EC3333 Tutorial 5 Suggested Answers

1. Your mortgage has 25 years left, and has an APR of 7.625% with monthly payments of $1449.

   a. What is the outstanding balance (the present value of the remaining mortgage payments)?

   b. Suppose you cannot make the mortgage payment and you are in danger of losing your house to foreclosure. The bank has offered to renegotiate your loan. The bank expects to get $150,000 for the house if it forecloses. They will lower your payment as long as they will receive at least this amount (in present value terms). If current 25-year mortgage interest rates have dropped to 5% (APR), what is the lowest monthly payment you could make for the remaining life of your loan that would be attractive to the bank (to avoid foreclosure)?

a.

$$\text{Interest Rate per Compounding Period} = \frac{\text{APR}}{k \text{ periods / year}}$$

The monthly discount rate is $\frac{0.07625}{12} = 0.635\%$.

$$PV = \underbrace{\frac{C}{r}\left(1 - \frac{1}{(1+r)^N}\right)}_{\text{The Annuity Formula}}$$

$$\text{Present Value} = \left(\frac{1449}{0.00635}\right) \times \left\{1 - \left(\frac{1}{1.00635^{25\times 12}}\right)\right\} = 194{,}024.13$$

b.

Here the present value is $150,000 and the monthly payment needs to be calculated.

$$C = \frac{P}{\left(\frac{1}{r}\right)\left(1 - \frac{1}{(1+r)^N}\right)}$$

$$r = 0.05/12 = 0.004167$$

$$\text{Payment} = \frac{150000 \times 0.004167}{\left(1 - \frac{1}{1.004167^{25\times 12}}\right)} = 876.88$$

2. You have an outstanding student loan with required payments of \$500 per month for the next four years. The interest rate on the loan is 9% APR (monthly compounding).

a. Calculate the present value of your student loan.

You are considering making an extra payment of \$100 today (that is, you will pay an extra \$100 today that you are not required to pay, on top of the \$500 per month that you are required to pay).

b. How does that reduce the present value of your loan?

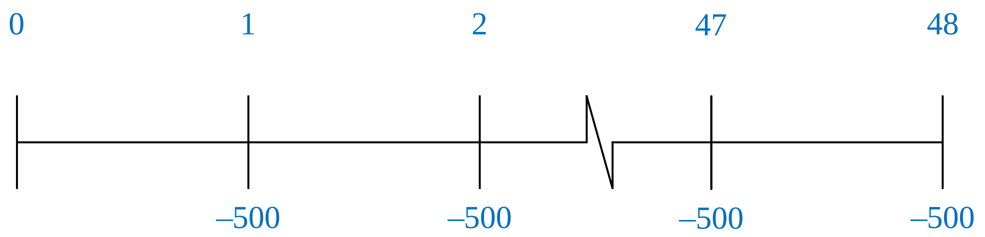
With this prepayment of \$100 today, your last monthly payment at the end of year 4 will be reduced by an amount X, with the interim payments remaining at \$500 per month.

c. Write down the present value of the revised cash flows and use that to solve for the amount X that you can get with this prepayment of \$100 today.

d. Finally, calculate the effective rate of return (expressed as an APR with monthly compounding) you have earned on the \$100 prepayment.

a.

The timeline of our required payments i



The remaining balance on the student loan equals the present value of the remaining payments. The loan interest rate is 9% APR, or 9% / 12 = 0.75% per month, so the present value of the payments is

$$PV = \frac{500}{0.0075}\left(1 - \frac{1}{1.0075^{48}}\right) = \$20{,}092.39$$

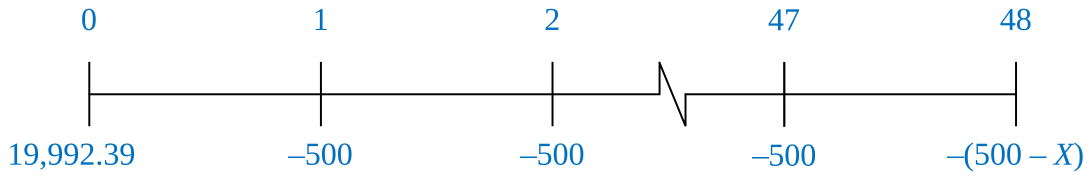
b.

If you prepay an extra \$100 today, your will lower your remaining balance to

\$20,092.39 – 100 = \$19,992.39.

c.

Though your balance is reduced, your required monthly payment does not change. Instead, it will reduce the payments you need to make at the very end of the loan. With the extra payment, the timeline changes:



That is, we will pay off the loan by paying \$500 per month for 47 months, and some smaller amount, \$500 – X, in the last month. To solve for X, recall that the PV of the remaining cash flows equals the outstanding balance when the loan interest rate is used as the discount rate:

$$19{,}992.39 = \frac{500}{0.0075}\left(1 - \frac{1}{(1+0.0075)^{48}}\right) - \frac{X}{1.0075^{48}}$$

Solving for X gives

$$19{,}992.39 = 20{,}092.39 - \frac{X}{1.0075^{48}}$$

$$X = \$143.14$$

So, the final payment will be lower by \$143.14.

d.

The extra payment effectively lets us exchange \$100 today for \$143.14 in four years. We claimed that the return on this investment should be the loan interest rate. Let's see if this is the case:

$\$100 \times (1.0075)^{48} = \$143.14$, so it is.

Thus, you earn a 9% APR (the rate on the loan).

3. You are 25 years old and decide to start saving for your retirement. You plan to save \$5000 at the end of each year (so the first deposit will be one year from now), and will make the last deposit when you retire at age 65. Suppose you earn 8% per year on your retirement savings.

   a. How much will you have saved for retirement at age 65?

   b. How much will you have saved at age 65 if you wait until age 35 to start saving (again, with your first deposit at the end of the year)?

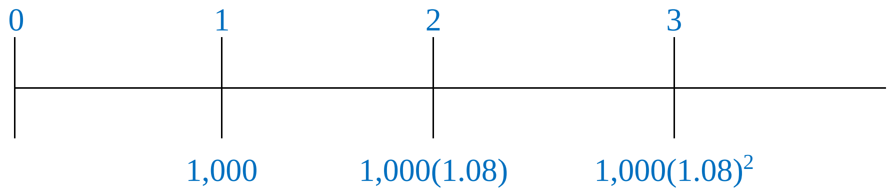
Future value of annuity is given by: $FV = \frac{C}{r}((1+r)^N - 1)$

amount	\$5,000	
rate	8%	
retirement age	65	
start age	**a. 25**	**b. 35**
Savings	**1,295,282.59**	**566,416.06**
=\$5,000*(1/0.08)*((1+0.08)^(65-Start Age)-1)		

4. A rich relative has bequeathed you a growing perpetuity. The first payment will occur in a year and will be \$1000. Each year after that, on the anniversary of the last payment you will receive a payment that is 8% larger than the last payment. This pattern of payments will go on forever. If the interest rate is 12% per year,
   a. What is today's value of the bequest?
   b. What is the value of the bequest immediately after the first payment is made?

$$PV \text{ (growing perpetuity) } = \frac{C}{r-g}$$

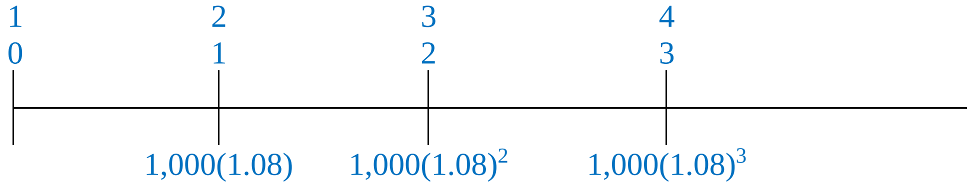
a. Timeline:



Using the formula for the PV of a growing perpetuity gives:

$$\text{PV} = \left(\frac{1{,}000}{0.12 - 0.08}\right) = \$25{,}000.$$

b. Timeline:

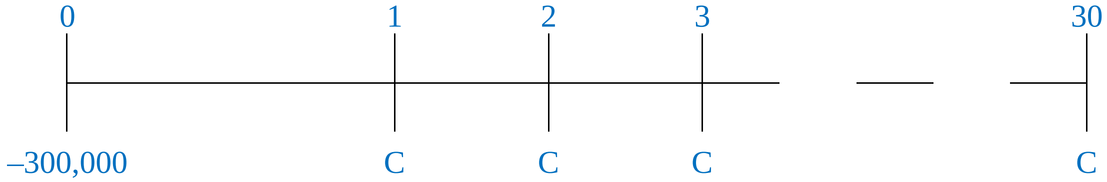


Using the formula for the PV of a growing perpetuity gives:

$$\text{PV} = \frac{1{,}000(1.08)}{0.12 - 0.08} = \$27{,}000.$$

5. You are thinking of purchasing a house. The house costs $350,000. You have $50,000 in cash that you can use as a down payment on the house, but you need to borrow the rest of the purchase price. The bank is offering a 30-year mortgage that requires annual payments and has an interest rate of 7% per year. What will your annual payment be if you sign up for this mortgage?

Timeline: (From the perspective of the bank)



Loan or annuity payments is:

$$C = \frac{P}{\left(\frac{1}{r}\right)\left(1 - \frac{1}{(1+r)^N}\right)}$$

$$\mathrm{C} = \frac{300{,}000}{\frac{1}{0.07}\left(1 - \frac{1}{1.07^{30}}\right)} = \$24{,}176$$