Practice Problem Set 5

Game Theory (C.29)

Question 5.1

Evangeline and Gabriel met at Orientation. They want desperately to meet each other again, but they forgot to exchange names or phone numbers when they met the first time. There are two possible strategies available for each of them. These are *Go to the Big Party* or *Stay Home and Study*. They will surely meet if they both go to the party, and they will surely not otherwise. The payoff to meeting is 1,000 for each of them. The payoff to not meeting is zero for both of them. The payoffs are described by the matrix below.

		Gabriel	
		Go to Party	Stay Home
Evangeline	Go to Party	$1000, 1000$	$0, 0$
	Stay Home	$0, 0$	$0, 0$

(i) Is there an equilibrium with strictly dominant strategies in this game? (Note: A strategy is strictly dominant if, regardless of what any other players do, the strategy earns a player a strictly higher payoff than other strategies.)

(ii) Find all of the pure-strategy Nash equilibria for this game. Do any of the pure Nash equilibria that you found seem more reasonable than others? Why or why not?

Let us change the game a little bit. Evangeline and Gabriel are still desperate to find each other. But now there are two parties that they can go to. There is a little party at which they would be sure to meet if they both went there and a huge party at which they might never see each other. The expected payoff to each of them is 1,000 if they both go to the little party. Since there is only a 50-50 chance that they would find each other at the huge party, the expected payoff to each of them is only 500. If they go to different parties, the payoff to both of them is zero. The payoff matrix for this game is:

		Gabriel	
		Little Party	Big Party
Evangeline	Little Party	$1000, 1000$	$0, 0$
	Big Party	$0, 0$	$500, 500$

(iii) Does this new game have an equilibrium with strictly dominant strategies?

(iv) Find all the Nash equilibria. Is there one equilibrium that is Pareto superior to others? Does the concept of Nash equilibrium predict that Evangeline and Gabriel will converge to this equilibrium?

Answer

(i) There is no equilibrium with strictly dominant strategies. For example, if Gabriel stays home, Evangeline would be indifferent between going to the party and staying home (0 is not strictly larger than 0).

(ii) There are two: (Go, Go) and (Stay, Stay).

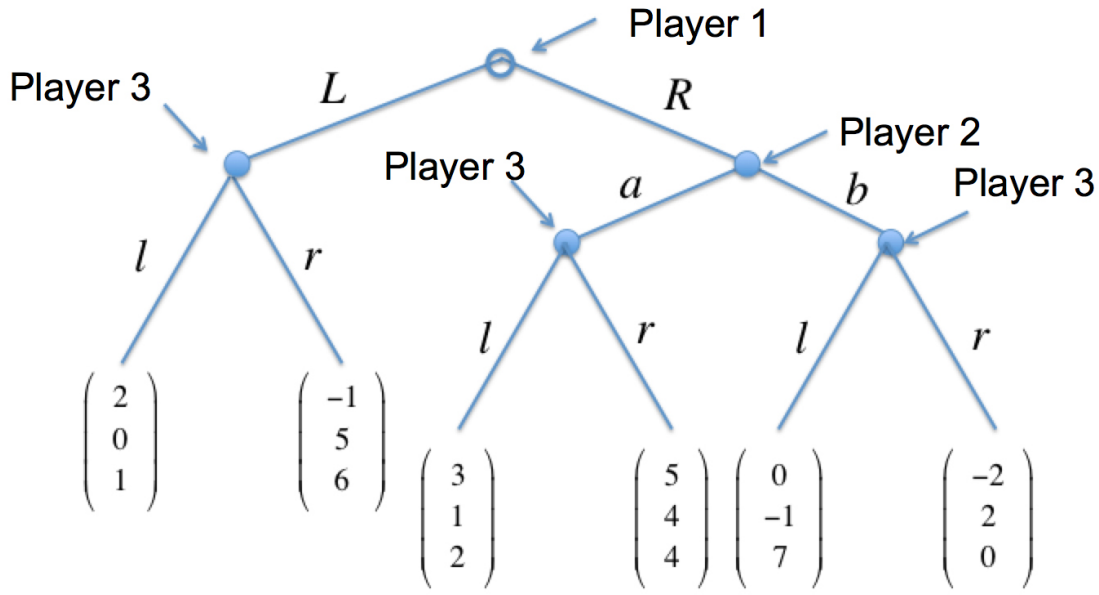
Although (Stay, Stay) is a Nash equilibrium, it seems a silly one. If either player believes that there is any chance that the other will go to the party, he or she will also go. Here, the take-away message is that the concept of Nash equilibrium is sometimes useful and sometimes less so. As a student of economics, it is important for you to be able to judge when the concept is useful and when it is not. But to do so, you have to be able to identify the Nash equilibriums before making a judgment on whether an equilibrium make sense (or not).

(iii) No.

(iv) Both go to the little party, or both go to the big party. The first equilibrium is Pareto superior to the second one, but the concept of Nash equilibrium does not predict that Evangeline and Gabriel will select the Pareto superior outcome.

Question 5.2

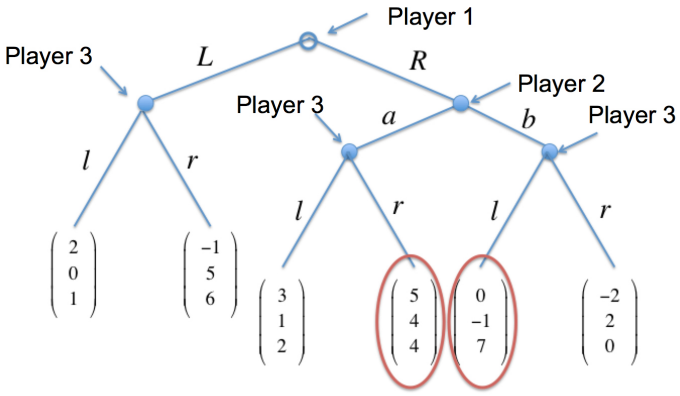
Solve the game represented by the following game tree using backward induction:



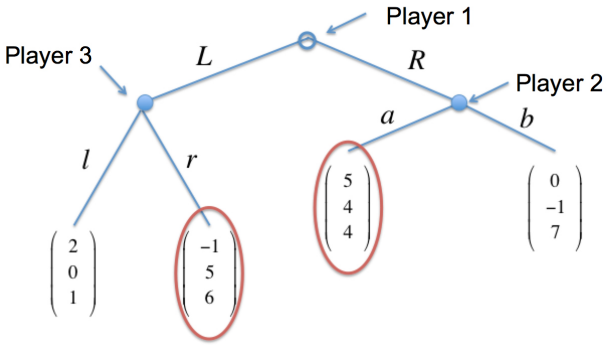
Answer

By backward induction, Player 1 should choose R, Player 2 should choose a if Player 1 chooses R; Player 3 should choose r if Player 1 chooses R and Player 2 chooses a.

Last stage



Second last stage



First stage