EC2101: Microeconomic Analysis I

Lecture 7

General Equilibrium Analysis: Exchange Economy

- First Fundamental Theorem
- Second Fundamental Theorem
- Walras' Law

First Fundamental Theorem of Welfare Economics

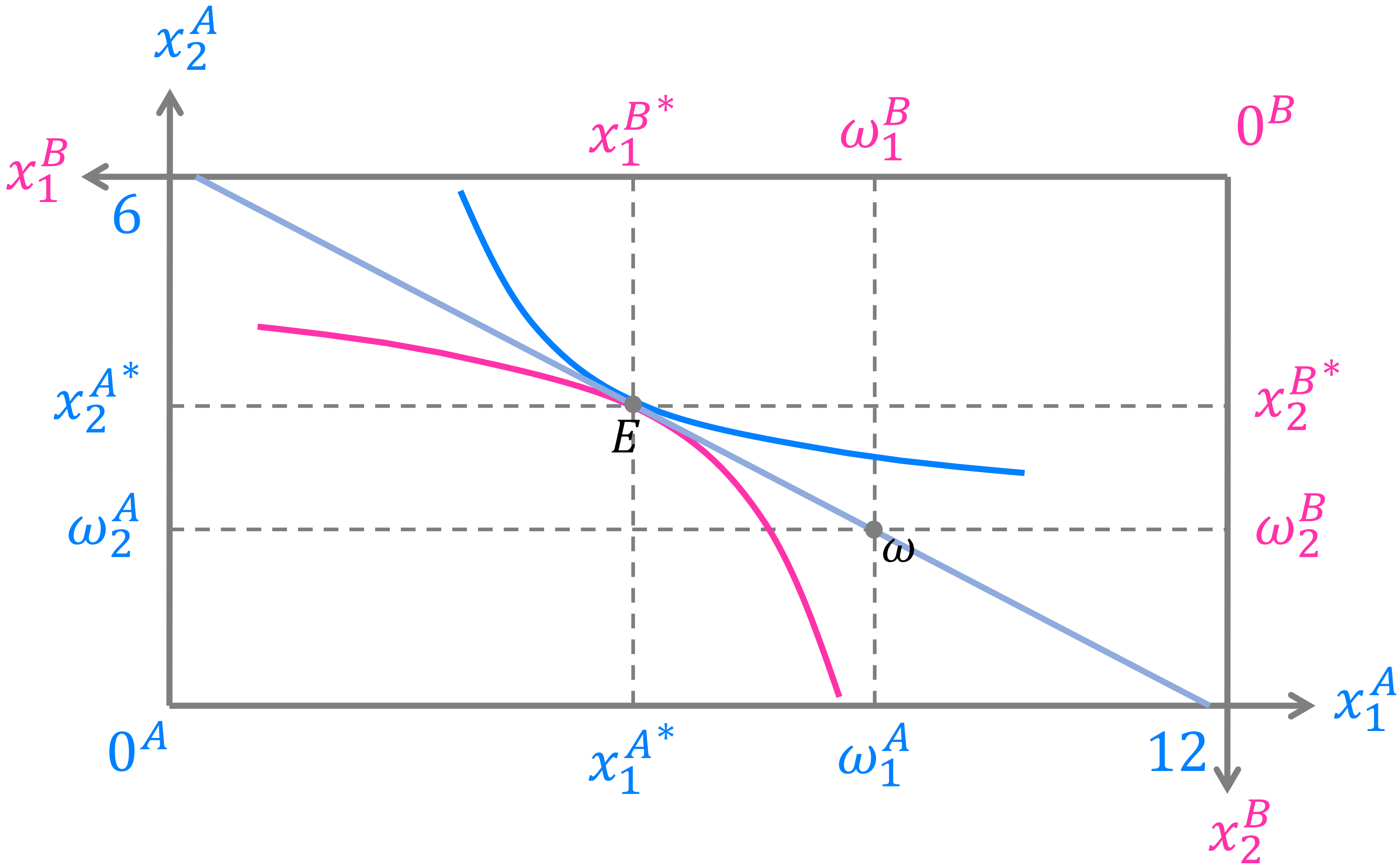
Competitive Equilibrium

- A competitive equilibrium comprises
  an allocation $\left(\left(x_1^{A^*}, x_2^{A^*}\right), \left(x_1^{B^*}, x_2^{B^*}\right)\right)$ and
  a pair of prices (p_1^*, p_2^*) such that:
  - Each consumer maximizes her utility
    given her budget constraint.
    - Let $\left(\left(x_1^{A^*}, x_2^{A^*}\right), \left(x_1^{B^*}, x_2^{B^*}\right)\right)$ denote each consumer's optimal choice given the equilibrium prices (p_1^*, p_2^*).
  - The markets for both goods clear:

$$x_1^{A^*} + x_1^{B^*} = \omega_1^A + \omega_1^B$$

$$x_2^{A^*} + x_2^{B^*} = \omega_2^A + \omega_2^B$$

Competitive Equilibrium

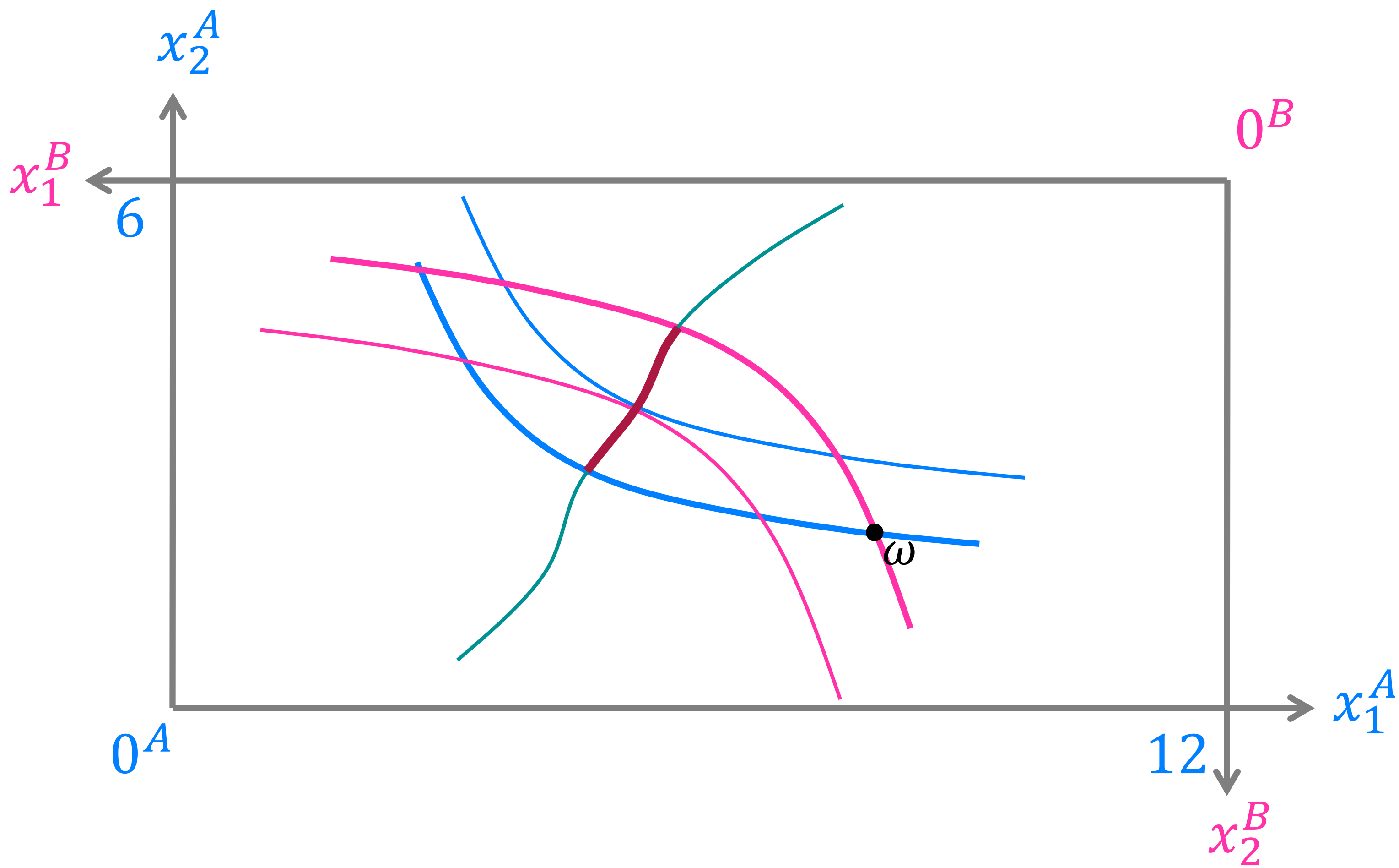


At point *E*, the two consumers' indifference curves are tangent to each other.

First Fundamental Theorem of Welfare Economics

- Suppose that:
  - There are markets and market prices for all goods.
  - All buyers and sellers are competitive price-takers.
  - Each consumer's utility depends only on her own consumption.
- Then any competitive equilibrium allocation is Pareto efficient.
  - In fact, any competitive equilibrium allocation is in the core.

Core



The core is the part of the contract curve where both consumers are at least as well off as they were at the endowment ω.

First Fundamental Theorem of Welfare Economics

- A competitive equilibrium allocation is Pareto efficient.
  - Suppose the equilibrium prices are (p_1^*, p_2^*), and the allocation $\left((x_1^{A^*}, x_2^{A^*}), (x_1^{B^*}, x_2^{B^*})\right)$ is the equilibrium allocation given the equilibrium prices.
  - Then the allocation $\left((x_1^{A^*}, x_2^{A^*}), (x_1^{B^*}, x_2^{B^*})\right)$ is Pareto efficient.

First Fundamental Theorem of Welfare Economics: Proof

- Suppose at the equilibrium prices (p_1^*, p_2^*),
  the equilibrium allocation is $\left((x_1^{A^*}, x_2^{A^*}), (x_1^{B^*}, x_2^{B^*})\right)$.
- Proof by contradiction:
  Suppose this equilibrium allocation is not Pareto efficient.
- Then there must exist another feasible allocation
  $\left((y_1^A, y_2^A), (y_1^B, y_2^B)\right)$
  where at least one consumer is better off
  and no one is worse off
  compared to the equilibrium allocation.

First Fundamental Theorem of Welfare Economics: Proof

- Suppose consumer *A* strictly prefers $\left(y_1^A, y_2^A\right)$ to $\left(x_1^{A^*}, x_2^{A^*}\right)$ while consumer *B* weakly prefers $\left(y_1^B, y_2^B\right)$ to $\left(x_1^{B^*}, x_2^{B^*}\right)$.

- By definition, the equilibrium allocation
  $$\left(\left(x_1^{A^*}, x_2^{A^*}\right), \left(x_1^{B^*}, x_2^{B^*}\right)\right)$$
  is the utility-maximizing basket for each consumer given the budget constraint.
  - Thus by revealed preference,

$$p_1 y_1^A + p_2 y_2^A > p_1 \omega_1^A + p_2 \omega_2^A \qquad (1)$$
$$p_1 y_1^B + p_2 y_2^B \geq p_1 \omega_1^B + p_2 \omega_2^B \qquad (2)$$

First Fundamental Theorem of Welfare Economics: Proof

- Sum up (1) and (2):

$$p_1\left(y_1^A + y_1^B\right) + p_2\left(y_2^A + y_2^B\right) > p_1\left(\omega_1^A + \omega_1^B\right) + p_2\left(\omega_2^A + \omega_2^B\right) \quad (3)$$

- Allocation $\left(\left(y_1^A, y_2^A\right), \left(y_1^B, y_2^B\right)\right)$ must also be feasible:

$$y_1^A + y_1^B = \omega_1^A + \omega_1^B \quad (4)$$

$$y_2^A + y_2^B = \omega_2^A + \omega_2^B \quad (5)$$

- Plug in (4) and (5) into (3):

$$p_1\left(\omega_1^A + \omega_1^B\right) + p_2\left(\omega_2^A + \omega_2^B\right) > p_1\left(\omega_1^A + \omega_1^B\right) + p_2\left(\omega_2^A + \omega_2^B\right),$$

which is a contradiction.

First Fundamental Theorem of Welfare Economics: Implications

- Each consumer maximizes her own utility.
  - There is no central planner.
- Yet a society that relies on competitive markets will achieve Pareto efficiency.
- How should we allocate scarce resources?
  - The market mechanism requires only publicly known prices that move in response to excess demand or excess supply.

First Fundamental Theorem of Welfare Economics: Comments

- The theorem holds only in competitive markets under certain conditions.
- The theorem does not hold if:
  - Consumers or firms have price-setting power.
  - There is externality.
  - There is asymmetric information.

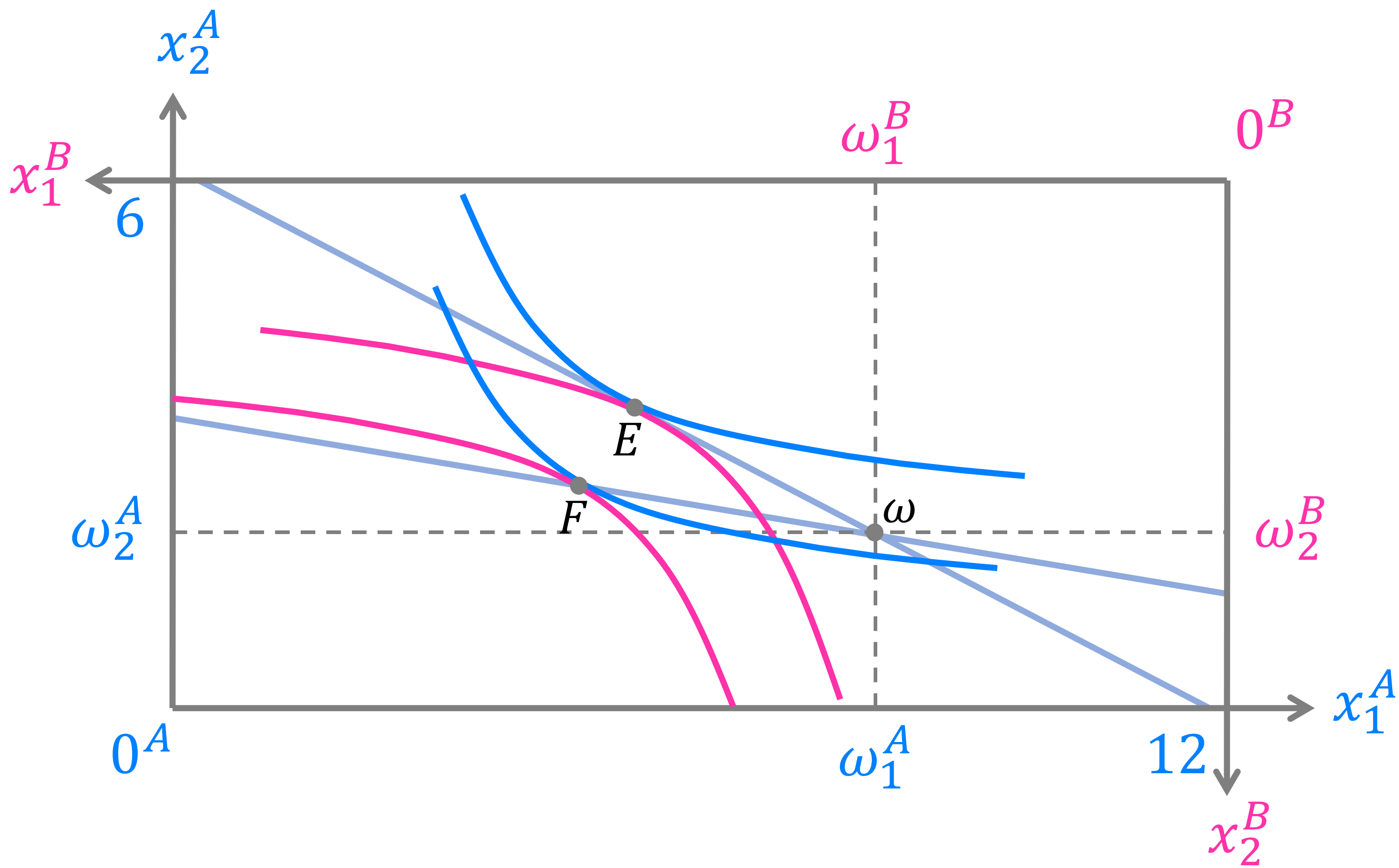
First Fundamental Theorem of Welfare Economics: Comments

- The location of the competitive equilibrium allocation is highly dependent on the location of the initial endowment allocation.
    - If we start at an initial endowment allocation where Consumer *A* has most of good 1 and good 2, we will end up at a competitive equilibrium allocation where Consumer *A* has most of good 1 and good 2.

First Fundamental Theorem of Welfare Economics: Comments

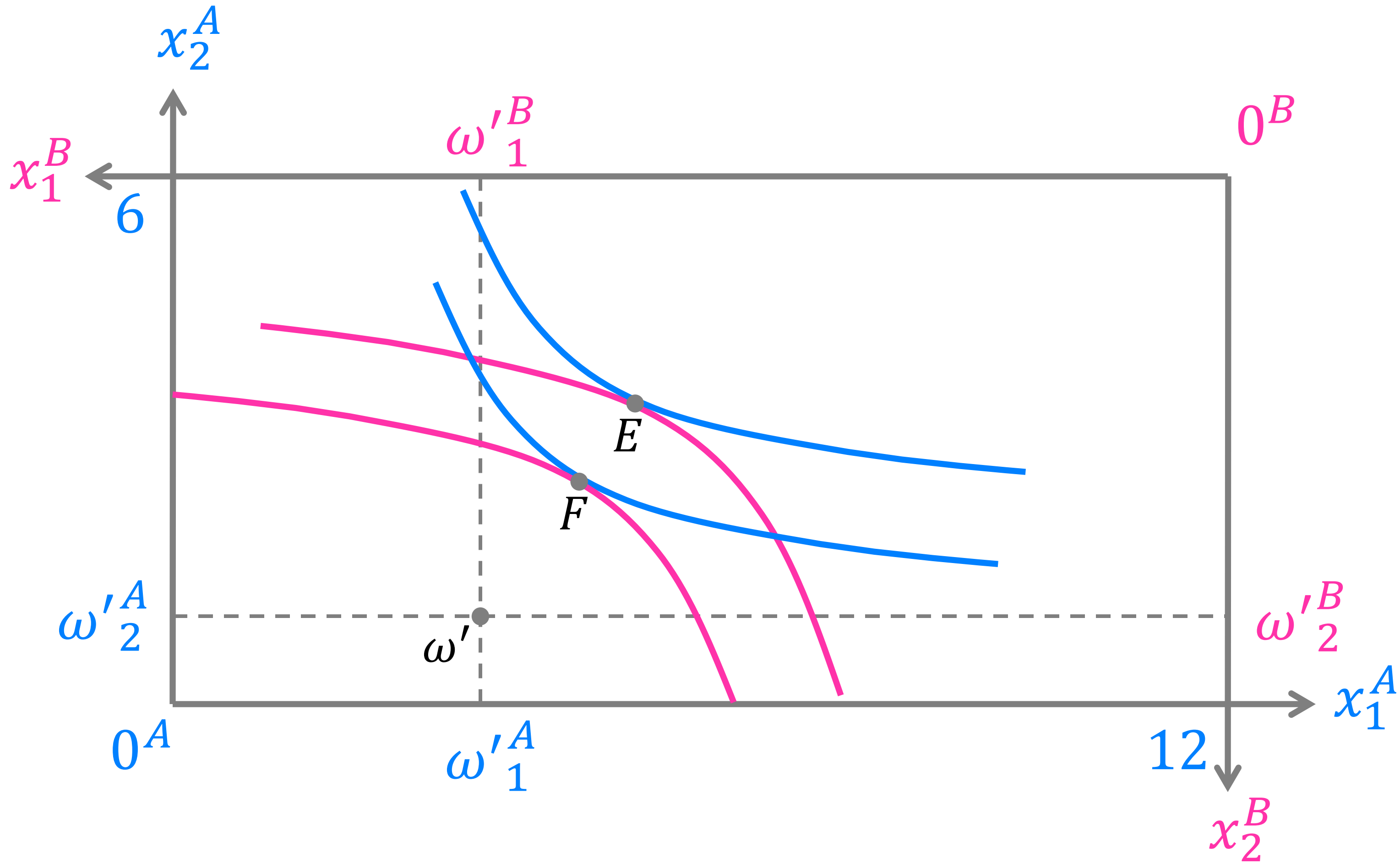
- Efficiency does not mean equity.
  - A Pareto-efficient allocation may or may not be an equitable allocation.
  - E.g., an allocation where one consumer has everything and the other consumer has nothing can be Pareto efficient.

E and *F* are always Pareto Efficient regardless of prices or endowments



Given a particular endowment allocation,
a Pareto-efficient allocation may not be a competitive equilibrium.

E and F are always Pareto Efficient regardless of prices or endowments



If the endowment allocation is ω',
neither E nor F is a competitive equilibrium.

Competitive Equilibrium and Pareto Efficiency

- First Fundamental Theorem:
  - A competitive equilibrium allocation is Pareto efficient.
- Is a Pareto-efficient allocation a competitive equilibrium?
- Since the location of the competitive equilibrium allocation is highly dependent on the location of the initial endowment allocation, not every Pareto-efficient allocation can be achieved in equilibrium.

Second Fundamental Theorem of Welfare Economics

Second Fundamental Theorem of Welfare Economics

- Suppose that:
  - There are markets and market prices for all goods.
  - All buyers and sellers are competitive price-takers.
  - Each consumer's utility depends only on her own consumption.
  - All consumers have convex indifference curves.

Second Fundamental Theorem of Welfare Economics

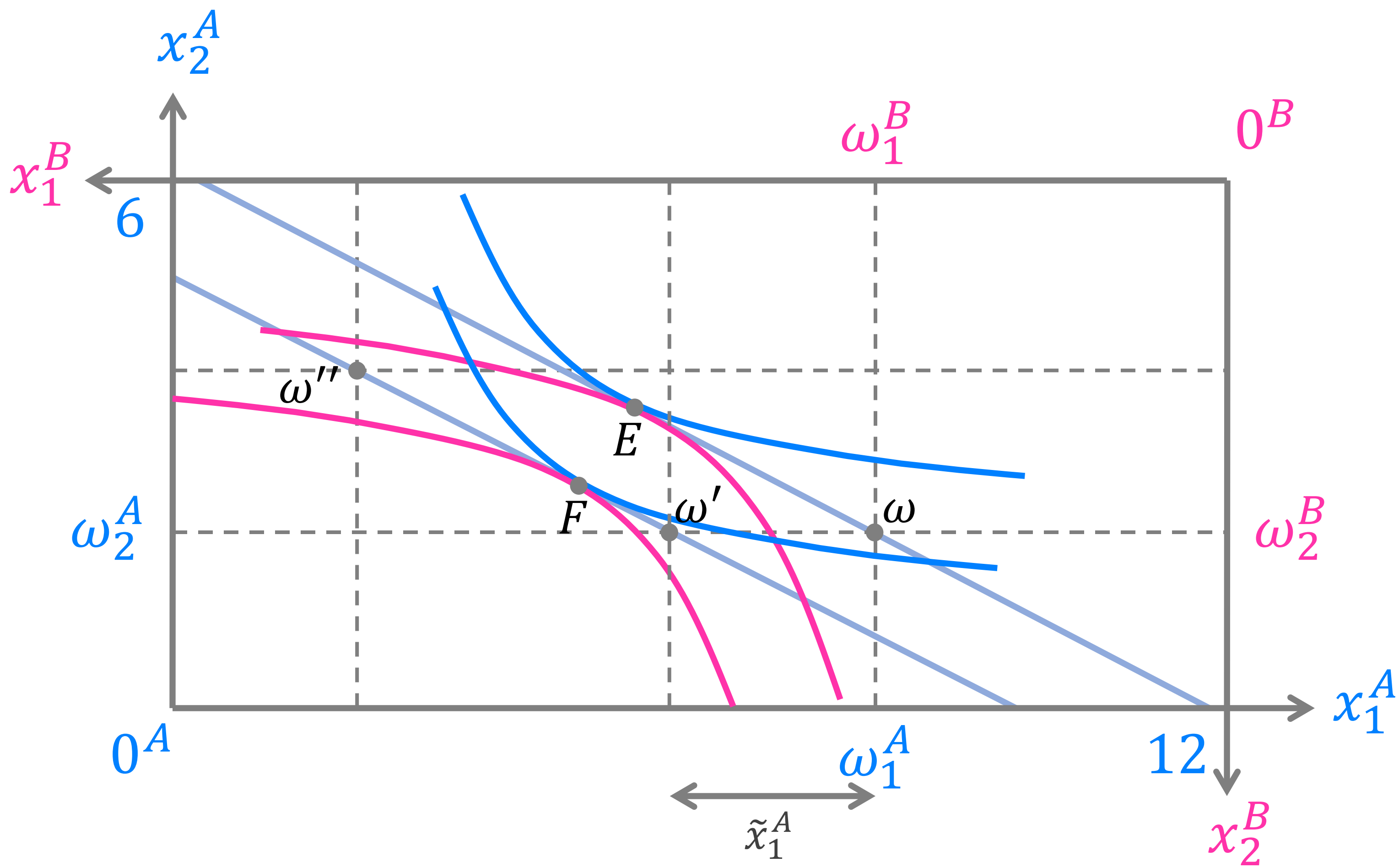
- Let the Target be any Pareto-efficient allocation.

- Then there are:
  - Competitive equilibrium prices for the goods.
  - A vector of lump-sum transfers that sum to zero.

- When the budget constraints based on these prices
  are modified with these transfers,
  the Target is the resulting competitive equilibrium allocation.

- Any Pareto-efficient allocation can be made
  a competitive equilibrium.

Second Fundamental Theorem of Welfare Economics



F will be a competitive equilibrium
if the endowment allocation is ω' or ω''.

Lump-Sum Transfer

- A transfer is defined as lump-sum if no change in a consumer's behavior can affect the size of the transfer.

- A lump-sum transfer can be expressed as:

$$T^h = p_1 \tilde{x}_1^h + p_2 \tilde{x}_2^h$$

  - If $T^h > 0$, it is a subsidy.
  - If $T^h < 0$, it is a tax.

- If there are two consumers, then

$$T^A = -T^B$$

Lump-Sum Transfer

- The consumers' budget constraints are modified with the transfers:
  - $p_1 x_1^A + p_2 x_2^A = p_1 \omega_1^A + p_2 \omega_2^A + T^A$
  - $p_1 x_1^B + p_2 x_2^B = p_1 \omega_1^B + p_2 \omega_2^B + T^B$

First and Second Fundamental Theorem of Welfare Economics

- First Fundamental Theorem of Welfare Economics:
  - Under certain assumptions,
    any competitive equilibrium allocation
    is Pareto efficient.

- Second Fundamental Theorem of Welfare Economics:
  - Under certain assumptions,
    with lump-sum transfers,
    any Pareto-efficient allocation
    can be made a competitive equilibrium.

Application

First and Second Fundamental Theorems

- Without referring to the lecture notes, write down the First and Second Fundamental Theorems of Welfare Economics.

- How is the First Fundamental Theorem relevant in the real world? Can you think of examples that support or oppose the First Fundamental Theorem?

- How is the Second Fundamental Theorem relevant in the real world? Can you think of examples that support or oppose the Second Fundamental Theorem?

Exercise 7.1

Lump-Sum Transfer

- Chip and Dale's preferences for walnuts (x_1) and pecans (x_2) are $U^C = x_1^C x_2^C$ and $U^D = x_1^D x_2^D$ respectively.
- Chip has stored 7 walnuts and 7 pecans for winter while Dale has stored 3 walnuts and 3 pecans.
- Find the lump-sum transfer necessary to ensure a competitive equilibrium allocation of $\left(x_1^{C^*}, x_2^{C^*}\right), \left(x_1^{D^*}, x_2^{D^*}\right) = (5{,}5), (5{,}5)$. I.e., find T^C and T^D.
- *Hint:*
    - *What must be true in a competitive equilibrium?*
    - *Find the new price ratio. Let $p_1 = p$ and $p_2 = 1$.*
    - *Write the modified budget constraint.*

Exercise 7.1

Lump-Sum Transfer

Exercise 7.2

Pareto Efficiency and Competitive Equilibrium

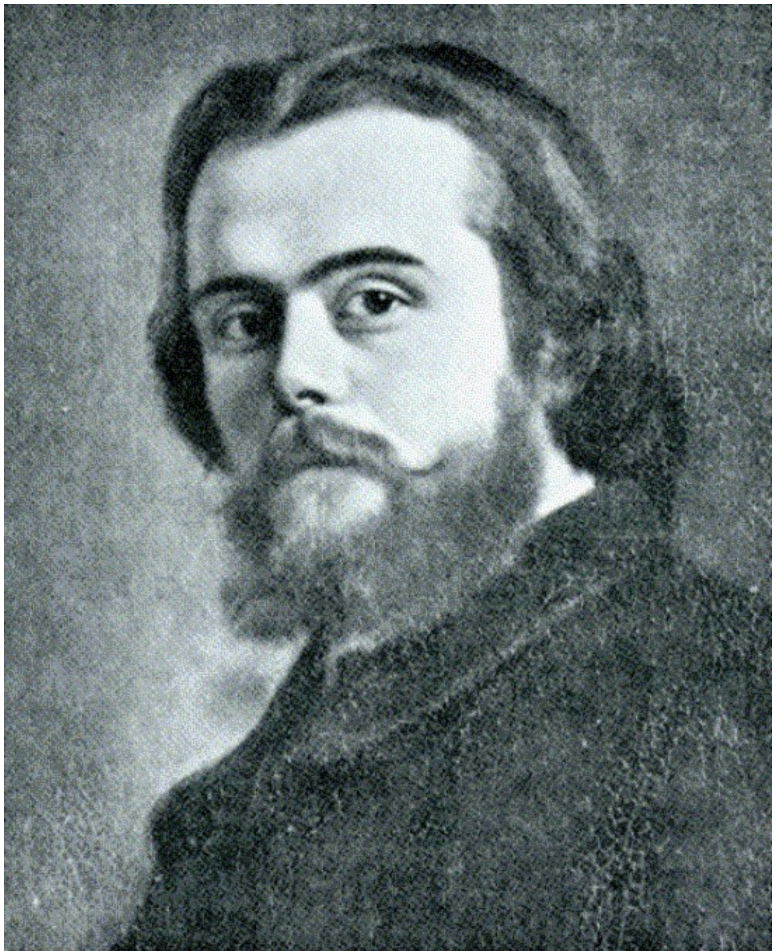
- Indicate whether the following statements are True or False. Explain briefly.
  - If an allocation is Pareto efficient, then it must be a competitive equilibrium.
  - If an allocation is a competitive equilibrium, then it must be Pareto efficient.
  - If an allocation is not a competitive equilibrium, then it must not be Pareto efficient.
  - If an allocation is not Pareto efficient, then it must not be a competitive equilibrium.

Exercise 7.2

Pareto Efficiency and Competitive Equilibrium

Walras' Law

Léon Walras



Léon Walras
1834–1910

- Developed the idea of marginal utility (independently of William Stanley Jevons and Carl Menger).
- Pioneered general equilibrium theory.
- "May now be the most widely-read nineteenth-century economist after Ricardo and Marx."

Gross Demand at Any Given Prices

- A consumer's gross demand is
  the utility-maximizing quantity of each good
  at the given prices.

Gross Demand at Any Given Prices

- Let p_1, p_2 be any pair of strictly positive prices.
  - These prices may or may not be the equilibrium prices.
- Given p_1, p_2, let
  (x_1^A, x_2^A) be consumer *A*'s gross demand and
  (x_1^B, x_2^B) be consumer *B*'s gross demand.
- Since p_1, p_2 may not be the equilibrium prices, it is possible that:

$$x_1^A + x_1^B \neq \omega_1^A + \omega_1^B$$
$$x_2^A + x_2^B \neq \omega_2^A + \omega_2^B$$

Net Demand

- A consumer's net demand for a good is
  the difference between
  her gross demand for that good and
  her endowment of that good.

- Consumer *A*'s net demand for good 1 is:

$$x_1^A - \omega_1^A$$

- Consumer *A*'s net demand for good 2 is:

$$x_2^A - \omega_2^A$$

Aggregate Net Demand

- The aggregate net demand for a good is
  the sum of the consumers' net demand for that good:

$$(x_1^A - \omega_1^A) + (x_1^B - \omega_1^B) = x_1^A + x_1^B - \omega_1^A - \omega_1^B$$

$$(x_2^A - \omega_2^A) + (x_2^B - \omega_2^B) = x_2^A + x_2^B - \omega_2^A - \omega_2^B$$

- If the aggregate net demand for a good is positive,
  there is excess demand for that good.

- If the aggregate net demand for a good is negative,
  there is excess supply of that good.

Value of Net Demand

- Consumer *A*'s gross demand $\left(x_1^A, x_2^A\right)$ lies on her budget line:

$$p_1 x_1^A + p_2 x_2^A = p_1 \omega_1^A + p_2 \omega_2^A$$

- Rearranging:

$$p_1\left(x_1^A - \omega_1^A\right) + p_2\left(x_2^A - \omega_2^A\right) = 0$$

  - $p_1\left(x_1^A - \omega_1^A\right)$ is the value of consumer *A*'s net demand for good 1
  - $p_2\left(x_2^A - \omega_2^A\right)$ is the value of consumer *A*'s net demand for good 2

Value of Net Demand

- The total value of consumer *A*'s net demand for the two goods is zero:

$$p_1\left(x_1^A - \omega_1^A\right) + p_2\left(x_2^A - \omega_2^A\right) = 0 \qquad (1)$$

- Likewise, the total value of consumer *B*'s net demand for the two goods is zero:

$$p_1(x_1^B - \omega_1^B) + p_2(x_2^B - \omega_2^B) = 0 \qquad (2)$$

- Summing up (1) and (2),

$$p_1\left(x_1^A + x_1^B - \omega_1^A - \omega_1^B\right) + p_2\left(x_2^A + x_2^B - \omega_2^A - \omega_2^B\right) = 0$$

Walras' Law

- Walras' Law:

$$p_1\left(x_1^A + x_1^B - \omega_1^A - \omega_1^B\right) + p_2\left(x_2^A + x_2^B - \omega_2^A - \omega_2^B\right) = 0$$

- The total value of
  the aggregate net demand
  for the two goods
  is zero.

Walras' Law: Implications

- In the two-good exchange economy,
  if one market is in equilibrium,
  the other market must also be in equilibrium.

- Suppose the market for good 1 clears:

$$x_1^A + x_1^B - \omega_1^A - \omega_1^B = 0$$

- By Walras' law,

$$p_1\left(x_1^A + x_1^B - \omega_1^A - \omega_1^B\right) + p_2\left(x_2^A + x_2^B - \omega_2^A - \omega_2^B\right) = 0$$

- The market for good 2 must clear as well:

$$x_2^A + x_2^B - \omega_2^A - \omega_2^B = 0$$

Walras' Law: Implications

- In the two-good exchange economy, excess supply in one market implies excess demand in the other market.
- Suppose there is excess supply of good 1:

$$x_1^A + x_1^B - \omega_1^A - \omega_1^B < 0$$

- By Walras' law,

$$p_1\left(x_1^A + x_1^B - \omega_1^A - \omega_1^B\right) + p_2\left(x_2^A + x_2^B - \omega_2^A - \omega_2^B\right) = 0$$

- There must be excess demand for good 2:

$$x_2^A + x_2^B - \omega_2^A - \omega_2^B > 0$$

Walras' Law vs. Competitive Equilibrium

- Walras' Law holds for ANY prices,
  not just the equilibrium prices.

- At the equilibrium prices,
  the aggregate net demand for each good is zero:

$$p_1\underbrace{\left(x_1^A + x_1^B - \omega_1^A - \omega_1^B\right)}_{=0} + p_2\underbrace{\left(x_2^A + x_2^B - \omega_2^A - \omega_2^B\right)}_{=0} = 0$$

- At non-equilibrium prices,
  the aggregate net demand for each good is not zero:

$$p_1\underbrace{\left(x_1^A + x_1^B - \omega_1^A - \omega_1^B\right)}_{\neq 0} + p_2\underbrace{\left(x_2^A + x_2^B - \omega_2^A - \omega_2^B\right)}_{\neq 0} = 0$$

Exercise 7.3

Walras' Law

- Indicate whether the following statements are True or False. Explain briefly.
  - If Walras' law holds,
    then we are at a competitive equilibrium.
  - If we are at a competitive equilibrium,
    then Walras' law must hold.

Exercise 7.3

Walras' Law

Summary

General Equilibrium Analysis: Exchange Economy

- Edgeworth box:
- Endowment allocation:
- Affordable consumption plan:
- Budget line:
- Feasible allocation:
- Competitive equilibrium:

Summary

General Equilibrium Analysis: Exchange Economy

- Pareto dominate:
- Pareto move/improvement:
- Pareto efficiency/optimality:
- Contract curve:
- Core:
- Lump-sum transfer:

Summary

General Equilibrium Analysis: Exchange Economy

- Gross demand:
- Net demand:
- Aggregate net demand:
- Value of net demand:
- Total value of net demand:
- Total value of aggregate net demand:

Summary

General Equilibrium Analysis: Exchange Economy

- First Fundamental Theorem:

- Second Fundamental Theorem:

- Walras' Law: