

LECTURE 2

PREFERENCE

BUDGET CONSTRAINT

CONSUMER CHOICE



# Where are we?

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- Preference
  - ▣ Indifference curves
    - Do not cross
    - Downward sloping if “more is better” is satisfied for both goods
    - Slope of indifference curve
  - ▣ Utility functions
  - ▣ Special preferences
- Budget constraint
- Consumer choice
  - ▣ Which basket will the consumer choose to buy?

Part 1

# Preference

# Marginal Rate of Substitution

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- Definition 2.1 *Marginal rate of substitution* of  $x$  for  $y$  is the rate at which the consumer is willing to give up  $y$  to get more of  $x$ , maintaining the same level of satisfaction

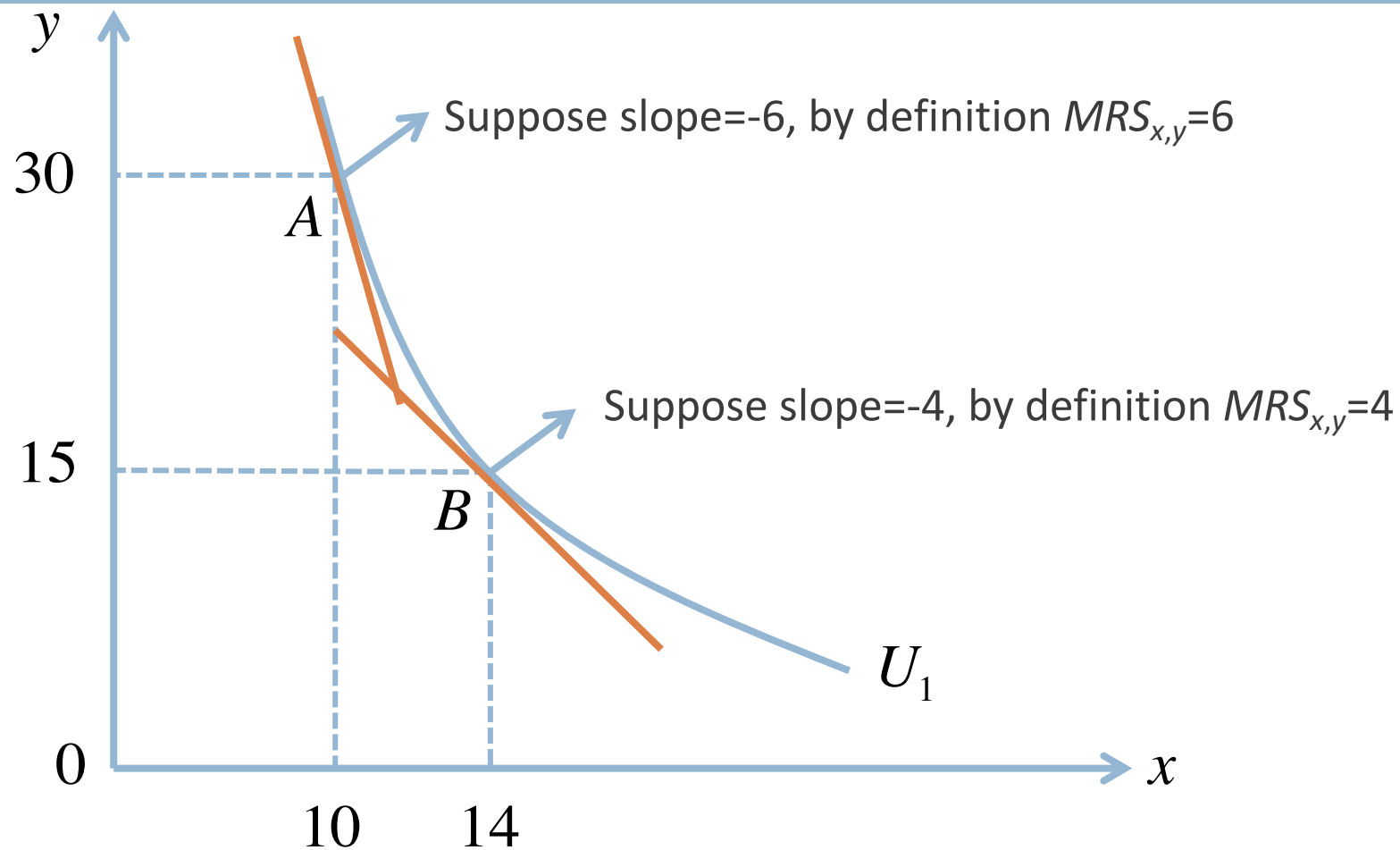
$$MRS_{x,y} = - \left. \frac{dy}{dx} \right|_{\text{Same } U} = - \left. \frac{\Delta y}{\Delta x} \right|_{\text{Same } U}$$

where  $\Delta x$  is extremely small

- $MRS_{x,y}$  is the negative of the slope of the indifference curve (with  $x$  on the horizontal axis and  $y$  on the vertical axis)

# MRS and the Shape of Indifference Curve

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# Diminishing Marginal Rate of Substitution

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- *Diminishing marginal rate of substitution* means  $MRS_{x,y}$  decreases as the consumer gets more  $x$  and less  $y$  along the same indifference curve
  - ▣ Holding satisfaction level fixed, as the consumer gets more of  $x$ , the willingness to give up  $y$  and get additional  $x$  reduces
- If diminishing marginal rate of substitution holds
  - ▣ And if the three assumptions hold
  - ▣ Then indifference curves are convex to the origin

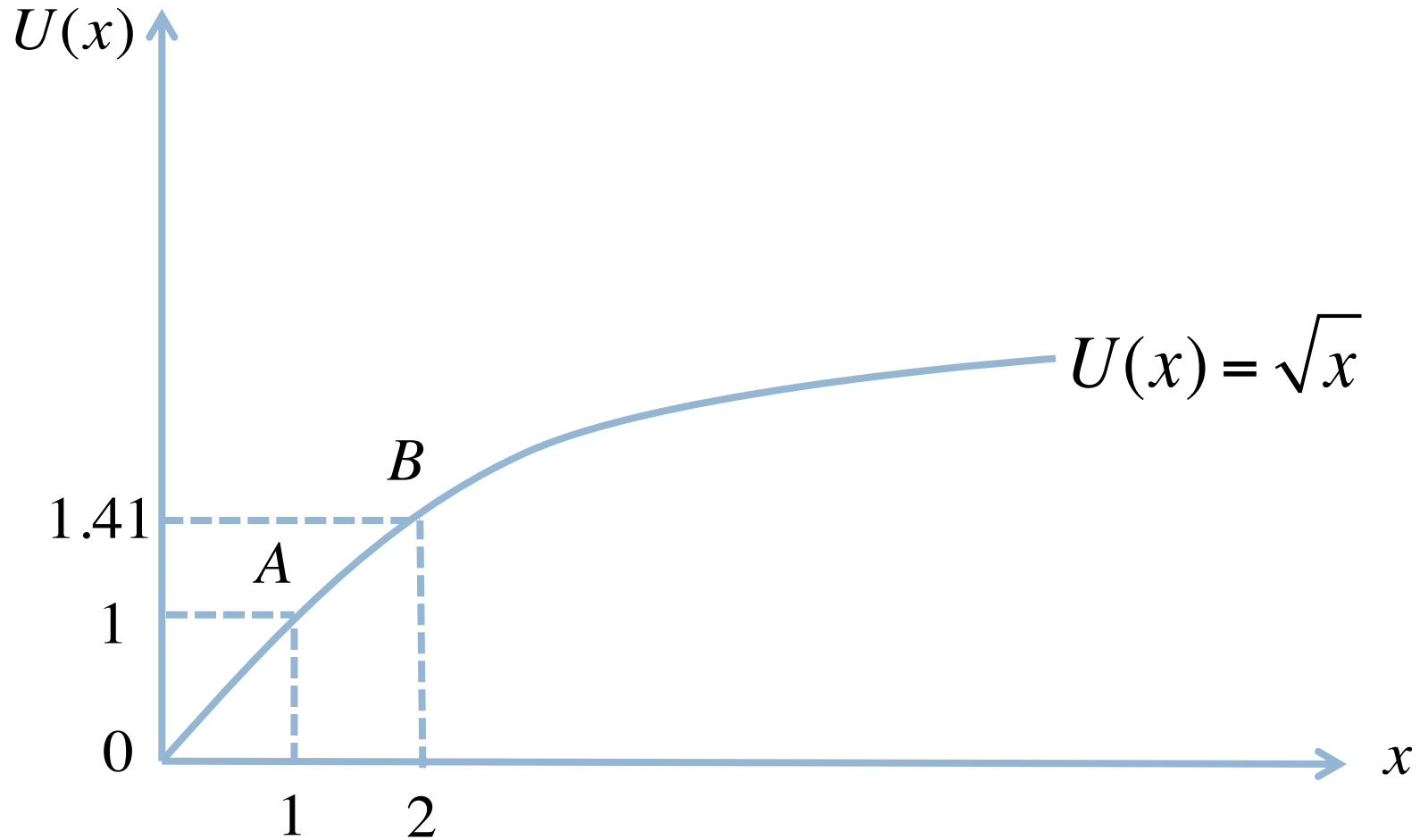
# Utility Function

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- *Utility* is a numeric value indicating the consumer's level of satisfaction
- Definition 2.2 *Utility function* assigns a level of utility to each consumption basket so that  $A \succ B$  is equivalent to  $U(A) > U(B)$ 
  - ▣ Utility function represents preference
  - ▣ Higher utility = higher the level of satisfaction

# Utility Function with One Good: An Example

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# Marginal Utility

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- Definition 2.3 *Marginal utility* is the rate at which utility changes as the level of consumption of a good changes

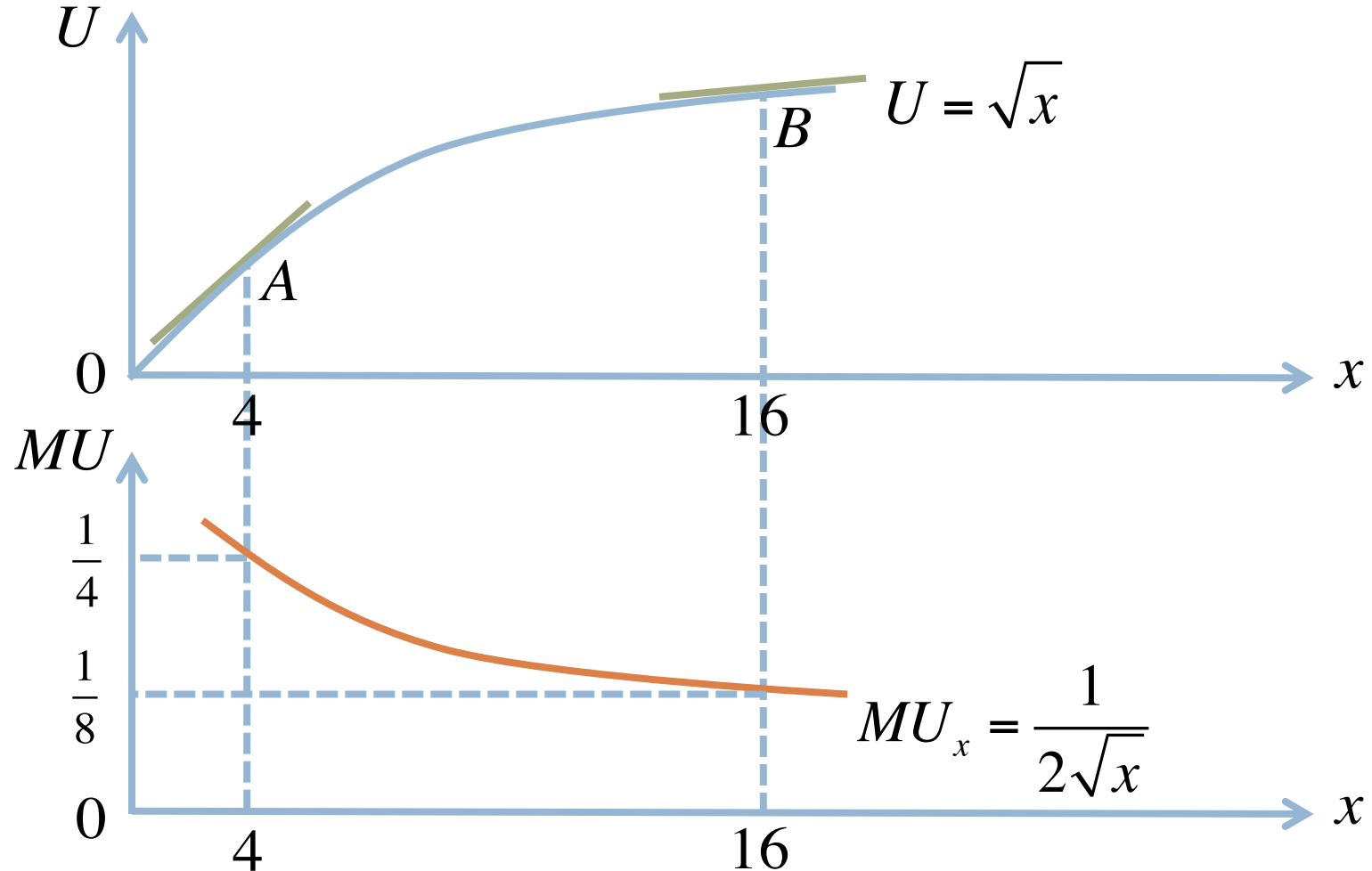
$$MU_x = \frac{dU}{dx} = \frac{\Delta U}{\Delta x}$$

where  $\Delta x$  is extremely small

- $MU$  is the slope of the utility function
- What does the sign of marginal utility tell us?

# Marginal Utility in Graph

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# Principle of Diminishing Marginal Utility

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- *Principle of diminishing marginal utility*
  - ▣ Marginal utility decreases as consumption level rises
  - ▣ Utility increases slower as consumption level rises
  - ▣ Utility function becomes flatter as consumption level rises

# Marginal Utility with Two Goods

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- More often, we will deal with utility functions with two goods  $U(x, y)$
- Marginal utility of  $x$

$$MU_x = \frac{\partial U}{\partial x}$$

- Marginal utility of  $y$

$$MU_y = \frac{\partial U}{\partial y}$$

- Diminishing marginal utilities
  - ▣  $MU_x$  decreases with  $x$ , holding  $y$  constant
  - ▣  $MU_y$  decreases with  $y$ , holding  $x$  constant

# Utility Function with Two Goods: An Example

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- Suppose the utility function of the consumer is

$$U(x, y) = \sqrt{xy}$$

- “More is better” is satisfied for both goods

$$MU_x = \frac{\sqrt{y}}{2\sqrt{x}} > 0, \quad MU_y = \frac{\sqrt{x}}{2\sqrt{y}} > 0$$

- Diminishing marginal utility is satisfied for both goods

# From Utility Function to Indifference Curves

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- Indifference curves can be drawn from the utility function
- (2,2) indifferent to (1,4) and (4,1)

$$U(2,2) = U(1,4) = U(4,1) = \sqrt{4} = 2$$

- (3,3) preferred to (2,2)

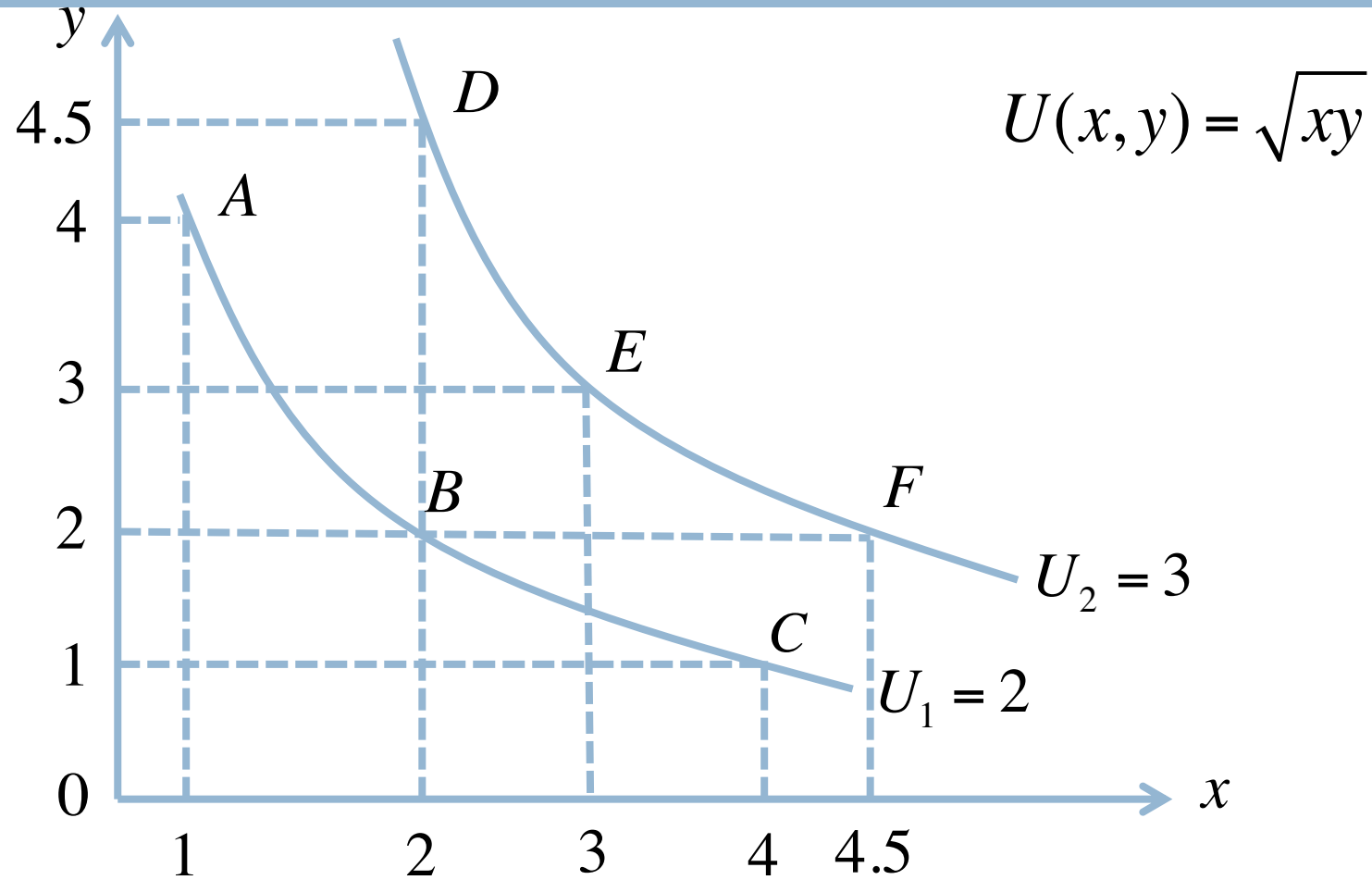
$$U(3,3) = 3 > U(2,2) = 2$$

- (3,3) indifferent to (2,4.5) and (4.5,2)

$$U(3,3) = U(2,4.5) = U(4.5,2) = 3$$

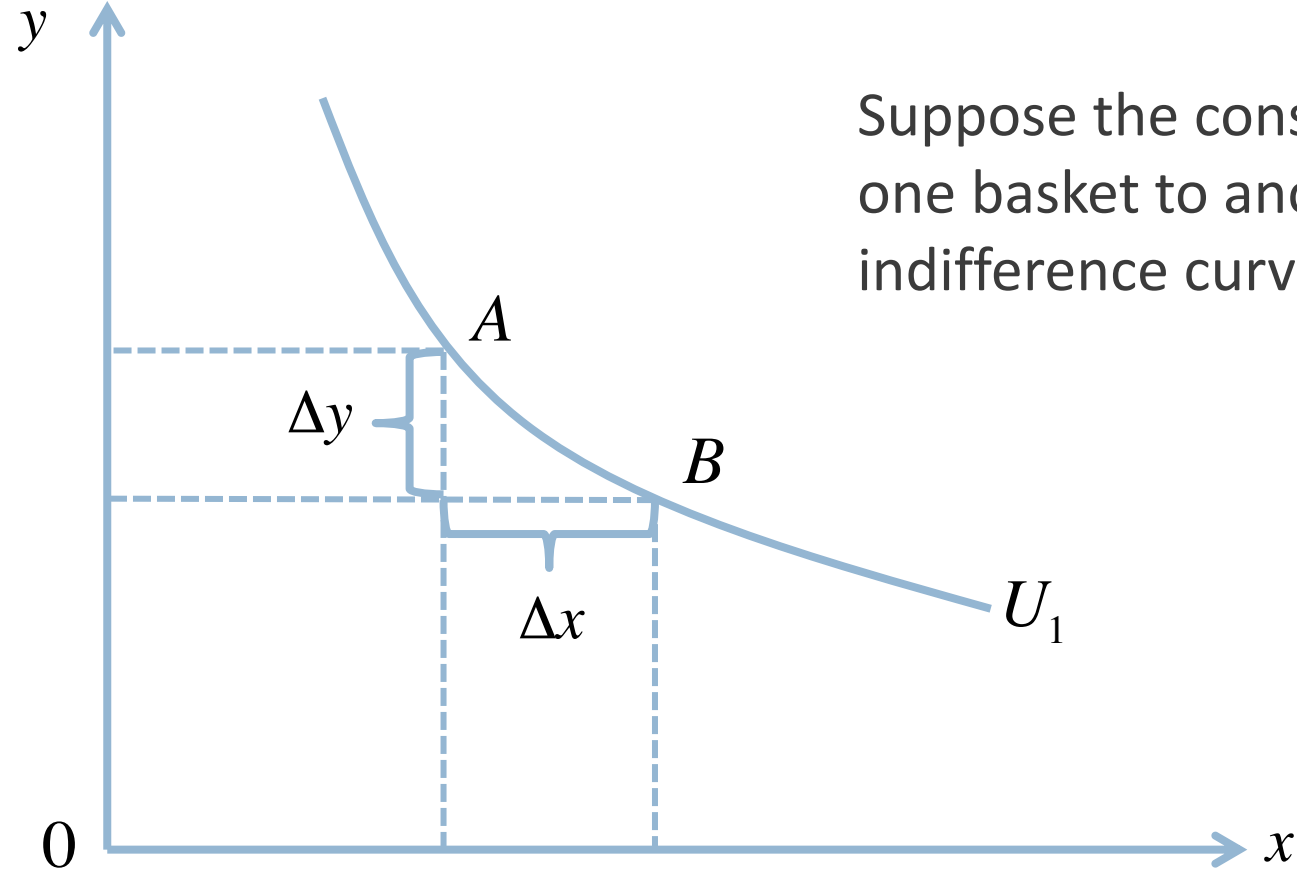
# Plotting Indifference Curves

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# MRS and Utility Function

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Suppose the consumer moves from one basket to another along the same indifference curve



# MRS and Utility Function Cont'

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- The total change in utility is

$$\Delta U = MU_x(\Delta x) + MU_y(\Delta y)$$

- The total change in utility must be 0

$$0 = MU_x(\Delta x) + MU_y(\Delta y)$$

- Thus

$$\frac{MU_x}{MU_y} = -\frac{\Delta y}{\Delta x} = MRS_{x,y}$$

- ▣ The rate at which the consumer is willing to substitute between the two goods holding utility constant is equal to the ratio of the marginal utilities of the two goods

# Special Preferences

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- Definition 2.4 Perfect substitutes
  - ▣ Two goods are *perfect substitutes* if  $MRS$  is constant
  - ▣ Indifference curves are linear
- Definition 2.5 Perfect complements
  - ▣ Two goods are *perfect complements* if  $MRS$  is 0 or infinity
  - ▣ Indifference curves are L-shaped

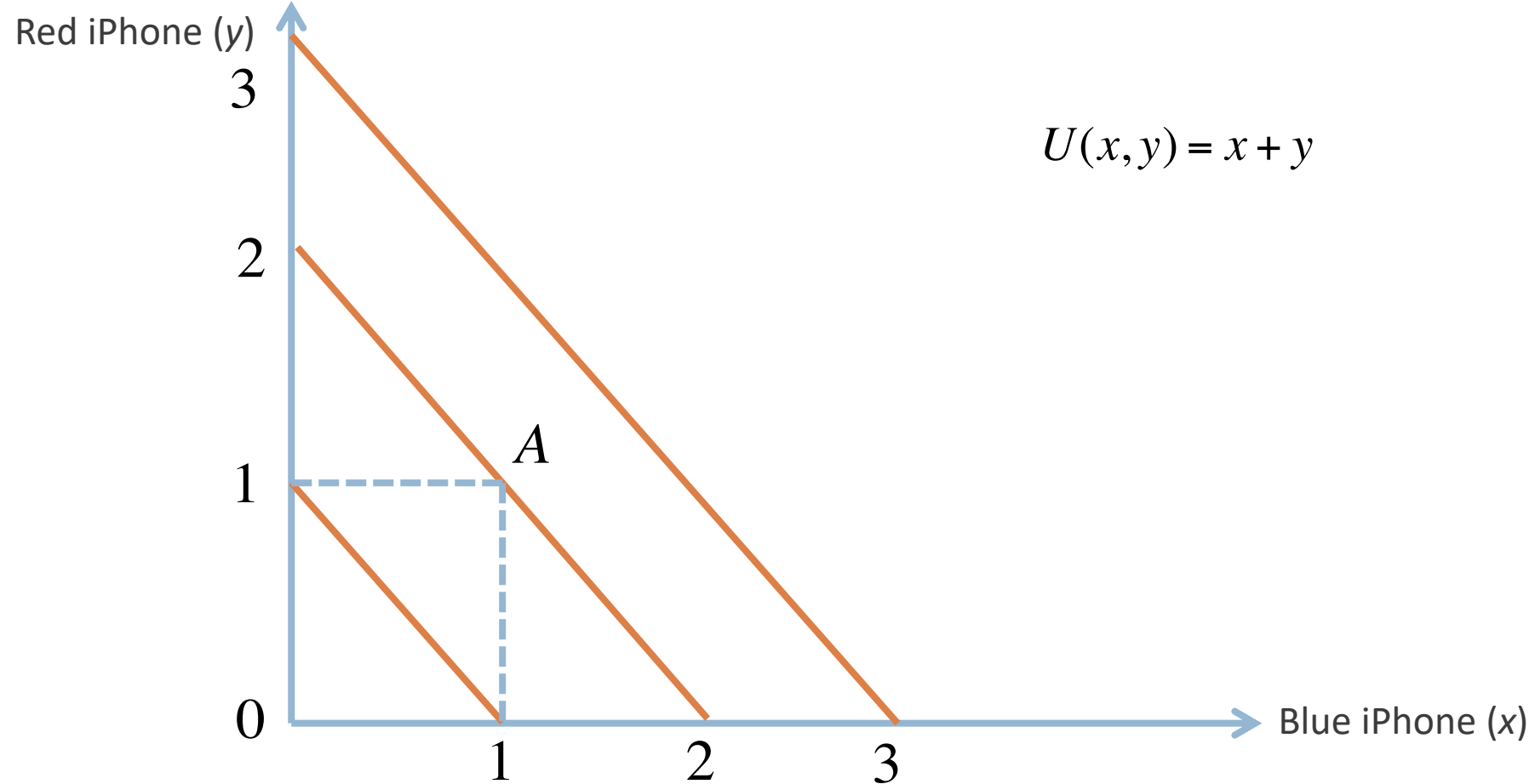
# Example of Perfect Substitutes

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- Suppose the consumer has the following preference
  - ▣ “Blue iPhones and red iPhones are equivalent”
  - ▣ 1 blue iPhone always brings the same utility as 1 red iPhone
- To the consumer, blue iPhone and red iPhone are perfect substitutes

# Example of Perfect Substitutes: iPhones

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# Example of Perfect Substitutes: iPhones Cont'

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- We know from the utility function for iPhones
  - ▣ Marginal utility of blue iPhone:  $MU_x = 1$
  - ▣ Marginal utility of red iPhone:  $MU_y = 1$
  - ▣  $MRS_{x,y} = 1/1 = 1$ 
    - Verify from graph, slope of indifference curve is -1
- For perfect substitutes
  - ▣ Utility functions are linear

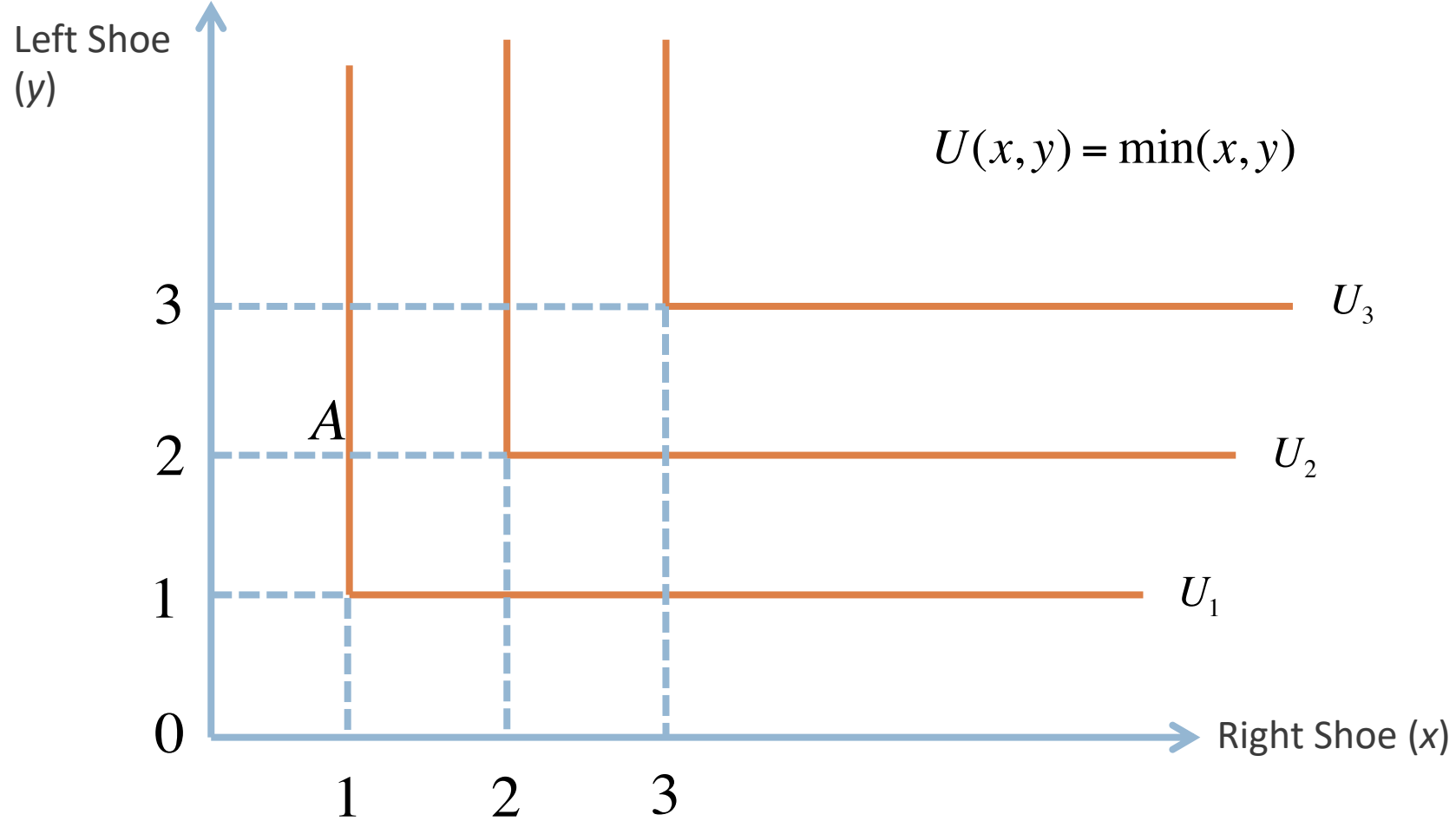
# Example of Perfect Complements

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- Suppose the consumer has the following preference
  - ▣ “For every right shoe, I need exactly one left shoe”
  - ▣ 2 right shoes and 1 left shoe brings the same utility as 1 right shoe and 1 left shoe
- To the consumer, right shoes and left shoes are perfect complements

# Example of Perfect Complements: Shoes

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# Example of Perfect Complements: Shoes Cont'

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- How does the “min” function work?
  - ▣  $\min(x, y)$  = the smaller of  $x$  and  $y$
  - ▣ E.g. if  $x=1$  and  $y=2$ ,  $U(x, y) = \min(1, 2) = 1$
  - ▣ E.g. if  $x=1$  and  $y=1$ ,  $U(x, y) = \min(1, 1) = 1$
- $MRS_{x,y}$  is
  - ▣ Infinity in the vertical part
  - ▣ 0 in the horizontal part
- For perfect complements
  - ▣ Utility functions are the “min” functions



Part 2

# Budget Constraint

# Budget Constraint

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- Suppose consumer chooses  $F$  units of food and  $C$  units of clothing
- The price of food is  $P_F$
- The price of clothing is  $P_C$
- Consumer has limited income  $I$
- *Budget constraint is*

$$P_F F + P_C C \leq I$$

# Budget Set vs. Budget Line

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## □ Budget set

- ▣ The set of all baskets the consumer can afford, that is, all baskets that satisfy the budget constraint

## □ Budget line

- ▣ The set of all baskets consumer can afford by spending all income

$$P_F F + P_C C = I$$

- ▣ Rearranging the budget line

$$C = \frac{I}{P_C} - \frac{P_F}{P_C} F$$

# Example of Budget Constraint

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- Suppose
  - ▣ The price of food is 20
  - ▣ The price of clothing is 40
  - ▣ Consumer's income is 600
- Budget constraint (and budget set) is

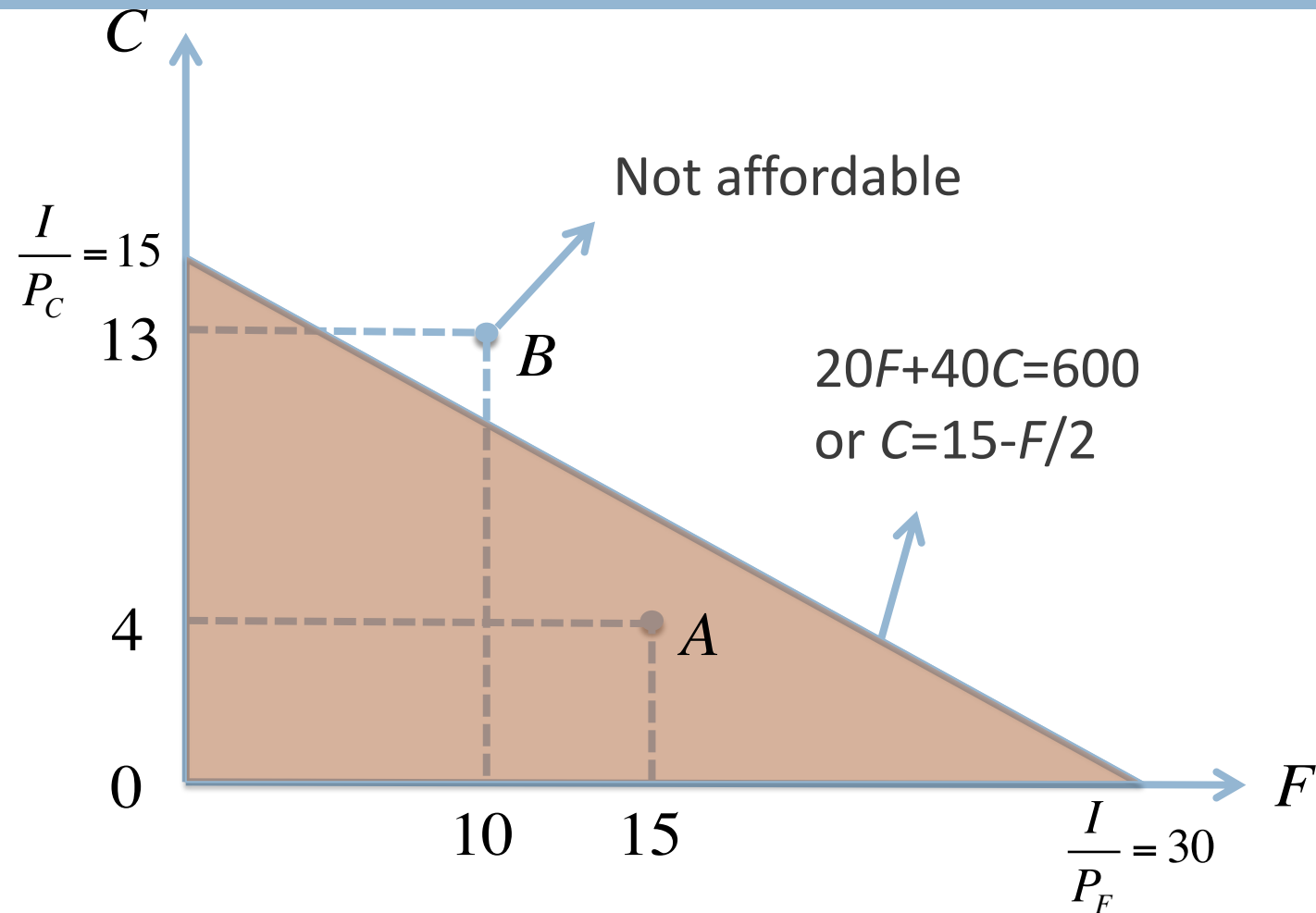
$$20F + 40C \leq 600$$

- Budget line is

$$20F + 40C = 600$$

# Budget Constraint in Graph

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# Slope of Budget Line

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- The budget line has a slope of

$$-\frac{P_F}{P_C} = -\frac{20}{40} = -\frac{1}{2}$$

- Slope of budget line represents the rate at which two goods can be substituted in the market, that is, based on the prices
  - ▣ Because clothing is twice as expensive as food
  - ▣ To get 1 additional unit of food, the consumer *must* give up 0.5 unit of clothing

Part 3

# Consumer Choice

# Optimal Choice

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- What is the optimal basket?
  - ▣ Consumer chooses the basket that gives him/her the highest utility given the budget constraint
- Let  $U(F,C)$  be the utility function of  $F$  units of food and  $C$  units of clothing, the consumer solves

$$\max_{F,C} U(F,C)$$

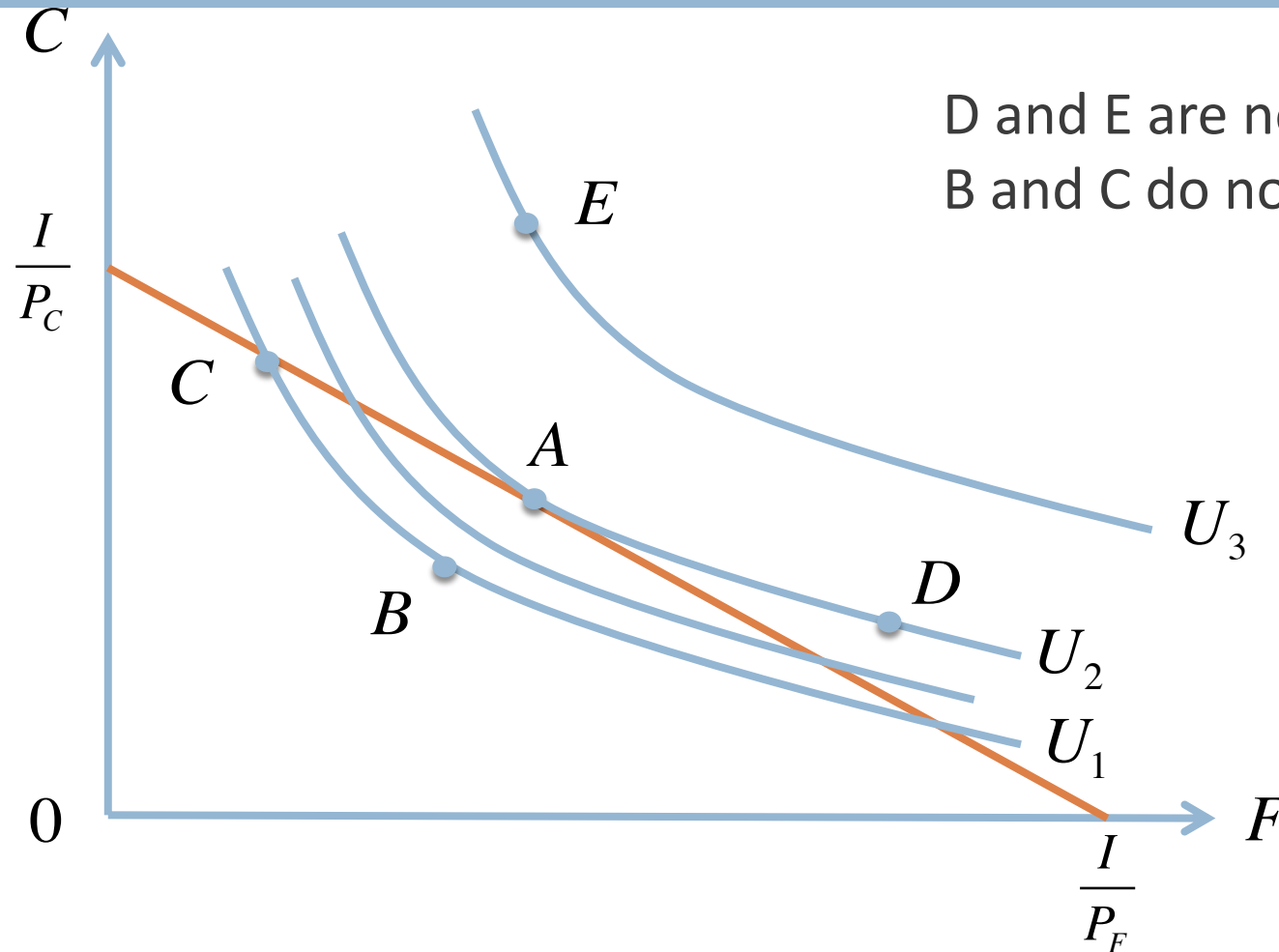
$$s.t. \quad P_F F + P_C C \leq I$$

- ▣  $F$  and  $C$  are the choice variables
- ▣  $P_F$ ,  $P_C$ , and  $I$  are the parameters



# Finding the Optimal Basket from the Graph

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D and E are not affordable  
B and C do not maximize utility

# Two Conditions for Optimal Choice

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- Consumer chooses the basket
  - ▣ On the budget line
  - ▣ On the highest indifference curve
- At the optimal basket
  - ▣ Consumer spends all the money

$$P_F F + P_C C = I$$

- ▣ Budget line tangent to indifference curve

$$-\frac{P_F}{P_C} = -MRS_{F,C} \Rightarrow \frac{P_F}{P_C} = MRS_{F,C}$$

# The Tangency Condition

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$$MRS_{F,C} = \frac{P_F}{P_C}$$

- To maximize utility, the amount of  $F$  and  $C$  consumed should be such that
  - ▣ The rate at which the consumer is willing to substitute between the two goods holding utility constant is equal to the rate at which the two goods are exchanged in the market
  - ▣ Otherwise the consumer is not maximizing utility

# The Equal Marginal Principle

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- Since

$$MRS_{F,C} = \frac{MU_F}{MU_C}$$

- The tangency condition can be rewritten as

$$\frac{MU_F}{MU_C} = \frac{P_F}{P_C} \Rightarrow \frac{MU_F}{P_F} = \frac{MU_C}{P_C}$$

- To maximize utility, consumer sets marginal utility per dollar of expenditure equal for both goods
  - ▣ Extra utility per dollar spent on food is the same as the extra utility per dollar spent on clothing

# What if $MU$ per dollar are not the same?

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- The equal marginal principle requires

$$\frac{MU_F}{P_F} = \frac{MU_C}{P_C}$$

- Suppose

$$\frac{MU_F}{P_F} < \frac{MU_C}{P_C}$$

- Assume the current basket is on the budget line
- To maximize utility, the consumer should buy
  - ▣ \$1 spent on  $C$  brings higher extra utility than \$1 spent on  $F$

# Finding the Optimal Basket using the Lagrangian Method

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- Recall the general form of the constrained maximization problem

$$\max_{x,y} f(x,y)$$

$$s.t. \quad g(x,y) = 0$$

- In consumer theory, it is

$$\max_{x,y} U(x,y)$$

$$s.t. \quad I - P_x x - P_y y = 0$$

# Finding the Optimal Basket using the Lagrangian Method

## Cont'

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- The Lagrangian function is

$$\Lambda(x, y, \lambda) = U(x, y) + \lambda(I - P_x x - P_y y)$$

- The first-order conditions are

$$\frac{\partial \Lambda}{\partial x} = MU_x - \lambda P_x = 0$$

$$\frac{\partial \Lambda}{\partial y} = MU_y - \lambda P_y = 0$$

$$\frac{\partial \Lambda}{\partial \lambda} = I - P_x x - P_y y = 0$$

- The third equation is just the budget line

# Meaning of the Lagrange Multiplier

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- Rearranging the first two equations we have the tangency condition

$$\lambda = \frac{MU_x}{P_x} = \frac{MU_y}{P_y} \Rightarrow MRS_{x,y} = \frac{P_x}{P_y}$$

- Using the Lagrangian method, we still end up solving
  - ▣ The tangency condition
  - ▣ The budget line
- What is the meaning of the Lagrange multiplier?
  - ▣ The multiplier is the extra utility of one extra dollar of consumption



# Price and Willingness to Pay

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- Rearranging the equation

$$\lambda = \frac{MU_x}{P_x} = \frac{MU_y}{P_y}$$

- We have

$$P_x = \frac{MU_x}{\lambda}, \quad P_y = \frac{MU_y}{\lambda}$$

- The right-hand side of the equation represents how much the consumer is willing to pay for one more unit of the good
  - ▣ The extra utility from consuming one more unit of the good divided by the extra utility of one dollar spending
- Price is equal to the consumer's willingness to pay for one more unit of the good