Welfare (Social Choice)

Week 12

(Chapter 34.1 only)

The Limits of Markets

- Suppose G is a public good (non-rivalrous; non-excludable)
- E.g. how much street lighting in neighbourhood where *n* individuals live?

- Pareto Efficiency requires:

$$\sum_{i=1}^{n} MRS_i = \frac{p_G}{p_X}$$

- Free market unlikely to deliver Pareto Efficiency due to free riders

Social Choice

- If there exists benevolent social planner who knows everyone's preference, she can compute the efficient level of public good
- But she needs the right to solicit contributions, by coercion if necessary, which makes her a dictator
- In reality, "social planners" are seldom benevolent (= dictators…)
- Can we design a non-dictatorial mechanism (for example, voting) that mimics a benevolent social planner?
- Preview of Conclusion: Even if individuals are rational, aggregating their preferences by voting may lead to outcomes with undesirable properties

Aggregating Preferences

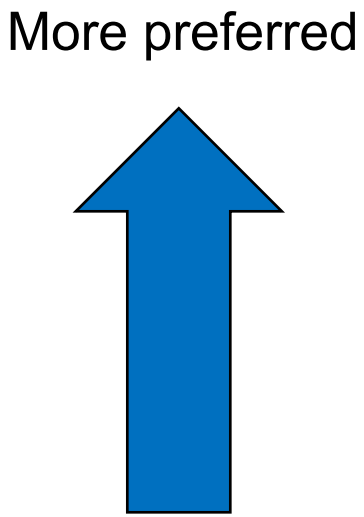
- *x*, *y*, *z* denote different alternatives

- 3 voters:
  - Tom
  - Jerry
  - Spike

- What decision will simple majority voting generate?

Majority Voting

Tom	Jerry	Spike
x	y	z
y	z	x
z	x	y



Majority Voting

Tom	Jerry	Spike
x	y	z
y	z	x
z	x	y

If we pick 2 out of 3 choices, and ask Tom, Jerry, and Spike to vote between them, then

- x beats y
- y beats z
- z beats x

- *Condorcet Paradox*: collective preferences may be cyclical, even if the preferences of individuals are transitive
- Majority wishes can be self-contradictory

How about a Rank-order vote?

Tom	Jerry	Spike
x (1)	y (1)	z (1)
y (2)	z (2)	x (2)
z (3)	x (3)	y (3)

Suppose voters vote honestly and lowest score wins

- x-score = 6
- y-score = 6
- z-score = 6

- Rank-order voting indecisive
- In reality, strategic voting likely (eg. Jerry may declare untruthfully that z is his most preferred choice)

Four Desirable Properties of a Social Decision Mechanism

1. If all individuals' preferences are rational (complete and transitive), so should the social preference (i.e., outcome generated by the mechanism)

- Preferences are complete if for any two alternatives x and y: either $x \succ y$, or $x \prec y$, or $x \sim y$
- Preferences are transitive if $x \succ y$ and $y \succ z$ implies that $x \succ z$
- Completeness + Transitivity $\Longrightarrow$ Rational Preference

Four Desirable Properties of a Social Decision Mechanism

2. (Pareto Property) If all individuals rank x before y, then so should the social preference

Four Desirable Properties of a Social Decision Mechanism

3. Social preference between x and y should depend on individuals' preferences between x and y only

<u>Independence of irrelevant alternatives, IIA</u>

If there are two alternatives, x and y, and $x \succcurlyeq y$, then introducing a third alternative z should not change that preference

(either $z \succcurlyeq x \succcurlyeq y$, or $x \succcurlyeq z \succcurlyeq y$, or $x \succcurlyeq y \succcurlyeq z$)

Four Desirable Properties of a Social Decision Mechanism

4. Social preference should not be dictatorial

- A social preference is dictatorial if there exists an individual such that the society strictly prefers x to y whenever the individual prefers x to y

Four Desirable Properties of a Social Decision Mechanism

1. *If all individuals' preferences are rational (complete and transitive), then so should be the social preference*

2. *If all individuals rank x before y then so should the social preference*

3. *Social preference between x and y should depend on individuals' preferences between x and y only*

4. *Social preference should not be dictatorial*

Arrow’s Impossibility Theorem

- (Kenneth Arrow, 1921–2017)
- The only Social Decision Mechanism that respects properties 1, 2 and 3 is dictatorial
- A non-dictatorial voting rule requires giving up at least one of properties 1, 2 or 3

Majority Voting

Tom	Jerry	Spike
x	y	z
y	z	x
z	x	y

- x beats y
- y beats z
- z beats x

- Preferences of Tom, Jerry, and Spike are complete and transitive
- Majority voting does not always aggregate transitive individual preferences into a transitive social preference