Le Van Minh - Tan San Xuin

Question 1

$$U(x,y) = 6\sqrt{x} + y$$

$$MU_x = \frac{\delta U}{\delta x} = \frac{3}{\sqrt{x}}; MU_y = \frac{\delta U}{\delta y} = 1$$

$$MRS_{x,y} = \frac{MU_x}{MU_y} = \frac{3}{\sqrt{x}}$$

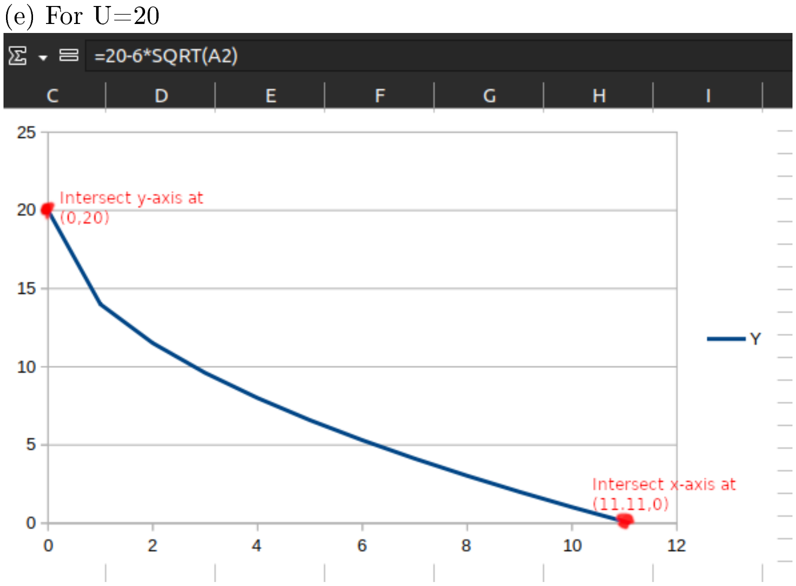
(a) Yes, since U increases when either x or y increases

(b) As x inceases, MU_x diminishes

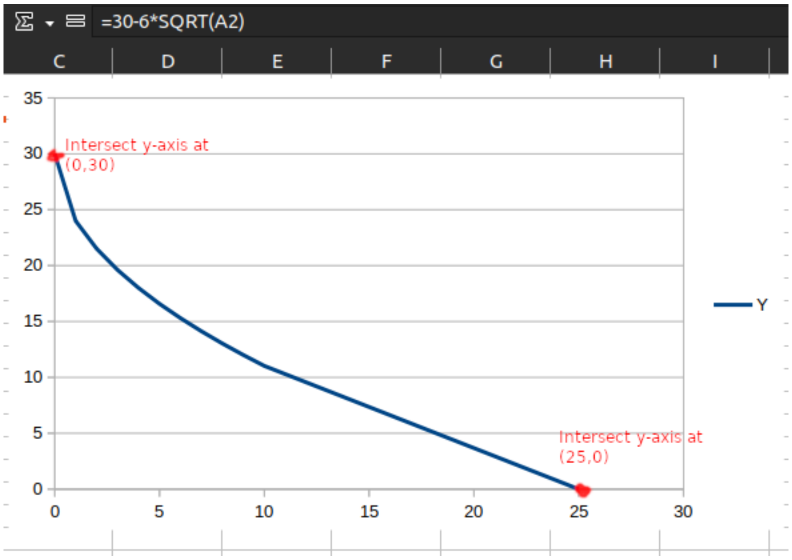
(c) As y increases, MU_y remains constant

(d) $MRS_{x,y} = \frac{3}{\sqrt{x}}$, MRS only depends on x, as x increases along the indifference curve, $MRS_{x,y}$ diminishes

(e) For U=20

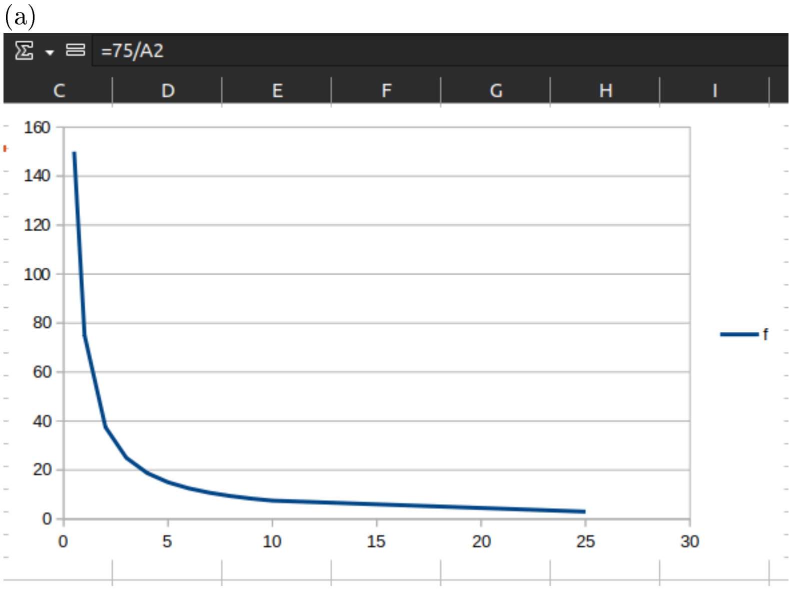


For U=30

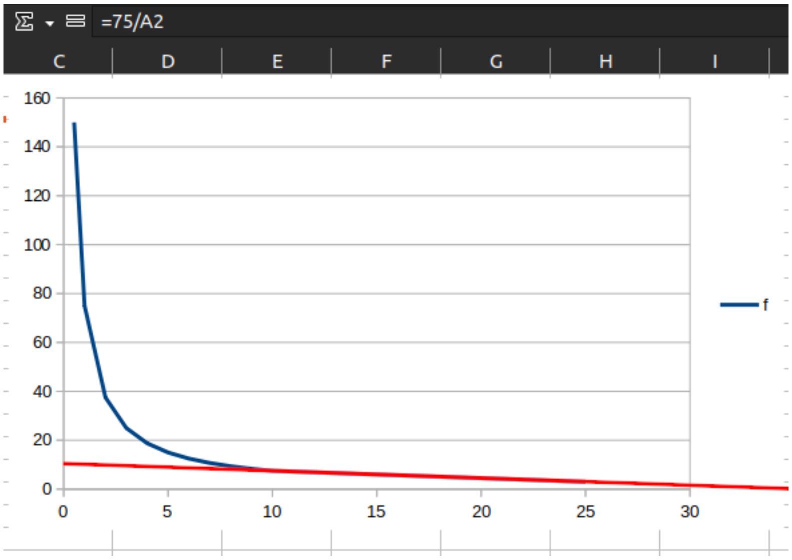


Question 2

(a)



(b)



(c) Moana's utility maximization problem

$$\max_{d,f} U(d,f) = df$$

subject to: $g(d,f) = 100d + 400f - 4000 = 0$
Lagrangian function: $\Lambda(d,f,\lambda) = U(d,f) + \lambda g(d,f)$

$$= df + \lambda(100d + 400f - 4000)$$

Solving Lagrangian function:

$$\begin{cases} \frac{\delta\Lambda}{\delta d} = f + 100\lambda & = 0 \\ \frac{\delta\Lambda}{\delta f} = d + 400\lambda & = 0 \\ \frac{\delta\Lambda}{\delta\lambda} = 100d + 400f - 4000 & = 0 \end{cases}$$

$$\begin{cases} 4(f + 100\lambda) & = 0 \\ d + 400\lambda & = 0 \\ 100d + 400f - 4000 & = 0 \end{cases}$$

$$\begin{cases} d - 4f & = 0 \\ d + 400\lambda & = 0 \\ 100d + 400f - 4000 & = 0 \end{cases}$$

$$\begin{cases} d & = 20 \\ f & = 5 \\ \lambda & = -0.05 \end{cases}$$

Question 3

(a)

$$\underset{x,y}{max} U(x,y) = \sqrt{xy}$$

subject to: $g(x,y) = 4x + 4y - 400 = 0$
Lagrangian function: $\Lambda(x,y,\lambda) = U(x,y) + \lambda g(x,y)$

$$= \sqrt{xy} + \lambda(4x + 4y - 400)$$

Solving Lagrangian function:

$$\begin{cases} \frac{\delta\Lambda}{\delta x} = \frac{1}{2}\sqrt{\frac{y}{x}} + 4\lambda & = 0 \\ \frac{\delta\Lambda}{\delta y} = \frac{1}{2}\sqrt{\frac{x}{y}} + 4\lambda & = 0 \\ \frac{\delta\Lambda}{\delta\lambda} = 4x + 4y - 400 & = 0 \end{cases}$$

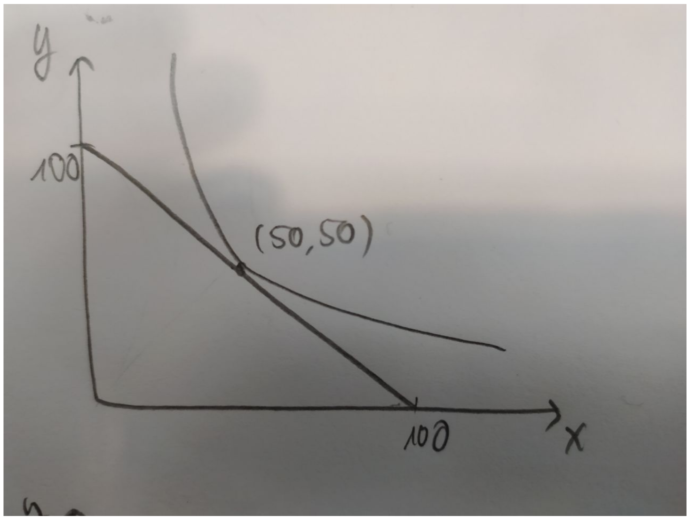
$$\begin{cases} \frac{1}{2}\sqrt{\frac{y}{x}} - \frac{1}{2}\sqrt{\frac{x}{y}} & = 0 \\ \frac{1}{2}\sqrt{\frac{x}{y}} + 4\lambda & = 0 \\ 4x + 4y - 400 & = 0 \end{cases}$$

$$\begin{cases} \sqrt{\frac{y}{x}} & = \sqrt{\frac{x}{y}} \\ \lambda & = -\frac{1}{8}\sqrt{\frac{x}{y}} \\ 4x + 4y - 400 & = 0 \end{cases}$$

$$\begin{cases} x & = y \\ \lambda & = -\frac{1}{8}\sqrt{\frac{x}{y}} \\ 4x + 4y - 400 & = 0 \end{cases}$$

$$\begin{cases} x & = y \\ \lambda & = -\frac{1}{8}\sqrt{\frac{x}{y}} \\ 4x + 4y - 400 & = 0 \end{cases}$$

$$\begin{cases} \lambda & = -0.125 \\ x & = 50 \\ y & = 50 \end{cases}$$



(b)

$$\max_{x,y} U(x,y) = \sqrt{xy}$$

subject to: $g(x,y) = 4x + 4y - 720 = 0$
Using the same process in (a)

$$\begin{cases} \lambda & = -0.125 \\ x & = 90 \\ y & = 90 \end{cases}$$

(c)

$$\max_{x,y} U(x,y) = \sqrt{xy}$$

subject to: $\begin{cases} 4x + 4y - 720 = 0 & x \geq 80 \\ 4y = 100 & x < 80 \end{cases}$
Hypothetically $x \geq 80$
Solve similarly to (b):

$$\begin{cases} \lambda & = -0.125 \\ x & = 90 \\ y & = 90 \end{cases}$$

Hypothesis is satisfied, accepting this basket as optimal.
(d) With the cash subsidy:

$$\max_{x,y} U(x,y) = \sqrt{xy}$$

subject to: $g(x,y) = 4x + 4y - 880 = 0$

Solve the optimization using Lagrage's method:

$$\begin{cases} \lambda & = -0.125 \\ x & = 110 \\ y & = 110 \end{cases}$$

$$U(x,y)_{cash} = 110$$

With the coconut (x) voucher:

$$\underset{x,y}{max} U(x,y) = \sqrt{xy}$$

subject to: $\begin{cases} 4x + 4y - 880 = 0 & x \geq 120 \\ 4y = 100 & x < 120 \end{cases}$

Assuming $x \geq 120$

Solve the optimization using Lagrage's method:

$$\begin{cases} \lambda & = -0.125 \\ x & = 110 \\ y & = 110 \end{cases}$$

Assumption not satisfied, optimal basket occurs at $x = 120$

$$\Rightarrow \begin{cases} x & = 120 \\ 4x + 4y - 880 & = 0 \end{cases}$$

$$\begin{cases} x & = 120 \\ y & = 100 \end{cases}$$

$$U(x,y)_{voucher} = 109.55$$

Therefore, Pue's optimal utility of when given cash subsity is greater than coconut voucher

Question 4

(a)

B lies below $BL_1 => A \succ B$

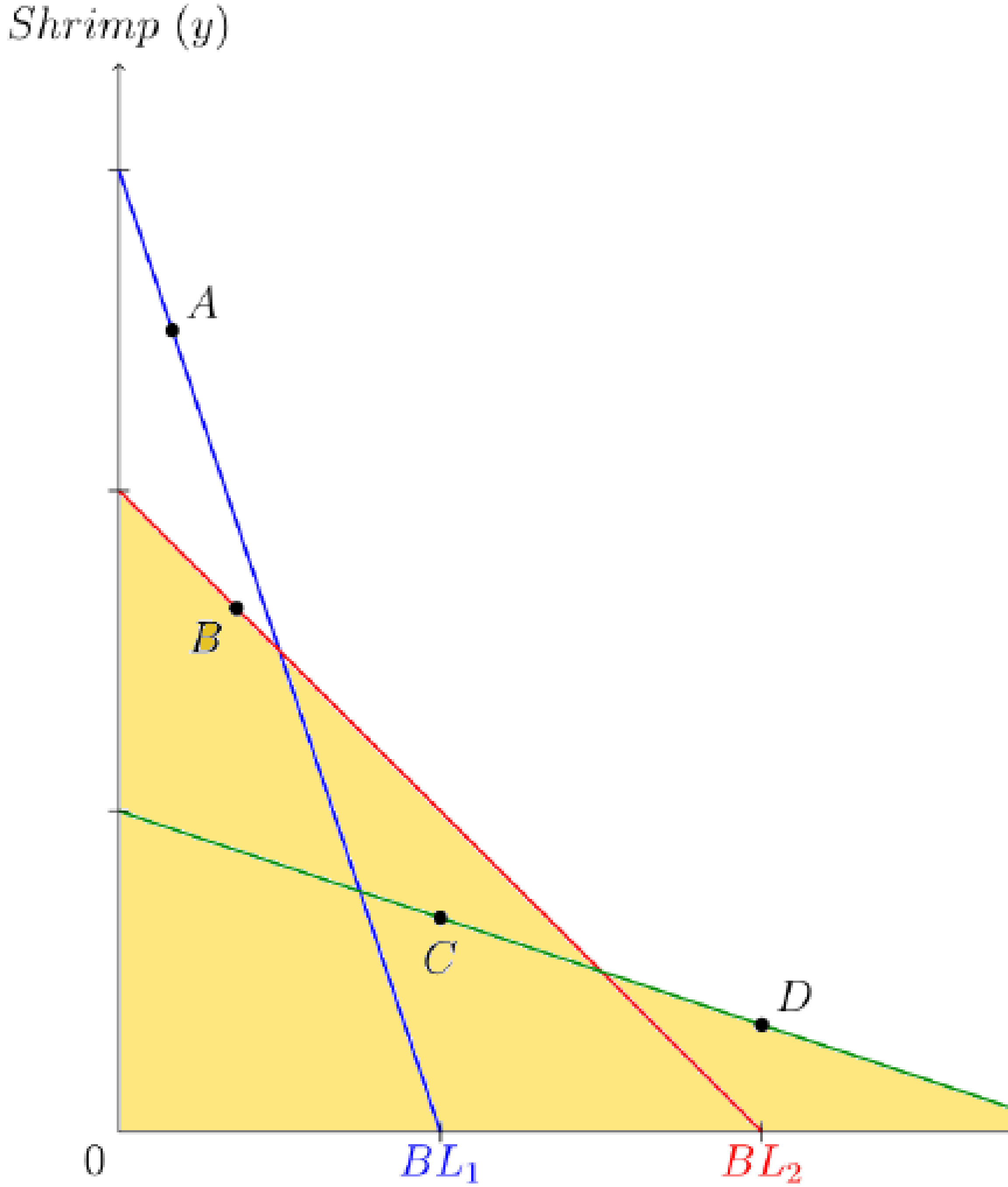
C lies below $BL_2 \Rightarrow B \succ C$

D is on the budget line with C $\Rightarrow C \succeq D$

Using these informations: $A \succ B \succ C \succeq D$

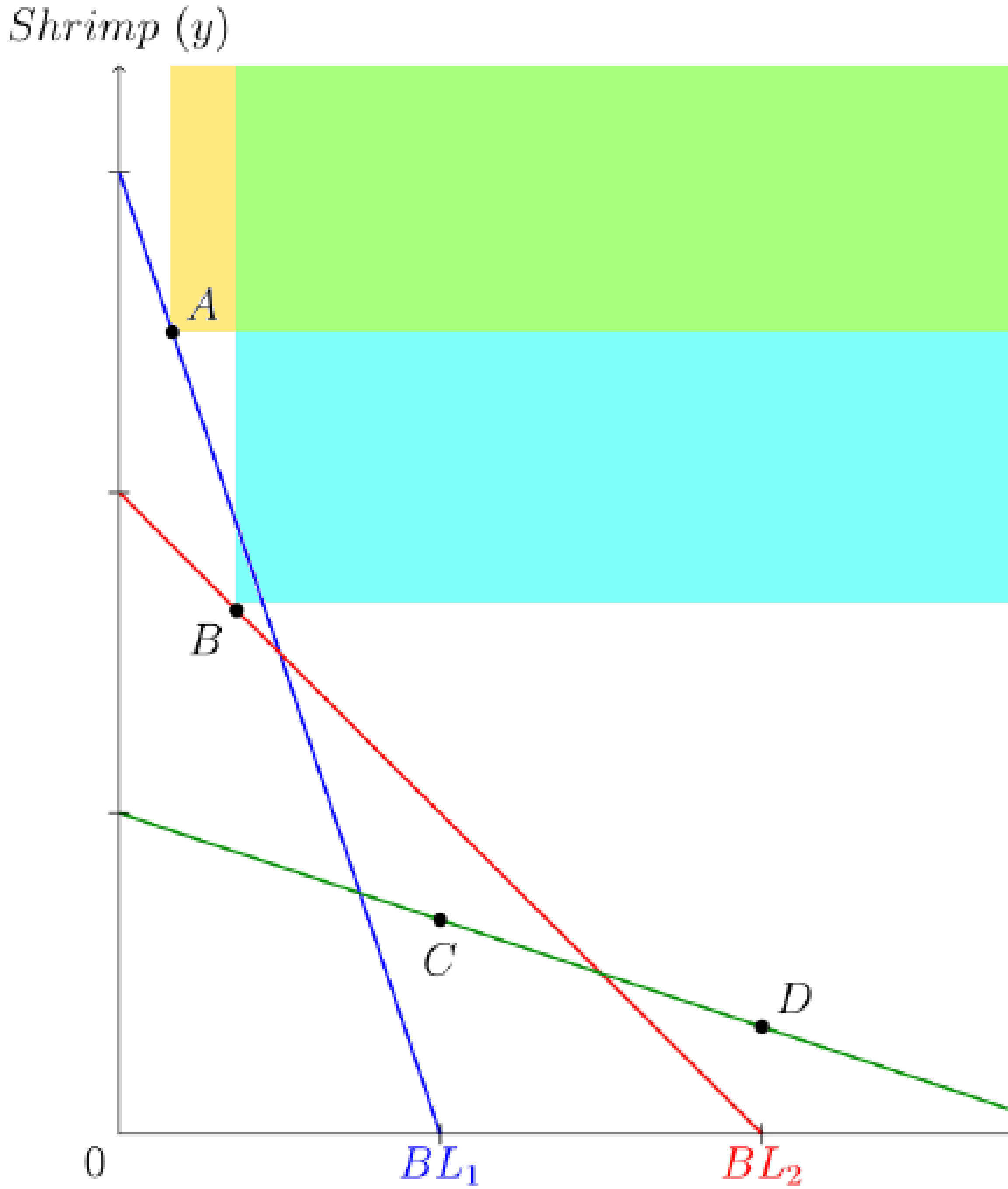
(b) The area under BL_2 is strictly less prefered to B since those basket cost less and should not be able to achieve same utility as B.

The area on and under BL_3 is strictly less prefered to B, since they are strictly less prefered to C and C is less prefered to B



(c) The rectangle above and to the right of B is prefered to B since the two goods satisfy monoticity.

The rectangle above and to the right of A is prefered to B, since they are prefered to A and $A \succ B$


$Shrimp\ (y)$
A
B
C
D
0
BL_1
BL_2