1. A profit-maximizing monopoly faces the inverse demand function $P = 15 - \sqrt{Q}$. Suppose that the marginal cost of production is $\$c$ per unit and the fixed cost is \$128. If the monopoly chooses to produce 64 units of the good, then c is equal to

A. 3
B. 5
C. 7
D. 9
E. None of the other options

Answer

$\pi = (15 - \sqrt{Q})Q - cQ$, so differentiating gives $15 - \frac{3}{2}\sqrt{Q} - c = 0$. Setting $Q = 64$ gives $c = 3$.

2. Marius has a utility function $U_M(w) = \ln(w^2 + 2w + 1)$, while Enjolras has a utility function $U_E(w) = \ln(3w + 3)$. Then whenever Marius and Enjolras have the same wealth of $w \geq 0$,

A. Marius is more risk averse than Enjolras
B. Marius is less risk averse than Enjolras
C. Marius and Enjolras are equally risk averse
D. None of the other options

Answer

The Arrow-Pratt measure of absolute risk aversion (ARA) is $A(w) = -\frac{u''(w)}{u'(w)} = \frac{1}{w+1}$ for both, so they are equally risk averse. Observe that $U_E(w) = \frac{1}{2}U_M(w) + \ln(3)$, and in fact, $U(w)$ and $aU(w) + b$ have the same ARA for any constants a and b.

3. Firms 1 and 2 are duopolists in a market selling a homogeneous good. The demand function for the good is $Q = 3101 - P$, where P is the market price and Q is the total quantity of the good available in the market. The marginal cost per unit of good for both firms is \$70, and the firms compete by setting prices simultaneously. If firm 2 develops a new technology that decreases its own marginal cost of production to \$60, then

A. The Herfindhal index of the market decreases
B. The Lerner index of firm 2 decreases
C. The equilibrium quantity of the good decreases
D. None of the other options

Answer

At $c_2 = 70$, the Bertrand equilibrium is $p_1 = p_2 = 70$, so $q_1 + q_2 = 3031$. When c_2 decreases to 60, the new Bertrand equilibrium is $p_1 = 70, p_2 = 70 - \epsilon, q_1 = 0, q_2 = 3031 + \epsilon$. Therefore, the HHI increases to 1, the Lerner index increases from 0 to $\frac{1}{7}$, and equilibrium quantity increases by ϵ.

4. Standard Oil is a monopoly facing an inverse demand function $P = 80 - Q$. Standard Oil can purchase as many factories as it wants at a cost of \$25 each. Each factory i independently produces the good with a cost function $c_i(q_i) = q_i^2$. How many factories should Standard Oil purchase to maximize profits?

A. 3
B. 5
C. 7
D. 9
E. None of the other options

Answer

Let n be the number of factories. Since the cost function is convex, the monopoly sets the marginal cost of production at each factory to be equal, so q_i is the same for all i. The profit function is

$$\pi = PQ - nc(q_i) - 25n = (80 - nq_i)nq_i - nq_i^2 - 25n$$

Differentiating w.r.t. q_i and n, the first order conditions are

$$\frac{\partial \pi}{\partial q_i} = 80n - 2n^2 q_i - 2nq_i = 0$$

$$\frac{\partial \pi}{\partial n} = 80q_i - 2nq_i^2 - q_i^2 - 25 = 0$$

Solving the two equations gives $q_i = 5$ and $n = 7$.

5. Consider a two-period consumption model where Fantine receives an income of \$60 in each period. Suppose that when the interest rate is $r = 20\%$, the optimal consumption for Fantine is (c_1, c_2), while when the interest rate increases to $r' = 25\%$, the optimal consumption for Fantine is (c_1', c_2'). Which of the following statements is/are true?

A. If $c_1 > 60$, then $c_2' \geq c_2$.
B. If $c_1 = 60$, then $c_2' \geq c_2$.
C. If $c_1 < 60$, then $c_2' \geq c_2$.
D. All of the other options (except "none of the other options").
E. None of the other options.

Answer

A is false: If Fantine is a borrower, then the income effect is negative when interest increases. If the income effect dominates the substitution effect, then $c_2' < c_2$, for example, see slide 49 of lecture 1.

C is false: Even when Fantine is a lender, if period 2 consumption is not a normal good, then it is possible for $c_2' < c_2$. For example, when $U(x_1, x_2) = \min\{0.5x_1, x_1 + x_2 - 120\}$, then $(c_1, c_2) = (17.14, 111.43)$, while $(c_1', c_2') = (20, 110)$.

B is true: Any consumption bundle (c_1', c_2') with $c_2' < c_2$ lies within the intertemporal budget constraint at $r = 20\%$. Since Fantine prefers (c_1, c_2) to (c_1', c_2') at $r = 20\%$, she will still prefer prefers (c_1, c_2) to (c_1', c_2') at $r' = 25\%$, so (c_1', c_2') cannot be optimal.