

Practice Problem Set 9
Public Goods (C.37) Asymmetric Information (C.38)

Question 9.1

(Public Goods) Two individuals W(illiam) and M(ary) have identical utility functions defined on X private good (expenditure on candies) and Y public good (expenditures on CDs). The utility of individual i is $U_i = 2 \log X_i + \log Y$, where $Y = Y_W + Y_M$, Y_i is the contribution of individual i , and \log denotes natural log. The budget of individual i is $X_i + Y_i = 100$.

- (a) Compute the amount of public good privately produced.
(b) Compute the socially optimal level of public good.

Answer

(a) W chooses Y_W to maximize $U_W = 2 \log X_W + \log(Y_W + Y_M)$. The FOC is $-2/(100 - Y_W) + 1/(Y_W + Y_M) = 0$.

By symmetry, $-2/(100 - Y_M) + 1/(Y_W + Y_M) = 0$. Hence, $Y_W = Y_M = 20$.

(b) The socially optimal condition is $MRS_W + MRS_M = MC$ where $MC = 1$ and $MRS_i = MU_i(Y)/MU_i(X)$. We have $\frac{1/Y}{2/X_W} + \frac{1/Y}{2/X_M} = 1$. Since $X_W + X_M + Y = 200$, $Y = 66.7$.

Question 9.2

Bonnie and Donna are business partners. Whenever they work, they have to work together. Their only source of income is profit from their partnership. Their total profit per year is $50H$, where H is the number of hours that they work per year. Since they must work together, they both must work the same number of hours. Both hate working, so we may treat the variable “hours of labor” like a public “bad” for the two person community consisting of Bonnie and Donna (nonetheless, they work because they get an income in return). Bonnie’s utility function is $u_B = c_B - 0.02H^2$ and Donna’s utility function is $u_D = c_D - 0.005H^2$, where c_B and c_D are the annual amounts of money spent on consumption for Bonnie and for Donna. Let the number of hours that they both work be H . Find the Pareto optimal number of hours for Bonnie and Donna to work.

Answer

In this question, we have a public bad instead of a public good, but the way to solve the problem is conceptually the same.

For Pareto efficiency, we max u_B subject to $u_D = \text{constant}$ and $c_B + c_D = 50H$. Differentiating the Lagrangian function with respect to c_B , c_D , and H (as shown in class) gives $H = 1000$.

Alternatively, suppose that there exists a benevolent social planner that seeks to maximize the utility of Bonnie and Donna. The problem that she confronts is to maximize $u_B + u_D = c_B - 0.02H^2 + c_D - 0.005H^2$ subject to $c_B + c_D = 50H$. Substitute $c_B + c_D = 50H$ into $u_B + u_D$ gives $u_B + u_D = 50H - 0.025H^2$. The first order condition is $50 - 0.05H = 0$, which gives us $H = 1000$.

Question 9.3

In Pot Hole, there are numerous used cars for sale. These cars vary in quality. Original owners know exactly what their cars are worth. All used cars look the same to potential buyers until they have bought them; then they find out the truth. Suppose that the quality of cars is uniformly distributed between 0 and 2,000. If a car is of quality X , its original owner will be willing to sell it for any price greater than X . If a buyer knew that a car was of quality

X , she would be willing to pay $X + 500$ for it. When buyers are not sure of the quality of a car, they are willing to pay its expected value, given their knowledge of the distribution of qualities on the market. Let X^* be some number between 0 and 2,000 and suppose that all cars of quality lower than X^* are sold, but original owners keep all cars of quality higher than X^* . Find X^* .

Answer

$(0 + X^*)/2 + 500 = X^*$. Hence, $X^* = \$1,000$.

Question 9.4

There are two types of electric pencil-sharpener producers. “High-quality” manufacturers produce very good sharpeners that consumers value at \$14. “Low-quality” manufacturers produce less good ones that are valued at \$8. At the time of purchase, customers cannot distinguish between a high-quality product and a low-quality product; nor can they identify the manufacturer. However, they can determine the quality of the product after purchase. The consumers are risk neutral. Let the probability of them getting a high-quality product be q . Each type of manufacturer can manufacture the product at a constant unit cost of \$11.50. The market is competitive and there are a large number of potential entrants.

- (a) Suppose that the sale of low-quality electric pencil-sharpeners is illegal, so that the only items allowed to appear on the market are of high quality. What will be the equilibrium price?
- (b) Suppose that there were no high-quality sellers. How many low-quality sharpeners would you expect to be sold in equilibrium?
- (c) Could there be an equilibrium in which equal (positive) quantities of the two types of pencil sharpeners appear in the market?
- (d) Now we change our assumptions about the technology. Suppose that each producer can choose to manufacture either a high-quality or a low-quality pencil-sharpener, with a unit cost of \$11.50 for the former and \$11 for the latter, what would we expect to happen in equilibrium?
- (e) Assuming that each producer is able to make the production choice described in the last question, what good would it do if the government banned production of low-quality electric pencil-sharpeners?

Answer

- (a) \$11.50 since the market is competitive.
- (b) Sellers won't sell for less than \$11.50. Consumers won't pay that much for low-quality product. So in equilibrium there would be no sales.
- (c) No. Average willingness to pay would be $14q + 8(1-q) = 14(0.5) + 8(0.5) = \11 , which is less than the cost of production. So there would be zero trade.
- (d) No trade. Producers would produce the low-quality product since it has a lower production cost. If all producers produce low-quality output, costs will be \$11 and the willingness-to-pay for low quality is \$8.
- (e) If there is no ban, there will be no output and no consumers' surplus. If low-quality products are banned, then in equilibrium there is output and positive consumers' surplus.