Supplementary Notes on Bond Prices and Yields

Current Yield

- The simplest measure of the yield on a bond is the current yield (or flat yield, interest yield, income yield or running yield). This is defined as:

$$y_c = \frac{Annual\ Coupon\ Payment}{Clean\ Price}$$

Yield to Maturity (YTM)

- It is the interest rate that makes the present value of the bond's payments equal to its price.
- It is the solution to (*T* is # of semester):

$$P = \sum_{t=1}^{T} \frac{F_t}{(1+YTM)^t}$$

- YTM is the IRR of cash-flows delivered by bonds.
- Each future cash-flow, F_t, is discounted using the same rate.
- IRR is an average discount rate assumed to be constant over the different maturities, so that the present value of the cashflows generated by the asset equals to the price of the asset now.
- YTM may easily be computed by trial-and-error (numerical methods e.g. using solver function on Excel).
- If coupons are paid semi-annually, the YTM calculated above is a semi-annual rate.
- To express the semi-annual rate computed from a semi-annual coupon paying bond to an annual basis, multiple the semi-annual YTM rate by 2. This is also known as the Bond Equivalent Yield (an annual percentage rate), and this is how the market usually express the YTM (in APR terms) for semi-annual coupon paying bonds.
- As seen on Slide 17 of Lecture 6, the bond equivalent yield (BEY) is expressed as an annual percentage rate (simple interest); hence it can differ from the effective annual yield (EAR), which accounts for the compound interest.

Spot Zero-Coupon (or Discount) Rate

- Spot Zero-Coupon (or Discount) Rate, $R_{0,t}$ is the annualized rate on a pure discount bond.

$$\frac{1}{(1+R_{0,t})^t} = B(0,t)$$

  - where *B(0,t)* is the market price at date 0 of a bond paying off $1 at date t, Thus, *B(0,t)* can be also used as a discount factor to get the present value (time=*0*) of an equivalent of $1 at time *t*.

- The spot rate is calculated by finding the discount rate that makes the present value (PV) of a zero-coupon bond equal to its price. That is, the YTM or IRR of a zero-coupon bond with maturity t.
- Spot rates term structure can show different interest rates for different year-to-maturity.
- Since each zero-coupon bond has only one cashflow, which is on maturity, the spot zero-coupon (discount) rate is thus very useful for us to infer what is the discount rate implied by the market for various maturities (that corresponds to the zero-coupon bond). Therefore, the spot zero-coupon rate is also known as the discount rate. This is used in the general pricing formula of a coupon bond, by discounting each corresponding cashflow as if it is a zero-coupon bond of the same maturity, as in Slide 36 of Lecture 6.

  - General pricing formula

$$P = \sum_{t=1}^{T} \frac{F_t}{(1+R_{0,t})^t} = \sum_{t=1}^{T} F_t B(0,t)$$

Forward Rates

- One may represent the term structure of interest rates as set of implicit forward rates.
- Consider two choices for a 2-year horizon:
  - Choice A: Buy 2-year zero
  - Choice B: Buy 1-year zero and rollover for 1 year
  - What yield from year 1 to year 2 will make you indifferent between the two choices?

$$1+F_{1,1} = \frac{(1+R_{0,2})^2}{(1+R_{0,1})}$$

- They are ‘implicit’ in the term structure.
- Rates that explain the relationship between spot rates of different maturity.

***Bond Par (or Swap) Yield**

- Recall that a par bond is a bond with a coupon identical to its yield to maturity.
- The bond's price is therefore equal to its principal.
- Then we define the par yield *c(n)* so that a n-year maturity fixed bond paying annually a coupon rate of *c(n)* with a $100 face value quotes at par
- Typically, the par yield curve is used to determine the coupon level of a bond issued at par

$$100 = \sum_{i=1}^{n} \frac{100c(n)}{(1+R_{0,i})^i} + \frac{100}{(1+R_{0,n})^n} \Rightarrow c(n) = \frac{1 - \frac{1}{(1+R_{0,n})^n}}{\sum_{i=1}^{n} \frac{1}{(1+R_{0,i})^i}}$$

- Par yields are useful for pricing other interest rate derivatives such as an interest rate swap (a forward contract in which one stream of future interest payments is exchanged for another based on a specified principal amount).

Term Structure and Yield Curves

- The financial markets usually express the term structure of interest rates using a variety of rates:

a. Spot Zero-Coupon (or Discount) Rate

The spot (or zero-coupon) yield curve is a plot of spot yields (or zero-coupon yields) against term to maturity.

b. Forward yield curve

The forward (or forward-forward) yield curve is a plot of forward yields against term to maturity.

c. *Bond Par (or Swap) Yield

The par (or swap) yield curve is a plot of the yield to maturity against term to maturity for bonds priced at par.

d. *Coupon yield curve

The coupon yield curve is a plot of the yield to maturity against term to maturity for a group of bonds with the same coupon.

*** extra not required in this course**