Practice Problem Set 2

Uncertainty (C.12), Monopoly (C.25)

Question 2.1

Penny is a businesswoman who has invested in a foreign country. Voters in the foreign country recently elected a new president who threatens to close the country's economy and impose autarky. If the president does not carry out his threat, Penny will earn 36 bitcoins. If he carries out his threat, Penny's overseas assets will be expropriated and she will earn nothing.

Penny is considering whether to buy insurance to cover this political risk. Suppose the probability of the foreign president carrying out his threat is 0.1. Penny is an expected utility maximizer with a utility function $U = C^{1/2}$, where C is the amount of money that Penny receives (in bitcoins).

i) Plot a diagram with C_A as the x-axis and C_{NA} as the y-axis, where C_A is Penny's earnings if the foreign country becomes autarkic and C_{NA} is Penny's earnings otherwise. Label Penny's endowment point on the diagram.

ii) Suppose every 1 bitcoin of insurance costs 0.25 bitcoins. Write down the equation of Penny's state-contingent budget constraint and draw it on the diagram. Compute Penny's optimal C_A and C_{NA}. How much insurance premium will she pay?

iii) Suppose every 1 bitcoin of insurance costs 0.1 bitcoins instead. Write down the equation of Penny's state-contingent budget constraint and draw it on the diagram. Compute Penny's optimal C_A and C_{NA}. How much insurance premium will she pay?

ANSWER

If Penny buys K unit (in bitcoins) of insurance, which costs γ per unit, she pays an insurance premium of γK.

Now $C_{NA} = 36 - \gamma K$ and $C_A = K - \gamma K = (1-\gamma)K$.
Rearranging, we have $C_{NA} = 36 - \frac{\gamma}{1-\gamma} C_A$.

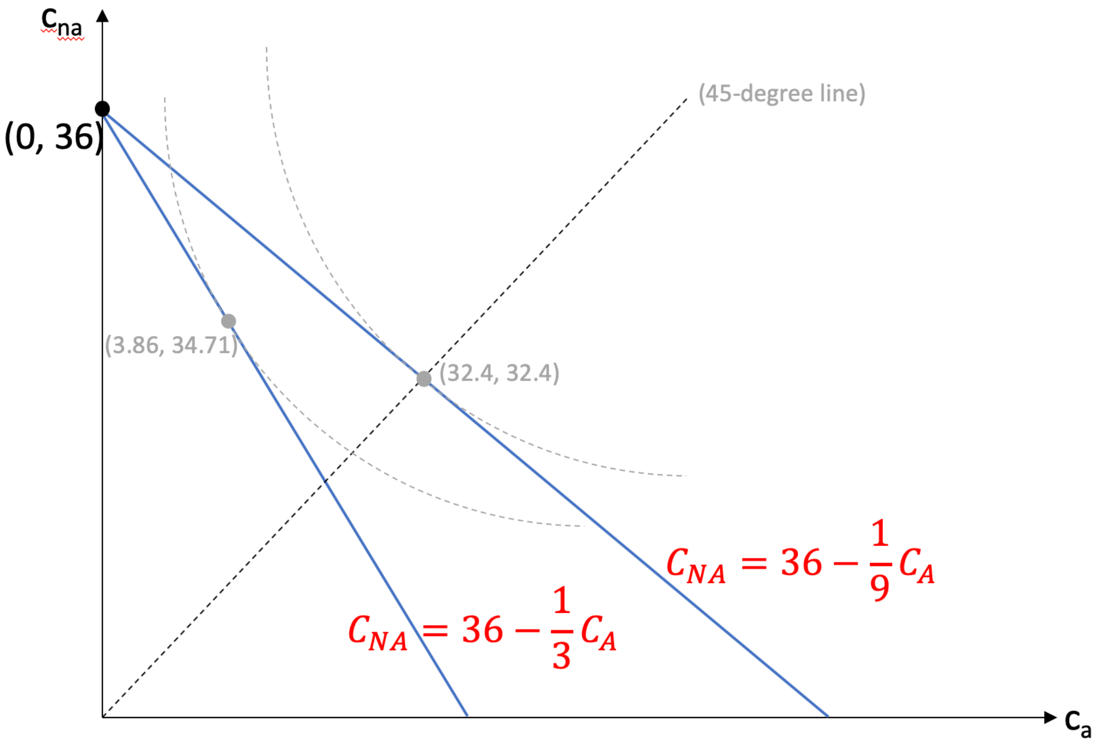
The expected utility of Penny is

$$EU = \pi\, U(C_A) + (1-\pi)U(C_{NA}) = \pi\sqrt{C_A} + (1-\pi)\sqrt{36 - \frac{\gamma}{1-\gamma}C_A}$$

The optimality condition is given by

$$\frac{dEU}{dC_A} = 0 \Rightarrow \frac{\pi}{1-\pi} = \frac{\gamma}{1-\gamma} \times \frac{\sqrt{C_A}}{\sqrt{36 - \frac{\gamma}{1-\gamma}C_A}}$$

i)



ii) Now $\gamma = 0.25$, so the state-contingent budget constraint is $C_{NA} = 36 - \frac{1}{3}C_A$. From the optimality condition, we can derive that $C_A = 3.86, C_{NA} = 34.71, \gamma K = 1.29$.

iii) Now $\gamma = 0.1$, so the state-contingent budget constraint is $C_{NA} = 36 - \frac{1}{9}C_A$. From the optimality condition, we can derive that $C_A = C_{NA} = 32.4, \gamma K = 3.6$. Note that then insurance is fair, and indeed as shown in class, Penny is fully insured.

Question 2.2

Orange is the only firm that sells consumer good X. Show that if the demand for good X is elastic, increasing the price of X decreases the revenue of Orange.

Answer

We need to show that $\frac{dTR}{dP} < 0$ when demand is elastic (in other words, when $\epsilon < -1$).

$$TR \quad = P \cdot Q$$

$$\frac{dTR}{dP} \quad = P \cdot \frac{DQ}{dP} + Q = Q\left(\frac{P}{Q} \cdot \frac{DQ}{dP} + 1\right) = Q(\epsilon + 1)$$

Since Q is non-negative, $Q(\epsilon + 1)$ is negative as long as $\epsilon < -1$.

Question 2.3

A drug firm is considering whether to sell a new drug, which it owns the patent. The demand for the drug is given by $Q = 2000 - 100P$. To sell this drug, the firm needs to purchase a machine, which costs \$7000. Once the machine is acquired, the marginal cost of producing a tablet of the drug is \$4.

i) What are the firm's optimal price-quantity decisions?

ii) Suppose now that the firm can acquire the machine for free. What are the firm's optimal price-quantity decisions?

iii) The firm has now acquired the machine. However, due to some design flaw, the machine can only produce 500 tablets. What are the firm's optimal price-quantity decisions?

Answer

i) Set MR=MC, we get P=12 and Q=800. However, profit is 12*800-7000-4*800=-600. The firm should not acquire the machine. So the optimal output is 0.

ii) P=12 and Q=800. The firm now makes a profit of \$6400.

iii) Since the firm has already acquired the machine, regardless of whether the machine was acquired at a cost or for free, the firm will produce the drug (any sunk cost should not affect present decisions).

The firm should set Q=500. Accordingly, P=\$15.