Practice Problem Set 4
Oligopoly (C.28)

Question 4.1

In Pulau Kowai, consumers purchase smartphones without a phone plan. There are only two suppliers of smartphones: Apple and Samsung. They compete by setting quantity sequentially. Samsung is the Stackelberg leader and Apple is the follower. The inverse demand function of smartphone in this island is given by $P = 200 - A - S$, where P is price, A is the amount of iPhones sold, and S the amount of Galaxy phones sold. The marginal cost of Apple is \$50, and that of Samsung is \$100. Consumers in this island cannot tell the difference between an Apple phone and a Samsung phone. Find the equilibrium A, S, and P. Do you think that Samsung's position as the Stackelberg leader allows it to sell more than Apple, and earn a bigger profit? Why or why not?

Answer

Solving by backward induction:

In stage 2, Apple will take whatever S that Samsung announces in stage 1 as given, and proceeds to maximize its profits. It will set $MR = MC$, where $MR = 200 - 2A - S$ and $MC = 50$. Hence, Apple's reaction function is given by $A = 75 - \frac{S}{2}$.

Now consider Samsung's problem in stage 1. It knows that $P = 200 - A - S = 200 - \left(75 - \frac{S}{2}\right) - S = 125 - \frac{S}{2}$.

Hence, for Samsung, $MR = 125 - S$. Set $MR = MC$, we know that $S = 25$, $A = 62.5$, and $P = 112.5$. Samsung's profit is lower than that of Apple (312.5 compared to 3906.25).

Notice that in this case, even though Samsung is the Stackelberg leader, it sells less than Apple. This is because it has higher production cost.

Question 4.2

In Pulau Kawai, consumers purchase smartphones without a phone plan. There are only two suppliers of smartphones: Apple and Samsung. Consumers in this island cannot tell the difference between an Apple phone and a Samsung phone.

Apple and Samsung compete by setting price simultaneously. The inverse demand function of smartphone in this island is given by $P = 200 - A - S$, where P is price, A is the amount of iPhones sold, and S the amount of Samsung phones sold. The marginal cost of Apple is \$50, and that of Samsung is \$100. What are the equilibrium prices and quantities of smartphones sold in Pulau Kawai?

Answer

First, take note that Apple and Samsung are engaged in Bertrand competition. Since Apple and Samsung both set prices, you will need to specify both P^A and P^S.

We know that Apple will never set $P^A < 50$ and Samsung will never set $P^S < 100$. Hence if Apple sets $P^A < 100$, it will capture the entire market. But does Apple want to do so? To check, Suppose Apple is a profit maximizing monopoly, it will seek to maximize

$$\pi_A = P^A \cdot A - 50A = (200 - A)A - 50A$$

Set $\frac{d\pi_A}{dA} = 0 \Longrightarrow 200 - 2A - 50 = 0 \Longrightarrow A = 75 \Longrightarrow P^A = 125$

We know that Apple would want to set its price to be as close to \$125 as possible. But if it sells above \$100, Samsung can undercut Apple by charging a price that is lower than Apple's but higher than \$100. Hence $P^A > 100$ cannot be supported as an equilibrium.

An equilibrium would be $P^A = 100 - \epsilon$, $P^S = 100$ (Note that in the exams, you will have full credits if you say that $P^A = 99$, $P^A = 99.9$, $P^A = 99.99$, $P^A = 100$ or $P^A = 100 - \epsilon$). Apple will capture the whole market, selling $100 + \epsilon$ units of phone. Since $TR - TC \approx 100 \cdot 100 - 100 \cdot 50 = 5000$, Apple makes a profit. Notice also that $P > MC$ even though Apple and Samsung are engaged in Bertrand competition.

This is an equilibrium because: If Samsung deviates and sets another price that is above \$100, Apple will still capture the entire market; If Samsung sets any price that is below \$100, it will make losses; If Apple sets any price that is above \$100, it will lose the market to Samsung; If Apple sets any price that is below \$100, its profits decrease—hence there is no incentive for Samsung or Apple to deviate from $P^A = 100 - \epsilon$ and $P^S = 100$.

Question 4.3

Two companies, Company 1 and Company 2, sell competing products. Their products are substitutes (but not perfect substitutes), so that the number of units that either company sells is a decreasing function of its own price and an increasing function of the other product's price. Let p_1 be the price and x_1 the quantity sold of product 1 and let p_2 and x_2 be the price and quantity sold of product 2. Then $x_1 = 1000(90 - 0.5p_1 + 0.25p_2)$ and $x_2 = 1000(90 - 0.5p_2 + 0.25p_1)$. For each company, the cost of selling an extra unit is zero. Each company chooses the ***price*** that maximize its profits.

Solve for the equilibrium prices p_1 and p_2.

Answer

(The important thing to note about this question is that while the two companies are engaged in price competition, their products are not homogenous. This is why there are two, instead of one, demand equations. Note that each company earns a positive profit in equilibrium. If, instead, the two companies are selling a homogenous product, we are back to the basic Bertrand model that you have seen in lecture, price will be equal to marginal cost and hence zero in equilibrium.)

Firm 1 max $\pi_1 = p_1 x_1 = 1000p_1(90 - 0.5p_1 + 0.25p_2)$. Differentiate π_1 w.r.t. p_1, we get the first order condition $p_1 = 90 + 0.25p_2$. Do the same for firm 2, we have $p_2 = 90 + 0.25p_1$. Hence $p_1 = 120, p_2 = 120$.

Question 4.4

Toyota and Honda are duopolists in the market for cars, a homogenous good. Toyota chooses the quantity x_1 and Honda chooses the quantity x_2, and the demand for cars is given by $X = 40 - P$, where P is the price of a car and $X = x_1 + x_2$. The (total) cost function of each firm is $c_i = 10x_i$ (where $i = 1$ if the firm is Toyota and $i = 2$ if the firm is Honda.)

i) If Honda does not exist and Toyota monopolizes the market, how much will Toyota produce and at what price? What will its profit be?

ii) From now onwards, suppose that both firms are in the market. Find the equilibrium outputs and price if Toyota and Honda make their decisions sequentially, with Toyota setting its output before Honda does.

iii) Suppose instead of quantities, Toyota chooses price p_1 and Honda chooses price p_2 simultaneously. The game is repeated infinitely many times between them and the two firms have a common discount factor δ. Suppose the two firms collude as a joint monopoly and divide the profits equally between themselves in the current period as long as no firm has cheated before the current period. Otherwise, they revert to Bertrand competition. Recall that $0<\delta<1$. There exists a $\bar{\delta}$ such that for any $\delta>\bar{\delta}$, no cheating by both firms in every period can be sustained as an equilibrium. What is the value of $\bar{\delta}$?

Answer

i) Set $x_2 = 0$, $P = 40 - x_1 \Rightarrow MR_1 = 40 - 2x_1$. Set $MR = MC$, we have $x_1 = 15$, $P = 25$. $\pi = 225$.

ii) We solve the question by backward induction. After Toyota set x_1, Honda will set $MR_2 = MC \Rightarrow 40 - x_1 - 2x_2 = 10 \Rightarrow x_2 = 15 - 0.5x_1$. Now consider Toyota's problem: knowing that once x_1 is determined, Honda will set $x_2 = 15 - 0.5x_1$, Toyota faces the inverse demand function $P = 40 - x_1 - x_2 = 25 - 0.5x_1 \Rightarrow MR_1 = 25 - x_1 \Rightarrow x_1 = 15 \Rightarrow x_2 = 7.5 \Rightarrow P = 17.5$.

iii) Toyota (and likewise Honda) will not cheat if $112.5 + 112.5\delta + 112.5\delta^2 + \cdots \geq 225 + 0\delta + 0\delta^2 + \cdots \Longrightarrow \frac{112.5}{1-\delta} \geq 225 \Longrightarrow \delta \geq 0.5$. Hence, $\bar{\delta} = 0.5$.