

Macroeconomics Analysis II, EC3102

Tutorial 8 (Solow model)¹

Question 1 *Growth rates in the Solow model.*

Given the production function is

$$Y_t = \bar{A} K_t^{\frac{1}{3}} \bar{L}^{\frac{2}{3}}.$$

a.

Express the growth rate of per capita GDP as a function of the growth rate of the capital stock. (Hint: You may use the results in Question 4 for previous tutorial. Note that in this question, labour force is fixed.)

b.

By dividing the capital accumulation equation $\Delta K_{t+1} = \bar{s}Y_t - \bar{d}K_t$ by K_t , we obtain:

$$\frac{\Delta K_{t+1}}{K_t} = \bar{s} \frac{Y_t}{K_t} - \bar{d}.$$

From the capital-output ratio in steady state, we obtain:

$$\frac{K^*}{Y^*} = \frac{\bar{s}}{\bar{d}}.$$

Combining these results, show that the growth rate of per capita GDP is $\frac{1}{3} \cdot \bar{s} \frac{Y^*}{K^*} \cdot \left(\frac{K^{*\frac{2}{3}}}{K_t^{\frac{2}{3}}} - 1 \right)$.

¹Questions 1 is adapted from the adopted text, Macroeconomics by Jones (2018), Chapter 5 Q.9

Question 2 Changes in model parameter: An increase in research productivity.²

Suppose an economy is on a balanced growth path in a simple Romer model, as described in the text, where aggregate output is:

$$Y_t = A_t L_{yt},$$

law of motion of ideas (idea production function) is:

$$\Delta A_{t+1} = \bar{z} A_t^\alpha L_{at},$$

labour resource constraint is:

$$L_{yt} + L_{at} = \bar{L},$$

and the allocation of labour is:

$$L_{at} = \bar{l} \bar{L},$$

where \bar{z} is a productivity parameter of the researchers, A_0 is the existing stock of ideas at time, $t = 0$, \bar{L} is the total population and assumed to be constant, and \bar{l} is the constant fraction of population who works in the research sector.

Suppose in year 2030, research productivity \bar{z} rises immediately and permanently to the new level \bar{z}' .

a.

How would the growth rate of ideas behave if $\alpha < 1$?

b.

Assuming $\alpha = 1$, solve for the new growth rate of knowledge and growth rate of output per capita.

c.

Assuming $\alpha = 1$, sketch the graph of $\ln y_t$ against time t .

d.

Why might research productivity increase in an economy?

²Question 2 is adapted from the adopted text, Macroeconomics by Jones (2018), Chapter 6, Q.8.