Question 1

(15 marks) ***[Start this question on a new sheet of paper. Write down your matric card on all scripts. Do NOT write anything into examsoft.]***

Consider a two-period consumption-savings model, augmented with a government sector. Each consumer has preferences described by the utility function $u(c_1, c_2) = 0.4\ln(c_1) + 0.6\ln(c_2)$, c_1 is consumption in period one, c_2 is consumption in period two.

Suppose that both households and the government start with zero initial assets (i.e., $a_0 = 0$, and $b_0 = 0$), and that the real interest rate is 5 percent. Assume that the government purchases in the first period are 14 ($g_1 = 14$), the second period are 21 ($g_2 = 21$). In the first period, the government levies a lump-sum tax of 22 ($t_1 = 22$). Also assume that the tax collection is carried out at the beginning of each period. Finally, the real incomes of the consumer in the two periods are $y_1 = 50$ and $y_2 = 63$.

a) **What are lump-sum taxes in the period two (t_2), given the above information? [4 marks]**

$$g_1 + \frac{g_2}{1+r} = t_1 + \frac{t_2}{1+r}$$
$$14 + \frac{21}{1.05} = 22 + \frac{t_2}{1.05}$$
$$\frac{(21 - t_2)}{1.05} = 8$$
$$21 - t_2 = 8.4$$
$$t_2 = 12.6$$

b) **Compute the optimal levels of consumption in all two periods, as well as national savings in period one.** ***(your results should be rounded to 2 decimal places). (Note: it would be easier if you work out the general formulas for c_1 and c_2 before plugging in the numerical values).*** **[5 marks]**

$$L(c_1, c_2, \lambda) = 0.4\ln(c_1) + 0.6\ln(c_2) + \lambda\left[c_1 + \frac{c_2}{1+r} - (y_1 - g_1) - \frac{y_2 - g_2}{1+r}\right] \quad (7)$$

Maximize $L(c_1, c_2, \lambda)$:

$$\frac{\partial L(c_1, c_2, \lambda)}{\partial c_1} = \frac{0.4}{c_1} + \lambda = 0 \quad (8)$$

$$\frac{\partial L(c_1, c_2, \lambda)}{\partial c_2} = \frac{0.6}{c_2} + \frac{\lambda}{1+r} = 0 \quad (9)$$

$$\frac{\partial L(c_1, c_2, \lambda)}{\partial \lambda} = \left[c_1 + \frac{c_2}{1+r} - (y_1 - g_1) - \frac{y_2 - g_2}{1+r}\right] = 0 \quad (10)$$

From (8) and (9) we have:

$$\begin{cases} \dfrac{0.4}{c_1} + \lambda = 0 \Rightarrow \dfrac{0.4}{c_1} = -\lambda & (11) \\ \dfrac{0.6}{c_2} + \dfrac{\lambda}{1+r} = 0 \Rightarrow \dfrac{0.6}{c_2} = -\dfrac{\lambda}{1+r} & (12) \end{cases}$$

$$\Rightarrow \frac{0.4\,c_2}{0.6\,c_1} = \frac{-\lambda}{-\dfrac{\lambda}{1+r}} = 1+r \qquad (13)$$

$$\Rightarrow \frac{2}{3}\frac{\boldsymbol{c_2}}{\boldsymbol{c_1}} = \mathbf{1} + \boldsymbol{r}$$

$$\Rightarrow \boldsymbol{c_2} = \mathbf{1.5}(\mathbf{1} + \boldsymbol{r})\boldsymbol{c_1} \qquad (\mathbf{14})$$

$$c_1 + \frac{1.5(1+r)c_1}{1+r} - (y_1 - g_1) - \frac{y_2 - g_2}{1+r} = 0$$

$$c_1 + 1.5c_1 - (y_1 - g_1) - \frac{y_2 - g_2}{1+r} = 0$$

$$2.5c_1 = (y_1 - g_1) + \frac{y_2 - g_2}{1+r}$$

$$c_1 = \frac{1}{2.5}\left[(y_1 - g_1) + \frac{y_2 - g_2}{1+r}\right] \qquad (15)$$

And thus, $c_2 = 1.5 \cdot \frac{1+r}{2.5}\left[(y_1 - g_1) + \frac{y_2 - g_2}{1+r}\right]$

$$= \frac{3}{5}(1+r)\left[(y_1 - g_1) + \frac{y_2 - g_2}{1+r}\right] (16)$$

Plugging the values for r, y_1, g_1, g_2 into (15) and (16), we have:

$$c_1 = \frac{1}{2.5}\left[(50 - 14) + \frac{63 - 21}{1 + 0.05}\right] = \frac{1}{2.5}[36 + 40] = 30.4$$

$$c_2 = 1.5(1+r)c_1 = 1.5(1 + 0.05) \cdot 30.4 = 47.88$$

*always write c1 and c2 in terms of the variables first before plugging in numbers

c) Suppose that the government decides to maintain t_1 at 22, g_1 at 14 and g_2 at 21. However, the government chooses to balance its lifetime budget constraint by obtaining its period 2's tax revenue from a consumption tax of 20% on the period 2 consumption.

1. **For this tax plan to work, what would the government require c_2 and c_1 to be?**

2. **Without working out the math, do you think the combination of c_2 and c_1 found in part c.1 is the optimal choice from the household point of view? Explain.**

(1) The government budget is now:

$$g_1 + \frac{g_2}{1+r} = t_1 + \frac{0.2c_2}{1+r}$$

$$\Rightarrow c_2 = \frac{1+r}{0.2}\left(g_1 + \frac{g_2}{1+r} - t_1\right)$$

$$\Rightarrow c_2 = \frac{1.05}{0.2}\left(14 + \frac{21}{1.05} - 22\right) = 5.25(12) = \mathbf{63}$$

Thus the amount of c_2 required for the tax plan to work is 63.

The consumer budget is:

$$c_1 + \frac{c_2(1+0.2)}{1+0.05} = (50-22) + \frac{63}{1+0.5}$$

$$c_1 + \frac{63(1+0.2)}{1+0.05} = (50-22) + \frac{63}{1+0.5}$$

$$c_1 = 28 - 60 * (1.2) + 60 = 28 - 60 * 0.2 = 28 - 12 = \mathbf{16}$$

(2) No it is NOT optimal. These amounts are what the government wants to see in order for their budget in balance. Students can work out to see that from the household point of view, the optimal c1 and c2 are different.

***Not optimal, rmb the credit constraint problem from tutorials**

***consumption tax is distortionary, distorts consumers preferences for each period or btwn diff goods**

Question 2

(15 marks) ***[Start this question on a new sheet of paper. Write down your matric card on all scripts. Do NOT write anything into examsoft.]***

Consider a representative consumer, who lives forever, at time t seeking to maximize the sum of discounted lifetime utility from t onwards. Suppose that the consumer's utility function for a period, t, is given by $u(c_t, M_t/P_t) = \ln(c_t) + \theta \ln(M_t/P_t)$; where θ is a positive constant.

In this model, the notation is as in class: at time t, a_t is the holdings of real asset (a "stock") at the end of period t, S_t is its nominal price in period t (information on dividends is provided below), B_t is the amount of bonds issued by the government purchased in period t which matures in one year with the face value normalized to 1, $P^b{}_t$ is the nominal price of a bond, M_t is the nominal money holdings at beginning of period t+1/end of period t, Y_t is the nominal income, c_t is the real consumption, P_t is the unit price of consumption, β is the subjective discount rate, i_t is the nominal interest rate, and π_t is the inflation rate.

In any period, t, the consumer receives Y_t as nominal income, consumes c_t units of goods, and holds a_t real assets at the end of period t, B_t bonds and M_t amount of nominal money.

For part a, b and c, assume that the dividend paid out per stock in period t is D_t.

a) Set up the consumer's maximization problem. [3 marks]

***combination of tutorial 2 and 3**

$$\max_{\{c_t, a_t, B_t, M_t^D\}_{t=0}^{\infty}} \sum_{t=0}^{\infty} \beta^t u\left(c_t, \frac{M_t^D}{P_t}\right)$$

$$\text{subject to} \quad \underbrace{P_t c_t + S_t a_t + M_t^D + P_t^b B_t}_{\text{outflows}} = \underbrace{Y_t + (S_t + D_t) a_{\overset{t-1}{\cancel{t+1}}} + M_{t-1}^D + B_{t-1}}_{\text{inflows}} \qquad (1)$$

problem

Where $u\left(c_t, \frac{M_t^D}{P_t}\right) = \ln(ct) + \theta \ln(Mt/Pt);$

b) Using Lagragian method, derive the money demand curve from this consumer's maximization problem. (*Note that you have to show the maths, not just the result.*) [5 marks]

$$\mathcal{L}(\dots) = \sum_{t=0}^{\infty} \beta^t u\left(c_t, \frac{M_t^D}{P_t}\right) - \sum_{t=0}^{\infty} [P_t c_t + S_t a_t + M_t^D + P_t^b B_t - Y_t - (S_t + D_t) a_{t+1} - M_{t-1}^D - B_{t-1}] \qquad (2)$$

$\lambda^{t+1} \beta^t$

problem

And w.r.t B_t,

$$\mathcal{L}_{B_t} = -\lambda_t \beta^t P_t^b + \beta^{t+1} \lambda_{t+1} = 0$$
$$\Rightarrow \lambda_t P_t^b = \beta \lambda_{t+1}$$
$$\Rightarrow \frac{\lambda_t}{\lambda_{t+1}} = \beta / P_t^b \qquad (3)$$

And w.r.t M_t^D,

$$\mathcal{L}_{M_t^D} = \frac{u_2\left(c_t, \frac{M_t^D}{P_t}\right)}{P_t} - \lambda_t + \beta \lambda_{t+1} = 0$$

$$\Rightarrow u_2\left(c_t, \frac{M_t^D}{P_t}\right) = P_t\lambda_t - P_t\beta\lambda_{t+1} \tag{4}$$

From equation (3) and (6), we have:

$$\frac{u_2\left(c_t, \frac{M_t^D}{P_t}\right)}{u_1\left(c_t, \frac{M_t^D}{P_t}\right)} = \frac{P_t\lambda_t - P_t\beta\lambda_{t+1}}{\lambda_t P_t}$$

$$= 1 - \frac{\beta\lambda_{t+1}}{\lambda_t} \tag{5}$$

Substituting (5) into (7):

$$\frac{u_2\left(c_t, \frac{M_t^D}{P_t}\right)}{u_1\left(c_t, \frac{M_t^D}{P_t}\right)} = 1 - P_t^b$$

$$= 1 - \frac{1}{1+i_t^b} = \frac{i_t^b}{1+i_t^b} = \frac{i_t}{1+i_t} \qquad \because i_t^b = i_t \tag{6}$$

where i_t is the market interest rate and i_t^b is the bond interest rate.

Since $u\left(c_t, \frac{M_t^D}{P_t}\right) = \ln(c_t) + \theta \ln(\frac{M_t}{P_t})$;

$$u_2 = \theta \cdot \left(\frac{P_t}{M_t}\right)$$

$$u_1 = \frac{1}{c_t}$$

$$\frac{u_2\left(c_t, \frac{M_t^D}{P_t}\right)}{u_1\left(c_t, \frac{M_t^D}{P_t}\right)} = \frac{\theta \cdot \left(\frac{P_t}{M_t}\right)}{\frac{1}{c_t}} = \left[\frac{\theta c_t}{\left(\frac{M_t}{P_t}\right)}\right] = \frac{i_t}{1+i_t}$$

$$\Rightarrow \frac{M_t}{P_t} = \theta\left(\frac{1+i_t}{i_t}\right)c_t$$

c) Suppose that, thinking they can increase the sales of bonds by giving discount on the bond price, the government plans on giving a 10% discount on whatever the market bond price is. [marks missing]

a. What is the money demand curve if the government carries out this plan?

b. How is price of bonds related to the nominal interest rate in this case?

The money demand curve is the same. We can refer back to the Lagrange function:

$$\mathcal{L}(\dots) = \sum_{t=0}^{\infty} \beta^t u\left(c_t, \frac{M_t^D}{P_t}\right)$$
$$- \sum_{t=0}^{\infty} \left[P_t c_t + S_t a_t + M_t^D + (\mathbf{1-0.1}) P_t^b B_t - Y_t - (S_t + D_t) a_{t+1} - M_{t-1}^D - B_{t-1}\right] \quad (7)$$

Then

$$\mathcal{L}_{B_t} = -\lambda_t \beta^t (\mathbf{1-0.1}) P_t^b P_t^b + \beta^{t+1} \lambda_{t+1} = 0$$
$$\Longrightarrow \lambda_t \cdot 0.9 \cdot P_t^b = \beta \lambda_{t+1}$$
$$\Longrightarrow \frac{\lambda_t}{\lambda_{t+1}} = \beta / 0.9 P_t^b \quad (8)$$

So:

$$\frac{u_2\left(c_t, \frac{M_t^D}{P_t}\right)}{u_1\left(c_t, \frac{M_t^D}{P_t}\right)} = 1 - 0.9 P_t^b$$

Now, $\frac{Facevalue - 0.9 P_t^b}{0.9 P_t^b} = i$

$$\frac{1 - 0.9 P_t^b}{0.9 P_t^b} = i$$
$$\Longrightarrow 0.9 P_t^b = \frac{1}{1+i}$$

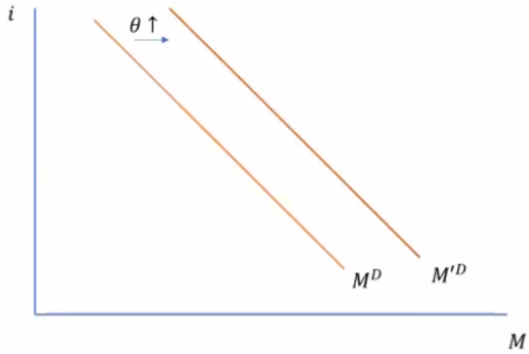
So

$$\frac{u_2\left(c_t, \frac{M_t^D}{P_t}\right)}{u_1\left(c_t, \frac{M_t^D}{P_t}\right)} = 1 - \frac{1}{1+i}$$

This is exactly the same expression as in part b

Intuition: Discount on bond price would not change anything since the people will rush to buy bonds and that pushes the price of bond up so that $0.9P_t^b$, which is the effective price of bond, would be the same as before. This is due to no arbitrage condition.

d) Suppose that θ increases for some reason, draw a diagram to show what happens to the Money Demand curve. Explain the economic intuition for your diagram. [marks missing]



Please note that showing the math to prove why M^D shifts right is not intuition.

Intuition: The preference towards money is also captured by θ. An increase in θ means the preference for money increases. As such, the demand for money increases.

Question 3:

(15 marks) *[Start this question on a new sheet of paper. Write your matric number on all scripts. Do NOT type anything in Examsoft.]*

(NOTE: before part a, everything is the same as tutorial 4 except for the value of interest rate; so you don't have to spend too much time reading).

In studying the Fiscal Theory of Inflation (FTI) and the Fiscal Theory of the Price Level (FTPL), the condition around which the analysis revolves is the present-value (lifetime) consolidated government budget constraint (GBC). As studied in Chapter 16, starting from the beginning of period *t*, the present-value consolidated GBC is

$$\frac{B_{t-1}}{P_t} = sr_t + \frac{sr_{t+1}}{1+r_t} + \frac{sr_{t+2}}{(1+r_t)(1+r_{t+1})} + \frac{sr_{t+2}}{(1+r_t)(1+r_{t+1})(1+r_{t+2})}\dots$$
$$+(t_t - g_t) + \frac{t_{t+1}-g_{t+1}}{1+r_t} + \frac{t_{t+2}-g_{t+2}}{(1+r_t)(1+r_{t+1})} + + \frac{t_{t+3}-g_{t+3}}{(1+r_t)(1+r_{t+1})(1+r_{t+2})}\dots$$

in which all of the notations are just as in Chapter 16.

In the steady state, suppose the following:

$r_t = r_{t+1} = r_{t+2} = \dots = r > 0$ (*i.e. steady state*)

$t_t - g_t = t_{t+1} - g_{t+1} = t_{t+2} - g_{t+2} = \dots = t - g$ (*i.e. steady state*)

$sr_t = sr_{t+1} = sr_{t+2} = \dots = sr$ (*i.e. steady state*)

$\mu_t = \mu_{t+1} = \mu_{t+2} = \dots = \mu$ (*i.e. steady state; μ is the nominal money growth*)

With these steady state assumptions, the present-value consolidated GBC from above simplifies considerably, to

$$\frac{B_{t-1}}{P_t} = sr + \frac{sr}{1+r} + \frac{sr}{(1+r)^2} + \frac{sr}{(1+r)^3}\dots$$

$$\frac{M_{t-1}}{P_{t-1}} = \frac{M_t}{P_t} = \frac{M_{t+1}}{P_{t+1}} = \frac{M_{t+2}}{P_{t+2}} = ... = 2\ trillions \qquad [eq.2]$$

a) Derive mathematically the long run inflation rate (denote it as π) ? [3 marks]

$$\frac{M_t}{P_t} = \frac{M_{t+1}}{P_{t+1}}$$

$$\frac{P_{t+1}}{P_t} = \frac{M_{t+1}}{M_t}$$

$$\frac{P_{t+1} - P_t}{P_t} = \frac{M_{t+1} - M_t}{M_t} = \mu = 0.1 = 10\%$$

b) Suppose further that

$M_t = 4$

Work out the long run seignorage revenue, i.e. *sr*._[4 marks]

$$sr_t = \frac{M_t - M_{t-1}}{P_t} = \frac{4 - \frac{4}{1.1}}{P_t}$$

Since: $\frac{M_t}{P_t} = 2 \rightarrow P_t = 2 \because M_t = 4$

Thus:

$$sr_t = \frac{M_t - M_{t-1}}{P_t} = \frac{4 - \frac{4}{1.1}}{2} = 2 - \frac{2}{1.1} = \frac{0.2}{1.1} = 0.182$$

c) Suppose that the Congress decided to increase real government spending in period t without changing tax plan and the future government spending plan. Upon hearing the news, the Central Bank announced that they were not willing to do anything else to accommodate the Congress' decision. What will happen to price level? Explain your answer. [marks missing]

The price has to increase to balance the intertemporal budget constraint. (Student should show the equation to show why price has to increase).

$$\frac{B_{t-1}}{\cancel{P_t}} \;Pt = \left(sr_t + \frac{sr_{t+1}}{1+r_t} + \frac{sr_{t+2}}{(1+r_t)(1+r_{t+1})} + \frac{sr_{t+2}}{(1+r_t)(1+r_{t+1})(1+r_{t+2})} ... \right)$$

$$pt + \left((t_t - g_t) + \frac{t_{t+1} - g_{t+1}}{1+r_t} + \frac{t_{t+2} - g_{t+2}}{(1+r_t)(1+r_{t+1})} + + \frac{t_{t+3} - g_{t+3}}{(1+r_t)(1+r_{t+1})(1+r_{t+2})} ... \right)$$

d) Continued from part C. A student claims that the decision of the Congress and the Central Bank does not have any effect on the seignorage revenue. Do you agree? Explain your answer. [marks missing]

We cannot agree. This is because the price level has increases and the change in nominal money supply remains the same as before (since the Central Bank is not going change anything), it should lower the real seignorage revenue.