LECTURE 3

CONSUMER CHOICE
REVEALED PREFERENCE
INDIVIDUAL DEMAND

Where are we?

- Preference
- Budget constraint
- Consumer's optimal choice
  - The tangency case
  - Other cases
- Revealed preference
  - What if we observe choice but not preference?
- Demand function
  - How does the optimal choice change with prices and income?

Part 1

Consumer Choice

Optimal Choice

- Consumer's optimal choice
  - On the budget line
  - On the highest indifference curve
- The optimal choice is the point of tangency
  - Tangency condition + budget line
  - Or the Lagrangian method
- Optimal basket is not always a point of tangency

What is the optimal basket?

- Suppose the consumer has utility function

$$U(F,C) = FC + 10F$$

- Price of food is 1, price of clothing is 2, consumer's income is 10
- The utility maximization problem is

$$\max_{F,C} \quad FC + 10F$$

$$s.t. \quad F + 2C = 10$$

What is the optimal basket? Cont'

- The tangency condition is

$$\frac{C+10}{F}=\frac{1}{2}$$

- The budget line is

$$F+2C=10$$

- The solution is *F*=15, *C*=-2.5
- Is it the optimal basket?

Rewriting the Utility Maximization Problem

- In fact, there should be two more constraints to any utility maximization problem
  - The consumption of each good cannot be negative
- The true utility maximization problem is

$$\max_{F,C} \quad FC + 10F$$

$$\begin{aligned} & F + 2C = 10 \\ s.t. \quad & F \geq 0 \\ & C \geq 0 \end{aligned}$$

Solving the Problem

- How to solve this problem?
- Assuming the two constraints are satisfied, we just need to solve

$$\max_{F,C} \quad FC + 10F$$

$$s.t. \quad F + 2C = 10$$

- Check if the solution indeed satisfies F>=0 and C>=0
  - If yes, we are done
- The solution F=15, C=-2.5 violates C>=0
  - This means our assumption is wrong

Solving the Problem Cont'

- The consumer wants -2.5 units of clothing
  - As C=-2.5 is not possible, C=0 is the best/closest we can get
- Thus the solution is F=10, C=0
- In this case the constraint C>=0 *binds*
  - That is, it holds with equality, C=0
- When there are inequality constraints, the constraints may or may not bind
  - In this example, the constraint C>=0 binds while the constraint F>=0 does not bind

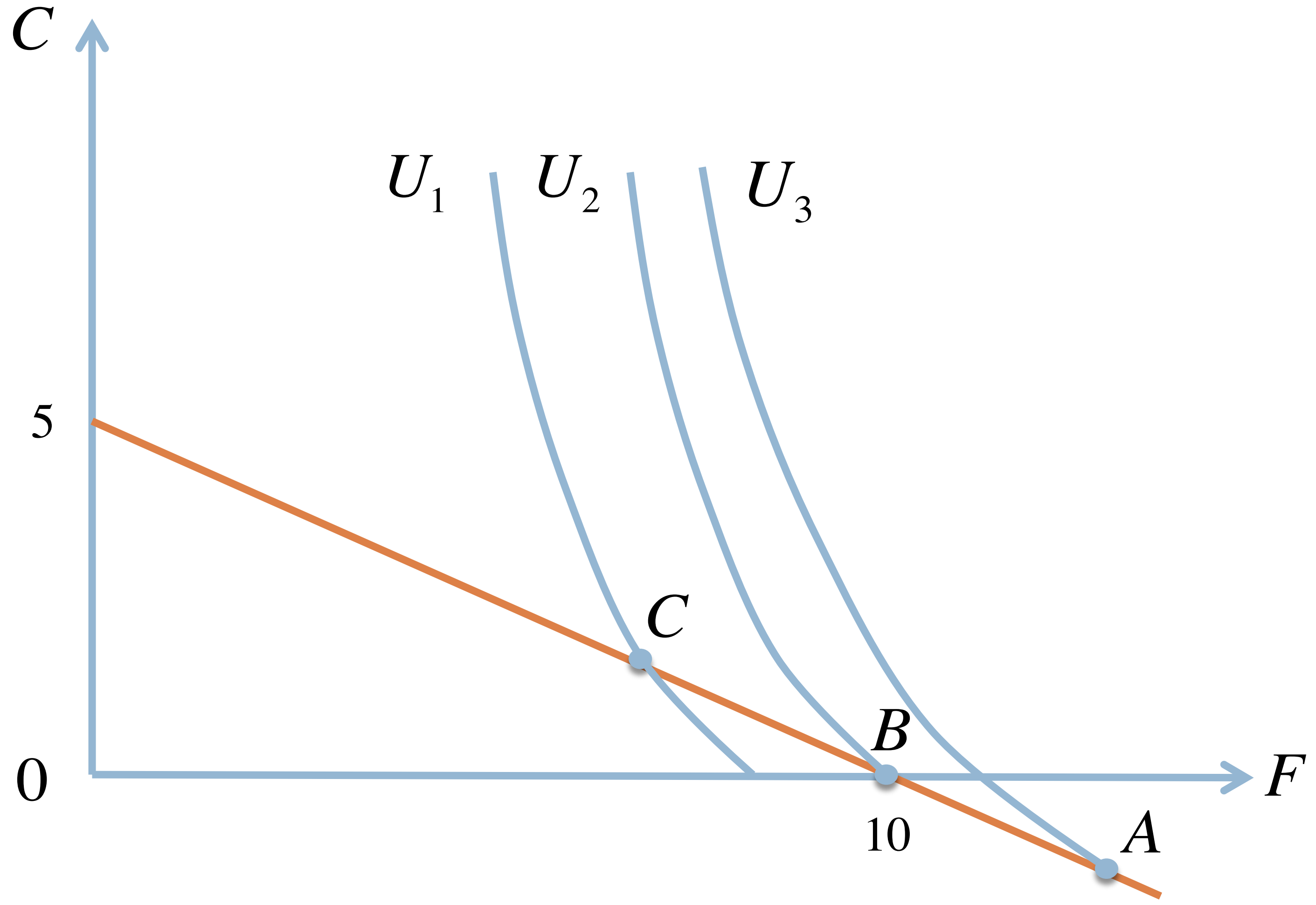
How do we know F=10, C=0 is optimal?

- At this basket, consumer spends all the money on food
- Comparing the per dollar marginal utilities at this point

$$\frac{MU_F}{P_F} = \frac{C+10}{P_F} = 10 > \frac{MU_C}{P_C} = \frac{F}{P_C} = \frac{10}{2} = 5$$

- If possible, consumer wants to buy more F and less C to increase utility
- But consumption of C is already 0

The Scenario in Graph



Corner Solution

- At optimal basket, it is *not* always true that both (all) goods are consumed
- Definition 3.1 *Corner solution* is an optimal basket at which the consumption of at least one good is 0
  - Optimal basket either on the horizontal or vertical axis
- Definition 3.2 An optimal basket in which both goods are consumed is an *interior solution*
- At corner solutions
  - Indifference curve may not be tangent to the budget line

Part 2

Revealed Preference

What is revealed preference?

- What we have been doing so far
  - Given preference (indifference curves/utility functions)
  - Given budget constraint
  - We can find consumer's optimal choice
- Can we go the other way round?
  - Given budget constraint
  - Given consumer's optimal choice
  - Can we get any information on preference?
- *Revealed preference* is the analysis that enable us to infer preference based on observed prices and choices

Strictly Preferred vs. Weakly Preferred

- A is *strictly preferred* to B

$$A \succ B$$

- Definition 3.3 A is *weakly preferred* to B if
  - Either

$$A \succ B$$

  - Or

$$A \approx B$$

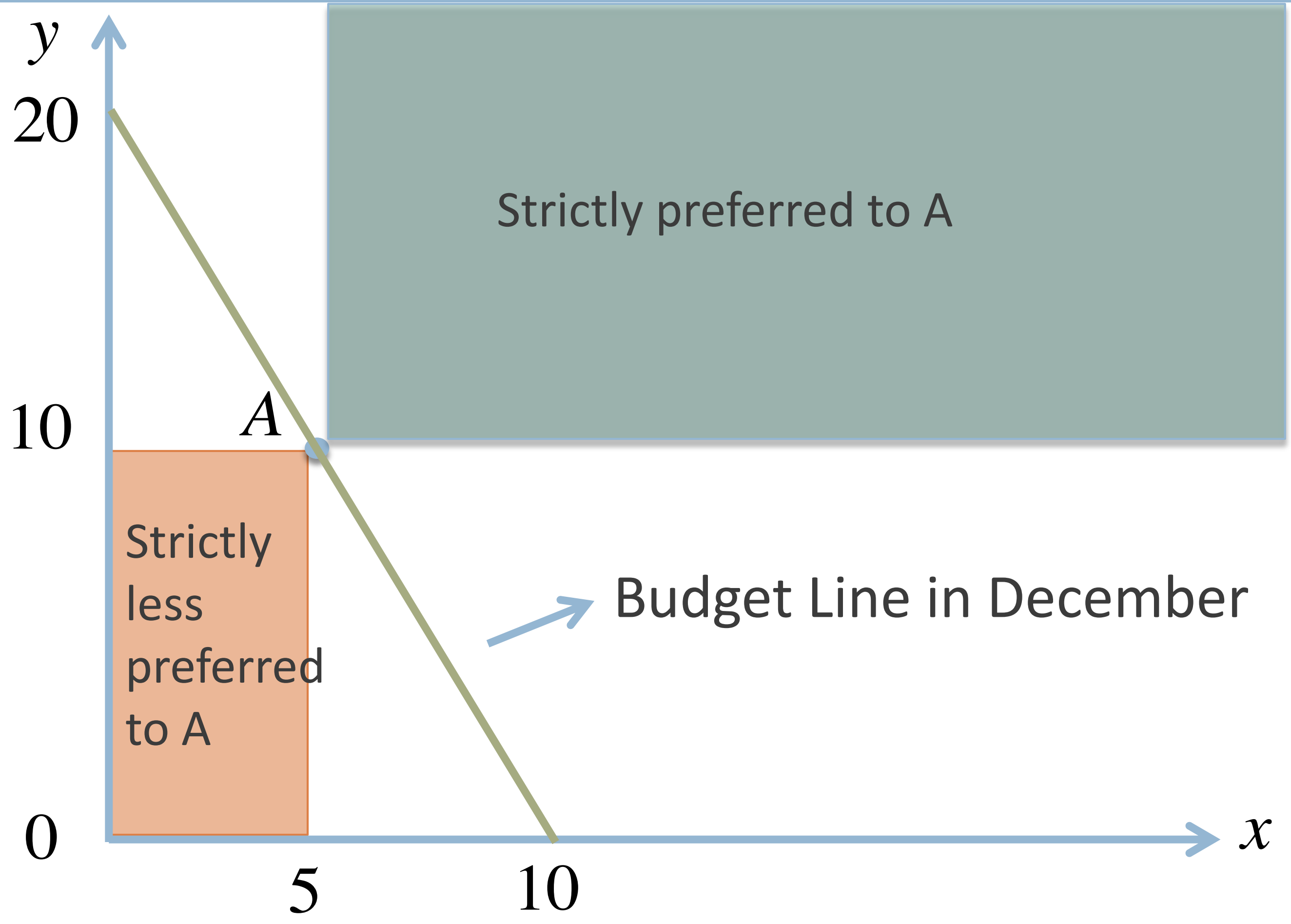
  - We use the notation

$$A \geq B$$

From Choice to Preference

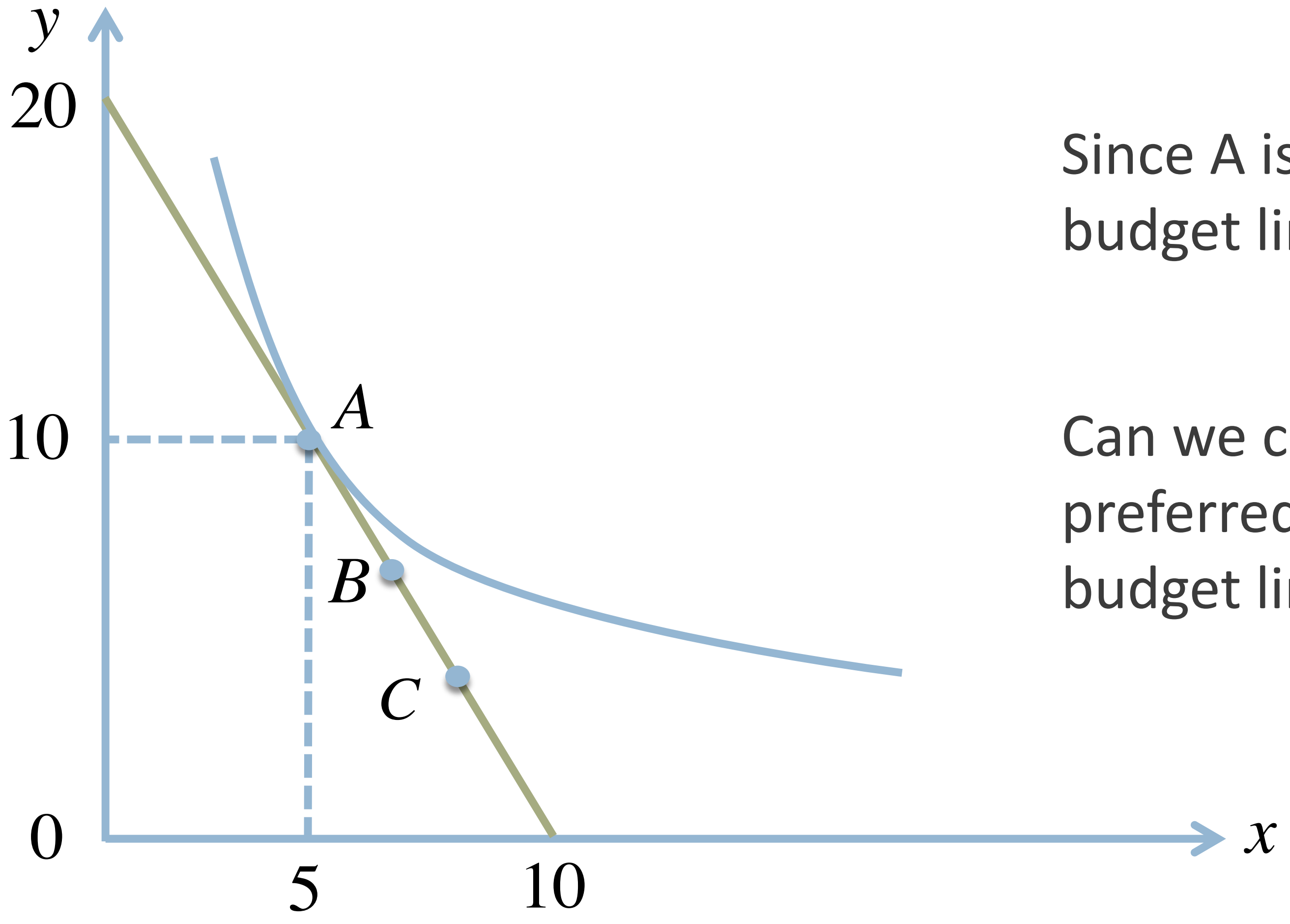
- Suppose we observe the budget constraint of a consumer
- We also know the optimal basket chosen given the budget constraint
- But we do not know his preference
  - We know his preference satisfies the three assumptions
  - We also know his preference does not change with prices or income
- Our goal
  - To infer preference – how he ranks different baskets

What we already know from “more is better”



Suppose A is the optimal choice in December

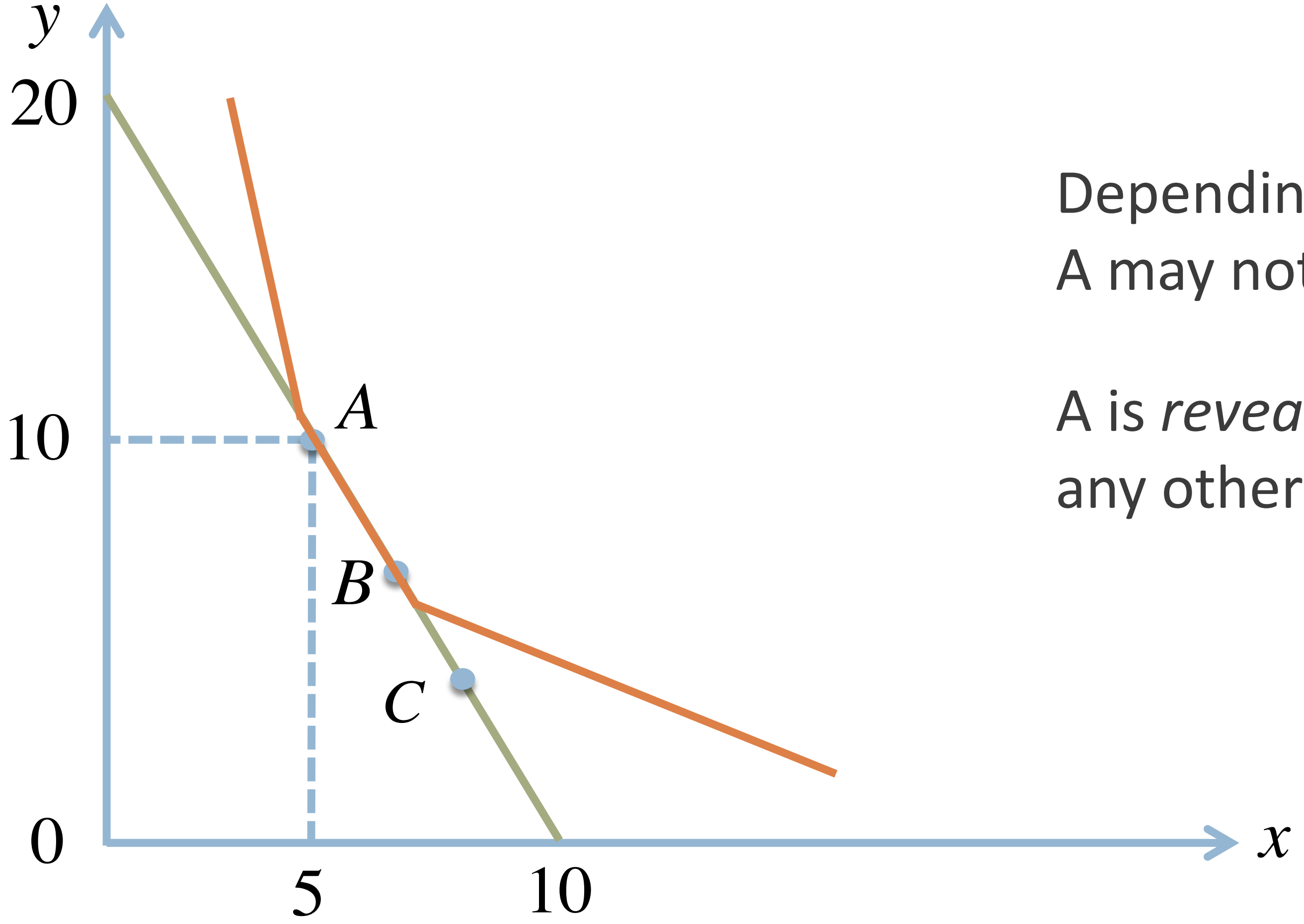
A vs. Other Points on the Budget Line



Since A is optimal, no other point on the budget line is strictly preferred to A

Can we conclude that A is strictly preferred to any other basket on the budget line?

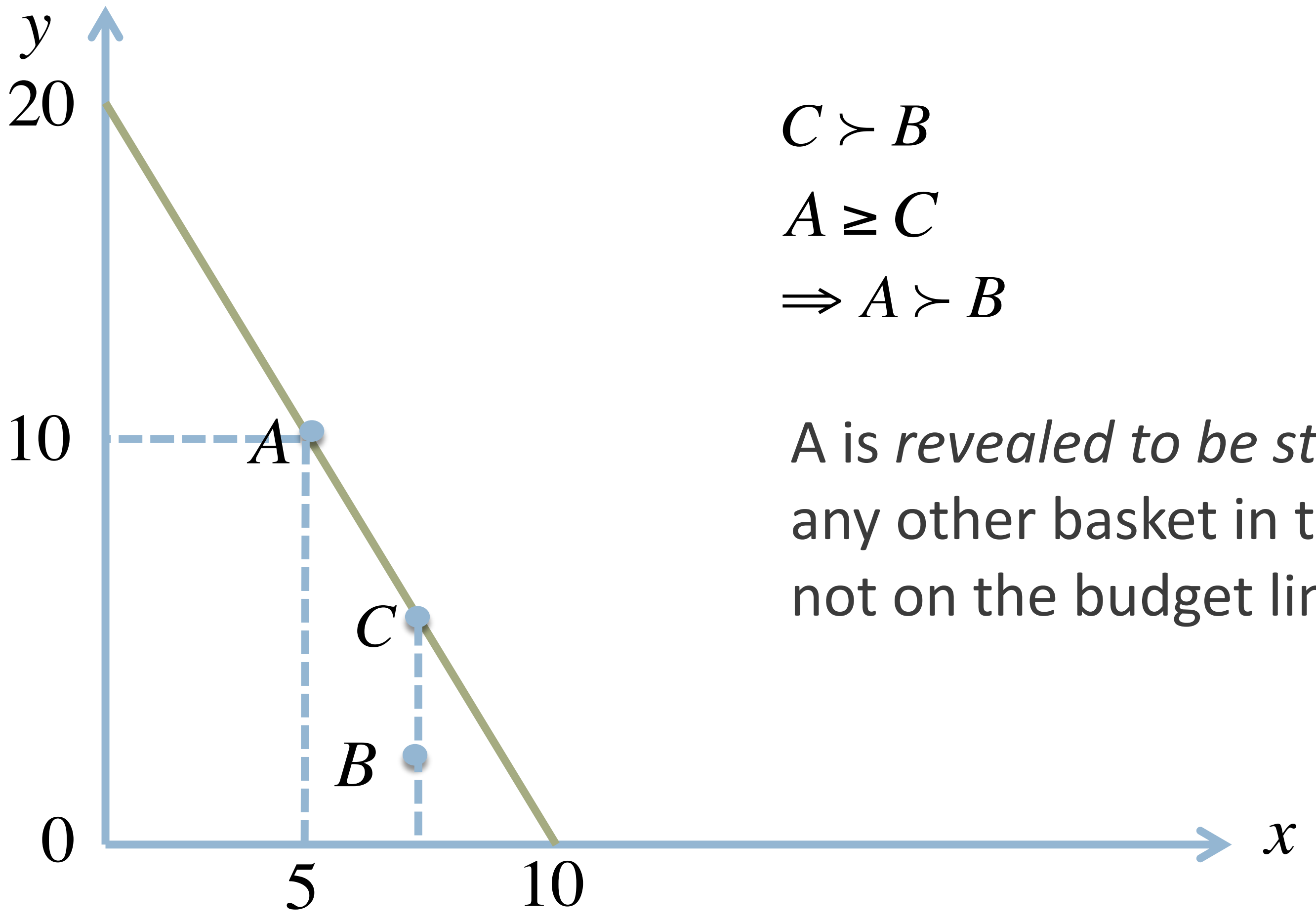
A vs. Other Points on the Budget Line Cont'



Depending on the consumer's preference, A may not be the only optimal basket

A is *revealed to be weakly preferred* to any other basket on the budget line

A vs. Other Points below the Budget Line



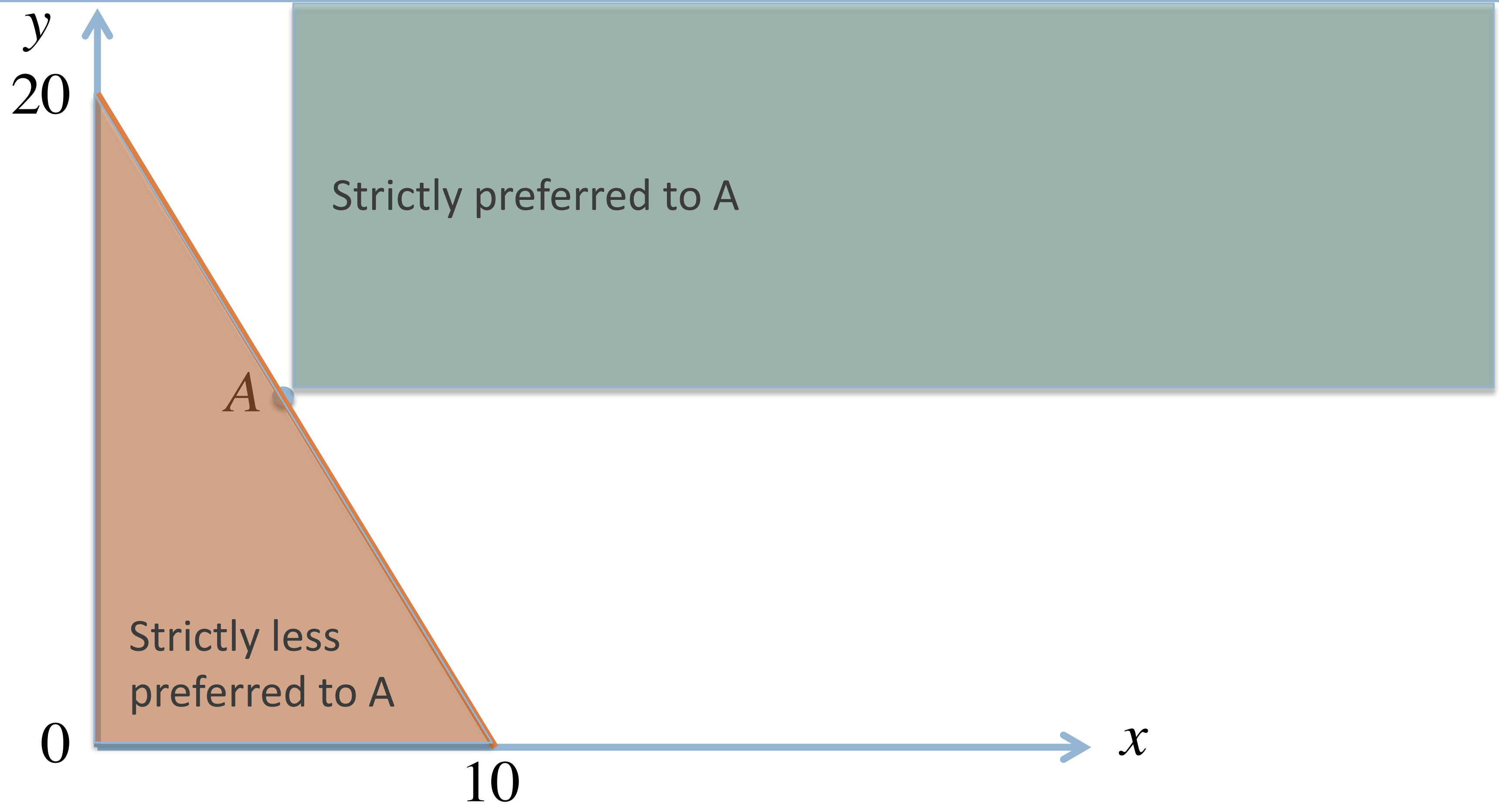
$C \succ B$

$A \geq C$

$\Rightarrow A \succ B$

A is *revealed to be strictly preferred* to any other basket in the budget set (but not on the budget line)

How Optimal Choice "Reveals" Preference



Another Way to Understand Revealed Preference

- Suppose basket A=(x_A, y_A) is the optimal basket given prices P_x, P_y, and income I
  - Basket A must be on the budget line

$$P_x x_A + P_y y_A = I$$

- No other affordable basket is strictly preferred to A
- Therefore, if basket B=(x_B, y_B) is strictly preferred to basket A, it must be that

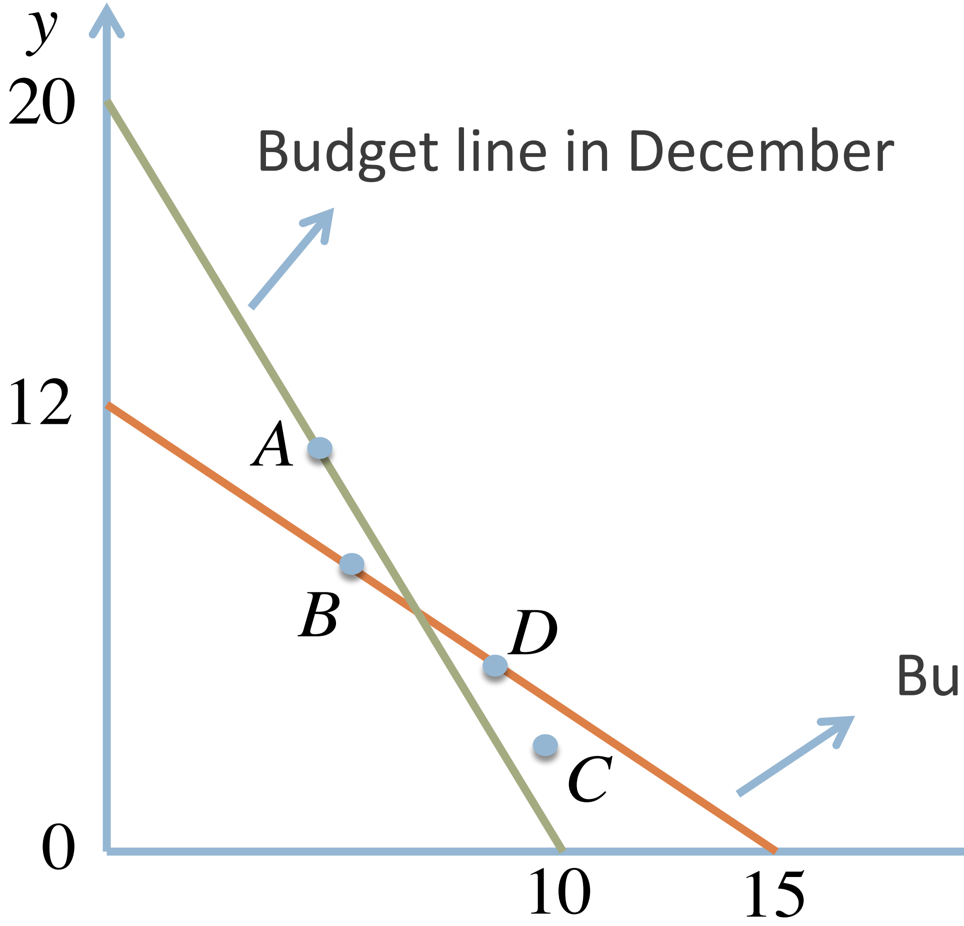
$$P_x x_B + P_y y_B > P_x x_A + P_y y_A = I$$

Another Way to Understand Revealed Preference Cont'

- Similarly, if basket C=(x_C, y_C) is indifferent to basket A, it must be that

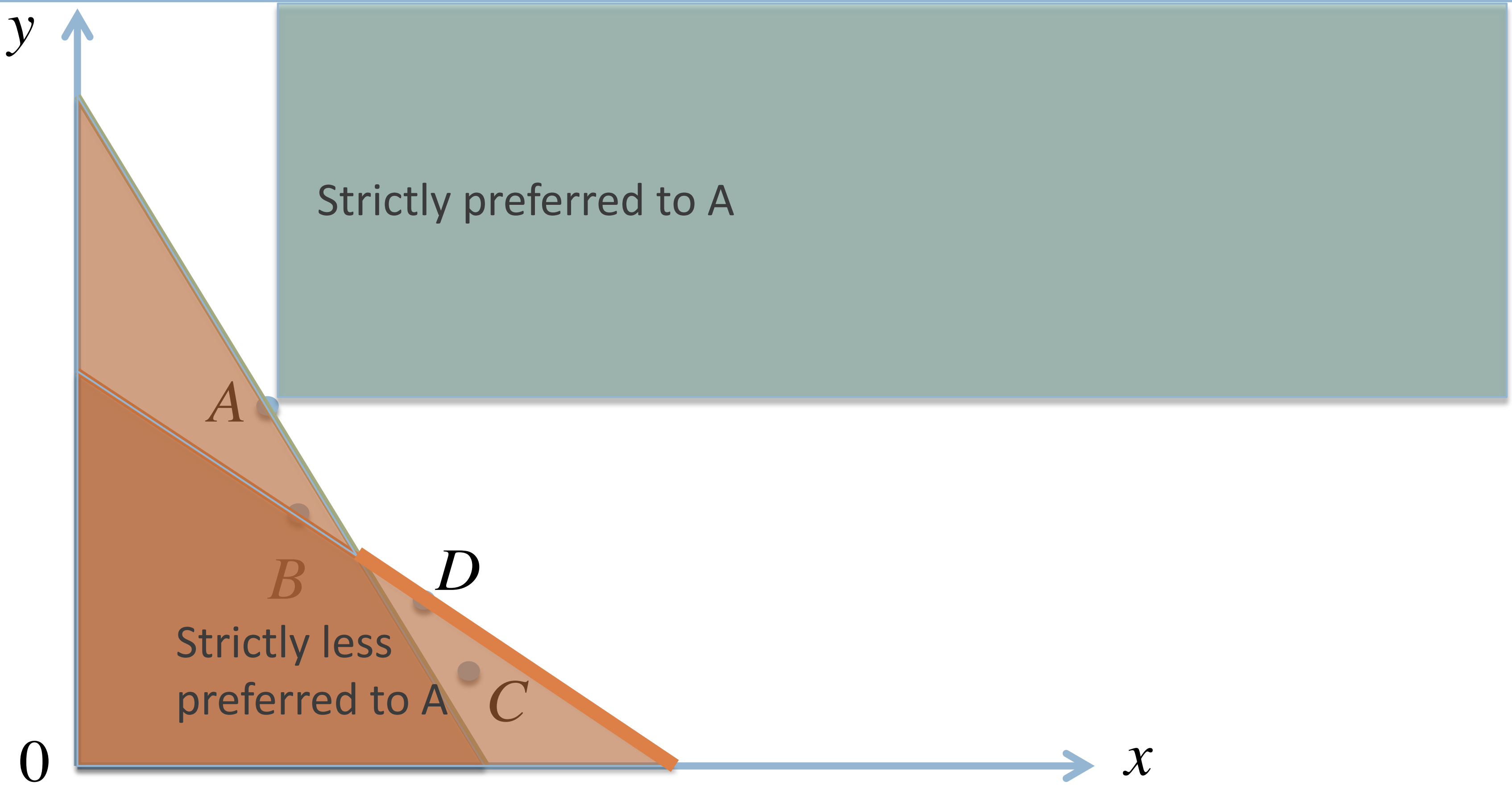
- To summarize
  - If A is the optimal basket given the budget constraint
  - Any basket that is strictly preferred to A cannot be affordable
  - Any basket that is indifferent to A cannot cost less than A

B is the Optimal Choice in January



$$A \succ B$$
$$B \succ C, B \geq D$$
$$\Rightarrow A \succ C, A \succ D$$

More Choices Observed, More Information Revealed on Preference



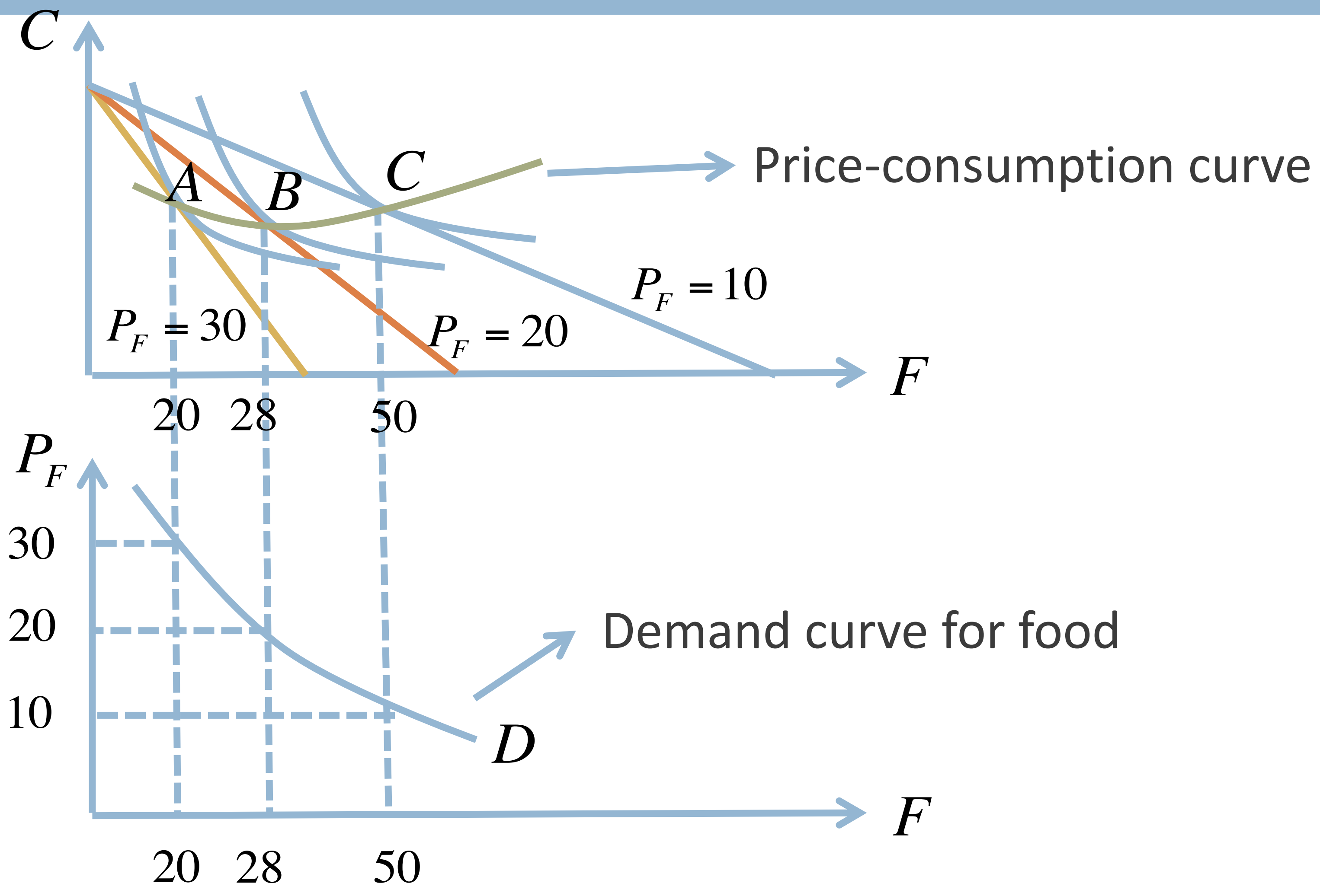
Part 3

Individual Demand

From Optimal Baskets to Individual Demand Curve

- Assume the consumer chooses food and clothing
- Suppose the price of food changes
  - The price of clothing and income are fixed
- How does the optimal basket change?
  - In particular, how does the consumption of food change?
- An individual consumer's demand curve for food captures the relationship between the optimal consumption of food for the consumer and the price of food

Example: Demand Curve for Food in Graph



Demand Curve

- Definition 3.4 A consumer's *demand curve* for a good is the optimal consumption of the good as a function of its price
  - Holding all other factors fixed
- Law of demand
  - Demand curve is downward sloping
  - Higher price, lower quantity demanded

Example: Deriving Demand Curve

- Suppose the consumer has utility function

$$U(F,C) = FC$$

- Suppose price of clothing is 2, income is 10
- What is the demand curve for food?
- The consumer solves

$$\max_{F,C} \quad FC$$

$$s.t. \quad P_F F + 2C = 10$$

Example: Deriving Demand Curve Cont'

- Tangency condition

$$\frac{P_F}{2} = \frac{C}{F}$$

- Or

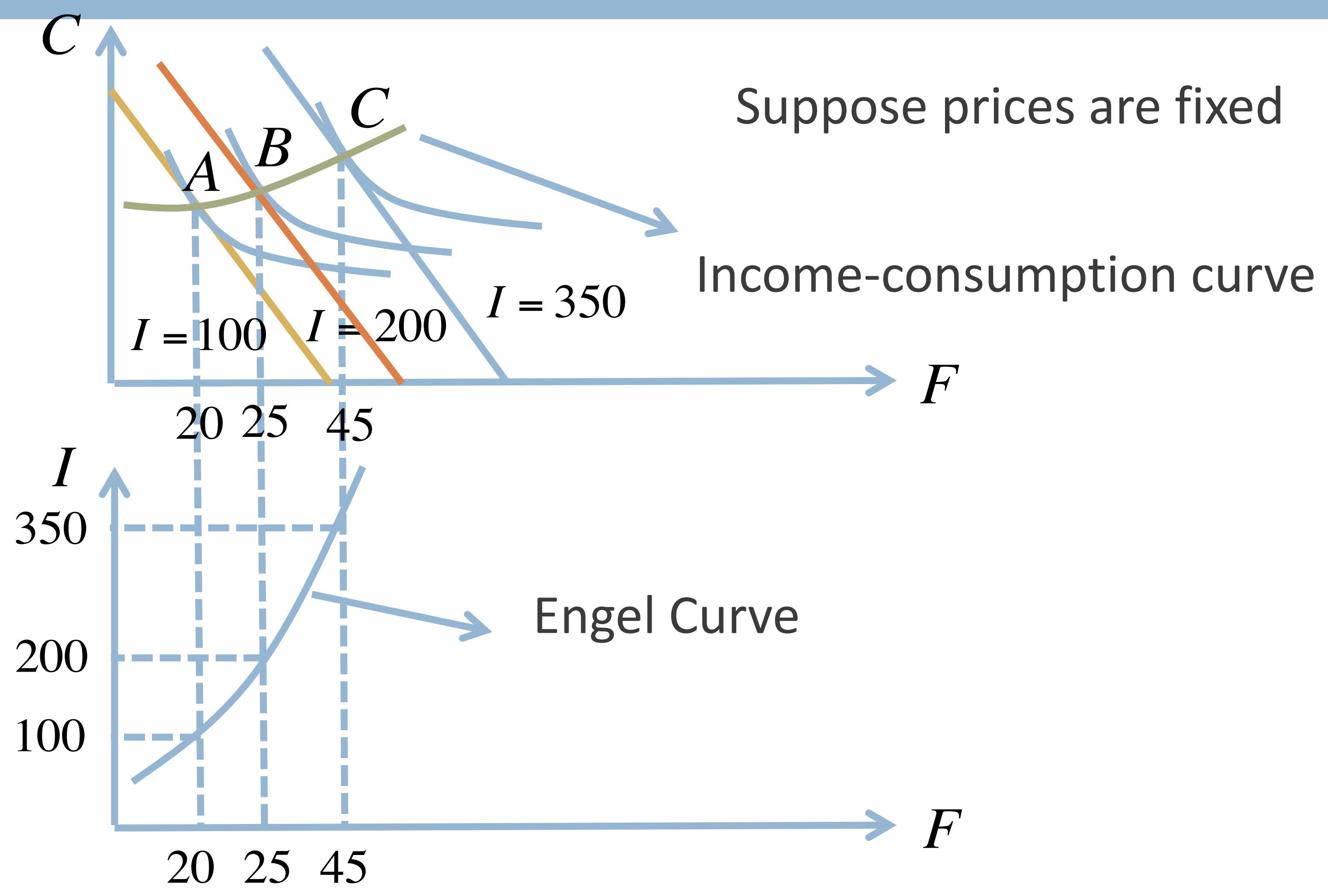
$$P_F F = 2C$$

- Budget line

$$P_F F + 2C = 10$$

- Demand curve for food is $F = \dfrac{5}{P_F}$

What if income changes?



Engel Curve

- Definition 3.5 A consumer's *Engel curve* of a good is the curve that shows the relationship between income and optimal consumption
  - Holding other factors fixed
- Definition 3.6 If the good is a *normal good*
  - Engel curve is upward sloping
- Definition 3.7 If the good is an *inferior good*
  - Engel curve is downward sloping

Demand Function

- Quantity demanded (optimal consumption) depends on
  - Price of the good
  - Income
  - Prices of other goods
- Can we write down a general formula?
  - Quantity demanded as a function of all parameters (income and all prices)
- Definition 3.8 A consumer's *demand function* for a good is quantity demanded as a function of income and all prices

Cobb-Douglas Utility Function

- Definition 3.9 A utility function is called a *Cobb-Douglas utility function* if it takes the following form

$$U(x,y) = Ax^{\alpha}y^{\beta}, A > 0, \alpha > 0, \beta > 0$$

- Examples of Cobb-Douglas utility function

$$U(x,y) = xy$$

$$U(x,y) = \frac{1}{3}x^2y^3$$

$$U(x,y) = \sqrt{xy}$$

$$U(x,y) = 4x^{\frac{1}{3}}y^5$$

Marginal Utilities of Cobb-Douglas Utility Functions

- Partially differentiating the utility function

$$MU_x = A\alpha x^{\alpha-1}y^{\beta}$$

$$MU_y = A\beta x^{\alpha}y^{\beta-1}$$

- Both marginal utilities are always positive
- "More is better" satisfied for both goods
- Indifference curves are downward sloping

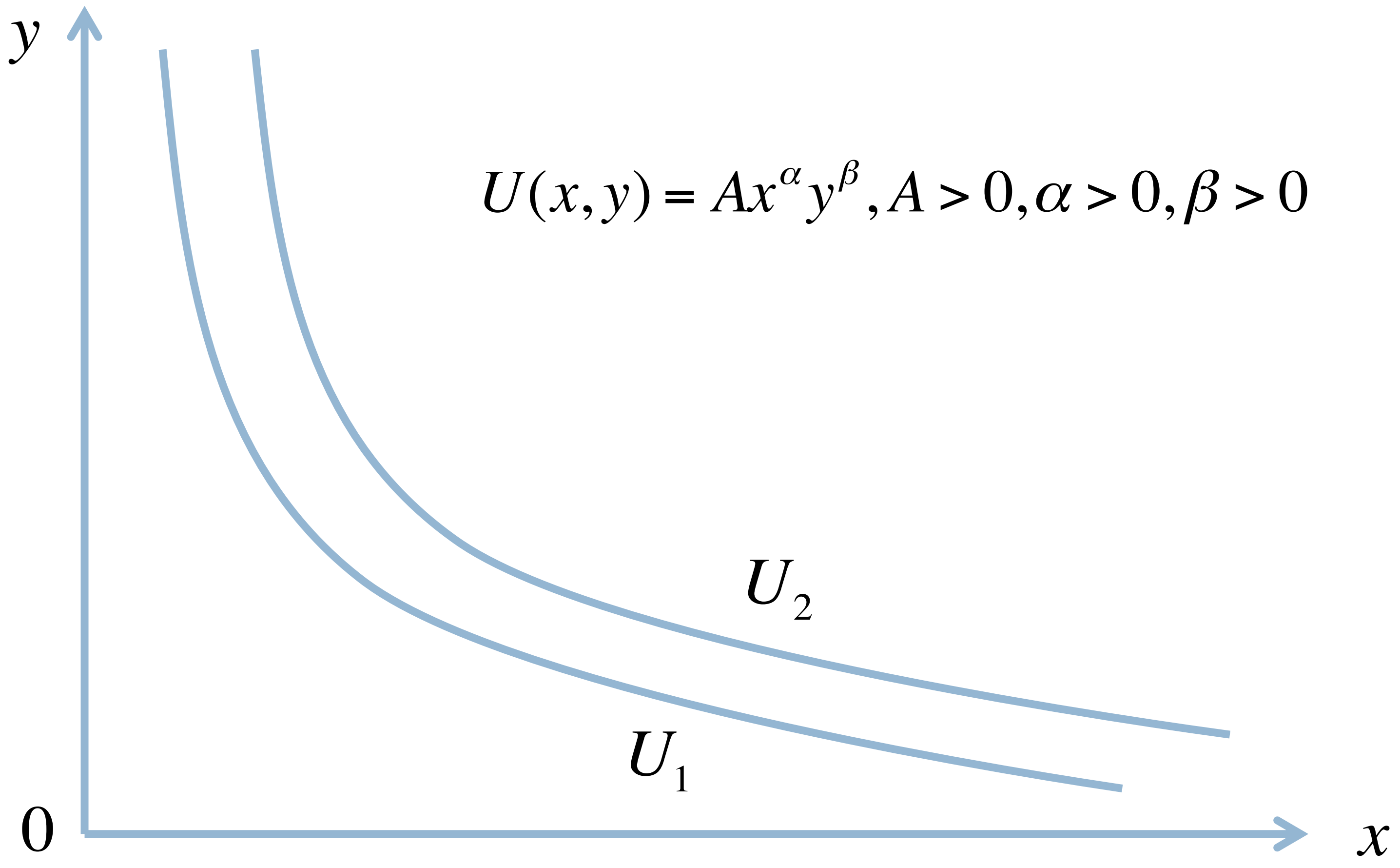
Marginal Rate of Substitution of Cobb-Douglas Utility Functions

- The marginal rate of substitution is

$$MRS_{x,y} = \frac{MU_x}{MU_y} = \frac{A\alpha x^{\alpha-1}y^{\beta}}{A\beta x^{\alpha}y^{\beta-1}} = \frac{\alpha y}{\beta x}$$

- As the consumer gets more *x* and less *y* along the same indifference curve
  - $MRS_{x,y}$ diminishes
- Indifference curves are convex

Typical Indifference Curves for Cobb-Douglas Utility Functions



Demand Function for Cobb-Douglas Utility Function

- The consumer solves

$$\max_{x,y} \quad Ax^{\alpha}y^{\beta}$$

$$s.t. \quad P_x x + P_y y = I$$

- The tangency condition is

$$\frac{\alpha y}{\beta x} = \frac{P_x}{P_y}$$

- Tangency condition can be written as

$$P_y y = \frac{\beta}{\alpha} P_x x$$

Demand Function for Cobb-Douglas Utility Function Cont'

- Plugging into the budget line

$$P_x x + \frac{\beta}{\alpha} P_x x = I$$

- Thus the demand function for *x* is

$$x = \frac{\alpha}{\alpha + \beta} \times \frac{I}{P_x}$$

- And the demand function for *y* is

$$y = \frac{\beta}{\alpha + \beta} \times \frac{I}{P_y}$$

Properties of Cobb-Douglas Utility Function

- Demand for one good does not depend on

- Consumer always spends a fixed proportion of income on each good
  - The total expenditure on *x* is

$$P_x x = P_x \times \frac{\alpha}{\alpha+\beta} \times \frac{I}{P_x} = \frac{\alpha I}{\alpha+\beta}$$

  - The total expenditure on *y* is

$$P_y y = P_y \times \frac{\beta}{\alpha+\beta} \times \frac{I}{P_y} = \frac{\beta I}{\alpha+\beta}$$