EC2101: Microeconomic Analysis I

Where Are We?

- The firm's production function, $Q(L, K)$
- The firm in the short run:
  - Optimal choice of L and K
  - Cost curves, $C(Q_0)$
- The firm in the long run:
  - Optimal choice of L and K
  - Cost curves, $C(Q_0)$
- The firm's optimal choice of Q

Lecture 9

Theory of the Producer

- Cost in the Long Run
  - Isoquant & Isocost
  - Long-Run Cost-Minimizing Input Choice
  - Long-Run Cost Curves
  - Economies of Scale
- Short-Run Cost vs. Long-Run Cost

Short-Run vs. Long-Run Input Choice

- The price of labor L is w per unit.
- The price of capital K is r per unit.

- In the short run, capital is fixed at K_0.
  - Solve for the cost-minimizing quantity of L.

- In the long run, both L and K are variable.
  - Solve for the cost-minimizing quantity of L and K.

Cost in the Long Run

Isoquant & Isocost

Optimal L and K in the Long Run

- For any output level Q_0,
  we need to find the quantity of L and K that
  minimizes the total cost of production.

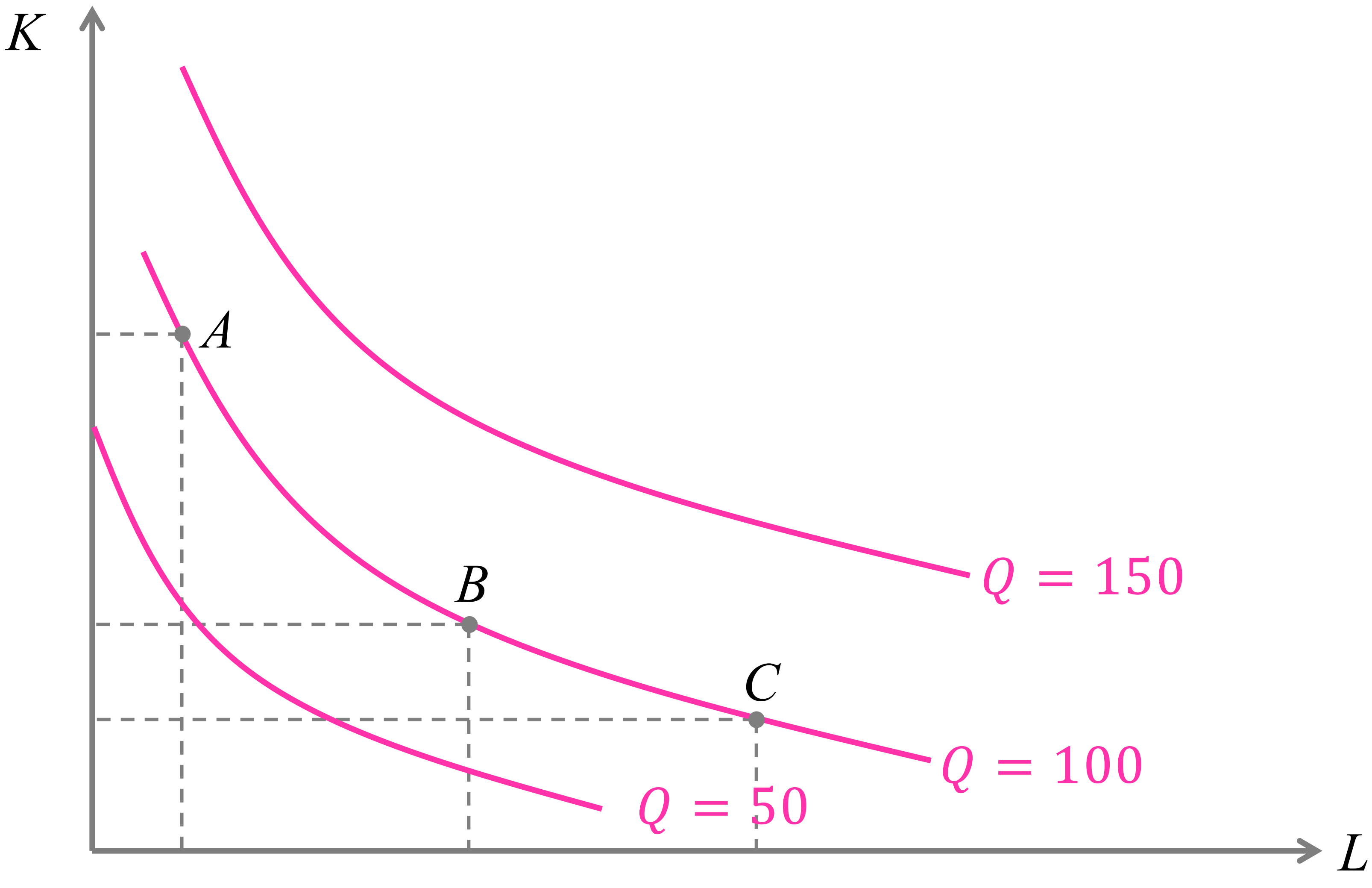
- We need a curve that represents output:
  - Something analogous to the indifference curve —
    isoquant.

- We need another curve that represents cost:
  - Something analogous to the budget line —
    isocost.

Isoquant

- Isoquant:
  - The set of all combinations of L and K that generate the same level of output Q.
  - The graphical representation of the production function, $Q(L, K)$, for a given level of output.

Isoquant: Graphical Representation

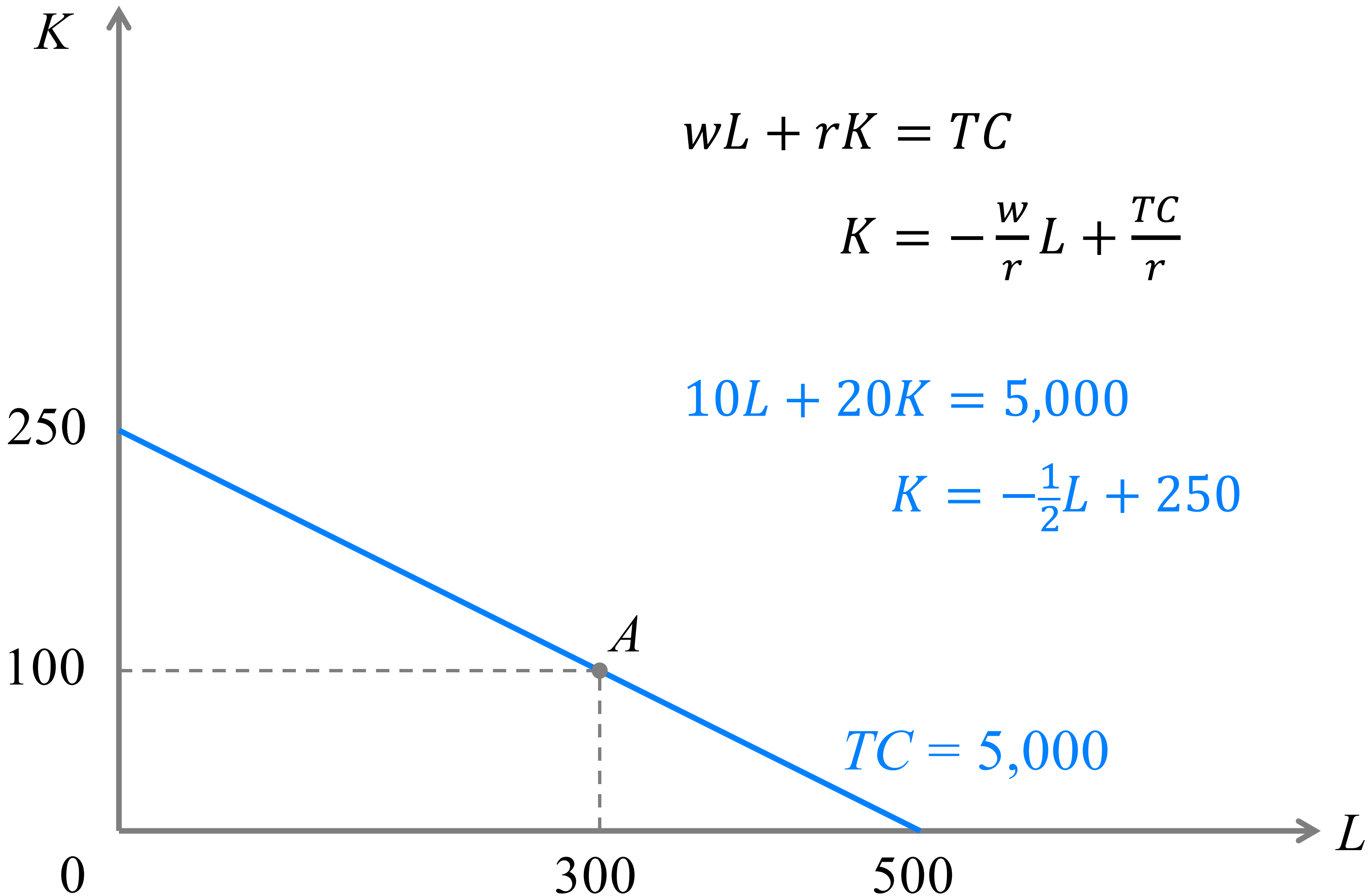


Isocost

- Isocost:
  - The set of all combinations of L and K that cost the firm the same amount of money.
  - $wL + rK = TC$
- E.g., suppose $w = 10$ and $r = 20.$
  - The isocost for a total cost of $\$5,000$ is:

$$10L + 20K = 5,000$$

Isocost: Graphical Representation



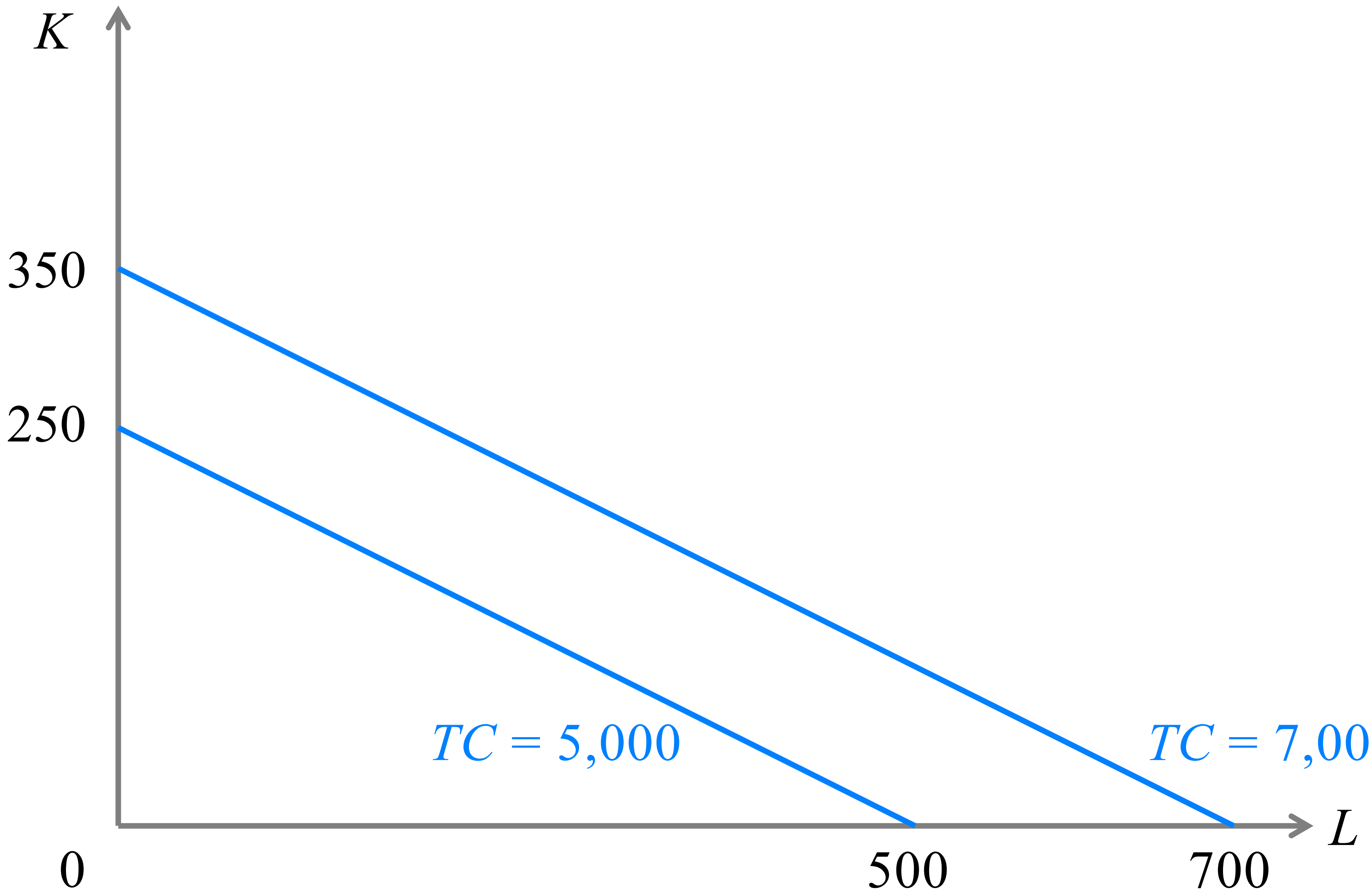
$$wL + rK = TC$$

$$K = -\frac{w}{r}L + \frac{TC}{r}$$

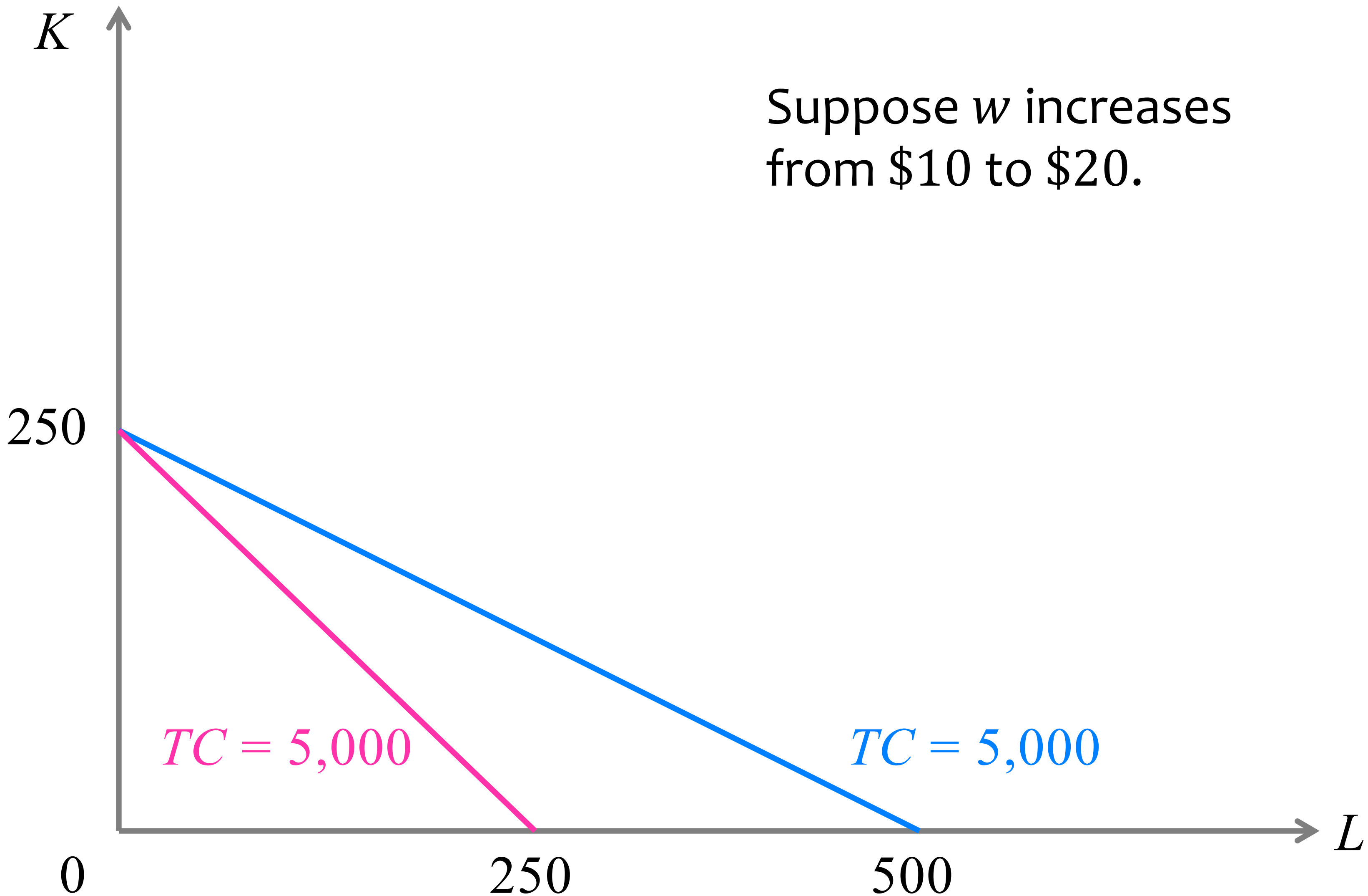
$$10L + 20K = 5{,}000$$

$$K = -\frac{1}{2}L + 250$$

Higher Isocost, Higher Total Cost



What if labor becomes more expensive?



Isoquant vs. Isocost

- If two points are on the same isoquant:
  - They generate the same amount of output.
- If two points are on the same isocost:
  - They cost the firm the same amount of money.
- Two points that are on the same isoquant are not necessarily on the same isocost.
- Likewise, two points that are on the same isocost are not necessarily on the same isoquant.

Summary

Consumer Theory vs. Producer Theory

Consumer Theory	**Producer Theory**
Indifference curve $U(x,y) = 10$	
Slope: $MRS_{x,y} = \frac{MU_x}{MU_y}$	
Budget line $p_x x + p_y y = M$	
Slope: $-\frac{p_x}{p_y}$	

Long-Run Cost-Minimizing Input Choice

How much labor and capital should the firm use?

- Assume the firm maximizes profit.

Profit = Total Revenue – Total Cost

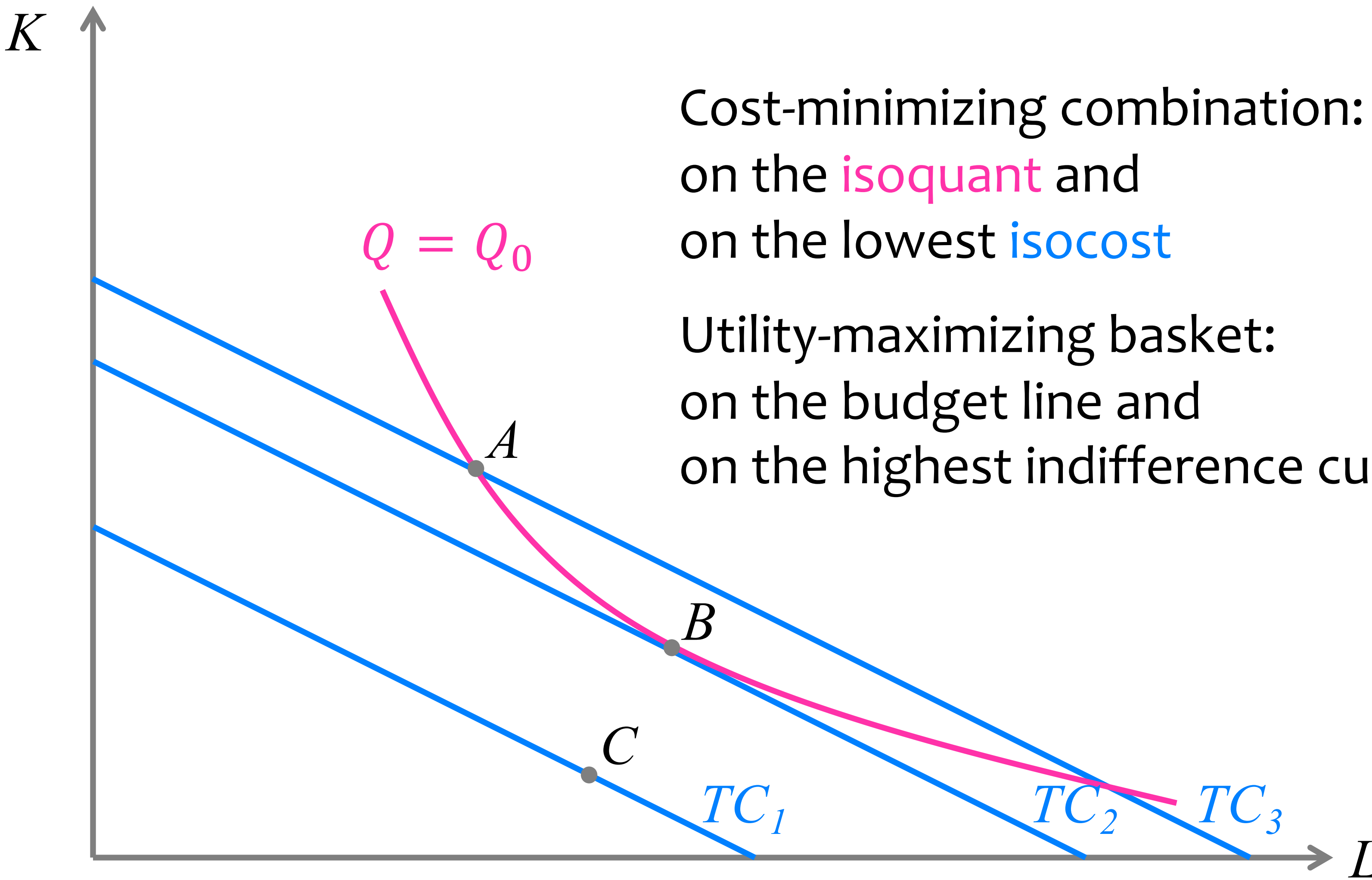
- For any output level Q_0,
  the firm chooses L and K to
  minimize the total cost of production.

- The constrained optimization problem is:

$$\min_{L,K} LRTC = wL + rK$$

$$subject\ to\ \ f(L,K) = Q_0$$

Which combination is cost-minimizing?



Cost-minimizing combination:
on the isoquant and
on the lowest isocost

Utility-maximizing basket:
on the budget line and
on the highest indifference curve

Cost-Minimizing Input Choice

- The cost-minimizing input choice
  - must be on the isoquant
  - must be on the lowest isocost
- Isoquant: $f(L, K) = Q_0$ (i)

- Tangency condition: $MRTS_{L,K} = \dfrac{w}{r}$

$$\frac{MP_L}{MP_K} = \frac{w}{r} \qquad \text{(ii)}$$

Equivalently, $$\frac{MP_L}{w} = \frac{MP_K}{r}$$

Cost-Minimizing Input Choice: Example

- Suppose the production function is $Q = KL$.
- Input prices are $w = 1$ and $r = 2$.
- What is the cost-minimizing choice of inputs if the firm wants to produce $Q = 8$?

Cost-Minimizing Input Choice: Example

- Isoquant:

$$Q = 8$$

$$KL = 8 \qquad \text{(i)}$$

- Tangency condition:

$$\frac{MP_L}{MP_K} = \frac{w}{r}$$

$$\frac{K}{L} = \frac{1}{2} \qquad \text{(ii)}$$

- Solving the two equations, we get

$$L = 4, K = 2$$

Summary

Consumer Theory vs. Producer Theory

Consumer Theory	**Producer Theory**
$\max_{x,y} U(x,y)$ s.t. $p_x x + p_y y = M$	
Budget line $p_x x + p_y y = M$	
Tangency condition $\frac{MU_x}{MU_y} = \frac{p_x}{p_y}$	

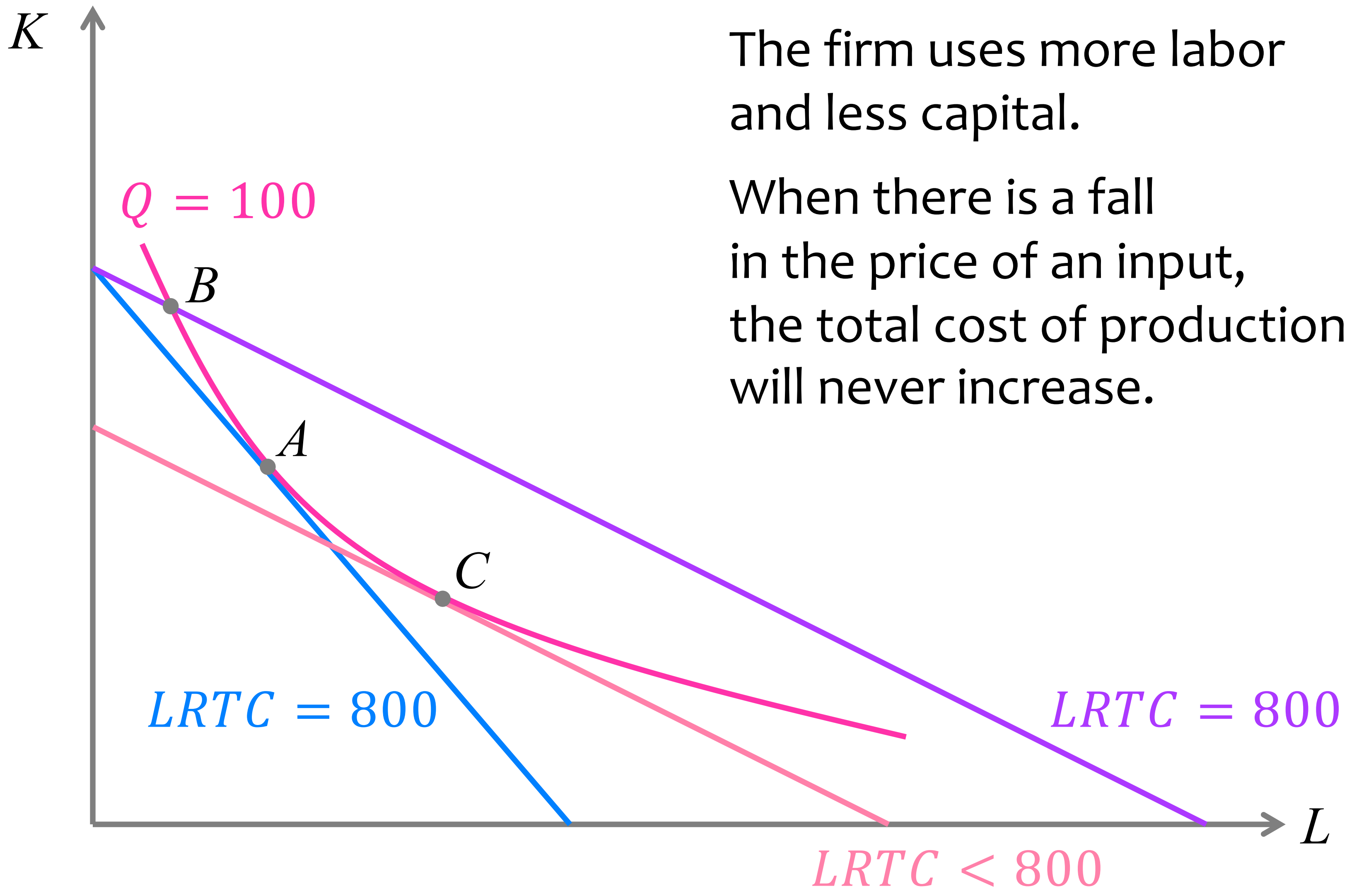
Long-Run Cost-Minimizing Input Choice:

Comparative Statics

Comparative Statics:
Changes in Input Prices and Output Level

- When input prices w and/or r change:
  - How does the cost-minimizing choice of L and K change?
- When the output level Q changes:
  - How does the cost-minimizing choice of L and K change?
- This analysis is called comparative statics.

Suppose the price of labor (w) falls

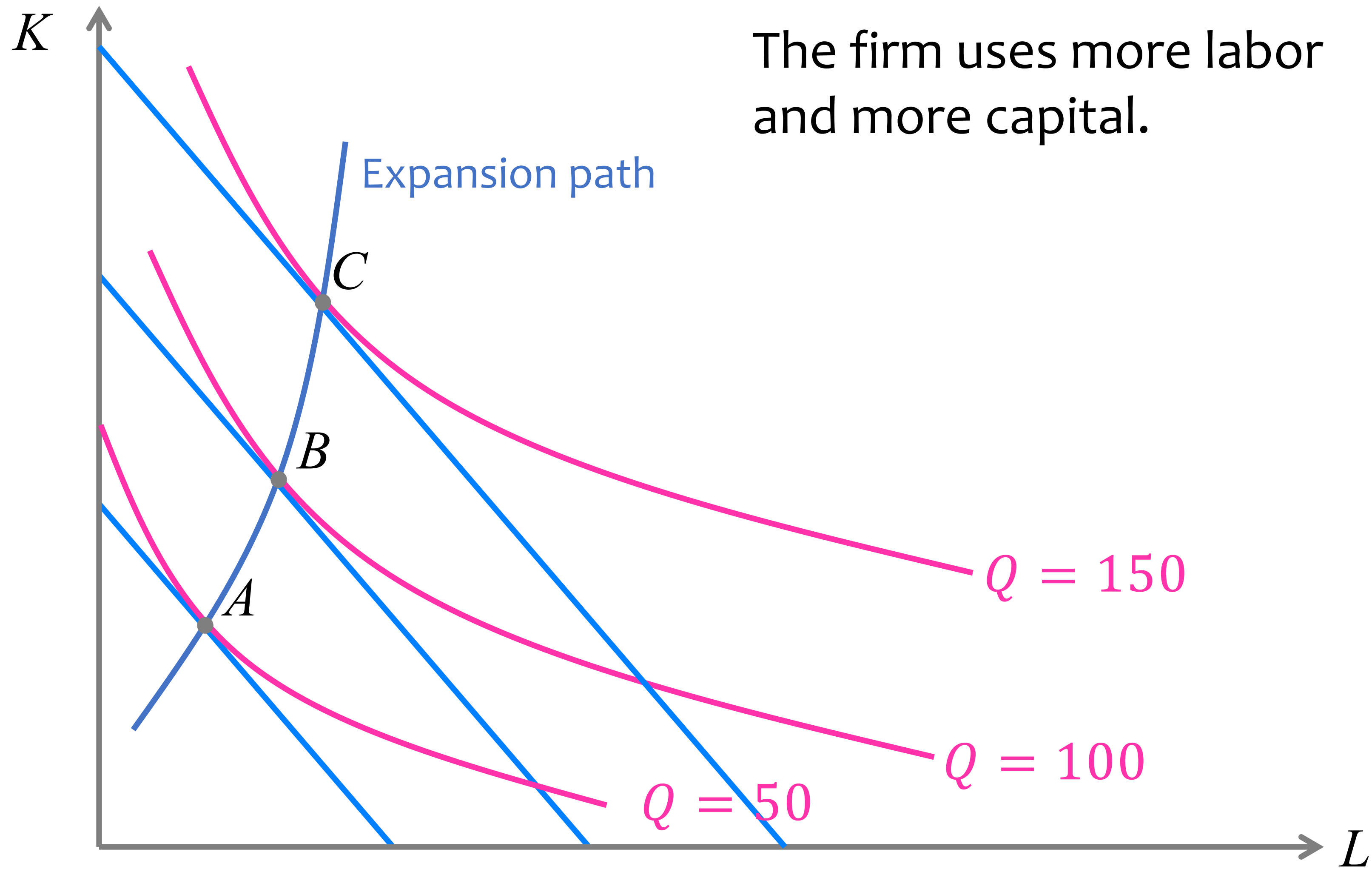


The firm uses more labor and less capital.

When there is a fall in the price of an input, the total cost of production will never increase.

Suppose output level (Q) increases

The firm uses more labor and more capital.



Long-Run Cost-Minimizing Input Choice:

Normal Input & Inferior Input

Normal Input and Inferior Input

- An input is a normal input if:
  - The cost-minimizing quantity of the input increases when output increases, holding input prices fixed.
- An input is an inferior input if:
  - The cost-minimizing quantity of the input decreases when output increases, holding input prices fixed.

Exercise 9.1

Expansion Path with an Inferior Input

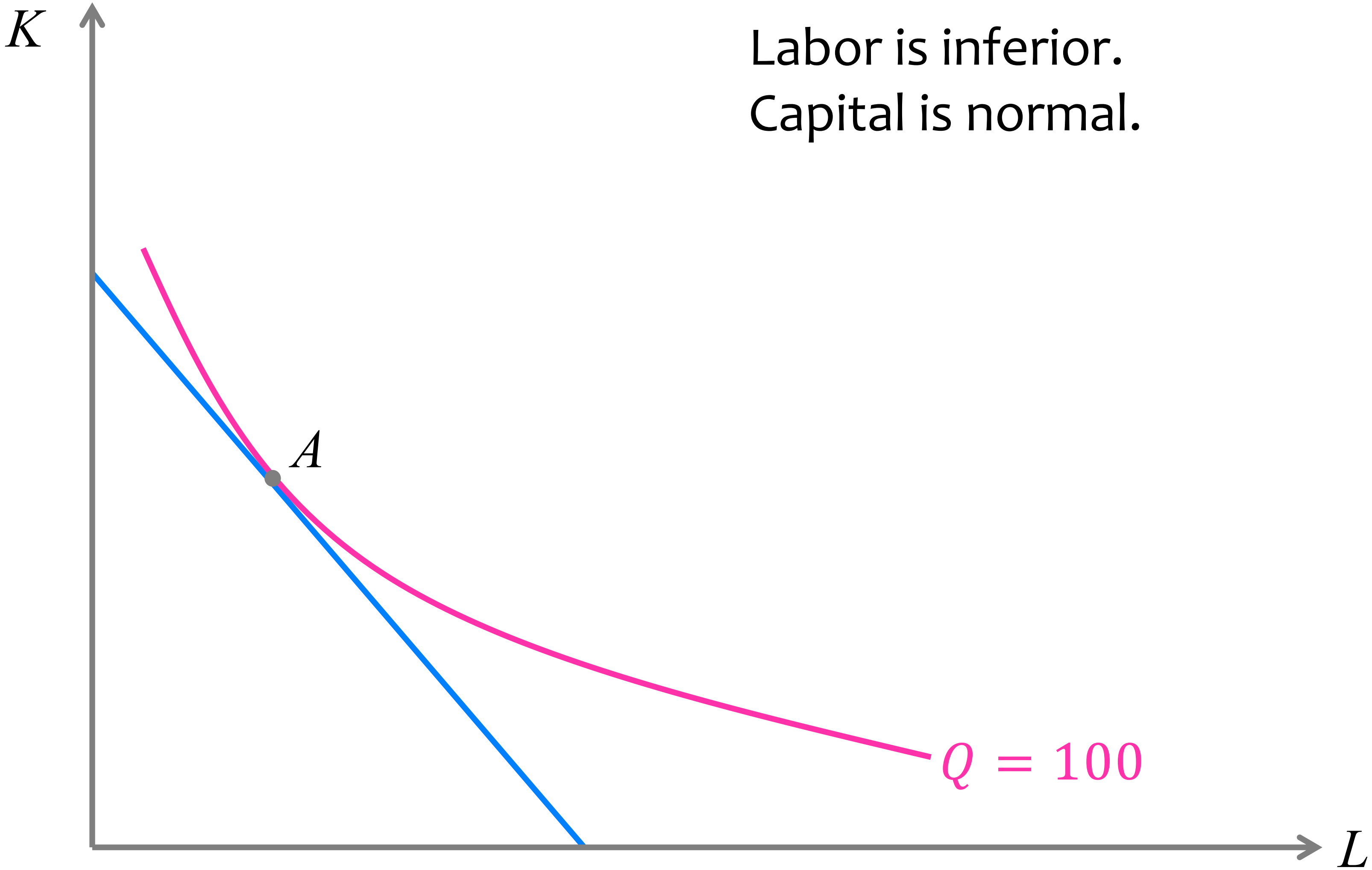
Suppose labor is inferior and capital is normal.

Is the long-run expansion path
downward sloping or upward sloping?

Exercise 9.1

Expansion Path with an Inferior Input

Labor is inferior.
Capital is normal.



Exercise 9.2

Two Inferior Inputs

Is it possible for both inputs — L and K — to be inferior?

Exercise 9.3

Giffen Input

Is it possible for labor to be a Giffen input?
I.e., when labor becomes cheaper,
to produce the same quantity of output,
the firm uses less labor.

A. Yes

B. No

C. I have no idea

Hint: What is a Giffen good in consumer theory?
Draw a graph showing the isoquant and isocost.

Exercise 9.3

Giffen Input

Long-Run Cost-Minimizing Input Choice:

Demand Functions for Labor and Capital

Demand Functions for Labor and Capital

- As the input prices w and/or r or the output level Q changes, the firm's cost-minimizing choice of L and K may also change.

- Demand function for labor L:
  - The cost-minimizing choice of labor L as a function of w, r, and Q.
  - $L(w, r, Q)$

- Demand function for capital K:
  - The cost-minimizing choice of capital K as a function of w, r, and Q.
  - $K(w, r, Q)$

Deriving Input Demand Functions: Example

- Suppose the production function is $Q = KL$.
- Input prices are w and r.
- To minimize cost, the firm chooses K and L such that

$$\frac{MP_L}{MP_K} = \frac{w}{r}$$

$$\frac{K}{L} = \frac{w}{r}$$

- This gives us:

$$K = \frac{w}{r}L \quad \text{and} \quad L = \frac{r}{w}K$$

Deriving Input Demand Functions: Example

- Substitute $K = \frac{w}{r}L$ into the production function $Q = KL$.

$$Q = KL$$
$$= \left(\frac{w}{r}L\right)L$$
$$Q = \frac{w}{r}L^2$$

- The demand function for labor is:

$$L(w, r, Q) = \sqrt{\frac{rQ}{w}}$$

Deriving Input Demand Functions: Example

- Substitute $L = \frac{r}{w}K$ into the production function $Q = KL$.

$$Q = KL$$
$$= K\left(\frac{r}{w}K\right)$$
$$Q = \frac{r}{w}K^2$$

- The demand function for capital is:

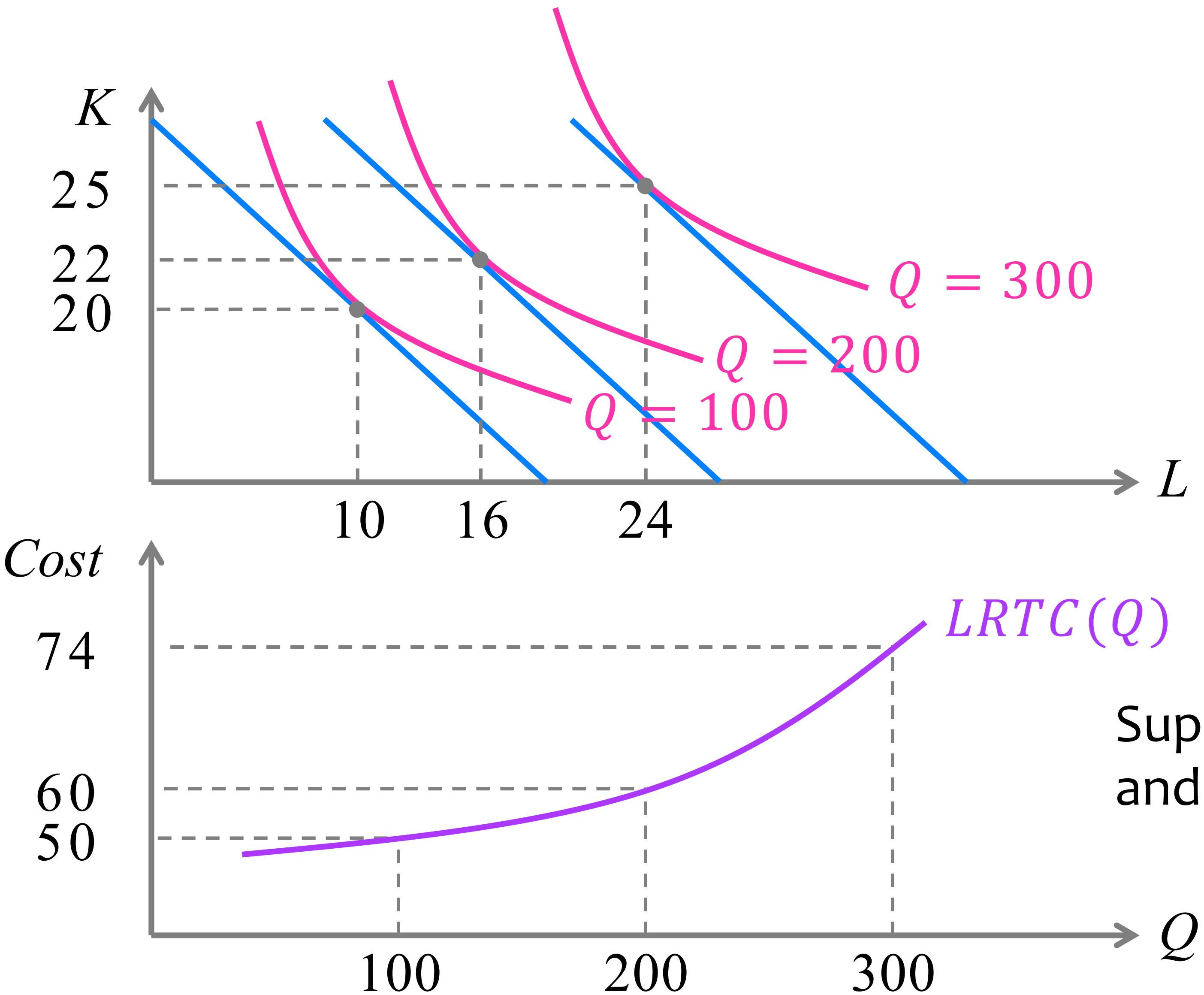
$$K(w, r, Q) = \sqrt{\frac{wQ}{r}}$$

Long-Run Cost Curves

Long-Run Cost Curves:

Long-Run Total Cost

Long-Run Total Cost Curve ($LRTC$)



Suppose $w = 1$ and $r = 2$.

Long-Run Total Cost Curve

- Long-run total cost curve:
  - Total cost in the long run as a function of Q, holding w and r fixed.
  - $LRTC(Q)$
- Every point on the long-run total cost curve represents the firm's minimized total cost for a given level of output, holding input prices fixed.
- There are no fixed costs in the long run.
  - $LRTC = 0$ when $Q = 0$.

Long-Run Total Cost Function

- Long-run total cost function:
  - Total cost in the long run as a function of Q, w, and r.
  - $LRTC(Q, w, r)$

Deriving Long-Run Total Cost Function: Example

- Suppose the production function is $Q = KL$.
- Input prices are w and r.
- We have already derived the cost-minimizing choice of labor and capital:

$$L(w, r, Q) = \sqrt{\frac{rQ}{w}}$$

$$K(w, r, Q) = \sqrt{\frac{wQ}{r}}$$

Deriving Long-Run Total Cost Function: Example

- The long-run total cost function is:

$$LRTC(Q,w,r) = wL + rK$$

$$= w\sqrt{\frac{rQ}{w}} + r\sqrt{\frac{wQ}{r}}$$

$$= \left(w\sqrt{\frac{rQ}{w}}\right) \times \frac{\sqrt{w}}{\sqrt{w}} + r\sqrt{\frac{wQ}{r}} \times \frac{\sqrt{r}}{\sqrt{r}}$$

$$= \frac{w\sqrt{wrQ}}{w} + \frac{r\sqrt{wrQ}}{r}$$

$$= 2\sqrt{wrQ}$$

Long-Run Cost Curves:

Long-Run Average Total Cost & Marginal Cost

Average Total Cost and Marginal Cost

- Long-Run Average Total Cost ($LRATC$)

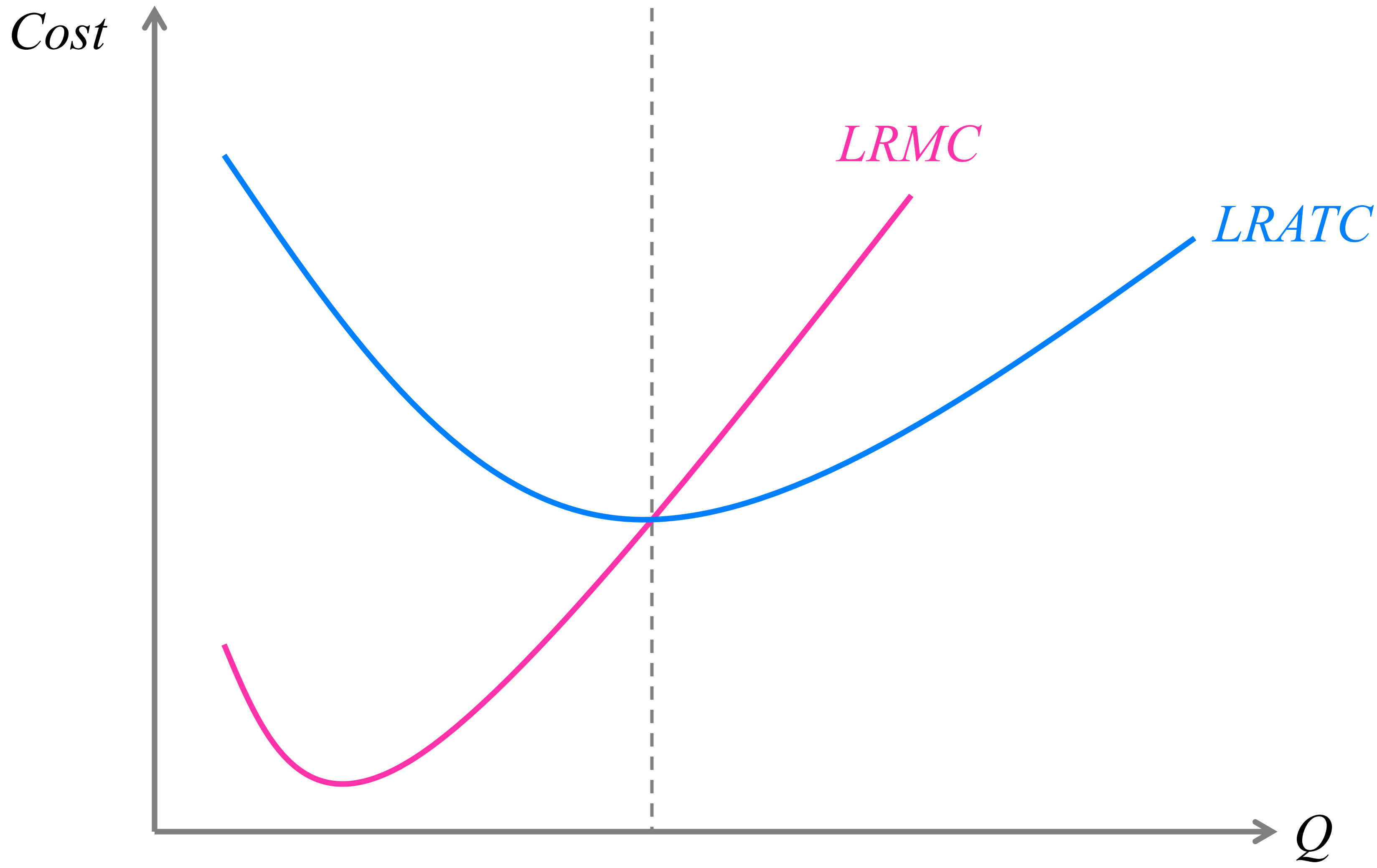
$$LRATC(Q) = \frac{LRTC(Q)}{Q}$$

- Long-Run Marginal Cost ($LRMC$)

$$LRMC(Q) = \frac{dLRTC(Q)}{dQ} = \frac{\Delta LRTC(Q)}{\Delta Q}$$

where ΔQ is extremely small.

$LRMC$ intersects $LRATC$ at the minimum of $LRATC$



Exercise 9.4

Cost in the Long Run

Suppose the production function is $Q = KL^2$. The price of labor is $w = 1$ and the price of capital is $r = 1$.

(a) Suppose the firm wants to produce $Q = 256$. What is the cost-minimizing choice of labor and capital in the long run?

(b) Derive the demand function for labor and the demand function for capital.

(c) Find the firm's long-run total cost curve, $LRTC(Q)$.

(d) Find the firm's long-run marginal cost curve, $LRMC(Q)$ and long-run average total cost curve, $LRATC(Q)$.

Exercise 9.4(a)

Cost in the Long Run

Exercise 9.4(b)

Cost in the Long Run

Exercise 9.4(c)

Cost in the Long Run

Exercise 9.4(d)

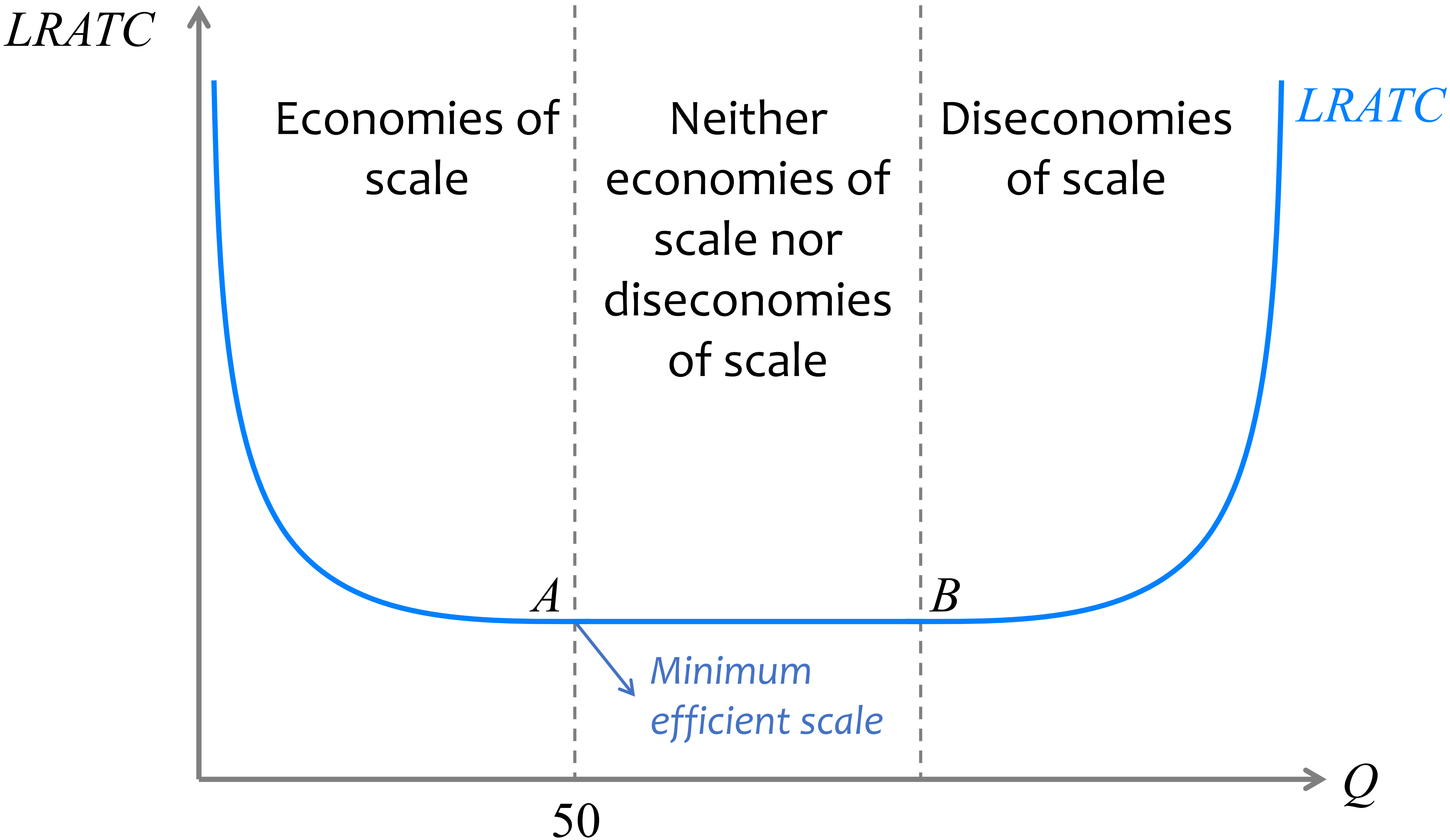
Cost in the Long Run

Economies of Scale

Economies of Scale

- Economies of Scale
  - If $LRATC$ is decreasing in Q.
- Diseconomies of Scale
  - If $LRATC$ is increasing in Q.

Economies of Scale



Sources of Economies of Scale

- Indivisible input
  - The size of some input cannot be scaled down.
  - The cost of the input gets spread out
    as the quantity of output increases.

Sources of Economies of Scale

- Returns to specialization
  - More workers can lead to specialization.
  - Specialization improves productivity.
  - E.g., suppose $w = 1$ and $r = 1$.
    - When $L = 2$ and $K = 1 \quad \Rightarrow \quad Q = 2$.
      - Then $LRTC(Q = 2) = 3$ and $LRATC(Q = 2) = 1.5$.
    - When $L = 3$ and $K = 1 \quad \Rightarrow \quad Q = 4$ because of increased specialization of labor.
      - Then $LRTC(Q = 4) = 4$ and $LRATC(Q = 4) = 1$.

Sources of Diseconomies of Scale

- Managerial diseconomies of scale
  - An increase of $\alpha\%$ in Q requires
    an increase of $\beta\% > \alpha\%$ in the firm's spending on managers.

Exercise 9.5

Economies of Scale

The following table shows long-run total costs for Firms *A*, *B*, and C. Does each firm experience economies of scale, diseconomies of scale, or neither?

Q	Firm A	Firm B	Firm C
1	$40	$11	$16
2	$50	$24	$32
3	$60	$39	$48
4	$70	$56	$64
5	$80	$75	$80

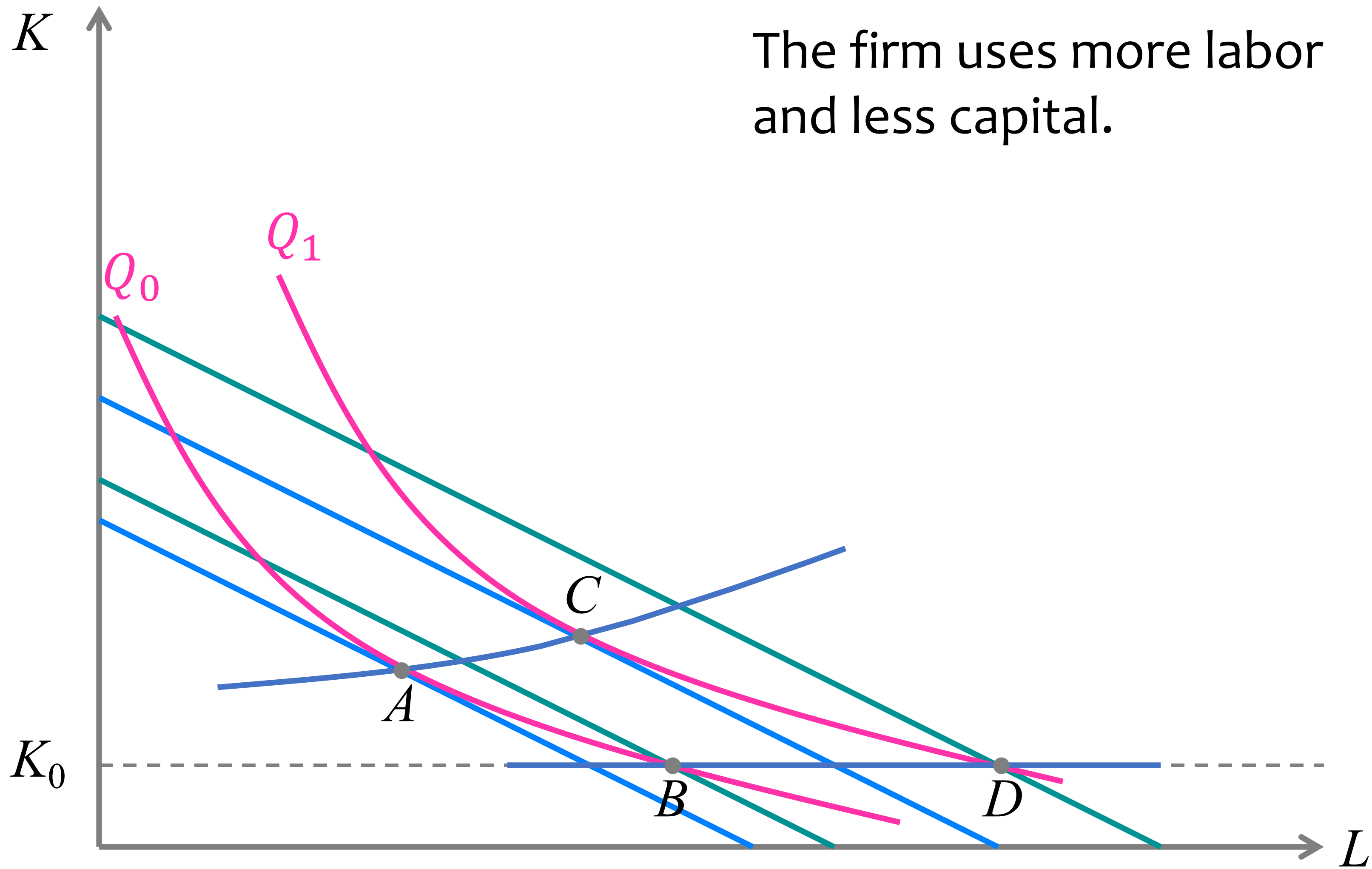
Exercise 9.5

Economies of Scale

Short-Run Cost vs. Long-Run Cost

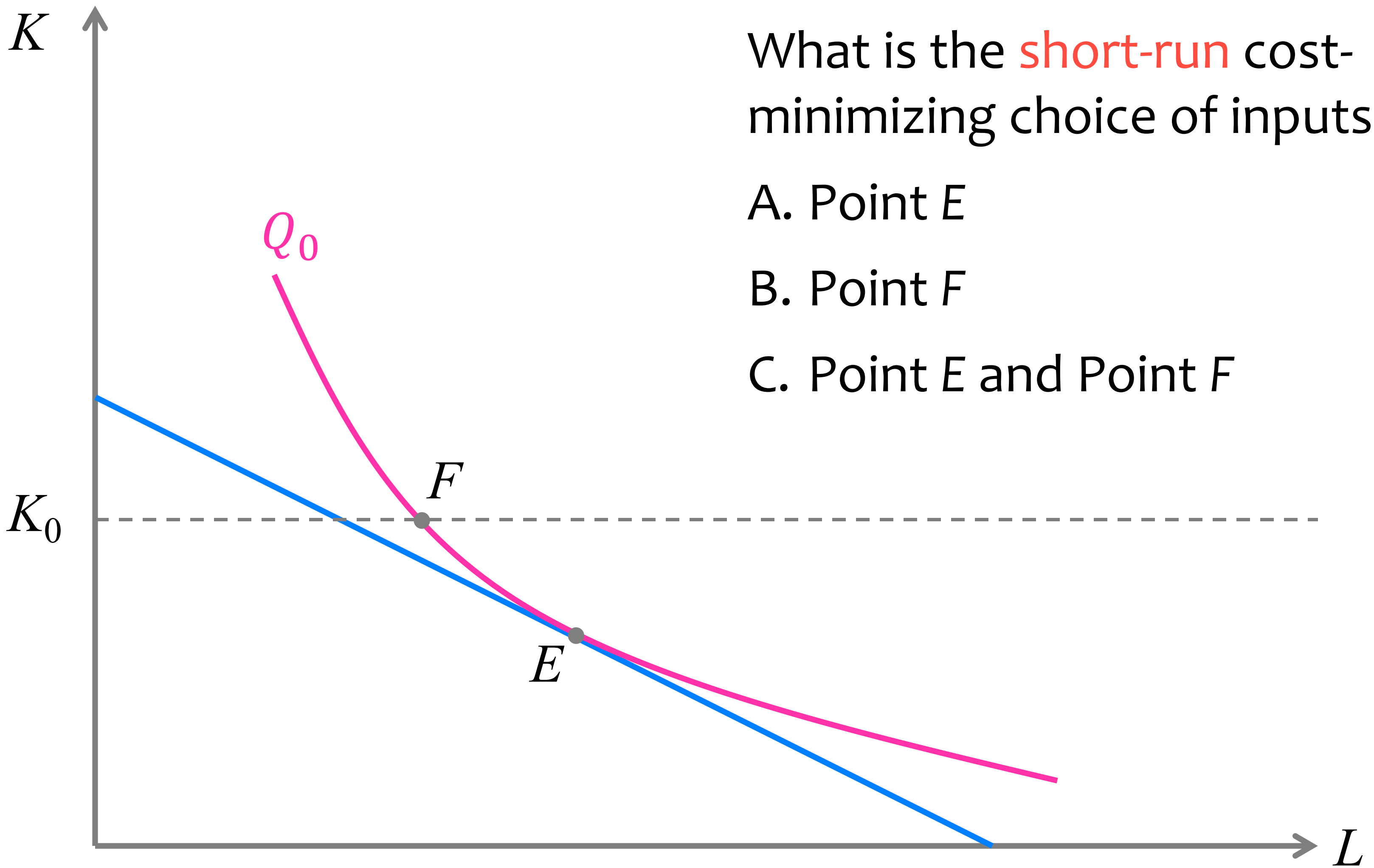
Short-run Expansion Path

The firm uses more labor and less capital.



Exercise 9.6

Short-Run Cost Minimization



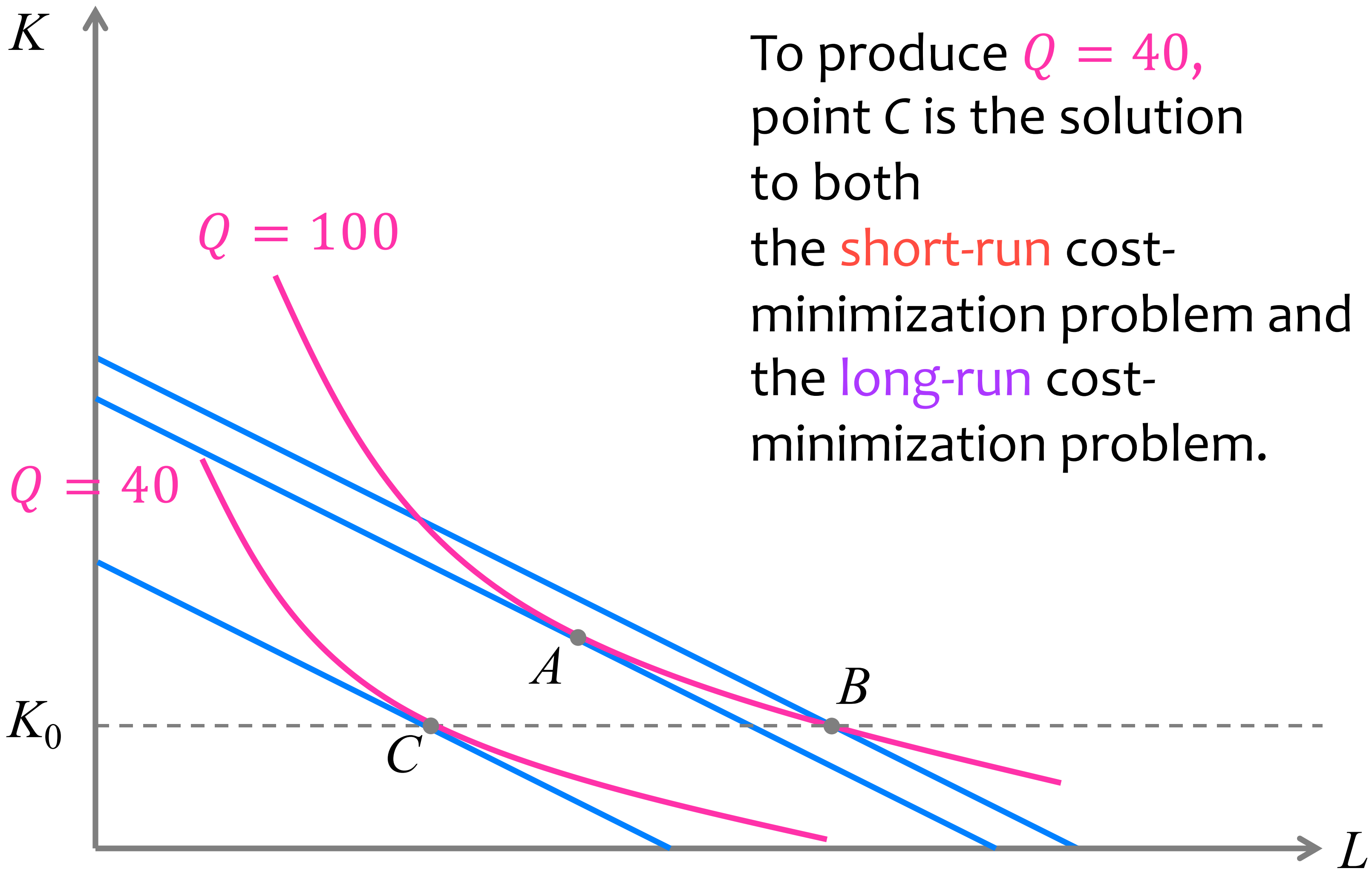
What is the short-run cost-minimizing choice of inputs?

A. Point *E*

B. Point *F*

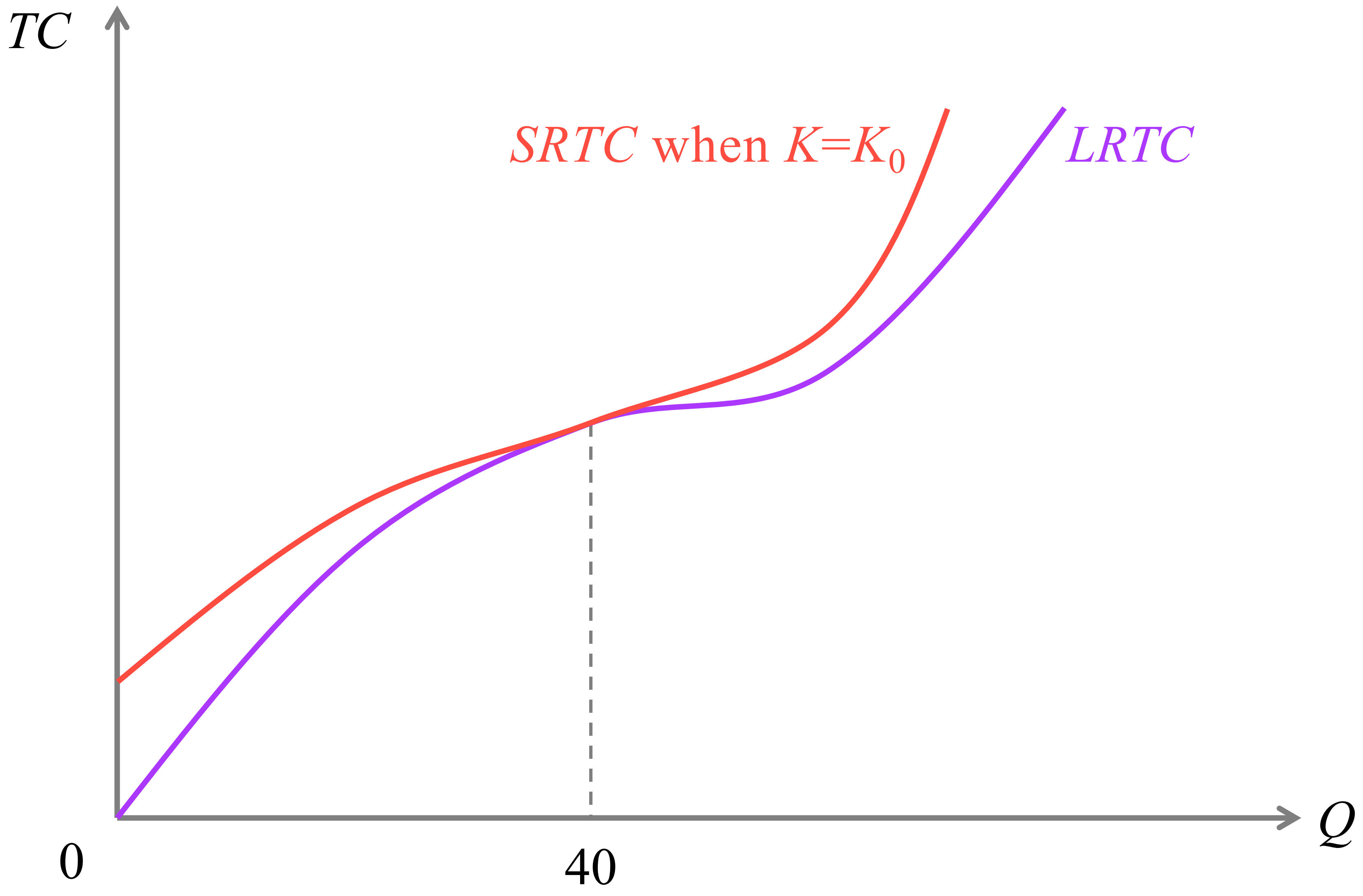
C. Point *E* and Point *F*

Is $SRTC = LRTC$ possible?

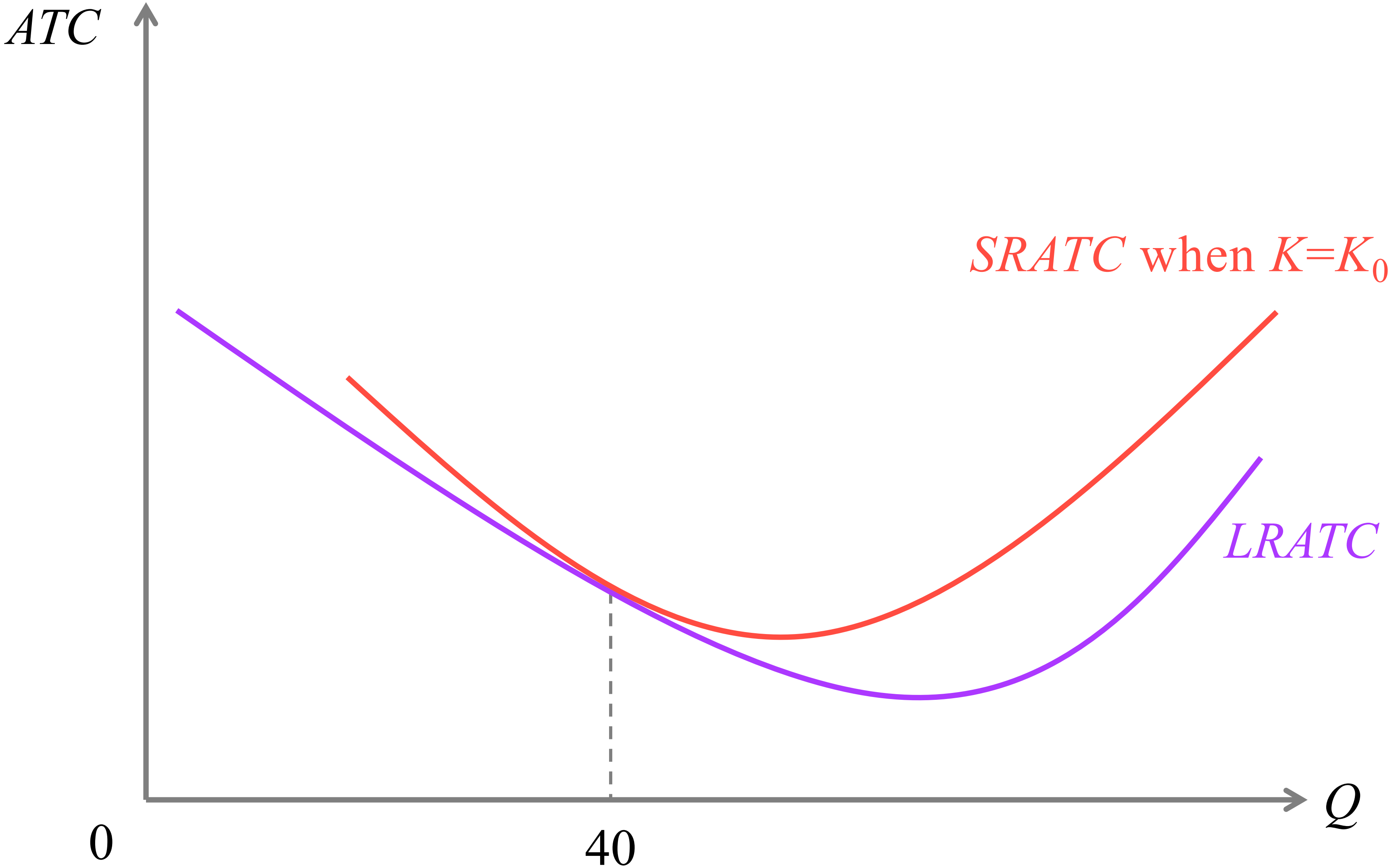


To produce $Q = 40$, point C is the solution to both the short-run cost-minimization problem and the long-run cost-minimization problem.

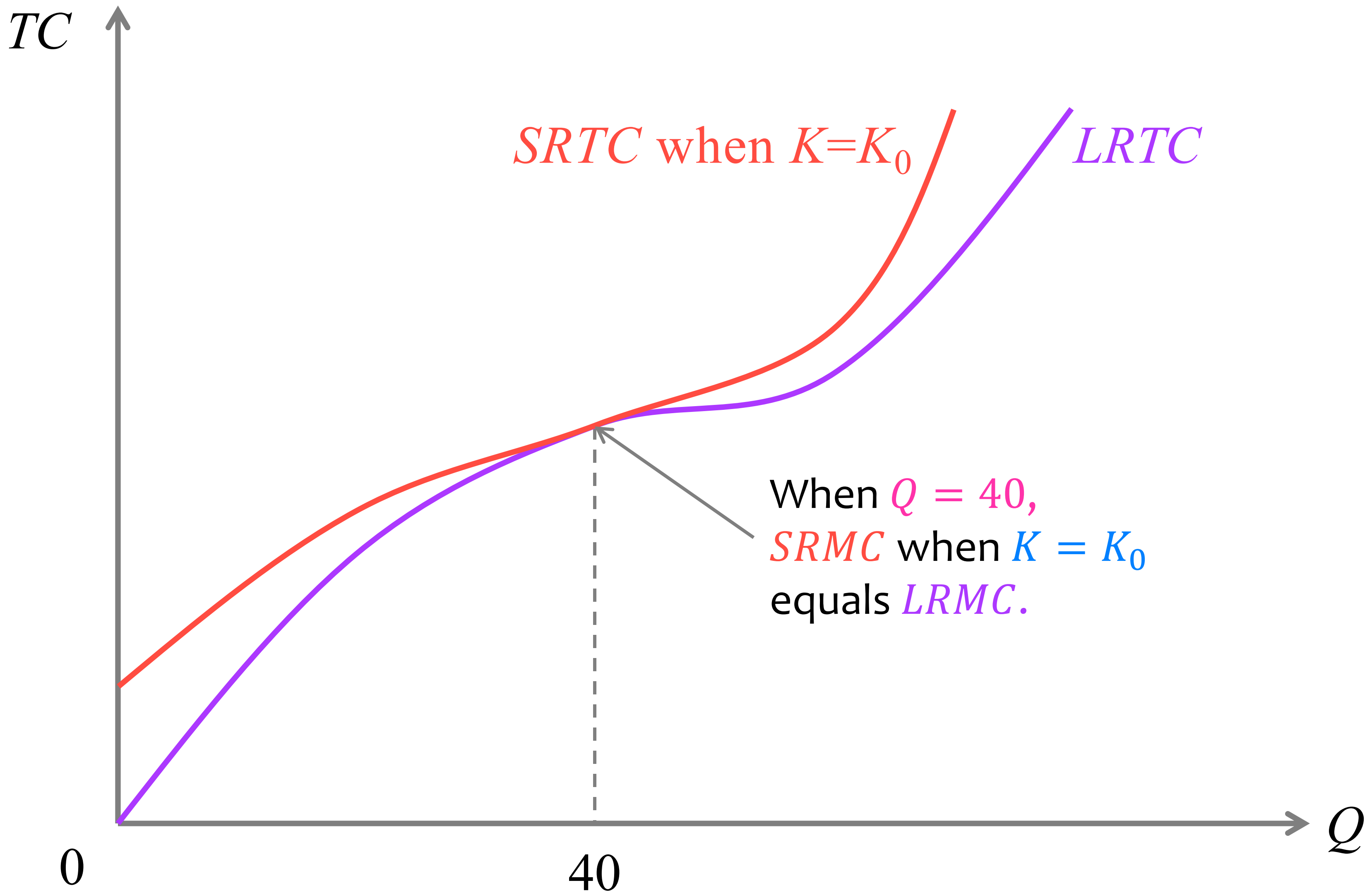
$SRTC$ cannot be lower than $LRTC$



$SRATC$ cannot be lower than $LRATC$



How about Marginal Cost?



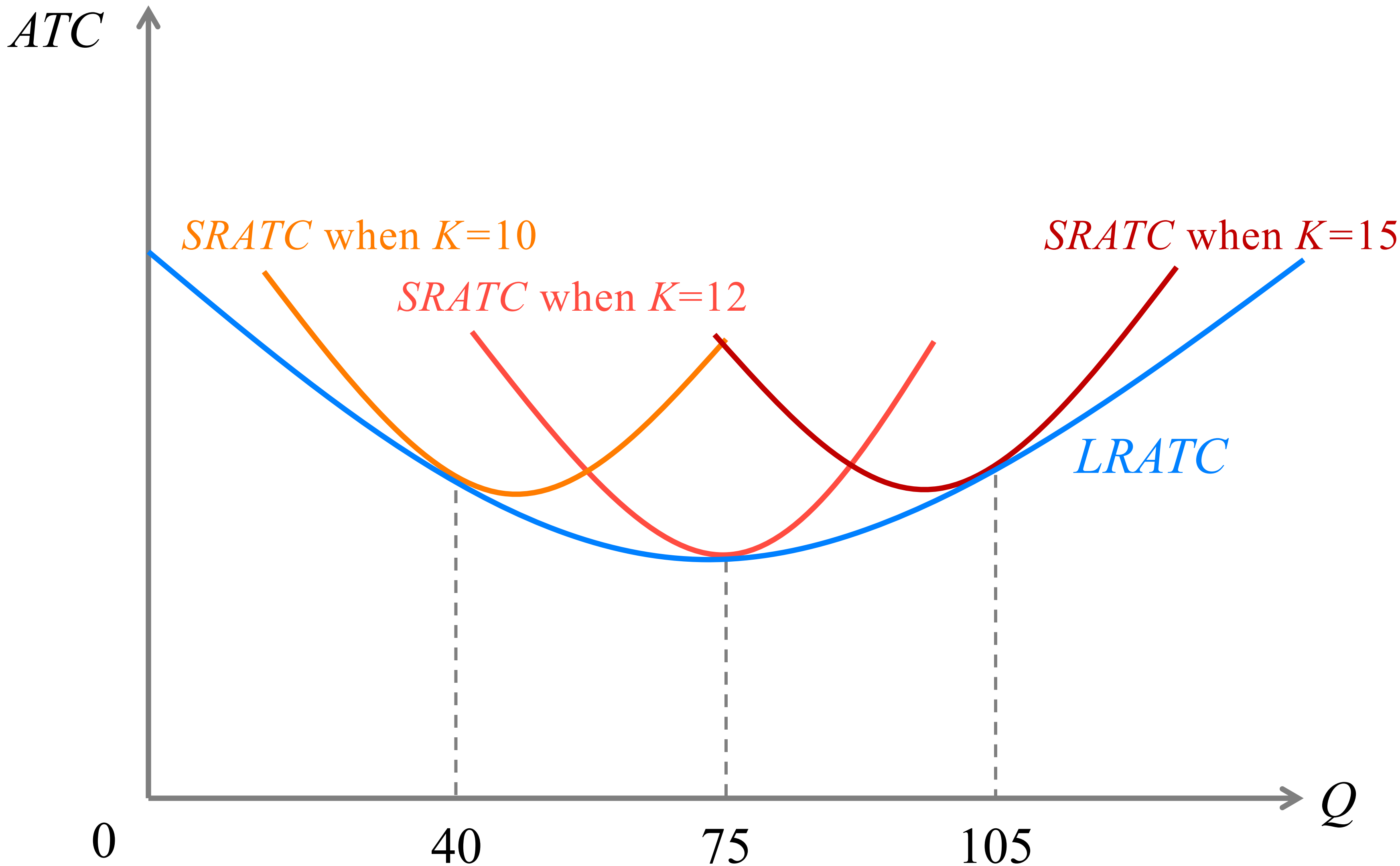
When is Long-Run Cost Equal to Short-Run Cost?

- Suppose in the short run, capital is fixed at K_0.
- Suppose when the firm produces Q_0,
  K_0 is the cost-minimizing choice of capital in the long run.
- When $Q = Q_0$:
  - The choice of inputs in the long run and
    the choice of inputs in the short run are the same.
    - $SRTC = LRTC$
    - $SRATC = LRATC$
    - $SRMC = LRMC$

$LRATC$ vs. $SRATC$

- Suppose if the firm produces $Q = 40$:
  - Its optimal choice of capital in the long run is $K = 10$.
- Suppose if the firm produces $Q = 75$:
  - Its optimal choice of capital in the long run is $K = 12$.
- Suppose if the firm produces $Q = 105$:
  - Its optimal choice of capital in the long run is $K = 15$.

$LRATC$ is the lower envelope of $SRATC$



When $LRATC$ is at its minimum

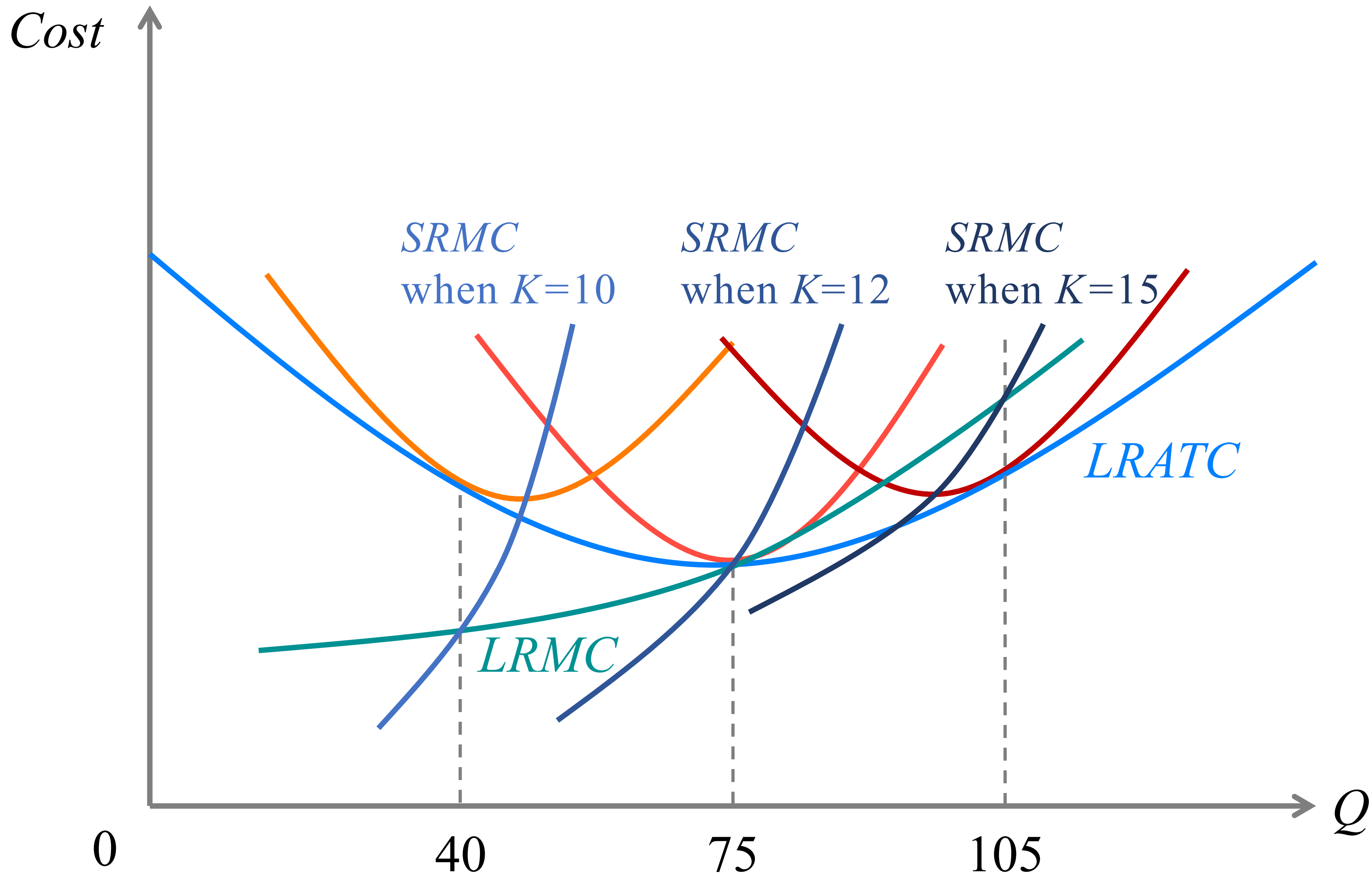
- When the firm produces $Q = 75$,
  its $LRATC$ is the lowest across all possible levels.

- At this output level,
  $SRATC$ when $K = 12$ must also reach its minimum.
  - When $LRATC$ is at its minimum, its slope is 0.
  - At the point where $SRATC$ is tangent to $LRATC$,
    $SRATC$ and $LRATC$ have the same slope.
  - Therefore, $SRATC$'s slope is also 0 at that tangency.
  - Thus, $SRATC$ is also at its minimum.

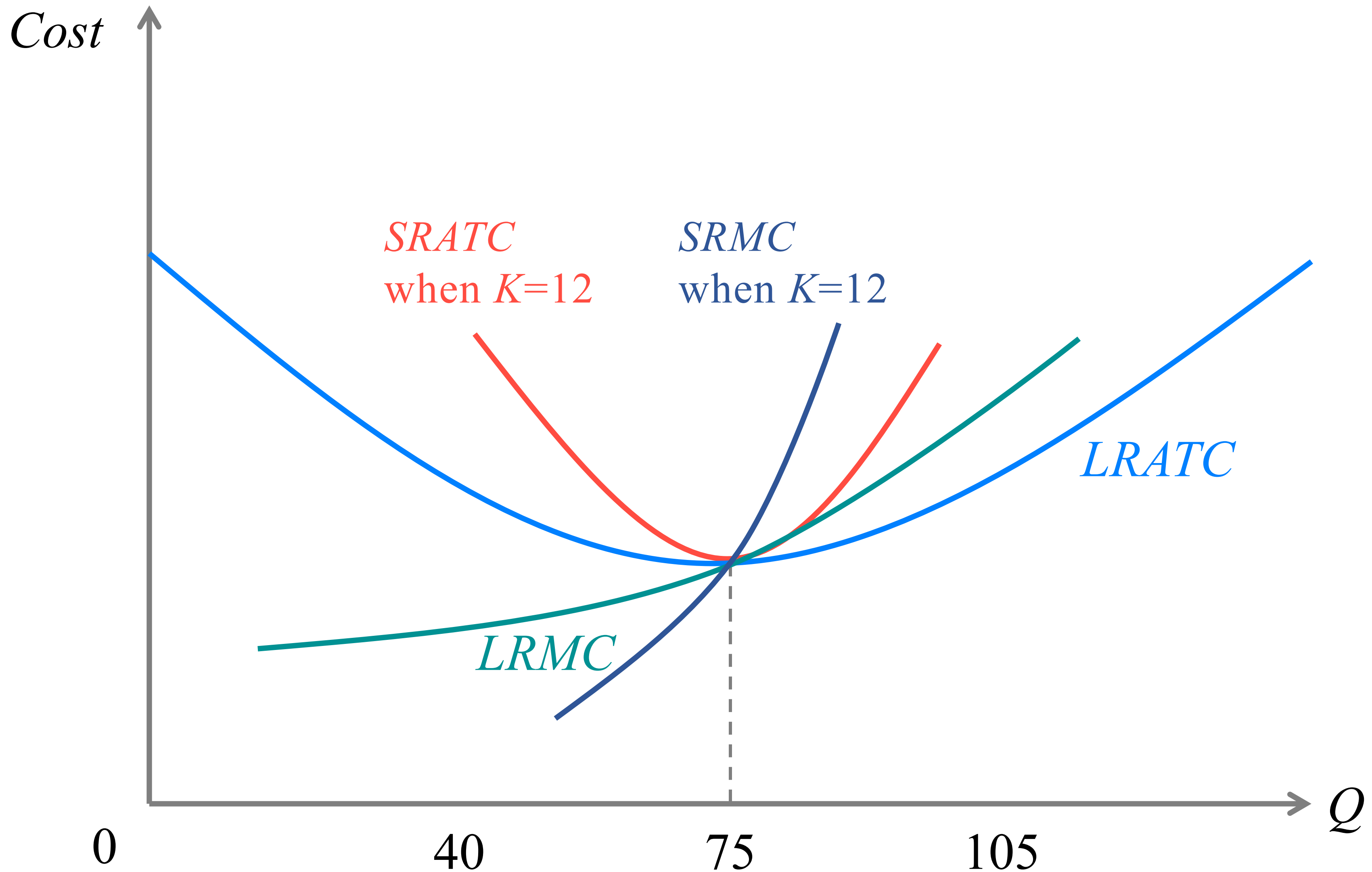
When $LRATC$ is not at its minimum

- $SRATC$ is not tangent to $LRATC$ at $SRATC$'s minimum point.
  - When $LRATC$ is not at its minimum,
    it is either decreasing or increasing,
    i.e., its slope is either negative or positive.
  - At the point where $SRATC$ is tangent to $LRATC$,
    $SRATC$ and $LRATC$ have the same slope.
  - Therefore, $SRATC$'s slope is also either negative or positive at that tangency.
  - Thus, $SRATC$ is also not at its minimum.

$LRMC$ vs. $SRMC$


Cost
SRMC
when K=10
SRMC
when K=12
SRMC
when K=15
LRATC
LRMC
Q
0
40
75
105

The Minimum Point of $LRATC$



The Big Picture

Demand

$$\max_{x,y} U(x, y)$$
$$\text{s.t. } p_x x + p_y y = M$$

$+$

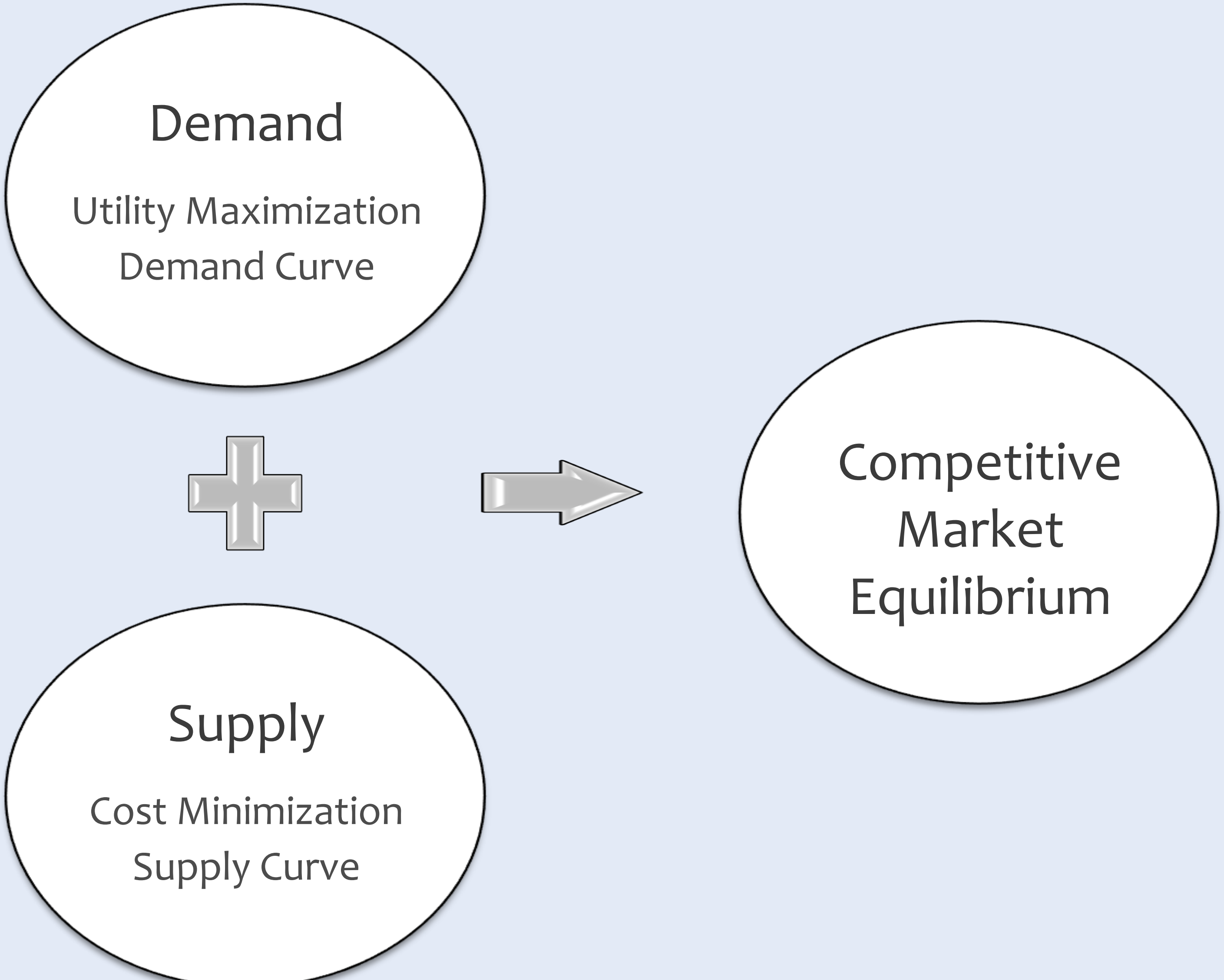
Supply

$$\min_{L,K} LRTC = wL + rK$$
$$\text{s.t. } f(L, K) = Q_0$$

$\Rightarrow$

Market Equilibrium

Where are we?


Demand
Utility Maximization
Demand Curve
Supply
Cost Minimization
Supply Curve
Competitive
Market
Equilibrium