Government and Fiscal Policy in the Consumption-Savings Model

Chapter 7

OUTLINES

- CONSUMER (representative household):
  - Utility
  - Budget constraint
- GOVERNMENT:
  - Government spending and tax
  - Ricardian Equivalence concept
  - Government budget constraint
- Economy-wide resource frontier
- Math review (in order to solve the model): Lagrange Method
- Solve the model using Lagrange Method
- Effects of tax policy (How taxation affects National Savings, interest rate and consumption pattern)
  - Explaining Ricardian Equivalence in the model
- DISCUSSION ON TAXATION: Lumpsum vs Proportional

CONSUMER

Basics

- Notation
  - c_1: consumption in period 1
  - c_2: consumption in period 2
  - P_1: nominal price of consumption in period 1
  - P_2: nominal price of consumption in period 2

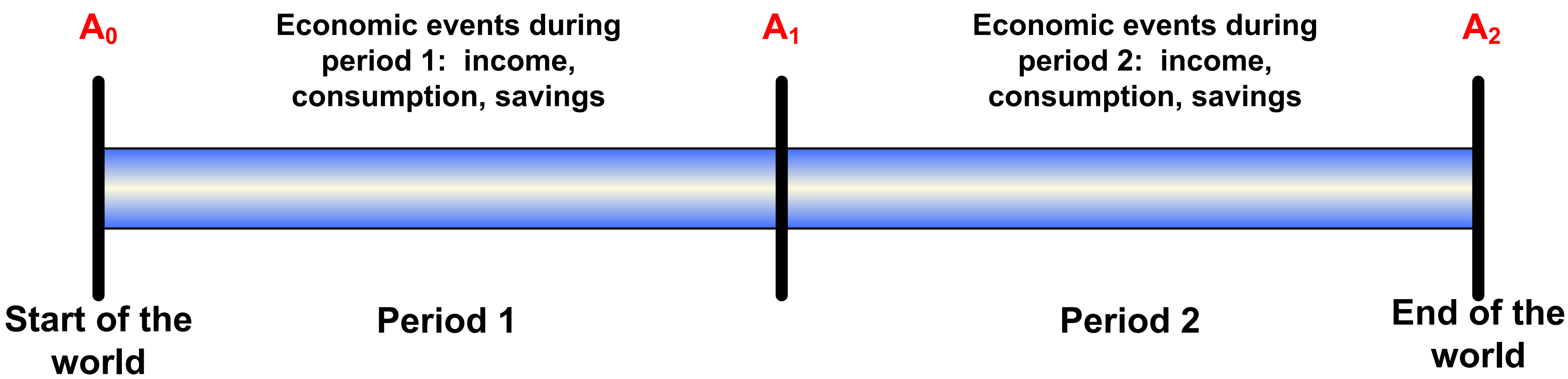
$$\pi_2 = \frac{P_2 - P_1}{P_1} \left(= \frac{P_2}{P_1} - 1 \right)$$

  - Y_1: nominal income in period 1 (“falls from the sky”)
  - Y_2: nominal income in period 2 (“falls from the sky”)
  - ...

Basics

- Notation
  - …
  - A_0: nominal wealth at the beginning of period 1/end of period 0
  - A_1: nominal wealth at the beginning of period 2/end of period 1
  - A_2: nominal wealth at the beginning of period 3/end of period 2

BASICS



Start of economic planning horizon

End of economic planning horizon

Basics

- Notation
  - ...
  - i: nominal interest rate between periods
  - r: real interest rate between periods
  - π_2: net inflation rate between period 1 and period 2
  - y_1: real income in period 1 ($= \frac{Y_1}{P_1}$)
  - y_2: real income in period 2 ($= \frac{Y_2}{P_2}$)
  - T_1: lump sum nominal tax in period 1
  - T_2: lump sum nominal tax in period 2
  - t_1: lump sum real tax in period 1
  - t_2: lump sum real tax in period 2

UTILITY FUNCTIONS

- Two-good case: $u(c_1, c_2)$, with $u_i(c_1, c_2) > 0$ and $u_{ii}(c_1, c_2) < 0$ for each of $i = 1,2$
  - Utility strictly increasing in each good individually (partial)
  - Diminishing marginal utility in each good individually

Easily extends to *N*-good case: $u(c_1, c_2, c_3, c_4,\ldots, c_N)$

UTILITY FUNCTIONS

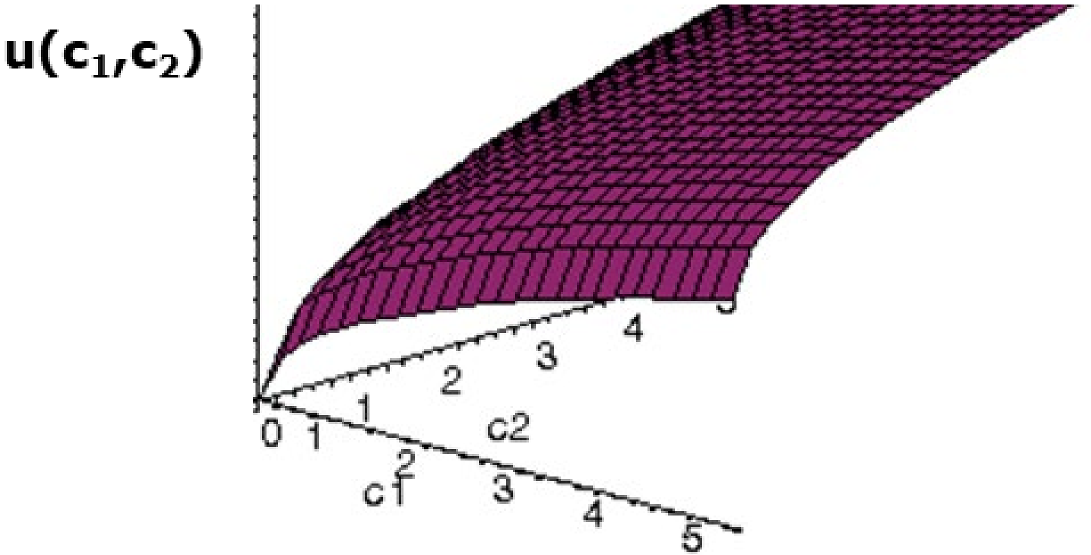
- ❑ **Two-good case**

Example:

$$u(c_1, c_2) = \ln c_1 + \ln c_2$$

or

$$u(c_1, c_2) = \sqrt{(c_1)} + \sqrt{(c_2)}$$



UTILITY FUNCTIONS

Example:

$$\boldsymbol{u}(\boldsymbol{c_1}, \boldsymbol{c_2}) = \ln \boldsymbol{c_1} + \ln \boldsymbol{c_2}$$

$$u_{\mathbf{1}}(\boldsymbol{c_1}, c_2) = \frac{\partial u(c_1, c_2)}{\partial c_1} = \frac{1}{c_1} > 0$$

$$u_{\mathbf{11}}(\boldsymbol{c_1}, c_2) = \frac{\partial\left(u_{\mathbf{1}}(\boldsymbol{c_1}, c_2)\right)}{\partial \boldsymbol{c_1}}$$

$$= \frac{\partial\left(\frac{\boldsymbol{\partial u(c_1, c_2)}}{\boldsymbol{\partial c_1}}\right)}{\partial c_1} = \frac{\partial\left(\frac{\mathbf{1}}{\boldsymbol{c_1}}\right)}{\partial c_1} = -\frac{1}{c_1^2} < 0$$

Utility Functions

Example: $\boldsymbol{u}(\boldsymbol{c_1}, \boldsymbol{c_2}) = \ln \boldsymbol{c_1} + \ln \boldsymbol{c_2}$

$$u_2(c_1, \boldsymbol{c_2}) = \frac{\partial u(c_1, c_2)}{\partial c_2} = \frac{1}{c_2} > 0$$

$$u_{\boldsymbol{22}}(c_1, \boldsymbol{c_2}) = \frac{\partial\left(u_{\boldsymbol{2}}(c_1, \boldsymbol{c_2})\right)}{\partial \boldsymbol{c_2}}$$

$$= \frac{\partial\left(\frac{\boldsymbol{\partial u(c_1, c_2)}}{\boldsymbol{\partial c_2}}\right)}{\partial c_2} = \frac{\partial\left(\frac{\boldsymbol{1}}{\boldsymbol{c_2}}\right)}{\partial c_2} = -\frac{1}{c_2^2} < 0$$

Utility Functions

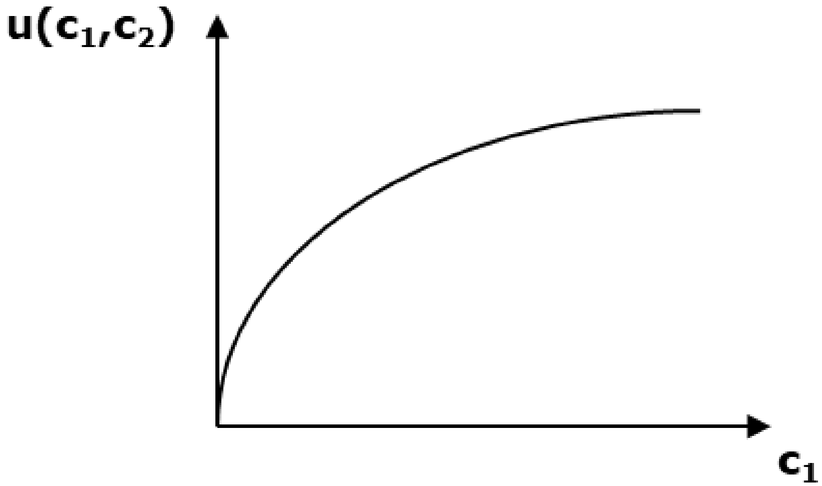
Example: $u(c_1, c_2) = \ln c_1 + \ln c_2$

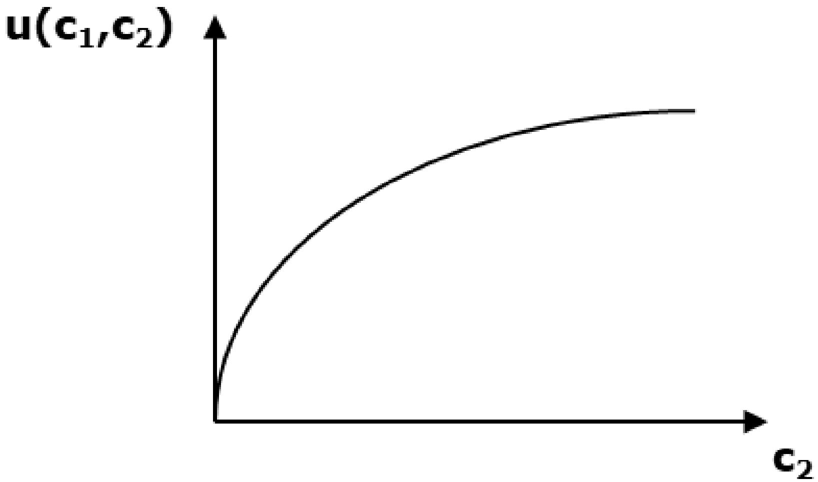
$$\left.\begin{array}{l} u_2(c_1, c_2) > 0 \\ u_1(c_1, c_2) > 0 \end{array}\right\} \rightarrow u_i(c_1, c_2) > 0 \quad \text{for i=1,2}$$

$$\left.\begin{array}{l} u_{22}(c_1, c_2) < 0 \\ u_{11}(c_1, c_2) < 0 \end{array}\right\} \rightarrow u_{ii}(c_1, c_2) < 0 \quad \text{for i=1,2}$$

UTILITY FUNCTIONS

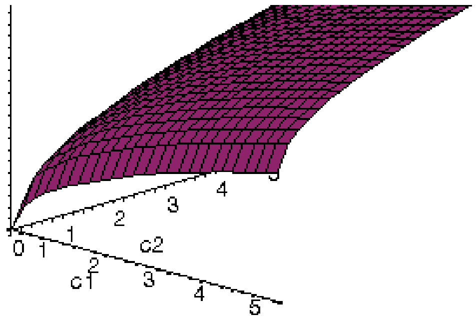
Viewed in good-by-good space



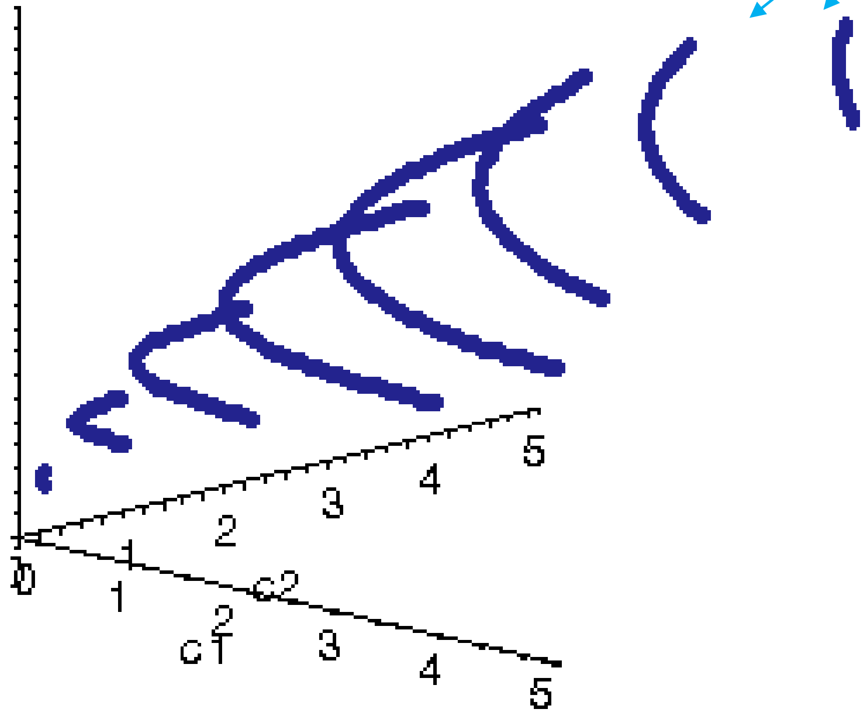


UTILITY FUNCTIONS

Alternative views



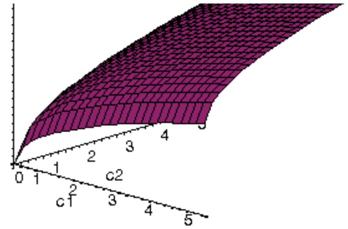
Emphasizing the contours

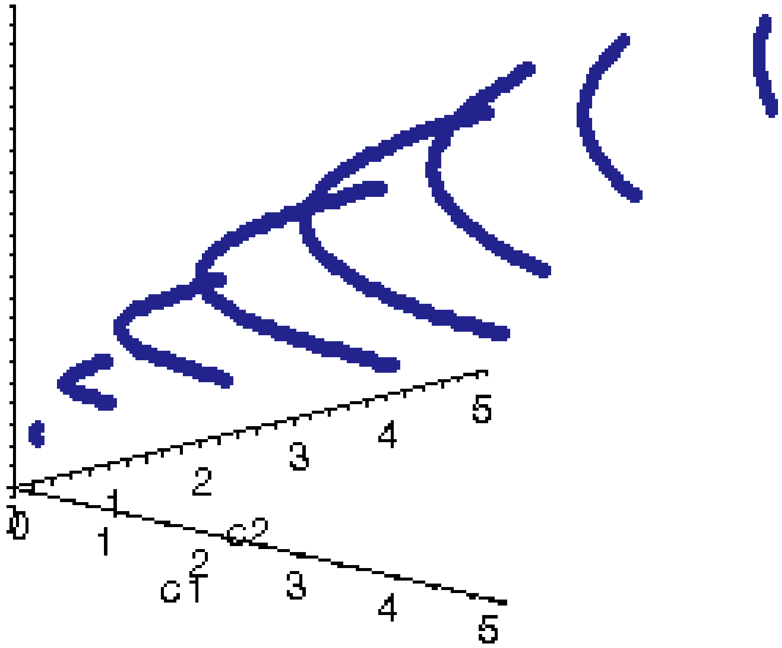


Indifference Curve: the set of all consumption bundles that deliver a particular level of utility/happiness

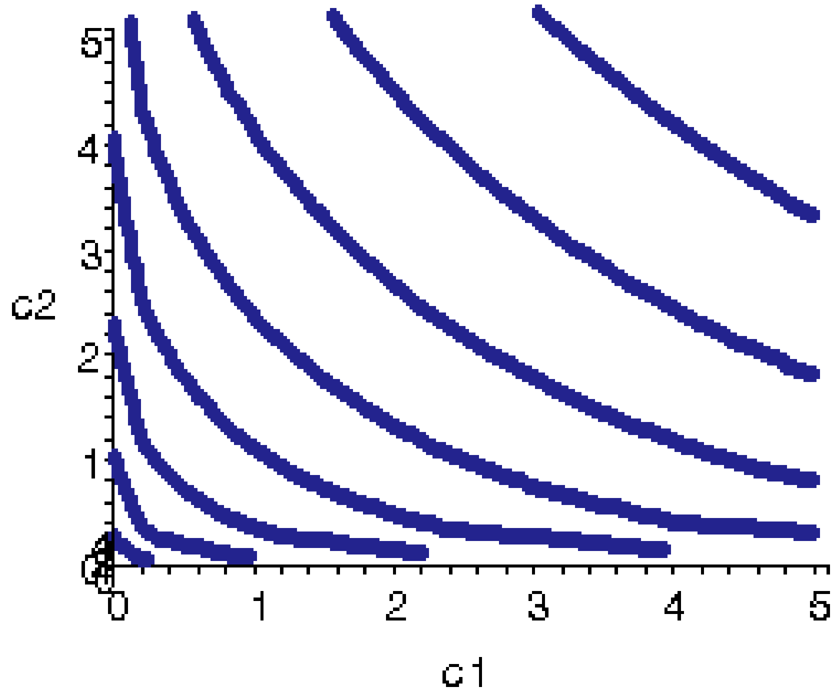
UTILITY FUNCTIONS

Alternative views





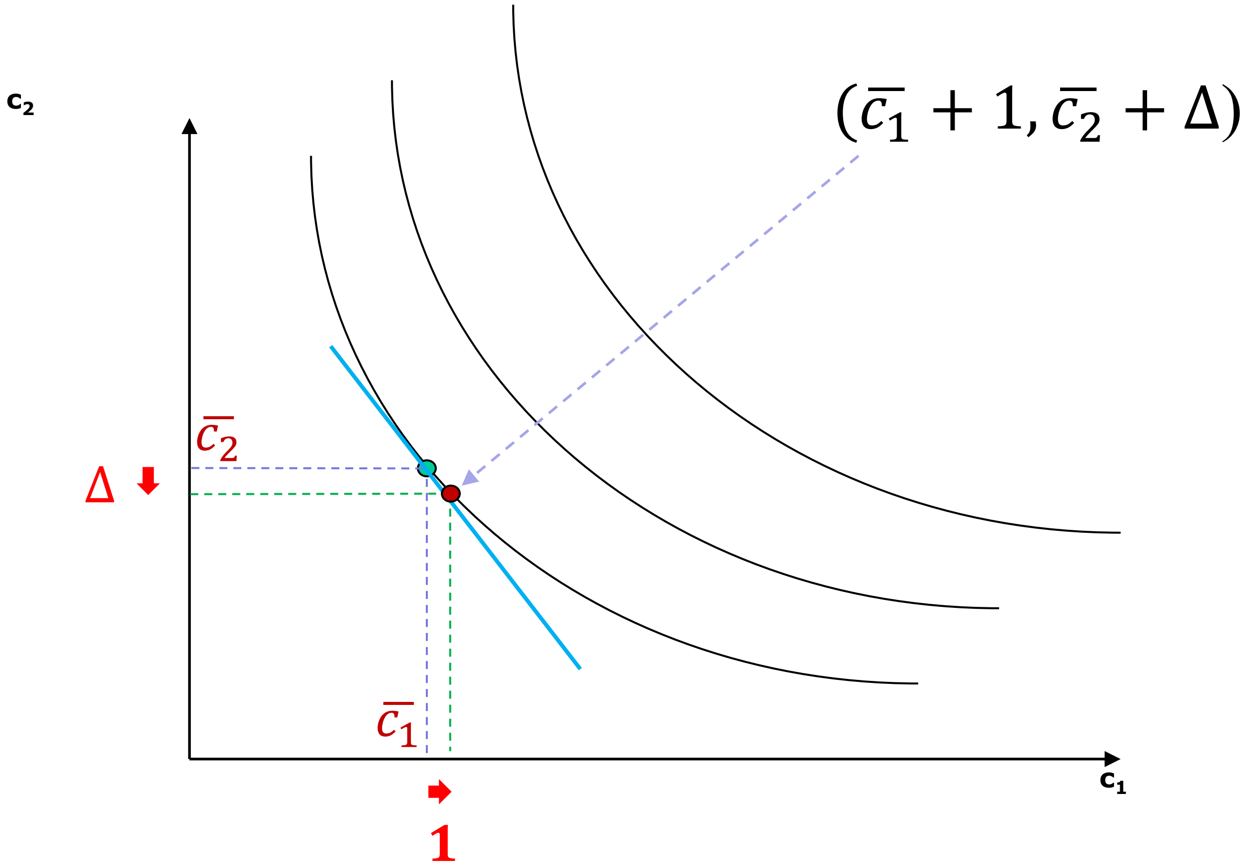
Top-down view of contour



Utility Functions

- Marginal Rate of Substitution (MRS)
  - Maximum quantity of one good consumer is willing to give up to obtain one extra unit of the other good
  - Graphically, the (negative of the) slope of an indifference curve
  - MRS is itself a function of c_1 and c_2 (i.e., $MRS(c_1, c_2)$)

UTILITY FUNCTIONS



- ❑ MRS is itself a function of c_1 and c_2 (i.e., $MRS(c_1, c_2)$)

Utility Functions

- MRS equals ratio of marginal utilities

  - $$MRS(c_1, c_2) = \frac{u_1(c_1, c_2)}{u_2(c_1, c_2)}$$

$$\frac{u_1(c_1,c_2)}{u_2(c_1,c_2)} = \frac{\frac{\partial u(c_1,c_2)}{\partial c_1}}{\frac{\partial u(c_1,c_2)}{\partial c_2}} = \frac{\cancel{\partial u(c_1,c_2)}}{\partial c_1} \times \frac{\partial c_2}{\cancel{\partial u(c_1,c_2)}} = \frac{\partial c_2}{\partial c_1}$$

  - Using Implicit Function Theorem

CONSUMER BUDGET CONSTRAINT(S)

- Introduce tax payments into consumer side of framework
  - To get budget constraints in real terms by multiplying by dividing both sides by *P*
  - Period-1 budget constraint in nominal terms

$$P_1 c_1 + P_1 \cdot t_1 + A_1 - A_0 = Y_1 + i \cdot A_0$$

  - Period-2 budget constraint in nominal terms

$$P_2 c_2 + P_2 \cdot t_2 + A_2 - A_1 = Y_2 + i \cdot A_1$$

Period-1 budget constraint in real terms

I am not testing this slide

$$P_1 c_1 + P_1 \cdot t_1 + A_1 - A_0 = Y_1 + i \cdot A_0$$

$$c_1 + t_1 + \frac{A_1}{P_1} - \frac{A_0}{P_1} = \frac{Y_1}{P_1} + i \cdot \frac{A_0}{P_1}$$

$$c_1 + t_1 + a_1 = y_1 + (1+i) \cdot \frac{A_0}{P_0} \cdot \frac{P_0}{P_1}$$

Here: assume

$$\frac{P_1 - P_0}{P_0} = \pi$$

$$c_1 + t_1 + a_1 = y_1 + \frac{A_0}{P_0} \cdot \frac{(1+i)}{1+\pi}$$

Fisher equation

$$\frac{1+i}{1+\pi} = 1 + r$$

$$c_1 + t_1 + a_1 = y_1 + a_0 \cdot (1+r)$$

or

$$\boxed{c_1 + t_1 + a_1 - a_0 = y_1 + r \cdot a_0}$$

Consumer Budget Constraint(s)

- Period-1 budget constraint in real terms

$$\boxed{c_1 + t_1 + a_1 - a_0 = y_1 + r \cdot a_0}$$

- Period-2 budget constraint in real terms

$$\boxed{c_2 + t_2 + a_2 - a_1 = y_2 + r \cdot a_1}$$

CONSUMER BUDGET CONSTRAINT(S)

- Combine into **lifetime budget constraint (LBC)**
  - Solve period-2 budget constraint for a_1…
  - …and substitute into period-1 budget constraint

$$\underbrace{c_1 + \frac{c_2}{1+r}}_{\text{Present discounted value (PDV) of all lifetime expenditure}} = \underbrace{y_1 - t_1 + \frac{y_2 - t_2}{1+r} + (1+r)a_0}_{\text{Present discounted value (PDV) of all lifetime disposable income (i.e., after-tax income)}}$$

Combine into lifetime budget constraint (LBC)

- Solve period-2 budget constraint for a_1…
  - …and substitute into period-1 budget constraint

$$\boxed{c_1 + t_1 + a_1 - a_0 = y_1 + r \cdot a_0}$$

$$\Downarrow$$

$$a_1 = y_1 + (1 + r) \cdot a_0 - c_1 - t_1 \qquad (1)$$

$$\boxed{c_2 + t_2 + a_2 - a_1 = y_2 + r \cdot a_1}$$

$$\Downarrow$$

$$c_2 + t_2 + a_2 = y_2 + (1 + r) \cdot a_1 \qquad (2)$$

Combine into lifetime budget constraint (LBC)

- Subtitute the expression of a_1 in (1) into a_1 in (2)

$$c_2 + t_2 + a_2 = y_2 + (1+r)\cdot \underbrace{(y_1 + (1+r)\cdot a_0 - c_1 - t_1)}_{a_1}$$

$$\frac{c_2}{1+r} + \frac{t_2}{1+r} + \frac{a_2}{1+r} = \frac{y_2}{1+r} + y_1 + (1+r)\cdot a_0 - c_1 - t_1$$

$$c_1 + \frac{c_2}{1+r} = y_1 - t_1 + \frac{y_2}{1+r} + (1+r)\cdot a_0 - \frac{t_2}{1+r} - \frac{a_2}{1+r}$$

$Assume\ a_2 = 0$

$$\boxed{c_1 + \frac{c_2}{1+r} = y_1 - t_1 + \frac{y_2 - t_2}{1+r} + (1+r)\cdot a_0}$$

GOVERNMENT

Government purchases —1

- Government purchases of goods and services are an important source of demand
  - Transfer payments are not considered as government purchase
  - It is just a part of government SPENDING which also include government transfers or paying debts.
- Government purchases can be:
  - A source of short-run fluctuations
  - An instrument to reduce fluctuations

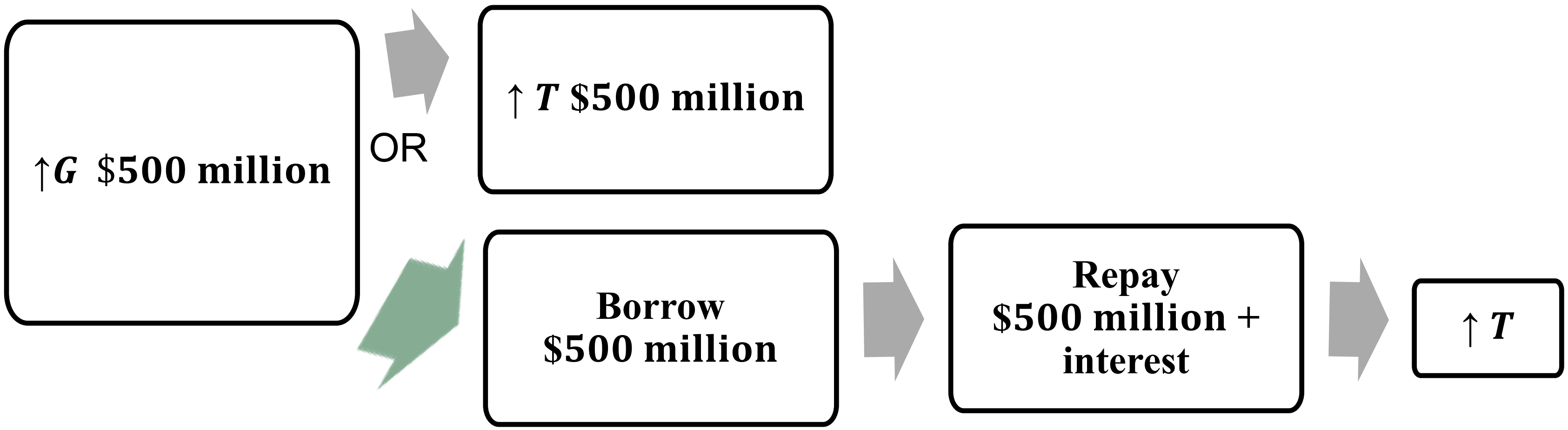
Government Purchases—2

- Discretionary fiscal policy
  - Purchases of new goods or services
    - American Recovery and Reinvestment Act of 2009
    - Workfare training support (Singapore)
    - "Government purchases" is just a part of fiscal policy
  - Tax rate changes
    - Investment tax credit of 1961 (US)
    - Economic Growth and Tax Relief Reconciliation Act of 2001 (US)
    - Corporate Income Tax (CIT) Rebate, Property Tax Rebate for non-Residential Properties … (Singapore, 2020)

Ricardian Equivalence

- According to Ricardian equivalence:
  - The timing of tax changes does not matter for consumer behavior
  - The present value of government tax collection determines behavior
- Consider an example:
  - Suppose Congress decides to hire more teachers, increasing government purchases by $500 million

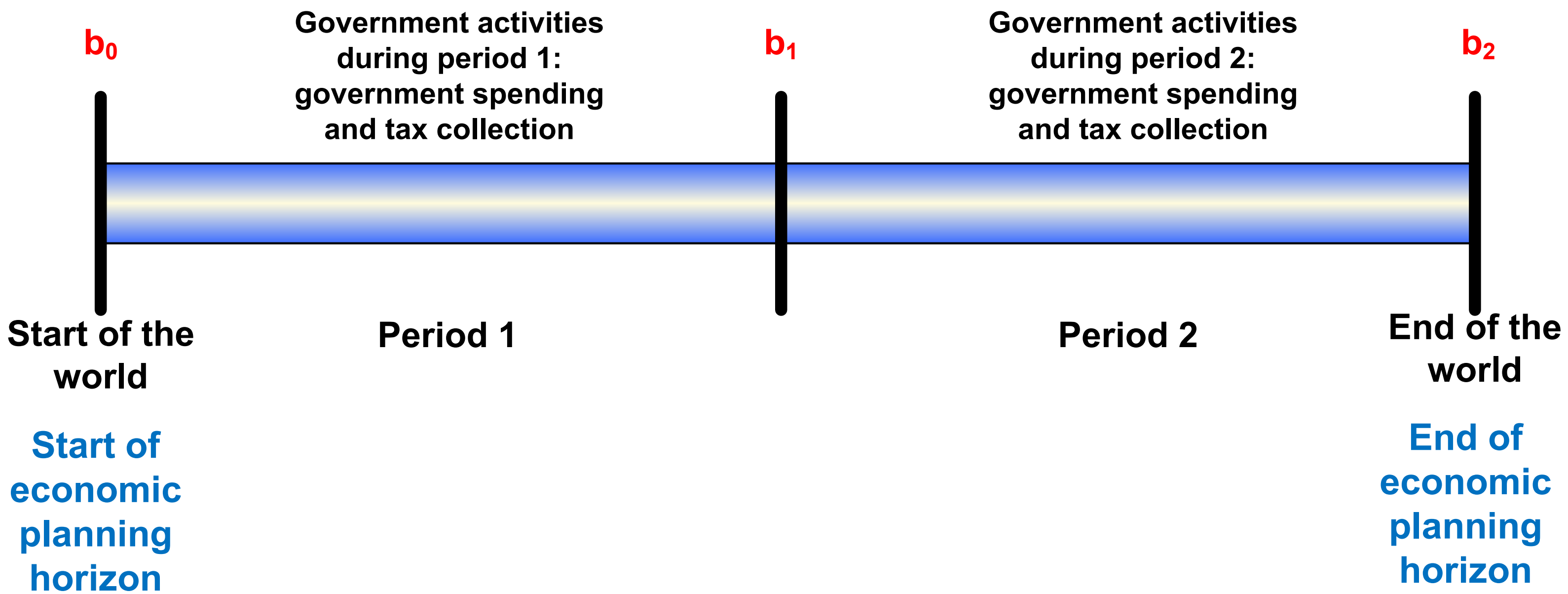
Ricardian Equivalence



A Dynamic Model of the Government

- So far only consumers in our two-period framework
- Introduce government in very simple form
  - Exists for both periods
  - Has spending in each period it needs to finance – can be financed via
    - Taxes
    - Issuing government debt/assets

A Dynamic Model of the Government



A Dynamic Model of the Government

- Notation
  - g_1: real government spending in period 1
  - g_2: real government spending in period 2
  - b_0: government asset position at beginning of period 1/end of period 0
  - b_1: government asset position at beginning of period 2/end of period 1
  - b_2: government asset position at beginning of period 3/end of period 2

A Dynamic Model of the Government

- Economic activities/actions described by period budget constraints
  - Period-1 government budget constraint

$$\underbrace{g_1 + b_1}_{\text{Total expenditure in period 1}} = \underbrace{(1+r)b_0 + t_1}_{\text{Total income in period 1}}$$

Total expenditure in period 1: period-1 spending + wealth to *carry into period* 2

Total income in period 1: period-1 tax collections + income from wealth *carried into period* 1 (inclusive of interest)

$$g_1 + \underbrace{b_1 - b_0}_{\text{Savings; Change in asset/debt}} = t_1 + rb_0$$

Savings; Change in asset/debt

Asset income during period 1

A Dynamic Model of the Government

- Period-2 government budget constraint

$$\underbrace{g_2 + b_2}_{\text{Total expenditure in period 2}} = \underbrace{(1+r)b_1 + t_2}_{\text{Total income in period 2}}$$

Total expenditure in period 2: period-2 spending + wealth to *carry into period* 3

Total income in period 2: period-2 tax collections + income from wealth *carried into period* 2 (inclusive of interest)

$$g_2 + \underbrace{b_2 - b_1}_{\text{Change in asset/debt}} = t_2 + rb_1$$

Change in asset/debt

Asset income during period 2 (a flow)

A Dynamic Model of the Government

- Definition: A government's savings during a given period is the ***change*** in its wealth during that period
  - "Fiscal surplus" if government savings is positive
  - "Fiscal deficit" if government savings is negative

Government Budget Constraint(s)

- Adopt a lifetime view of the budget constraint(s)
  - All analysis conducted from perspective of beginning of period 1
  - Period-1 government budget constraint

$$g_1 + b_1 = t_1 + (1+r)b_0$$

Asset position at end of period 1/beginning of period 2 the key link

  - Period-2 government budget constraint

Assume = 0 (no defaults)

$$g_2 + b_2 = t_2 + (1+r)b_1$$

Government Budget Constraint(s)

- Combine into lifetime budget constraint (LBC)
  - Solve period-2 budget constraint for b_1…
  - …and substitute into period-1 budget constraint

IMPORTANT: Government must balance budget over its *lifetime*, not necessarily in each period

$$\underbrace{g_1 + \frac{g_2}{1+r}}_{\text{Present discounted value (PDV) of all lifetime government expenditure}} = \underbrace{t_1 + \frac{t_2}{1+r} + (1+r)b_0}_{\text{Present discounted value (PDV) of all lifetime government income}}$$

For graphical simplicity, will often assume $b_0 = 0$ (i.e., government begins life with zero net wealth).

Note this is a *different* assumption than $b_2 = 0$.

ECONOMY-WIDE RESOURCE FRONTIER

Economy-Wide Resource Frontier

- Consumer lifetime budget constraint

$$c_1 + \frac{c_2}{1+r} = y_1 - t_1 + \frac{y_2 - t_2}{1+r} + (1+r)a_0$$

- Government lifetime budget constraint

$$g_1 + \frac{g_2}{1+r} = t_1 + \frac{t_2}{1+r} + (1+r)b_0$$

ECONOMY-WIDE RESOURCE FRONTIER (Derivation)

$$c_1 + \frac{c_2}{1+r} = y_1 - t_1 + \frac{y_2 - t_2}{1+r} + (1+r)a_0$$

$$c_1 + \frac{c_2}{1+r} = y_1 + \frac{y_2}{1+r} - \left(t_1 + \frac{t_2}{1+r}\right) + (1+r)a_0$$

Since $g_1 + \frac{g_2}{1+r} = t_1 + \frac{t_2}{1+r} + (1+r)b_0$, we have:

$$c_1 + \frac{c_2}{1+r} = y_1 + \frac{y_2}{1+r} - \left(g_1 + \frac{g_2}{1+r} - (1+r)b_0\right) + (1+r)a_0$$

Suppose = 0

$$c_1 + \frac{c_2}{1+r} = y_1 - g_1 + \frac{y_2 - g_2}{1+r} + (1+r)(a_0 + b_0)$$

ECONOMY-WIDE RESOURCE FRONTIER

- Summing the two yields economy-wide resource frontier
  - The GDP accounting equation in two-period form

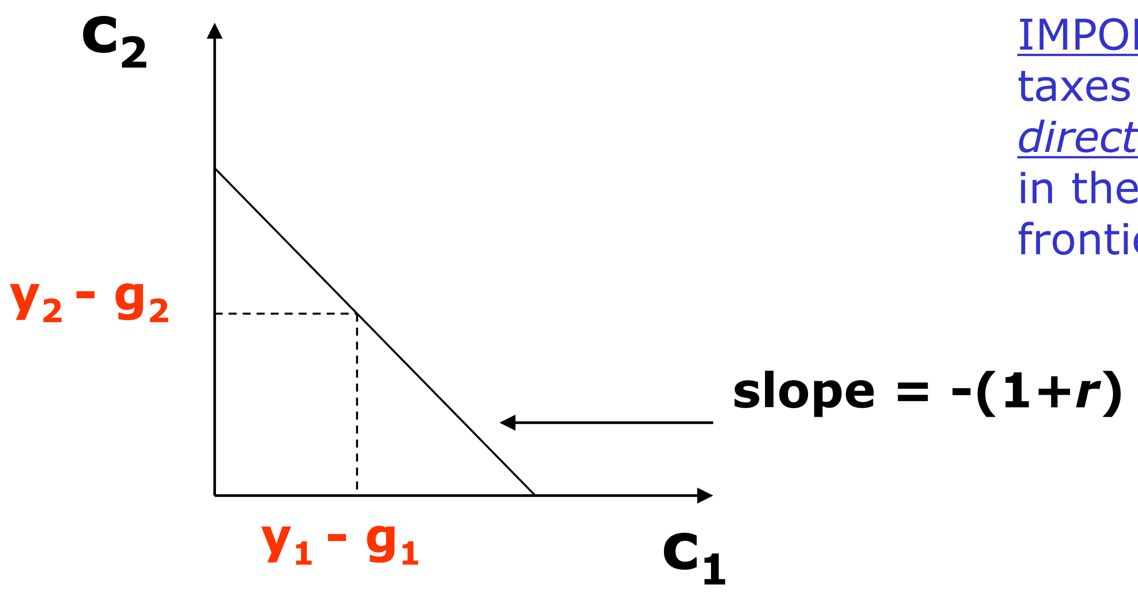
Suppose = 0 for graphical simplicity

$$c_1 + \frac{c_2}{1+r} = y_1 - g_1 + \frac{y_2 - g_2}{1+r} + (1+r)\cancel{(a_0 + b_0)}$$

$$c_1 + \frac{c_2}{1+r} = y_1 - g_1 + \frac{y_2 - g_2}{1+r}$$

$$\boxed{\boldsymbol{c_2 = \left(y_1 - g_1 + \frac{y_2 - g_2}{1+r}\right)(1+r) - (1+r)c_1}}$$

ECONOMY-WIDE RESOURCE FRONTIER



Resource Frontier

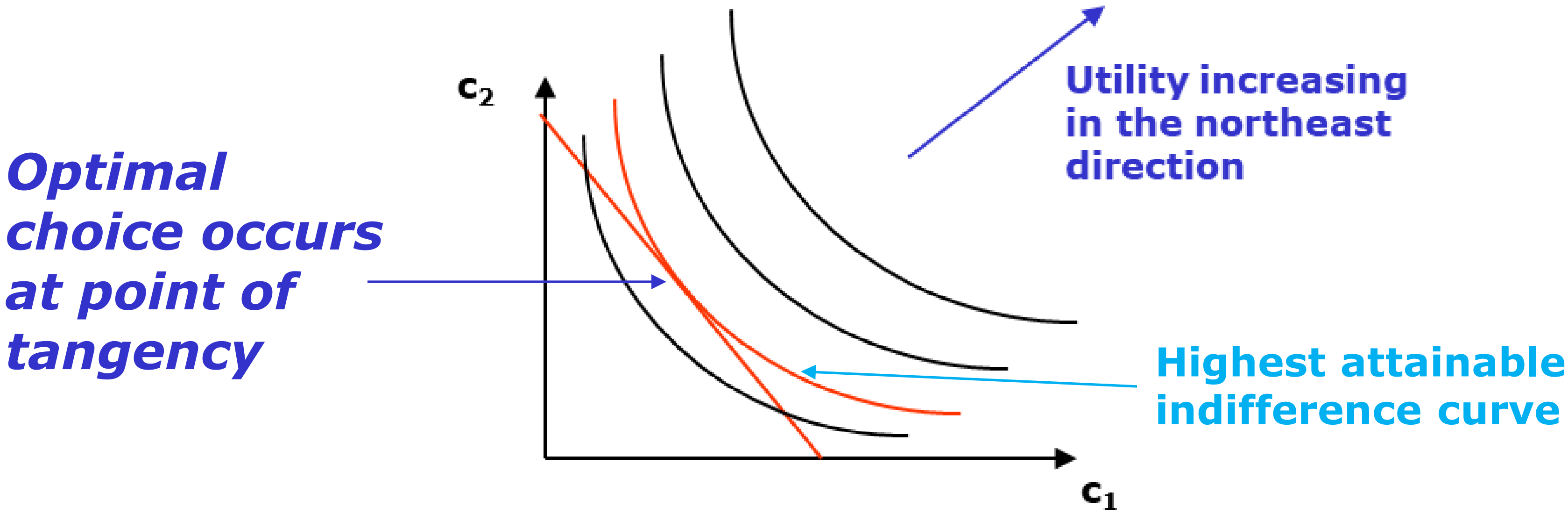
IMPORTANT: taxes do not *directly* appear in the resource frontier…

THEOREM (intermediate micro): If taxes are *lump-sum*, then consumer optimal choices can be analyzed using *either* the consumer LBC *or* the economy-wide resource frontier (superimpose indifference map), and either approach will yield the same predictions.

An important theoretical result for the analysis of tax policy.

CONSUMER OPTIMIZATION (Graph solution)

- How to optimally allocate *the PDV disposable income*, $y_1 - g_1 + \frac{y_2 - g_2}{1+r}$, across the two goods c_1 and c_2?



CONSUMER OPTIMIZATION (Graph solution)

OPTIMALITY CONDITION:

At the optimal choice,

MRS = slope of frontier line

ratio of marginal utilities = price ratio

MATH REVIEW:
LAGRANGE METHOD

LAGRANGE METHOD

- Consumer optimization a constrained optimization problem
  - Maximize some function (economic application: utility function)…
  - …taking into account some restriction on the objects to be maximized over (economic application: budget constraint/resource frontier)

- Lagrange Method: ***mathematical tool*** to solve constrained optimization problems

LAGRANGE METHOD (math solution for general case)

Original setting

Max Objective Function subject to: $\begin{cases} Constraint\ 1 = 0 \\ Constraint\ 2 = 0 \\ \quad\quad\ldots \\ Constraint\ n = 0 \end{cases}$

Set up another function

$$L(\ldots) = \text{Objective Function} + \lambda_1(Constraint\ 1) + \lambda_2(Constraint\ 2) + \ldots + \lambda_n(Constraint\ n)$$

$\lambda_1, \lambda_2 \ldots \lambda_n$ are the Lagrangian multipliers. They are created as the $L(\ldots)$ function is set up.

LAGRANGE METHOD (math solution for 2-constraint case)

Lagrange (the mathematician) proved a very useful result:

The solutions to the
maximization of $L(\ldots)$
are also the solutions
to the original setting.

LAGRANGE METHOD (math solution for 2-constraint case)

Example:

Max $f(x, y)$

subject to: $\begin{cases} g(x, y) = 0 \\ h(x, y) = 0 \end{cases}$

Two constraints

STEP 1: Set up the Lagrangian function for this problem, we have:

$$L(x, y, \lambda_1, \lambda_2) = \underbrace{f(x, y)}_{\text{obj. func}} + \lambda_1 \cdot \underbrace{g(x, y)}_{\text{constraint 1}} + \lambda_2 \cdot \underbrace{h(x, y)}_{\text{constraint 2}}$$

Function with 4 variables

LAGRANGE METHOD (math solution for 2-constraint case)

STEP 2: Work out FOCs e do the usual FOCs:

$$\frac{\partial L(x, y, \lambda_1, \lambda_2)}{\partial x} = 0$$

$$\frac{\partial L(x, y, \lambda_1, \lambda_2)}{\partial y} = 0$$

$$\frac{\partial L(x, y, \lambda_1, \lambda_2)}{\partial \lambda_1} = 0$$

$$\frac{\partial L(x, y, \lambda_1, \lambda_2)}{\partial \lambda_2} = 0$$

STEP 3: Solve for the choice variables

$$\boxed{x^\star, y^\star, \lambda_1^\star, \lambda_2^\star}$$

Solution to

Max $L(x, y, \lambda_1, \lambda_2)$

LAGRANGE METHOD (math solution for 2-constraint case)

Then: $\boxed{\underbrace{x^{\star}, y^{\star}}, \lambda_1^{\star}, \lambda_2^{\star}}$

*$x^{\star}, y^{\star}$ are **<u>also</u>** the solution to the original maximization problem, **<u>which is:</u>***

$$\text{Max } f(x, y)$$

$$\text{subject to: } \begin{cases} g(x, y) = 0 \\ h(x, y) = 0 \end{cases}$$

SOLVE THE MODEL USING
LAGRANGE METHOD

LAGRANGE ANALYSIS (applied to current problem)

- Consumer maximization problem:

Max $U(c_1, c_2)$ *subject to:*

$$\boldsymbol{c_1 = y_1 - g_1 + \frac{y_2 - g_2}{1+r} - \frac{1}{1+r} c_2}$$

- *So* $\boldsymbol{U(c_1, c_2)}$ *is the objective function; and*
- $\mathbf{c_1 = y_1 - g_1 + \frac{y_2 - g_2}{1+r} - \frac{1}{1+r} c_2}$ *is the constraint*

LAGRANGE ANALYSIS (applied to current problem)

- Step 1: Set up the Lagrangian function for this problem

$$\boxed{L(c_1, c_2, \lambda) = U(c_1, c_2) + \lambda\left(y_1 - g_1 + \frac{y_2 - g_2}{1+r} - \frac{1}{1+r}c_2 - c_1\right)}$$

Note your constraint was written as $c_1 = y_1 - g_1 + \frac{y_2 - g_2}{1+r} - \frac{1}{1+r}c_2$

But you need to bring all terms to one side (zero on the other):

$$\underbrace{y_1 - g_1 + \frac{y_2 - g_2}{1+r} - \frac{1}{1+r}c_2 - c_1}_{\text{This side will go into the Lagrangian function}} = \mathbf{0}$$

LAGRANGE ANALYSIS (applied to current problem)

- Step 2: Solve for the maximization of $L(c_1, c_2, \lambda)$

$$\frac{\partial L(c_1, c_2, \lambda)}{\partial c_1} = U_1(c_1, c_2) - \lambda = 0 \qquad (1)$$

$$\frac{\partial L(c_1, c_2, \lambda)}{\partial c_2} = U_2(c_1, c_2) - \frac{\lambda}{1+r} = 0 \qquad (2)$$

$$\frac{\partial L(c_1, c_2, \lambda)}{\partial \lambda} = y_1 - g_1 + \frac{y_2 - g_2}{1+r} - \frac{1}{1+r} c_2 - c_1 = \mathbf{0} \qquad (3)$$

LAGRANGE ANALYSIS (applied to current problem)

- Step 3: Solve (1), (2) and (3) for c_1, c_2 and λ. But here we are not given a specific function for $U(c_1, c_2)$ so we cannot solve for them specifically. However, we can also derive the optimality condition which was graphically shown. From (1) and (2):

$$U_1(c_1, c_2) - \lambda = 0 \qquad (1)$$

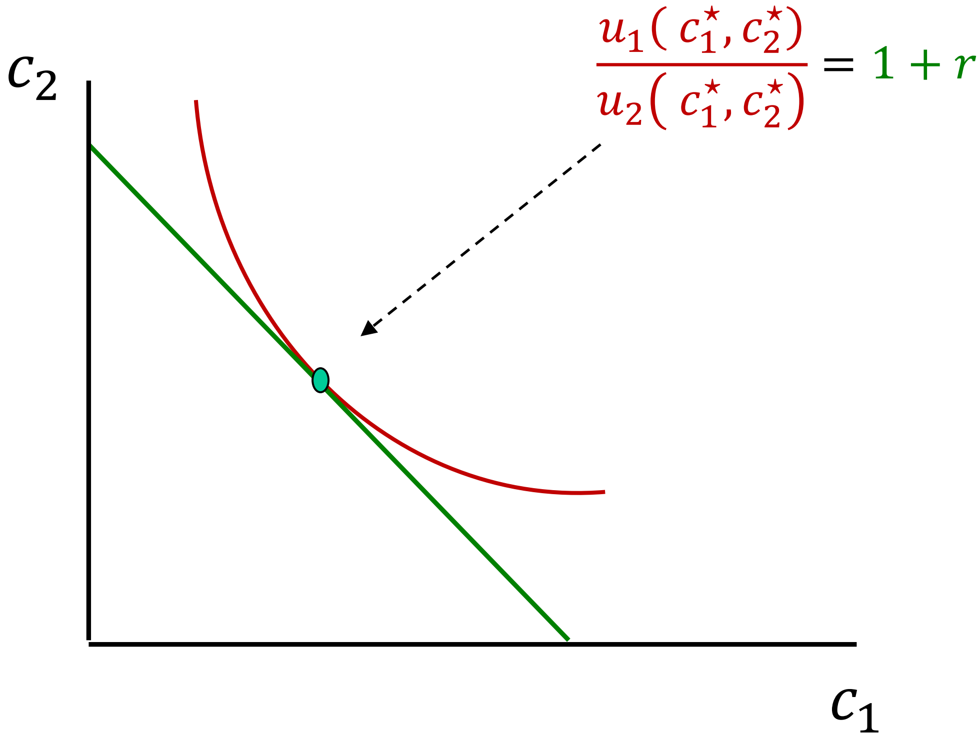
$$U_2(c_1, c_2) - \frac{\lambda}{1+r} = 0 \qquad (2)$$

We have:

$$\frac{U_1(c_1, c_2)}{U_2(c_1, c_2)} = \frac{1+r}{1} \qquad (2)$$

Left-hand side is the MRS (slope of the indifference curve) and right hand side is the slope of budget constraint

LAGRANGE ANALYSIS (applied to current problem)



i.e., MRS = price ratio

LAGRANGE ANALYSIS (applied to current problem)

- A specific function for $U(c_1, c_2)$:
  - Suppose that $U(c_1, c_2) = \ln(c_1) + \ln c_2$
- *Then:*

$$U_1(c_1, c_2) = \frac{1}{c_1}$$

$$U_2(c_1, c_2) = \frac{1}{c_2}$$

So:

$$\frac{U_1(c_1, c_2)}{U_2(c_1, c_2)} = \frac{\frac{1}{c_1}}{\frac{1}{c_2}} = \frac{c_2}{c_1} = 1 + r$$

LAGRANGE ANALYSIS (applied to current problem)

$$\frac{c_2}{c_1} = 1 + r \Rightarrow c_2 = (1 + r)c_1 \qquad (4)$$

Sub (4) into (3):

$$y_1 - g_1 + \frac{y_2 - g_2}{1 + r} - \frac{1}{1 + r}\overbrace{(1 + r)c_1}^{c_2} - c_1 = 0$$

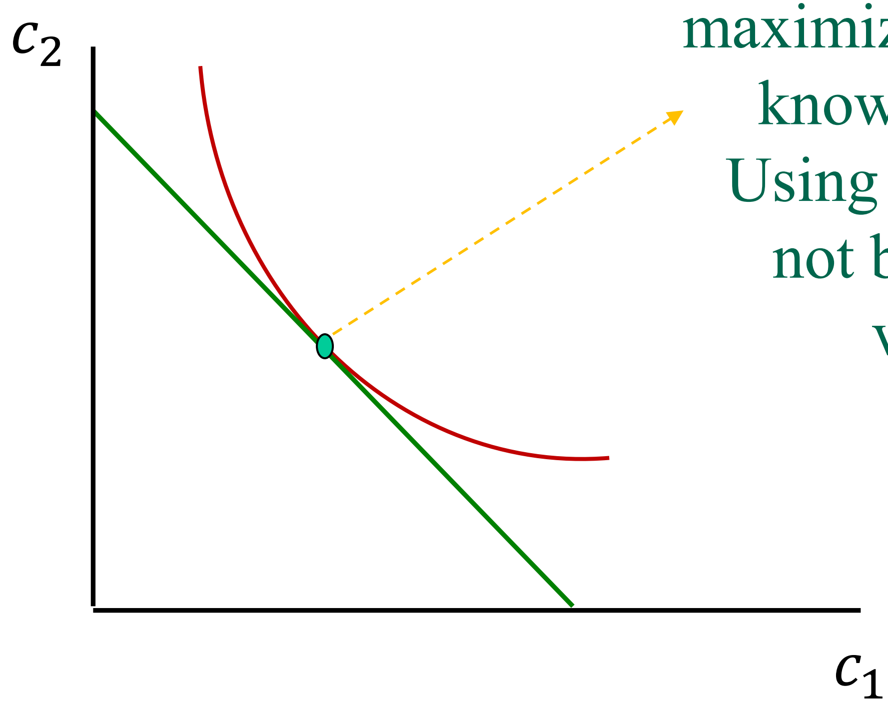
$$y_1 - g_1 + \frac{y_2 - g_2}{1 + r} - 2 \cdot c_1 = 0$$

$$\Rightarrow c_1 = \frac{1}{2}\left(y_1 - g_1 + \frac{y_2 - g_2}{1 + r}\right)$$

$$\Rightarrow c_2 = \frac{1 + r}{2}\left(y_1 - g_1 + \frac{y_2 - g_2}{1 + r}\right)$$

We can confidently claim that c_1 and c_2 worked out here are the solutions to the consumer maximization problem

LAGRANGE ANALYSIS (applied to current problem)



With a specific function, now we solve the maximization problem and know what this point is. Using just graph, we will not be able to know the values of c_1 and c_2.

EFFECTS OF TAX POLICY

NATIONAL SAVINGS

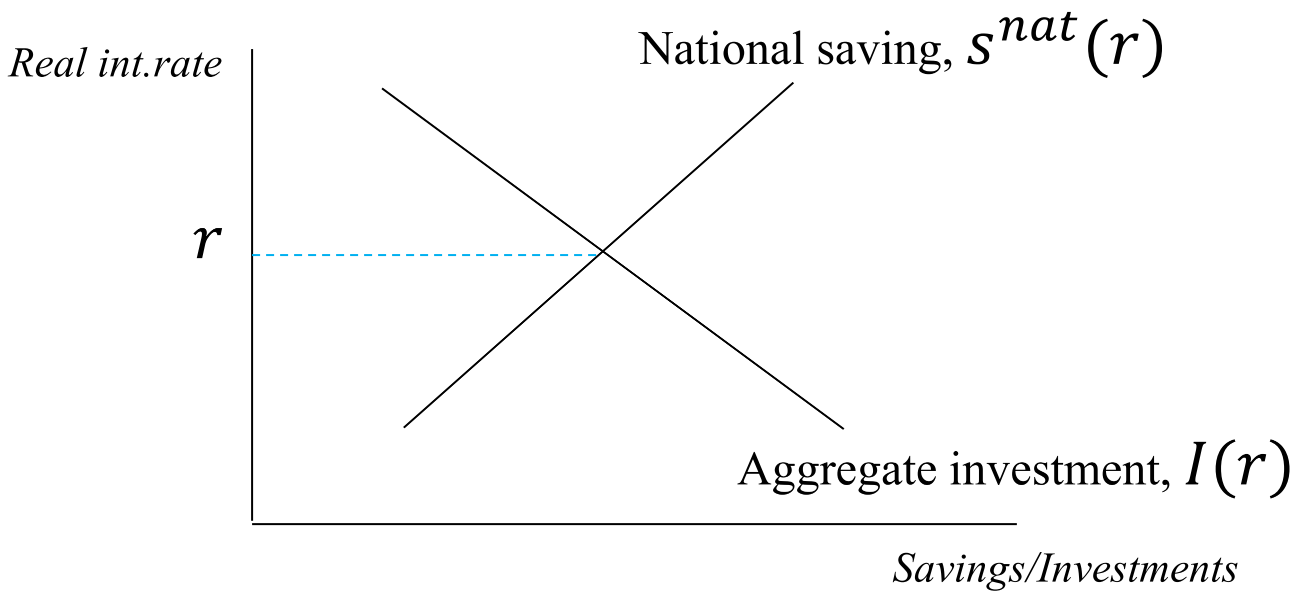
- National savings = savings by consumers + savings by government + savings by firms

  - No firms in our model (yet..), so $s_1^{firm} = 0$
  - $s_1^{priv} = y_1 - t_1 - c_1$
  - $s_1^{govt} = t_1 - g_1$
  - $s_1^{nat} = s_1^{priv} + s_1^{govt} = y_1 - t_1 - c_1 + t_1 - g_1 = y_1 - c_1 - g_1$

  - Suppose there is a policy change in taxes. If consumption is the same after the implementation of policy then national saving, s_1^{nat} remains the same; since g_1 and y_1 remain unchanged.

NATIONAL SAVINGS

- Why do we need to consider national savings?



The national savings will determine the market equilibrium real interest rate, r, which appears in the consumer's budget constraint.

Effects of Tax Policy

- Policy Experiment: Is national savings affected by a decrease in t_1?
  - Suppose g_1 and g_2 do not change;
  - Question 1: Effect on t_2?
    - t_2 must rise

$$g_1 + \frac{g_2}{1+r} = t_1 - \Delta t_1 + \frac{t_2 + \Delta t_2}{1+r}$$

$$g_1 + \frac{g_2}{1+r} = t_1 + \frac{t_2}{1+r} - \Delta t_1 + \frac{\Delta t_2}{1+r}$$

$$\boxed{\Longrightarrow \Delta t_1 (1+r) = \Delta t_2}$$

EFFECTS OF TAX POLICY

- …
  - Question 2: Effect of tax changes on consumers' optimal choice of period-1 consumption?
    - NO EFFECT ON optimal c_1

Effects of Tax Policy

- …
  - To see that the results for c_1 and c_2 are not affected after $t_1 \downarrow$, we need to answer:
    - Is the utility function affected? Ans: NO. Utility function captures a person's preference. Changes in tax will not change the taste of individuals.
    - Is the budget constraint affected? Ans: NO. Why? (we will look into this)

Effects of Tax Policy

- Examine budget constraint
  - Recall before tax change:

$$c_1 + \frac{c_2}{1+r} = y_1 - t_1 + \frac{y_2 - t_2}{1+r}$$

  - With tax changes:

$$c_1 + \frac{c_2}{1+r} = y_1 - t_1 - \Delta t_1 + \frac{y_2 - t_2 + \Delta t_2}{1+r}$$

$$\boxed{c_1 + \frac{c_2}{1+r} = y_1 - t_1 + \frac{y_2 - t_2}{1+r}} \; -\Delta t_1 + \frac{\Delta t_2}{1+r}$$

Effects of Tax Policy

$$\Rightarrow c_1 + \frac{c_2}{1+r} = y_1 - t_1 + \frac{y_2 - t_2}{1+r} \underbrace{- \Delta t_1 + \frac{\Delta t_2}{1+\mathrm{r}}}_{= 0 \text{ (argued)}}$$

$$\Rightarrow c_1 + \frac{c_2}{1+r} = y_1 - t_1 + \frac{y_2 - t_2}{1+r} \qquad (\star)$$

- Thus, under the assumption that real interest rate, r, does not change:

Same budget constraint.

Same set of indifference curves (argued just now)

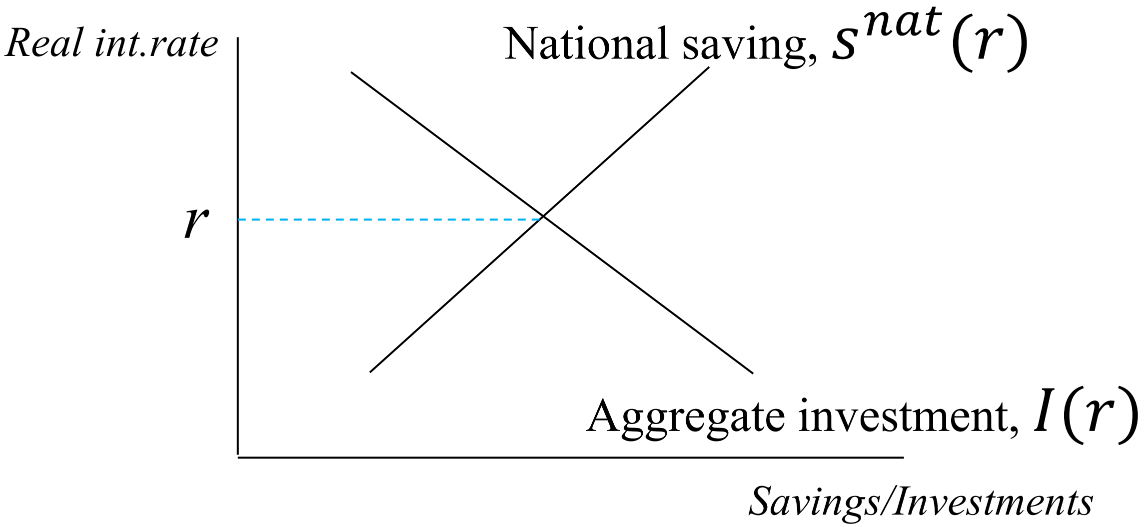
Same optimal consumption choice (c_1, c_2) as before.

Effects of Tax Policy

- Thus, under the assumption r does not change, national savings $s_1^{nat} = y_1 - c_1 - g_1$ is unchanged.

- So, no shift in national saving curve ➔ same real interest rate as before the tax changes.

EFFECTS OF TAX POLICY



- Thus, **the assumption that real interest rate, r, does not change** is consistent with all equations & graphs in the model. Thus it is indeed a solution (or our assumption is indeed true)

EFFECTS OF TAX POLICY

- Summary of the result:
  - With no change in g_1 and g_2, when lump-sum t_1 falls, lump-sum t_2 will increase by the same discounted value.
  - The consumption and real interest rate remain unchanged after the tax change.
  - The national savings also remain unchanged.

RICARDIAN EQUIVALENCE

- Ricardian Equivalence Theorem: For a given PDV of government spending, neither consumption nor national savings is affected by the precise *timing* of lump-sum taxes

- A benchmark result/concept in the theory of macroeconomic policy

- PROBLEM with Ricardian Equivalence: Prediction relies crucially on lump-sum taxes

Ricardian Equivalence

- <u>Economic Interpretation</u>: Rational consumers understand that a tax cut today means a tax increase in the future (because total government spending is unchanged)
  - Thus <u>*entire tax cut is saved by consumers*</u> in order to pay higher taxes in the future
  - Private savings and government savings move in exactly offsetting ways

$$s_1^{priv} = y_1 - t_1 - c_1$$

$+\Delta t_1$

Rises when t_1 decreases, *<u>GIVEN</u>* that we have *<u>CONCLUDED</u>* that c_1 does not change

$$s_1^{govt} = t_1 - g_1$$

$-\Delta t_1$

Decreases when t_1 decreases

DISCUSSION ON TAXATION

Nature of Taxation

- Lump-Sum Tax
  - A tax whose total incidence (i.e., total amount paid) does not depend in any way on any decisions/choices an individual makes
  - Real-world examples: not that we know of

- Taxes in our two-period framework so far
  - Lump-sum! Total amounts t_1 and t_2 paid by consumer are independent of any of their decisions/choices

Period-1 budget constraint

$$c_1 + t_1 + a_1 - a_0 = y_1 + ra_0$$

Period-2 budget constraint

$$c_2 + t_2 + a_2 - a_1 = y_2 + ra_1$$

Nature of Taxation

- Proportional (aka distortionary) Tax
  - A tax whose total incidence depends on decisions/choices an individual makes
  - In simple two-period framework: consumers only make consumption choices c_1 and c_2

τ is consumption tax *rate* (aka sales tax rate)

Period-1 budget constraint

$$(1+\tau_1)c_1 + a_1 - a_0 = y_1 + ra_0$$

$$\boxed{c_1 + c_1\tau_1}$$

Period-2 budget constraint

$$(1+\tau_2)c_2 + a_2 - a_1 = y_2 + ra_1$$

Proportional Taxation

τ is consumption tax rate (aka sales tax rate)

Period-1 budget constraint

$$(1+\tau_1)c_1 + a_1 - a_0 = y_1 + ra_0$$

Period-2 budget constraint

$$(1+\tau_2)c_2 + a_2 - a_1 = y_2 + ra_1$$

- Combine into consumer LBC

$$(1+\tau_1)c_1 + \frac{(1+\tau_2)c_2}{1+r} = y_1 + \frac{y_2}{1+r} + (1+r)a_0$$

- Slope is

$$-\left(\frac{1+\tau_1}{1+\tau_2}\right)(1+r)$$

Proportional Taxation

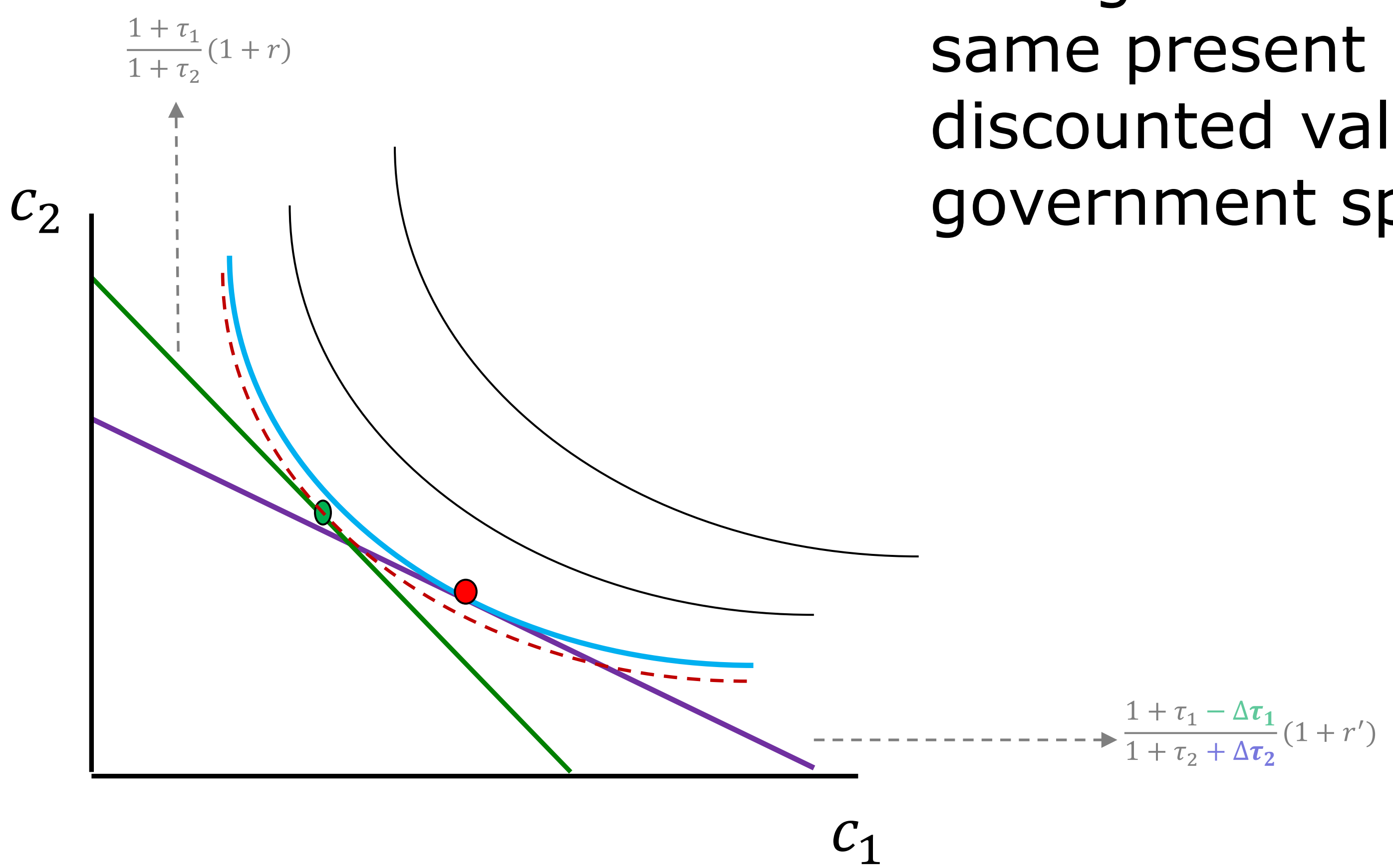
$$(1+\tau_1)c_1 + \frac{(1+\tau_2)c_2}{1+r} = y_1 + \frac{y_2}{1+r} + (1+r)a_0$$

$$c_2 = \left\{-(1+\tau_1)c_1 + y_1 + \frac{y_2}{1+r}\right\} \cdot \frac{1+r}{1+\tau_2}$$

$$c_2 = -\frac{(1+\tau_1)}{1+\tau_2}(1+r) \cdot c_1 + \left\{y_1 + \frac{y_2}{1+r}\right\} \cdot \frac{1+r}{1+\tau_2}$$

- Slope is $-\left(\frac{1+\tau_1}{1+\tau_2}\right)(1+r)$

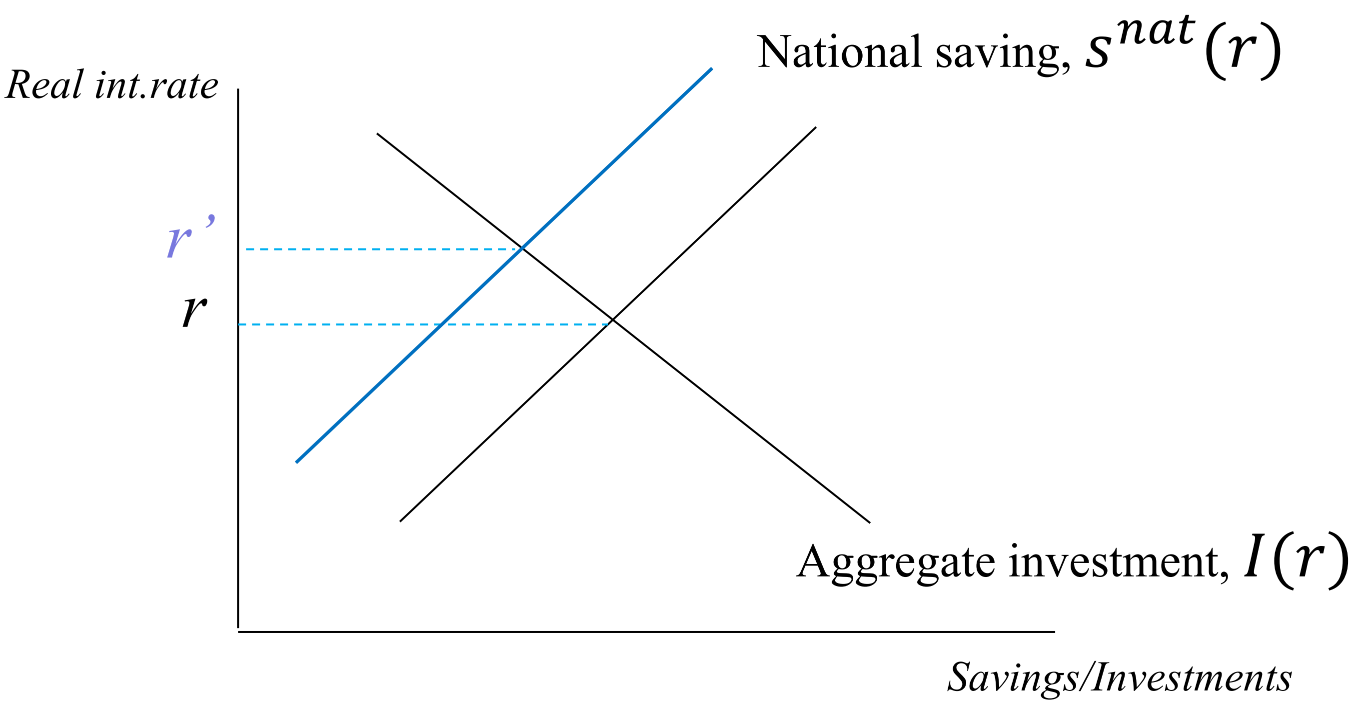
Proportional Taxation



- Change in taxes but same present discounted value of government spending

PROPORTIONAL TAXATION

$$s^{nat} = y_1 - c_1 - g_1$$



Proportional Taxation

- Non-lump-sum taxes: optimal consumption choices are determined using consumer LBC
- Changes in tax *rates* do affect optimal consumption choices because they change slope of consumer LBC
- Ricardian Equivalence Theorem does not apply
  - Changes in tax rates *do* affect national savings

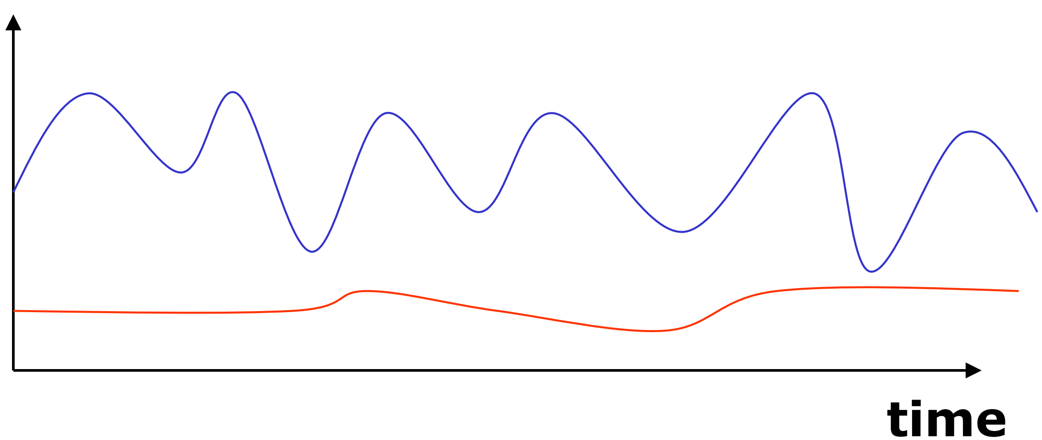
RICARDIAN EQUIVALENCE?

- So why the fascination with Ricardian Equivalence?
- A benchmark result/concept in the theory of macroeconomic policy
  - Effects of actual policy proposals can be compared to the Ricardian Equivalence benchmark
  - In practice, *does* seem like tax rebates are sometimes saved

RICARDIAN EQUIVALENCE?

- At aggregate level, total tax collections sometime "seem" lump-sum (i.e., independent of aggregate macroeconomic activity)

At least partly due to due "automatic stabilizers" built into the tax code…



GDP (or any aggregate measure of real activity…ie, consumption) moves up and down over time…

…but total tax revenue of the government fluctuates very little

Ricardian Equivalence?

- Ricardian Equivalence
  - Is a theoretical benchmark
  - Is an empirical benchmark

Ricardian Equivalence is about the (lack of) effects of *changes in tax policy*, holding total government liabilities fixed. If g_1 and/or g_2 change, Ric. Equiv. does *not* apply.

Singapore context

- Why Singapore's personal income tax (PIT) are so low compared to other economies?
  - PIT rates are low. Top rate is reduced from 28% in 2002 to 20% in 2007, and now 22%.
  - No "western style" social welfare: instead of "consumption welfare", "development welfare"
  - Fiscal prudence.
  - Government as a big investor: Investing through Gov Linked Corp (GLCs) - e.g. Gov Investment Corp (GIC) to manage funds, to generate revenues.
- Singapore fiscal policy is not like other economies.