EC3333 Tutorial 7 Suggested Answers

1. The following table shows the yields to maturity of zero-coupon Treasury securities.

Term to Maturity (years)	Yield to Maturity (%)
1	3.5
2	4.5
3	5.0
4	5.5
5	6.0

a. Calculate the forward 1-year rate for year 2,3,4, and 5 respectively.

b. Describe the conditions under which the calculated forward 1-year rate would be an unbiased estimate of the 1-year spot rate of interest for that year.

c. Assume that the conditions in part b hold, what does the yield curve here imply?

d. Assume that a few months earlier, the forward 1-year rate of interest for that year had been significantly higher than it is now. What factors could account for the decline in the forward rate?

a.

$$(1+f_n) = \frac{(1+y_n)^n}{(1+y_{n-1})^{n-1}}$$

f_n = One-year forward rate for period n

y_n = Yield for a security with a maturity of n

$$(1+y_n)^n = (1+y_{n-1})^{n-1}(1+f_n)$$

f_2	5.510%
f_3	6.007%
f_4	7.014%
f_5	8.024%

b.

The conditions would be those that underlie the expectations theory of the term structure: the market is arbitrage free, and market participants are risk neutral and willing to substitute among maturities solely on the basis of yield differentials. This behavior would rule out liquidity or term premia relating to risk.

c.

If conditions in part b holds, the yield curve implies that the market expects to see higher and higher 1-year spot rate in future.

d.

Under the expectations hypothesis, lower implied forward rates would indicate lower expected future spot rates for the corresponding period. Since the lower expected future rates embodied in the term structure are nominal rates, either lower expected future real rates or lower expected future inflation rates would be consistent with the specified change in the observed (implied) forward rate.

2. A 9-year bond paying coupons annually has a yield to maturity of 10% and a duration of 7.194 years. If the market yield increases by 50 basis points (0.50%), what is the percentage change in the bond price?

$\frac{\Delta P}{P} = -D^* \times \Delta y$ where $D^* = \mathrm{D}/(1+y) =$ Modified duration

The percentage change in the bond's price is:

$$-\frac{\boldsymbol{D}}{\mathbf{1}+\boldsymbol{y}} \times \boldsymbol{\Delta y} = -\frac{\mathbf{7.194}}{\mathbf{1.10}} \times \mathbf{0.005} = -\mathbf{0.0327} = -\mathbf{3.27}\%, \text{ or a } \mathbf{3.27}\% \text{ decline}$$

3. Find the duration of a 6% coupon bond making *annual* coupon payments if it has 3 years until maturity and has a yield to maturity of 6%. What is the duration if the yield to maturity is 10%?

$$D = \sum_{t=1}^{T} \frac{PV(C_t)}{P} \times t$$

$$where\ PV(C_t) = \frac{C_t}{(1+y)^t} \text{ and } P = \sum_{t=1}^{T} \frac{C_t}{(1+y)^t}$$

a. YTM = 6%

(1) Time until Payment (Years)	(2) Cash Flow	(3) PV of CF (Discount Rate = 6%)	(4) Weight	(5) Column (1) × Column (4)
1	$ 60.00	$ 56.60	0.0566	0.0566
2	60.00	53.40	0.0534	0.1068
3	1,060.00	890.00	0.8900	2.6700
	Column sums	$1,000.00	1.0000	2.8334

Duration = 2.833 years

b. YTM = 10%

(1) Time until Payment (Years)	(2) Cash Flow	(3) PV of CF (Discount Rate = 10%)	(4) Weight	(5) Column (1) × Column (4)
1	$ 60.00	$ 54.55	0.0606	0.0606
2	60.00	49.59	0.0551	0.1102
3	1,060.00	796.39	0.8844	2.6532
	Column sums	$900.53	1.0000	2.8240

Duration = 2.824 years, which is less than the duration at the YTM of 6%.

4. Assume there are four default-free bonds with the following prices and future cash flows:

Bond	Price Today	Cash Flows Year 1	Cash Flows Year 2	Cash Flows Year 3
A	$934.58	1000	0	0
B	881.66	0	1000	0
C	1,118.21	100	100	1100
D	839.62	0	0	1000

Do these bonds present an arbitrage opportunity? If so, how would you take advantage of this opportunity? If not, why not?

To determine whether these bonds present an arbitrage opportunity, check whether the pricing is internally consistent. Calculate the spot rates implied by Bonds A, B, and D (the zero-coupon bonds), and use this to check Bond C. (You may alternatively compute the spot rates from Bonds A, B, and C, and check Bond D, or some other combination.)

$$934.58 = \frac{1000}{(1+YTM_1)} \Rightarrow YTM_1 = 7.0\%$$

$$881.66 = \frac{1000}{(1+YTM_2)^2} \Rightarrow YTM_2 = 6.5\%$$

$$839.62 = \frac{1000}{(1+YTM_3)^3} \Rightarrow YTM_3 = 6.0\%$$

Given the spot rates implied by Bonds A, B, and D, the price of Bond C should be

$$\frac{100}{(1+0.07)} + \frac{100}{(1+0.065)^2} + \frac{1100}{(1+0.06)^3} = \$1105.21$$

Its price really is $1118.21, so it is overpriced by $13 per bond. Yes, there is an arbitrage opportunity.

To take advantage of this opportunity, you want to (short) Sell Bond C (since it is overpriced). To match future cash flows, one strategy is to sell 10 Bond Cs (it is not the only effective strategy; any multiple of this strategy is also arbitrage). This complete strategy is summarized in the table below.

	Today	1 Year	2Years	3Years
Sell 10 Bond C	11,182.10	–1,000	–1,000	–11,000
Buy Bond A	–934.58	1,000	0	0
Buy Bond B	–881.66	0	1,000	0
Buy 11 Bond D	–9,235.82	0	0	11,000
Net Cash Flow	$130.04	0	0	0

Notice that your arbitrage profit equals 10 times the mispricing on each bond (subject to rounding error).