Practice Problem Set 8
Externalities (C.35)

Question 8.1

You have learnt in the lecture that a market failure arising from negative production externalities can be corrected through (a) a merger of the firms involved, or by (b) an assignment of property rights. Actually, there exist other solutions, one of which is for the government to tax the market activity that generates the negative externality. We call this tax a Pigouvian tax, named after Arthur Pigou who developed the concept of externalities.

Consider the following example: Firm 1 produces output x with a cost function $c_1(x) = x^2 + 10$. Firm 2 produces output y with a cost function $c_2(y, x) = y^2 + x$. Thus, the more that firm 1 produces, the greater are firm 2's costs. Both firms face competitive product markets. The competitive price of x is \$20 and the competitive price of y is \$40. No new firms can enter the industry and the old ones must remain. What is the Pigouvian tax on x that the government should impose to achieve economic efficiency?

Answer

A quick way to solve this question is to note that for every additional unit of x that firm 1 produces, the cost of firm 2 increases by 1. Hence, the negative externality is \$1 per unit and the government should set a Pigouvian tax of \$1 per unit of x produced.

A longer (perhaps safer) way to solve this question is to find out the socially efficient output of x. We know that a merger between firms 1 and 2 will lead to the socially efficient level of x being produced. When the two firms are merged, their combined profit is $\pi = 20x + 40y - x^2 - 10 - y^2 - x$. Differentiating π w.r.t. x gives $x^* = 9.5$.

The Pigouvian tax on x should ensure that firm 1 will choose $x^* = 9.5$ on its own, without the need of a merger. With the tax (t), firm 1's profit is $\pi_1 = 20x - x^2 - 10 - tx$. Differentiating π_1 w.r.t. x gives $20 - 2x^* - t = 0$. Hence $t = 1$ if $x^* = 9.5$.

Question 8.2

(Externality) This question is an illustration of how the concept of externality can be applied to study important, non-economic issues.

Suppose there are contiguous regions, U(pstream) and D(ownstream). It has been raining heavily and region U is flooding. The only way to mitigate flooding is for region U to destroy its dikes and release water downstream. Suppose that region U suffers a welfare loss equivalent to \$60 billion if it does not destroy its dikes. If it releases floodwaters downstream, region D suffers a welfare loss equivalent to \$100 billion. Consider the scenarios described in (a), (b), and (c) below.

(a) Suppose that regions U and D are sovereign countries and they can communicate and make agreement at zero cost. There exists an international court of justice to uphold agreements made between countries. Will region U destroy its dikes? Is this outcome efficient?

(b) Suppose that regions U and D are sovereign countries in a world without an international court of justice to enforce agreements between countries. To prevent U from discharging water downstream, D can launch a surprise preemptive military attack. The campaign will cost the two regions $80 billion in total and will end in victory for D. There is no other benefit or cost to either region should an attack take place. Will D launch an attack?

(c) Suppose instead that regions U and D together form a unified country and there is a central government that seeks to maximize the total welfare of both regions. Each region can communicate with the central government at zero cost and both regions obey all instructions from the central government. Now, will region U destroy its dikes? Is this outcome efficient?

Answer

(a) Since the international court upholds any agreement between U and D and the cost of negotiation is zero, Coase theorem holds and we expect the two regions to negotiate an outcome whereby region D pays region U an amount between $60 billion and $100 billion and region U will not blow up the dikes. This is an efficient outcome. The two regions collectively suffer $60 billion.

(b) Now transaction cost is exorbitantly high because the cost of contract enforcement is one of its components. The Coase theorem no longer holds and an agreement will not be reached because it cannot be enforced. If region U blows up its dikes, it will cost region D $100 billion. By comparison, a war will cost region D at most $80 billion. Hence region D will attack. This is not efficient because the two regions collectively suffer $140 (=80+60) billion.

(c) Since the central government seeks to maximize the total welfare of both regions, it will order region U not to blow up the dikes (and region D can always remind the central government to do so at zero cost). Hence, region U will not blow up the dikes. This is an efficient outcome. The two regions collectively suffer $60 billion.

Question 8.3

El Carburetor has a population size of 1001. In El Carburetor, there is not much to do except to drive your car around town. Everybody in town is just like everybody else. While everybody likes to drive, everybody complains about the congestion, noise, and pollution caused by traffic. A typical resident's utility function is $U = m + 16d - d^2 - \frac{6h}{1000}$, where m is the resident's daily consumption of Big Macs, d is the number of hours per day that he, himself, drives, and h is the total amount of driving (measured in person-hours per day) done by all other residents of El Carburetor. The price of Big Macs is $1 each. Every person in El Carburetor has an income of $40 per day. To keep calculations simple, suppose it costs nothing to drive a car.

(a) If an individual believes that the amount of driving he does won't affect the amount that others drive, how many hours per day will he choose to drive? If everyone does the same, what is the total amount h of driving by other persons? What will be the utility of each resident?

(b) Suppose that the residents decide to pass a law restricting the total number of hours that anyone is allowed to drive. How much driving should everyone be allowed if the objective is to maximize the utility of the typical resident?

(c) The same objective could be achieved with a tax on driving. How much would the tax have to be per hour of driving?

Answer

(a) Differentiate U w.r.t. d, we find that $d^* = 8$. Hence $h = 8000$ and utility of each resident is 56.

(b) If the residents of El Carburetor act collectively and internalize the externalities of driving, h becomes a choice variable and, in fact, $h = 1000d$. Rewriting the utility function gives $U = m + 16d - d^2 - 6d = m + 10d - d^2$. Differentiate U w.r.t. d, we find that $d^* = 5$. Hence, everyone drives less.

(c) The tax reduces the utility of a resident by reducing his consumption of Big Macs. With the tax, the resident's utility function becomes $U = \frac{(40-td)}{p_{bigmac}} + 16d - d^2 - \frac{6h}{1000}$, where $p_{bigmac} = 1$. Differentiate U w.r.t. d, we get $16 - 2d^* = t$. If we want $d^* = 5$, then $t = 6$.