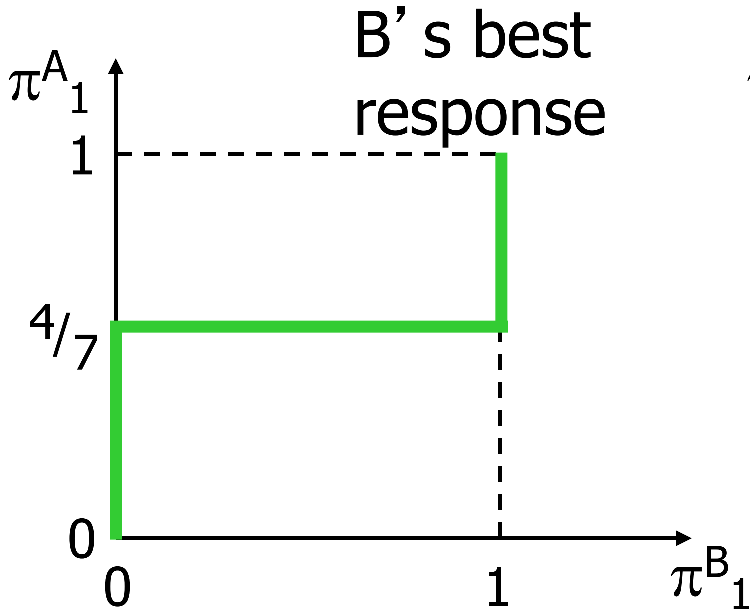
GAME APPLICATIONS II

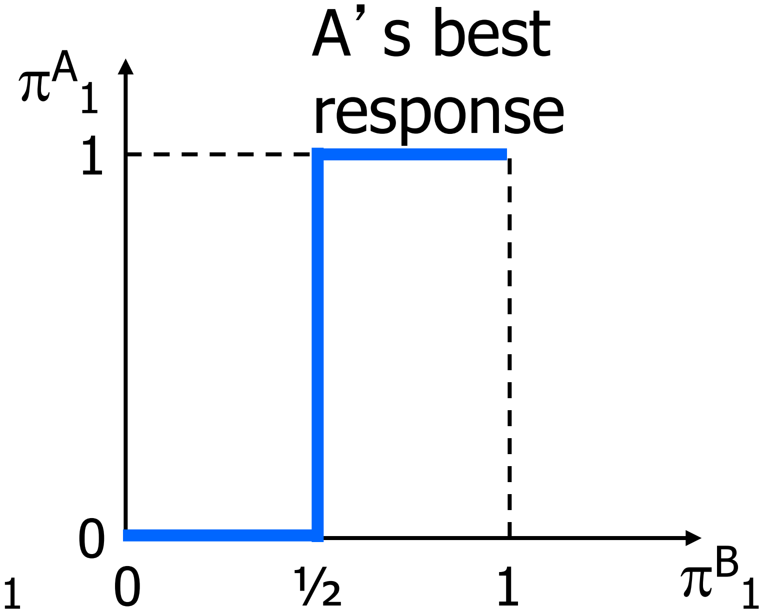
Week 9

(Chapter 30)

Best Responses Curves

No need to know how to draw best response curves





Some Common Types of Games

- Games of coordination
- Games of competition
- Games of coexistence
- Games of commitment
- Bargaining games

Coordination Games

- Simultaneous play games in which the payoffs to the players are largest when they coordinate their actions

- Famous examples are:
  1. The Prisoner’s Dilemma
  2. The Battle of the Sexes
  3. Assurance Games
  4. Chicken

(1) The Prisoner’s Dilemma

- A simultaneous play game in which each player has a strictly dominant action
- The only NE is for each player to choose her strictly dominant action
- Yet both players can achieve strictly larger payoffs than in the NE by coordinating with each other on another pair of actions

(1) The Prisoner’s Dilemma

		Player 2	
		Cheat	Cooperate
Player 1	Cheat	C , C	A , D
	Cooperate	D , A	B , B

- A>B>C>D and A+D<2B
- How to overcome?
  - Future punishments
  - Enforceable contracts

(2) The Battle of the Sexes

Two activities: “Soccer (S)” and “Dating Show (D)”

- Wife prefers “D” to “S”
- Husband prefers watching “S” to “D”
- Both prefer watching something together to being apart

(2) The Battle of the Sexes

		Husband	
		D	S
Wife	D	8,4	2,2
	S	2,2	4,8

- Two PSNEs
- Any (other) MSNE?

(2) The Battle of the Sexes

		Husband	
		D	S
Wife	D	8,4	2,2
	S	2,2	4,8

- Let π_W represent prob. that wife chooses “D”
- Let π_H represent prob. that husband chooses “D”

- $EV_W(D) = 8\pi_H + 2(1 - \pi_H) = 2 + 6\pi_H$
- $EV_W(S) = 2\pi_H + 4(1 - \pi_H) = 4 - 2\pi_H$
- Indifferent if $EV_W(D) = EV_W(S)$, i.e. if $\pi_H = 0.25$

(2) The Battle of the Sexes

		Husband	
		D	S
Wife	D	8,4	2,2
	S	2,2	4,8

- Let π_W represent prob. that wife chooses "D"
- Let π_H represent prob. that husband chooses "D"

- $EV_H(D) = 4\pi_W + 2(1 - \pi_W) = 2 + 2\pi_W$
- $EV_H(S) = 2\pi_W + 8(1 - \pi_W) = 8 - 6\pi_W$
- Indifferent if $EV_H(D) = EV_H(S)$, i.e. if $\pi_W = 0.75$

(2) The Battle of the Sexes

		Husband	
		D	S
Wife	D	8,4	2,2
	S	2,2	4,8

- Besides the two PSNEs, there exists another MSNE:
- For wife (and husband), the expected value of this NE is

(3) Assurance Games

- A simultaneous play game with two PSNE, one of the PSNE gives strictly greater payoffs to **each** player than does the other PSNE

- Challenge: How can each player give the other an “assurance” that will cause the better NE to prevail?

(3) Assurance Games

- A common example is the “arms race” problem
- India and Pakistan can both increase their stockpiles of nuclear weapons. This is very costly
- Having nuclear superiority over the other gives a higher payoff, but the worst payoff to the other
- Not increasing the stockpile is best for both

(3) Assurance Games

		Pakistan	
		Don’t	Stockpile
India	Don’t	5,5	1,4
	Stockpile	4,1	3,3

- Two PSNE:

- Which is more likely?
- What if India moves first?

The role of communications

		Gabriel	
		Little Party	Big Party
Evangeline	Little Party	10,10	0,0
	Big Party	0,0	5,5

- Two PSNE: (Little, Little), (Big, Big)
- The concept of NE does not help us predict which NE will prevail
- What if they can communicate before each decides?
- Communication is welfare enhancing here (but not in all situations)

The role of communications

		Tony	
		Split	**Steal**
Lucy	**Split**	66885/2, 66885/2	0,66885
	Steal	66885,0	0,0

- Lucy and Tony promises each other that they will split the money
- But best response is “steal” if the other player chooses “split”

Another grim example

		Sara	
		Split	**Steal**
Steve	**Split**	100150/2, 100150/2	0, 100150
	Steal	100150,0	0,0

Steve and Sara promises each other that they will split the money

(4) A Game of Chicken

- A simultaneous play game with two “coordinated” NE in which each player chooses the action not chosen by the other player

- (Example) Two drivers race their cars at each other
  - A driver who swerves is a “chicken”
  - A driver who does not swerve is “macho”

- If both do not swerve, there is a crash and a low payoff to both

- If both swerve, there is no crash and a moderate payoff to both.

- If one swerves and the other does not, the swerver gets a low payoff and the non-swerver gets a high payoff

(4) A Game of Chicken

Rebel without a cause

		Buzz	
		Swerve	No Swerve
Jimmie	Swerve	**1,1**	**-2,4**
	No Swerve	**4,-2**	**-5,-5**

- PSNEs:
- A MSNE

Some Common Types of Games

- Games of coordination
- Games of competition
- Games of coexistence
- Games of commitment
- Bargaining games

Games of Competition

- Simultaneous play games in which any increase in the payoff to one player is exactly the decrease in the payoff to the other player
- These games are thus often called “constant (payoff) sum” games or “zero sum games”
- Example:

		Goal Keeper	
		Left	Right
Striker	Left	5,-5	8,-8
	Right	9,-9	2,-2

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Coexistence Games

- Simultaneous play games that can be used to model how members of a species act towards each other

- An important example is the hawk-dove game
  - “Hawk” means “be aggressive”
  - “Dove” means “don’t be aggressive”

Coexistence Games

The Hawk-Dove Game

Example

- Two bears come to a fishing spot
- Either bear can fight the other to try to drive it away to get more fish for itself but suffer battle injuries (Hawk)
- Or it can tolerate the presence of the other, share the fishing, and avoid injury (Dove)

Coexistence Games

The Hawk-Dove Game

		Bear 2	
		Hawk	Dove
Bear 1	Hawk	-5,-5	8,0
	Dove	0,8	4,4

- Let’s look for PSNE
- We found two:
- Notice that purely peaceful coexistence is not a NE
- How about MSNE?

Coexistence Games

The Hawk-Dove Game

Bear 2

		Hawk	Dove
Bear 1	Hawk	-5,-5	8,0
	Dove	0,8	4,4

- π_1: prob. that 1 chooses Hawk; π_2: prob. that 2 chooses Hawk
- $EV_1(H) = -5\pi_2 + 8(1 - \pi_2) = 8 - 13\pi_2$
- $EV_1(D) = 0 + 4(1 - \pi_2) = 4 - 4\pi_2$
- Bear 1 indifferent if $EV_1(H) = EV_1(D)$, i.e. if $\pi_2 = 4/9$
- By symmetry, Bear 2 indifferent between actions if $\pi_1 = 4/9$

Coexistence Games

The Hawk-Dove Game

		Bear 2	
		Hawk	Dove
Bear 1	Hawk	-5,-5	8,0
	Dove	0,8	4,4

- The game has a MSNE
- For each bear, the expected value of the mixed-strategy NE is

$$-5\left(\frac{4}{9}\right)\left(\frac{4}{9}\right)+8\left(\frac{5}{9}\right)\left(\frac{4}{9}\right)+0\left(\frac{4}{9}\right)\left(\frac{5}{9}\right)+4\left(\frac{5}{9}\right)\left(\frac{5}{9}\right)=\frac{180}{81}$$

Coexistence Games

The Hawk-Dove Game

- Games of Coexistence is useful when we think about the evolutionary process

- Main Idea:
  - Behaviors are genetically programmed
  - Evolution encourages growth of some types but not others

Coexistence Games

The Hawk-Dove Game

Instead of treating π_1 and π_2 as probabilities of individual bears choosing to be "Hawks", treat them as proportion of aggressive bears in the entire bear population

If $\pi_1=\pi_2 > (4/9)$

- $EV_1(D) = 4 - 4\pi_2 > 8 - 13\pi_2 = EV_1(H)$
- Doves are better off than Hawks
- Better chances in reproduction
- Percentage of Hawks will decrease until it reaches 4/9 of population

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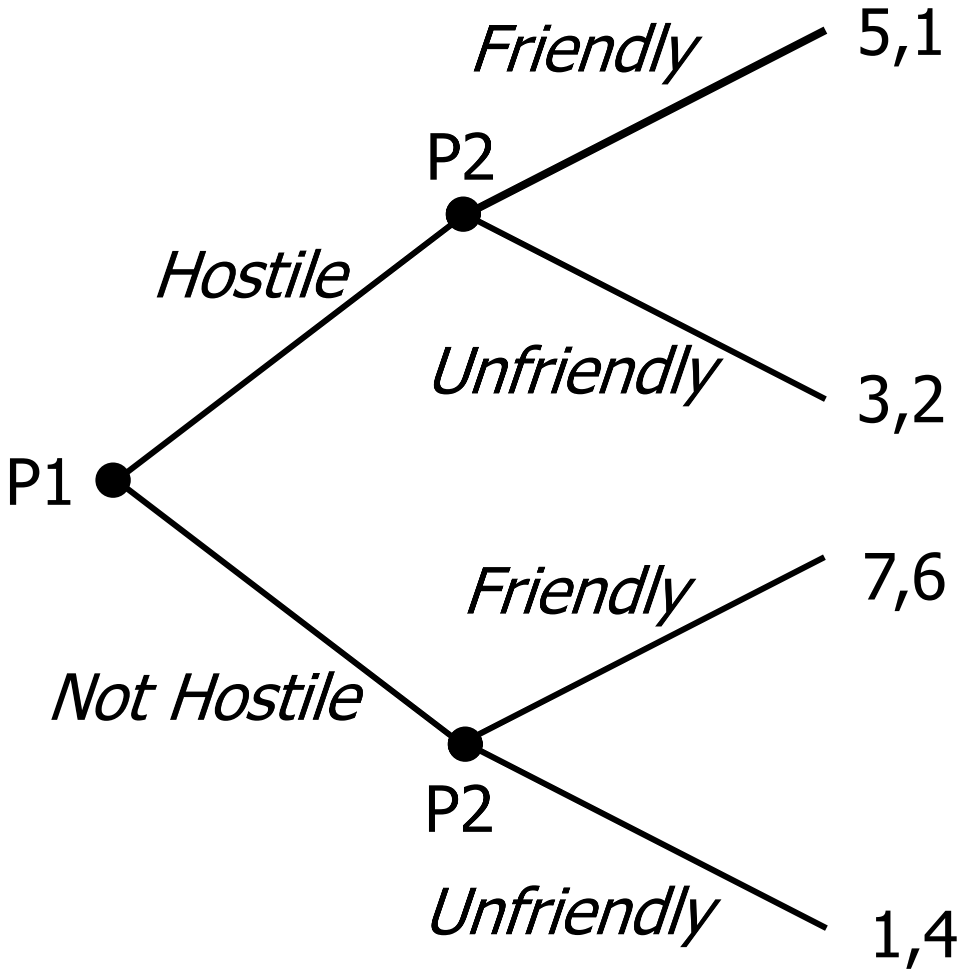
Commitment Games

- Sequential play games in which
  - One player chooses an action before the other player
  - The first player’s action is irreversible and observable
  - The first player knows that his action is observable

Commitment Games



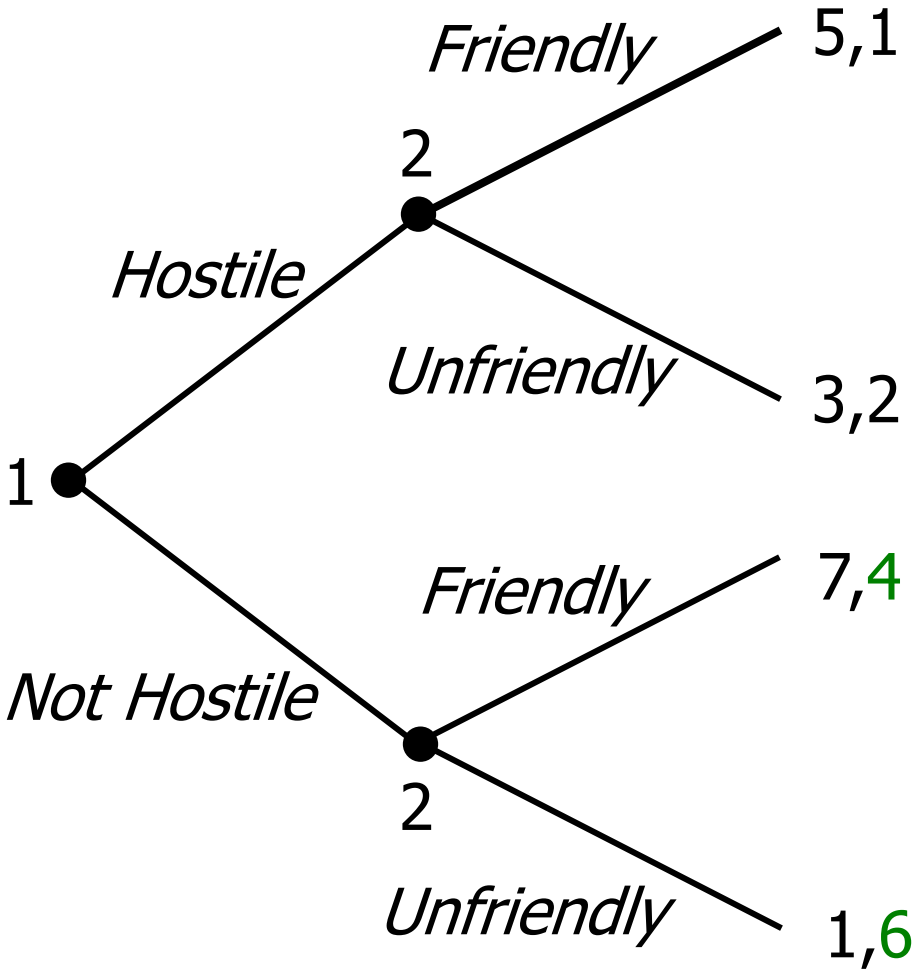
- P2 tries to persuade P1 not to be hostile
- If P1 is risk averse, may be tempted to be hostile

- Is a claim by P2 that she will commit to choosing action *UF* if P1 chooses *H* credible to P1?

- Is a claim by P2 that she will commit to choosing action *F* if P1 chooses *NH* credible to P1?

- So P1 should choose *NH*

Commitment Games



- We change the payoffs slightly

- Player 2 tries to persuade Player 1 not to be hostile

- Now, is a claim by Player 2 that she will commit to choosing action *F* if Player 1 chooses *NH* credible to Player 1?

- Hence, as much as Player 1 prefers to receive 7 instead of 3, she should choose *H*

Split or steal round 2: Committing to steal

		Nick	
		Split	**Steal**
Abraham	**Split**	**P/2, P/2**	**0,P**
	Steal	**P,0**	**0,0**

If Nick commits to steal, Abraham will actually be indifferent between split or steal

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Bargaining Games

- Two approaches:
    - ~~Nash bargaining~~
    - Rubinstein bargaining

Rubinstein Bargaining

- Two players, A and B, bargain over the division of a cake of size 1
- The players take turn in making offers, with Player A starting in period 1
- If the player who receives an offer accepts it, then the game ends immediately. Else the game continues to the next period
  - For simplicity, we assume that a player accepts when indifferent
  - They have k periods to agree; else both get nothing (Let k=3)
  - Player A discounts next period's payoffs by $0 \leq \alpha \leq 1$
  - Player B discounts next period's payoffs by $0 \leq \beta \leq 1$

Backward Induction

- Consider Stage 3
- A offers x_3 to herself
- How should B respond?
- If B rejects, gets 0
- If B accepts, get $1 – x_3$
    - Accept if $1 – x_3 \geq 0$
    - *i.e.* accept if $x_3 \leq 1$
    - Knowing this, A should set $x_3 = 1$
    - B will accept

Backward Induction

- Now consider Stage 2

- B offers x_2 to herself

- How should A respond?

- If A rejects, gets 1 in Stage 3, valued at α in Stage 2
- If A accepts, get $1 – x_2$
    - Accept if $1 – x_2 \geq \alpha$
    - *i.e.,* accept if $x_2 \leq 1-\alpha$

    - Knowing this, B should set $x_2 = 1-\alpha$
    - A will accept

Backward Induction

- Now Consider Stage 1

- A offers x_1 to herself

- How should B respond?

- If B rejects, gets $1-\alpha$ in Stage 2, valued at $\beta(1-\alpha)$ in Stage 1
- If B accepts, get $1 – x_1$
    - Accept if $1 – x_1 \geq \beta(1-\alpha)$
    - *i.e.,* accept if $x_1 \leq 1-\beta(1-\alpha)$

    - Knowing this, A should set $x_1 = 1-\beta(1-\alpha)$
    - B will accept

Rubinstein Bargaining

- Notice that the game ends immediately, in period 1

- Player A gets $1 - \beta(1 – \alpha)$ units of the cake
  Player B gets $\beta(1 – \alpha)$ units

- Which is the larger?
- Patience is an important determinant
  - Low α hurts player A
  - Low β hurts player B