LECTURE 4

CAPM

The Capital Asset Pricing Model

EC3333 Financial Economics I

Learning Objectives

- Explain the theory of the capital asset pricing model (CAPM), and be able to construct and use the security market line.
  - State the main assumptions underlying the Capital Asset Pricing Model.
  - Explain why the CAPM implies that the market portfolio of all risky securities is the efficient portfolio.
  - Compare and contrast the capital market line with the security market line.
  - Define beta for an individual stock and for a portfolio.
  - Define alpha for an individual stock and the implications of CAPM on alpha.

The Efficient Portfolio

- To identify the efficient portfolio, we need for every risky asset
  - Expected returns
  - Volatilities
  - Correlations
- A lot of parameters to estimate and a difficult estimation task
- But CAPM allows us to identify the efficient portfolio without knowledge of the expected return of each security

The CAPM Assumptions

- Investors can buy and sell all securities at competitive market prices (without incurring taxes or transactions costs) and can borrow and lend at the risk-free interest rate.

- All investors are rational mean-variance optimizers
  - Investors hold only efficient portfolios of traded securities (i.e. portfolios that yield the maximum expected return for a given level of volatility).

- Investors have homogeneous expectations regarding the volatilities, correlations, and expected returns of securities.

Equilibrium and the Efficiency of the Market Portfolio

- Homogeneous expectations + Mean-variance optimizers + borrowing and lending rate at the risk-free interest rate
  → All investors face identical efficient frontier and CAL
  → All investors hold the same efficient portfolio of risky assets (P)

- In equilibrium, demand = supply
  → The efficient portfolio of risky assets that all investors hold must equal the market portfolio
  → All investors hold the market portfolio in equilibrium

The Market Portfolio

- Market Capitalization
  - The total market value of a firm's outstanding shares

$$MV_i = (\text{ Number of Shares of } i \text{ Outstanding }) \times (\text{Price of } i \text{ per Share})$$

- Value-Weighted Portfolio
  - A portfolio in which each security is held in proportion to its market capitalization

$$x_i = \frac{\text{Market Value of } i}{\text{Total Market Value of All Securities}} = \frac{MV_i}{\sum_j MV_j}$$

CAPM: Resulting Equilibrium Conditions

- When the CAPM assumptions hold, all investors will hold combinations of only two portfolios: the risk- free asset and the market portfolio
  - The two mutual fund theorem
    - Mutual funds are financial intermediaries that sell shares to savers and use their funds to buy diversified pools of assets and manage them
  - When the tangent line goes through the market portfolio, it is called the capital market line (CML)

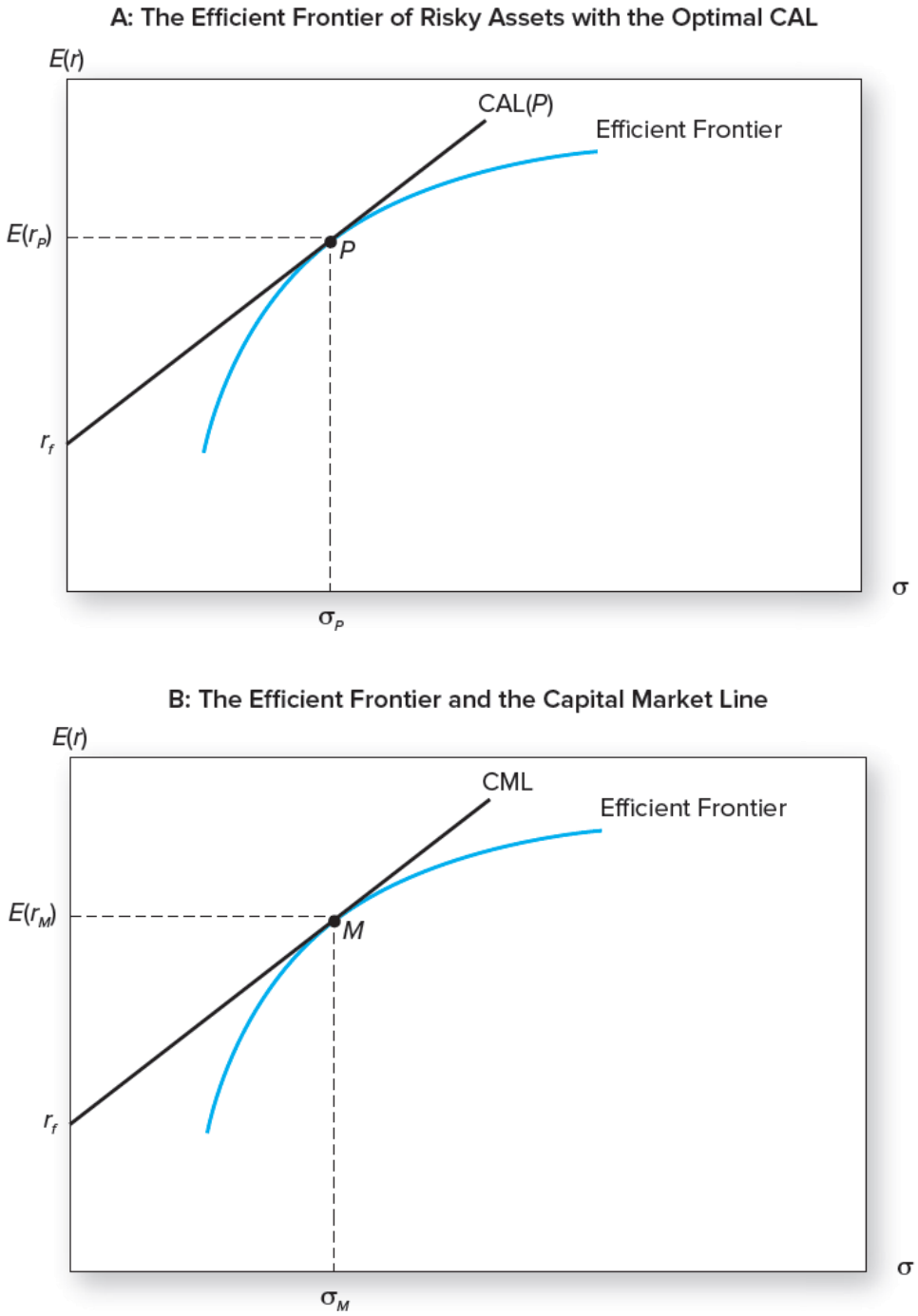


Figure 9.1 Capital allocation line and the capital market line (from adopted text, Bodie, Kane and Marcus, Investments, McGraw Hill, 12e)

The Capital Market Line (CML)

- The CML is a capital allocation line formed by investment in two passive portfolios:
  1. Risk-free short-term T-bills (or a money market fund)
  2. Fund of common stocks that mimics a broad market index
- Note that the CML is CAL with the optimal risky portfolio replaced by the market portfolio

Passive Strategies

- Passive strategy avoids any direct or indirect security analysis
- A natural candidate for a passively held risky asset would be a well-diversified portfolio of common stocks such as the S&P 500

Market Indexes

- Report the value of a particular portfolio of securities
  - S&P 500
  - Wilshire 5000
  - Dow Jones Industrial Average (DJIA)
  - Nasdaq Composite

The Capital Market Line (CML)

- Suppose
  - x = portfolio weight on the market portfolio, M
  - $(1 - x)$ = portfolio weight on the risk-free asset, F

- The expected return of a portfolio on the CML:

$$E(r_e) = (1 - x)r_f + xE(r_M)$$
$$E(r_e) = r_f + x[E(r_M) - r_f]$$

- With standard deviation:

$$\sigma_e = x\sigma_M$$

The Capital Market Line (CML)

- Rearrange and substitute $x = \sigma_e/\sigma_M$

$$E(r_e) = r_f + \frac{[E(r_M) - r_f]}{\sigma_M}\sigma_e$$

- This equation is called the capital market line (CML)

$$\text{Slope} = \frac{E(r_M) - r_f}{\sigma_M}$$

The Return and Risk For Individual Securities: Market Risk and Beta

- Given an efficient market portfolio, the expected return of an investment is:

$$E[R_i] = r_i = r_f + \underbrace{\beta_i^{\text{Mkt}} \left(E[R_{Mkt}] - r_f\right)}_{\text{Risk premium for security } i}$$

Beta

$$\beta_i^{Mkt} = \frac{\mathbf{Cov}(R_i, R_{Mkt})}{\mathbf{Var}(R_{Mkt})}$$

Covariance with the market

Variance of the market

Estimating Beta from Historical Returns

- Beta = the expected percent change in the excess return of the security for a 1% change in the excess return of the market portfolio
  - Consider Cisco Systems stock and how it changes with the market portfolio
  - Cisco tends to be up when the market is up, vice versa
  - A 10% change in the market's return corresponds to about a 15% change in Cisco's return
  - Thus, Cisco's return moves about one and a half times that of the overall market, so Cisco's beta is about 1.5
- Beta corresponds to the slope of the best-fitting line in the plot of the security's excess returns versus the market excess return

Figure 12.1 Monthly Returns for Cisco Stock and for the S&P 500, 2000-2017
(from adopted text, Berk and DeMarzo, Corporate Finance, Pearson, 5e)

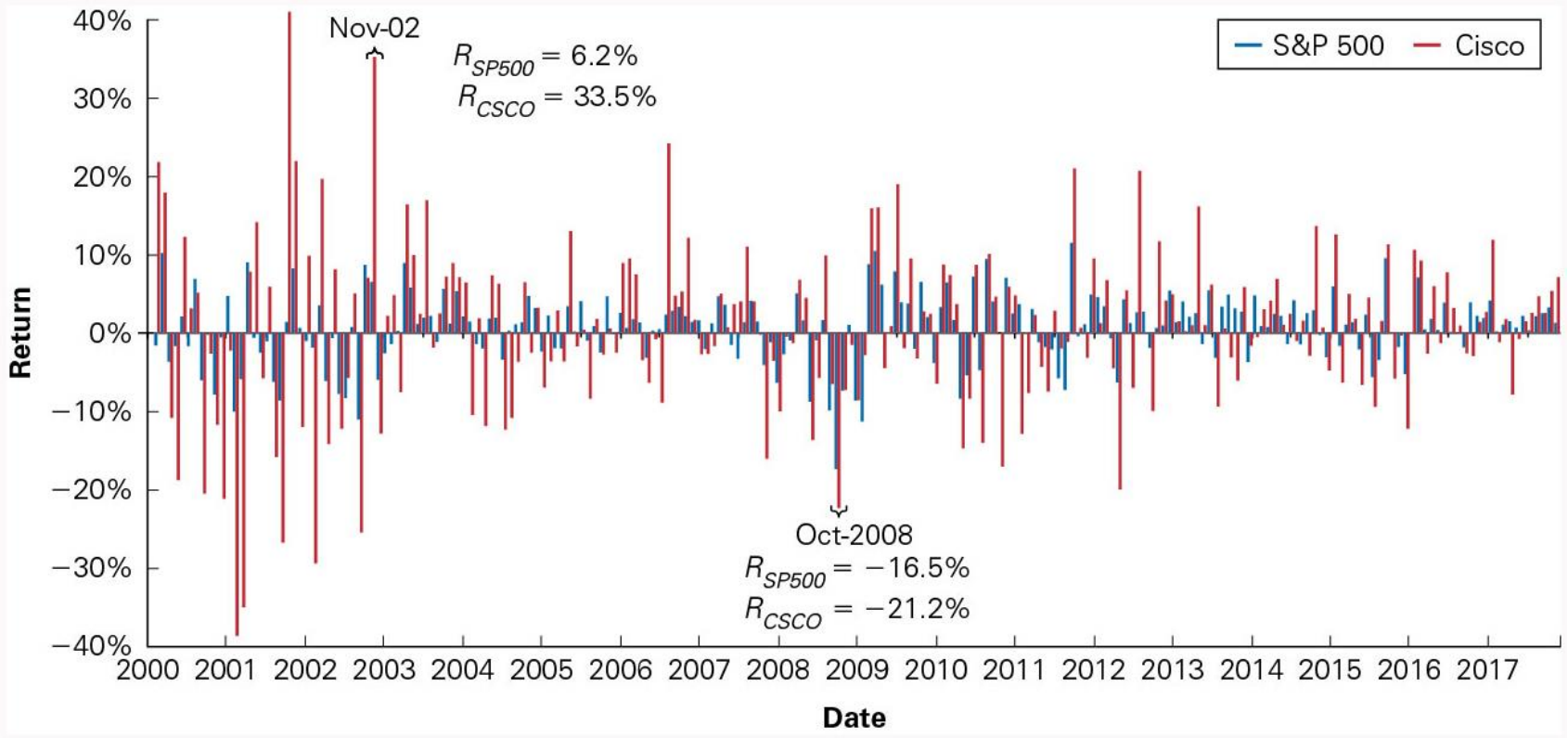
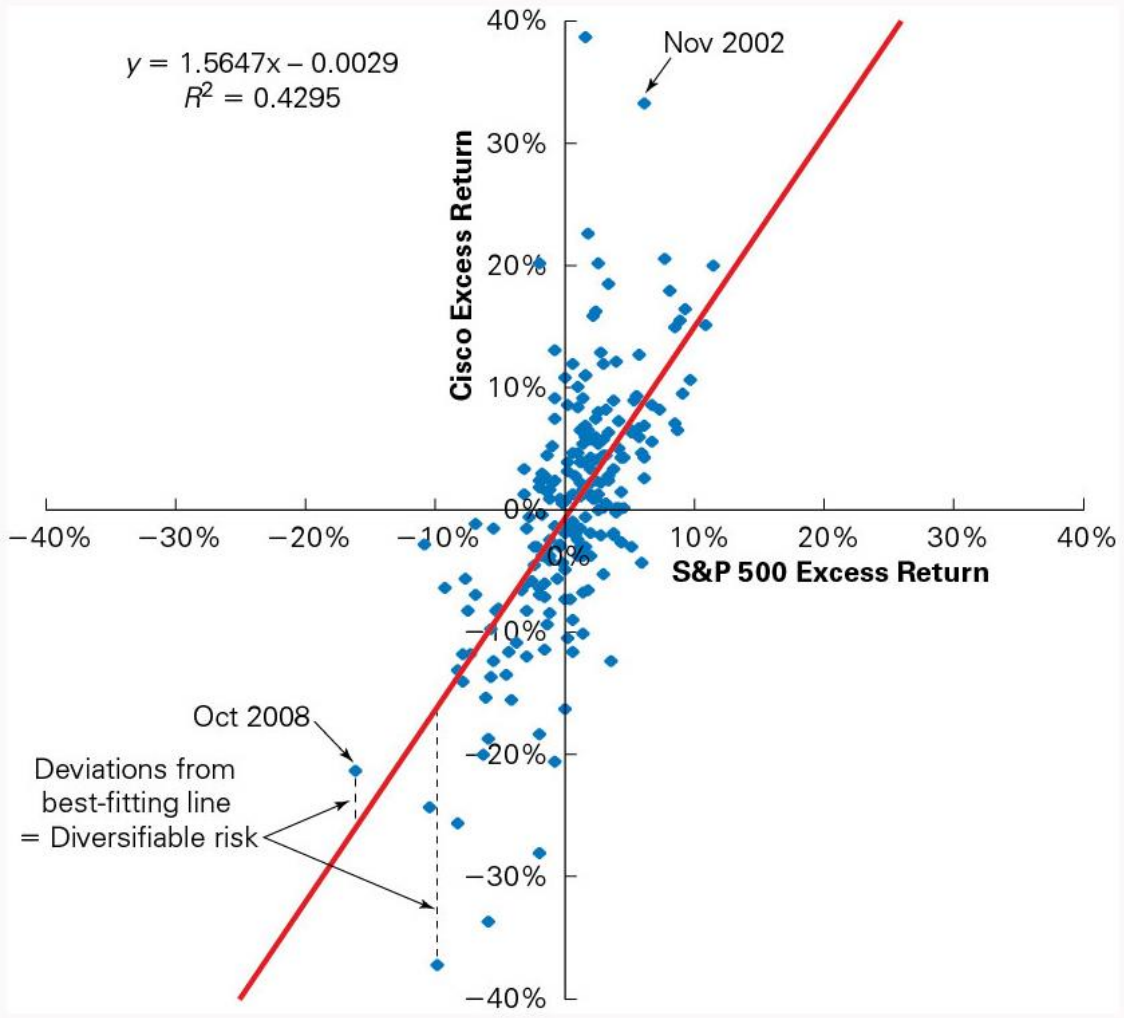


Figure 12.2 Scatterplot of Monthly Excess Returns for Cisco Versus the S&P 500, 2000-2017
(from adopted text, Berk and DeMarzo, Corporate Finance, Pearson, 5e)



Estimating Beta from Historical Returns

$$(R_i - r_f) = \alpha_i + \beta_i(R_M - r_f) + \varepsilon_i$$

- Using the monthly returns for Cisco during 2000–2017, estimate
  - β =1.56 with a 95% confidence interval of [1.3, 1.8]
  - α = –0.29% with a standard error of 0.5% (statistically insignificant)

Table 10.6 Betas with Respect to the S&P 500 for Individual Stocks (Based on Monthly Data for 2013–2018) (1 of 4)
(from adopted text, Berk and DeMarzo, Corporate Finance, Pearson, 5e)

Company	Ticker	Industry	Equity Beta
Edison International	EIX	Utilities	0.15
Tyson Foods	TSN	Packaged Foods	0.19
Newmont Mining	NEM	Gold	0.31
The Hershey Company	HSY	Packaged Foods	0.33
Clorox	CLX	Household Products	0.34
Walmart	WMT	Superstores	0.55
Procter & Gamble	PG	Household Products	0.55
McDonald's	MCD	Restaurants	0.63
Nike	NKE	Footwear	0.64
Pepsico	PEP	Soft Drinks	0.68
Williams-Sonoma	WSM	Home Furnishing Retail	0.71
Coca-Cola	KO	Soft Drinks	0.73
Johnson & Johnson	JNJ	Pharmaceuticals	0.73

Table 10.6 Betas with Respect to the S&P 500 for Individual Stocks (Based on Monthly Data for 2013–2018) (2 of 4)
(from adopted text, Berk and DeMarzo, Corporate Finance, Pearson, 5e)

Company	Ticker	Industry	Equity Beta
Macy's	M	Department Stores	0.75
Molson Coors Brewing	TAP	Brewers	0.78
Starbucks	SBUX	Restaurants	0.80
Foot Locker	FL	Apparel Retail	0.83
Harley-Davidson	HOG	Motorcycle Manufacturers	0.88
Pfizer	PFE	Pharmaceuticals	0.89
Sprouts Farmers Market	SFM	Food Retail	0.89
Philip Morris	PM	Tobacco	0.89
Intel	INTC	Semiconductors	0.93
Netflix	NFLX	Internet Retail	0.98
Kroger	KR	Food Retail	1.04
Microsoft	MSFT	Systems Software	1.04
Alphabet	GOOGL	Internet Software and Services	1.06

Table 10.6 Betas with Respect to the S&P 500 for Individual Stocks (Based on Monthly Data for 2013–2018) (3 of 4)
(from adopted text, Berk and DeMarzo, Corporate Finance, Pearson, 5e)

Company	Ticker	Industry	Equity Beta
eBay	EBAY	Internet Software and Services	1.11
Cisco Systems	CSCO	Communications Equipment	1.14
Southwest Airlines	LUV	Airlines	1.15
Apple	AAPL	Computer Hardware	1.24
salesforce.com	CRM	Application Software	1.25
Walt Disney	DIS	Movies and Entertainment	1.29
Marriott International	MAR	Hotels and Resorts	1.32
Amgen	AMGN	Biotechnology	1.37
Toll Brothers	TOL	Homebuilding	1.37
Wynn Resorts Ltd.	WYNN	Casinos and Gaming	1.38
Parker-Hannifin	PH	Industrial Machinery	1.43
Prudential Financial	PRU	Insurance	1.51
Nucor	NUE	Steel	1.57

Table 10.6 Betas with Respect to the S&P 500 for Individual Stocks (Based on Monthly Data for 2013–2018) (4 of 4)
(from adopted text, Berk and DeMarzo, Corporate Finance, Pearson, 5e)

Company	Ticker	Industry	Equity Beta
Amazon.com	AMZN	Internet Retail	1.62
General Motors	GM	Automobile Manufacturers	1.64
Autodesk	ADSK	Application Software	1.72
Hewlett-Packard	HPQ	Computer Hardware	1.77
Tiffany & Co.	TIF	Apparel and Luxury Goods	1.77
Brunswick	BC	Leisure Products	1.84
Chesapeake Energy	CHK	Oil and Gas Exploration	1.85
Netgear	NTGR	Communications Equipment	1.94
Ethan Allen Interiors	ETH	Home Furnishings	2.04
Trimble	TRMB	Electronic Equipment	2.44
Advanced Micro Devices	AMD	Semiconductors	2.83

The Security Market Line

- The security market line (SML) is graphed as the line through the risk-free investment and the market
  - According to the CAPM, if the expected return and beta for individual securities are plotted, they should all fall along the SML
- There is no clear relationship between an individual stock's volatility (= total risk) and its expected return
- Expected return is determined by only that part of an individual stock's volatility that cannot be diversified away

$$E[R_i] = r_f + \beta_i^{Mkt}(E[R_{Mkt}] - r_f)$$

Figure 9.2 The Security Market Line
(from adopted text, Bodie, Kane and Marcus, Investments, McGraw Hill, 12e)

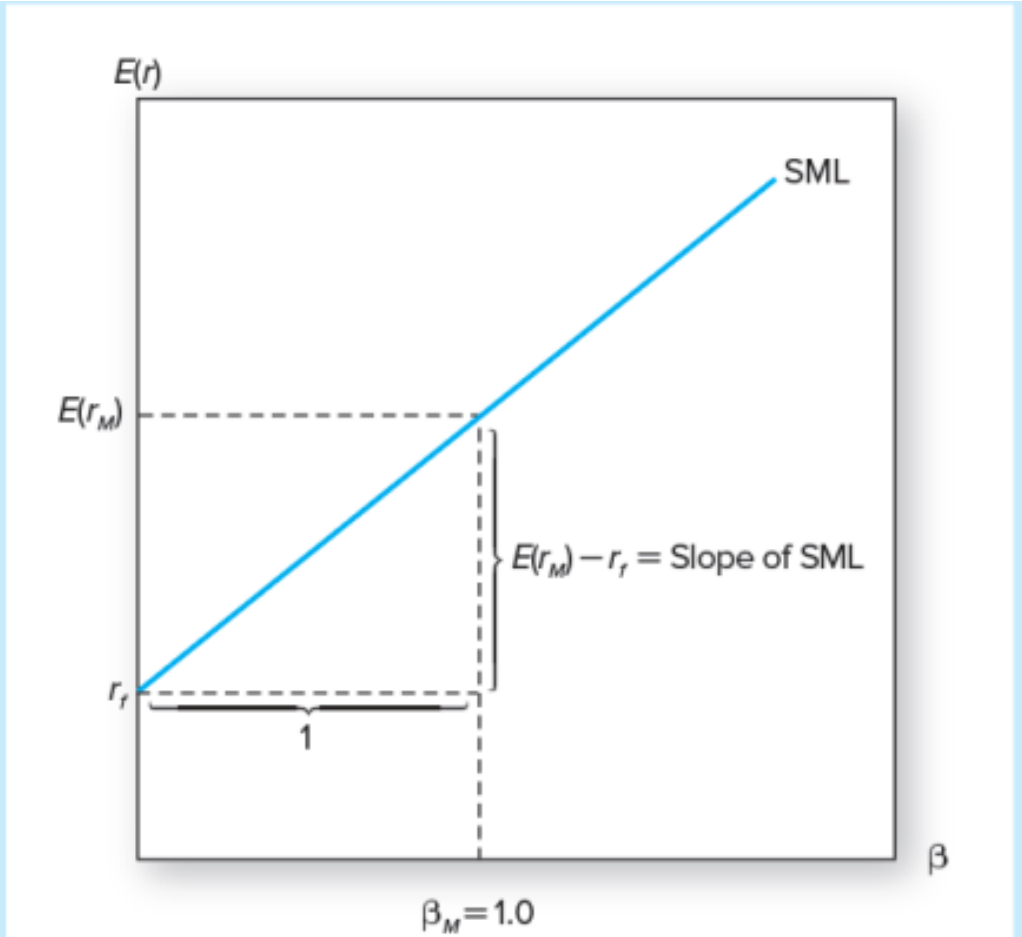


Figure 9.2 The security market line

Beta of a Portfolio

$$\beta_P = \frac{\mathrm{Cov}(R_p, R_M)}{\mathrm{Var}(R_M)}$$

$$= \frac{\mathrm{Cov}(\sum_i x_i R_i, R_M)}{\mathrm{Var}(R_M)}$$

$$= \sum_i x_i \frac{\mathrm{Cov}(R_i, R_M)}{\mathrm{Var}(R_M)}$$

$$\beta_P = \sum_i x_i \beta_i$$

- x_i is the portfolio weight on security i

SML for a Portfolio of Securities

- CAPM also holds for a portfolio of securities:

$$E(R_P) = \sum_i x_i E(R_i)$$

$$E(R_P) = \sum_i x_i (r_f + \beta_i [E(R_{Mkt}) - r_f])$$

$$E(R_P) = r_f + \beta_P [E(R_{Mkt}) - r_f]$$

$$\text{where } \beta_P = \sum_i x_i \beta_i$$

The SML and Alpha

- Alpha is the difference between a stock's expected return and its required return according to the SML

$$\alpha_i = \underbrace{E(R_i)}_{\text{Expected return}} - \underbrace{(r_f + \beta_i[E(R_M) - r_f])}_{\text{Required return according to SML}}$$

$$E[R_i] = \underbrace{r_f + \beta_i(E[R_M] - r_f)}_{\text{Expected return for } i \text{ from the SML}} + \underbrace{\alpha_i}_{\text{Distance above / below the SML}}$$

- Stocks with non-zero alpha's do not lie on the SML
- alpha ≠ 0 → market portfolio is inefficient

Figure 9.3 The SML and a Positive-Alpha Stock
(from adopted text, Bodie, Kane and Marcus, Investments, McGraw Hill, 12e)

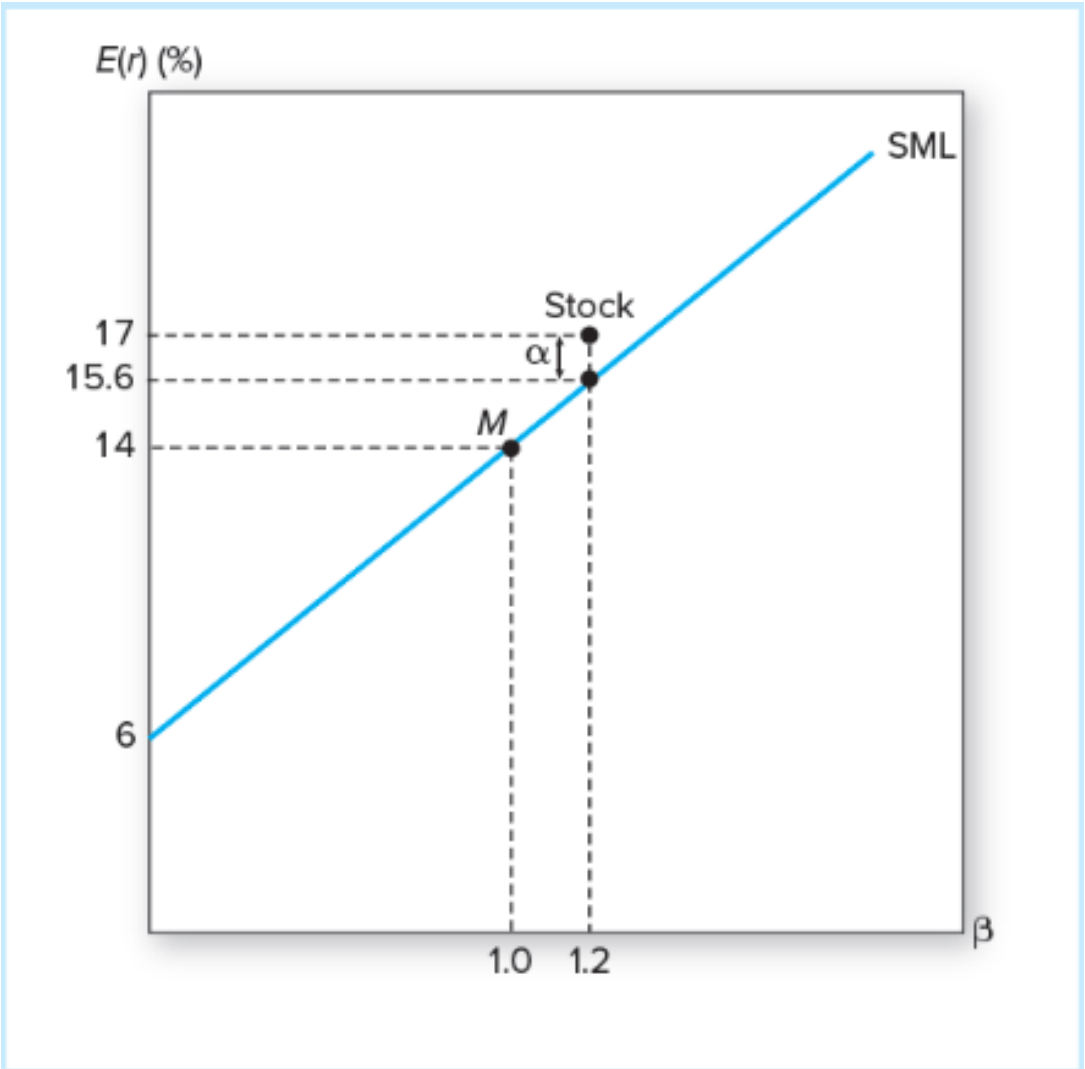


Figure 9.3 The SML and a positive-alpha stock

The SML and Alpha

$$(R_i - r_f) = \alpha_i + \beta_i(R_M - r_f) + \varepsilon_i$$

- Recall that using the monthly returns for Cisco during 2000– 2017, estimate α = –0.29% with a standard error of 0.5% (statistically insignificant)
- Because $E[\varepsilon_i] = 0$, take expectation, we get

$$\alpha_i = \underbrace{E(R_i)}_{\text{Expected return}} - \underbrace{(r_f + \beta_i[E(R_M) - r_f])}_{\text{Required return according to SML}}$$

The SML and Alpha

$$\alpha_i = \underbrace{E(R_i)}_{\text{Expected return}} - \underbrace{(r_f + \beta_i[E(R_M) - r_f])}_{\text{Required return according to SML}}$$

- Thus, α_i represents a risk-adjusted performance measure for the historical returns
  - CAPM $\rightarrow$ α_i should not be significantly different from zero
- Caveats:
  - Difficult to estimate with accuracy without a very long data series
  - The alphas for individual stocks have very little persistence
  - During 1996-2000, Cisco's return had an alpha of 3% per month
  - This positive alpha did not forecast superior future performance