PROBLEM SET 6 – Tutorial Week 9 (October 10–13)

Deadline: 11:59 p.m. two days before your tutorial. Please submit a PDF in groups of 2–3 within your tutorial group. On the first page, write your full names (as on the roster) in alphabetical order. Start each question on a new page. Name your PDF "PSet # – LastName LastName LastName," e.g., "PSet 6 – Banerjee Duflo Kremer." Points will be deducted for not adhering to the instructions.

QUESTION 1

Let us define an allocation $\left((x_1^A, x_2^A), (x_1^B, x_2^B)\right)$ as feasible if $x_1^A + x_1^B \leq \omega_1^A + \omega_1^B$ and $x_2^A + x_2^B \leq \omega_2^A + \omega_2^B$. Prove the First Fundamental Theorem of Welfare Economics using this definition of feasibility.

Hint: Consider a proof by contradiction, i.e., suppose the equilibrium allocation is not Pareto efficient. Do not just copy the proof in the lecture notes. Think!

QUESTION 2

There are 6 cups of espresso (x_1) and 6 bars of chocolate (x_2) in the house. Anton and Ego have identical preferences, $U^i = \alpha \log(x_1^i) + (1-\alpha)\log(x_2^i)$, where $i = A, E$ and $\alpha \in (0,1)$.

(a) Suppose the Target is where $U^A = U^E$. Find the ratio of equilibrium prices at the allocation where $U^A = U^E$.

(b) Determine the lump-sum transfer necessary to achieve the Target if Anton is initially endowed with $(4,1)$ and Ego with $(2,5)$. Set x_2 as the numeraire, i.e., assume $p_1 = p$ and $p_2 = 1$.

QUESTION 3

Auguste and Renata consume only cherries (x_1) and apples (x_2). Auguste is endowed with 3 pounds of cherries and 11 pounds of apples and Renata is endowed with 7 pounds of cherries and 5 pounds of apples. Suppose when the price of cherries (x_1) is $\$9$ a pound and the price of apples (x_2) is $\$3$ a pound, the aggregate gross demand (the sum of the gross demand of the two consumers) for cherries (x_1) is 8 pounds. Find the aggregate gross demand for apples (x_2).

QUESTION 4

Linguini and Colette consume only éclair (x_1) and profiterole (x_2). Linguini has utility function $U^L = x_1^L x_2^L$ and Colette has utility function $U^C = 2x_1^C x_2^C$. Linguini is endowed with 10 éclair (x_1) and 3 profiterole (x_2), while Colette is endowed with 20 éclair (x_1) and 9 profiterole (x_2).

(a) Derive the equation of the contract curve, i.e., find $x_2^L(x_1^L)$.

(b) Draw an Edgeworth box with x_1 on the horizontal axis and x_2 on the vertical axis. Position Linguini on the bottom left corner and Colette on the top right corner. Indicate the total number of units of x_1 and x_2. Label the endowment allocation. Draw the contract curve you found in (a).

Suppose the price of éclair (x_1) is $\$1$ and the price of profiterole (x_2) is $\$2$.

(c) Find each consumer's utility-maximizing basket.

(d) How much of each good does each consumer want to buy or sell? Are the markets in equilibrium at the given prices?

(e) Verify that Walras' law holds at these prices.

Now we will solve for the competitive equilibrium.

(f) Use the contract curve you derived in (a) to find the equilibrium price ratio, p_1/p_2.

(g) Set x_2 as the numeraire, i.e., assume $p_1 = p$ and $p_2 = 1$. Write each consumer's budget line equation given the equilibrium price ratio.

(h) Find the equilibrium allocation, $\left((x_1^L, x_2^L), (x_1^C, x_2^C)\right)$.