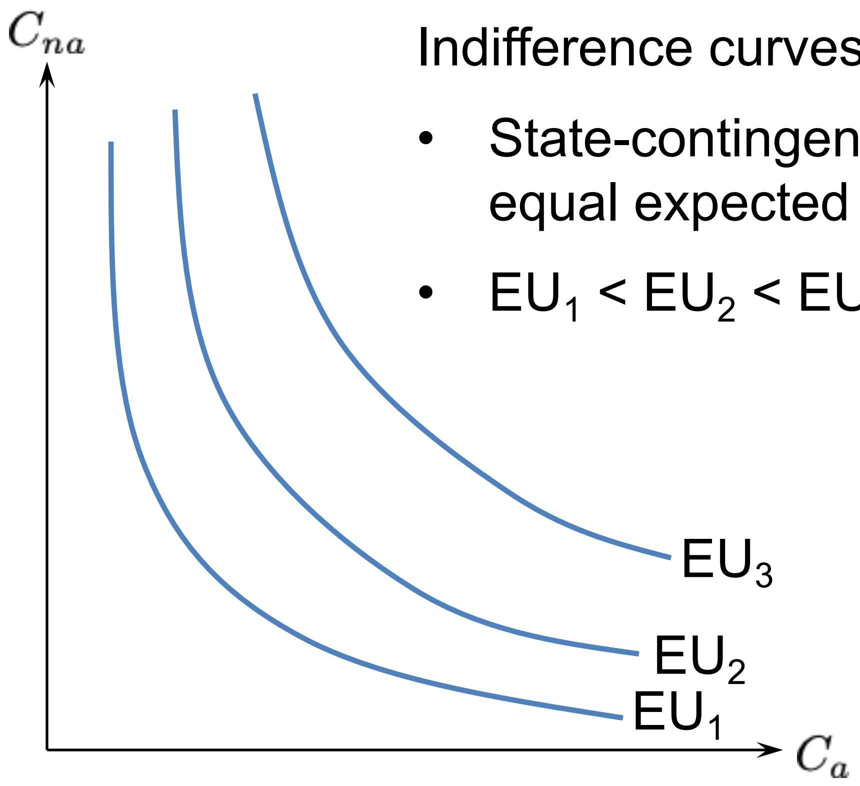
State-Contingent Budget Constraint



$$C_{na} = \underbrace{-\frac{\gamma}{1-\gamma}}_{\text{slope}} \cdot C_a + \underbrace{\frac{m - \gamma L}{1-\gamma}}_{\text{intercept}}$$

- How is a rational choice made under uncertainty?
  - Choose the most preferred, feasible state-contingent consumption plan
- Where is the most preferred, feasible state-contingent consumption plan?

Preferences (over State-Contingent Consumption Plans)



Indifference curves

- State-contingent consumption plans that give equal expected utility are equally preferred
- $EU_1 < EU_2 < EU_3$

Preferences (over State-Contingent Consumption Plans)

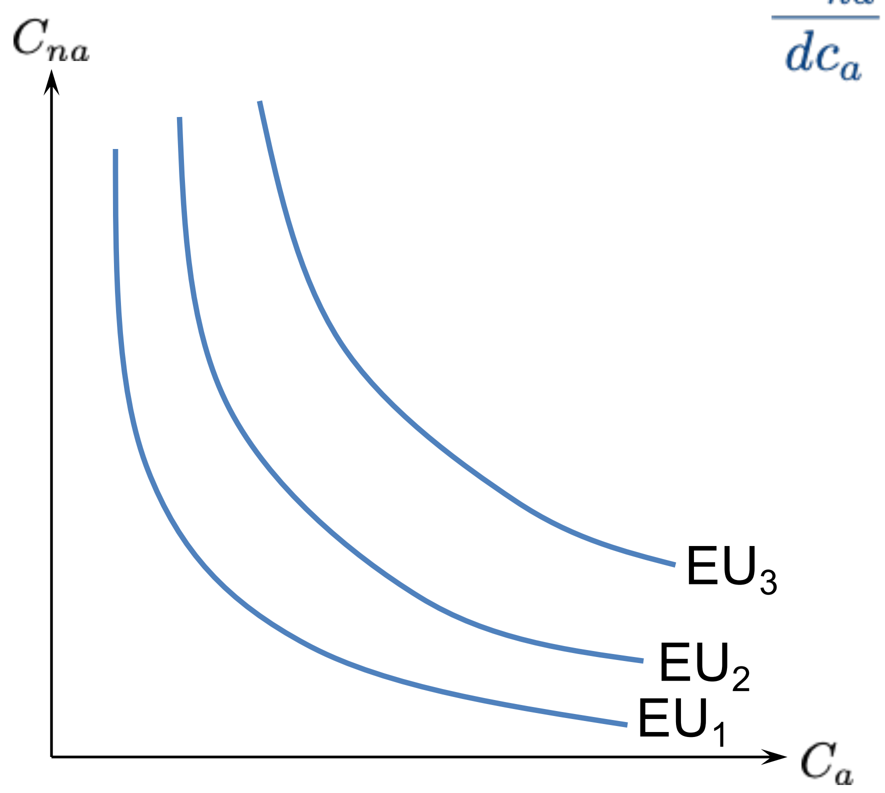
- What is the MRS of an indifference curve?
- Get c_1 with probability π_1 and c_2 with probability $\pi_2 = 1 - \pi_1$
- $EU = \pi_1 U(c_1) + \pi_2 U(c_2)$
- Along an indifference curve, EU constant, hence $dEU = 0$

Preferences (over State-Contingent Consumption Plans)

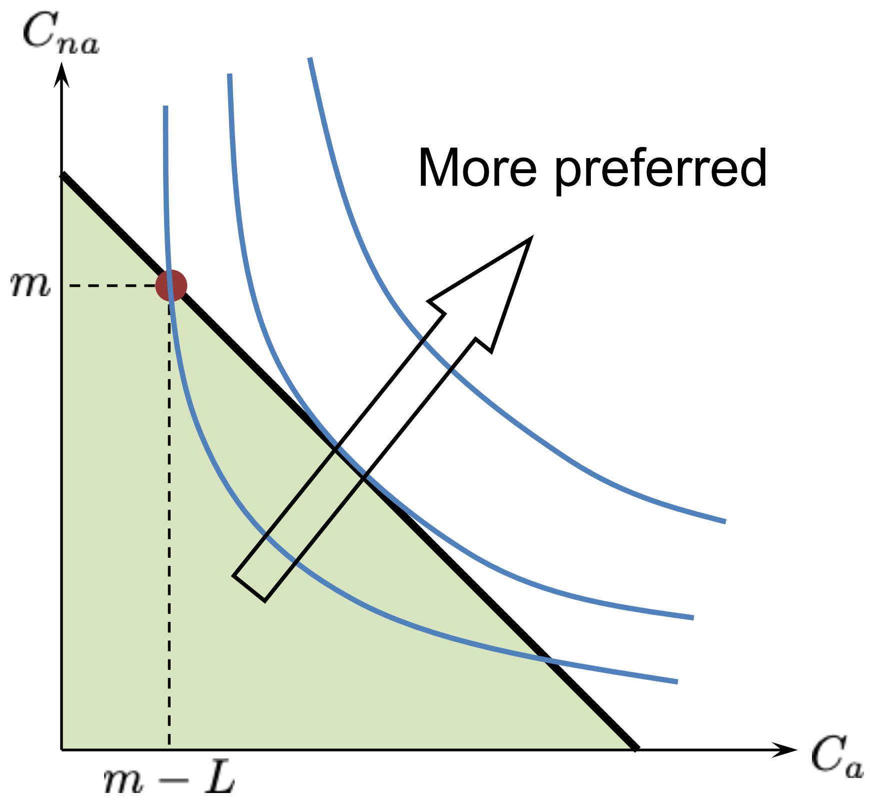
$$EU = \pi_1 U(c_1) + \pi_2 U(c_2)$$

Preferences (over State-Contingent Consumption Plans)

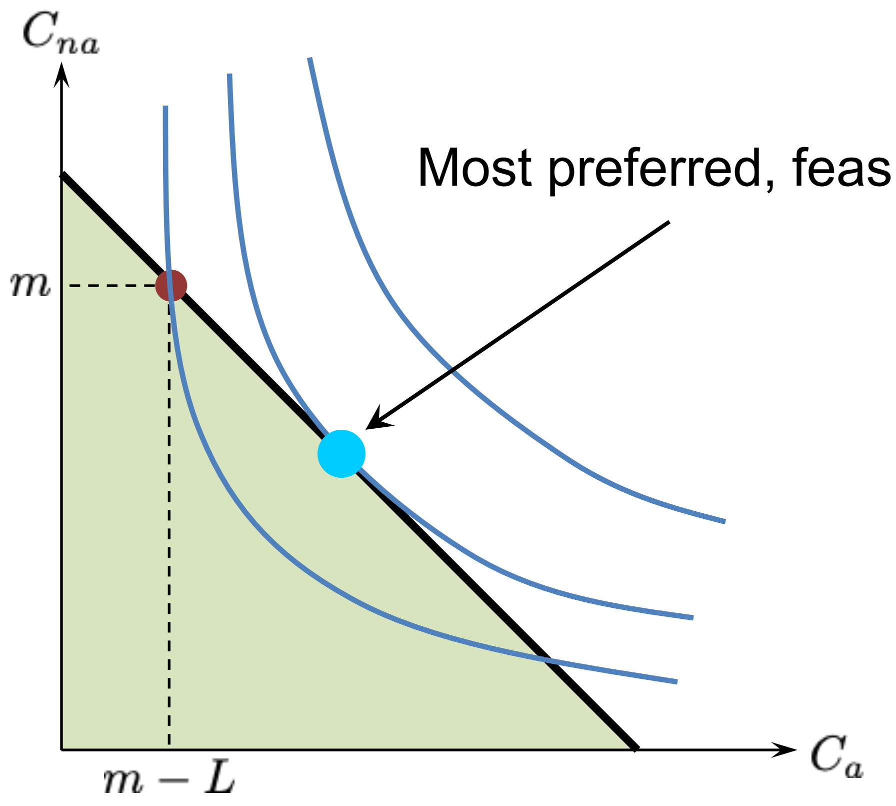
$$\frac{dc_{na}}{dc_a} = -\frac{\pi_a \cdot MU(c_a)}{(1-\pi_a) \cdot MU(c_{na})}$$



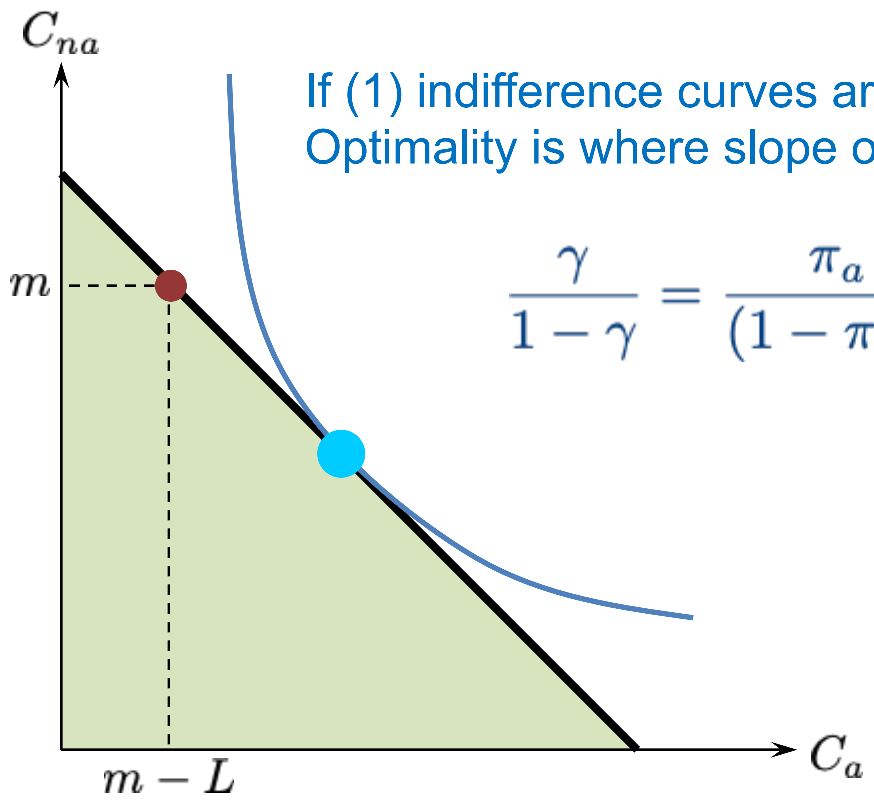
Optimal Consumption Plan



Optimal Consumption Plan



State-Contingent Budget Constraints



If (1) indifference curves are convex, and (2) solution is interior,
Optimality is where slope of budget constraint = MRS,

$$\frac{\gamma}{1-\gamma} = \frac{\pi_a \cdot MU(c_a)}{(1-\pi_a) \cdot MU(c_{na})}$$

Competitive Insurance

If entry to the insurance industry is free,

- Expected economic profit = 0

$$\implies \gamma K - \pi_a K - (1 - \pi_a)0 = (\gamma - \pi_a)K = 0$$

$$\implies \gamma = \pi_a$$

- If cost of $1 insurance = accident probability, insurance is fair

Competitive Insurance

- When insurance is fair, rational insurance choices satisfy

$$\frac{\gamma}{1-\gamma} = \frac{\pi_a \cdot MU(c_a)}{(1-\pi_a) \cdot MU(c_{na})} \implies \frac{\pi_a}{1-\pi_a} = \frac{\pi_a \cdot MU(c_a)}{(1-\pi_a) \cdot MU(c_{na})}$$

$$\implies MU(c_a) = MU(c_{na})$$

- Marginal utility of income must be the same in both states

Competitive Insurance

- How much fair insurance does a risk-averse consumer buy?
- Risk-aversion $\Rightarrow$ MU monotonically decreasing (MU(c) $\downarrow$ as c $\uparrow$)
- Hence, $$MU(c_a) = MU(c_{na}) \implies c_a = c_{na}$$
- i.e., full-insurance

“Unfair” Insurance

- Suppose insurers make positive expected economic profit

$$\implies \gamma K - \pi_a K - (1-\pi_a)0 = (\gamma - \pi_a)K > 0$$

$$\implies \gamma > \pi_a$$

$$\implies \frac{\gamma}{1-\gamma} > \frac{\pi_a}{1-\pi_a}$$

“Unfair” Insurance

- Rational choice requires

$$\frac{\gamma}{1-\gamma} = \frac{\pi_a \cdot MU(c_a)}{(1-\pi_a) \cdot MU(c_{na})}$$

- Since

$$\frac{\gamma}{1-\gamma} > \frac{\pi_a}{1-\pi_a} \implies MU(c_a) > MU(c_{na}) \implies c_a < c_{na}$$

- i.e., a risk-averter buys less than full “unfair” insurance

Example: Willingness to Buy Insurance

- Suppose there is only full insurance available
  - Individual unable to pick optimal bundle
  - Either buy the available insurance, or live without insurance
- Suppose the individual's utility function is $U(X) = \sqrt{X}$
- Initial wealth $50,000
  - With probability 0.05, negative income shock of $10,000
- If no insurance, expected income 49500, and

$$EU = 0.95\sqrt{50000} + 0.05\sqrt{40000} = 222.43$$

- With full insurance, receives $10,000 when shock occurs
- How much is the individual willing to pay for this insurance?

Willingness to Pay for Full Insurance

- Let the price of the insurance be *P*
- If the consumer buys the insurance, $$EU = \sqrt{50000 - P}$$
- *EU* with insurance should be no smaller than *EU* without insurance,

$$\sqrt{50000 - P} \geq 222.43$$

- Thus $$P \leq 526$$

Mutual Insurance

- 100 consumers each independently risk a $10,000 loss
- Loss probability = 0.01. On average, 1 consumer realizes loss
- Each person's initial wealth is $40,000
- With no insurance, expected wealth is

$$0 \cdot 99 \times \$40{,}000 + 0 \cdot 01(\$40{,}000 - \$10{,}000)$$
$$= \$39{,}900.$$

Mutual Insurance

- Suppose consumers agree to insure one another
  - If someone realizes loss, each person pays her $100
  - Each person expected to pay $100
  - Expected income is $39,900 (as before)
- But now, risk is spread
  - Good if one is risk averse (Higher expected utility)
- Examples
  - Medishield
  - Obamacare

Diversification

Besides buying insurance, diversification can also mitigate risk

Example

- Two firms, A and B. Shares cost $10
- With probability 0.5, A's profit is $100 and B's profit is $20
- With probability 0.5, A's profit is $20 and B's profit is $100
- You have $100 to invest. What should you do?

Diversification

- Buy only firm A’s stock?
- \$100/10 = 10 shares.
- You earn \$1000 with probability 0.5 and \$200 with probability 0.5.
- Expected earning: \$500 + \$100 = \$600

- Buy only firm B’s stock?
- \$100/10 = 10 shares.
- You earn \$200 with probability 0.5 and \$1000 with probability 0.5.
- Expected earning: \$100 + \$500 = \$600

Diversification

- Buy 5 shares in each firm?

- You earn $600 for sure.

- In this example, diversification maintains expected earning and lowers risk.

- Typically, diversification lowers expected earnings in exchange for lowered risk.

Measuring Risk

- Suppose there are two bonds in the market:
  - Bond 1:
    - With probability 0.5, \$1,500
    - With probability 0.5, \$500
  - Bond 2:
    - With probability 0.1, \$10,000
    - With probability 0.9, \$0
- They have the same expected value (\$1,000)
- Are they equally risky?

Measuring Risk

- We use standard deviation to measure degree of risk
- Bond 1 $\sqrt{0.5\times(1500-1000)^2+0.5\times(500-1000)^2} = 500$
- Bond 2 $\sqrt{0.1\times(10000-1000)^2+0.9\times(0-1000)^2} = 3000$
- Bond 2 is riskier
  - Same expected value
  - Higher standard deviation