EC3333 Tutorial 2

1. Consider a world with two possible states of the world, 1 and 2, which are equally likely.

   Consider two assets A and B whose returns are positively correlated. Their returns have the following probability distribution.

State	Probability	R_A	R_B	R_P
1	0.5	10.0%	12.0%	?
2	0.5	-2.0%	-1.0%	?

   Consider a parallel universe where the returns on the two assets A and B are negatively correlated instead. Their returns have the following probability distribution.

State	Probability	R_A	R_B	R_P
1	0.5	10.0%	-1.0%	?
2	0.5	-2.0%	12.0%	?

   Suppose we are interested in a portfolio *P*, where the portfolio weights on asset A and asset B are $x_A = 0.4$ and $x_B = 0.6$, respectively.

   a. Fill in the blank in the above tables (i.e., compute the portfolio returns in different states of the world).
   b. Calculate the expected values $E(R_A)$, $E(R_B)$, and $E(R_P)$ in both cases.
   c. Calculate the volatilities σ_A, σ_B, and σ_P in both cases.
   d. Calculate the covariances σ_{AB} and correlation coefficients ρ_{AB} in both cases.
   e. How are the expected values and volatilities related to the correlation coefficients?

2. Stocks offer an expected rate of return of 18%, with a standard deviation of 22%. Gold offers an expected return of 10% with a standard deviation of 30%.
   a. In light of the apparent inferiority of gold with respect to both mean return and volatility, would anyone hold gold? If so, demonstrate graphically why one would do so.
   b. Given the data above, redo part a with the additional assumption that the correlation coefficient between gold and stocks equals 1. Draw a graph illustrating why one would or would not hold gold in one's portfolio. Could this set of assumptions for expected returns, standard deviations, and correlation represent an equilibrium for the security market?

3. Historical data show that the average annual return on the S&P 500 portfolio over the past 90 years has averaged roughly 8% more than the Treasury bill return and that the standard deviation of the return on S&P 500 has been about 20% per year. Assume that these values are representative of the investors' expectations for future performance and that the current T-bill rate is 5%.

   a. Calculate the expected return and variance of portfolios invested in T-bills and the S&P 500 index with weights as follows:

x_{Bills}	x_{Index}
-0.2	1.2
0.0	1.0
0.2	0.8
0.4	0.6
0.6	0.4
0.8	0.2
1.0	0.0

   b. Plot the expected return and standard deviation of these portfolios on the E(r) versus σ plane. What do you observe? Do they fall on a straight line? If they do, what are the vertical intercept and the slope? Looking at your plot, when the portfolio weight is non-negative, where is the portfolio located in the E(r) versus σ plane? When the portfolio weight is negative for T-bill, where is the portfolio relative to the portfolio withweight equals zero for T-bill (and weight equals one for S&P500)?

   c. Suppose the utility function is given by $U = E(r) - 0.5 \times A\sigma^2$, where A is an index of the investor's risk aversion. Suppose we do not allow short selling.
      - Calculate the utility levels of each portfolio for an investor with A=2. What do you conclude?
      - Calculate the utility levels of each portfolio for an investor with A=3. What do you conclude?