Practice Problem Set 7
Game Applications (C.30)

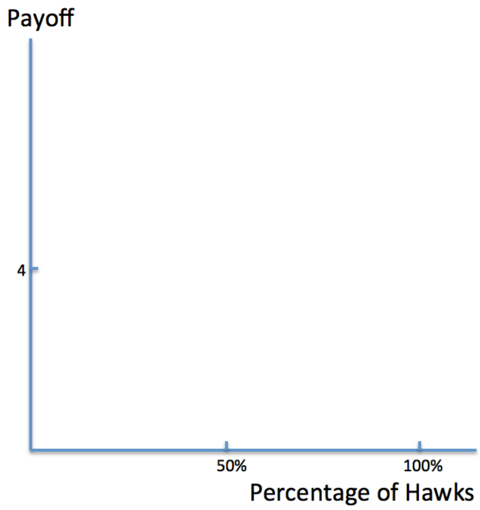
Question 7.1

This problem is an illustration of the Hawk-Dove game. It applies game theory to understand evolution. Males of a certain species frequently come into conflict with other males over the opportunity to mate with females. If a male runs into a situation of conflict, he has two alternative strategies. If he plays "Hawk," he will fight the other male until he either wins or is badly hurt. If he plays "Dove," he makes a bold display but retreats if his opponent starts to fight. If two Hawk players meet, they are both seriously injured in battle. If a Hawk meets a Dove, the Hawk gets to mate with the female and the Dove slinks off. If a Dove meets another Dove, they both strut their stuff but neither chases the other away. Eventually the female may select one of them at random or may get bored and wander off. The expected payoffs to each male are shown in the box below.

		Animal B	
		Hawk	Dove
Animal A	Hawk	$-5, -5$	$10, 0$
	Dove	$0, 10$	$4, 4$

(i) Suppose that there is a large male population and the fraction p are Hawks. Then the fraction of any player's encounters that are with Hawks is about p and the fraction of encounters that are with Doves is about $1 - p$. Find the value of p such that at this value Hawks do exactly as well as Doves.

(ii) On the axes (right), use blue ink to graph the average payoff to the strategy Dove when the proportion of Hawks in the male population who is p. Use red ink to graph the average payoff to the strategy, Hawk, when the proportion of the male population who are Hawks is p. Label the equilibrium proportion in your diagram by E.

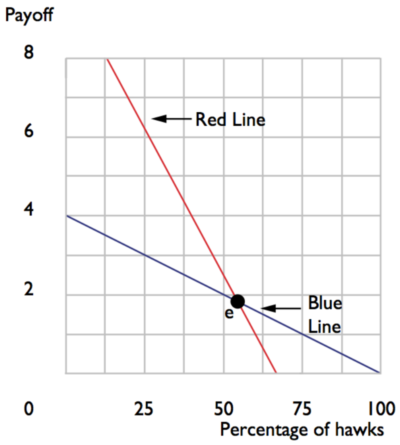


(iii) If the proportion of Hawks is slightly greater than E, which strategy does better? Suppose $\frac{3}{5}$ of the male animals are Hawks initially, if the more profitable strategy tends to be adopted more frequently in future plays, what will the proportion of Hawks in the male population be in the long run?

Answer

(i) The value of being a Dove is $4 - 4p$. The value of being a Hawk is $10 - 15p$. In equilibrium, $p = \frac{6}{11}$.

(ii)



(iii) If the proportion of Hawks is slightly greater than E, acting like a dove brings a higher payoff. Suppose $\frac{3}{5}$ of the male animals are Hawks initially, acting like a dove brings a higher payoff (Since $\frac{3}{5} > \frac{6}{11}$), the proportion of Hawks will decrease until the equilibrium (where $\frac{6}{11}$ of the male animals are Hawks) is restored.

Question 7.2

		Player 2	
		L	**R**
Player 1	**U**	0, 0	2, -2
	D	x, -x	1, -1

Consider the above game matrix. (i) Is this a zero-sum game? Find the Nash Equilibriums when (ii) $x < 0$, (iii) $0 < x < 1$, (iv) $x > 1$.

Answer

(i) Yes.

(ii) If $x < 0$, Up strictly dominates Down and Left strictly dominates Right. Player 1 will always play U, and will not randomize (since the expected payoff of U is always higher than the expected payoff D). For the same reason, Player 2 will always play L. Hence both players will not randomize and we have only one PSNE, which is (Up, Left).

(iii) If $0 < x < 1$, Left strictly dominates Right. Player 2 will always play L and will not randomize. Knowing this, Player 1 will always play D and will not randomize too. Hence both players will not randomize and we have only one PSNE, which is (Down, Left).

(iv) If $x > 1$, there is no PSNE.

Player 1 is indifferent between playing U and D if $\pi_L = \frac{1}{1+x}$, Player 2 is indifferent between playing L and R if $\pi_U = \frac{x-1}{1+x}$. Notice that **if $\boldsymbol{x > 1}$**, $0 < \pi_L = \frac{1}{1+x} < 0.5$ and $0 < \pi_U = \frac{x-1}{1+x} < 1$. Hence, there exists a MSNE when both plays randomize: Player 1 plays U with probability $\pi_U = \frac{x-1}{1+x}$ and Player 2 plays L with probability $\pi_L = \frac{1}{1+x}$.

(Additional Notes)

We can use the results in iv) to check the earlier claim that players will not randomize in ii) and iii).

In iii), **if $\mathbf{0 < x < 1}$**, Player 2 is indifferent playing L or R only if $\pi_U = \frac{x-1}{1+x} < 0$, but π_U is a probability and must take a value between 0 and 1. Hence Player 2 will not randomize. We can check that for $0 < x < 1$, the expected payoff for Player 2 to play L is always higher than the payoff of playing R.

Now consider ii). If $\mathbf{-1 < x < 0}$, Player 1 is indifferent playing U or D only if $\pi_L = \frac{1}{1+x} > 1$, but π_L is a probability and must take a value between 0 and 1. Hence Player 1 will not randomize.

If $\mathbf{x < -1}$, Player 1 is indifferent playing U or D only if $\pi_L = \frac{1}{1+x} < 0$, but π_L is a probability and must take a value between 0 and 1. Hence Player 1 will not randomize.

<u>Question 7.3</u>

Today is Sunday. Mona is going to be out of town tomorrow for three days (Monday to Wednesday) and will not need her car during this time. Lisa is interested in renting her car. The value to Lisa of having the car during this time is \$50 per day. The total cost to Mona of letting Lisa use the car is \$20, regardless of how many days Lisa uses it. Lisa can send a message to Mona tonight, offering to rent her car for three days for a specified price. Mona can either accept the offer or reject the offer and make a counteroffer. The only problem is that it takes 24 hours for a counteroffer to be made and accepted. If Mona's counteroffer (made on Monday) is accepted, Lisa could use the car only for two days. If not, Lisa can make a final offer on Tuesday, and if accepted, she could use the car for one day.

Assume that Mona and Lisa would accept an offer when they are indifferent. Furthermore, they do not discount the future (i.e., their discount factors are 1). Find the Rubinstein bargaining solution to this problem (find the value of offer made, the day the offer is made, and the net payoffs of the players).

<u>Answer</u>

We start by working backward from Tuesday. If Lisa offers \$20 to rent the car for the last day, Mona will accept. In this case, Lisa will get a profit of \$50 − \$20 = \$30.

On Monday, when Mona makes her offer, she is aware that Lisa will reject the offer unless it gives Lisa a profit of \$30. Mona is also aware that the value to Lisa of renting the car for two days is \$100. Therefore the highest price for two days car rental that Lisa will accept is \$70. Since Mona's cost for renting the car is \$20, Mona would make a net profit of \$50 and Lisa would make a net profit of \$30.

Now consider what Lisa would do today. She will offer Mona \$70 for the three days rental, and Mona will accept the offer. Mona's net payoff would be \$50 (=70-20), and Lisa's net payoff would be \$80 (=150-70).