

Question 2

(a) - At the equilibrium allocation (of Cobb-Douglas utility function), we can use the tangency condition between two utility function:

$$MRS_{x_1 x_2}^i = \frac{MU_{x_1}^i}{MU_{x_2}^i} = \frac{\frac{\alpha}{x_1^i \times \ln(10)}}{\frac{1-\alpha}{x_2^i \times \ln(10)}} = \frac{\alpha x_2^i}{(1-\alpha)x_1^i}$$

$$MRS_{x_1 x_2}^A = MRS_{x_1 x_2}^E$$

$$\Rightarrow \frac{\alpha x_2^A}{(1-\alpha)x_1^A} = \frac{\alpha x_2^E}{(1-\alpha)x_1^E}$$

$$\Rightarrow \frac{x_2^A}{x_1^A} = \frac{x_2^E}{x_1^E} = \frac{x_2^A + x_2^E}{x_1^A + x_1^E} = \frac{6}{6} = 1$$

- We have the tangency condition with the price/budget line:

$$MRS_{x_1 x_2}^A = \frac{\alpha x_2^A}{(1-\alpha)x_1^A} = \frac{p_{x_1}}{p_{x_2}}$$

$$\Rightarrow \frac{p_1}{p_2} = \frac{\alpha}{1-\alpha}$$

(b) - From the equal utility condition;;

$$U^A = U^E$$

$$\Rightarrow \alpha \log(x_1^A) + (1-\alpha) \log(x_2^A) = \alpha \log(x_1^E) + (1-\alpha) \log(x_2^E)$$

$$\Rightarrow x_1^A = x_1^E$$

$$\Rightarrow x_1^A = x_1^E = x_2^A = x_2^E = 3$$

- From (a):

$$p_1 = \frac{\alpha}{1-\alpha} p_2$$

- The lump-sum transfer Anton will receive is:

$$T^A = p_1(x_1^A - \omega_1^A) + p_2(x_2^A - \omega_2^A)$$

$$= \frac{\alpha}{1-\alpha}(3-4) + (3-1) = 2 - \frac{\alpha}{1-\alpha}$$

Question 4

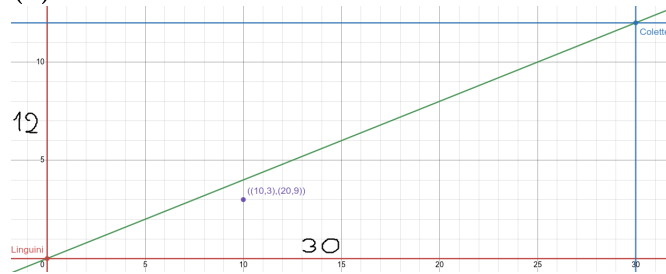
(a) Contract curve satisfies tangency condition.

$$MRS_{12}^L = MRS_{12}^C$$

$$\frac{x_2^L}{x_1^L} = \frac{x_2^C}{x_1^C} = \frac{x_2^L + x_2^C}{x_1^L + x_1^C} = \frac{12}{30} = \frac{2}{5}$$

$$x_2^L = \frac{2}{5}x_1^L$$

(b)



(c) - For Linguini:

$$MRS_{12}^L = \frac{x_2^L}{x_1^L} = \frac{p_1}{p_2} = \frac{1}{2}$$

$$\Rightarrow x_1^L = 2x_2^L$$

While satisfying budget constraint:

$$p_1 x_1^L + p_2 x_2^L = p_1 \omega_1^L + p_2 \omega_2^L = 16$$

$$x_2^L = 4$$

$$x_1^L = 8$$

- Do the same to Colette:

$$x_2^C = 9.5$$

$$x_1^C = 19$$

(d) - Linguini wants to buy 1 profiterole and sell 2 éclair.

- Colette wants to buy 0.5 profiterole and sell 1 éclair

- The market is not in equilibrium

(e) Walras's law equation:

$$p_1(x_1^L + x_1^C - \omega_1^L - \omega_1^C) + p_2(x_2^L + x_2^C - \omega_2^L - \omega_2^C) = 0$$

$$1(8 + 19 - 10 - 20) + 2(4 + 9.5 - 3 - 9) = 0$$

$$0 = 0$$

(f) - Using tangency condition:

$$\frac{p_1}{p_2} = MRS_{12}^L = \frac{x_2^L}{x_1^L} = \frac{2}{5}$$

(g) $p_2 = 1 \Rightarrow p_1 = 0.4$

Linguini's BL: $0.4x_1^L + x_2^L = 7$

Colette's BL: $0.4x_1^C + x_2^C = 17$

(h) - Tangency condition:

$$x_2^L = \frac{2}{5}x_1^L$$

- Budget constraint:

$$0.4x_1^L + x_2^L = 0.4x_1^L + 0.4x_1^L = 7$$

$$\begin{cases} x_1^L &= 8.75 \\ x_2^L &= 3.5 \end{cases}$$

- Equilibrium allocation:

$$((8.75, 3.5), (21.25, 8.5))$$