Tutorial 1 - Submission

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Question 1

1.
   - Player set $N = \{1, 2, \dots, n\}$
   - Each player i has a set of strategies $S_i = \{q \in \mathbb{N} : q \leq a\}$, from which they choose their strategy s_i
   - Strategy profile $s = (s_1, s_2, \dots, s_n)$ is the combination of each player's tragegy
   - Utility for each player is their total profit:

$$Q = \sum_{i \in N} s_i$$
$$u_i(s) = s_i(a - c - Q)$$

2. Each player best response is:

$$u_i(s_i, s_{-i}) = s_i(a - c - \sum s_{-i} - s_i)$$
$$u_i'(s_i, s_{-i}) = a - c - \sum s_{-i} - 2s_i$$
$$u_i' = 0 \Leftrightarrow s_i = \frac{a - c - \sum s_{-i}}{2}$$
$$u_i'' = 2 > 0 \Rightarrow \text{ utility at maximum}$$
$$B_i(s_{-i}) = \frac{a - c - \sum s_{-i}}{2}$$

   We have:

$$2B_i(s_{-i}) + \sum s_{-i} = a - c$$

   Nash equilibrium $s^* = (s_1^*, s_2^*, \dots, s_n^*)$ is the solution of system:

$$s_1^* + \sum s_i = a - c$$
$$s_2^* + \sum s_i = a - c$$
$$\dots$$

   The system is consistent with only one solution $s = \left(s_i = \frac{a-c}{n+1} : i \in N\right)$

3.

$$\begin{aligned} p^* = a - Q = a - n\frac{a-c}{n+1} &= a\left(1 - \frac{n}{n+1}\right) + \frac{cn}{n+1} \\ &= \frac{a}{n+1} + c\left(1 - \frac{1}{n+1}\right) \\ &\Rightarrow \lim_{n\to+\infty} p^* = c \end{aligned}$$

Profit of each firm is $\pi_i = \pi^*$:

$$\begin{aligned} \pi^* &= s_i^*(p^* - c) \\ \Rightarrow \lim_{n\to+\infty} \pi^* &= \lim_{n\to+\infty} \frac{a-c}{n+1} \times (\lim_{n\to+\infty} p^* - c) \\ &= 0 \times (c - c) = 0 \end{aligned}$$

Question 2

A seller profit monotonically increases with their demand.

- If $l_i < l_j$, then $q_i \uparrow\uparrow l_i \Rightarrow$ not Nash equilibrium
- If $l_i > l_j$, then $q_i \uparrow\downarrow l_i \Rightarrow$ not Nash equilibrium
- If $l_i = l_j$, then q_i can not increase at any other l_i

$$\Rightarrow B_i(l_j) = l_j$$

The set of pure-strategy Nash equilibria is $E = \{s = (l_i, l_j) \in S : l_i = l_j\}$