# Macroeconomics Analysis II, EC3102

# Tutorial 4 Solution

<u>Question 1 (sol)</u>

a) This is simply an application of our idea that an economic agent cannot end its life with anything other than zero assets, because for utility-maximization purposes it would not make sense for it to die with strictly positive assets and if everyone knows the agent will not be around in the next period to pay its debts, it cannot die with strictly negative assets (i.e., cannot die in debt). Hence, we have $B_T = 0$ .

b) A useful rearrangement of the government budget constraint (GBC) is

$$(1 + i_{T-1})B_{T-1} = P_T(t_T - g_T) + M_T - M_{T-1} \tag{1}$$

in which we have imposed $B_T$ . A second useful way of writing this expression is

$$(1 + i_{T-1})B_{T-1} = P_T\left[(t_T - g_T) + \frac{M_T - M_{T-1}}{P_T}\right] \tag{2}$$

Comment: *In (2), we now have as the second term inside square brackets real seignorage revenue in period $T$. This expression states that the nominal value of government debt outstanding (inclusive of interest payments) – which is the left-hand-side of this expression – must equal the nominal value of the fiscal surplus plus the nominal value of seignorage revenue.*

*If there is zero inflation between period $T-1$ and period $T$ , then clearly $P_T = P_{T-1} = 1$. To compute real seignorage revenue, we must first find $M_T$, the amount of money the monetary authority decides for the end of period $T$ .*

From the question, we have $t_T - g_T = 9$, $i_{T-1} = 0.1$, $B_{T-1} = 10$, $P_{t-1} = 1$ *and* $M_{T-1} = 10$. *Thus:*

$$(1 + 0.1) \cdot 10 = 1 \cdot \left[9 + \frac{M_T - 10}{1}\right] \tag{3}$$

With the given values, the previous expression immediately gives us that $M_T = 12$. Real seignorage revenue in period T is thus $\frac{M_T - M_{T-1}}{1} = \frac{12-10}{1} = 2$.

<u>Intuition:</u> From equation (2), we can see that if nothing changes, the equation ***cannot balance*** with the info $t_T - g_T = 9$, $i_{T-1} = 0.1$, $B_{T-1} = 10$, $P_{t-1} = 1$ *and* $M_{T-1} = 10.$ And since the price level does not change at all, this means that the action that the monetary authority happens to choose helps to balance the GBC

without affecting the price level. *(Note well that here the two authorities are acting independently but it just happens that the price level does not change. So here it looks like the monetary authority is providing the full help.)*. So, we can say that the rest of the imbalance are 'absorbed' by the monetary authority. In other words, monetary authority needs to print more money to provide seignorage revenues to balance the GBC.

*Comment: Some of you might do it this way which is totally alright:*

$$\frac{M_T - M_{T-1}}{P_T} = (g_T - t_T) + \frac{1 + i_{T-1}}{P_T} \cdot B_{T-1} - \frac{B_T}{P_T}$$

*From here, you can just substitute the values into the RHS to find* $\frac{M_T - M_{T-1}}{P_T} = 2$. *This is good.*

$$\frac{M_T - M_{T-1}}{P_T} = 9 + \frac{1 + 0.1}{1} \cdot 10 - \frac{0}{1} \quad (\because B_T = 0\,,\ \text{part a})$$
$$\frac{M_T - M_{T-1}}{P_T} = 2$$

Comment: *Some might wonder why we have more money printed and yet the price level is kept unchanged. Yes, if all others kept unchanged, the printing of money will cause inflation but note that the question mentions that the "*suppose that the monetary authority chooses a value for $M_T$ which when coupled with this fiscal policy implies that there is **zero inflation** between period $T-1$ and period $T$". *This means that fiscal and monetary interactions somehow lead to this unchanged price level (it is an assumption).*

c) The monetary authority continues to choose $M_T = 12$ , as found in part b above. The GBC of course must continue to hold – let's now use the first form of the GBC derived in part b.
Inserting the given values, the GBC becomes

$$(1\ +\ 0.10) \cdot 10 = P_T(8) + 12 - 10 \qquad (4)$$
$$\Rightarrow P_T = 1.125,$$

which means that there is 12.5 percent inflation between period T −1 and period T.

Real seignorage revenue is thus $\frac{M_T - M_{T-1}}{P_T} = \frac{12-10}{1.125} = 1.777$, (less than the what we found in part b.)

<u>Intuition:</u> The reason for the difference is that the price level adjusts between period T −1 and period T while the monetary authority sticks to a nominal policy of $M_T = 12$.

The generation of a smaller real fiscal surplus in the final period of the economy would mean it needs more real seignorage revenue if it had to repay a fixed real amount of debt. So fiscal authority, in order to balance the budget, needs monetary authority to help. But since the monetary side has chosen independently that $M_T = 12$. Here you can say that the monetary authority 's action does not fully help to balance the GBC, and thus the price has to be higher (this is a pressure on current price). *[side comment: this can be considered as  a partial help from monetary side]*

By generating inflation, with the same nominal $B_{T-1}$ but higher price level, the real terms $\frac{B_{T-1}}{P_T}$ is lower. And thus, in that sense, the government is able to reduce the real amount of debt $\frac{B_{T-1}}{P_T}$ – it must repay, which offsets the smaller real seignorage revenue.

<u>Question 2 (sol)</u>

a) Using the given numerical values in the PVGBC,

$$14 \text{ trillion} = B_{t-1} = \left(\frac{1+0.05}{0.05}\right)(t-g)$$

from which it obviously follows that $(t-g) = \frac{2}{3}\text{trillion} = 667$ billion. Intuitively, if the entire debt has to be repaid using a constant fiscal surplus (and zero seignorage) over time, that surplus has to $667 billion in every time period.

b) Using the given numerical values in the PVGBC,

$$14 \text{ trillion} = B_{t-1} = \left(\frac{1+0.025}{0.025}\right)(t-g)$$

from which it obviously follows that $(t - g)$ = 0.3415 trillion = 341.5 billion. Intuitively, if the entire debt has to be repaid using a constant fiscal surplus (and zero seignorage) over time, that surplus has to $341.5 billion in every time period. The required surplus in this case is much smaller than in part a because of the lower interest rate, which implies smaller interest payments on the debt that has to be repaid.

c) Using the given numerical values in the PVGBC,

$$14 \text{ trillion} = B_{t-1} = \left(\frac{1+0.05}{0.05}\right)sr$$

from which it obviously follows that $sr = \frac{2}{3}\text{trillion} = 667$ billion. Intuitively, if the entire debt has to be repaid using a constant quantity of seignorage revenue over time (because $t - g = 0$ in every time period), seignorage revenue has to $667 billion in every time period.

d) Using the given numerical values in the PVGBC,

$$14 \text{ trillion} = B_{t-1} = \left(\frac{1+0.025}{0.025}\right)sr$$

from which it obviously follows that sr = 0.3415 trillion = 341.5 billion. Intuitively, if the entire debt has to be repaid using a constant quantity of seignorage revenue over time (because $t - g = 0$ in every time period), seignorage revenue has to $341.5 billion in every time period. As in part b, the required seignorage revenue is much smaller than in part c because of the lower interest rate, which implies smaller interest payments on the debt that has to be repaid.