Macroeconomics Analysis II, EC3102
Tutorial 4

Question 1. *Unpleasant Monetarist Arithmetic.*[1]

Consider a **finite** period economy, the final period of which is period T (so that there is no period $T\ +1$) – every agent in the economy knows that period T is the final period of the economy. In this economy, the government conducts both fiscal policy (engaging in government spending and collecting taxes) and monetary policy (expanding or contracting the money supply). **The timing of fiscal policy and monetary policy will be described further below.** The economy has now arrived at the very beginning of period T, and the period-T consolidated government budget constraint is

$$M_T - M_{T-1} + B_T + \ P_T t_T = P_T g_T + (1 + i_{T-1})B_{T-1}$$

where the notation is as follows:

- M_t is the **nominal** money supply at the end of period $t;$
- B_t is the **nominal** quantity of government debt outstanding at the end of period t (i.e., a **positive** value of B_t here means that the government is in **debt** at the end of period t);
- t_t is the **real** amount of lump-sum taxes the government collects in period t (and there are no distortionary taxes);
- i_{t-1} is the **nominal** interest rate on government assets held between period $t-1$ and t, and it is **known with certainty in period** $t-1$**;**
- g_t **is** the **real** amount of government spending in period t;
- P_t is the nominal price level of the economy in period t.

Thus, once period T begins, the economic objects yet to be determined are t_T , g_T , M_T , and B_T.

a. Compute the numerical value of B_T ? Show any important steps in your computations/logic. The remainder of this question is independent of part a.

   For the remainder of this question, suppose that for some reason $B_T = 0$ -- *the fiscal authority is committed to this decision about bonds and will never deviate from it. Also suppose for the remainder of this question that* $i_{T-1} = 0.1, B_{T-1} = 10$ *(i.e., the government is in debt at the beginning of period* T*, given the definition of* B_t*),* $P_{t-1} = 1$ *(notice the time subscripts here), and* $M_{T-1} = 10$.

   The timing of fiscal policy and monetary policy is as follows.
   - *At the beginning of any period t, the monetary authority and the fiscal authority independently decide on monetary policy (the choice of* M_t *) and fiscal policy (the choices of* t_t *and* g_t *), respectively.*

   Finally, in parts b and c, suppose that the nominal price level is flexible (i.e., it is not at all "sticky").

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[1] *This problem is based on a classic work in macroeconomic theory by Thomas Sargent and Neil Wallace "*Some Unpleasant Monetarist Arithmetic,*" Federal Reserve Bank of Minneapolis Quarterly Review, Vol. 5, 1981).*

b. Suppose the fiscal side of the government decides to run a **primary real fiscal surplus of $t_T - g_T = 9$** in period T. Also suppose that the monetary authority chooses a value for M_T which when coupled with this fiscal policy implies that there is **zero inflation** between period $T-1$ and period T. Compute numerically **the real value of seignorage revenue** the government earns in period T, clearly explaining the key steps in your computations/logic. Also provide brief economic intuition for **why** the government needs to generate this amount of seignorage revenue in period T?

c. Suppose the monetary authority sticks to its monetary policy (i.e., its choice of M_T) you found in part b above. However, the fiscal authority decides instead to run a primary real fiscal surplus of $t_T - g_T = 8$. Compute numerically the real value of seignorage revenue the government must earn in period T as well as the inflation rate between period T −1 and period T. Clearly explain the key steps in your computations/logic. In particular, why is real seignorage revenue here different or not different from what you computed in part b?

<u>Question 2</u>

In studying the Fiscal Theory of Inflation (FTI) and the Fiscal Theory of the Price Level (FTPL), the condition around which the analysis revolves is the present-value (lifetime) consolidated government budget constraint (GBC). As studied in Chapter 16, starting from the beginning of period t, the present-value consolidated GBC is

$$\frac{B_{t-1}}{P_t} = sr_t + \frac{sr_{t+1}}{1+r_t} + \frac{sr_{t+2}}{(1+r_t)(1+r_{t+1})} + \frac{sr_{t+3}}{(1+r_t)(1+r_{t+1})(1+r_{t+2})} \cdots$$
$$+(t_t - g_t) + \frac{t_{t+1} - g_{t+1}}{1+r_t} + \frac{t_{t+2} - g_{t+2}}{(1+r_t)(1+r_{t+1})} + + \frac{t_{t+3} - g_{t+3}}{(1+r_t)(1+r_{t+1})(1+r_{t+2})} \cdots$$

in which all of the notation is just as in Chapter 16.

In this problem, you will study a "steady state" version of the FTI, which enables the FTI to generate some "long run" predictions about the inflationary consequences of government indebtedness. To operationalize the steady state version of the FTI, suppose the following:

- $r_t = r_{t+1} = r_{t+2} = r > 0$ (i.e., steady state).
- $t_t - g_t = t_{t+1} - g_{t+1} = t_{t+2} - g_{t+2} = \cdots = t - g$ (i.e., steady state).
- $sr_t = sr_{t+1} = sr_{t+2} = \cdots = sr$ (i.e., steady state).
- $\mu_t = \mu_{t+1} = \mu$ (i.e., steady state).
- Suppose $P_t = 1$ and does not change under any circumstances (this assumption effectively allows us to leave aside the FTPL, as stated above). (Note: this assumption does **not** necessarily imply that P_{t+1} or P_{t+2} or P_{t+3}, etc, are equal to one.)

With these steady state assumptions, the present-value consolidated GBC from above simplifies considerably, to

$$B_{t-1} = sr + \frac{sr}{1+r} + \frac{sr}{(1+r)^2} + \frac{sr}{(1+r)^3} \dots + (t-g) + \frac{t-g}{1+r} + \frac{t-g}{(1-r)^2} + \frac{t-g}{(1-r)^3} \dots,$$

which can be simplified to

$$\begin{aligned} B_{t-1} &= [sr + (t-g)]\left\{1 + \frac{1}{1+r} + \frac{1}{(1+r)^2} + \frac{1}{(1+r)^3} \dots\right\} \\ &= \left(\frac{1+r}{r}\right)[t - g + sr] \end{aligned}$$

Suppose that $B_{t-1} = \$14$ trillion. Second, the steady state value of real money balances equals \$2 trillion in every time period. That is,

$$\frac{M_t}{P_t} = \frac{M_{t+1}}{P_{t+1}} = \frac{M_{t+2}}{P_{t+2}} = \cdots = \$2 \text{ trillion}$$

The entire analysis is then based on either of these last two representations of the present value GBC, which from here we will refer to as the PVGBC.

a. Suppose that seignorage revenue will be always be zero (sr = 0 in every time period). If the real interest rate is five percent (r = 0.05), compute the numerical value of (t-g) that the fiscal side of the government must set in every time period to ensure the present-value consolidated GBC is satisfied. Be clear about the sign and the magnitude. Present your logic, and provide brief economic explanation.
b. Re-do the analysis in part a, assuming instead that r = 0.025. Compare the conclusion here with the conclusion in part a, providing brief economic explanation for why the conclusions do or do not differ.
c. Suppose that the fiscal side of the government is able to balance its budget in every period (t-g = 0 in every time period), but (perhaps because of political considerations) is never able to run a surplus (but also never runs a deficit). If the real interest rate is five ercent (r = 0.05) and assuming that r cannot be affected by monetary policy, compute the numerical value of sr that the monetary side of the government must generate in every time period to ensure the present-value consolidated GBC is satisfied. Be clear about the sign and the magnitude. Present your logic, and provide brief economic explanation.
d. Re-do the analysis in part c, assuming instead that r = 0.025. Compare the conclusion here with the conclusion in part c, providing brief economic explanation for why the conclusions do or do not differ.