PROBLEM SET 3 — Tutorial Week 5 (September 5-8)

QUESTION 1

Dory consumes only plankton (x) and algae (y), and her preferences are well-behaved. Dory regards algae (y) as a Giffen good. Suppose the price of algae (p_y) falls.

(a) Is the substitution effect positive or negative? Is the income effect positive or negative? Is the total effect positive or negative?
   Note: "Effect" refers to the change in quantity demanded.

(b) Draw a Hicks-style graph, showing the substitution and income effects.

QUESTION 2

Marlin's utility function for plankton (x) and algae (y) is $U(x, y) = (x - 4)y$ where $x \geq 4$ and $y \geq 0$. The price of plankton (x) is p_x, the price of algae (y) is p_y, and Marlin's income is $M > 4p_x$. *(Note: x and y need not be integers.)*

(a) Write down the utility maximization problem. Derive the demand functions for plankton (x) and for algae (y).

(b) How does the consumption of algae (y) change with the price of algae (p_y)? I.e., does the consumption of algae (y) increase, decrease, or remain constant? What can you infer about algae (y)?
   Hint: Use the demand function for algae (y) that you derived in (a).

(c) How does the consumption of algae (y) change with the price of plankton (p_x)? What can you infer about plankton (x) and algae (y)?
   Hint: Use the demand function for algae (y) that you derived in (a).

(d) Suppose $p_x = 1$ and $p_y = 2$. Draw the Engel curve for plankton (x). Is plankton (x) a normal good?

QUESTION 3

Squirt consumes molluscs (x) and jellyfish (y); his utility function is $U(x,y) = min\{x, 3y\}$. Suppose Squirt has an income of $24, and the price of molluscs (p_x) is $1 and the price of jellyfish (p_y) is $5.

(a) Find Squirt's optimal basket.
    Hint: Graph the budget line and several indifference curves.

(b) Suppose the price of jellyfish (p_y) falls to $3. Calculate the substitution and income effects (with respect to jellyfish (y)).
    Hint: Graph the hypothetical budget line and find the intermediate basket.

QUESTION 4

In the Slutsky decomposition, the hypothetical budget line is parallel to the new budget line (just like the Hicks decomposition) but passes through the original optimal choice. We shall go through the Slutsky decomposition in this question.

Nemo buys only plankton (x) and algae (y). His utility function is $U(x,y) = xy$. He gets a weekly allowance of $100. Both goods are priced at $10 each.

(a) What is his **optimal basket**? Call this basket A. Calculate his utility from basket A. Draw the budget line and the indifference curve going through basket A. Indicate basket A on the graph.

(b) Suppose p_y increases to $20. What is his **new optimal basket**? Call this basket C. Calculate his utility from basket C. On the graph in (a), draw the new budget line ($p_x = $10, p_y = 20) and the indifference curve going through basket C. Indicate basket C on the graph.

(c) In order for him to afford basket A at $p_x = $10, p_y = 20, how much would his allowance have to increase by? On the graph in (a), draw the **hypothetical budget line** (where $p_x = $10, p_y = 20), which goes through basket A.

(d) Nemo manages to wheedle his father into increasing his allowance by the amount calculated in (c). Now with his increased allowance, and at $p_x = $10, p_y = 20, what will his optimal basket be? Call this basket B. Calculate his utility from the **intermediate basket**, basket B.

(e) On the graph in (a), indicate basket B, and draw the indifference curve going through basket B.

(f) Calculate the substitution effect (from basket A to basket B) and the income effect (from basket B to basket C) for plankton (x) and for algae (y).

(g) How is the Slutsky decomposition different from the Hicks decomposition?