EC3333 Tutorial 6 Suggested Answers

*For this module, unless otherwise stated, the par value of the bond is assumed to be $1000.

1. A 30-year bond with a face value of $1000 has a coupon rate of 5.5%, with semiannual payments.

   a. What is the coupon payment for this bond?

   b. Draw the cash flows for the bond on a timeline.

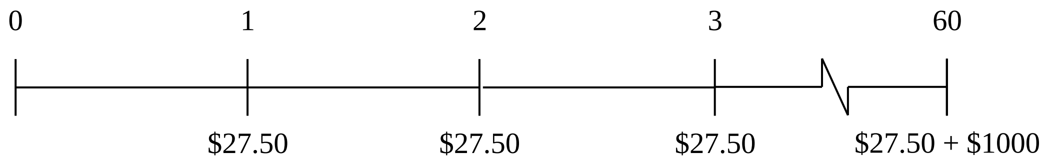
a.

The coupon payment is:

$$CPN = \frac{\text{Coupon Rate} \times \text{Face Value}}{\text{Number of Coupons per Year}} = \frac{0.055 \times \$1000}{2} = \$27.50.$$

b.

The timeline for the cash flows for this bond is (the unit of time on this timeline is six-month periods):



2. Suppose a 10-year, $1000 bond with an 8% coupon rate and semiannual coupons is trading for a price of $1034.74.

   a. What is the bond's yield to maturity (expressed as an APR with semiannual compounding)? (You can use the Excel solver function to do this.)

   b. If the bond's yield to maturity changes to 9% APR, what will the bond's price be?

a.

$$P_B = \sum_{t=1}^{T} \frac{C}{(1+r)^t} + \frac{\text{Par Value}}{(1+r)^T}$$

or

$$P_B = \underbrace{\frac{C}{r}\left(1 - \frac{1}{(1+r)^T}\right)}_{\text{The Annuity Formula}} + \frac{\text{Par Value}}{(1+r)^T}$$

Yield to maturity: Using the Excel solver function, enter the formula for the bond price and solve for r, with T=20, Par Value = $1000, C=40, and set the bond price = $1034.74.

This results in: YTM per ½ year = r = 3.75%.

YTM (expressed as an APR with semiannual compounding) = 3.75% × 2 = 7.50%.

b. YTM per ½ year = r = 4.5% per 6 months, the new price is $934.96

3. Suppose the current zero-coupon yield curve for risk-free bonds is as follows:

Maturity (years)	1	2	3	4	5
YTM	5.00%	5.50%	5.75%	5.95%	6.05%

a. What is the price per $1000 face value of a two-year, zero-coupon, risk-free bond?

b. What is the price per $1000 face value of a four-year, zero-coupon, risk-free bond?

c. What is the risk-free interest rate for a five-year maturity?

$$P_B = \sum_{t=1}^{T} \frac{C}{(1+r)^t} + \frac{\text{Par Value}}{(1+r)^T}$$

$$P_{zero-cpn} = \frac{\text{Par Value}}{(1+r)^T}$$

a. $P = 1000/(1.055)^2 = \$898.45$

b. $P = 1000/(1.0595)^4 = \$793.59$

c. 6.05% per year.

4. The following table summarizes prices of various default-free, zero-coupon bonds (expressed as a percentage of face value):

Maturity (years)	1	2	3	4	5
Price (per $100 face value)	$95.51	$91.05	$86.38	$81.65	$76.51

a. Compute the yield to maturity for each bond.
b. Plot the zero-coupon yield curve (for the first five years).
c. Is the yield curve upward sloping, downward sloping, or flat?

a.

$$P_{zero-cpn} = \frac{\text{Par Value}}{(1+r)^T}$$

Use the following equation.

$$1 + \text{YTM}_T = \left(\frac{\text{FV}_T}{P}\right)^{1/T}$$

$$1 + \text{YTM}_1 = \left(\frac{100}{95.51}\right)^{1/1} \Rightarrow \text{YTM}_1 = 4.70\%$$

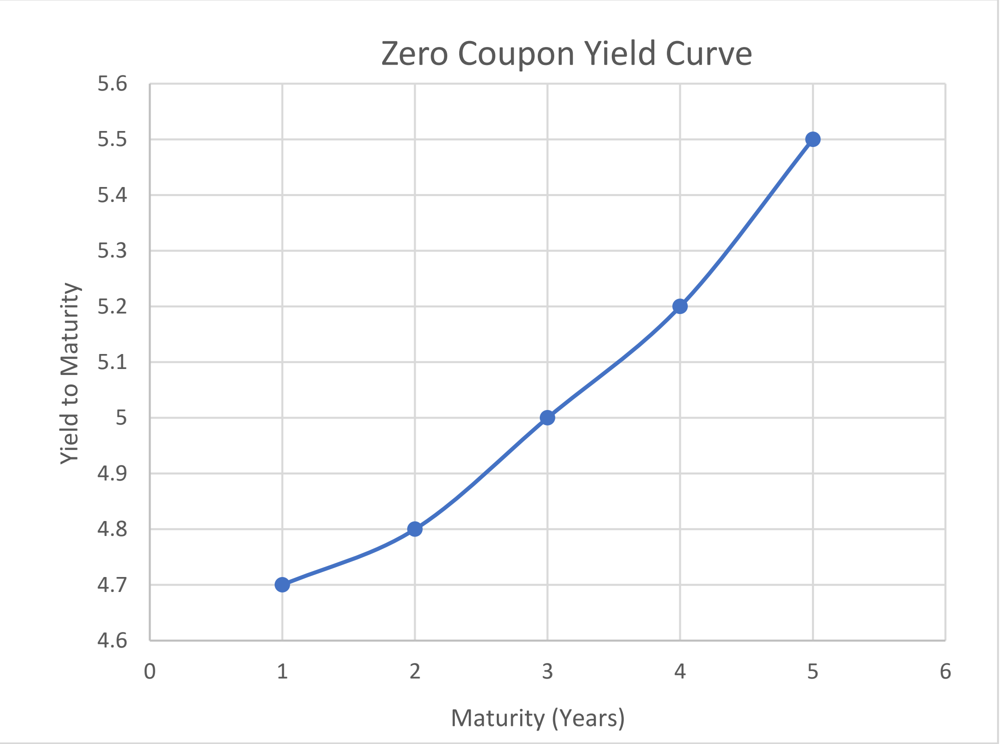
$$1 + \text{YTM}_1 = \left(\frac{100}{91.05}\right)^{1/2} \Rightarrow \text{YTM}_1 = 4.80\%$$

$$1 + \text{YTM}_3 = \left(\frac{100}{86.38}\right)^{1/3} \Rightarrow \text{YTM}_3 = 5.00\%$$

$$1 + \text{YTM}_4 = \left(\frac{100}{81.65}\right)^{1/4} \Rightarrow \text{YTM}_4 = 5.20\%$$

$$1 + \text{YTM}_5 = \left(\frac{100}{76.51}\right)^{1/5} \Rightarrow \text{YTM}_5 = 5.50\%$$

b. The yield curve is as shown below.



c.

The yield curve is upward sloping.

5. A bond with an annual coupon rate of 4.8%, sells for $970. What is the bond's current yield?

Annual coupon rate: 4.80% → $48 Coupon payments
Current yield = Bond's annual coupon payment divided by the bond price

$$\left(\frac{\$48}{\$970}\right) = 4.95\%$$

6. Consider an 8% coupon bond selling for $953.10 with 3 years until maturity making annual coupon payments. Calculate the yield to maturity of the bond. (You can use the excel solver function to do this.)

Now assume that the interest rates in the next three years will be, with certainty, r_1=8%, r_2 = 10%, and r_3 = 12%, respectively. Calculate the realized compound yield of the bond (assuming that you reinvest all the coupon payments at the prevailing interest rates).

$$P_B = \sum_{t=1}^{T} \frac{C}{(1+r)^t} + \frac{\text{Par Value}}{(1+r)^T}$$

or

$$P_B = \underbrace{\frac{C}{r}\left(1 - \frac{1}{(1+r)^T}\right)}_{\text{The Annuity Formula}} + \frac{\text{Par Value}}{(1+r)^T}$$

Yield to maturity: Using the Excel solver function, enter the formula for the bond price and solve for r, with T=3, Par Value = \$1000, C=80, and set the bond price = \$953.10.

This results in: YTM = 9.88%.

Realized compound yield: First, find the future value (FV) of reinvested coupons and principal:

FV = (\$80 × 1.10 ×1.12) + (\$80 × 1.12) + \$1,080 = \$1,268.16

Then find the rate (r_{realized}) that makes the FV of the purchase price equal to \$1,268.16:

\$953.10 × $(1 + r_{\text{realized}})^3$ = \$1,268.16 ⇒ r_{realized} = 9.99% or approximately 10%

7. Consider a bond paying a coupon rate of 10% per year semiannually when the market interest rate is only 4% *per half-year*. The bond has 3 years until maturity.
   a. Find the bond's price today and 6 months from now after the next coupon is paid.
   b. What is the 6-month holding period return on the bond?

a.

$$P_B = \underbrace{\frac{C}{r}\left(1 - \frac{1}{(1+r)^T}\right)}_{\text{The Annuity Formula}} + \frac{\text{Par Value}}{(1+r)^T}$$

$$P_{today} = \underbrace{\frac{50}{0.04}\left(1 - \frac{1}{(1+0.04)^{3x2}}\right)}_{\text{The Annuity Formula}} + \frac{1000}{(1+0.04)^{3X2}} = \$1052.42$$

$$P_{6m\ from\ now} = \underbrace{\frac{50}{0.04}\left(1 - \frac{1}{(1+0.04)^{5}}\right)}_{\text{The Annuity Formula}} + \frac{1000}{(1+0.04)^{5}} = \$1044.52$$

b.

The 6-month holding period return on the bond is

$$R_{t+1} = \frac{C_{t+1} + P_{t+1}}{P_t} - 1 = \frac{C_{t+1}}{P_t} + \frac{P_{t+1} - P_t}{P_t}$$

$$= \text{Current Yield} + \text{Capital Gain/Loss Rate}$$

$$R_{t+6m} = \frac{C_{t+6m} + P_{t+6m}}{P_t} - 1 = \frac{C_{t+6m}}{P_t} + \frac{P_{t+6m} - P_t}{P_t}$$

$$= \text{Current Yield} + \text{Capital Gain/Loss Rate}$$

$$R_{t+6m} = \frac{\$50 + (\$1{,}044.52 - \$1{,}052.42)}{\$1{,}052.42} = \frac{\$50 - \$7.90}{\$1{,}052.42} = 4.0\%$$

8. A 2-year bond with par value of $1,000 making annual coupon payments of $100 is priced at $1,000. What is the yield to maturity of the bond? What will be the realized compound yield to maturity if the 1-year interest rate next year turns out to be (a) 8%, (b) 10%, (c) 12% (assuming that you reinvest all the coupon payments at the prevailing interest rates)?

The bond is selling at par value. Its yield to maturity equals the coupon rate, 10%.

If the first-year coupon is reinvested at an interest rate of *r* percent, then total proceeds at the end of the second year will be: [$100 * (1 + *r*)] + $1,100

Therefore, realized compound yield to maturity is a function of *r*, as shown in the following table:

r	Total proceeds	Realized YTM = $\sqrt{\text{Proceeds}/1000} - 1$
8%	$1,208	$\sqrt{1208/1000} - 1 = 0.0991 = 9.91\%$
10%	$1,210	$\sqrt{1210/1000} - 1 = 0.1000 = 10.00\%$
12%	$1,212	$\sqrt{1212/1000} - 1 = 0.1009 = 10.09\%$

9. Consider a bond with a 10% coupon rate and with yield to maturity = 8%. If the bond's yield to maturity remains unchanged, then in one year, will the bond price be higher, lower, or unchanged? Why?

The bond price will be lower. As time passes, the bond price, which is now above par value, will approach par.

10. Suppose the bond has a current yield of 9% and a yield to maturity of 10%. Is the bond selling at a premium or at a discount? Why?

If the yield to maturity is greater than the current yield, then the bond offers the prospect of price appreciation as it approaches its maturity date. Therefore, the bond must be selling below par value.