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1. $((x_1^A, x_2^A), (x_1^B, x_2^B))$

Suppose consumer A strictly prefers (y_1^A, y_2^A) to (x_1^{*A}, x_2^{*A})

consumer B weakly prefers (y_1^B, y_2^B) to (x_1^{B*}, x_2^{B*})

Thus, $p_1 y_1^A + p_2 y_2^A > p_1 w_1^A + p_2 w_2^A$ — ①

$p_1 y_1^B + p_2 y_2^B \geq p_1 w_1^B + p_2 w_2^B$ — ②

However, it is impossible.

Contradiction:

① + ②,

$p_1(y_1^A + y_1^B) + p_2(y_2^A + y_2^B) > p_1(w_1^A + w_1^B) + p_2(w_2^A + w_2^B)$

feasible

$y_1^A + y_1^B \leq w_1^A + w_1^B$ — ③

$y_2^A + y_2^B \leq w_2^A + w_2^B$ — ④

③ p_1 + ④ p_2,

$p_1(y_1^A + y_1^B) + p_2(y_2^A + y_2^B) \leq p_1(w_1^A + w_1^B) + p_2(w_2^A + w_2^B)$

(contradiction!)

Question 2

(a) - At the equilibrium allocation (of Cobb-Doughlas utility function), we can use the tangency condition between two utility function:

$$MRS^i_{x_1x_2} = \frac{MU^i_{x_1}}{MU^i_{x_2}} = \frac{\frac{\alpha}{x^i_1 \times ln(10)}}{\frac{1-\alpha}{x^i_2 \times ln(10)}} = \frac{\alpha x^i_2}{(1-a)x^i_1}$$

$$MRS^A_{x_1x_2} = MRS^E_{x_1x_2}$$

$$\Rightarrow \frac{\alpha x^A_2}{(1-a)x^A_1} = \frac{\alpha x^E_2}{(1-a)x^E_1}$$

$$\Rightarrow \frac{x^A_2}{x^A_1} = \frac{x^E_2}{x^E_1} = \frac{x^A_2 + x^E_2}{x^A_1 + x^E_1} = \frac{6}{6} = 1$$

- We have the tangency condition with the price/budget line:

$$MRS^A_{x_1x_2} = \frac{\alpha x^A_2}{(1-\alpha)x^A_1} = \frac{p_{x_1}}{p_{x_2}}$$

$$\Rightarrow \frac{p_1}{p_2} = \frac{\alpha}{1-\alpha}$$

(b) - From the equal utility condition:;

$$U^A = U^E$$

$$\Rightarrow \alpha log(x^A_1) + (1-\alpha)log(x^A_2) = \alpha log(x^E_1) + (1-\alpha)log(x^E_2)$$

$$\Rightarrow x^A_1 = x^E_1$$

$$\Rightarrow x^A_1 = x^E_1 = x^A_2 = x^E_2 = 3$$

- From (a):

$$p_1 = \frac{\alpha}{1-\alpha}p_2$$

- The lump-sum transfer Anton will receive is:

$$T^A = p_1(x^A_1 - \omega^A_1) + p_2(x^A_2 - \omega^2_2)$$

$$= \frac{\alpha}{1-\alpha}(3-4) + (3-1) = 2 - \frac{\alpha}{1-\alpha}$$

3. aggregate net demand for x_1 is

$$(x_1^A + x_1^B) - (w_1^A + w_1^B) = 8 - (3+7) = -2$$

$\therefore$ Walras' law,

$$9 \times -2 + 3\,[(x_2^A + x_2^B) - (w_2^A + w_2^B)] = 0$$

$$(x_2^A + x_2^B) - (w_2^A + w_2^B) = \frac{18}{3} = 6$$

$$(x_2^A + x_2^B) = 6 + (11 + 5)$$

$$= 22$$

Question 4

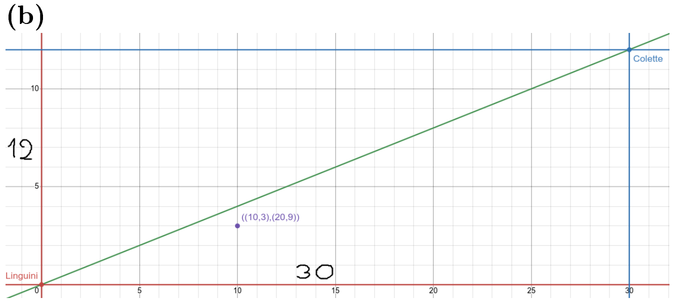
(a) Contract curve satisfies tangency condition.

$$MRS_{12}^L = MRS_{12}^C$$

$$\frac{x_2^L}{x_1^L} = \frac{x_2^C}{x_1^C} = \frac{x_2^L + x_2^C}{x_1^L + x_1^C} = \frac{12}{30} = \frac{2}{5}$$

$$x_2^L = \frac{2}{5} x_1^L$$

(b)



(c) - For Linguini:

$$MRS_{12}^L = \frac{x_2^L}{x_1^L} = \frac{p_1}{p_2} = \frac{1}{2}$$

$$\Rightarrow x_1^L = 2x_2^L$$

While satisfying budget constraint:

$$p_1 x_1^L + p_2 x_2^L = p_1 \omega_1^L + p_2 \omega_2^L = 16$$

$$x_2^L = 4$$

$$x_1^L = 8$$

- Do the same to Colette:

$$x_2^C = 9.5$$

$$x_1^C = 19$$

(d) - Linguini wants to buy 1 profiterole and sell 2 éclair.
- Colette wants to buy 0.5 profiterole and sell 1 éclair
- The market is not in equilibrium

(e) Walras's law equation:

$$p_1(x_1^L + x_1^C - \omega_1^L - \omega_1^C) + p_2(x_2^L + x_2^C - \omega_2^L - \omega_2^C) = 0$$

$$1(8 + 19 - 10 - 20) + 2(4 + 9.5 - 3 - 9) = 0$$

$$0 = 0$$

(f) - Using tangency condition:

$$\frac{p_1}{p_2} = MRS_{12}^L = \frac{x_2^L}{x_1^L} = \frac{2}{5}$$

(g) $p_2 = 1 \Rightarrow p_1 = 0.4$
Linguini's BL: $0.4x_1^L + x_2^L = 7$
Colette's BL: $0.4x_1^C + x_2^C = 17$
(h) - Tangency condition:

$$x_2^L = \frac{2}{5}x_1^L$$

- Budget constraint:

$$0.4x_1^L + x_2^L = 0.4x_1^L + 0.4x_1^L = 7$$

$$\begin{cases} x_1^L & = 8.75 \\ x_2^L & = 3.5 \end{cases}$$

- Equilibrium allocation:

$$((8.75, 3.5), (21.25, 8.5))$$