Macroeconomics Analysis II, EC3102
Tutorial 1

<u>Question 1.</u> *Government and Credit Constraints in the Two-Period Economy.*

Consider again our usual two-period consumption-savings model, augmented with a government sector. Each consumer has preferences described by the utility function $u(c_1, c_2) = \ln c_1 + \ln c_2$, where ln stands for the natural logarithm, c_1 is consumption in period one, and c_2 is consumption in period two.

Suppose that both households and the government start with zero initial assets (i.e., $a_0 = 0$ and $b_0 = 0$), and that the real interest rate is always 10 percent. Assume that government purchases in the first period are one ($g_1 = 1$) and in the second period are 9.9 ($g_2 = 9.9$). In the first period, the government levies lump-sum taxes in the amount of 8 ($t_1 = 8$). Finally, the real incomes of the consumer in the two periods are $y_1 = 9$ and $y_2 = 23.1$.

a) What are lump-sum taxes in period two (t_2), given the above information?

b) Compute the optimal level of consumption in periods one and two, as well as national savings in period one.1

c) Consider a tax cut in the first period of 1 unit, with government purchases left unchanged. What is the change in national savings in period one? Provide intuition for the result you obtain.

d) Now suppose again that $t_1 = 8$ and also that credit constraints on the consumer are in place, with lenders stipulating that consumers cannot be in debt at the end of period one (i.e., the credit constraint again takes the form $a_1 \geq 0$). Will this credit constraint affect consumers' optimal decisions? Explain why or why not. Is this credit constraint welfare enhancing, welfare-diminishing, or welfare-neutral?

e) Now with the credit constraint described above in place, consider again the tax cut of 1 unit in the first period, with no change in government purchases. (That is, t_1 falls from 8 units to 7 units.) What is the change in national savings in period one that arises due to the tax cut? Provide economic intuition for the result you obtain.

<u>Solution:</u>

a) Using the government's LBC,

$$g_1 + \frac{g_2}{1+r} = t_1 + \frac{t_2}{1+r}$$

Plugging the values of g_1, g_2 and t_1 into the LBC above, we have

$$1 + \frac{9.9}{1+0.1} = 8 + \frac{t_2}{1+0.1}$$

$$1 + 9 = 8 + \frac{t_2}{1+0.1}$$

$$2 = \frac{t_2}{1+0.1}$$

$$\Rightarrow \boldsymbol{t_2 = 2.2}$$

b)

Comment: We can either make use of the optimality condition which is

$$\frac{u_1(c_1, c_2)}{u_2(c_1, c_2)} = (1+r) \qquad (\star)$$

if we follow the intuition told in lecture. But here, I would like to go through with you the maximization solution in details again so that you can see the math behind.

First set up the objective function:

$$u(c_1, c_2) = \ln(c_1) + \ln(c_2) \qquad (1)$$

And consumer's life-time budget constraint:

$$c_1 + \frac{c_2}{1+r} = \underbrace{(y_1 - t_1)}_{\substack{\text{disposable income} \\ \text{in period 1}}} + \underbrace{\frac{y_2 - t_2}{1+r}}_{\substack{PV\ of \\ \text{disposable income} \\ \text{in period 2}}} \qquad (2)$$

Group the taxes in equation (2) together, we have:

$$c_1 + \frac{c_2}{1+r} = y_1 + \frac{y_2}{1+r} - \left(t_1 + \frac{t_2}{1+r}\right) \qquad (3)$$

And the government's life-time budget constraint is:

$$g_1 + \frac{g_2}{1+r} = t_1 + \frac{t_2}{1+r} \qquad (4)$$

Replacing $t_1 + \frac{t_2}{1+r}$ in (3) with $g_1 + \frac{g_2}{1+r}$ in (4), we get:

$$c_1 + \frac{c_2}{1+r} = y_1 + \frac{y_2}{1+r} - \left(g_1 + \frac{g_2}{1+r}\right) \qquad (5)$$

Grouping the terms on the right hand side of (5) with same time index together, we have:

$$c_1 + \frac{c_2}{1+r} = (y_1 - g_1) + \frac{y_2 - g_2}{1+r} \qquad (6)$$

<u>Comment:</u> Going through the steps from (2) to (5), you can see that it is NOT because $g_1 = t_1$ and $g_2 = t_2$ that we replace t_1 and t_2 in (2) with g_2 and g_2 in (6) respectively. But rather, it is because the PV of total government spending is the same with the PV of total tax collection - as in (4) and then we arrange the terms to get (6).

Now, set up the Lagrange function from the objective function and the life-time budget constraint:

$$L(c_1, c_2, \lambda) = \ln(c_1) + \ln(c_2) + \lambda\left[c_1 + \frac{c_2}{1+r} - (y_1 - g_1) - \frac{y_2 - g_2}{1+r}\right] \tag{7}$$

Comment:

Some might ask if the sign in front of lambda matters. The answer is mathematically NO. You can have:

$$\ln(c_1) + \ln(c_2) - \lambda\left[c_1 + \frac{c_2}{1+r} - (y_1 - g_1) - \frac{y_2 - g_2}{1+r}\right]$$

And it is perfectly fine because the value of the solution for λ will be reversed in sign. That is all. But we are not interested in the value of λ, are we? No! λ is just a temporarily created variable to serve our need. After we find what we want - which is the optimal choice of c_1 and c_2 - we don't need to know λ any more. Bye Lambda!

Maximize $L(c_1, c_2, \lambda)$:

$$\frac{\partial L(c_1, c_2, \lambda)}{\partial c_1} = \frac{1}{c_1} + \lambda = 0 \tag{8}$$

$$\frac{\partial L(c_1, c_2, \lambda)}{\partial c_2} = \frac{1}{c_2} + \frac{\lambda}{1+r} = 0 \tag{9}$$

$$\frac{\partial L(c_1, c_2, \lambda)}{\partial \lambda} = \left[c_1 + \frac{c_2}{1+r} - (y_1 - g_1) - \frac{y_2 - g_2}{1+r}\right] = 0 \tag{10}$$

Comment:

Equation (10) is just the life-time budget constraint.

From (8) and (9) we have:

$$\begin{cases} \dfrac{1}{c_1} + \lambda = 0 \Rightarrow \dfrac{1}{c_1} = -\lambda & (11) \\ \dfrac{1}{c_2} + \dfrac{\lambda}{1+r} = 0 \Rightarrow \dfrac{1}{c_2} = -\dfrac{\lambda}{1+r} & (12) \end{cases}$$

$$\Rightarrow \frac{\frac{1}{c_1}}{\frac{1}{c_2}} = \frac{-\lambda}{-\frac{\lambda}{1+r}} = 1 + r \tag{13}$$

$$\Rightarrow \boldsymbol{\frac{c_2}{c_1} = 1 + r}$$

$$\Rightarrow \boldsymbol{c_2 = (1 + r)c_1} \tag{\textbf{14}}$$

Comment:
Equation (14) is the same with ($\star$). On the left hand side of (14), we have the MRS (or $\frac{u_1(c_1,c_2)}{u_2(c_1,c_2)}$.

Substitute c_2 found in (14) into the life-time budget constraint (which is the equation (10), we have:

$$c_1 + \frac{\mathbf{(1+r)c_1}}{1+r} - (y_1 - g_1) - \frac{y_2 - g_2}{1+r} = 0$$

$$c_1 + \mathbf{c_1} - (y_1 - g_1) - \frac{y_2 - g_2}{1+r} = 0$$

$$2c_1 = (y_1 - g_1) + \frac{y_2 - g_2}{1+r}$$

$$c_1 = \frac{1}{2}\left[(y_1 - g_1) + \frac{y_2 - g_2}{1+r}\right] \qquad (15)$$

And thus, $c_2 = \frac{1+r}{2}\left[(y_1 - g_1) + \frac{y_2-g_2}{1+r}\right] \qquad (16)$

Plugging the values for r, y_1, g_1, g_2 into (15) and (16), we have:

$$c_1 = \frac{1}{2}\left[(9-1) - \frac{23.1 - 9.9}{1+0.1}\right] = \frac{1}{2}\left[8 + \frac{13.2}{1+0.1}\right] = 10$$

$$c_2 = (1+r)c_1 = (1+0.1)\cdot 10 = 11$$

National savings in period one is:

$$savings_1 = \text{national income} - \text{total expenditure} = y_1 - g_1 - c_1 = 9 - 1 - 10 = -2$$

The private savings (*not required for this but we will use it in part d*) is:

$$savings_1 = \text{disposable income} - \text{consumption} = y_1 - t_1 - c_1 = 9 - 8 - 10 = -9$$

Comment:
The solution I wrote is long because I wish to explain in details. But in exam or tutorial, you can just write as short as possible.

c) Now with the government purchases unchanged, but t_1 is now 7 (one less of 8), then equation (4) - the government budget constraint will be.

$$g_1 + \frac{g_2}{1+r} = (t_1 - 1) + \frac{t_2 + \Delta t_2}{1+r} \qquad (17)$$

Comment
Here, t_2 will have to increase to make up for the reduction of tax collection in the first period in order to ensure the same amount of present-value of tax collections in the whole life time. And thus Δt_2 is positive.

And the consumer budget constraint (equation (3)) will be:

$$c_1 + \frac{c_2}{1+r} = [y_1 - (t_1 - 1)] + \frac{y_2 - (t_2 + \Delta t_2)}{1+r} \tag{18}$$

$$\Longrightarrow c_1 + \frac{c_2}{1+r} = y_1 + \frac{y_2}{1+r} - \left((t_1 - 1) + \frac{t_2 + \Delta t_2}{1+r}\right)$$

So from (17), we can replace $(t_1 - 1) + \frac{t_2+\Delta t_2}{1+r}$ in (18) with $g_1 + \frac{g_2}{1+r}$, to get:

$$c_1 + \frac{c_2}{1+r} = y_1 + \frac{y_2}{1+r} - \left(g_1 + \frac{g_2}{1+r}\right)$$

$$\Rightarrow c_1 + \frac{c_2}{1+r} = (y_1 - g_1) + \frac{y_2 - g_2}{1+r} \tag{19}$$

And thus, we get the same consumer's life-time budget constraint as before (compare (19) with (5)). And since it is the same consumer and nothing has been done to suggest that his utility function is different. Therefore, we have the same objective function and the same budget constraint, we should get the same optimal choices of c_1 and c_2 as before.

Intuition: With government purchases unchanged, a change in the timing of lump-sum taxes will not fool the consumers. They will save the amount of tax cut, knowing that in the future, the government will take back that amount again in order to ensure enough tax collections for their spending plans. This leads to no change in consumption and hence no change in national savings. ***This is the Ricardian Equivalence proposition*** – consumers increase their private savings after the tax cut in anticipation of the tax increase that must occur in period two.

d) Examine the period-one budget constraint of the consumer:

$$c_1 + a_1 = y_1 - t_1$$

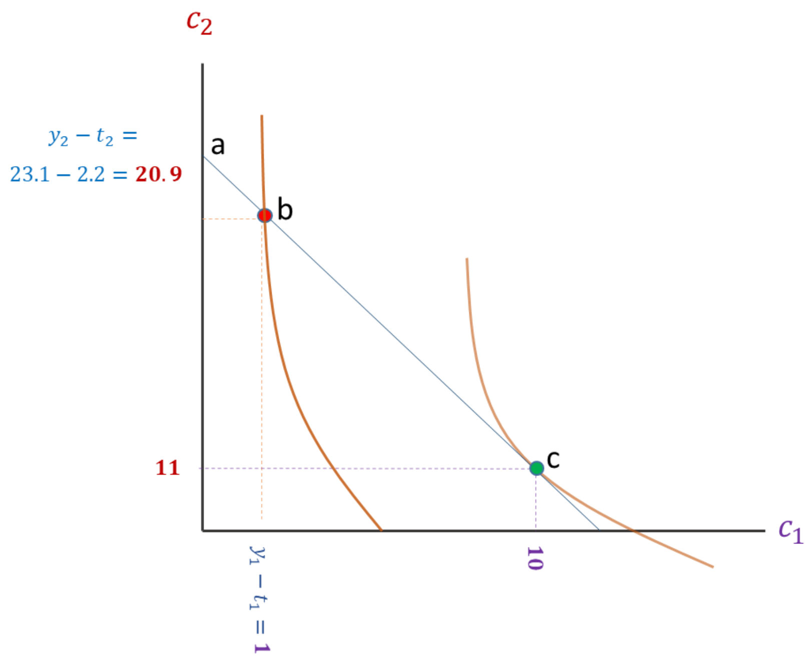
This expression, along with the value of $c_1 = 10$ you found in part b above can be used to determine that $a_1 = -9$ (this is private savings). Thus, consumers optimally (i.e., under no credit constraints) **want to be debtors** at the end of period one.

With the imposition of the credit constraints, consumers can no longer do so, and will choose $a_1 = 0$ because that is the closest they can get to their unrestricted choice while also satisfying the credit constraint. The period one budget constraint, with $a_1 = 0$, yields:

$$c_1 = y_1 - t_1 = 1$$

So what this means is that the consumer will have to choose to consume whatever he has got to consume. In other words, he is consuming at the **endowment point**.

The credit constraint diminishes welfare because consumers are being forced to choose a consumption allocation different from the one they would otherwise choose – graphically (see diagram), they are on a lower indifference curve than the one that maximizes utility subject to the LBC of the economy.



<u>*Comment on the graph*</u>
The green dot (point c) is where, if there is no constraint, the consumer will choose to consume with $c_1 = 10$ and $c_2 = 11$. *But with the constraint in place, the consumer has no choice but to consume at the red point. This red point is the endowment point (point b), which means that the consumer will consume whatever that he/she has in each period, there is no borrowing/lending.*
Given the constraint, the consumer cannot choose any better point then the red point (point b). Anywhere between point a and point b will do worse than point b.
<u>*Note:*</u> *You do not need to draw the graph unless the question explicitly tells you to do so.*

e) With $t_1 = 7$, the credit constraint is still binding, and $c_1 = y_1 - t_1 = 2$. Thus, because $s_1^{nat} = y_1 - c_1 - g_1$, national savings falls by exactly the amount by which consumption rises, which is one. This means that the national saving no longer remains the same.

This occurs because Ricardian Equivalence fails[1] if capital controls/borrowing constraints are binding (another reason, beyond distortionary taxes as described in the Lecture Notes, why Ricardian Equivalence fails). The reason here is that consumers were not at their unrestricted optimal choice to begin with – they wanted to consume more in period one than they were restricted to. Thus, any relaxation of their period one budget

[1] Note that (as taught in lecture), without any financial constraint (limit to borrowing or lending) the timing of tax change with no government spending plan change will result in no change in the national savings because there is no change in consumption (c_1) and government spending in period 1 (g_1). And this is a part of Ricardian Equivalence result. Is it not? But in part e, the context is different. Now we have financial constraint - cannot borrow; thus the consumption is affected if there is a change of tax timing. And since c_1 is now different while y_1 and g_1 remains the same, we have $s_1^{nat} = y_1 - c_1 - g_1$ to be different. Or we say that the Ricardian Equivalence does not hold in this case. Thus in this part e, we have another case where the RE fails besides the distortionary taxes as described in the Lecture Notes.

constraint (ie, in the form of lower taxes in period one) induces them to increase their consumption, dragging down national savings.

<u>Comment on the graph below</u>
This is graph for part e. Now the constraint is relaxed, the consumer will do better by making use of this relaxation and consume 1 more. As such he/she will move from point b to b' and c_1 increase from 1 to 2. As for c_2, your professor is lazy to calculate what is t_2' thus he just left it as $y_2 - t_2'$. You can do it yourself.
<u>Note:</u> You do not need to draw the graph unless the question explicitly tells you to do so.