Macroeconomics Analysis II, EC3102
Tutorial 5

(Understanding economic disparity through production model)[1]

Question 1 ***Returns to Scale in Production***

(will not be covered in tutorials, solutions will be provided)

Do the following production functions exhibit increasing, constant, or decreasing returns to scale in K and L?

a. $Y = F(K,L) = K^{\frac{2}{3}}L^{\frac{2}{3}}$

b. $Y = F(K,L) = K + L$

c. $Y = F(K,L) = K + K^{\frac{1}{3}}L^{\frac{1}{3}}$

Question 2 ***Production functions in intensive form: "per person" version***

(will not be covered in tutorials, solutions will be provided)

Write each production function given below in terms of output per person $y \equiv Y/L$ and capital per person $k \equiv K/L$. Show what these "per person" versions look like in a graph with k on the horizontal axis and y on the vertical axis. (Assume $\bar{A}$ is some fixed positive number.)

a. $Y = K^{\frac{1}{3}}L^{\frac{2}{3}}$ and $Y = K^{\frac{3}{4}}L^{\frac{1}{4}}$ using the same axes
b. $Y = K$
c. $Y = K + \bar{A}L$
d. $Y = K - \bar{A}L$

Question 3 ***The empirical fit of the production model (using Cobb Douglas production function)***

In the table below, the per capita GDP and capital per person in the year 2014 for 4 countries are shown. Fill in the missing columns of the table. (You might find it helpful to use a spreadsheet for this question. Take this as a chance to get familiar with Excel.) Note that all the values of US have been normalized to 1 in the table columns 3 onwards. So the values for other countries would be rescaled according to that of US. The reason for doing this is so that we can see how other countries are doing relative to US.

[1]Questions 1 to 3 are adapted from adopted text: Macroeconomics by Charles Jones (2018), Chapter 4 Questions 1, 2, 5, 6 and 7.

a.

Given the values in columns 1 and 2, fill in columns 3 and 4. That is, compute per capita GDP and capital per person relative to the US values.

b.

In column 5, use the production model: $Y = \bar{A}K^{\frac{1}{3}}L^{\frac{2}{3}}$, or equivalently: $y = \bar{A}k^{\frac{1}{3}}$, to compute predicted per capita GDP for each country relative to the US, assuming there are no TFP differences.

c.

In column 6, compute the level of TFP for each country that is needed to match up the model and the data. That is, in this part, we relax the assumption that there are no TFP differences across countries.

d.

Divide the values of US in columns 4, 5 and 6 by the values of other economies in the respective columns. Store the result in columns 7, 8 and 9 respectively. Explain how to interpret columns 7, 8 and 9.

e.

Comment on the general results you find.

f.

Suppose we modify the production function to $Y = \bar{A}K^{\frac{3}{4}}L^{\frac{1}{4}}$. That is, we now assume that the exponent on capital is $\frac{3}{4}$ instead of $\frac{1}{3}$, so that the diminishing returns on capital are less. How would you expect the general results to change?

	In 2011 USD		Relative to the US values (US=1)						
	1	2	3	4	5	6	7	8	9
Country	Capital per person	Per capita GDP	Capital per person	Per capita GDP	Predicted y*=k*^1/3	Implied TFP to match data	1/(column 4)	1/(column 5)	1/(column 6)
US	141841	51958	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000
South Korea	120472	34961							
Indonesia	41044	9797							
Ethopia	3227	1505							

Figure 1: Empirical fit of the production model: the importance of TFP versus capital

Question 4 ***Math Review: Basic properties of growth rate.***

By definition, the growth rate of a variable is its proportional rate of change. Math-ematically, the growth rate of a variable equals to the rate of change of its natural

log. That is, growth rate of a variable X, $\frac{\dot{X}(t)}{X(t)}$ equals $\frac{dlnX(t)}{dt}$. Since *lnX* is a function of X and X is a function of t, we can use the chain rule to write:

$$\frac{dlnX(t)}{dt} = \frac{dlnX(t)}{dX(t)}\frac{dX(t)}{dt} = \frac{1}{X(t)}\dot{X}(t).$$

Using the fact that the growth rate of a variable equals to the time derivative of its natural log, show the following.

a.

Why is the growth rate of variable X is $\frac{\dot{X}(t)}{X(t)}$?

b.

i. The growth rate of the product of two variables equals the sum of their growth rates. That is, if $Z(t) = X(t)Y(t)$, then $\dot{Z}(t)/Z(t) = [\dot{X}(t)/X(t)] + [\dot{Y}(t)/Y(t)]$.

ii. The growth rate of the ratio of two variables equals the difference of their growth rates. That is, if $Z(t) = X(t)/Y(t)$, then $\dot{Z}(t)/Z(t) = [\dot{X}(t)/X(t)] - [\dot{Y}(t)/Y(t)]$

iii. If $Z(t) = X(t)^{\alpha}$, then $\dot{Z}(t)/Z(t) = \alpha\dot{X}(t)/X(t)$.