Practice Problem Set 10
Asymmetric Information (C.38)

Question 10.1

You are the CEO of a private school and your school hires many instructors. There are two kinds of instructors: good instructors, who generate $3000 in revenue, and average instructors, who generate $2500 in revenue. The workforce at large has equal numbers of good and average instructors. You are willing to pay a good instructor $3000, and an average instructor $2500. You cannot tell a good instructor from an average one by their looks, but you would like to identify them. So you design a test, and promise to pay an instructor $3000 if they get 60 answers right. Studying for the test is costly for the instructors: a good instructor and an average instructor suffer utility losses equivalent to $5 and $10, respectively, to answer each question right in the test.

(a) Can the test lead to a separating equilibrium? How many answers would good and average instructors answer correctly in the test?

(b) Suppose instead that a good instructor and an average instructor suffer utility losses equivalent to $5 and $8, respectively, to answer each question right in the test. Do your answers in (a) change?

Answer

(a) Yes. Suppose a separating equilibrium exists and every good instructor studies for 60 questions (at a cost of 5*60=$300) and receives $3000 while every average instructor studies for 0 questions and receives $2500. For this equilibrium to exist, no one should have the incentive to deviate. Check: If a good instructor deviates and abandons studying, she will be treated as an average instructor and her net gain is $300-500=-$200. Hence she will not deviate; If an average instructor deviates and studies for 60 questions, she will be treated as a good instructor and her net gain is -$600+500=-$100. Hence she will not deviate.

(b) Now the separating equilibrium cannot be supported because an average instructor has the incentive to deviate: if she studies, she gains -$480+500=$20 in the process. Lesson: If a signaling tool is cheap for everyone, it loses its usefulness.

Question 10.2

Consider the principal-agent model with two effort levels and risk-neutral agent in the lecture notes. Suppose the cost of exerting high effort is 2000 (instead of 20).

(a) What is the first-best effort level? What is the expected net profit for the principal in the first best? What is the expected wage for the agent?

(b) Suppose the principal "sells the firm to the agent", that is, the principal pays the agent the entire profit less a fee. How much should the fee be? Verify that the effort choice, the expected net profit, and the expected wage are the same as in part a).

Answer

(a) Suppose effort is observable. If the principal chooses low effort, he should pay the agent 0 for low effort and 0 otherwise. The expected net profit is 0.5*1000+0.5*2000=$1500. If the principal chooses high effort, he should pay the agent $2000 for high effort and 0 otherwise. His expected net profit is 0.5*2000+0.5*4000-2000=$1000. Thus low effort is the first-best effort level. The expected net profit is $1500. The expected wage is 0.

(b) Suppose the fixed fee is a. If the agent chooses low effort, his expected utility is 0.5*1000+0.5*2000-a=1500-a. If the agent chooses high effort, his expected utility is 0.5*2000+0.5*4000-a-2000=1000-a. The agent will choose low effort and the principal should set a=1500. The expected net profit is $1500. The expected wage is 0. By selling the firm to the agent, the first best outcome is achieved.