

LECTURE 6
FIRST WELFARE THEOREM
WALRAS' LAW
MIDTERM REVIEW



Where are we?

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- Edgeworth box
- Pareto efficiency
- Competitive equilibrium
- First (and second) welfare theorems
 - ▣ What is the relationship between Pareto efficiency and competitive equilibrium?
- Walras' Law
 - ▣ A result derived from budget lines and optimal baskets
- Midterm Review

Part 1

First Welfare Theorem

General Competitive Equilibrium

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- In a competitive equilibrium,
 - ▣ Each consumer maximizes his/her utility given the budget constraint

$$x_1^{*A}, x_2^{*A}, x_1^{*B}, x_2^{*B}$$

denotes the optimal consumption for each consumer given the equilibrium prices

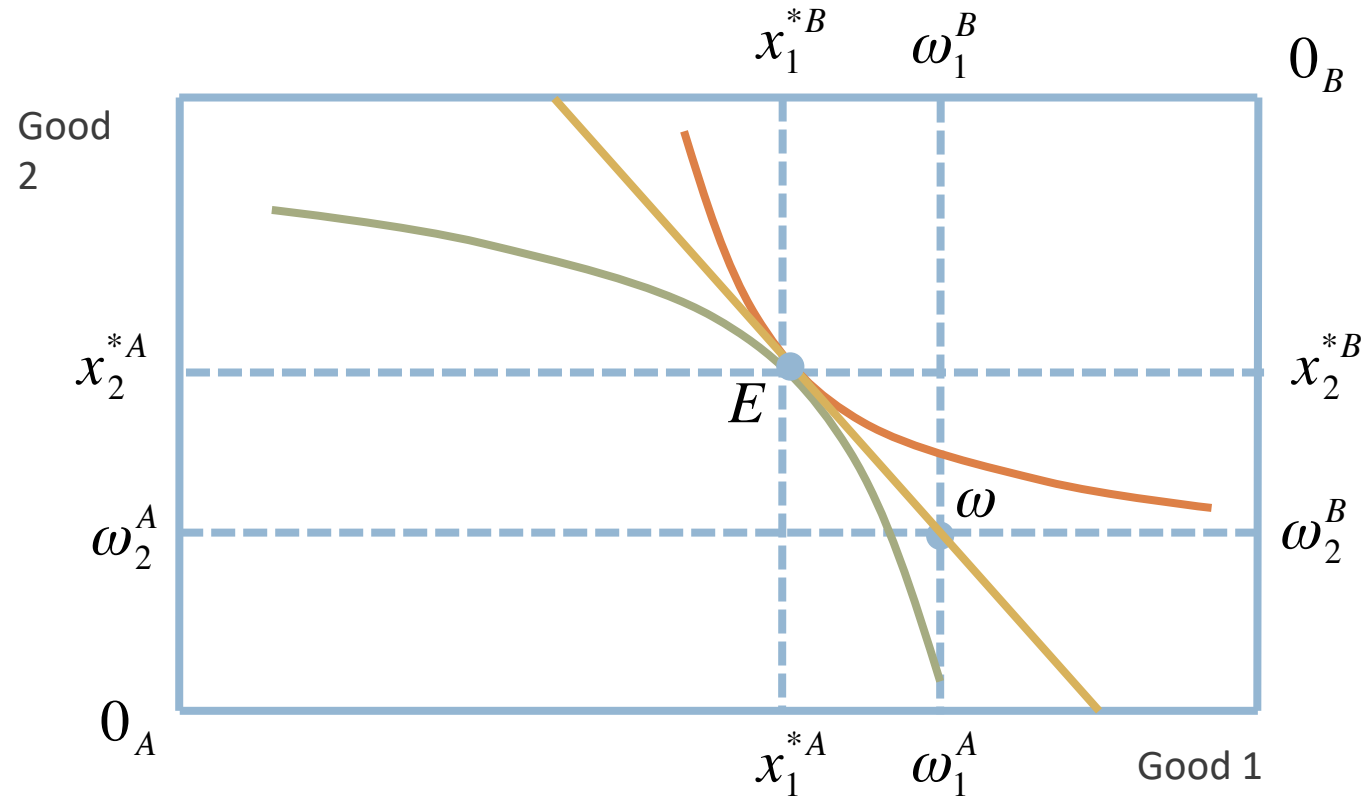
- ▣ Markets for both goods clear

$$x_1^{*A} + x_1^{*B} = \omega_1^A + \omega_1^B$$

$$x_2^{*A} + x_2^{*B} = \omega_2^A + \omega_2^B$$

Competitive Equilibrium in Graph

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At point E, the two consumers' indifference curves are tangent to each other

First Welfare Theorem

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- Definition 6.1 The *First Fundamental Theorem of Welfare Economics* states that a competitive equilibrium allocation is Pareto efficient

- Suppose the equilibrium prices are (P_1, P_2) and the allocation

$$x_1^{*A}, x_2^{*A}, x_1^{*B}, x_2^{*B}$$

is the equilibrium allocation

- Then the allocation

$$x_1^{*A}, x_2^{*A}, x_1^{*B}, x_2^{*B}$$

is Pareto efficient

Proof of First Welfare Theorem

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- Suppose at the equilibrium prices P_1 and P_2 , the equilibrium allocation is $x_1^{*A}, x_2^{*A}, x_1^{*B}, x_2^{*B}$
- Proof by contradiction: suppose this allocation is not Pareto efficient
- Then there must exist another feasible allocation

$$y_1^A, y_2^A, y_1^B, y_2^B$$

- ▣ where at least one consumer is better off
- ▣ and no one is worse off
- ▣ compared to the equilibrium allocation

Proof of First Welfare Theorem Cont'

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- Suppose consumer A strictly prefers (y^A_1, y^A_2) to (x^{*A}_1, x^{*A}_2) while consumer B weakly prefers (y^B_1, y^B_2) to (x^{*B}_1, x^{*B}_2)
- By definition, the equilibrium allocation is the utility-maximizing basket for each consumer given the budget constraint, thus by revealed preference,

$$P_1 y^A_1 + P_2 y^A_2 > P_1 \omega^A_1 + P_2 \omega^A_2 \quad (1)$$

Proof of First Welfare Theorem Cont'

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- Add up (1) and (2), we have

$$P_1(y_1^A + y_1^B) + P_2(y_2^A + y_2^B) > P_1(\omega_1^A + \omega_1^B) + P_2(\omega_2^A + \omega_2^B) \quad (3)$$

- Allocation $y_1^A, y_2^A, y_1^B, y_2^B$ must also be feasible

$$y_1^A + y_1^B = \omega_1^A + \omega_1^B$$

$$y_2^A + y_2^B = \omega_2^A + \omega_2^B$$

- Substituting into (3), we have a contradiction

$$P_1(\omega_1^A + \omega_1^B) + P_2(\omega_2^A + \omega_2^B) > P_1(\omega_1^A + \omega_1^B) + P_2(\omega_2^A + \omega_2^B)$$

The Invisible Hand

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- Each consumer maximizes his/her own utility
- No central planner
- Yet competitive market leads to a Pareto efficient allocation

Implication of First Welfare Theorem

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- How should we allocate limited resources in the economy?
 - ▣ E.g., land, masks
- What is a “good” way to allocate resources?
 - ▣ There are many ways to define “good”
 - ▣ Let’s suppose “good” means Pareto efficient
- Is there a mechanism we can rely on to allocate resources efficiently?
 - ▣ Yes, FWT tells us that we just need to create a competitive market and the market will allocate resources efficiently

Comments on First Welfare Theorem

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- It only holds in competitive markets
 - ▣ Not true if consumers or firms have price setting power
 - ▣ Not true if there is externality
 - ▣ Not true if there is asymmetric information
- Efficiency does not mean equity
 - ▣ A Pareto efficient allocation may or may not be an equitable allocation
 - ▣ E.g., one consumer has everything and the other consumer has nothing can be Pareto efficient

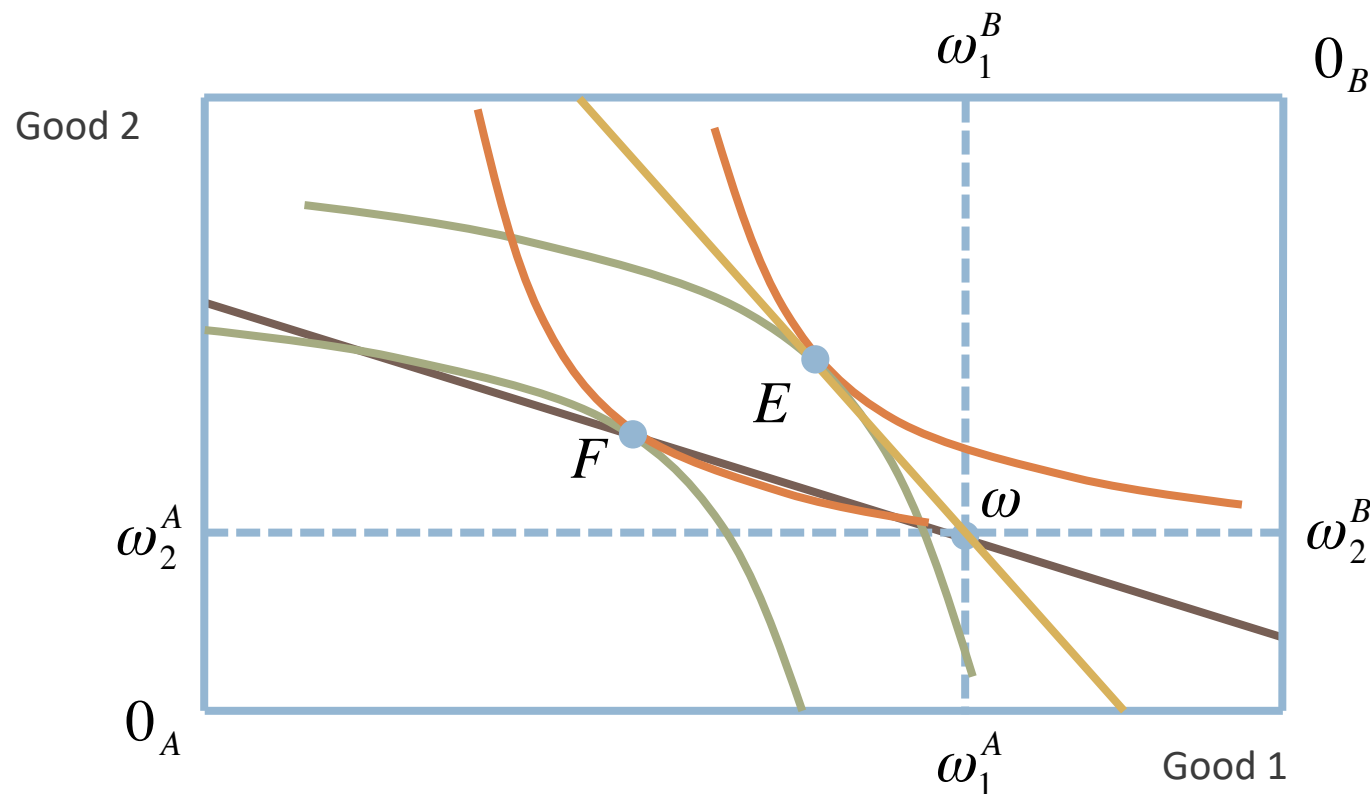
Pareto Efficiency vs. Competitive Equilibrium

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- Pareto Efficiency
 - ▣ An allocation where it is impossible to make someone better off without making someone else worse off
 - ▣ Does not depend on prices
 - ▣ Does not depend on endowment
- Competitive Equilibrium
 - ▣ A pair of prices such that
 - Markets clear
 - Everyone maximizes utility given budget constraint
 - ▣ Depend on endowment
 - Endowment allocation (and prices) determine budget constraints

E and F are Always Pareto Efficient Regardless of Prices

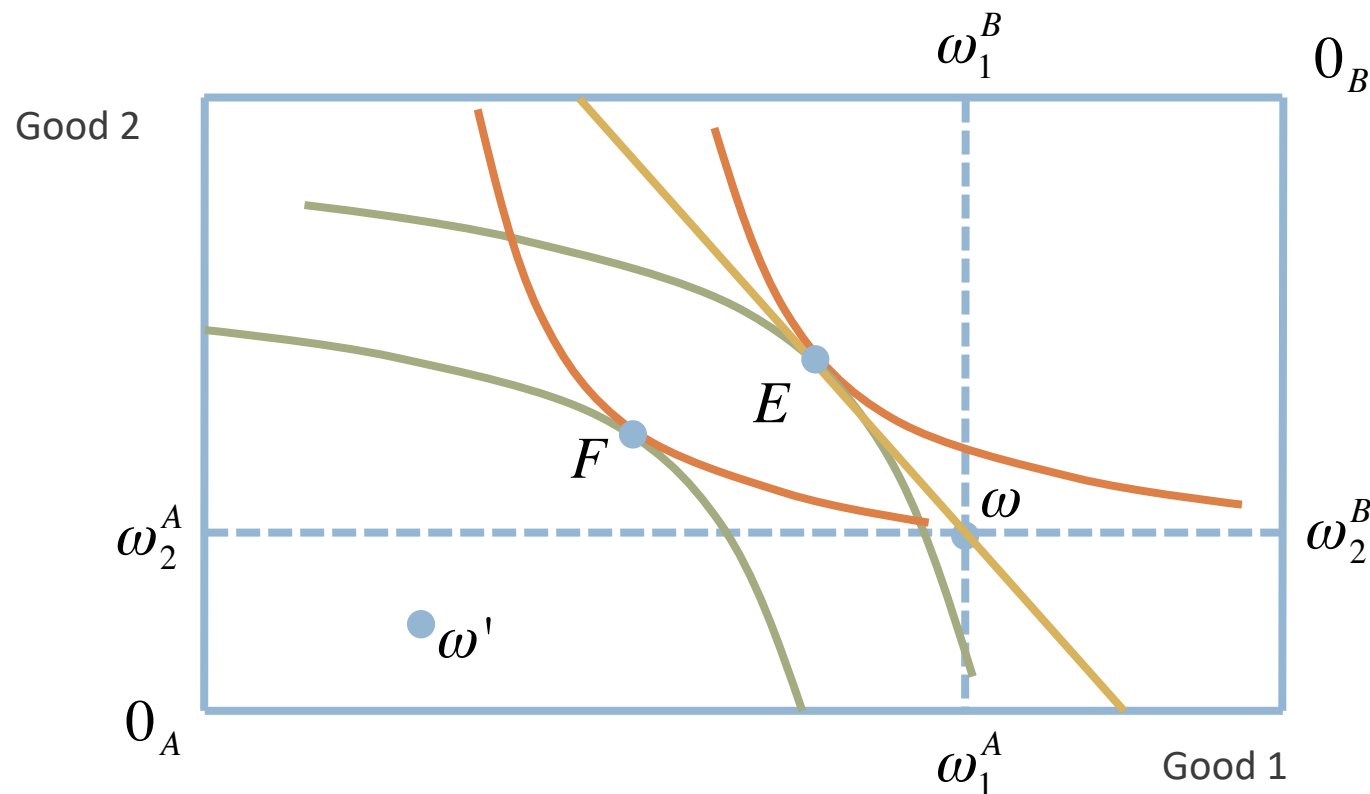
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A Pareto efficient allocation may not be an equilibrium allocation

E and F are Always Pareto Efficient Regardless of Endowment

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E or F is not the equilibrium allocation if the endowment is at point w'

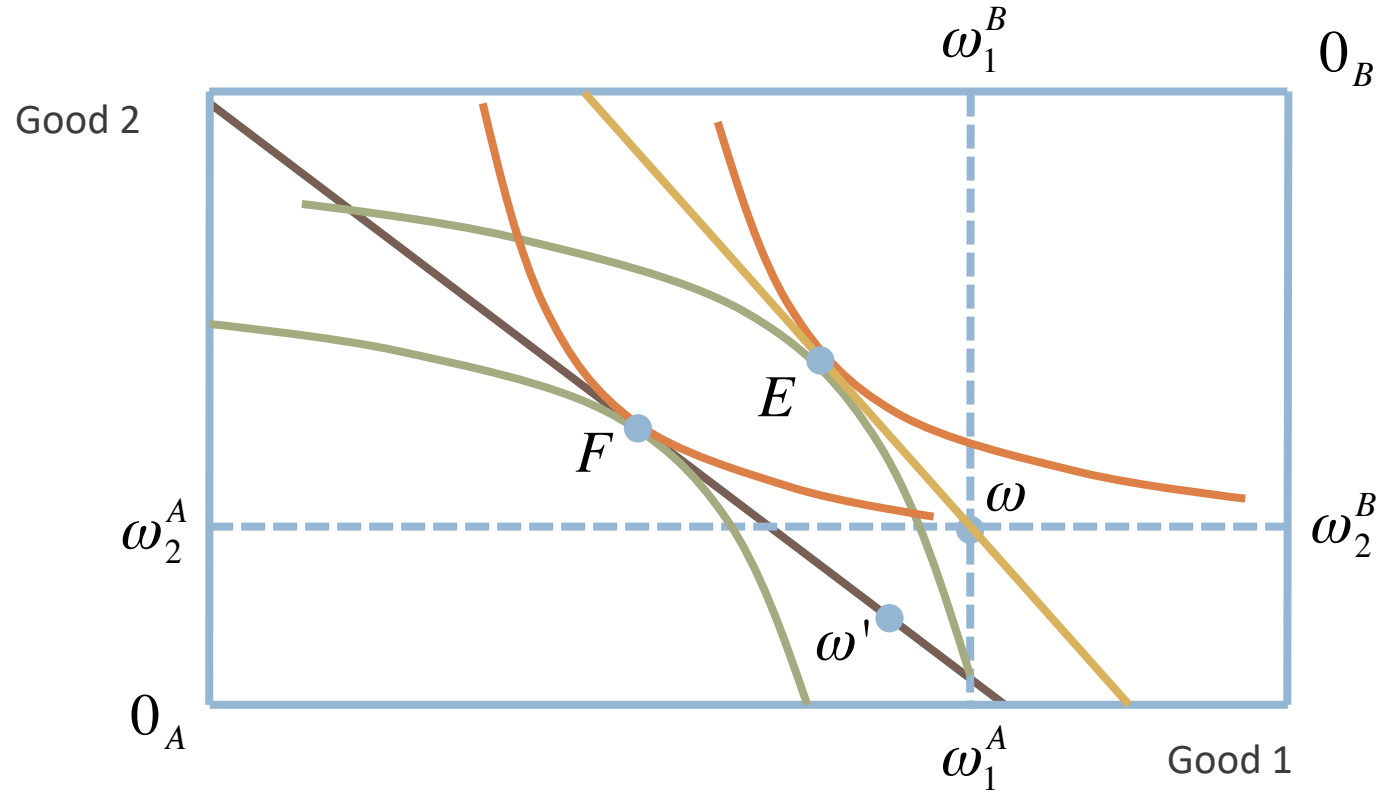
Second Welfare Theorem

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- First welfare theorem
 - ▣ Competitive equilibrium allocation is Pareto efficient
- How about the reverse?
 - ▣ We know that not every Pareto efficient allocation can be achieved in equilibrium given a particular endowment allocation
- The *Second Fundamental Theorem of Welfare Economics* states that any Pareto efficient allocation can be achieved in a competitive equilibrium through redistribution of endowments

Second Welfare Theorem in Graph

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F will be an equilibrium allocation if the endowment is w'

Part 2

Walras' Law

Relationship between the Two Markets

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- In a general equilibrium
 - ▣ Market for each good clear
 - ▣ For each of the two markets, demand equals supply
- Are the two markets independent of each other?
 - ▣ If market for one good clears, how about the market for the other good?
 - ▣ If there is excess demand or supply in one market, how about the other market?

Gross Demand at Any Given Prices

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- Let P_1, P_2 be any pair of prices
 - ▣ May or may not be the equilibrium prices
- Let (x_1^A, x_2^A) be A's gross demand and (x_1^B, x_2^B) be B's gross demand given P_1, P_2
 - ▣ The utility-maximizing quantity of each good for each consumer at the given prices
- Since P_1, P_2 may not be the equilibrium prices, it is possible that

$$x_1^A + x_1^B \neq \omega_1^A + \omega_1^B$$

$$x_2^A + x_2^B \neq \omega_2^A + \omega_2^B$$

Net Demand

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□ Definition 6.2 The *net demand* of a consumer for a good is the difference between the gross demand for that good and his/her endowment for that good

□ A's net demand for good 1 is

$$x_1^A - \omega_1^A$$

□ A's net demand for good 2 is

$$x_2^A - \omega_2^A$$

Aggregate Net Demand

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- Definition 6.3 The *aggregate net demand* for a good is the sum of the net demand for that good for the two consumers

$$x_1^A + x_1^B - \omega_1^A - \omega_1^B, \quad x_2^A + x_2^B - \omega_2^A - \omega_2^B$$

- When the aggregate net demand for a good is positive
 - ▣ There is excess for that good
- When the aggregate net demand for a good is negative
 - ▣ There is excess for that good

Value of Net Demand

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- (x_1^A, x_2^A) lies on consumer A's budget line

$$P_1 x_1^A + P_2 x_2^A = P_1 \omega_1^A + P_2 \omega_2^A$$

- Rearranging

$$P_1(x_1^A - \omega_1^A) + P_2(x_2^A - \omega_2^A) = 0$$

- The total value of consumer A's net demand for the two goods is 0

- ▣ The value of A's net demand for good 1 is

$$P_1(x_1^A - \omega_1^A)$$

- ▣ The value of A's net demand for good 2 is

$$P_2(x_2^A - \omega_2^A)$$

Walras' Law

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- Similarly, the total value of consumer B's net demand for the two goods is 0

$$P_1(x_1^B - \omega_1^B) + P_2(x_2^B - \omega_2^B) = 0$$

- Adding up the equation for A and B

$$P_1(x_1^A + x_1^B - \omega_1^A - \omega_1^B) + P_2(x_2^A + x_2^B - \omega_2^A - \omega_2^B) = 0$$

- Definition 6.4 The equation above is the *Walras' Law*
 - ▣ The total value of the aggregate net demand for the two goods is 0

Implications of Walras' Law

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- In the two-good exchange economy, if one market is in equilibrium, the other market must also be in equilibrium
- Suppose the market for good 1 clears

$$x_1^A + x_1^B - \omega_1^A - \omega_1^B = 0$$

- By the Walras' law

$$P_1(x_1^A + x_1^B - \omega_1^A - \omega_1^B) + P_2(x_2^A + x_2^B - \omega_2^A - \omega_2^B) = 0$$

- Market for good 2 clears as well

$$x_2^A + x_2^B - \omega_2^A - \omega_2^B = 0$$

Implications of Walras' Law Cont'

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- In the two-good exchange economy, an excess supply in one market implies an excess demand in the other market
- Suppose there is excess supply of good 1

$$x_1^A + x_1^B - \omega_1^A - \omega_1^B < 0$$

- By the Walras' law

$$P_1(x_1^A + x_1^B - \omega_1^A - \omega_1^B) + P_2(x_2^A + x_2^B - \omega_2^A - \omega_2^B) = 0$$

- There will be excess demand of good 2

$$x_2^A + x_2^B - \omega_2^A - \omega_2^B > 0$$

Walras' Law vs. Competitive Equilibrium

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- Walras' law holds for ANY prices
 - ▣ Not just the equilibrium prices
- At the equilibrium prices, the aggregate net demand for each good is 0

$$P_1(\underbrace{x_1^A + x_1^B - \omega_1^A - \omega_1^B}_{=0}) + P_2(\underbrace{x_2^A + x_2^B - \omega_2^A - \omega_2^B}_{=0}) = 0$$

- At non-equilibrium prices, the aggregate net demand is not 0

$$P_1(\underbrace{x_1^A + x_1^B - \omega_1^A - \omega_1^B}_{\neq 0}) + P_2(\underbrace{x_2^A + x_2^B - \omega_2^A - \omega_2^B}_{\neq 0}) = 0$$

Part 3

Midterm Review

Basic Information

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- 3 March, 6:30 pm to 7:45 pm
 - ▣ MPSH2A, 2B, 2C, MPSH5A
- Format
 - ▣ Open book
 - ▣ Pen and paper
- Coverage
 - ▣ Lecture 1 – 5, practice problems 1 – 4, quiz 1 – 5, additional questions on consumer theory, midterm practice problems
- Type of questions
 - ▣ MCQ
 - ▣ Structured questions

Exam Requirements

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- Do NOT write in pencil
- Memorize your tutorial number
- Bring your student ID card
- Bring your own draft paper
- You are allowed to
 - ▣ Refer to any printed material
 - ▣ Use a non-programmable calculator
- You are not allowed to
 - ▣ Communicate with others
 - ▣ Use any other electronic device

Logistics

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- Topics in the textbook that are not covered in lectures or tutorials are not required
- There will be no lecture or tutorial in week 7
- My consultation hours
 - ▣ By appointment only (Use the Zoom link for “consultation by appointment”)
 - 23 Feb, 28 Feb, and 1 Mar
 - Sign up on LumiNUS (no more than 3 slots per person)
 - Let me know in advance if you would like to meet f2f
 - ▣ Open consultation (Use the Zoom link for “consultation by appointment”)
 - 25 Feb: 3 pm to 5 pm
 - 2 Mar: 1:30 pm to 3:30 pm

Safety Measures

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- You need to show the NUSafe green pass on the uNivUS app to be admitted to the hall
 - ▣ The invigilator will check your pass at the entrance
- To generate a NUSafe green pass, you need to
 - ▣ Have a valid negative FET result uploaded on uNivUS
 - ▣ Connect to NUS Wifi
- Do NOT congregate outside the venue before or after the exam
 - ▣ You will be allowed to enter the hall 15 minutes before the exam starts
- Keep your masks on during the exam

Green Pass

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- You will NOT be allowed to enter the exam hall if you do not have a green pass
- If you forget to take the FET
 - ▣ Go to the nearest vending machine to get a kit and take the test
 - ▣ You will NOT be given extra time for the exam
- You MAY need to change your password for your NUS ID
 - ▣ If you have not been on campus for a long time
 - ▣ Otherwise you may not be able to connect to NUS Wifi

What if you are FET positive?

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- If you test positive and you are unwell (MOH Protocol 1)
 - ▣ Seek medical help
- If you test positive with mild or no symptoms (MOH Protocol 2)
 - ▣ You need to self-isolate for 72 hours
 - ▣ Test again after 72 hours
 - ▣ You may stop self isolation when you test negative, but no sooner than 72 hours
- If you receive a Health Risk Warning (MOH Protocol 3)
 - ▣ You can continue normal activities if you are FET negative for the day

What if you are FET positive Cont'

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- Check this page for more information on the protocols
 - ▣ <https://www.covid.gov.sg/>
- If you fall under P1/P2/P3
 - ▣ You need to declare it on uNivUS
 - ▣ Go to Health Declaration and then Protocol
- Do NOT take the exam if you fall under P1 or P2 on the exam day
 - ▣ You test positive on the day of the exam
 - ▣ Or you are within 72-hour self-isolation period

What if you are FET positive? Cont'

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- Let me know asap if you are under P1 or P2 on or after 28 Feb (Monday of week 7)
 - ▣ With valid supporting documents, e.g., doctor's note/MC, PCR test result, MOH notification, uNivUS health declaration etc.
 - ▣ At the very least, show screenshots of the uNivUS health declaration
 - ▣ A photo of a FET positive result is not a valid document
- You will take a makeup midterm in week 8
- If you are FET negative but unwell
 - ▣ You can take the makeup midterm if you have a valid MC

Tips for Midterm Preparation

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- Prepare the midterm as if it is closed book
 - ▣ Open book does not help much!
- Do NOT print random notes or questions from the internet because they could be wrong
- Consider making a cheat sheet
 - ▣ Flipping through course materials is time consuming
 - ▣ The process of making a cheat sheet is useful
- Reflect on the questions you have done
 - ▣ What is the question about? What have you learned from this question?

Our Topics

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- Consumer Theory
 - ▣ Optimal choice
 - ▣ Demand curve/demand function
 - ▣ Revealed preference
 - ▣ Substitution and income effects
 - ▣ Consumer welfare
- Exchange
 - ▣ Pareto efficiency
 - ▣ Competitive equilibrium

Common Utility Functions

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- Cobb Douglas
- Perfect substitutes
- Perfect complements
- Quasi linear

Answers to the Questions in Lectures

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- Lecture 1: what do Shin-chan's indifference curves look like?
 - ▣ Answer: practice problems 1 Q1
- Lecture 2: is the budget line always a straight line?
 - ▣ Answer: practice problems 2 Q1, practice problems 3 Q1
- Lecture 4: when is the substitution effect 0?
 - ▣ Answer: practice problems 3 Q2, additional question Q1