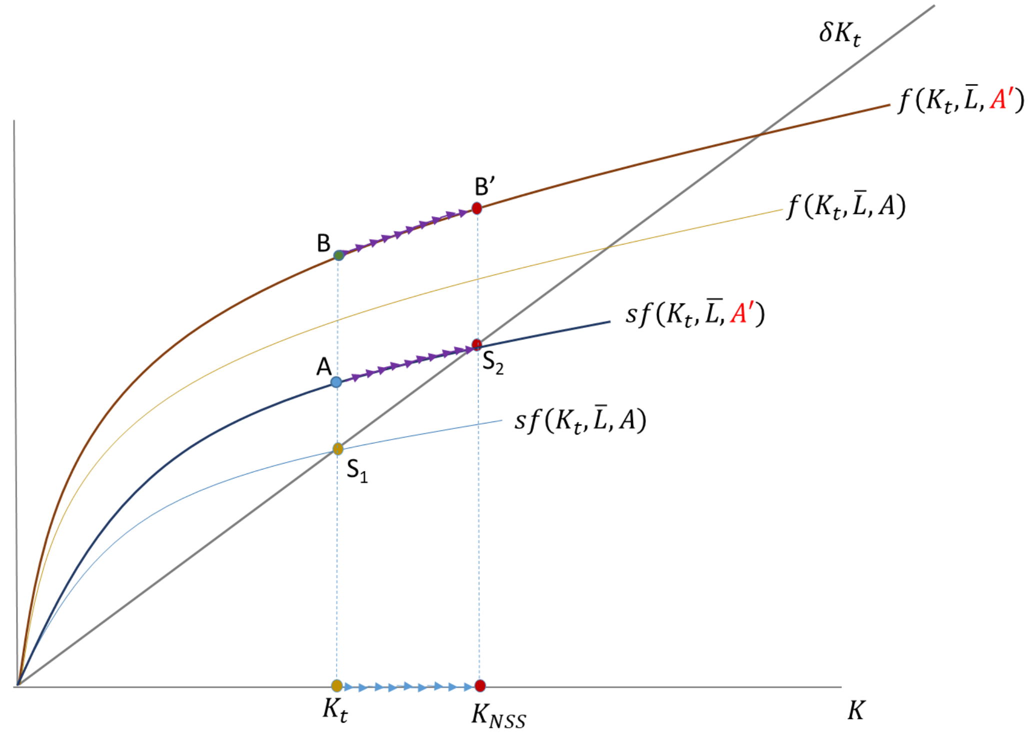
Macroeconomics analysis II, EC3102

Tutorial 7 Solution

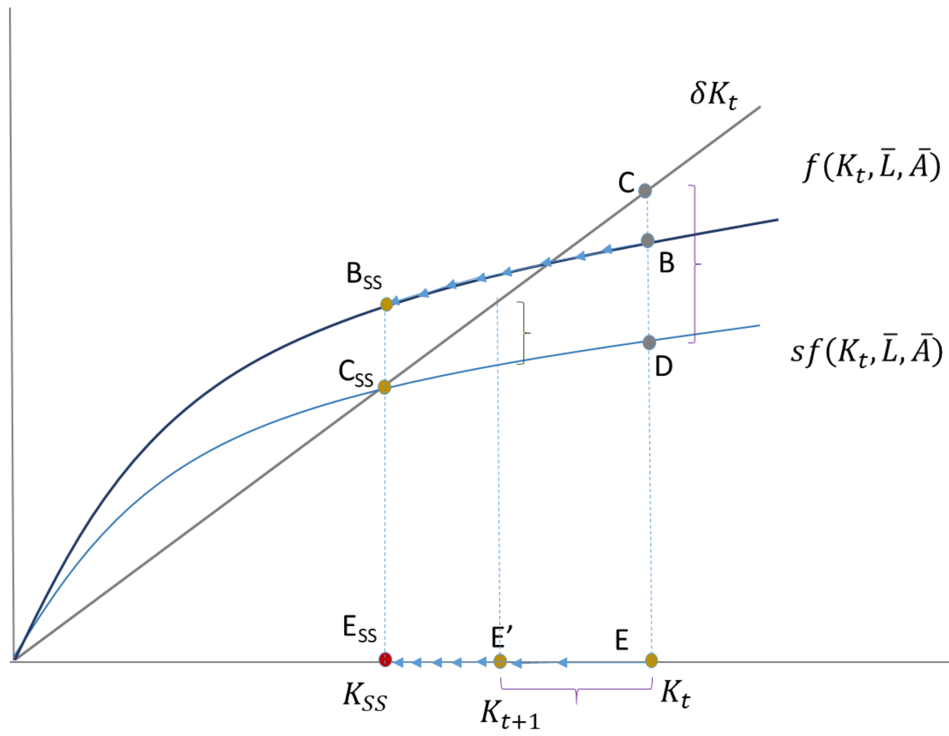
<u>Question 1:</u>



Upon opening up, the economy will experience higher technological state, as such the productivity level increases and so the production and investment (saving) per worker function will shift up also. Thus, this economy has a new steady state point (at S_2). However, the economy is having a capital per worker level lower than that in the new steady state.

After the opening up, the economy can produce more than it used to with the same amount of capital per worker level. Thus, the amount that is saved is higher – more than the depreciation, so the economy is having positive net investment. So there is capital accumulation. The positive net investments go on for many periods, leading to more capital accumulation until the economy reaches the new steady state.

Question 2



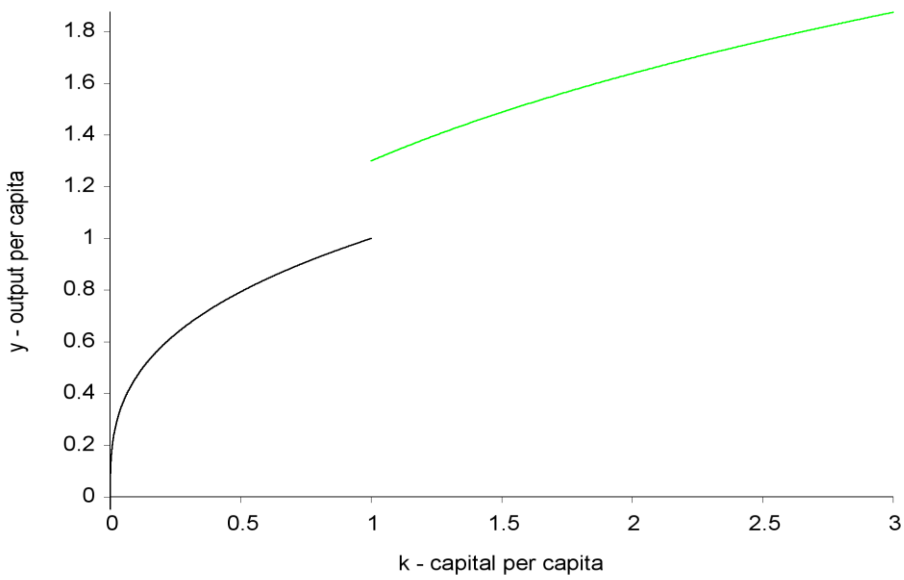
*A*s the economy A gets more capital from economy B, the amount of capital per worker level in country A will be higher than the steady state (the steady state capital level is at point E_{SS}). Assume that the capital per worker level is at point E in the graph above. Then, the amount of output produced is EB and the amount of saving (or investment) is ED but the amount of capital depreciation is EC. That means that the amount of investment is not enough to make up for the depreciation, so the net investment is negative and the magnitude of the net investment is CD (see graph). In the next period, the capital per worker will fall fromK_t toK_{t+1} by CD [or $K_{t+1} = K_t + CD$.

In time $t + 1$, there is still a negative net investment (see graph). That means we continue to see depletion in capital per worker in $t + 2$.
The same process will go on for periods of time until the capital per worker is back to the steady-state level at E_{SS}. At this point, the investment is just enough to cover for depreciation.

Question 3:

Part a.

The production function would look like this:



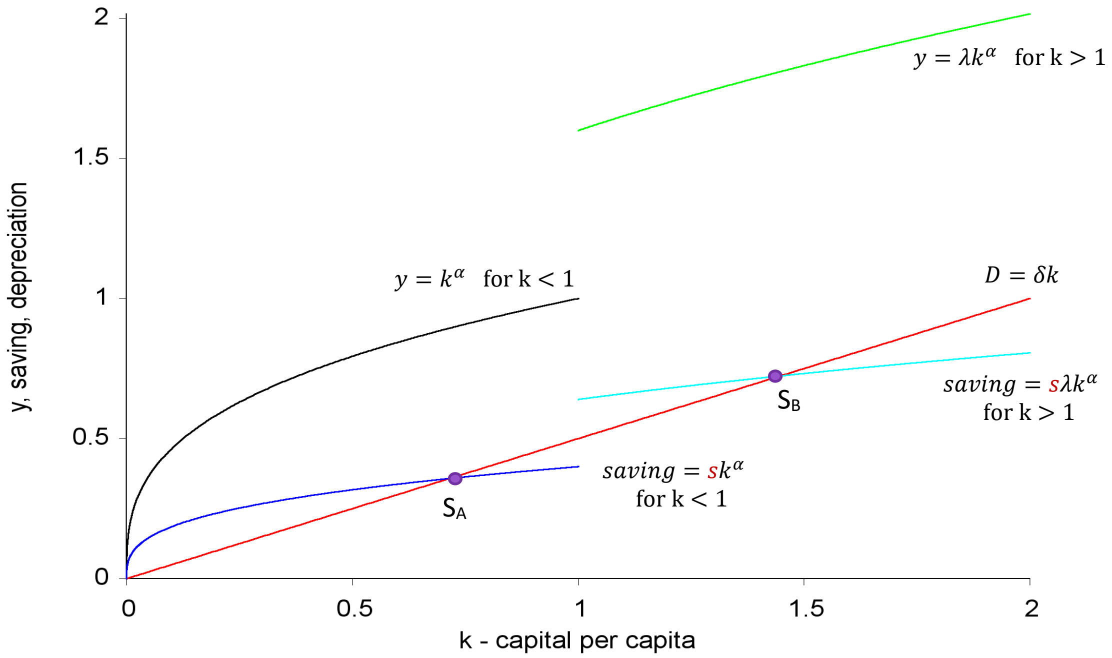
This production has a structural break at $k = 1$. The TFP before the break is 1 and after the break is $\lambda, \lambda > 1$.

1. The general aggregate production function of the form $Y_t = F(K, L)$ is:

$$\begin{cases} Y = 1.K^{\alpha}L^{1-\alpha} & \text{for } \frac{\mathrm{K}}{\mathrm{L}} < 1 \\ Y = \lambda K^{\alpha}L^{1-\alpha} & \text{for } \frac{\mathrm{K}}{\mathrm{L}} > = 1 \end{cases}$$

2. The economic effect:
   - The economy might need a critical threshold for $\frac{K}{L}$ (capital per person; or capital per capita) in order for productivity to be higher. That is, on average, each person needs to have a certain amount of capital in order to break out of the old productivity level. This is reasonable because with low capital level, individuals might not be more productive. For example, think of a poor economy where university students may not on average have enough money to buy a computer. Suppose now that the economy accumulates enough capital so that all students can own a computer. What do you think the effect might be on the productivity of the economy?
   - Anything else?

Part b.



So for $0 < k < 1$, S_A is the steady state. The reason is for capital level that is below S_A, savings is higher than the break-even investment. As such there is extra investment which would lead to capital accumulation, which in turns increase the capital level in the subsequent periods until the economy reaches the point S_A. On the other hand, for the capital level that is beyond the point S_A, saving is lower than the break-even investment. This means that the saving is not enough to cover for the loss of capital due to depreciation. As such, capital will be de-cumulated until the economy reach S_A.

For $k \geq 1, S_B$ is the steady state. The reason is for capital level that is below S_B, savings is higher than the break-even investment. As such there is extra investment which would lead to capital accumulation, which in turns increases the capital level in the subsequent periods until the economy reaches the point S_A. On the other hand, for the capital level that is beyond the point S_B, saving is lower than the break-even investment. This means that the saving is not enough to cover for the loss of capital due to depreciation. As such, capital will be de-cumulated until the economy reach S_B.

Part c.

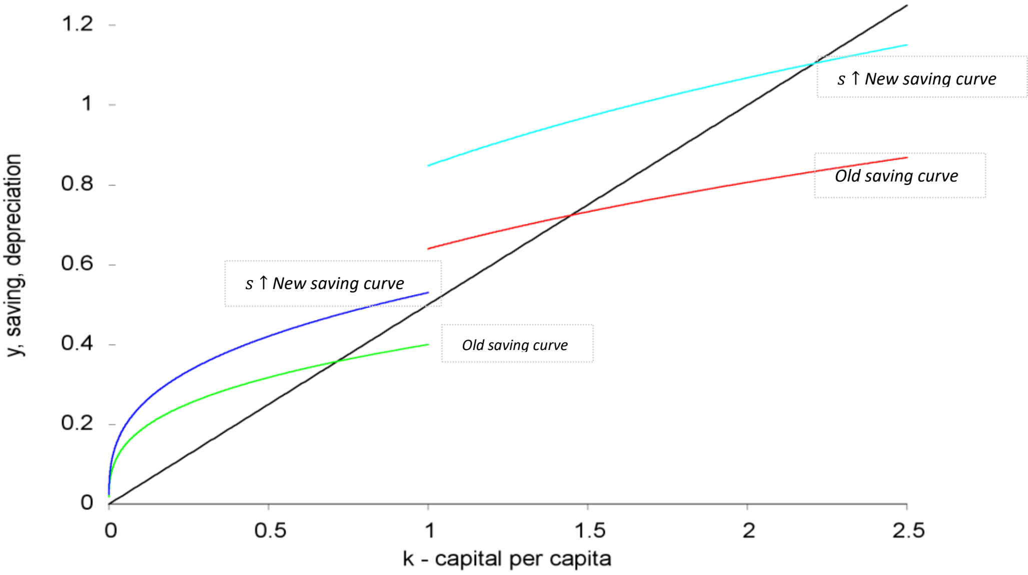
This model implies that countries may not converge to the similar per capita income due to different capital endowments to start with.

Part d.

Yes, a donation, *according to the model*, can help. The minimum is $1 - k_{econ}$, where k_{econ} is the capital that the economy is having. If the country is already in the lower steady state, the minimum is $1 - k_{SA}$, where k_{SA} is the capital level at the steady state S_A.

Comment*: One caveat here is that this is just a model. From this model, we derive a policy to help the poverty-trap economy. But whether the model is right or not, we need empirical support. Having wrong model would lead to wrong policy.*

Part e.



Yes, to lift the economy out of its poverty trap without outside assistance, the economy can increase the saving rate. As the figure above shows us, the saving curve is raised up and now the first part of the new saving curve (the blue) does not cut the break-even line but the second part (where $k \geq 1$) does. In this case, there is only one steady state for this economy with new and higher saving rate. Please note also that the production curve,

$$\begin{cases} y = k^{\alpha} & \text{for } k < 1 \\ y = \lambda k^{\alpha} & \text{for } k > 1 \end{cases}$$

not draw in the figure, remains the same because the increase in saving rate only affects the saving curve.

Comment on part b:

Mathematically, we can also show that there are two steady state by proving that the break-even investment line (depreciation line) cuts the saving curve at two points.

Condition: $\boldsymbol{s < \delta < \lambda s}$

The saving function is:

$$\begin{cases} S = sk^{\alpha} & \text{for } k < 1 \\ S = s\lambda k^{\alpha} & \text{for } k > 1 \end{cases}$$

The depreciation function is:

$$dep = \delta k$$

The steady states are where Saving curve cuts the depreciation curve, thus, we have:

$$dep = S$$

$$\delta k = \begin{cases} sk^{\alpha} & \text{for } \mathrm{k} < 1 \\ s\lambda k^{\alpha} & \text{for } \mathrm{k} > 1 \end{cases}$$

For the first case:

$$\delta k = sk^{\alpha}$$
$$\frac{\delta}{s} = k^{\alpha-1}$$
$$k = \left(\frac{\delta}{s}\right)^{\frac{1}{\alpha-1}}$$

$\text{Since it is the first case where } \mathrm{k} < 1, \text{we need to verify whether } \left(\frac{\delta}{s}\right)^{\frac{1}{\alpha-1}} < 1 \text{ or not}$

$\text{Since } \delta > \mathrm{s} \text{ (see question's assumption)}$

$$\Longrightarrow \frac{\delta}{s} > 1$$
$$\Longrightarrow \left(\frac{\delta}{s}\right)^{\frac{1}{\alpha-1}} < 1 \qquad (\text{because } \alpha < 1)$$
$$(\mathit{Satisfying\ the\ condition})$$

For the second case:

$$\delta k = s\lambda k^{\alpha}$$
$$\frac{\delta}{\lambda s} = k^{\alpha-1}$$
$$k = \left(\frac{\delta}{\lambda s}\right)^{\frac{1}{\alpha-1}}$$

$\text{Since it is the second case where } \mathrm{k} < 1, \text{we need to verify whether } \left(\frac{\delta}{\lambda s}\right)^{\frac{1}{\alpha-1}} > 1 \text{ or not}$

$\text{Since } \delta < \lambda\mathrm{s} \text{ (see question's assumption)}$

$$\Longrightarrow \frac{\delta}{\lambda s} > 1$$
$$\Longrightarrow \left(\frac{\delta}{\lambda\, s}\right)^{\frac{1}{\alpha-1}} > 1 \quad (\text{because } \alpha < 1)$$
$$(\mathit{Satisfying\ the\ condition})$$

So if the capital per capita level falls below 1, the economy will be stuck at the lower steady state, S_A.