EC3312: Game Theory & Applications to Economics

Tutorial 1

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1 Rock–paper–scissors

Find all the Nash equilibria of the following game.

		B: R	B: P	B: S
A	R	0 0	1 -1	-1 1
	P	-1 1	0 0	1 -1
	S	1 -1	-1 1	0 0

2 Nash equilibria of 2×2 games

Consider the following general 2×2 game.

		2: A	2: B
1	A	α a	γ b
	B	β c	δ d

The payoffs a, b, c, d, α, β, γ, and δ are all potentially different real numbers.

1. Given an example of payoffs such that the game does not have a pure-strategy equilibrium. What inequalities must the payoffs satisfy for this to occur?

2. If a mixed-strategy equilibrium exists, what are the players' strategies?

3. Using your answers from above, prove that a Nash equilibrium always exists.

3 The tragedy of the commons[1]

On an island there are 2 lakes and 16 fishermen. Each fisherman can fish on either lake. When L_1 fishermen fish on lake 1, the total number of fish caught there is

$$F_1(L_1) = 8L_1 - \frac{L_1^2}{2},$$

and when L_2 fishermen fish on lake 2, the total number of fish caught there is

$$F_2(L_2) = 4L_2.$$

Assume that all fishermen on a given lake catch the same number of fish.

1. Formulate this setting as a normal-form game.
2. Find the unique Nash equilibrium (up to permutations of the players). How many fish will be caught altogether?
3. The island chief believes that she can increase the total number of fish caught by restricting the number of fishermen fishing on lake 1. What is the number of fishermen fishing on lake 1 that maximises the number of fish caught altogether, and what is the number of fish caught in this situation? Explain why the chief is correct.
4. Since she is opposed to coercion, the chief decides to issue permits in order to fish on lake 1. If she is to bring about the optimal allocation of labour, how much should a permit cost (in terms of fish)?

4 Beauty contest

There are n players with $S_i = \{0, 1, \ldots, 100\}$. Given a strategy profile s, the players who chose actions closest to

$$\frac{1}{3n} \sum_{i \in N} s_i$$

split \$1,000 equally; the other players receive nothing.

1. Show that $s^0 = (0, 0, \ldots, 0)$ is a Nash equilibrium.
2. Show that there are no other pure-strategy Nash equilibria.
3. Show that there are no mixed-strategy Nash equilibria.

1 See Hardin, 1968, 'The tragedy of the commons', *Science*. Elinor Ostrom won a Nobel Prize for her work on this topic.

5 Mixed-strategy equilibrium

Consider a mixed strategy profile $\sigma \in \Sigma$. Suppose there is some $i \in N$ and $\sigma_i' \in \Sigma_i$ such that

$$u_i(\sigma_i', \sigma_{-i}) > u_i(\sigma).$$

1. Argue that there must be some $s_i \in S_i$ such that

$$u_i(s_i, \sigma_{-i}) > u_i(\sigma).$$

2. Explain the definition of mixed-strategy equilibrium in light of this.