Practice Problem Set 6
Game Applications (C.29)

Question 6.1

Tom is an employee who works for Jerry. Tom can either work (W) or shirk (S), while Jerry can either monitor Tom (M) or ignore him (I). As you can imagine, if Tom is working then Jerry prefers not to monitor, but if Jerry is not monitoring then Tom prefers to shirk. The game is represented in the following matrix:

		Jerry M	Jerry I
	W	1,1	1,2
Tom	S	0,2	2,1

Let p be the probability that Tom chooses W and q the probability that Jerry chooses M. Find the Nash equilibrium(s) of this game.

Answer

There is no PSNE. The unique MSNE is (p,q) = (0.5, 0.5).

Question 6.2

Consider the following game.

		Player 2 **Cheat**	*Player 2* **Cooperate**
Player 1	**Cheat**	1, 1	4, 0
	Cooperate	0, 4	2, 2

(i) What is the unique Nash equilibrium of this game if the game is played only once?
(ii) Suppose the game is repeated infinitely many times between the same two players. The two players have a common discount factor, δ=0.8. Suppose the two players play the Nash reversion strategy we discussed in class. At some period *T*, player 1 is contemplating on cheating. If player 1's objective is to maximize the present value of his total discounted payoff from period *T* onwards, should player 1 cheat in period *T*?
(iii) Recall that $0<\delta<1$. There exists a $\underline{\delta}$ such that for any $\delta>\underline{\delta}$, every player playing "cooperate" in every period can be supported using Nash reversion strategy. What is the value of $\underline{\delta}$ in this game?

Answer

(i) (Cheat, Cheat).
(ii) If player 1 does not cheat, then he receives 2 in every period on, his total payoff is $2+2\delta+2\delta^2+2\delta^3+\ldots=2/(1-\delta)=2/(1-0.8)=10$

If player 1 cheats, he gets a high payoff, 4, in this period, but will be punished from next period on, and his per period payoff will 1, his total payoff is
$4+\delta+\delta^2+\delta^3+\ldots=4+\delta(1+\delta+\delta^2+\ldots)=4+\delta/(1-\delta)=4+0.8/(1-0.8)=8$

Since payoff from not cheating is higher than payoff from cheating, player 1 will not cheat.

(iii) To make sure that each player cooperates, it must be the case that in any period, a player will find cheating unprofitable, that is, the total discounted payoff of not cheating must be higher than the total discounted payoff of cheating. This implies $2/(1-\delta) > 4+\delta/(1-\delta)$, thus $\delta>2/3$. Therefore, $\underline{\delta} = 2/3$.

Question 6.3

Consider a soccer game with the payoff matrix is as follows.

The Free Kick

		Kicker: Kick Left	Kicker: Kick Right
Goalie	Jump Left	1, 0	0, 1
	Jump Right	1-p,p	1, 0

The probability (and payoff) that the kicker will score if he kicks to the left and the goalie jumps to the right is p, where $0 < p < 1$. We want to see how the equilibrium probabilities change as p changes. Let π_G denotes the probability of the goalie jumping left. Let π_K denotes the probability of the kicker kicking left.

(i) Find all the MSNEs of this game.

(ii) The variable p tells us how good the kicker is at kicking the ball into the left side of the goal when it is undefended. As p increases, does the equilibrium probability that the kicker kicks to the left increase or decrease? Why?

[Hint: there are two types of probability in this question, probability of scoring/conceding (e.g., p) and probability of going left/right (π_G, π_K). The first type are "payoffs" in the game theory context. The second is the "probability" that we use to find MSNE. Do not let these "labels" confuse you.]

Answer

(i) Checking mutual best responses (which there are none) lead us to conclude that there is no PSNE in this game.

The goalie is indifferent between jumping left and right if

$$EV(left) = EV(right)$$
$$\Longrightarrow \pi_K = (1-p)\pi_K + (1-\pi_K)$$
$$\Longrightarrow \pi_K = \frac{1}{1+p}$$

The kicker is indifferent between kicking left and right if

$$EV(left) = EV(right)$$
$$\Longrightarrow 0\cdot\pi_G + p(1-\pi_G) = \pi_G + 0(1-\pi_G)$$
$$\Longrightarrow \pi_G = \frac{p}{1+p}$$

We found a MSNE in which the goalie jumps left with probability $\pi_G = \frac{p}{1+p}$ and the kicker kicks left with probability $\pi_K = \frac{1}{1+p}$.

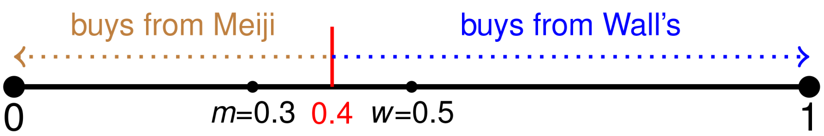
(ii) π_K decreases as p increases. Note that "Left" is the kicker's weak side. The better the kicker's weak side gets, the less often the goalie defends the kicker's good side. So kicker can kick to good side more often.

Question 6.4

In this question we look at a Hotelling's linear city model, which is often employed in economics to study location competition. Consider a population of consumers uniformly distributed along a linear line of length 1, represented by the black line in the diagram below.

In this linear city, the population has homogeneous tastes. By law, ice cream stores can only sell ice creams at p per unit. Each consumer values ice creams at v per unit, and $v > p$. Moreover, each consumer would consume one ice cream at most.

There are only two stores, Meiji and Wall's, selling ice creams. The cost of making an extra unit of ice cream is zero. The two stores simultaneously choose their respective locations along the linear line. Their location choices would determine the fraction of consumers each store captures, as consumers always buy from the closest store. Let m denote the location of Meiji and w denote the location of Wall's along this linear line. If, for example, $m = 0.3$ and $w = 0.5$, Meiji will capture 40% of the consumers and Wall's will capture the remaining 60%, as shown in the diagram below. If $m = w$, the two stores split the demand equally.



(i) Can m=0.1, w=0.75 be supported as a Pure Strategy Nash Equilibrium? Explain.
(ii) Can m=0.5, w=0.5 be supported as a Pure Strategy Nash Equilibrium? Explain.
(iii) Can m=0.3, w=0.6 be supported as a Pure Strategy Nash Equilibrium? Explain.
(iv) Can m=0, w=1 be supported as a Pure Strategy Nash Equilibrium? Explain.

Answer

(i) No. Given the strategy of the other player, each player can increase its profit (has the incentive to deviate) by moving toward the other player.
(ii) Yes.
(iii) No. Given the strategy of the other player, each player can increase its profit (has the incentive to deviate) by moving toward the other player.
(iv) No. Given the strategy of the other player, each player can increase its profit (has the incentive to deviate) by moving toward the other player.