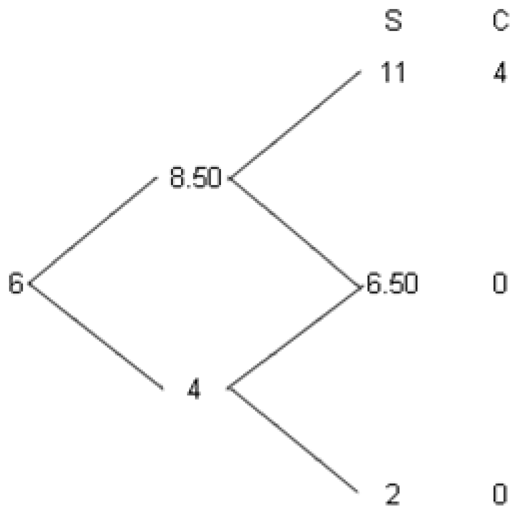
EC3333 Tutorial 10 Suggested Answers

1. The current price of the ABC Corporation stock is $6. In each of the next two years, this stock price can either go up by $2.50 or go down by $2. The stock pays no dividends. The one-year risk-free interest rate is 3% and will remain constant. Use the Binomial Option Pricing Model to find the replicating portfolio and the value of a two-year

   a. European call option on the ABC stock with a strike price of $7.

   b. European put option on the ABC stock with a strike price of $7.

a.



Up state at time 1:

$\Delta = (\$4 - 0)/(\$11 - \$6.50) = 0.889$,

$B = (0 - \$6.50 \times 0.889)/1.03 = -\$5.61$,

therefore $C_u = 0.889 \times \$8.50 - \$5.61 = \$1.95$.

Down state at time 1:

the option is worth nothing.

The call option at time 0 is therefore equivalent to the replicating portfolio:

$\Delta = (\$1.95 - 0)/(\$8.50 - \$4) = 0.433$,

$B = (0 - \$4 \times 0.433)/1.03 = -\$1.68$ and so,

by the Law of One Price, the initial option price is $0.433 \times \$6 - \$1.68 = \$0.92$.

b.

If the stock goes up twice, the put is worth zero.

If the stock ends up at $6.50, the put is worth $0.50;

if the stock goes down twice, the put is worth $5.

Given these final values, we can calculate the value of the put at earlier dates using the binomial model.

Up state at time 1:

$\Delta = (0 - \$0.50)/(\$11 - \$6.50) = -0.111$ and

$B = (\$0.50 - \$6.50 \times (-0.111))/1.03 = \$1.19$.

Therefore, $P_u = -0.111 \times \$8.50 + \$1.19 = \$0.25$.

Down state at time 1:

$\Delta = (\$0.50 - \$5)/(\$6.50 - \$2) = -1$ and

$B = (\$5 - \$2 \times (-1))/1.03 = \$6.80$

Therefore, $P_d = -1 \times \$4 + \$6.80 = \$2.80$.

Time 0:

$\Delta = (\$0.25 - \$2.80)/(\$8.50 - \$4) = -0.567$ and

$B = (\$2.80 - \$4 \times (-0.567))/1.03 = \$4.92$

Therefore, $P = -0.567 \times \$6 + \$4.92 = \$1.52$.

2. Use the Black-Scholes Option Pricing Model and the table below for this question.

a. What is the value of a European call option on a stock with the following characteristics?

Time to expiration	6 months
Standard deviation	50% per year
Exercise price	\$50
Stock price	\$50
Annual Interest Rate	3%
Dividend	0

b. What is the value of a European put option on the same stock with the same exerciseprice and time to expiration?

c. Recalculate the value of the European call option, successively substituting one of the changes below while keeping the values of the other parameters stated above. Consider each scenario independently.

i. Time to expiration = 3 months.

ii. Standard deviation = 25% per year.

iii. Exercise price = \$55.

iv. Stock price = \$55.

v. Interest rate = 5%.

a.

$d_1 = 0.2192 \Rightarrow N(d_1) = 0.5868$

$d_2 = –0.1344 \Rightarrow N(d_2) = 0.4465$

$Xe^{-rT} = 49.2556$

C = \$50 × 0.5868 – 49.2556 × 0.4465 = \$7.34

b.

From put-call parity:

$P = C + \text{PV}(X) - S_0$ = \$7.34 + \$49.26 – \$50 = \$6.60

c.

i. C falls to \$5.1443

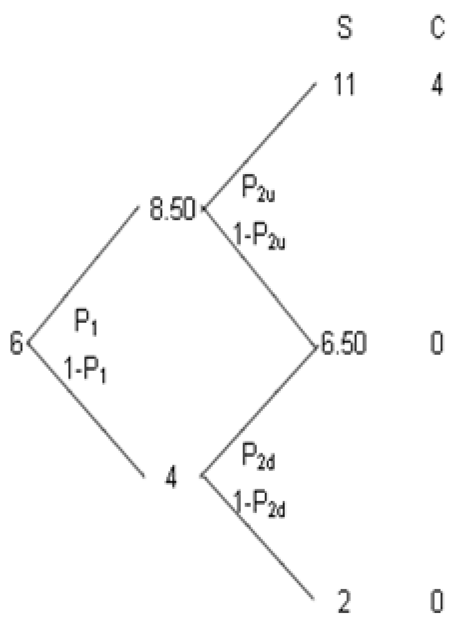
ii. C falls to \$3.8801

iii. C falls to \$5.4043

iv. C rises to \$10.5356

v. C rises to \$7.5636

3. Using the information in Problem 1, calculate the risk-neutral probabilities. Then use them to price the value of a two-year European call option on the ABC stock with a strike price of $7.



The risk neutral probabilities are

$$p_1 = \frac{(1.03)6-4}{8.5-4} = 48.44\%$$

$$p_{2u} = \frac{(1.03)8.5-6.5}{11-6.5} = 50.11\%$$

$$p_{2d} = \frac{(1.03)4-2}{6.5-2} = 47.11\%.$$

The value of the call is therefore

$$\frac{1}{1.03^2}\Big(4p_1p_{2u} + 0\big(p_1(1-p_{2u}) + (1-p_1)(p_{2d})\big) + 0(1-p_1)(1-p_{2d})\Big)$$

$$= \frac{4(0.4844)(0.5011)}{1.03^2} = \$0.9153.$$

4. Explain why risk-neutral probabilities can be used to price derivative securities in a world where investors are risk averse.

Risk neutral probabilities can be used to price derivative securities because the pricing of derivatives depends on the characteristics of the underlying asset. By careful construction of a replicating portfolio, one can create a risk-free portfolio involving the underlying asset and the derivative securities. Hence regardless of the risk preference of the investor, based on the law of one price, the value of this risk-free portfolio should generate the risk-free return. Therefore, we can infer the risk-neutral probabilities of the trajectory of the underlying asset and the value of the derivative securities can be computed using these probabilities. Risk-neutral probabilities are the easiest probabilities to work with, given that they simplify the calculations, and that is why we use them.