LECTURE 5

MARKET DEMAND

EXCHANGE ECONOMY

Question 1: Pareto Efficiency vs. Pareto Improvement

- Statement 1: if an allocation is Pareto efficient, there is no room for Pareto improvement.
  - True or false?
- Statement 2: if an allocation is not Pareto efficient, there is still room for Pareto improvement.
  - True or false?

Question 1: Solution

- Both are true
- By definition, from some allocation *X* to some other allocation *Y* is a Pareto improvement if from *X* to *Y*, at least one consumer is better off and no one else is worse off
- If an allocation is Pareto efficient, there is no way to make one consumer better off without making someone else worse off
  - In other words, there is no way to have a Pareto improvement
- If an allocation is not Pareto efficient, there is a way to make one consumer better off without making someone else worse off
  - In other words, there is room for Pareto improvement

Question 2: Feasible Allocation

- Recall an allocation is feasible if

$$x_1^A + x_1^B = \omega_1^A + \omega_1^B$$
$$x_2^A + x_2^B = \omega_2^A + \omega_2^B$$

- An alternative definition says that an allocation is feasible if

$$x_1^A + x_1^B \leq \omega_1^A + \omega_1^B$$
$$x_2^A + x_2^B \leq \omega_2^A + \omega_2^B$$

  - The total amount of each good consumed does not exceed the total amount available

Question 2: Feasible Allocation and Pareto Efficiency

- Tangency condition

$$MRS^A_{1,2} = MRS^B_{1,2} \quad (1)$$

- The allocation must be feasible

$$x^A_1 + x^B_1 = \omega^A_1 + \omega^B_1 \quad (2)$$

$$x^A_2 + x^B_2 = \omega^A_2 + \omega^B_2 \quad (3)$$

- Substituting (2) and (3) into (1), we can express the contract curve in terms of x^A_1 and x^A_2 or x^B_1 and x^B_2

Question 2: Feasible Allocation and Pareto Efficiency Cont’

- Suppose there are in total 8 units of good 1 and 4 units of good 2 in an economy
- Suppose the consumers’ preferences satisfy more is (strictly) better
- Is the allocation (1, 1, 1, 1) Pareto efficient?
  - Under the alternative definition, this allocation is feasible

Question 2: Solution

- No!
- For example, (2, 2, 2, 2) is a Pareto improvement
  - Both consumers get higher utility than at (1, 1, 1, 1)
- Is (2, 2, 2, 2) Pareto efficient?
- An allocation where the total consumption of a good is less than the total endowment is not Pareto efficient
  - When more is strictly better
- Pareto efficient allocations will still satisfy

$$x_1^A + x_1^B = \omega_1^A + \omega_1^B$$

$$x_2^A + x_2^B = \omega_2^A + \omega_2^B$$

Question 3: Meaning of Prices

- In the exchange economy
  - There is no income/money
- But the competitive equilibrium refers to
  - A pair of equilibrium prices
  - And an equilibrium allocation
- If there is no money, what do prices mean?
- For example, what does it mean if the price of good 1 is \$2 and the price of good 2 is \$1?

Question 3: Solution

- All we need to know is the relative price of the two goods
  - It tells us the relative scarcity of the two goods – how the two goods can be exchanged in the market
- If the price of good 1 is $2 and the price of good 2 is $1, it means
  - The consumer needs 2 units of good 2 to exchange for 1 unit of good 1
  - If the price of good 1 is $20 and the price of good 2 is $10, it does not make any difference

Question 4: Numerical Example of Competitive Equilibrium

- Suppose consumer A's utility function is

$$U^A(x_1^A, x_2^A) = x_1^A x_2^A$$

- Suppose consumer B's utility function is

$$U^B(x_1^B, x_2^B) = x_1^B x_2^B$$

- Consumer A's endowment is (10, 6) and consumer B's endowment is (10, 4)
- Find the equilibrium prices P_1 and P_2 and the equilibrium allocation

Question 4: Solution

- Consumer A's optimal choice

$$\frac{x_2^A}{x_1^A} = \frac{P_1}{P_2} \qquad (1)$$

$$P_1 x_1^A + P_2 x_2^A = 10P_1 + 6P_2 \qquad (2)$$

- Consumer B's optimal choice

$$\frac{x_2^B}{x_1^B} = \frac{P_1}{P_2} \qquad (3)$$

$$P_1 x_1^B + P_2 x_2^B = 10P_1 + 4P_2 \qquad (4)$$

- Market clearing

$$x_1^A + x_1^B = 10 + 10 = 20 \qquad (5)$$

$$x_2^A + x_2^B = 6 + 4 = 10 \qquad (6)$$

Question 4: Solution Cont'

- (1) and (3) give us

$$\frac{x_2^A}{x_1^A} = \frac{x_2^B}{x_1^B} = \frac{P_1}{P_2} \qquad (7)$$

- Plugging (5) and (6) into (7)

$$\frac{x_2^A}{x_1^A} = \frac{10 - x_2^A}{20 - x_1^A} = \frac{P_1}{P_2} \qquad (8)$$

- Solving

$$\frac{x_2^A}{x_1^A} = \frac{10 - x_2^A}{20 - x_1^A}$$

- We get

$$x_1^A = 2x_2^A \qquad (9)$$

Question 4: Solution Cont'

- Plugging (9) into (8)

$$\frac{P_1}{P_2} = 0.5 \quad (10)$$

- Plugging (9) and (10) into (2)

$$P_1 2x_2^A + 2P_1 x_2^A = 10P_1 + 12P_1 \quad \Rightarrow \quad x_2^A = 5.5$$

- The equilibrium allocation is

$$x_1^{*A} = 11, \quad x_2^{*A} = 5.5, \quad x_1^{*B} = 9, \quad x_2^{*B} = 4.5$$

Question 4: Comment

- In the previous example, we can only solve for the relative price P_1/P_2
- Relative price is what matters
  - In the previous example, we just need the price ratio to be $P_1/P_2=0.5$ in equilibrium
  - It does not matter if $P_1=2$, $P_2=4$ or $P_1=3$, $P_2=6$
- It is convenient to set one of the prices to 1
  - Such a price is called a *numeraire price*, such a good is called a *numeraire*
  - If we set good 2 as a numeraire in the example, then $P_1=0.5$

Q&A on Lecture 5