Practice Problem Set 1

Intertemporal Choice (C.10), Uncertainty (C.12)

Question 1.1

Margaret has a utility function $U(c_1, c_2) = c_1+c_2$ (Hint: this implies that she does not care whether she consumes in period 1 or period 2), where c_1 and c_2 are denoted in dollar units. Her initial endowment is \$20 in period 1 and \$40 in period 2.
Margaret is given an opportunity to buy a stock for \$12 in period 1, which she can sell for \$20 in period 2. Otherwise, she derives no utility from owning the stock. She can only buy one unit of the stock, if she buys at all.
i) If Margaret cannot borrow or lend, should she invest in the stock? Why or why not?
ii) Suppose that Margaret can borrow and lend at an interest rate of 50%. Should she invest in the stock? Why or why not?

Answer 1.1

i) She should buy the stock.
If she does not buy the stock, $c_1=20$ and $c_2=40$, so $U(c_1, c_2)=20+40=60$.
If she buys the stock, $c_1=8$, $c_2=60$, so $U(c_1, c_2) = 8+60=68$. Since 68>60, she should buy the stock.

ii) She should buy the stock.
If she does not buy the stock, she can maximize her utility by postponing all her consumption to period 2. This requires her to lend \$20 in period 1. For that, she will receive \$30 in period 2. So $U(c_1, c_2) = 0+(30+40) = 70$.
If she buys the stock, she can maximize her utility by lending \$8 in period 1 and receiving \$12 in period 2. So $U(c_1, c_2) = 0+(12+20+40) = 72$. Since 72>70, she should buy the stock.

Question 1.2

The ***certainty equivalent*** of a lottery is the amount of money you would have to be given with certainty to be just as well-off with that lottery. Suppose that your expected utility function over lotteries that give you an amount x if Event 1 happens and y if Event 1 does not happen is $EU = \pi\sqrt{x} + (1-\pi)\sqrt{y}$, where π is the probability that Event 1 happens and $1-\pi$ is the probability that Event 1 does not happen.

i) If $\pi = 0.5$, calculate the utility of a lottery that gives you \$10,000 if Event 1 happens and \$100 if Event 1 does not happen.

ii) If you were sure to receive \$4,900, what would your utility be?

iii) Calculate the certainty equivalent of receiving \$10,000 if Event 1 happens and \$100 if Event 1 does not happen.

Answer 1.2

i) $EU = \pi\sqrt{x} + (1-\pi)\sqrt{y} = 0.5 \times 100 + 0.5 \times 10 = 55.$

ii) $EU = \pi\sqrt{x} + (1-\pi)\sqrt{y} = 0.5\sqrt{4900} + 0.5\sqrt{4900} = \sqrt{4900} = 70.$

iii) Let the certainty equivalent be denoted by c. If I am indifferent between receiving the certainty equivalent and the lottery, it must be the case that $[\pi\sqrt{c} + (1-\pi)\sqrt{c}] = [\pi\sqrt{x} + (1-\pi)\sqrt{y}]$. Hence, $c = [\pi\sqrt{x} + (1-\pi)\sqrt{y}]^2$.

Now, $c = [\pi\sqrt{x} + (1-\pi)\sqrt{y}]^2 = [0.5\sqrt{10000} + (0.5)\sqrt{100}]^2 = 3025$.

Question 1.3

Suppose that you are a merchant in the ancient world. You have bought some goods from overseas and have been waiting a long time for your ship to arrive.

There is a 25% chance that it will arrive today. If it does arrive today, your wealth will be $1,600. If it does not come in today, it will never come and your wealth will be zero. Your utility function is $\sqrt{w}$, where w is wealth. What is the minimum price at which you should sell the rights to your ship?

Answer 1.3

First, note that the "ship" is a lottery in disguise. Now, if you sell your ship, say for $\$c$, it would not matter anymore whether the ship arrives or not. You enjoy $\$c$ for certain. You should only sell your ship if selling it and receiving $\$c$ for certain gives you an expected utility no less than your expected utility holding on to the ship (i.e., lottery). In other words, $\$c$ must be greater or equal to the certainty equivalent of the ship. Hence $0.25\sqrt{c} + 0.75\sqrt{c} \geq 0.25\sqrt{1600} + 0.75\sqrt{0}$. This implies that $c \geq [0.25\sqrt{1600}]^2 = 100$. The minimum price that you should sell is $100.