

A Model of Production

Chapter 4 of Macroeconomics by Charles I. Jones

Outline

- ❑ Introduction
- ❑ A model of production
 - ❑ Model's economic properties (using maths):
 - ❑ Cobb-Douglas Production function ([math](#))
 - ❑ Constant Returns to Scale, CRS & Intensive form ([math](#))
 - ❑ Allocating Resources: How firms allocate resources ([math](#)) using Marginal Productions of labor and capital ([math](#))
 - ❑ Why Cobb-Douglas function?
 - ❑ Diminishing marginal product ([math](#))
 - ❑ Solving the model: Deriving general equilibrium from labor market, capital market and production function (using 5 equations)
 - ❑ Interpreting the solution
- ❑ Analyzing the production model
 - ❑ Assuming equal TFP (parameter A), how does the model fit the data
 - ❑ Case study: Why doesn't capital flow from Rich to Poor Countries
 - ❑ Productivity (or TFP) differences: (to improve the model fit)
- ❑ Understand TFP differences: *what are the sources of TFP?*
 - ❑ Human capital, technology, institutions and misallocation
- ❑ Evaluating the production Model

These two show that
TFP differences are
crucial to explain cross-
country differences

4.1 Introduction

4.1 Introduction

- In this chapter, we learn:
 - How to set up and solve a macroeconomic model
 - The purpose of a production function and its use for understanding differences in GDP per capita across countries.
 - The **role of capital per person and technology** in explaining differences in economic growth.
 - The relevance of “**returns to scale**” and “**diminishing marginal products**”
 - How to look at **economic data** through the lens of a macroeconomic model

Introduction

- A model:
 - A mathematical representation of a hypothetical world that we use to study economic phenomena: Growth, employment, inflation, business cycles, interest rate...
 - Consists of equations and unknowns with real-world interpretations
- Macroeconomists:
 - Document facts
 - Build a model to understand the facts
 - Examine the model to see how effective it is

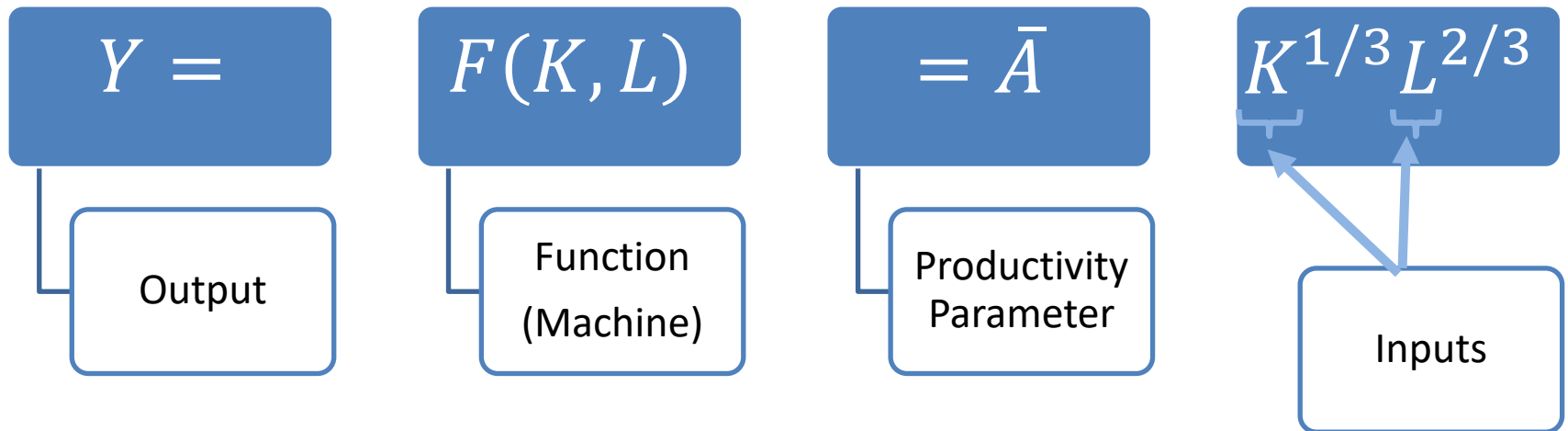
4.2 Model of production

4.2 A Model of Production

- Vast ***oversimplifications*** of the real world in a model can still allow it to provide important insights.
- Consider the following model:
 - Single, closed economy: no trade with outside
 - One consumption good/representative good: a basket of good
- Inputs in the production process:
 - Labor (\bar{L})
 - Capital (\bar{K}) (*The bar on top of L & K represents a fixed amount.*)
- Production function:
 - Shows how much output (Y) can be produced given any number of inputs

Production Function

$$Y = F(K, L) = \bar{A} K^{1/3} L^{2/3}$$



4.2.i Model's economic properties

Model

- Output growth corresponds to changes in Y .
- There are three ways that Y can change:
 1. Capital stock (K) changes
 2. Labor force (L) changes
 3. Ability to produce goods with given resources (K , L) changes
 - Technological advances occur (changes in A)
 - TFP is assumed to be exogenous in the Solow model

Cobb-Douglas Production Function

- The Cobb-Douglas production function is the particular production function that takes the form of:
 - $Y = K^\alpha L^{1-\alpha}$
 - α is assumed to be $1/3$
- And $F(K,L)$ is increasing in both K and L
 - More inputs yield more output.
 - $\frac{\partial F}{\partial K} > 0$ and $\frac{\partial F}{\partial L} > 0$
- A production function exhibits constant returns to scale if doubling each input exactly doubles output.

Mathematic derivation (1)

- If $0 \leq \alpha \leq 1$, partial differentiating $F()$ w.r.t K

$$\frac{\partial F}{\partial K} = \alpha K^{\alpha-1} L^{1-\alpha}$$

$$\Rightarrow \frac{\partial F}{\partial K} = \alpha \frac{K^{\alpha-1}}{L^{\alpha-1}}$$

$$\Rightarrow \frac{\partial F}{\partial K} = \alpha \left(\frac{K}{L} \right)^{\alpha-1} > 0$$

Mathematic derivation (2)

- Partial differentiating $F()$ w.r.t L

$$\frac{\partial F}{\partial L} = (1 - \alpha)K^{\alpha}L^{1-\alpha-1}$$

$$\Rightarrow \frac{\partial F}{\partial L} = (1 - \alpha)K^{\alpha}L^{-\alpha}$$

$$\Rightarrow \frac{\partial F}{\partial L} = (1 - \alpha) \frac{K^{\alpha}}{L^{\alpha}} = (1 - \alpha) \left(\frac{K}{L} \right)^{\alpha} > 0$$

Constant Returns to Scale (CRS)

- If increase K and L by $x\%$
 - ➔ Y also increases by $x\%$
- Mathematically,
 - $F(\beta K, \beta L) = \beta F(K, L)$
 - Homogeneous function (of degree 1)
- Standard replication argument

Output per Person (Intensive Form)

- Divide output by the number of workers

$$\frac{Y}{L} = F\left(\frac{K}{L}, \frac{L}{L}\right) = F\left(\frac{K}{L}, 1\right)$$

- per capita = per person = per worker
- Lowercase letters denote per capita

- We can rewrite output per person as

$$y = f(k)$$

where $y = \frac{Y}{L}$ and $k = \frac{K}{L}$

Mathematic derivation (3)

Here is an example:

$$Y = F(K, L) = K^{\alpha} L^{1-\alpha}$$

Divide both sides by L

$$\frac{Y}{L} = \frac{K^{\alpha} L^{1-\alpha}}{L}$$

$$y = K^{\alpha} \frac{L^{1-\alpha}}{L}$$

$$y = K^{\alpha} L^{-\alpha}$$

$$y = \left(\frac{K}{L}\right)^{\alpha}$$

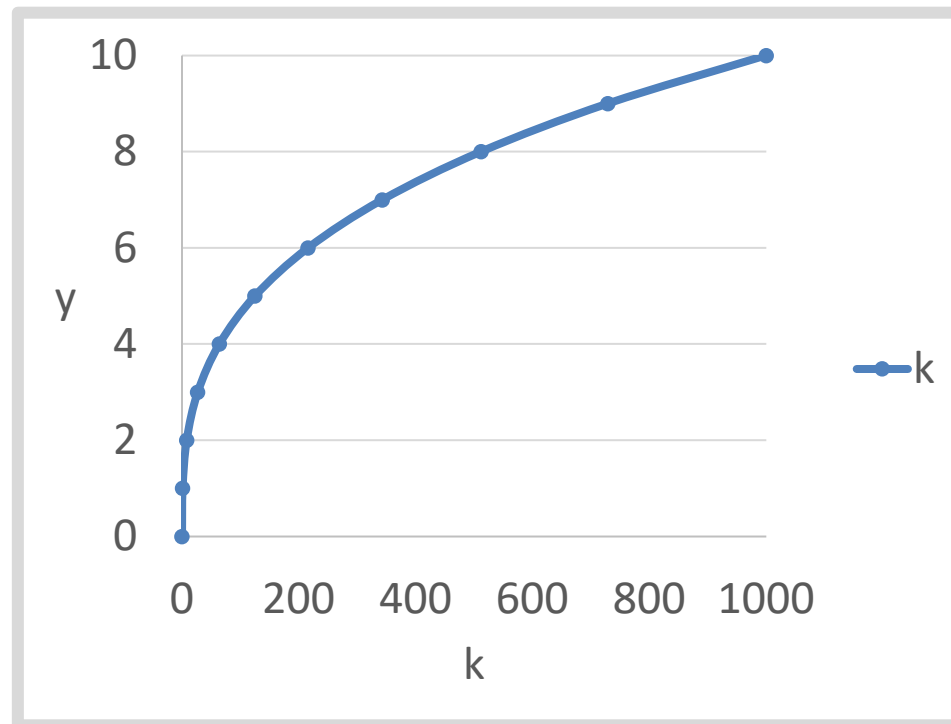
$$y = (k)^{\alpha}$$

Where:

$$y = \frac{Y}{L}; k = \frac{K}{L}$$

Typical Production Function

- Graph of $y = f(k) = k^{1/3}$
- Note: if $k = 0$ then $y = f(k) = 0$



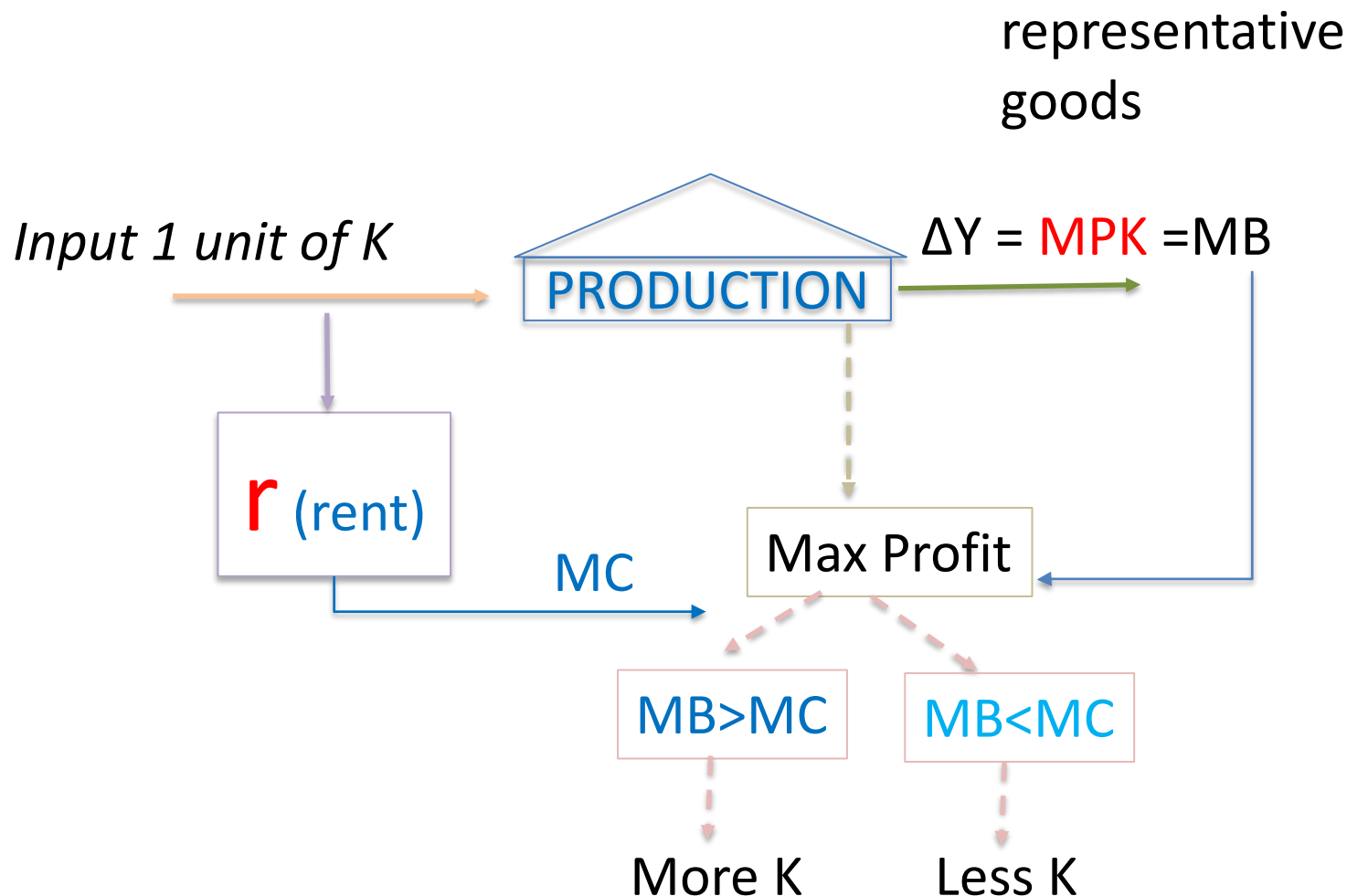
Returns to Scale Comparison (using Cobb-Douglas production)

- Sum of exponents
 - Sum to 1
 - Sum to > 1
 - Sum to < 1
- Result
 - Constant returns to scale
 - Increasing returns to scale
 - Decreasing returns to scale

Allocating Resources

- $\max_{K,L} \Pi = F(K,L) - rK - wL$
 - π : profits
 - r : rental rate of capital
 - w : wage rate
- The rental rate and wage rate are taken as given under perfect competition
 - Hire capital until the $MPK = r$
 - Hire labor until $MPL = w$
- For simplicity, the price of the output is normalized to one

Production firm



until $\text{MB} = \text{MC}$ or $\text{MPK} = r$

In economics, agents maximize any kind of benefit following a rule: $\text{MB} = \text{MC}$

Mathematic derivation (4)

$$\max_{K,L} \pi = 1K^{\alpha}L^{1-\alpha} - rK - wL$$

- Take the partial derivatives and set equal to zero to find the maximum (where the tangent line has a zero slope).

Mathematic derivation (4)

$$\begin{aligned}\frac{\partial F}{\partial K} &= \alpha K^{\alpha-1} L^{1-\alpha} - r = 0 \\ \frac{\partial F}{\partial L} &= (1 - \alpha) K^{\alpha} L^{1-\alpha-1} - w = 0\end{aligned}$$

- Solve for r and w .
- Note:

$$\alpha K^{\alpha-1} L^{1-\alpha} = MPK$$

$$(1 - \alpha) K^{\alpha} L^{1-\alpha-1} = MPL$$

Marginal Products

- The marginal product of labor (MPL)
 - *(Definition)* The additional output that is produced when one unit of labor is added, holding all other inputs constant

$$MPL = \frac{2}{3} \bar{A} \left(\frac{K}{L} \right)^{1/3} = \left(\frac{2}{3} \right) \left(\frac{Y}{L} \right)$$

- The marginal product of capital (MPK)
 - *(Definition)* The additional output that is produced when one unit of capital is added, holding all other inputs constant

$$MPK = \frac{1}{3} \cdot \bar{A} \cdot \left(\frac{L}{K} \right)^{2/3} = \frac{1}{3} \cdot \frac{Y}{K}$$

Mathematic derivation (5)

$$MPL \cdot L = \frac{2}{3} \bar{A} \left(\frac{K}{L} \right)^{1/3} L = wL$$

$$= \frac{2}{3} \bar{A} K^{1/3} L^{2/3}$$

$$(Oh behold: \bar{A} K^{1/3} L^{2/3} = Y)$$

- So:

$$MPL \cdot L = \frac{2}{3} Y = wL$$

Important note: MPL is not the same with w. They are equal only when the firm maximizes the profit.

Mathematic derivation (5)

- *Similarly:*

$$\begin{aligned}MPK \cdot K &= \frac{1}{3} \bar{A} \left(\frac{L}{K} \right)^{2/3} K = rK \\&= \frac{1}{3} \bar{A} L^{2/3} K^{1/3} = rK \\rK &= \frac{1}{3} Y\end{aligned}$$

- Important note: MPK is not the same with r. They are equal only when the firm maximizes the profits.

Why Cobb-Douglas function?

- $F(K,L)$ can be any function, but here, we specifically make use of Cobb-Douglas specification.

$$Y = F(K, L) = \underbrace{AK^\alpha L^\beta}$$

Cobb-Douglas form

- Cobb-Douglas by Charles Cobb and Paul Douglas (1927)

Why Cobb-Douglas function?

- Applying logarithmic on both sides:

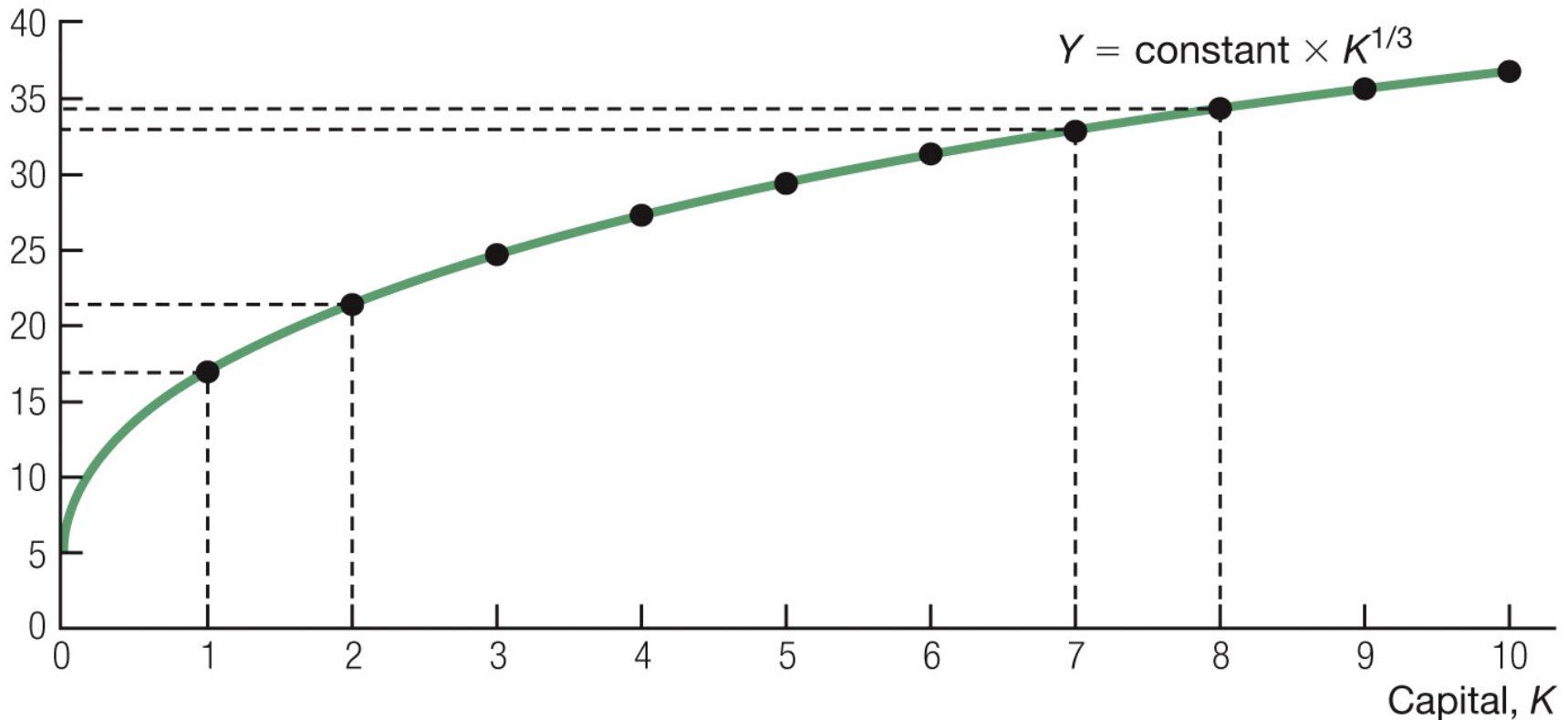
$$\ln(Y) = \ln(A) + \alpha \ln(K) + \beta \ln(L)$$

- If you have data for K , L and Y , we can run a regression of $\ln(Y)$ on $\ln(K)$ and $\ln(L)$, then the coefficients of the regressions are $\ln(A)$, α , β .
- This Cobb-Douglas specification is used widely in economics (e.g. utility)

Diminishing Marginal Product of Capital in Production

The Diminishing Marginal Product of Capital in Production

Tons of ice cream, Y



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Diminishing Returns

- Formally:

$$\frac{\partial^2 F}{\partial K^2} < 0 \quad \frac{\partial^2 F}{\partial L^2} < 0$$

Mathematic derivation (6)

$$\frac{\partial F}{\partial K} = \alpha K^{\alpha-1} L^{1-\alpha}$$
$$\frac{\partial^2 F}{\partial K^2} = \alpha(\alpha - 1) K^{\alpha-1-1} L^{1-\alpha}$$

□ If $0 < \alpha < 1$, then $\alpha(\alpha - 1) < 0$ and

$$\frac{\partial^2 F}{\partial K^2} = \alpha(\alpha - 1) K^{\alpha-1-1} L^{1-\alpha} < 0$$

Mathematic derivation (6)

$$\frac{\partial F}{\partial L} = (1 - \alpha)K^{\alpha}L^{-\alpha}$$

$$\frac{\partial^2 F}{\partial L^2} = -\alpha(1 - \alpha)K^{\alpha}L^{-\alpha-1}$$

$$\frac{\partial^2 F}{\partial L^2} = -\alpha(1 - \alpha)K^{\alpha}L^{-\alpha-1} < 0$$

4.2.ii Solving the model

Solving the Model: General Equilibrium—1

- ***Five Endogenous Variables***

- ❑ Output (Y)
- ❑ The amount of capital (K)
- ❑ The amount of labor (L)
- ❑ The wage (w)
- ❑ The rental price of capital (r)

Five Equations

- ❑ The production function
- ❑ The rule for hiring capital
- ❑ The rule for hiring labor
- ❑ Supply equals the demand for labor
- ❑ Supply equals the demand for capital

Five equations (what are they)

- The production function

$$Y = F(K, L) = AK^\alpha L^\beta$$

(So far, we assume $\beta = 1 - \alpha$)

- The rule for hiring capital:

$$MPK = r$$

- The rule for hiring labor:

$$MPL = w$$

Five equations (what are they)

- Supply equals the demand for labor

$$L = \bar{L}$$

($\bar{}$ stands for fixed amount)

- Supply equals the demand for capital

$$K = \bar{K}$$

($\bar{}$ stands for fixed amount)

So we have 5 equations with 5 unknown to solve for

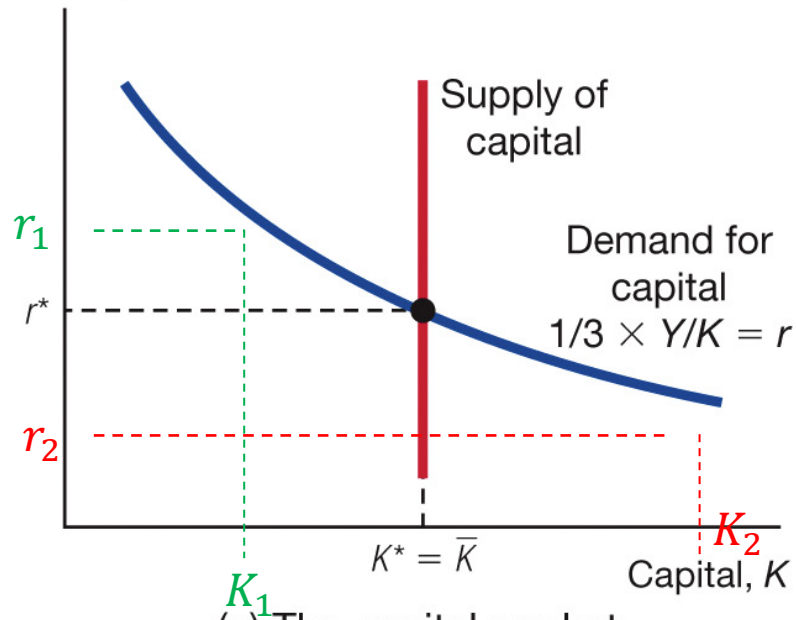
Solution—1

- A solution to the model
 - A new set of equations that express the five unknowns in terms of the parameters and exogenous variables
- General equilibrium
 - *(definition)* Solution to the model when more than a single market clears
 - In this context, there are two markets: capital and labor

Supply and Demand in the Capital and Labor Markets

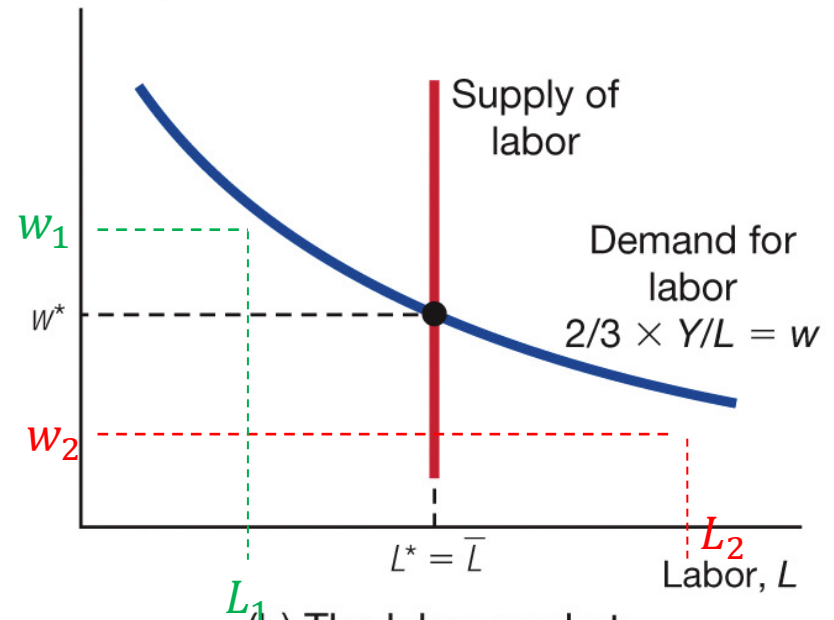
Supply and Demand in the Capital and Labor Markets

Rental price, r



(a) The capital market

Real wage, w



(b) The labor market

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Comments on demand curves

- The demand curves for capital and labor are based on the hiring rules:
 - *they trace out exactly the marginal product schedules for K and L*
- If the firm is offer rental rate r_1 , the amount of capital they wish to rent is K_1 .
- If the firm is offer rental rate r_2 , the amount of capital they wish to rent is K_2 .
- Tracing all the combinations of (r, K) we have the demand curve for capital. Similar for the labor demand curve.

Solution—2 for $\alpha = \frac{1}{3}$ and $\beta = \frac{2}{3}$

| | |
|--------------------------------------|---|
| The production function | $Y^* = \bar{A}K^{1/3}L^{2/3}$ |
| The rule for hiring capital | $r^* = \left(\frac{1}{3}\right)\left(\frac{Y^*}{K^*}\right) = \frac{1}{3}\bar{A}\left(\frac{\bar{L}}{\bar{K}}\right)^{2/3}$ |
| The rule for hiring labor | $w^* = \left(\frac{2}{3}\right)\left(\frac{Y^*}{L^*}\right) = \frac{2}{3}\bar{A}\left(\frac{\bar{K}}{\bar{L}}\right)^{1/3}$ |
| Supply equals the demand for labor | $L^* = \bar{L}$ |
| Supply equals the demand for capital | $K^* = \bar{K}$ |

4.2.iii Interpreting the solution

In This Model...

- The solution implies:
 - firms employ all the supplied capital and labor in the economy
 - the production function is evaluated with the given supply of inputs

$$Y^* = F(\bar{K}, \bar{L}) = \bar{A}\bar{K}^{1/3}\bar{L}^{2/3}$$

- $w = MPL$ evaluated at the equilibrium values of Y , K , and L
- $r = MPK$ evaluated at the equilibrium values of Y , K , and L

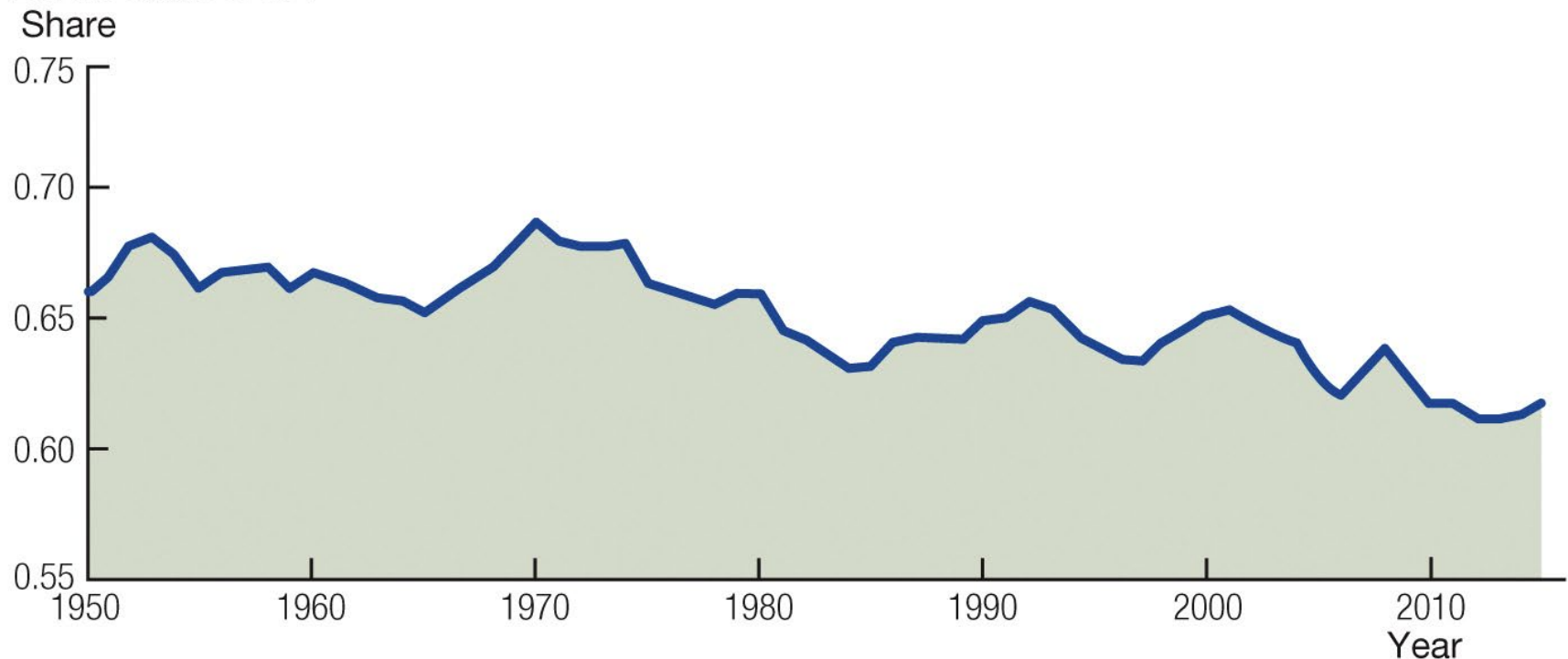
Interpreting the Solution

- The equilibrium wage is proportional to output per worker
 - Output per worker = (Y/L) $w^* = \frac{2}{3} \cdot \frac{Y^*}{L^*}$
- The equilibrium rental rate is proportional to output per capital
 - Output per capital = (Y/K) $r^* = \frac{1}{3} \cdot \frac{Y^*}{K^*}$
- In the United States, empirical evidence shows:
 - $\frac{2}{3}$ of production is paid to labor
 - $\frac{1}{3}$ of production is paid to capital
 - The factor shares of the payments are equal to the exponents on the inputs in the Cobb-Douglas function.

$$\frac{w^* L^*}{Y^*} = \frac{2}{3} \text{ and } \frac{r^* K^*}{Y^*} = \frac{1}{3}$$

Labor's Share of Income

Labor's Share of GDP



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Equilibrium

- All income is paid to capital or labor.
 - Results in zero profit in the economy
 - This verifies the assumption of perfect competition.
 - Also verifies that production equals spending equals income.

$$\underbrace{w^*L^*}_{\text{Labour Share}} + \underbrace{r^*K^*}_{\text{Capital Share}} = Y^*$$

Income=production

4.3 Analyzing the Production Model

4.3 Analyzing the Production Model

- Development accounting:
 - The use of a model to explain differences in incomes across countries

$$y^* = \bar{A}\bar{k}^{1/3}$$

- Setting the productivity parameter ($\bar{A} = 1$)

$$y^* = \bar{k}^{1/3}$$

4.3.i Empirical fit of the Production function

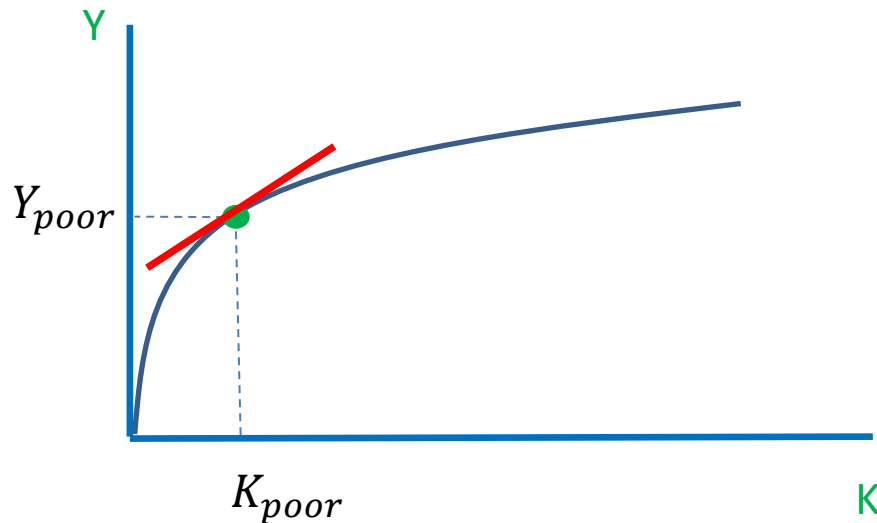
The Empirical Fit of the Production Function

- If the productivity parameter is 1 ($A = 1$), *the model over-predicts GDP per capita.*
- Diminishing returns to capital implies that:
 - Countries with low K will have a high MPK
 - Countries with a lot of K will have a low MPK, and cannot raise GDP per capita by much through more capital accumulation

Assume $A = 1$, our model says:

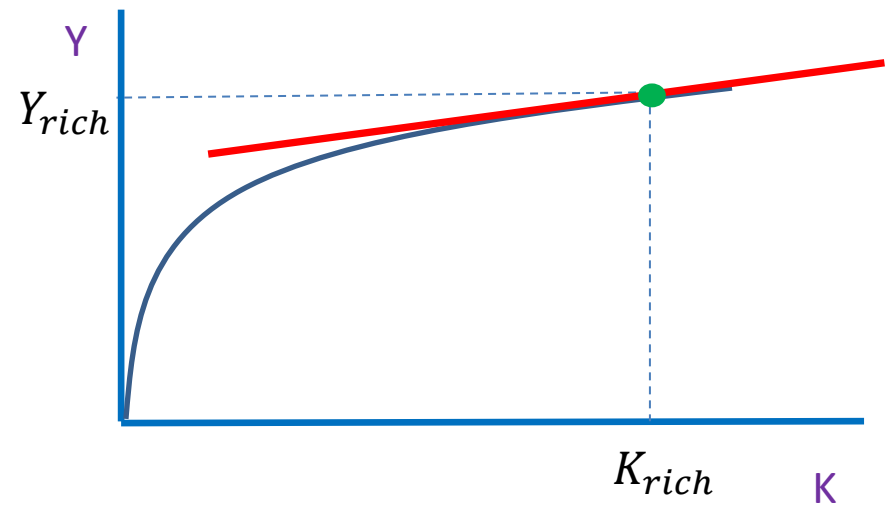
Poor Country:

Larger slope at K_{poor} \rightarrow higher rental



Rich Country:

Smaller slope at K_{rich} \rightarrow lower rental



Should we observe the flow of funds from rich to poor to seek for higher return?

The Model's Prediction for Per Capita GDP (United States = 1)

TABLE 4.3

The Model's Prediction for Per Capita GDP (U.S. = 1)

| Country | Observed capital per person, \bar{k} | Predicted per capita GDP $y = \bar{k}^{1/3}$ | Observed per capita GDP |
|----------------|--|--|-------------------------------|
| United States | 1.000 | 1.000 | 1.000 |
| Switzerland | 1.416 | 1.123 | 1.147 |
| Japan | 1.021 | 1.007 | 0.685 |
| Italy | 1.124 | 1.040 | 0.671 |
| Spain | 1.128 | 1.041 | 0.615 |
| United Kingdom | 0.832 | 0.941 | 0.733 |
| Brazil | 0.458 | 0.771 | 0.336 |
| China | 0.323 | 0.686 | 0.241 |
| South Africa | 0.218 | 0.602 | 0.232 |
| India | 0.084 | 0.437 | 0.105 |
| Burundi | 0.007 | 0.192 | 0.016 |

Predicted per capita GDP is computed as $\bar{k}^{1/3}$, that is, assuming no differences in productivity across countries. Data correspond to the year 2014 and are divided by the values for the United States.

Source: Penn World Tables, Version 9.0.

Using our model, assuming same $\bar{A} = 1$ for all countries

$$y_{us}^* = \bar{A} k_{us}^{1/3} \quad (\text{for US})$$

$$y_{chi}^* = \bar{A} k_{chi}^{1/3} \quad (\text{for China})$$

$$\Rightarrow \frac{y_{chi}^*}{y_{us}^*} = \frac{\bar{A} k_{chi}^{1/3}}{\bar{A} k_{us}^{1/3}} = \frac{k_{chi}^{1/3}}{k_{us}^{1/3}}$$

Using capital data (k) from previous slide:

$$\frac{y_{chi}^*}{y_{us}^*} = \frac{k_{chi}^{1/3}}{k_{us}^{1/3}} = \frac{0.323^{1/3}}{1^{1/3}} = 0.686$$

** is to denote that predicted value. So y^* is the predicted output*

0.686 is the GDP per capita of China relative to that of US

Using our model, assuming same \bar{A} for all countries

Since y_{us}^ is 1 using the proposed model (ie. $y_{us}^* = k_{us}^{*1/3} = 1, \because k_{us}^* = 1$)*

$$\frac{y_{chi}^*}{1} = 0.686 \Rightarrow y_{chi}^* = 0.686$$

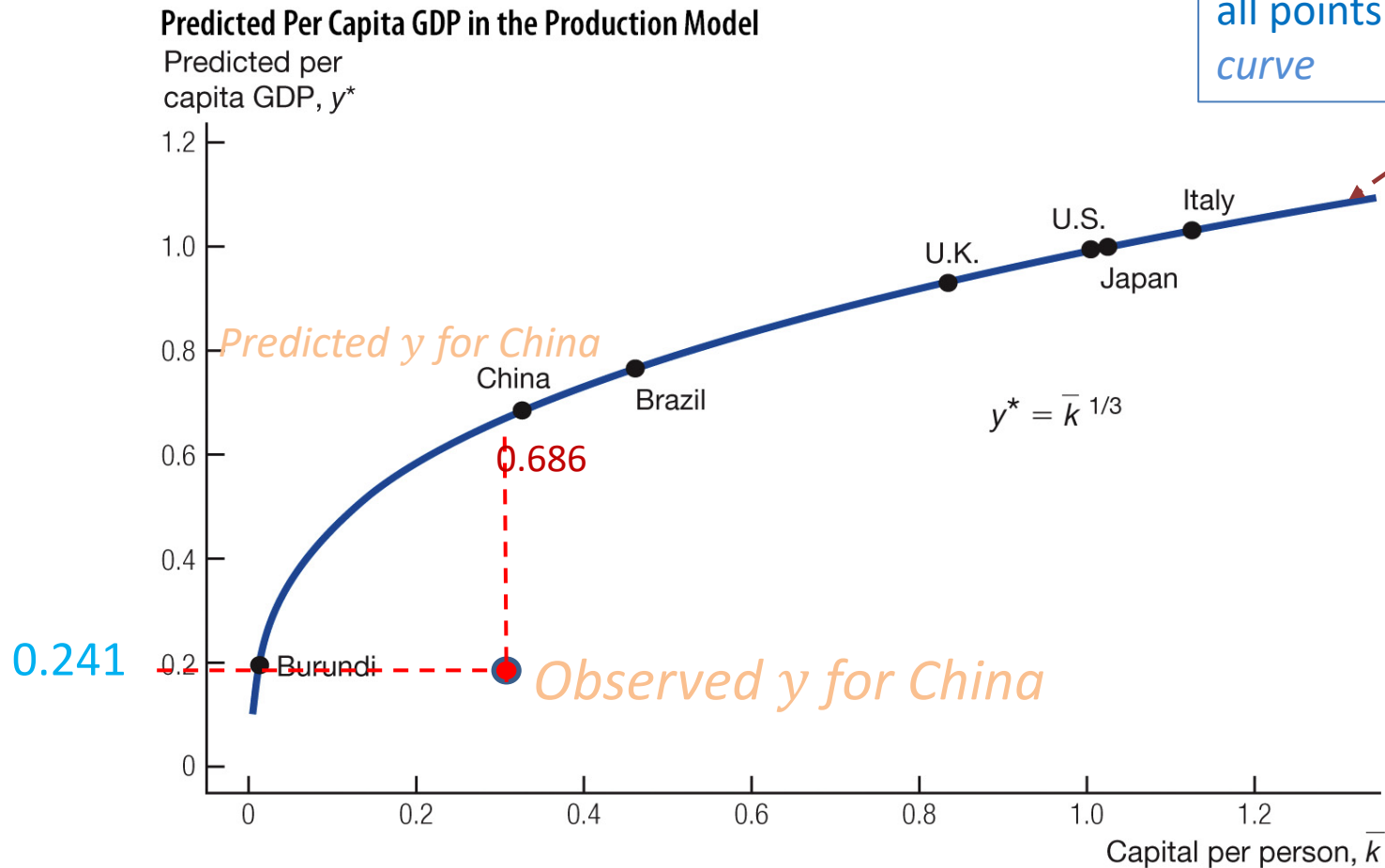
(under our model)

Note: *Note that $y_{chi}^* = 0.686$ is using our model assumption ($A=1$).*

*But what is the actual y_{chi} ? Only **0.241**. The model **over-estimates** the output*

Predicted Per Capita GDP in the Production Model

For this curve, we assumed, $\bar{A} = 1$, so all points are on *same curve*

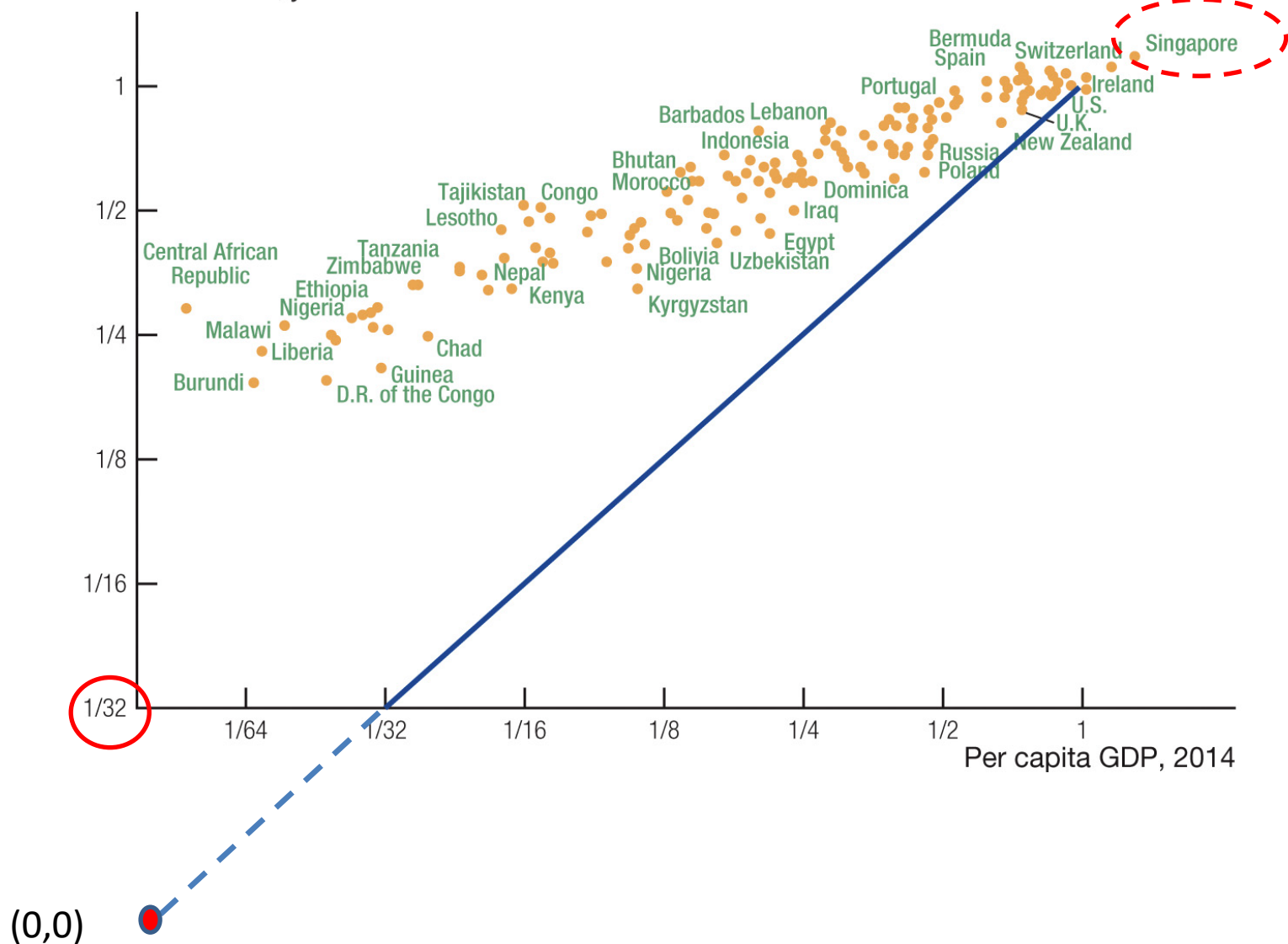


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The Model's Prediction for Per Capita GDP

The Model's Prediction for Per Capita GDP (U.S. = 1)

Predicted value, y^*



Comment on previous slide

- If the model were successful in explaining incomes,
 - Countries should lie close to the solid 45 degree line.
- Instead, the model predicts that most countries **should be substantially richer** than they are.

4.3.ii Case Study

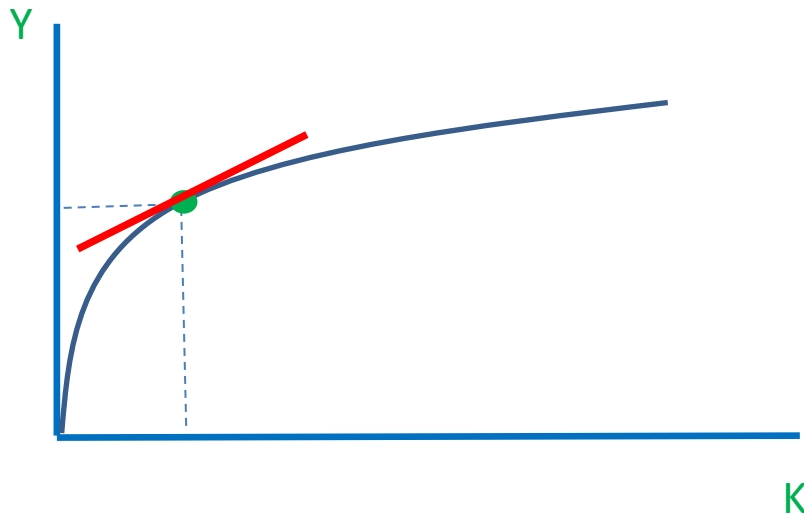
Case Study: Why Doesn't Capital Flow from Rich to Poor Countries?

- If MPK is higher in poor countries with low K , why doesn't capital flow to those countries?
 - Short Answer: Simple production model with no difference in productivity across countries is misguided
 - We must also consider the productivity parameter (*see next slide*)

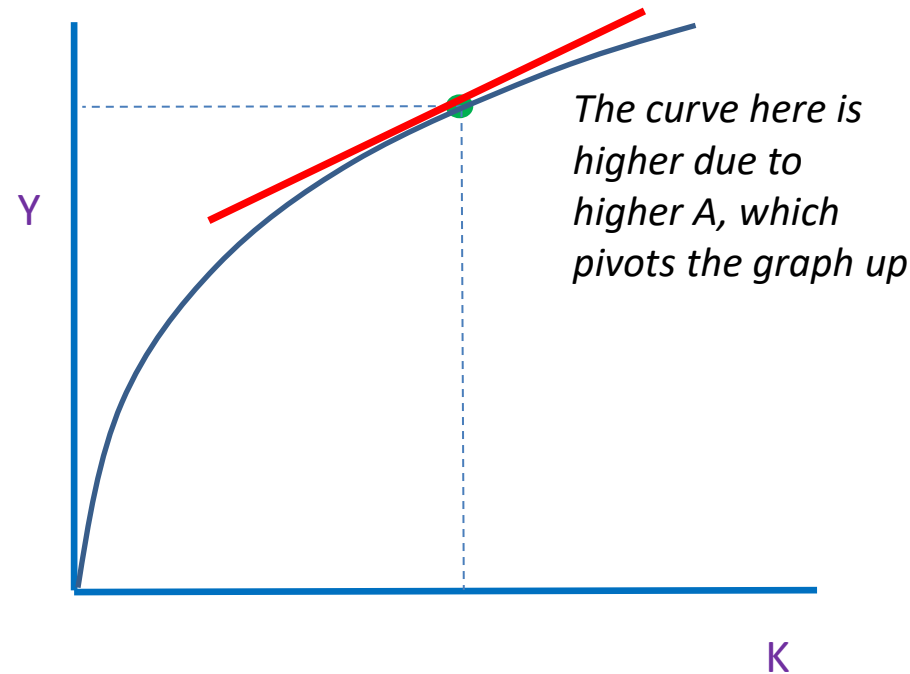
Assume $A_{poor} \neq A_{rich}$

$$Y = \bar{A}F(K, L)$$

Poor Country A_{poor} :



Rich Country A_{rich} :



No fund transfer from one to another because both have same rental rate for capital

4.3.iii Productivity Differences: Improving the Fit of the Model

Productivity Differences: Improving the Fit of the Model

- The productivity parameter measures how efficiently countries are using their factor inputs.
- Total factor productivity (TFP)
- If TFP is \neq to 1 \rightarrow better model

Total Factor Productivity

$$y = \bar{A}f(k)$$

- Data on TFP is not collected.
 - It can be calculated because we have data on output and capital per person.
 - TFP is referred to as the “residual.”
 - **That is:** if we know Y, K and L , we can **infer** \bar{A} using the functional specification (e.g. $Y = \bar{A}K^{1/3}L^{2/3}$)
- A lower level of TFP
 - implies that workers produce less output for any given level of capital per person.

Measuring TFP So the Model Fits Exactly—1

TABLE 4.4

Measuring TFP So the Model Fits Exactly

| Country | Per capita GDP (y) | $\bar{k}^{1/3}$ | Implied TFP (\bar{A}) |
|----------------|---------------------------|-----------------|------------------------------|
| United States | 1.000 | 1.000 | 1.000 |
| Switzerland | 1.147 | 1.123 | 1.022 |
| United Kingdom | 0.733 | 0.941 | 0.779 |
| Japan | 0.685 | 1.007 | 0.680 |
| Italy | 0.671 | 1.040 | 0.646 |
| Spain | 0.615 | 1.041 | 0.590 |
| Brazil | 0.336 | 0.771 | 0.436 |
| South Africa | 0.232 | 0.602 | 0.386 |
| China | 0.241 | 0.686 | 0.351 |
| India | 0.105 | 0.437 | 0.240 |
| Burundi | 0.016 | 0.192 | 0.085 |

Calculations are based on the equation $y = \bar{A} \bar{k}^{1/3}$. Implied productivity \bar{A} is calculated from data on y and \bar{k} for the year 2010, so that this equation holds exactly as $\bar{A} = y/\bar{k}^{1/3}$.

Using our model, assuming $y = \overline{A} k^{*1/3}$ form with different A's for different economies

$$\begin{aligned} y_{us} &= \overline{A}_{us} k_{us}^{1/3} \text{ (for US)} \\ y_{chi} &= \overline{A}_{chi} k_{chi}^{1/3} \text{ (for China)} \\ \Rightarrow \frac{y_{chi}}{y_{us}} &= \frac{\overline{A}_{chi} k_{chi}^{1/3}}{\overline{A}_{us} k_{us}^{1/3}} \end{aligned}$$

Note: We took out the * in y_{us} and y_{chi} because these are the actual outputs

Using per-capita capital data (k) and per-capita output data (y) from tables 4.3 and 4.4:

$$\frac{0.241}{1} = \frac{\overline{A}_{chi} 0.323^{1/3}}{1 \times 1^{1/3}}$$

$$(\because A_{us} = 1, k_{us} = 1, k_{chi} = 0.323, y_{chi} = 0.241, y_{us} = 1)$$

Using our model, assuming $y = \bar{A} k^{*1/3}$ form with different \bar{A} 's for different economies

$$\frac{0.241}{1} = \frac{\bar{A}_{chi} 0.323^{1/3}}{1 \times 1^{1/3}}$$

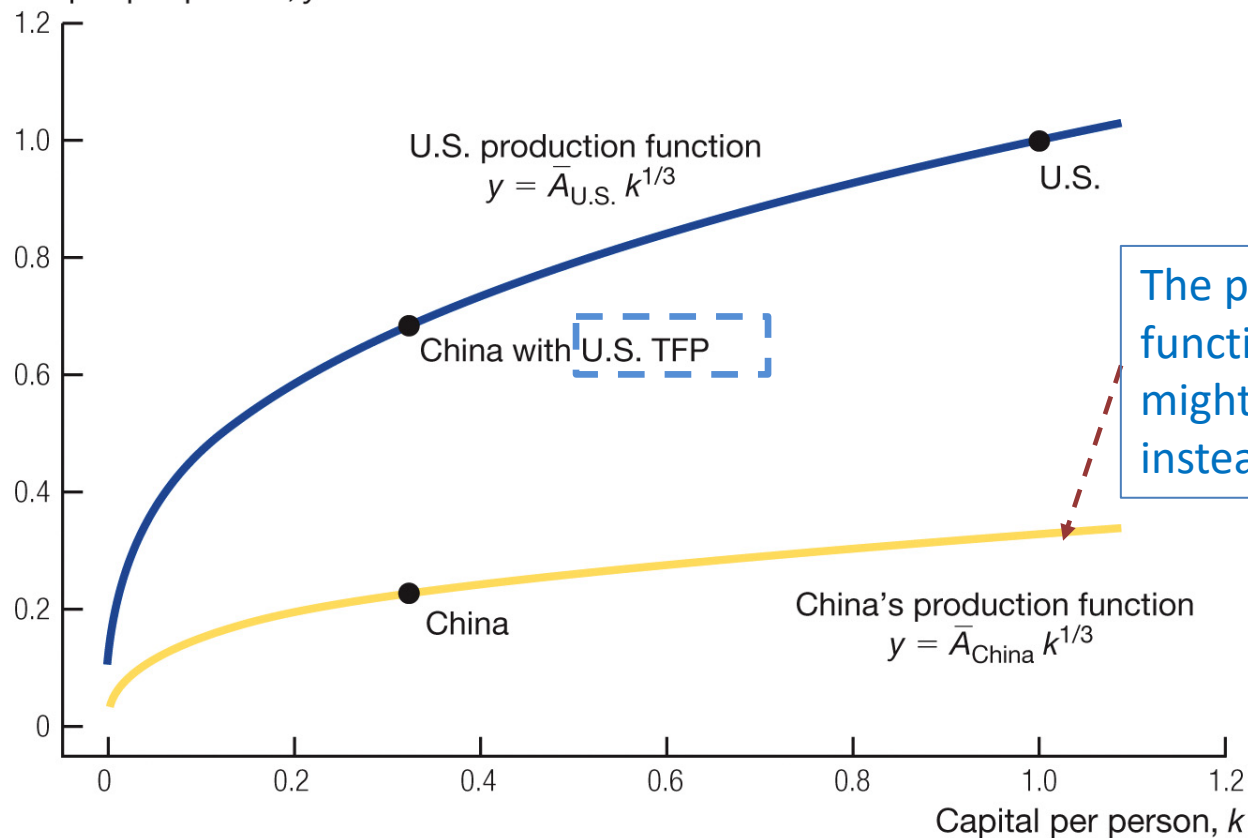
$$\Rightarrow \bar{A}_{chi} = \frac{0.241}{\frac{0.323^{1/3}}{1}} = 0.351$$

**Please note again that $\bar{A}_{chi} = 0.351$ is the implied TFP of China relative to that of US (\bar{A}_{us})*

United States and Chinese Production Functions

The U.S. and Chinese Production Functions

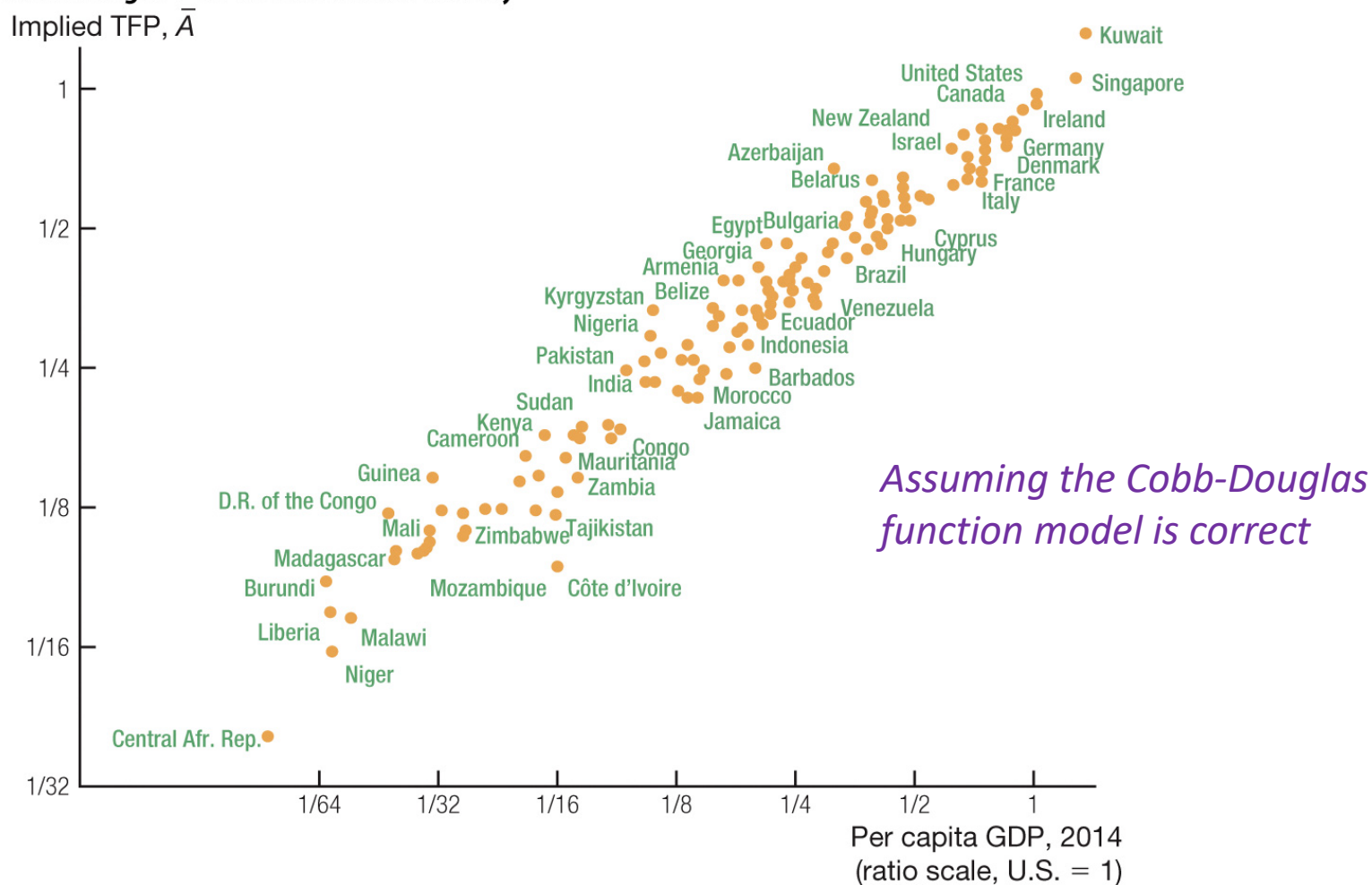
Output per person, y



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Measuring TFP So the Model Fits Exactly—2

Measuring TFP So the Model Fits Exactly



4.4 Understanding TFP Differences

4.4 Understanding TFP Differences

- Output differences between the richest and poorest countries?
 - Differences in capital per person explain about ***one-third*** of the difference.
 - TFP explains the remaining ***two-thirds***.
- Thus, rich countries are rich because:
 - They have **more capital per person**.
 - More importantly, they **use labor and capital more efficiently**.
 - Why are some countries more efficient at using capital and labor?

Understanding TFP Differences

- Human capital
- Technology
- Institutions
- Misallocation

Human Capital

- Human capital
 - Stock of skills that individuals accumulate to make them more productive
 - Education and training
 - people attend college
 - 1st graders learn to read
 - construction workers learn to operate a tower crane
 - doctors master a new surgical technique

Human Capital

- In the United States each year of education seems to increase future wages by 7 percent.
- A four-year education may raise wages by about 28 percent (over entire lifetime).
- In developing countries, returns can be even higher—up to 10 percent or even 13 percent per year.
- The typical student learning the basic skills associated with literacy and arithmetic may have higher returns than a college education.

Technology

- Richer countries may use more modern and efficient technologies than poor countries.
 - Increases productivity parameter
- Goods such as state-of-the-art computer chips, software, new pharmaceuticals, supersonic military jets, and skyscrapers are much more prevalent in rich countries than in poor.
 - As well as production techniques such as just-in-time inventory *methods*, *information technology*, and tightly *integrated transport networks*.

Institutions

- Even if human capital and technologies are better in rich countries, why do they have these advantages?
- Institutions are in place to foster human capital and technological growth.
 - Property rights
 - The rule of law
 - Government systems
 - Contract enforcement

Institutions

- Some challenges in countries with uncertain institutions:
 - No well-defined set of laws to follow to establish business
 - Rules are not the same for everyone
 - Licensing fees and taxes may vary over time and without warning
 - Corruption and bribes
 - Imports may be challenging to receive
 - Profits may be “taxed” away or stolen due to insufficient property rights
 - A coup or war could change the environment overnight

Misallocation

- Misallocation
 - Resources not being put to their best use
- Examples
 - Inefficiency of state-run resources
 - State-owned companies (SOE) versus Foreign-Joint Ventures
 - Funds are not allocated to the more-productive channels
 - Political interference

4.5 Evaluating the Production Model

- Per capita GDP is higher if capital per person is higher *and* if factors are used more efficiently.
- Constant returns to scale imply that output per person *can be written* as a function of capital per person.
- Capital per person is subject to strong diminishing returns because the exponent is much less than one.

Weaknesses of the Model

- In the absence of TFP, the production model incorrectly predicts differences in income.
- The model ***does not provide an answer as to why countries have different TFP levels.***