Multiple Choice Questions.

i) Choose the best answer.
ii) There are 6 questions. Question 6 is a bonus question. Each question carries 1 mark; the maximum aggregate score is 5 marks.

1. (Refer to Qn 5.1 of the Practice Problem Set. A strategy is weakly dominant if, regardless of what any other players do, the strategy earns a player a weakly higher payoff than other strategies. For instance, "Go To Party" is a weakly dominant strategy for Evangeline, and for Gabriel too.)

Which of the following is true?

A. A PSNE is a MSNE, the reverse is not always true
B. In a Prisoner's Dilemma, each player has a strictly dominant strategy
C. In a Prisoner's Dilemma, there is a unique PSNE
D. A strictly dominant strategy is also weakly dominant, the reverse is not always true
E. All of the other options

2. There is pool of \$8. Two players A and B take turns to take either \$2 or \$3 from the pool. The game terminates when there is (strictly) less than \$3 remaining. If player A goes first, what is the Nash equilibrium outcome of the game?

A. Player A gets \$6, player B gets \$2
B. Player A gets \$5, player B gets \$3
C. Player A gets \$4, player B gets \$3
D. Player A gets \$3, player B gets \$3
E. None of the other options

By backward induction, the Nash equilibrium strategy is that Player A takes \$2 in round 1, Player B takes \$3 in round 2, and Player A takes \$3 in round 3. Thus (B) is the Nash equilibrium outcome.

3. In a city with a population of 65, each person i chooses their desired amount of education e_i. Education offers a direct benefit and a positive externality on society, which we model using the utility function $U_i(e_i) = 24\sqrt{e_i} + 2\sqrt{f_i} - 4e_i$, where $f_i = \sum_{j \neq i} e_j$ represents the total education level across the rest of the population. What is the efficient level of education e_i^* for the city?

A. 8.81
B. 9
C. 10.56
D. 25
E. None of the other options

We want to internalize the externality by letting $f_i = 64e_i$, so we choose e_i to maximize $24\sqrt{e_i} + 2\sqrt{64e_i} - 4e_i = 40\sqrt{e_i} - 4e_i$. The first order condition is $\frac{20}{\sqrt{e_i}} - 4 = 0$, so $e_i = 25$.

4. X and Y each have $\$w$. They simultaneously allocate their budget to either private consumption at $\$p$ per unit, or a public good at a price of \$1 per unit. Both consumers have utility function $U(c, G) = cG$, where c is the private consumption and G is the total amount of the public good. If X and Y only maximize their own utility, the equilibrium amount of public good G^* is

A. $w/3$
B. $w/2$
C. $2w/3$
D. w
E. None of the other options

We choose G_X to maximize $U_X = c_X(G_X + G_Y) = \frac{w-G_X}{p}(G_X + G_Y)$. The first order condition is $\frac{w-G_Y-2G_X}{p} = 0$. By symmetry, we also have $G_X = G_Y$, hence $G_X = \frac{w}{3}$ and $G^* = G_X + G_Y = \frac{2w}{3}$.

5. In FASS, *teh tarik* can only be purchased at the Deck or the Café on the Ridge. *Teh tarik* can be made using different amounts of sugar s, which we assume to be in the interval [0,1]. Customers have sugar preferences c uniformly distributed along [0,1]. Each customer knows her sugar preference and gains a utility of $1.3 - |c - s| - p$ from purchasing a cup of *teh tarik* containing s amount of sugar at price p. Suppose both places are mandated to sell *teh tarik* at \$1 per cup, and they choose fixed sugar levels s_D and s_C simultaneously. Consumers will purchase exactly one cup of *teh tarik* from a place that provides the highest utility, if it is positive. [As an example, suppose that a consumer prefers a sugar level of $c = 0.1$. Then he will purchase *teh tarik* from the Deck if $(s_D, s_C) = (0, 0.3)$, but will not consume any if $(s_D, s_C) = (0.8, 0.6)$.] Given that both places want to maximize revenue, which of the following pair(s) of (s_D, s_C) is a Nash equilibrium?

A. (0.2,0.8)
B. (0.3,0.7)
C. (0.5,0.5)
D. More than one of the other options
E. None of the other options

At (A), a deviation to $s_D = 0.3$ will increase demand from 0.5 to 0.55, so it is not an equilibrium.

At (B), any change of s_D or s_c results in a decrease in demand (for the respective firm), so it is an equilibrium.

At (C), a deviation to $s_D = 0.3$ will increase demand from 0.3 to 0.4, so it is not an equilibrium.

6. (Bonus question) What is/are the MSNE(s) of the following game?

		Player 2	
		Left	Right
Player 1	Up	0,1	4,0
	Middle	1,0	2,3
	Down	2,1	1,2

A. Player 1 plays "Up" with probability 1/2 and "Down" with probability 1/2. Player 2 plays "Left" with probability 3/5 and "Right" with probability 2/5.
B. Player 1 plays "Up" with probability 3/4 and "Middle" with probability 1/4. Player 2 plays "Left" with probability 2/3 and "Right" with probability 1/3.
C. Player 1 plays "Middle" with probability 1/4 and "Down" with probability 3/4. Player 2 plays "Left" with probability 1/2 and "Right" with probability 1/2.
D. More than one of the other options
E. None of the other options

Check that Player 1 is better off deviating to playing "Down" in (B), and Player 2 is better off deviating to playing "Right" in (C), while there are no incentives to deviate in (A). Hence (A) is the only MSNE.

Alternatively, observe that the strategy "Middle" is strictly dominated by the mixed strategy 2/5 "Up" and 3/5 "Down": Player 1 gets a payoff of 6/5 and 11/5 against "Left" and "Right" respectively by playing this mixed strategy. Strictly dominated strategies cannot be played in a Nash equilibrium, so we can reduce the game to a standard 2 by 2 form to determine the Nash equilibria.