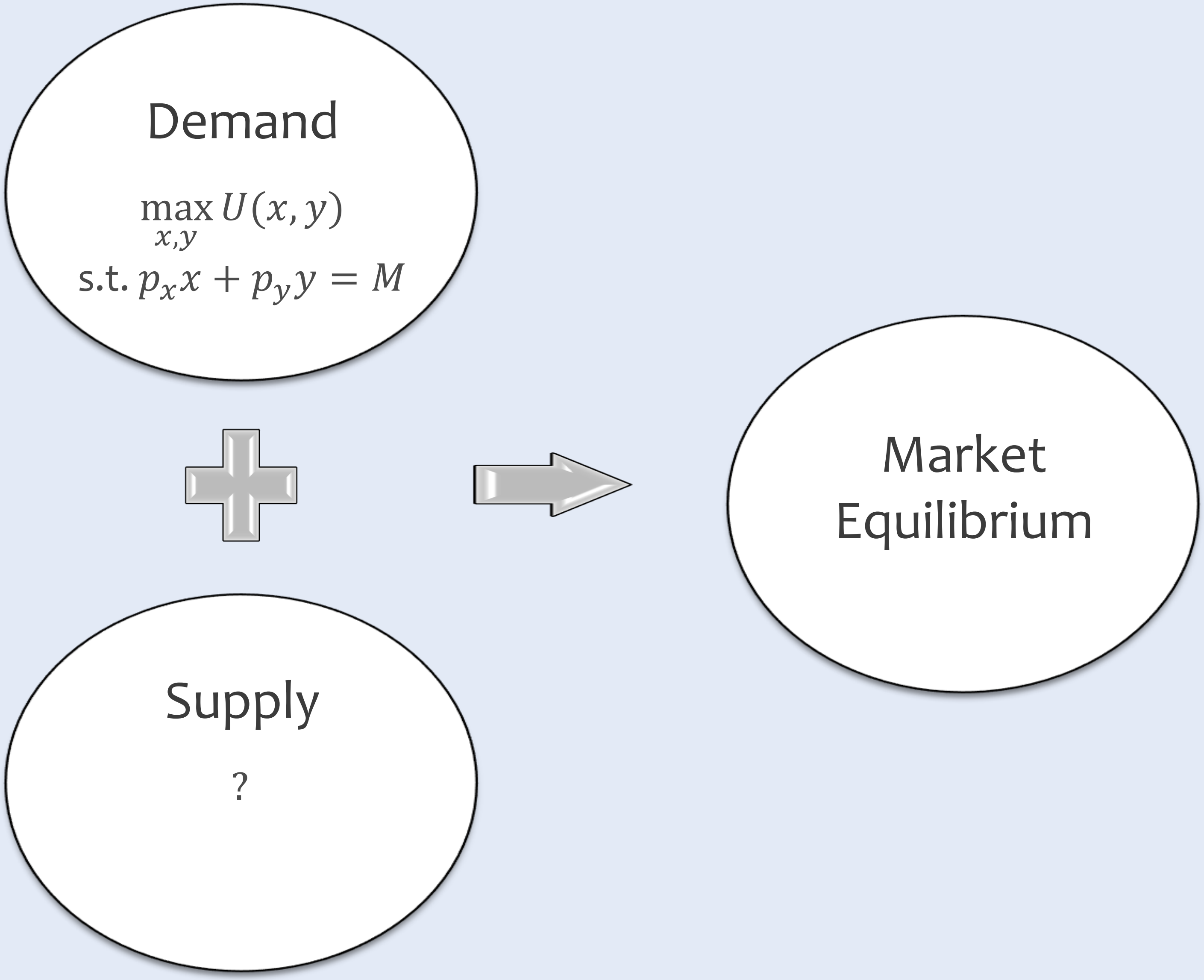
EC2101: Microeconomic Analysis I

The Big Picture



Producer Theory

- The firm's production function
- The firm in the short run:
  - Optimal choice of L and K
  - Cost curves
- The firm in the long run:
  - Optimal choice of L and K
  - Cost curves
- The firm's optimal choice of Q

Lecture 8

Theory of the Producer

- Production Function
  - With One Input
  - With Two Inputs
- Returns to Scale
- Technological Progress

What is Production?

- Firms transform inputs into outputs.
- Factors of production / Inputs:
  - Labor
  - Raw material
  - Equipment
  - Land
- A firm’s production technology tells us
  how the firm transforms inputs into outputs.

Production Function

- Suppose the firm needs two inputs
  — labor (L) and capital (K) —
  to produce an output.

- The production function tells us
  the maximum quantity of output (Q)
  that the firm can produce
  given the quantity of inputs, L and K.
  - $Q = f(L, K)$

Short Run vs. Long Run

- Production in the short run:
  - At least one input is fixed.
- Production in the long run:
  - All inputs are variable.

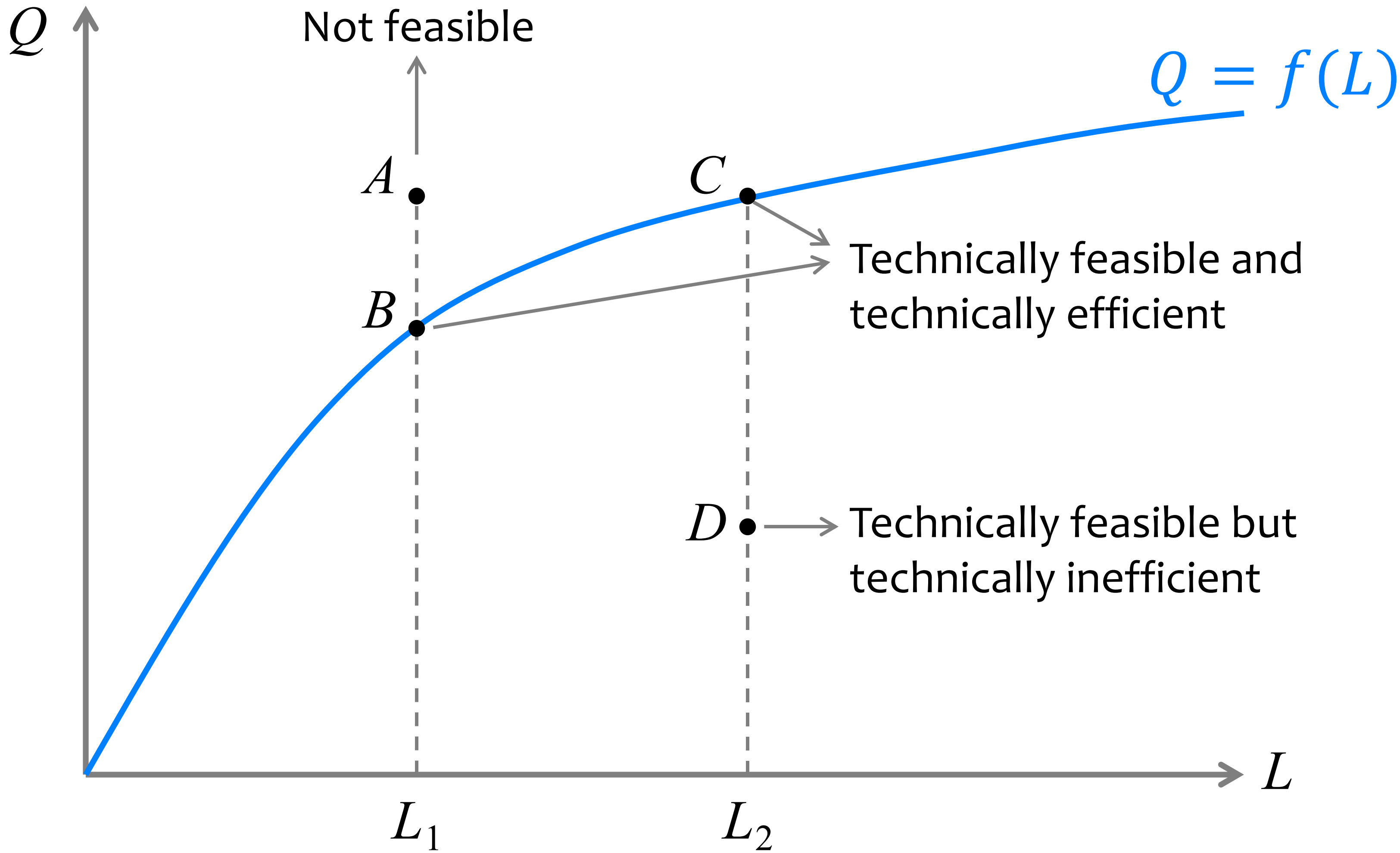
Production Function with One Input

Production Function with One Input

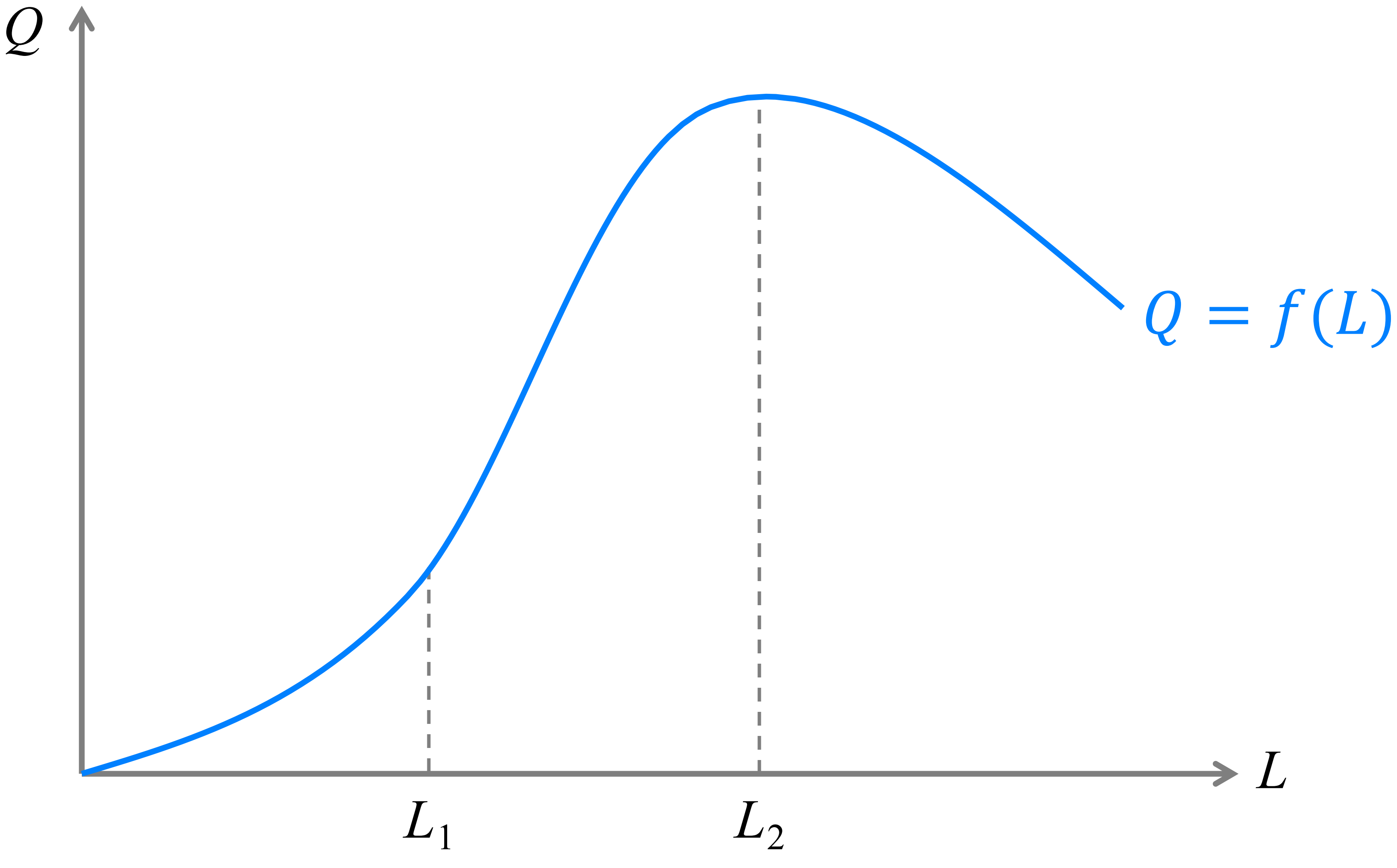
- Suppose that in the short run:
  - Capital is fixed.
  - The firm can adjust only the quantity of labor.
  - The production function is:

$$Q = f(L)$$

Technically Efficient and Technically Feasible



A Typical Production Function



Production Function:

Marginal Product

Marginal Product

- Marginal product of labor:
  - The rate at which the output level changes as the quantity of labor changes.

$$MP_L = \frac{dQ}{dL} = \frac{\Delta Q}{\Delta L}$$

  where ΔL is extremely small.

  - The slope of the production function.

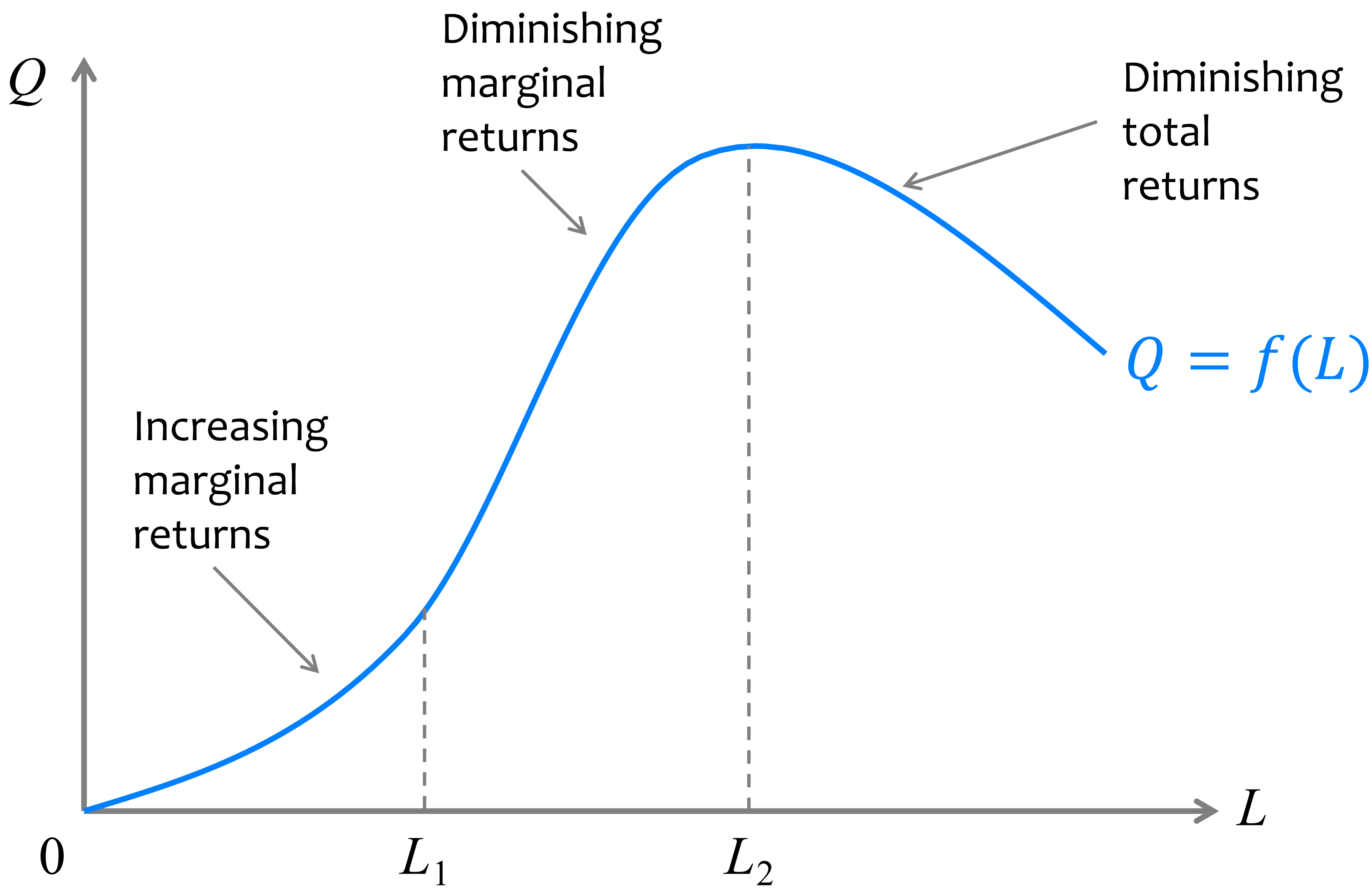
Increasing vs. Diminishing Total Returns

- Increasing total returns:
  - Q increases as L increases.
  - MP_L is positive.
  - $MP_L = \frac{dQ}{dL} > 0$
- Diminishing total returns:
  - Q decreases as L increases.
  - MP_L is negative.
  - $MP_L = \frac{dQ}{dL} < 0$

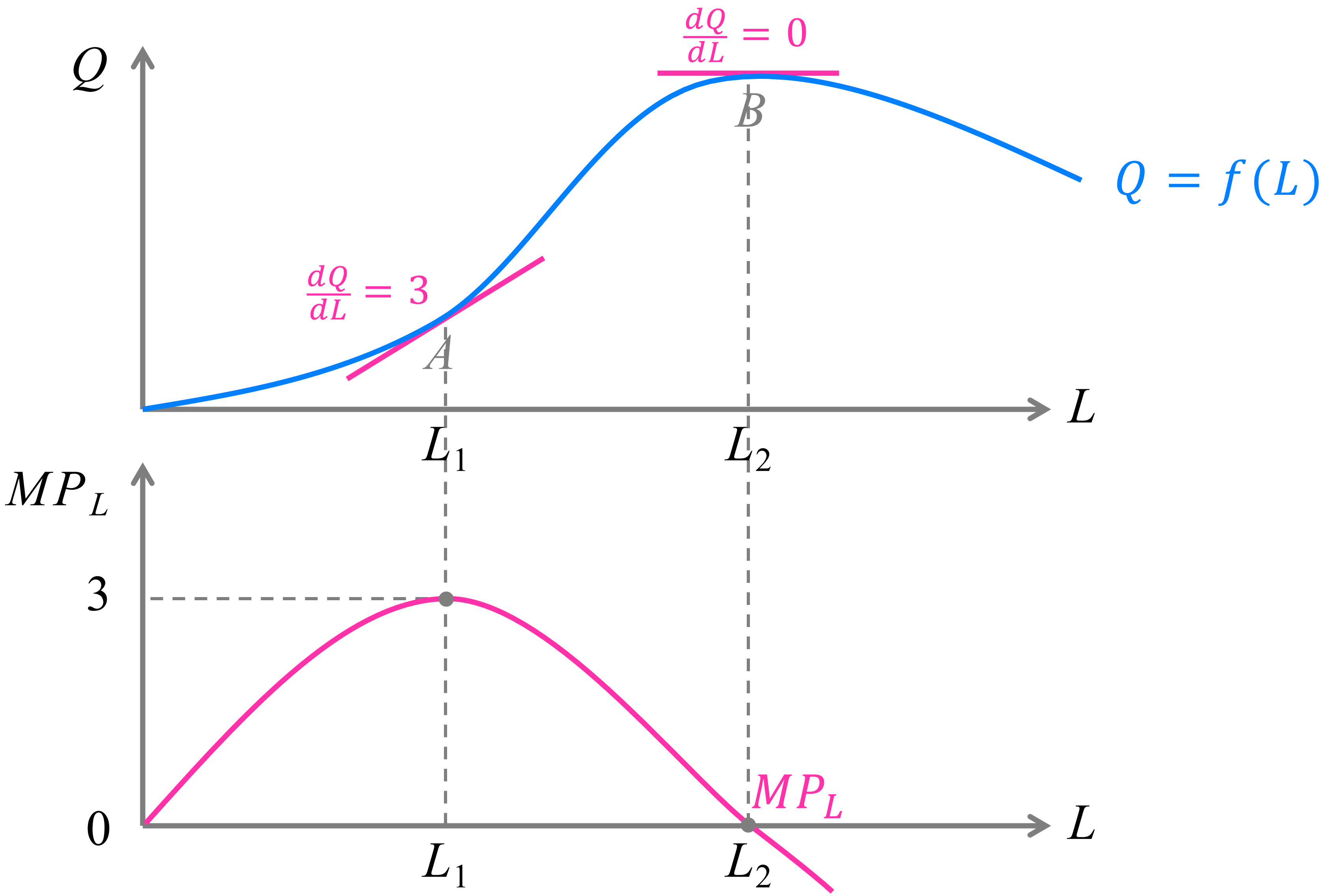
Increasing vs. Diminishing Marginal Returns

- Increasing marginal returns:
  - MP_L increases as L increases.
- Diminishing marginal returns:
  - MP_L decreases as L increases.
- Law of diminishing marginal returns:
  - Suppose capital is fixed.
    The marginal product of labor (MP_L)
    will eventually decline
    as the quantity of labor (L) increases.

A Typical Production Function



Marginal Product: Graphical Representation



Production Function:

Average Product

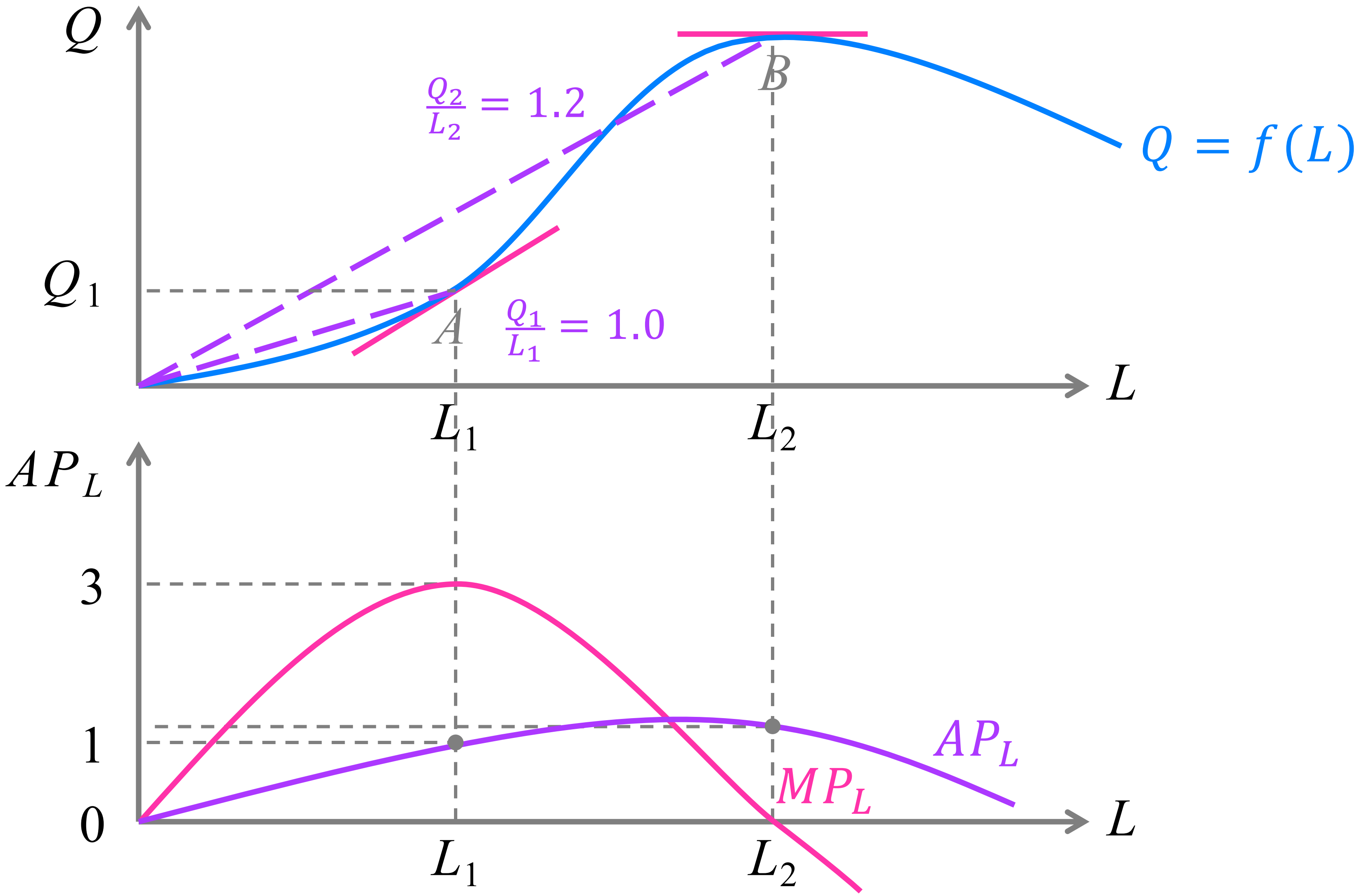
Average Product

- Average product of labor:
  - Output per unit of labor.

$$AP_L = \frac{Q}{L}$$

  - The slope of
    the ray connecting the origin and the point $(L, f(L))$.

Average Product: Graphical Representation



Production Function:

Marginal vs. Average

Marginal vs. Average

- Suppose you buy 5 apples, and pay a total of $5.00.
  - You pay an average price of $1.00 per apple.
- Now you decide to buy 1 additional apple.
  - Now you pay an average price of $0.90 per apple.
- Does the 6th apple cost you
  more than $1.00 or less than $1.00?

MP_L and AP_L

- When AP_L is rising as L increases:
  - As the quantity of labor increases (L), the average product of labor (AP_L) is rising.
  - The output generated by an additional unit of labor (MP_L) is pulling up the average (AP_L).
  - $MP_L > AP_L$

MP_L and AP_L

- When AP_L is falling as L increases:
  - As the quantity of labor increases (L), the average product of labor (AP_L) is falling.
  - The output generated by an additional unit of labor (MP_L) is pulling down the average (AP_L).
  - $MP_L < AP_L$

MP_L and AP_L

- To summarize:
  - When AP_L is rising as L increases, $MP_L > AP_L$.
  - When AP_L is falling as L increases, $MP_L < AP_L$.
- Therefore, AP_L intersects with MP_L at the highest point of AP_L.

MP_L and AP_L: Mathematical Explanation

- Since $AP_L = \frac{Q(L)}{L}$

- Take the derivative of AP_L with respect to L:

$$\frac{dAP_L}{dL} = \frac{d\left(\frac{Q(L)}{L}\right)}{dL}$$

$$= \frac{L \cdot MP_L - Q(L) \cdot 1}{L^2}$$

$$= \frac{MP_L - AP_L}{L}$$

MP_L and AP_L: Mathematical Explanation

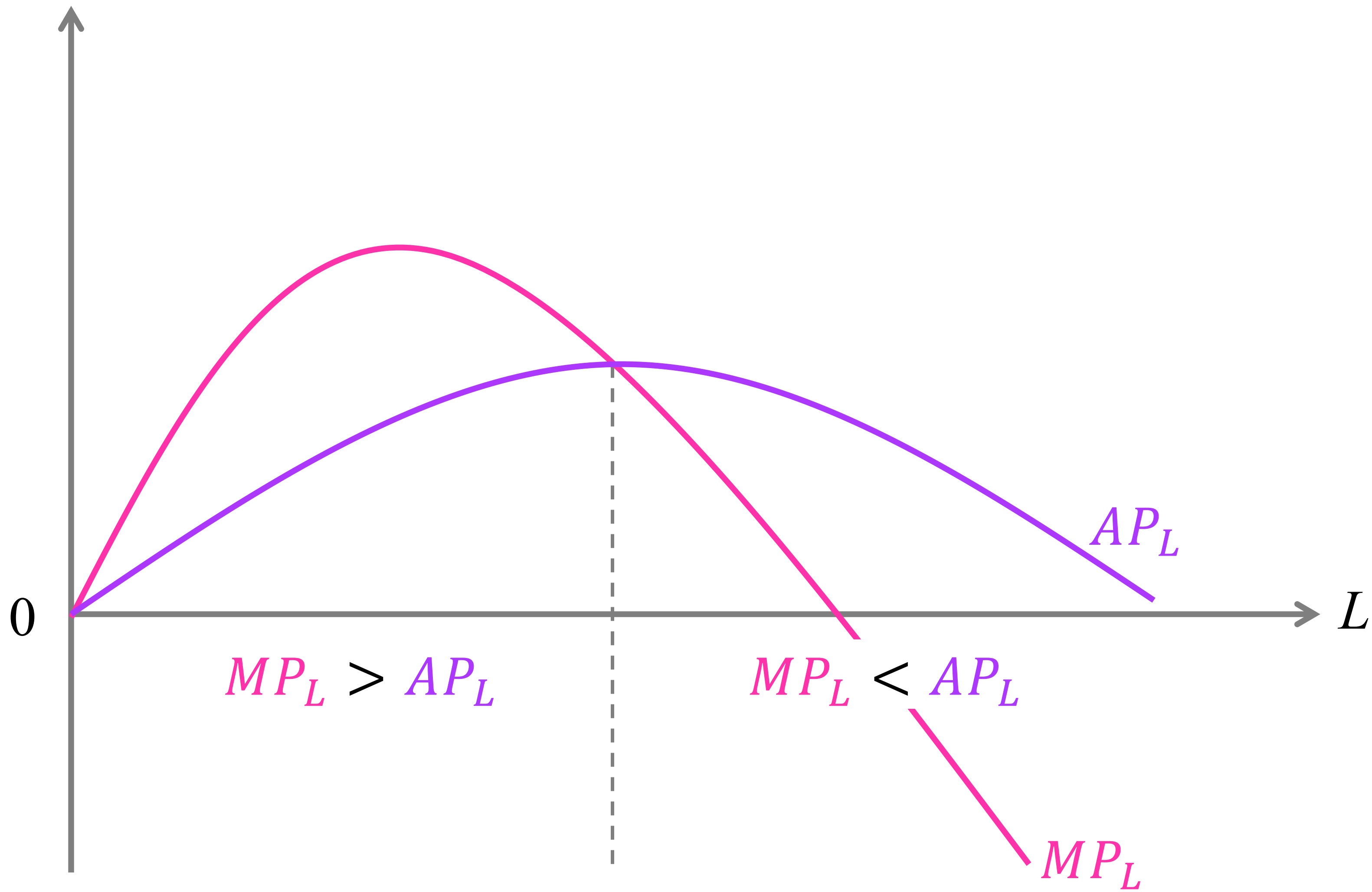
- If AP_L is rising as L increases, then:

$$\frac{dAP_L}{dL} > 0$$

$$\frac{MP_L - AP_L}{L} > 0$$

$$MP_L > AP_L$$

MP_L and AP_L: Graphical Representation



Summary

Consumer Theory vs. Producer Theory

Consumer Theory	**Producer Theory**
Utility function $U(x)$	
Marginal utility $MU_x = \frac{\partial U}{\partial x}$	
Diminishing marginal utility $\frac{\partial MU_x}{\partial x} \leq 0$	

Production Function with Two Inputs

Production Function with Two Inputs

- Suppose that in the long run:
  - The firm can adjust both the quantity of labor and the quantity of capital.
  - The production function is:

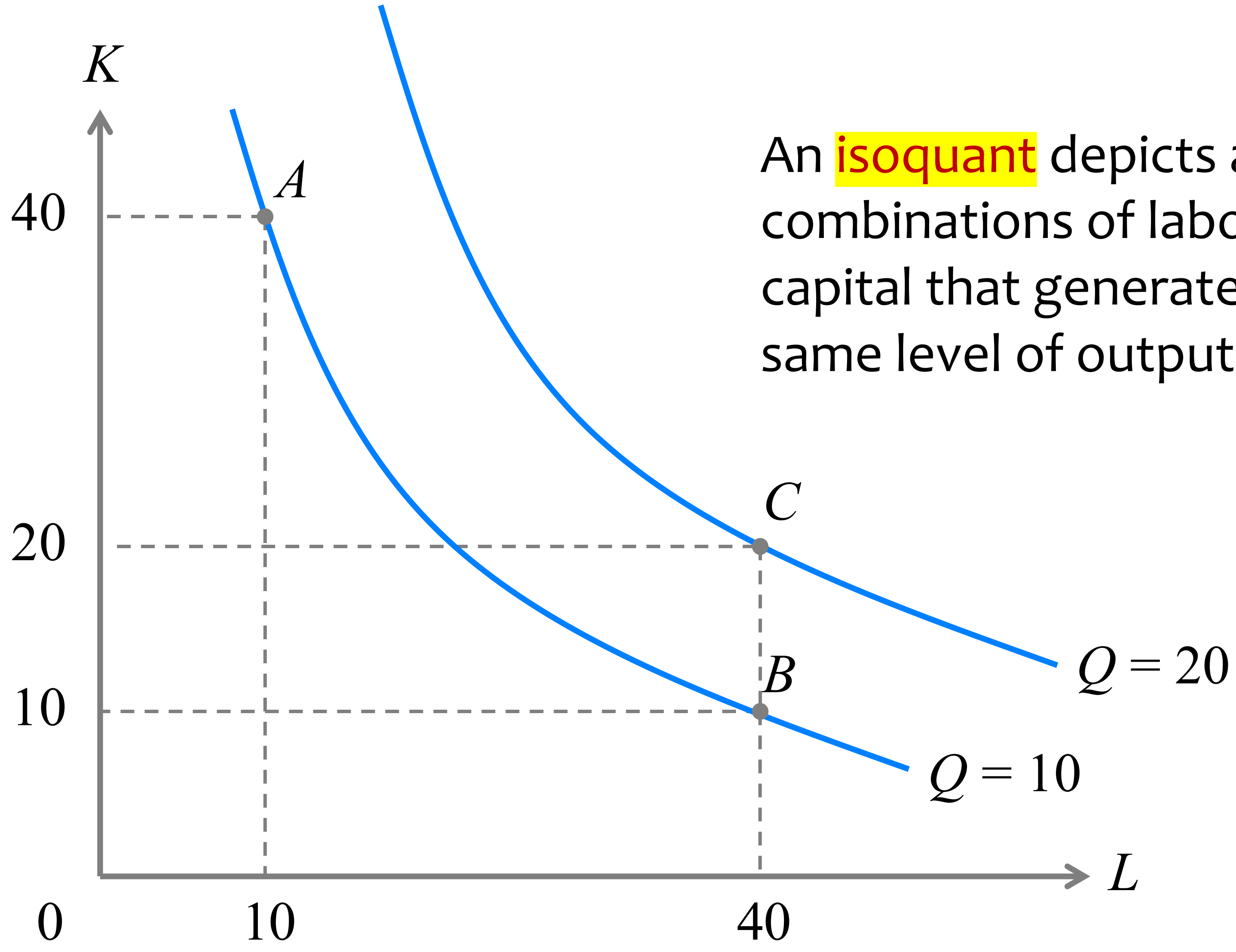
$$Q = f(L, K)$$

- Marginal products:

$$MP_L = \frac{\partial Q}{\partial L}$$

$$MP_K = \frac{\partial Q}{\partial K}$$

Isoquant



An isoquant depicts all the combinations of labor and capital that generate the same level of output.

Marginal Rate of Technical Substitution

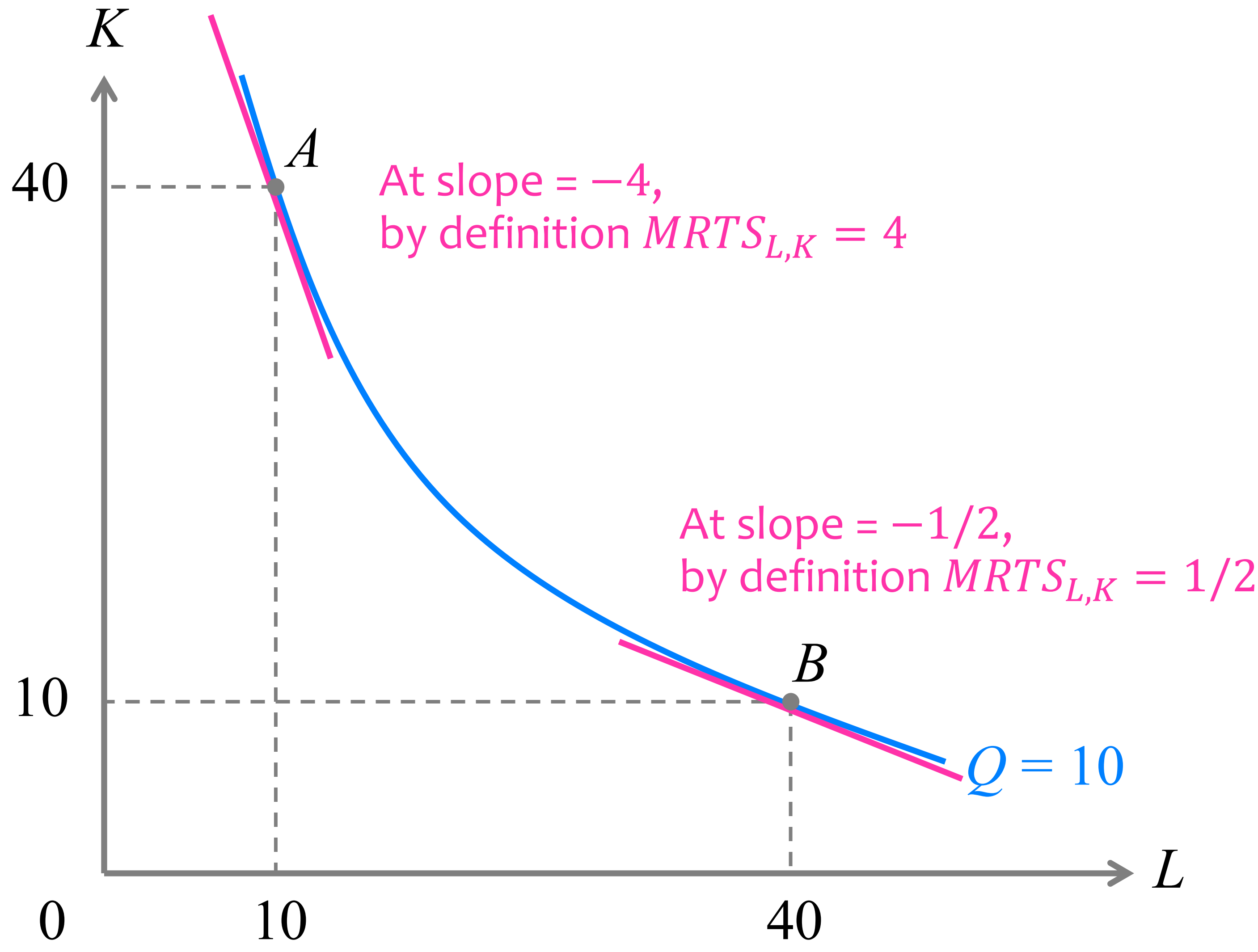
- Marginal rate of technical substitution of labor for capital:
  - The rate at which the firm can
    reduce the quantity of capital and
    increase the quantity of labor,
    maintaining the same level of output.

$$MRTS_{L,K} = -\left.\frac{dK}{dL}\right|_{same\ Q} = -\left.\frac{\Delta K}{\Delta L}\right|_{same\ Q}$$

where ΔL is extremely small.

  - The negative of the slope of the isoquant.

$MRTS$ and Slope



Diminishing Marginal Rate of Technical Substitution

- Diminishing marginal rate of technical substitution:
  - On an isoquant,
    $MRTS_{L,K}$ decreases
    as the firm uses more labor and less capital.
  - Holding the output level fixed,
    as the firm uses more labor,
    the ability to give up capital
    in exchange for an additional unit of labor falls.

$MRTS$ and MP_L

- Suppose the consumer moves from one point to another point on the same isoquant.
- The total change in output is:

$$\Delta Q = MP_L(\Delta L) + MP_K(\Delta K)$$

$$0 = MP_L(\Delta L) + MP_K(\Delta K)$$

$$MP_L(\Delta L) = -MP_K(\Delta K)$$

$$\frac{MP_L}{MP_K} = -\frac{\Delta K}{\Delta L}$$

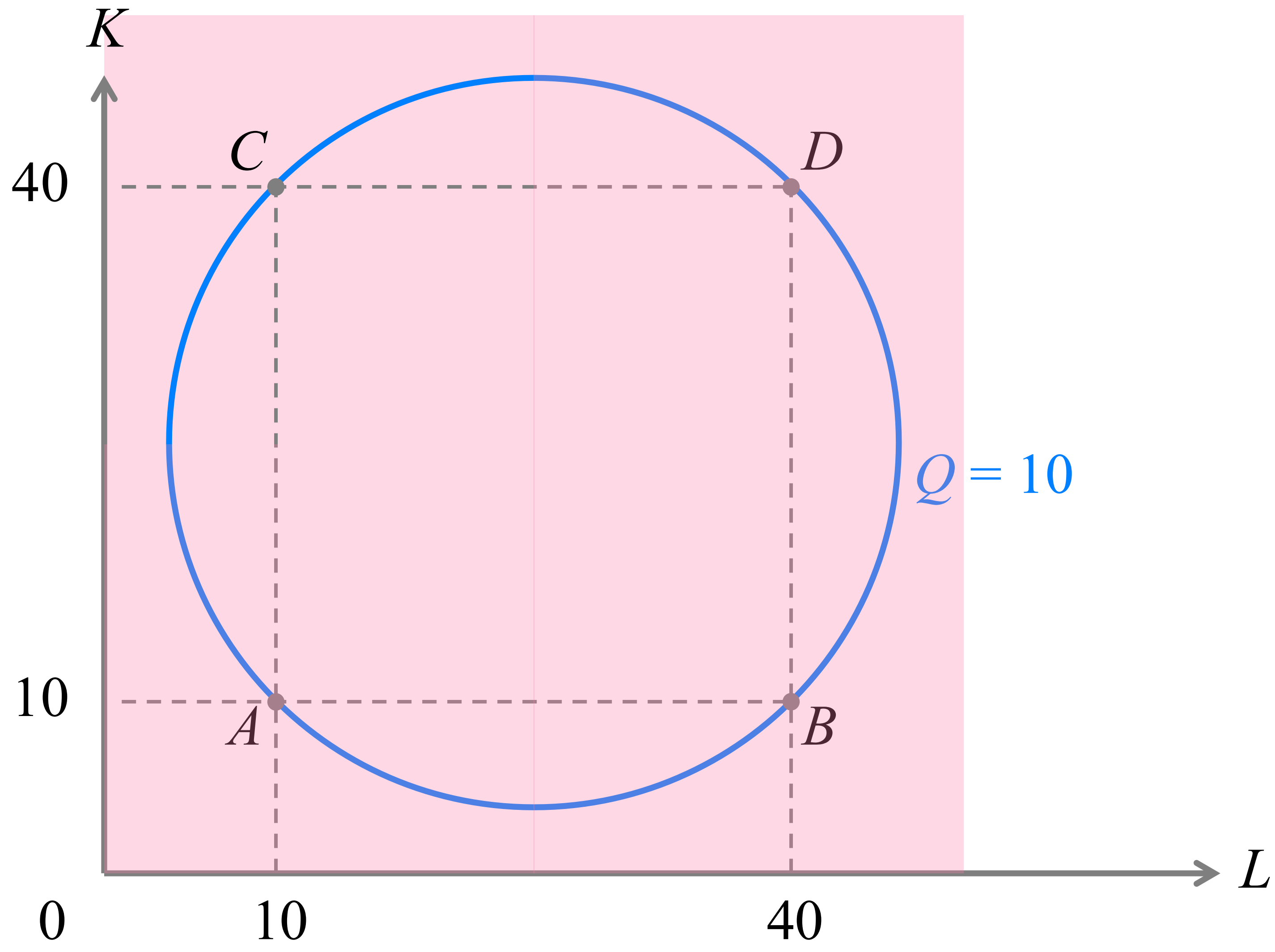
$$\frac{MP_L}{MP_K} = MRTS_{L,K}$$

Summary

Consumer Theory vs. Producer Theory

Consumer Theory	**Producer Theory**
Indifference curve $U(x, y) = 10$	
Marginal rate of substitution $MRS_{x,y} = \frac{MU_x}{MU_y}$	
Diminishing MRS $\frac{\partial MRS_{x,y}}{\partial x} \leq 0$	

Uneconomic Region of Production

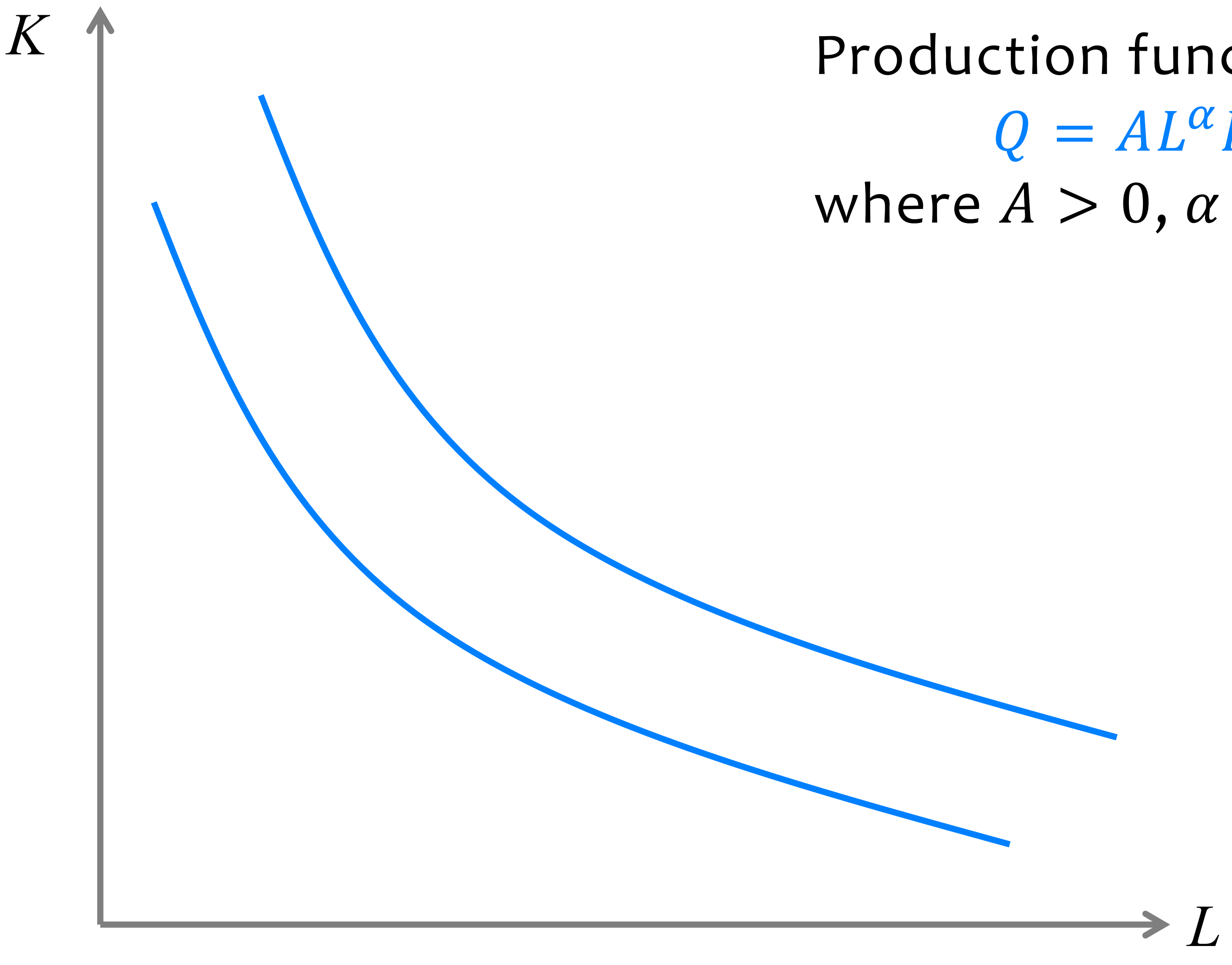


Marginal Product and Uneconomic Region of Production

- In the uneconomic region of production:
  - At least one marginal product is negative.

- Cost-minimizing firms never produce in the uneconomic region of production, e.g.,
  - If the firm produces at point *B*, it uses 40 units of labor and 10 units of capital.
  - The firm can produce the same quantity of output at point *A* using 10 units of labor and 10 units of capital.

Cobb-Douglas Production Function



Production function:

$$Q = AL^{\alpha}K^{\beta}$$

where $A > 0, \alpha > 0, \beta > 0$

Exercise 8.1

Linear Production Function

A firm uses two inputs in the production process: experienced worker *(E)* and new worker *(N)*. Suppose 1 experienced worker is equivalent to 2 new workers.

(a) Draw a graph of the firm's production function with *E* on the horizontal axis and *N* on the vertical axis.

(b) What is the slope of the isoquants?

(c) Write the mathematical expression of the production function.

Exercise 8.1

Linear Production Function

Exercise 8.2

Fixed-Proportion Production Function

A firm uses two inputs in the production process: bicycle tires (*T*) and bicycle frames (*F*). A bicycle requires exactly 1 frame and 2 tires.

(a) Draw a graph of the firm's production function with *T* on the horizontal axis and *F* on the vertical axis.

(b) Write the mathematical expression of the production function.

(c) Where are the kinks of the isoquants?

Exercise 8.2

Fixed-Proportion Production Function

Returns to Scale

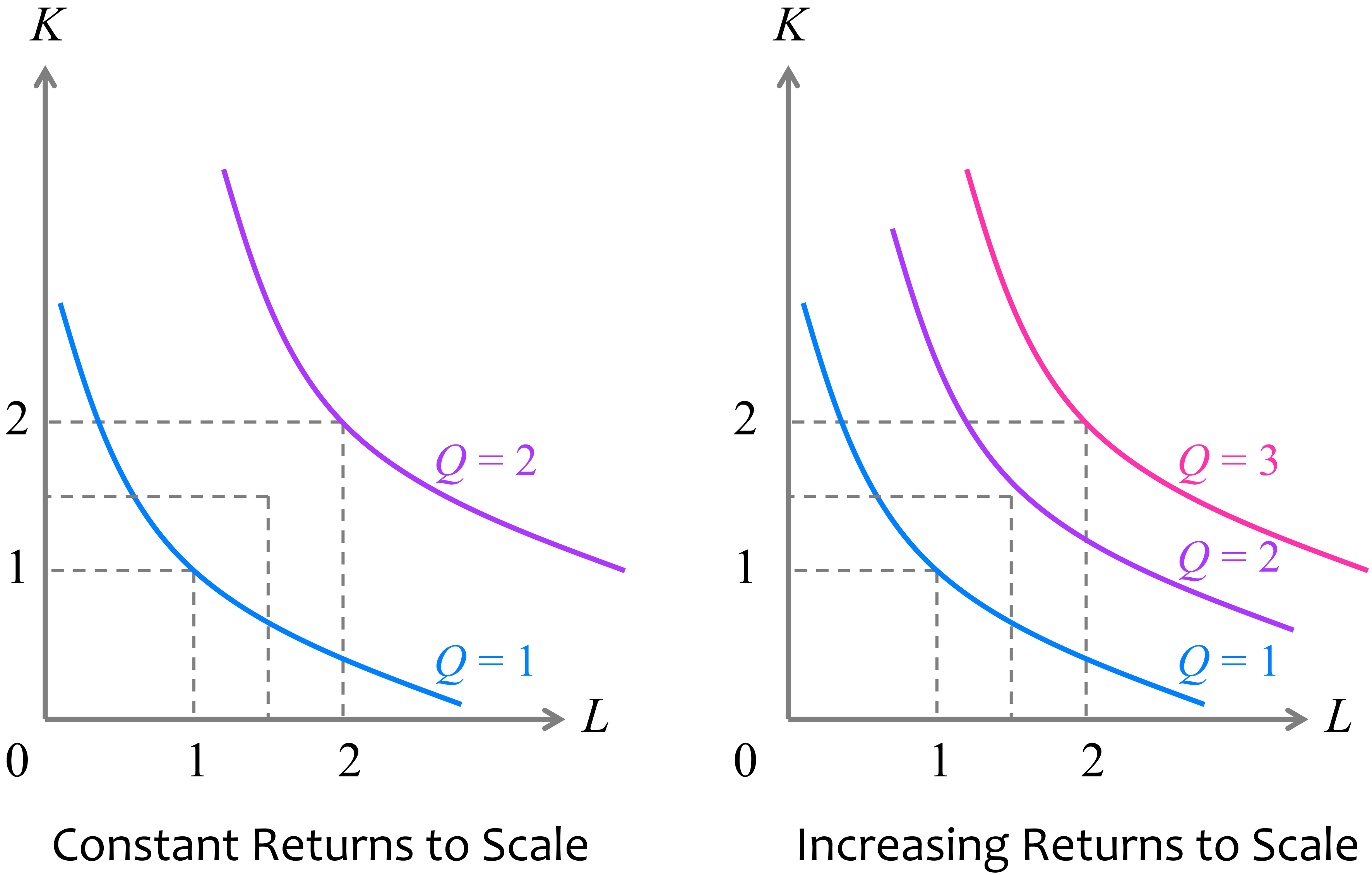
Returns to Scale

- How much more Q can the firm produce when it uses more L and more K?
- Returns to scale measure the rate at which output increases when all inputs increase proportionally, e.g.,
  - What is the increase in output if both labor and capital increase by 25%?
  - What is the increase in output if both labor and capital increase by 100%?

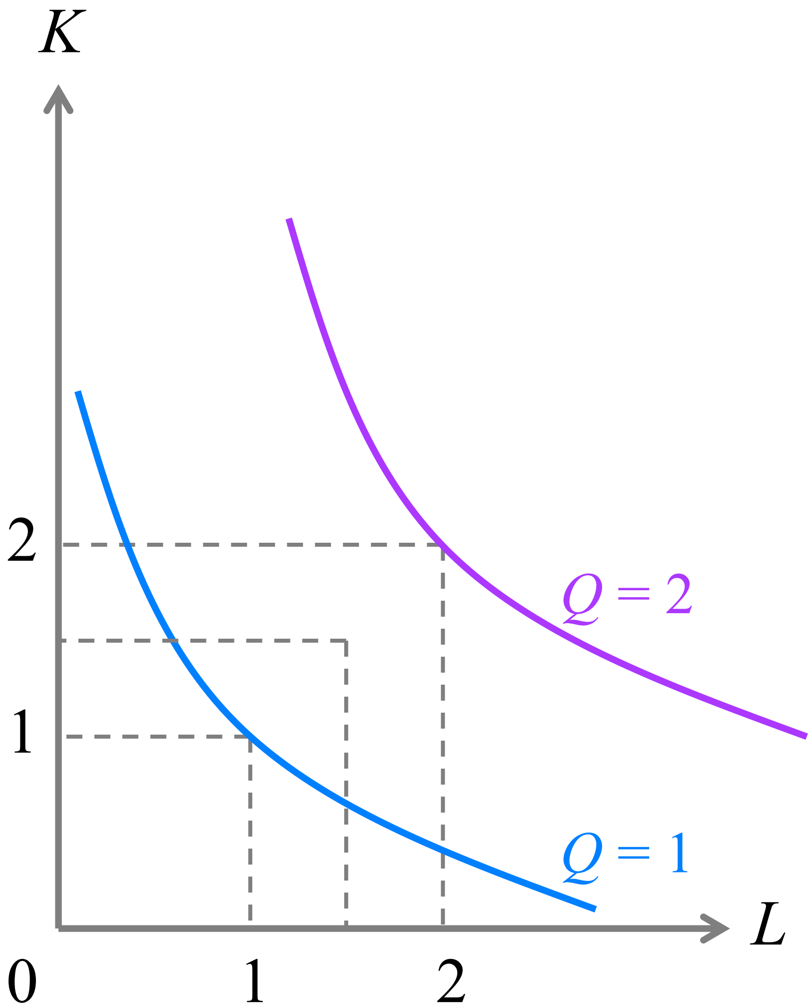
Returns to Scale

- Suppose L increases to αL and
  K increases to αK, where $\alpha > 1$.
- Output Q increases to βQ.
  - $\beta > \alpha$: Increasing returns to scale
  - $\beta = \alpha$: Constant returns to scale
  - $\beta < \alpha$: Decreasing returns to scale

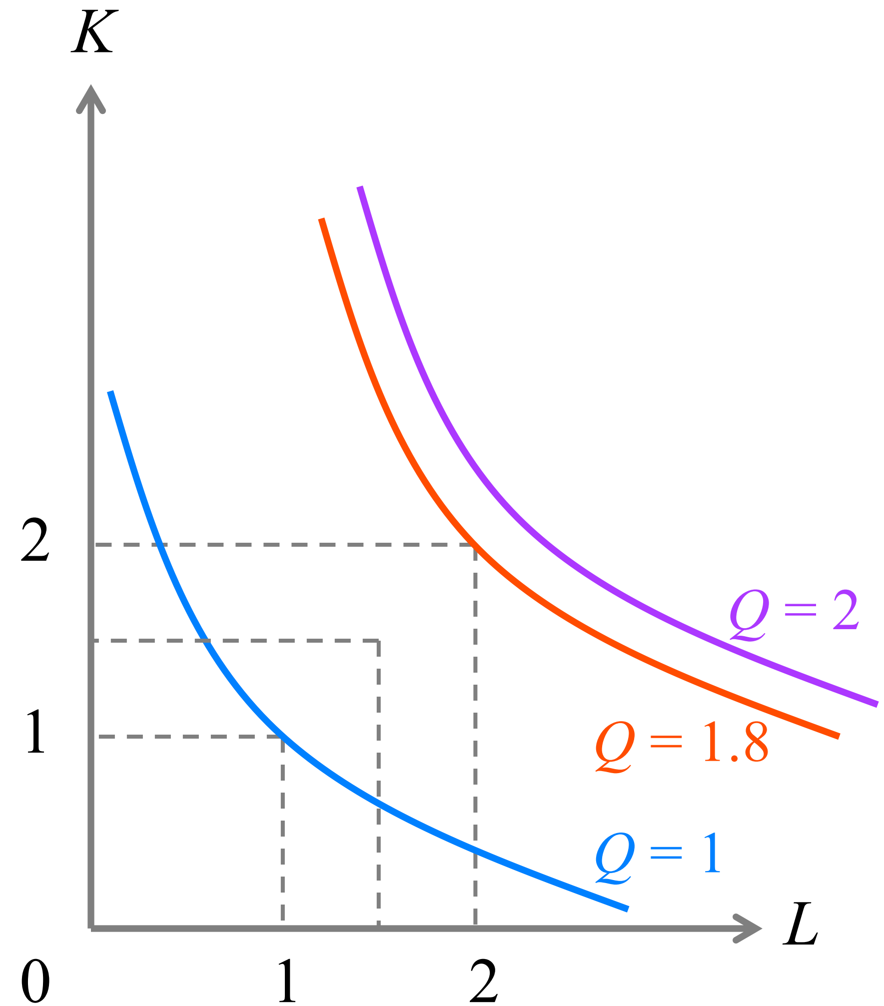
Returns to Scale



Returns to Scale



Constant Returns to Scale



Decreasing Returns to Scale

Technological Progress

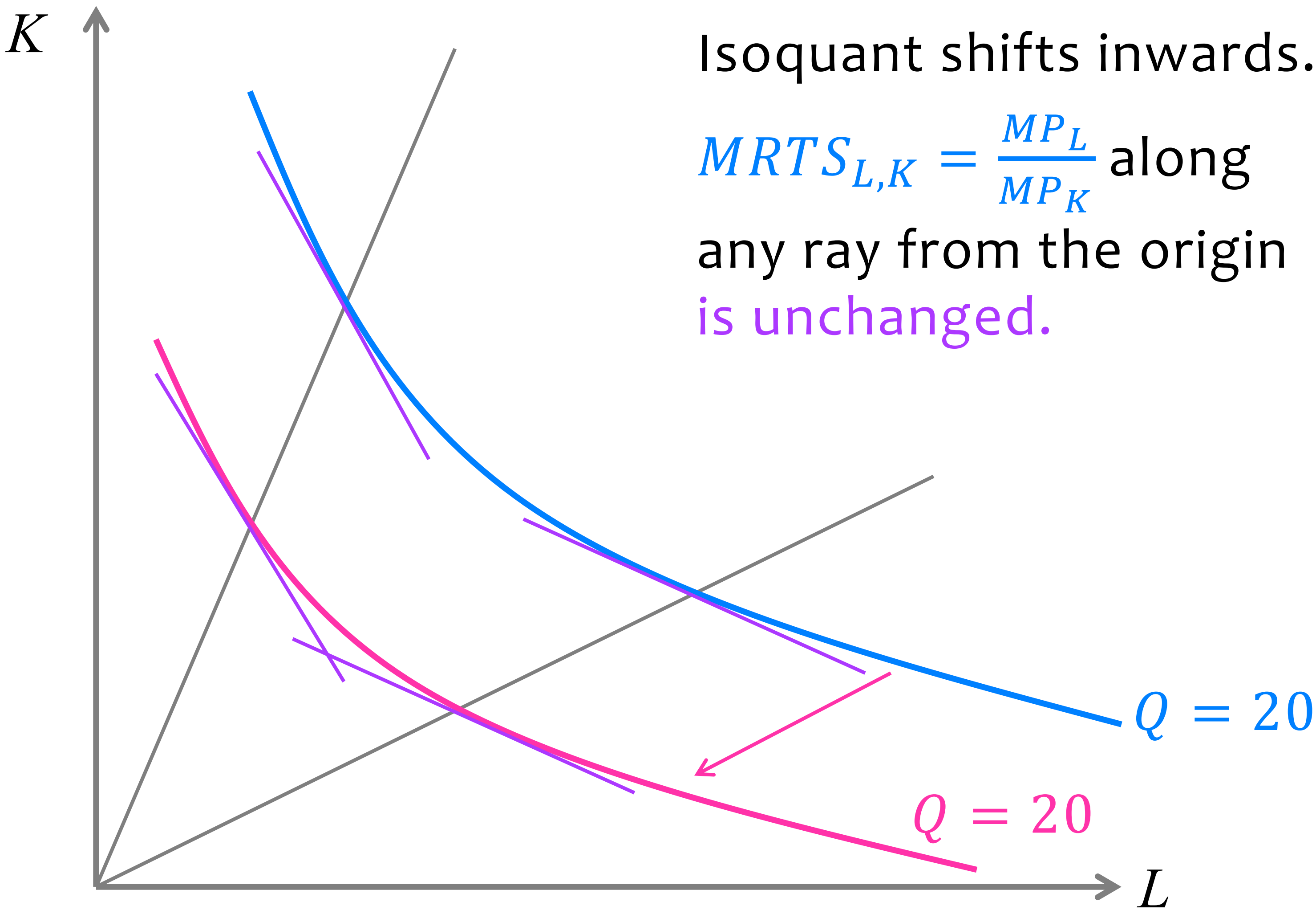
Technological Progress

- Thus far we have assumed a fixed production function, which implies that the production technology is fixed.
- What if there an improvement in technology?
- We have technological progress if:
  - For any given combination of inputs, the firm produces a higher level of output.

  or

  - To produce any level of output, the firm uses fewer inputs.

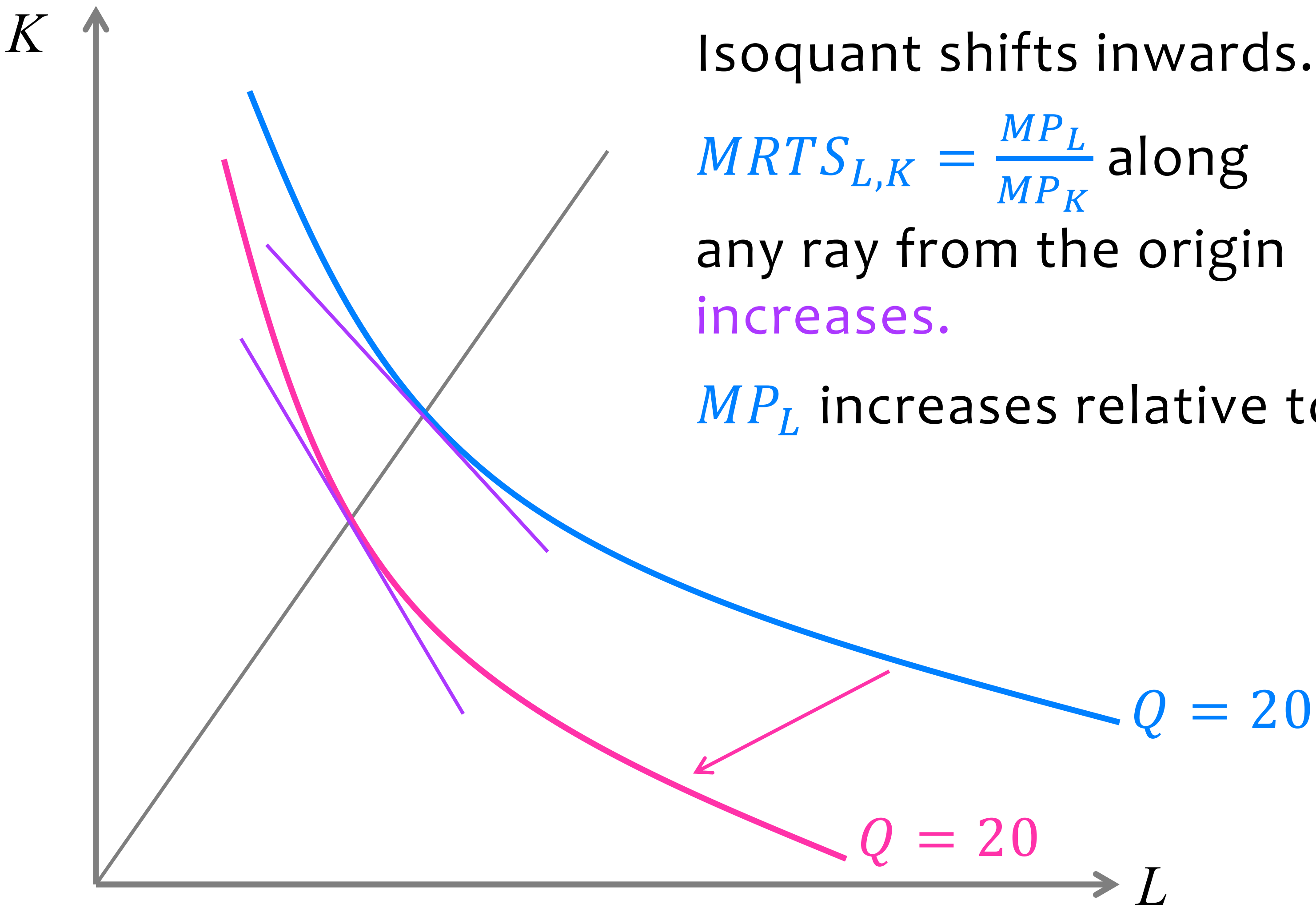
Neutral Technological Progress



Isoquant shifts inwards.

$MRTS_{L,K} = \frac{MP_L}{MP_K}$ along any ray from the origin is unchanged.

Capital-Saving Technological Progress

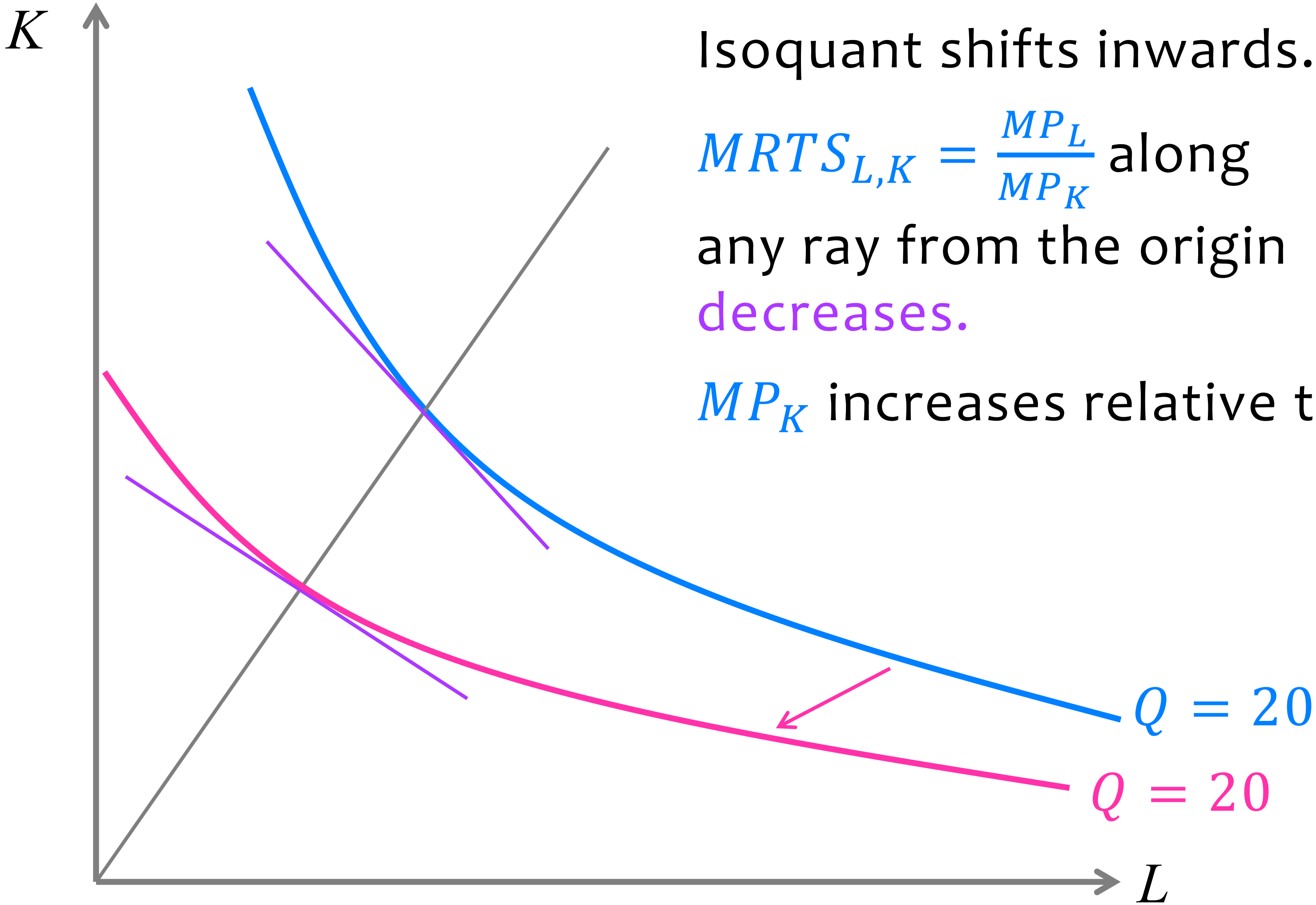


Isoquant shifts inwards.

$MRTS_{L,K} = \frac{MP_L}{MP_K}$ along any ray from the origin increases.

MP_L increases relative to MP_K.

Labor-Saving Technological Progress



Isoquant shifts inwards.

$MRTS_{L,K} = \frac{MP_L}{MP_K}$ along any ray from the origin decreases.

MP_K increases relative to MP_L.

Exercise 8.3

Technological Progress

- Suppose the initial production function is:

$$Q^I = KL + K$$

- The new production function is:

$$Q^N = 2(KL + K)$$

- $MRTS^I_{L,K} = \frac{K}{L+1}$ and $MRTS^N_{L,K} = \frac{K}{L+1}$

- $MRTS_{L,K}$ is the same.

- Is this neutral technological progress?

Exercise 8.3

Technological Progress

Exercise 8.4

Technological Progress: Cobb-Douglas

- Suppose the initial production function is

$$Q^I = KL$$

- The new production function is

$$Q^N = 2KL$$

- $MRTS^I_{L,K} = \frac{K}{L}$ and $MRTS^N_{L,K} = \frac{K}{L}$

- $MRTS_{L,K}$ is the same.

- Is this neutral technological progress?

Exercise 8.4

Technological Progress: Cobb-Douglas

Lecture 8

Theory of the Producer

- Concepts of Cost
- Cost in the Short Run
  - Short-Run Cost-Minimizing Input Choice
  - Short-Run Cost Curves

Concepts of Cost

Opportunity Cost and Economic Cost

- The opportunity cost of any choice is
  whatever must be given up when we make that choice.
- Economic cost is equivalent to opportunity cost.
- Economic cost / Opportunity cost comprises
  explicit costs and implicit costs.
  - Explicit costs require a cash outlay,
    e.g., paying wages to workers.
  - Implicit costs do not require a cash outlay,
    e.g., the opportunity cost of the business owner's time.

Explicit Cost vs. Implicit Cost: Example

- Suppose you own and run a small economic consulting firm.
- Your annual explicit costs are:
  - Wages to employees: $200,000
  - Rent: $60,000
  - Utilities and supplies: $40,000
- Your best alternative is to work for Google for $100,000 per year.
  - Your annual implicit cost is $100,000.

Economic Cost: Example

- Your economic cost / opportunity cost of running your own firm is:
  - Explicit costs + Implicit cost
  - $(\$200{,}000 + \$60{,}000 + \$40{,}000) + \$100{,}000 = \$400{,}000$
- By running your own firm,
  - You are incurring all the explicit costs.
  - And forgoing the salary you could have earned if you had chosen the best alternative instead — the implicit cost.

Sunk Cost: Example

- Suppose you own a restaurant.
- Because of the COVID-19 circuit breaker,
  you cannot serve meals in your restaurant,
  but you may prepare meals for delivery and take-away.
- You have to pay the rent
  regardless of whether you stay open or you shut down.

	Revenue	**Rent**	**Other costs**
Stay open	\$18,000	\$10,000	\$10,000
Shut down	\$0	\$10,000	\$0

Sunk Cost

- Sunk Cost:
  - A result of past decisions.
  - Cost that can never be recovered no matter what you do.
    - Decisions made in the present or future cannot change sunk costs.
  - Irrelevant when making decisions in the present or future.
- To determine sunk cost:
  - Ask "What costs do not vary regardless of which alternative I choose?"

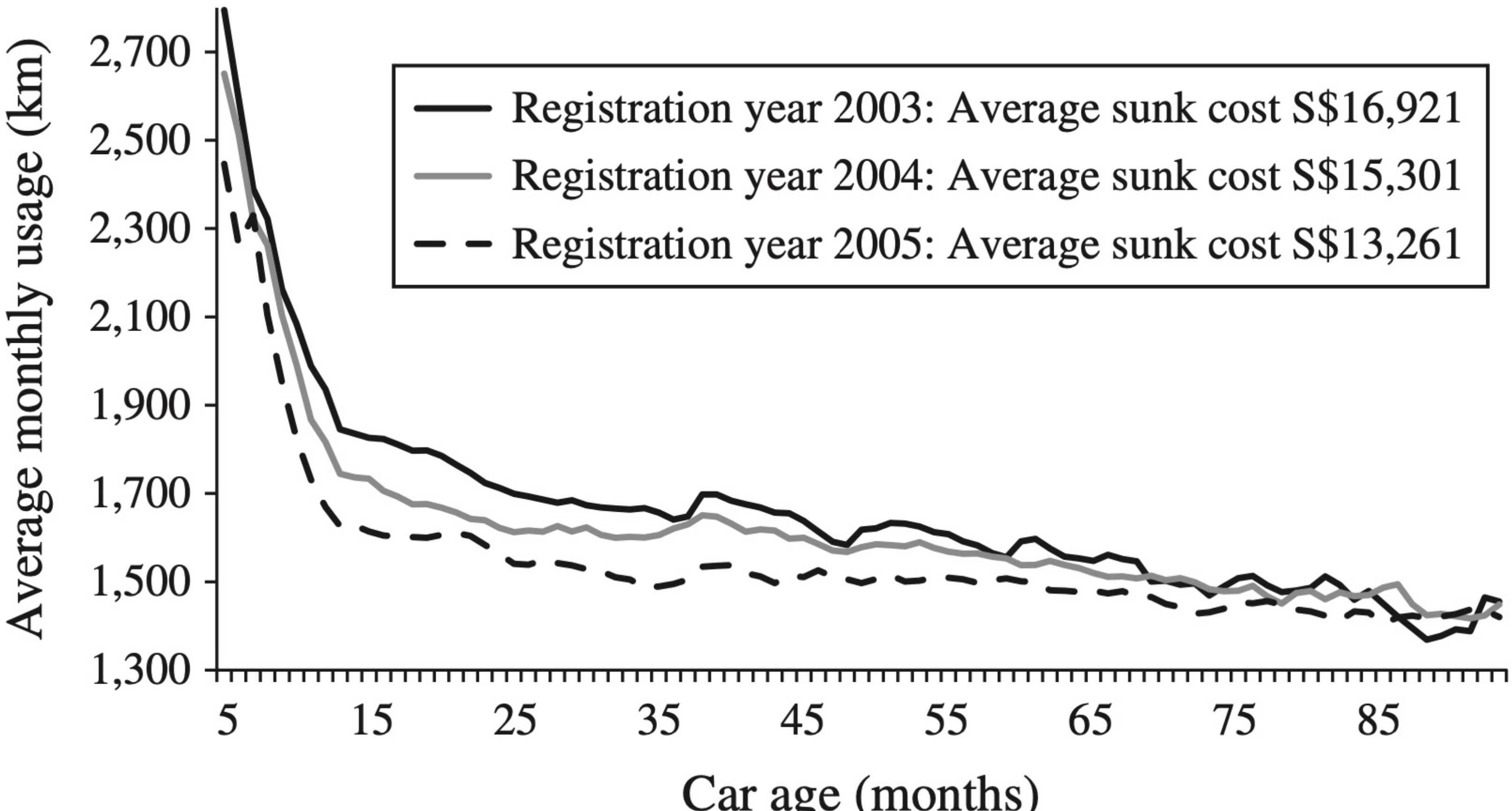
Sunk Cost Fallacy

- Do people drive more when they have paid more for their cars?

- Sunk costs associated with buying a car in Singapore:
  - Certificate of Entitlement (COE)
  - Additional Registration Fee (ARF)

- An increase in the sunk costs of $13,038 (the outcome of changes in government policy between 2009 and 2013)
  is associated with
  an increase in monthly driving of 86 km in the first four years of car ownership.

Average Monthly Usage by Car Age



Note: For the most popular model in the sample (3,403 cars).

Source: Ho, Png, and Reza. 2018. "Sunk Cost Fallacy in Driving the World's Costliest Cars." *Management Science,* 64:4, 1761–1778.

Cost in the Short Run

Short Run vs. Long Run in Production

- Suppose the firm uses labor L and capital K to produce Q.
- In the short run:
  - At least one input is fixed at a particular level.
    - Usually we assume that K is fixed.
- In the long run:
  - The firm is free to adjust the quantities of both L and K.

Short-Run Total Cost

- The price of labor L is w per unit.
- The price of capital K is r per unit.

- Suppose in the short run, capital is fixed at K_0.

- The firm's short-run total cost is:

$$SRTC = wL + rK_0$$

Short-Run Cost-Minimizing Input Choice

How much labor should the firm use?

- Assume the firm's goal is to maximize profit.

Profit = Total Revenue – Total Cost

- For any output level Q_0,
  the firm chooses L to minimize the total cost of production.

- The constrained optimization problem is:

$$\min_{L} SRTC = wL + rK_0$$

$$subject\ to\quad f(L, K_0) = Q_0$$

Short-Run Labor Choice: Example

- Suppose the production function is:

$$Q = KL$$

- In the short run, capital is fixed at $K = 2$.

- For any output level Q, the amount of labor required is:

$$L = \frac{Q}{K} = \frac{Q}{2}$$

- To produce $Q = 4$, the amount of labor required is:

$$L = \frac{Q}{2} = \frac{4}{2} = 2$$

Short-Run Cost Curves

Short-Run Cost Curves:

Short-Run Total Cost

Short-Run Total Cost: Example

- Short-run total cost:
  - $SRTC = wL + rK_0$
- Suppose $w = 2$ and $r = 3$.
- If $K_0 = 2$ and the firm wants to produce $Q = 4$, then $L = \frac{Q}{K_0} = 2$.
- The firm's short-run total cost is:

$$SRTC = wL + rK_0 = 2 \cdot 2 + 3 \cdot 2 = 10$$

Short-Run Total Cost Curve: Example

- Short-run total cost curve:
  - Total cost in the short run
    as a function of Q,
    holding w and r fixed.
  - $SRTC(Q)$
- The firm's short-run total cost curve is:

$$
\begin{aligned}
SRTC(Q) &= wL + rK_0 \\
&= 2\left(\frac{Q}{2}\right) + 3 \cdot 2 \\
&= Q + 6
\end{aligned}
$$

Short-Run Total Cost: Example

- Short-run total cost:
  - $SRTC = wL + rK_0$

- Suppose we do not know the values of w and r.

- If $K_0 = 2$ and the firm wants to produce $Q = 4$, then $L = \frac{Q}{K_0} = 2$.

- The firm's short-run total cost is:

$$SRTC = wL + rK_0 = 2w + 2r$$

Short-Run Total Cost Function: Example

- Short-run total cost function:
  - Total cost in the short run
    as a function of Q, w, and r.
  - $SRTC(Q, w, r)$
- The firm's short-run total cost function is:

$$
\begin{aligned}
SRTC(Q) &= wL + rK_0 \\
&= w\left(\frac{Q}{2}\right) + r \cdot 2 \\
&= \frac{wQ}{2} + 2r
\end{aligned}
$$

Short-Run Cost Curves:

Fixed Cost,
Variable Cost,
Sunk Cost

Fixed Cost vs. Variable Cost

- Fixed Cost (FC):
  - Cost that does not vary as Q changes, as long as $Q > 0$.
- Variable Cost (VC):
  - Cost that varies as Q changes.
  - When Q is 0, variable cost is 0.
- In the short run, for any $Q > 0$:
  - Fixed Cost $= rK_0$
  - Variable Cost $= wL$
  - $SRTC(Q) = wL + rK_0 = VC(Q) + FC$

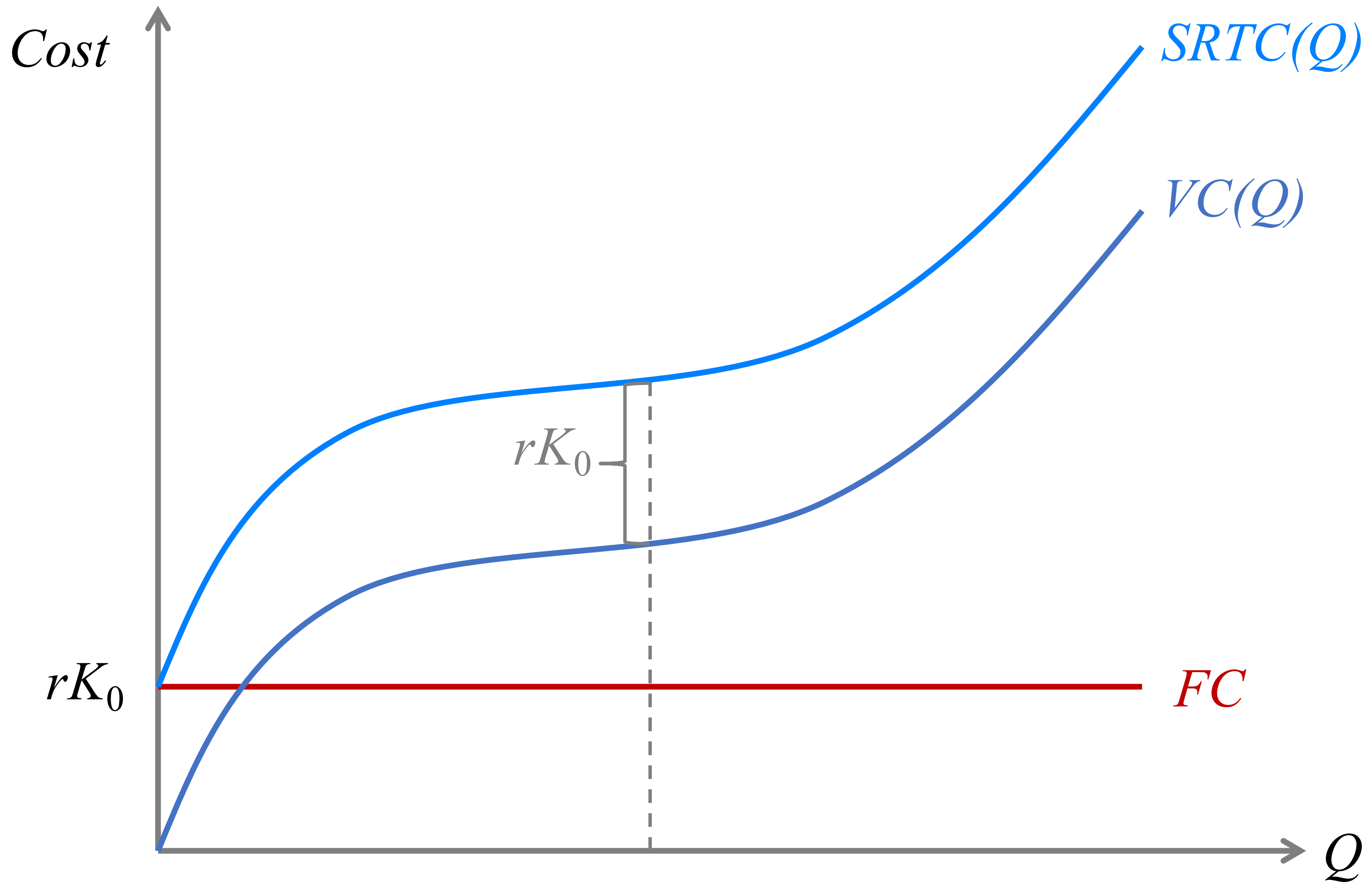
Fixed Cost vs. Sunk Cost

- Suppose you rent a plant/factory for production.
  - The monthly rent is $\$10{,}000$.
- Suppose you want to temporarily shut down the plant, i.e., produce $Q = 0$.
- Non-sunk fixed cost:
  - If you can sublet the plant to another firm at $\$10{,}000$ per month, then the rent is not sunk.
- Sunk fixed cost:
  - If you cannot sublet the plant, then the rent is sunk.

Sunk Cost and $SRTC$ at $Q = 0$

- Short-run total cost curve:
  - $SRTC(Q) = wL + rK_0 = VC(Q) + FC$
- If FC is non-sunk:
  - $SRTC(Q = 0) = 0$
- If FC is sunk:
  - $SRTC(Q = 0) = FC$
- If part of FC is sunk:
  - $SRTC(Q = 0) =$ the sunk part of FC

Fixed Cost, Variable Cost, Short-Run Total Cost Curves



Exercise 8.5

Fixed Cost vs. Sunk Cost

Suppose you rent an office space. The monthly rent is $\$10{,}000$.

Suppose you want to temporarily shut down,
i.e., produce $Q = 0$.

Suppose you can sublet the office space for $\$8{,}000$ a month.

(a) How much is your fixed cost?

(b) How much is your non-sunk fixed cost?

(c) How much is your sunk fixed cost?

Short-Run Cost Curves:

Short-Run Marginal Cost

Short-Run Marginal Cost

- Short-run Marginal Cost ($SRMC$):
  - The rate at which total cost changes as output changes.
  - $SRMC(Q) = \frac{dSRTC(Q)}{dQ} = \frac{\Delta SRTC(Q)}{\Delta Q}$

    where ΔQ is extremely small.
  - Slope of the short-run total cost curve.
  - $SRMC(Q) = \frac{dSRTC(Q)}{dQ} = \frac{d(VC(Q)+FC)}{dQ} = \frac{dVC(Q)}{dQ}$
  - Slope of the short-run variable cost curve.

Diminishing Marginal Return (of Labor) and Short-Run Marginal Cost

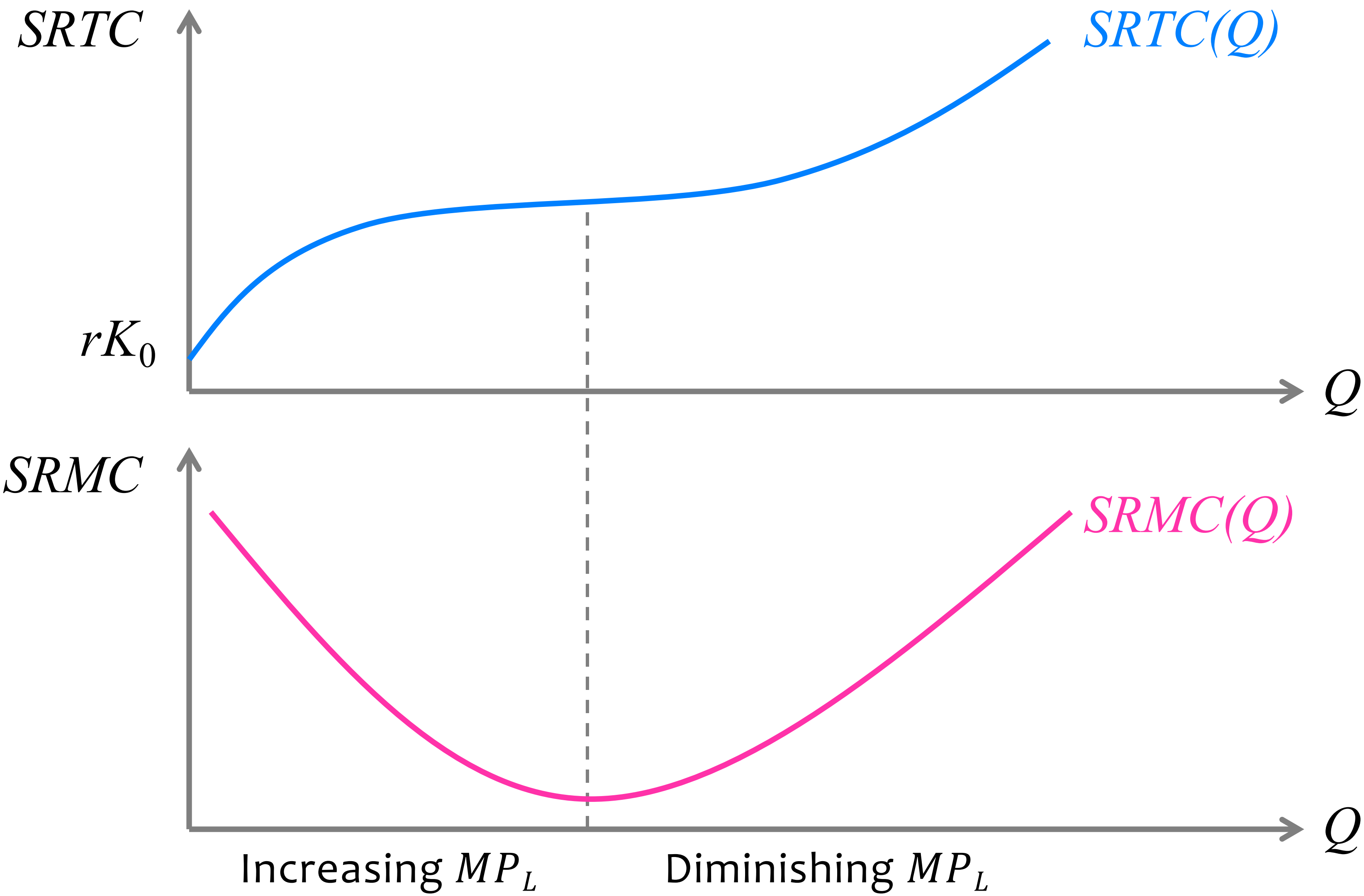
- Rewrite the short-run marginal cost:
  - $SRMC(Q) = \frac{\Delta VC}{\Delta Q} = \frac{w \cdot \Delta L}{\Delta Q} = w \cdot \frac{1}{\left(\frac{\Delta Q}{\Delta L}\right)} = \frac{w}{MP_L}$

    where ΔQ is extremely small.

- Recall diminishing marginal returns:
  - MP_L decreases as L increases.

- If we have diminishing marginal returns (assuming $MP_L > 0$),
  - $SRMC$ increases as Q increases.

Short-Run Total Cost,
Short-Run Marginal Cost Curves



Short-Run Cost Curves:

Short-Run Average Costs

Short-Run Average Costs

- Short-Run Average Total Cost ($SRATC$):

$$SRATC(Q) = \frac{SRTC(Q)}{Q}$$

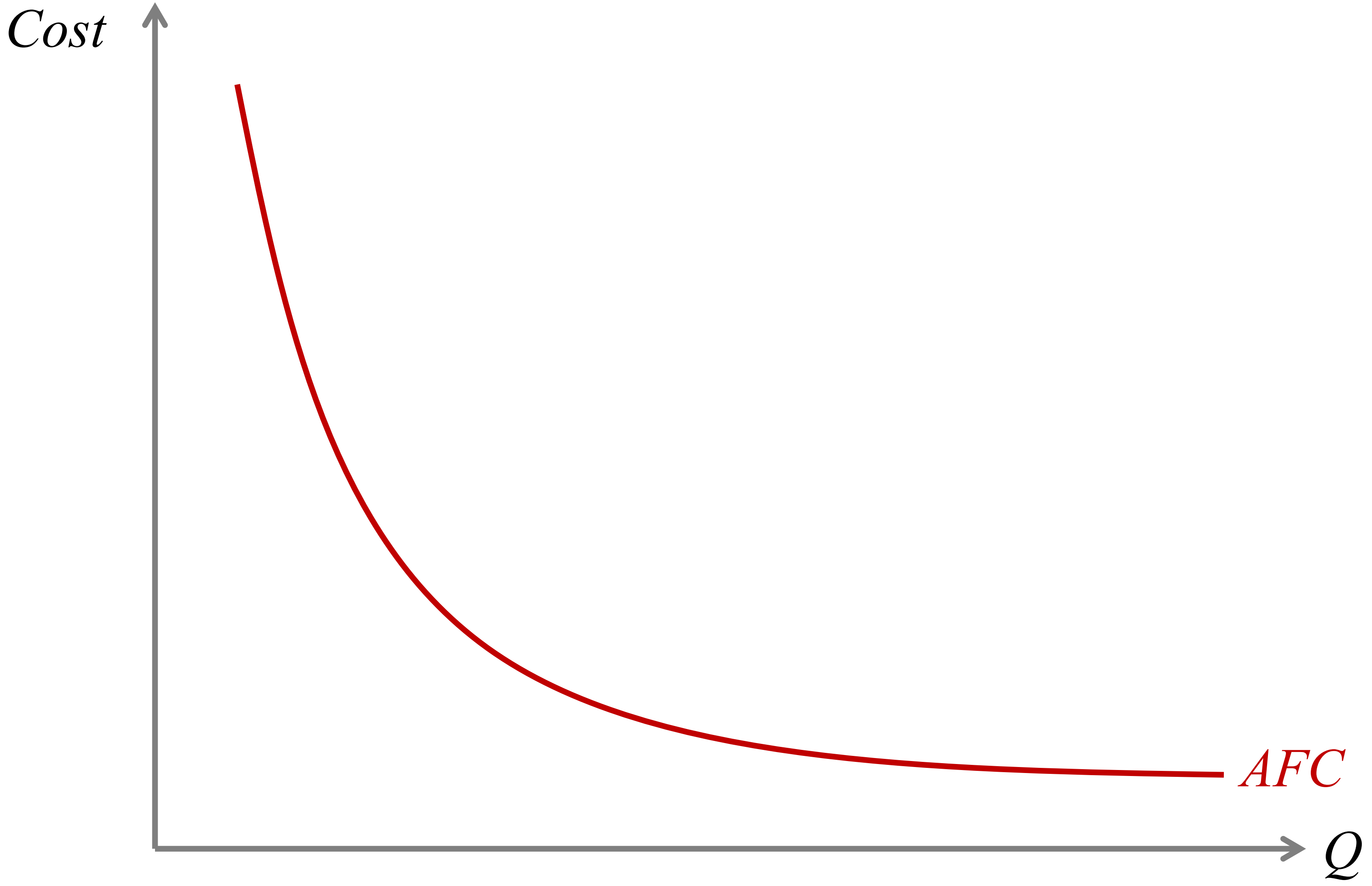
- Average Fixed Cost (AFC):

$$AFC(Q) = \frac{FC(Q)}{Q}$$

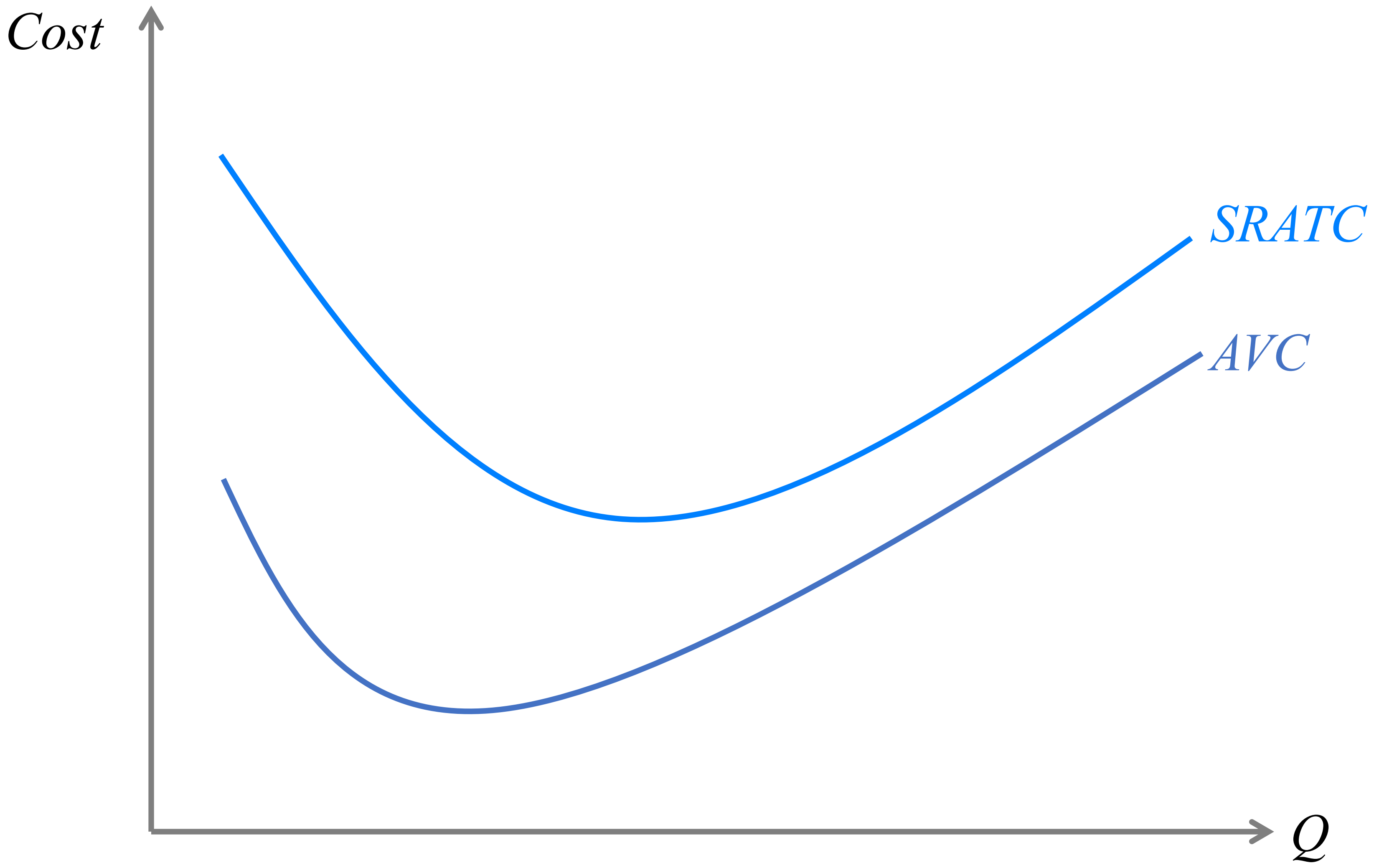
- Average Variable Cost (AVC):

$$AVC(Q) = \frac{VC(Q)}{Q}$$

Average Fixed Cost Curve



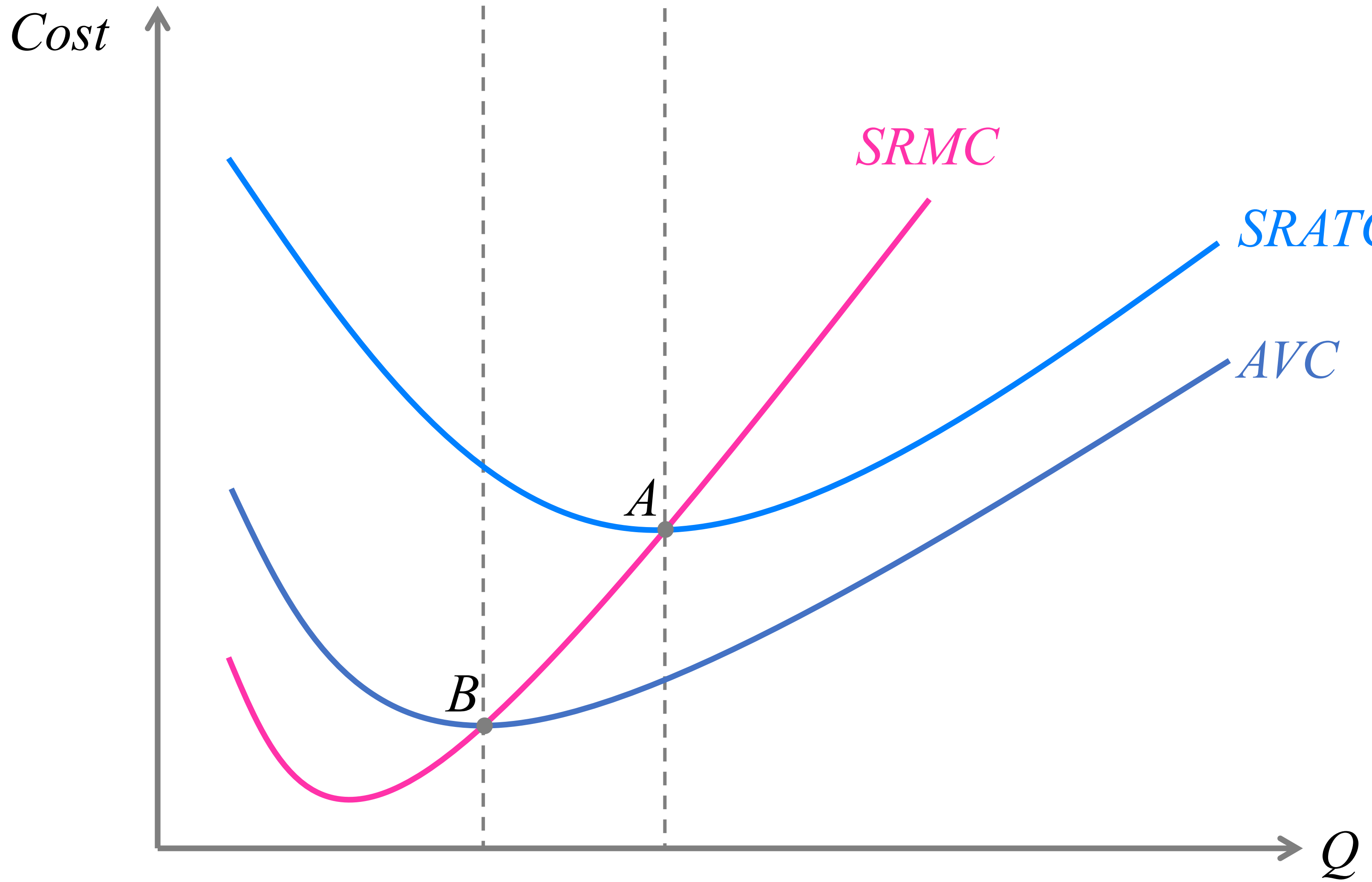
Short-Run Average Total Cost, Average Variable Cost Curves



MC vs. ATC

- When ATC is falling:
  - The cost of an additional unit of output (MC) is pulling down the average total cost (ATC).
  - $MC < ATC$
- When ATC is rising:
  - The cost of an additional unit of output (MC) is pulling up the average total cost (ATC).
  - $MC > ATC$

$SRMC$ intersects $SRATC$ and AVC at their minimums



Exercise 8.6

$SRMC$ vs. AVC

- Differentiate AVC with respect to Q.
- What does the mathematical expression tell you about the shape of the AVC curve?

Exercise 8.7

Cost in the Short Run

Suppose the production function is $Q = KL^2$. The price of labor is $w = 1$ and the price of capital is $r = 1$. In the short run, capital is fixed at $K = 16$.

(a) Suppose the firm wants to produce $Q = 256$. What is the cost-minimizing choice of labor in the short run?

(b) Find the firm's short-run total cost curve, $SRTC(Q)$.

(c) Find the firm's short-run marginal cost curve, $SRMC(Q)$; short-run average total cost curve, $SRATC(Q)$; average variable cost curve, $AVC(Q)$; and average fixed cost curve, $AFC(Q)$.

Exercise 8.7

Cost in the Short Run