GAME THEORY

Week 6

(Chapter 29; except 29.3)

Strategic Interaction

- *Strategic interaction*
  - A player’s payoff depends on other players’ strategies
  - What is “best” for a player may depend on what other players are doing

- No strategic interaction in
  - Perfectly competitive market
  - Monopoly market

Part 1

SIMULTANEOUS-MOVE GAMES

Prisoner’s Dilemma

- Two suspects of a crime, Raj and Howard are arrested and held in separate cells (i.e., communication is not possible)
- Each of them knows that
  - If he confesses and the other person does not, he will get a light sentence of 1 year in jail and the other person will go to jail for 10 years
  - If both confess, each will get 5 years in jail
  - If neither confesses, each will get 2 years in jail
- Each of them decides (simultaneously) what to do

Game Structure

- *Players*: Raj (Player 1) and Howard (Player 2)

- *Strategies*: "Confess" (C) and "Do Not Confess" (D)
  - Let S_i be the set of strategies for player i
  - Let s_i be a particular strategy chosen by player i
  - In this game, S_i ={C, D} for i = 1, 2

- *Payoffs*: Utilities of players given the strategies chosen
  - Let $u_i(s_i, s_{-i})$ be the payoff function for player i
    - s_i : strategy chosen by player i
    - s_{-i}: strategies chosen by all the other players
    - u_i(C, C)=-5; u_i(C, D)=-1; u_i(D, C)=-10; u_i(D, D)=-2

Normal Form Representation

- A *normal form* of a game is a payoff matrix that specifies all the strategies for each player and the associated payoffs for each player for every strategy profile
  - A *strategy profile* is a list of strategies chosen by each player

- In a two-player game
  - Player 1 (row player) chooses rows
  - Player 2 (column player) chooses columns

Normal Form of Prisoner's Dilemma

		Howard	
		Confess	**Do Not Confess**
Raj	**Confess**	-5, -5	-1, -10
	Do Not Confess	-10, -1	-2, -2

Raj's payoff if he does not confess but Howard does

Howard's payoff if he does not confess but Raj does

Assumptions

- Every player is rational
  - Everyone knows that everyone is rational
  - Everyone knows that everyone knows that everyone is rational…
  - (Rationality is common knowledge)

- (X is common knowledge if everyone knows that everyone knows that everyone knows that …[infinitely many]… that everyone knows X)

- Everyone knows the structure of the game
  - Everyone knows that everyone knows the structure…
  - (The game structure is common knowledge)

Best Response

- What is the "best" choice for each player?

- s_i is a *best response* for player *i* to rivals' strategies s_{-i} if

$$u_i(s_i, s_{-i}) \geq u_i(s'_i, s_{-i}) \qquad \forall s'_i \in S_i$$

- Raj's *best response* is the strategy that maximizes Raj's payoff given the strategy chosen by Howard
  - Given any strategy chosen by Howard, Raj has a best response
  - [Given any strategy chosen by Raj, Howard has a best response]

Best Response in Prisoner's Dilemma

		Howard	
		Confess	**Do Not Confess**
Raj	**Confess**	-5, -5	-1, -10
	Do Not Confess	-10, -1	-2, -2

Raj's best response if Howard plays Confess

Howard's best response if Raj plays Do Not Confess

Nash Equilibrium

- Each player chooses a strategy that maximizes his payoff given the strategy chosen by the other player
  - Each player selects his best response to the strategy actually chosen by the other player

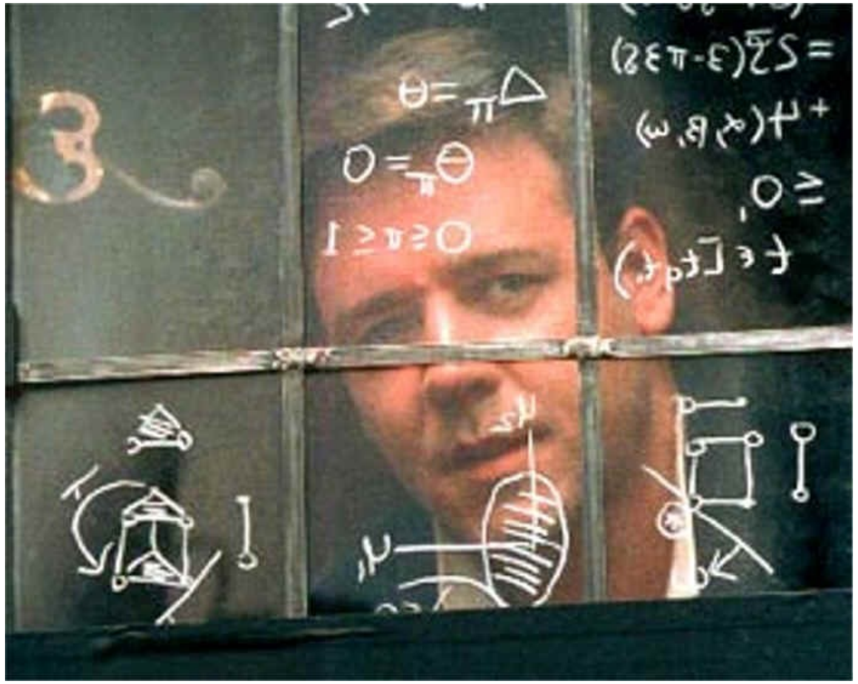
- In a two-player game, a strategy profile (s_1^*, s_2^*) is a *Nash equilibrium* if s_1^* and s_2^* are mutual best responses against each other:

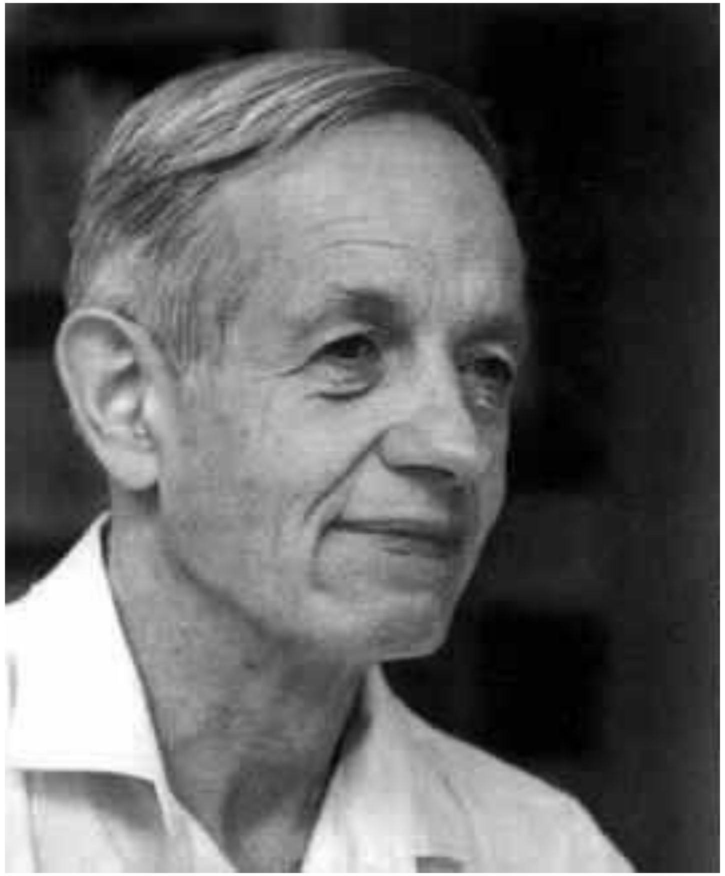
$$u_1(s_1^*, s_2^*) \geq u_1(s_1, s_2^*) \qquad \forall s_1 \in S_1$$

and

$$u_2(s_1^*, s_2^*) \geq u_2(s_1^*, s_2) \qquad \forall s_2 \in S_2$$

John Nash (1928 – 2015)





Nash Equilibrium in Prisoner's Dilemma

		Howard	
		Confess	**Do Not Confess**
Raj	**Confess**	-5, -5	-1, -10
	Do Not Confess	-10, -1	-2, -2

Interpreting Nash Equilibrium

- At Nash equilibrium, no one has an incentive to deviate to another strategy <u>unilaterally</u>
  - When Howard chooses “Confess”, Raj has no incentive to choose “Do Not Confess”

- “No regret” at Nash equilibrium
  - Looking back at his decision, Raj will not regret
  - Given that Howard chooses “Confess”, Raj should indeed choose “Confess”

Implications of Prisoner’s Dilemma

- Raj and Howard will be collectively better off if they do not confess
  - If they do not confess, each will get 2 years in jail
- (D, D) is the socially efficient outcome
- But (D, D) is not a Nash equilibrium
- Players’ rational pursuit of their individual best interest can lead to outcomes that are bad for everyone

Implications of Prisoner's Dilemma

- Individual rationality does not (necessarily) bring about social optimum
- But First Welfare Theorem suggests that individual rationality brings about social optimum
- *It is not from the benevolence of the butcher, the brewer, or the baker that we expect our dinner, but from their regard to their own interest.*

  – Adam Smith

- What drives the different conclusions?

  First Welfare Theorem assumes away strategic interactions (all players are price takers).

Prisoner’s Dilemma

		Player 2	
		Cheat	**Cooperate**
Player 1	**Cheat**	C, C	A, D
	Cooperate	D, A	B, B

Where A>B>C>D and A+D<2B

For a game to be a Prisoner’s dilemma,

- “Cheat” is a strictly dominant strategy for all players
- But it is socially optimal to cooperate

Bertrand Competition as Prisoner's Dilemma

- Two firms with same MC set price once and for all
- The only Nash equilibrium is to set *P*=*MC*
- Each firm can get much higher profit by charging the monopoly price
  - But this is not a Nash equilibrium

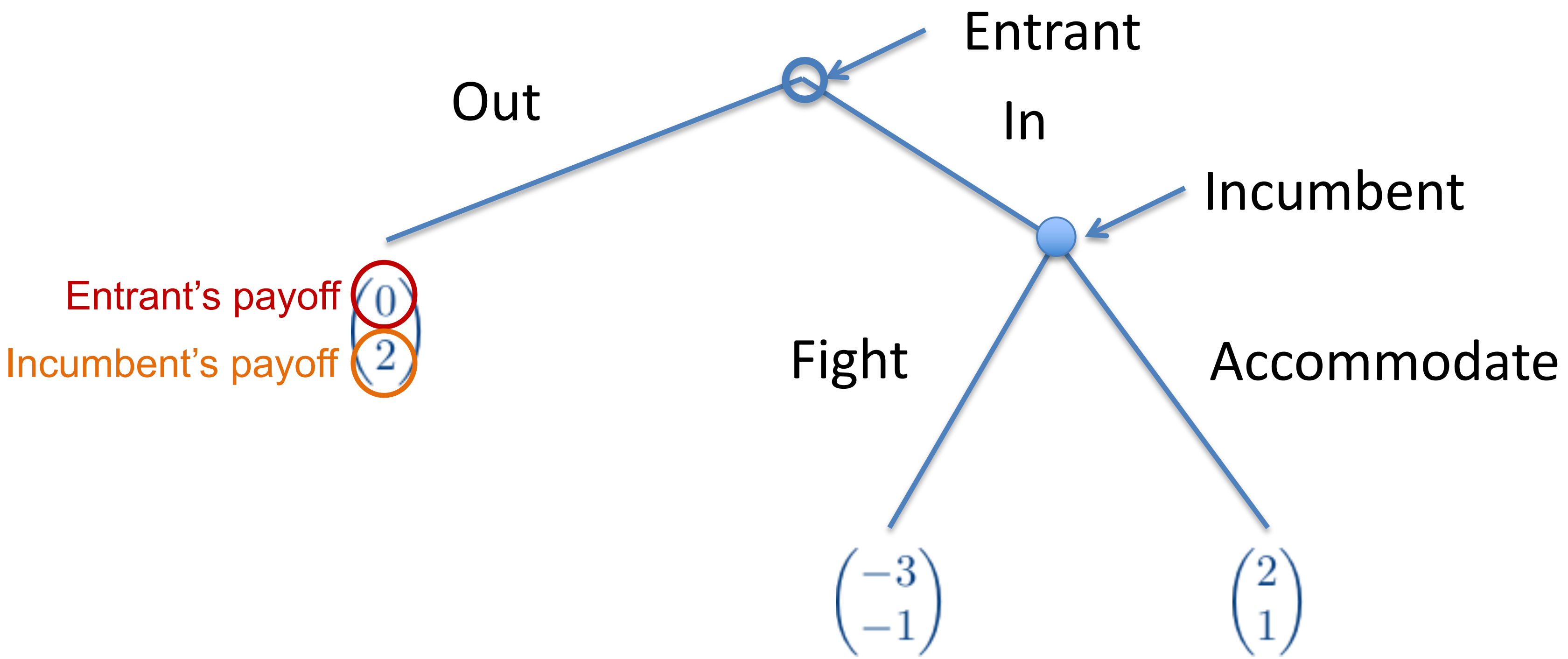
Part 2

(FINITE) SEQUENTIAL GAMES

Entry Game

- A market currently has one incumbent firm
- Period 1
  - An entrant chooses “out” or “in”
  - (The incumbent observes the entrant’s choice)
- Period 2
  - If the entrant chooses “in”, the incumbent can choose to “fight” or “accommodate”
  - If the entrant chooses “out”, the incumbent does nothing
  - (The incumbent’s strategy is a function of entrant’s strategy)

Extensive Form of the Entry Game



- The *extensive form* of a game uses a game tree to show the order of moves and payoffs
- Useful in sequential games

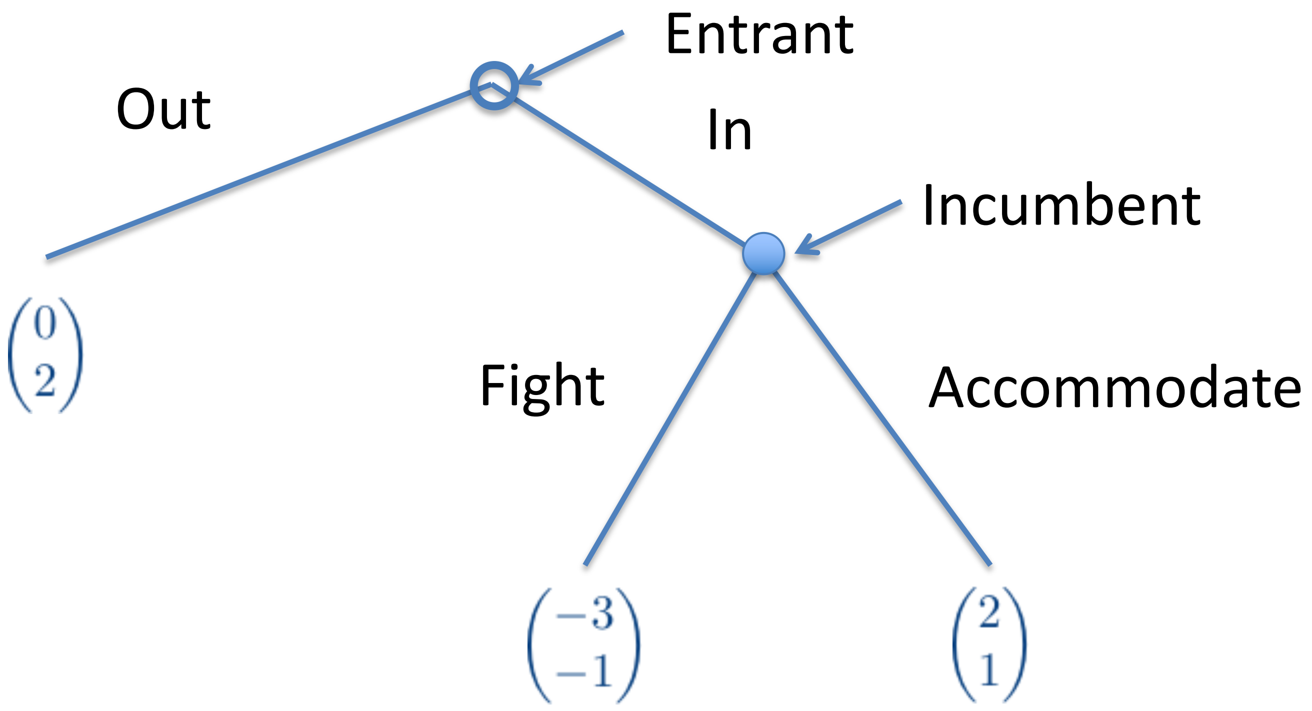
Strategies in Entry Game

- Entrant has 2 strategies
  - “In” and “Out”

- (Incumbent has one information set)

- Incumbent has 2 strategies
  - Fight if In
  - Accommodate if In

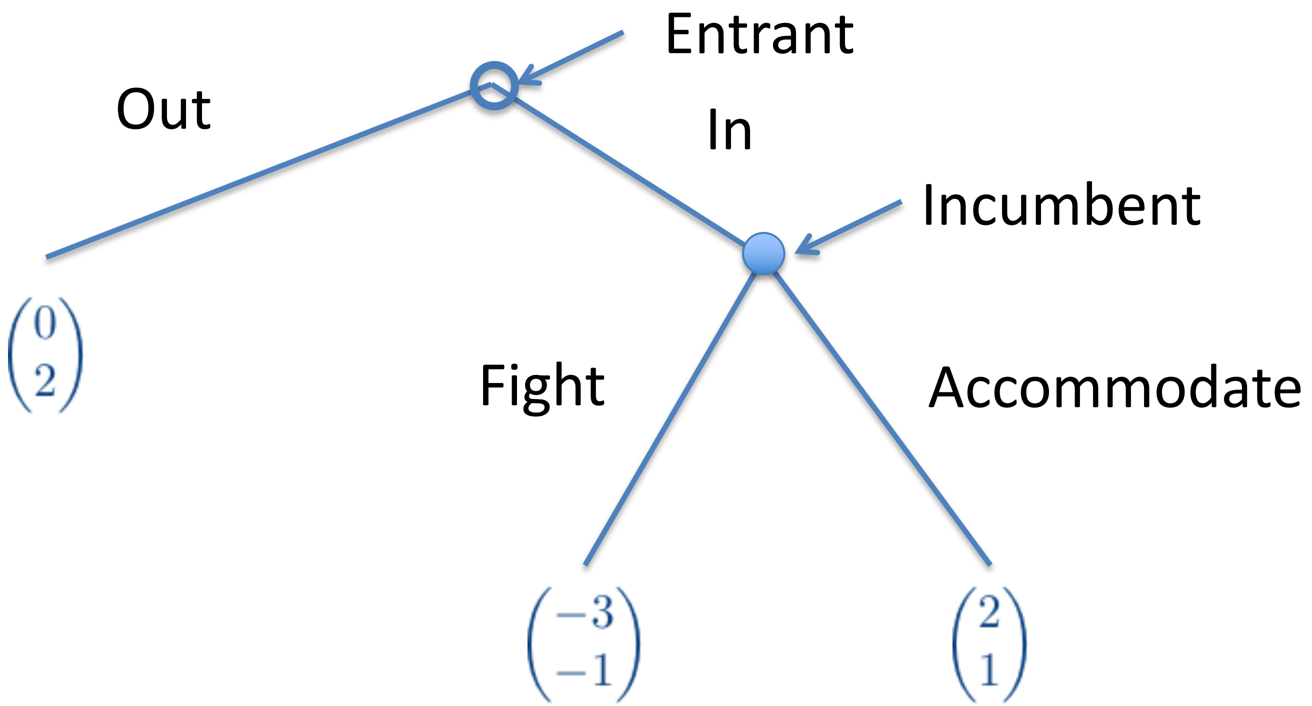
Will incumbent ever choose “Fight if In”?



If the entrant does choose “In”

- Payoff for the incumbent is higher if it chooses “Accommodate”
- The incumbent will not fight
- The entrant will not choose “Out”

Sequential Rationality



If a player finds herself at some information set in the game tree

- She should play the strategy that maximizes her payoff from that point on
- Incumbent choosing "Fight if In" is not sequentially rational

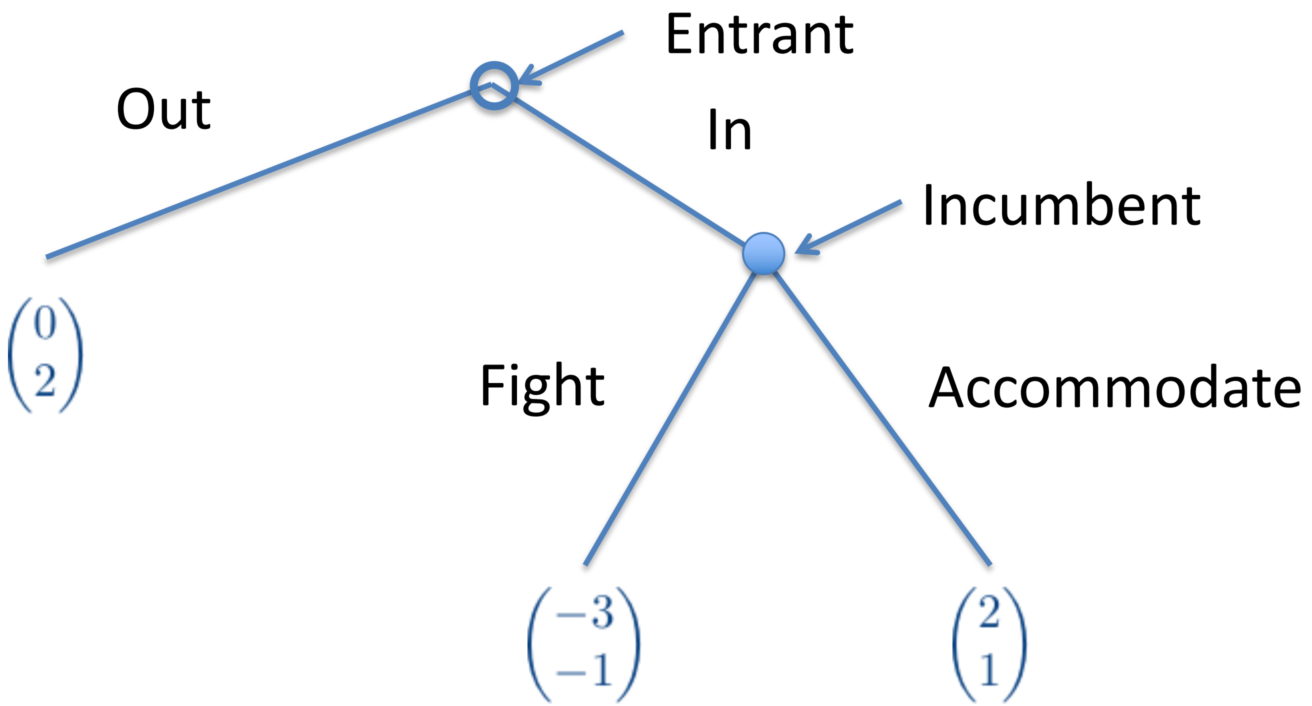
Subgame Perfect Nash Equilibrium

- A *subgame* is a subset of the game that begins with an information set containing a single decision node

- A profile of strategies constitute a *subgame perfect Nash equilibrium* (SPNE) if the strategies played in the SPNE constitute a Nash equilibrium in every subgame of the game

- (Every SPNE is a Nash equilibrium; not every Nash equilibrium is SPNE)

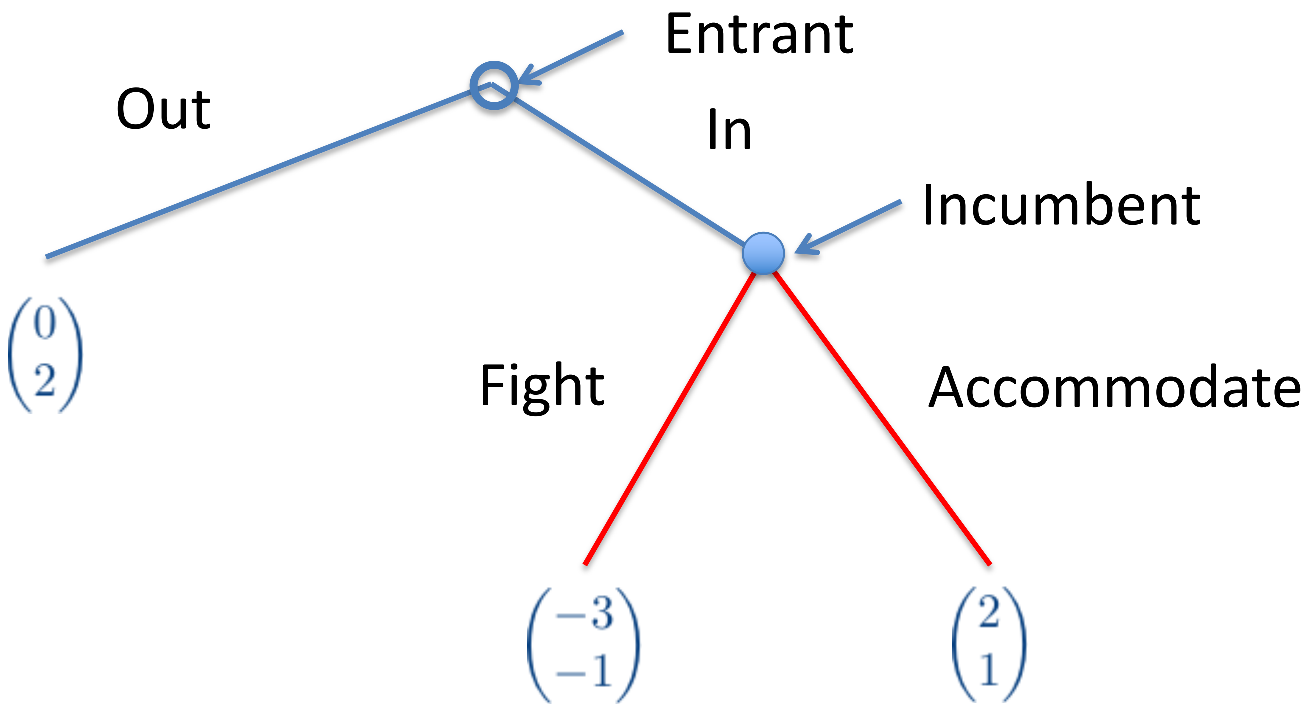
Subgames in the Entry Game



Backward Induction

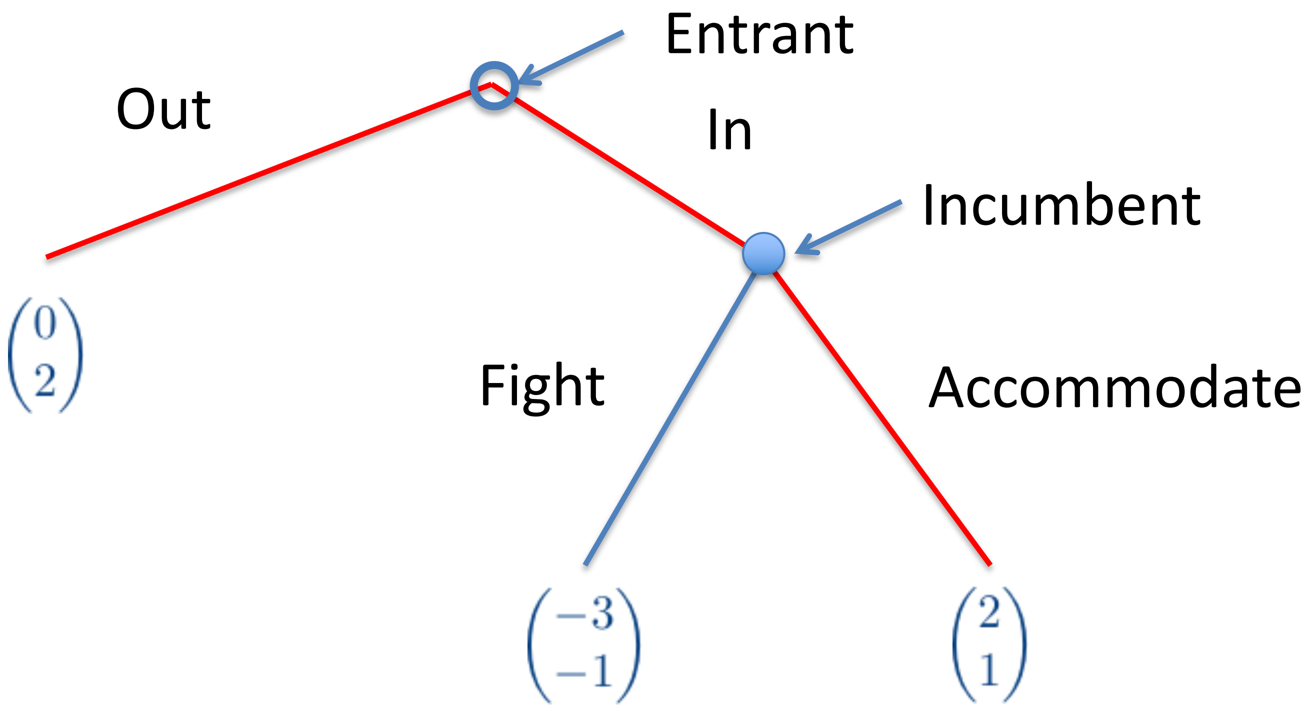
- Every finite sequential game has at least one SPNE
- SPNE can be found by backward induction
  - Solve for the optimal strategy in the last subgame
  - Solve for the optimal strategy in the second subgame
  - Solve for the optimal strategy in the third subgame…

Backward Induction



- If the entrant enters, the incumbent's optimal strategy is to accommodate

Backward Induction



- The entrant should enter, knowing that once she enters, the incumbent's optimal strategy is to accommodate
- (In, Accommodate if in) is the unique SPNE

Part 3

REPEATED GAMES

Prisoner's Dilemma

		Robin	
		Cheat	**Cooperate**
Wayne	**Cheat**	3, 3	10, 0
	Cooperate	0, 10	6, 6

- Wayne and Robin decides simultaneously and independently whether to "cheat" or "cooperate"
- (cheat, cheat) is the unique NE
- What if the game is played repeatedly?

Repeated Games

- In a one-shot game, players do not need to worry about the “consequence” of their actions
  - Retaliation is impossible
  - No punishment for cheating
- If players interact repeatedly, then one could base their action on what the other player has done to him in the previous periods
- It is <u>possible</u> that no one will cheat

Repeated Games

- The same players play the same game repeatedly
  - The game in each period is called the *stage game*
- *Finitely repeated game*
  - Repeat the stage game for T periods
- *Infinitely repeated game*
  - Repeat the stage game infinitely many times
- The players observe the outcomes of all previous stage games
- Every player maximizes the sum of her discounted payoffs in all periods

Nash Reversion Strategy

- Suppose both players adopt the Nash Reversion Strategy
  - I will start off choosing “cooperate”
  - If in the previous period, either player choses “cheat”, then from this period on, I will choose “cheat” in every period

- One period of cheating will trigger permanent punishment—reverting to the Nash equilibrium strategy in the one-shot game

Finitely Repeated Prisoner's Dilemma

		Robin	
		Cheat	**Cooperate**
Wayne	**Cheat**	3, 3	10, 0
	Cooperate	0, 10	6, 6

- Suppose each player maximizes the sum of his discounted payoff stream
- The game is repeated 100 times
- Suppose both players begin by cooperating
- If one cheats, revert to stage game NE

Should Wayne cheat in period 1?

- Suppose the discount factor of Wayne is $\delta = 1$
- If he cooperates in every period, his total payoff is $6 \times 100 = 600$
- If he cheats in period 1
  - He gets 10 in that period
  - But from period 2 on, both players will cheat
  - Wayne gets 3 from period 2 on
  - His total payoff will then be $10 + 3 \times 99 = 307 < 600$
- So Wayne should cooperate?

Should Wayne cheat in period 100?

		Robin	
		Cheat	**Cooperate**
Wayne	**Cheat**	3, 3	10, 0
	Cooperate	0, 10	6, 6

- This is the last stage game
- Game over after period 100
- It is as if they are playing the static game
- Both players will cheat in period 100

Should Wayne cheat in period 99?

- By backward induction
- Every player knows that each player will cheat in period 100
- Hence, no gain by choosing to cooperate in period 99. Every player chooses “cheat”
- By the same reasoning, every player chooses “cheat” in period 98
- …
- Every player chooses “cheat” in period 1

Why cooperation cannot be sustained?

- In this case, the “end game” ruins cooperation. In the final period every player will cheat, regardless of what have been chosen in previous period

- If the stage game is a prisoner’s dilemma, cooperation cannot be sustained if game is only finitely repeated

Infinitely Repeated Games

- Suppose players play the same static game for infinitely many times
- Each player maximizes the sum of discounted payoff over all periods
- Assume each player’s discount factor is $\delta = 0.9$

No Cheating if players are patient

- Assume each player's discount factor is $\delta = 0.9$

- If Wayne deviates in period *t*
  - He gets 10 in that period
  - But 3 in every period after
  - Total payoff is $10 + 0.9 \times 3 + 0.9^2 \times 3 + 0.9^3 \times 3 + \cdots = 37$

- If he cooperates, he gets $6 + 0.9 \times 6 + 0.9^2 \times 6 + 0.9^3 \times 6 + \cdots = 60$

- Wayne should cooperate

Cheating if players are impatient

- Assume each player's discount factor is $\delta = 0.1$

- If Wayne deviates in period *t*
  - He gets 10 in that period
  - But 3 in every period after
  - Total payoff is $10 + 0.1 \times 3 + 0.1^2 \times 3 + 0.1^3 \times 3 + \cdots = 10.33$

- If he cooperates, he gets $6 + 0.1 \times 6 + 0.1^2 \times 6 + 0.1^3 \times 6 + \cdots = 6.67$

- Wayne should cheat