Practice Problem Set 7
Game Applications (C.30)

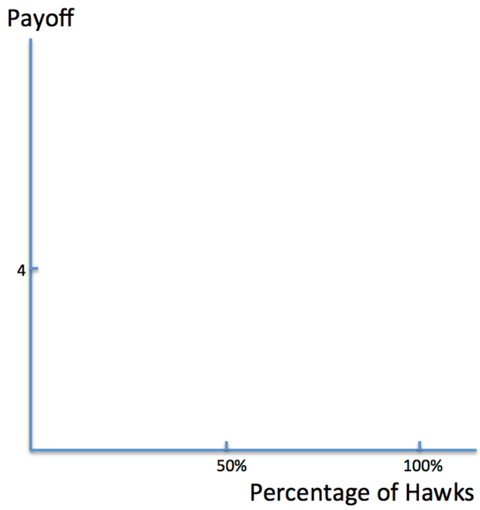
Question 7.1

This problem is an illustration of the Hawk-Dove game. It applies game theory to understand evolution. Males of a certain species frequently come into conflict with other males over the opportunity to mate with females. If a male runs into a situation of conflict, he has two alternative strategies. If he plays "Hawk," he will fight the other male until he either wins or is badly hurt. If he plays "Dove," he makes a bold display but retreats if his opponent starts to fight. If two Hawk players meet, they are both seriously injured in battle. If a Hawk meets a Dove, the Hawk gets to mate with the female and the Dove slinks off. If a Dove meets another Dove, they both strut their stuff but neither chases the other away. Eventually the female may select one of them at random or may get bored and wander off. The expected payoffs to each male are shown in the box below.

		Animal B	
		Hawk	Dove
Animal A	Hawk	$-5, -5$	$10, 0$
	Dove	$0, 10$	$4, 4$

(i) Suppose that there is a large male population and the fraction p are Hawks. Then the fraction of any player's encounters that are with Hawks is about p and the fraction of encounters that are with Doves is about $1 - p$. Find the value of p such that at this value Hawks do exactly as well as Doves.

(ii) On the axes (right), use blue ink to graph the average payoff to the strategy Dove when the proportion of Hawks in the male population who is p. Use red ink to graph the average payoff to the strategy, Hawk, when the proportion of the male population who are Hawks is p. Label the equilibrium proportion in your diagram by E.



(iii) If the proportion of Hawks is slightly greater than E, which strategy does better? Suppose $\frac{3}{5}$ of the male animals are Hawks initially, if the more profitable strategy tends to be adopted more frequently in future plays, what will the proportion of Hawks in the male population be in the long run?

Question 7.2

		Player 2	
		L	**R**
Player 1	**U**	0, 0	2, -2
	D	x, -x	1, -1

Consider the above game matrix. (i) Is this a zero-sum game? Find the Nash Equilibriums when (ii) $x < 0$, (iii) $0 < x < 1$, (iv) $x > 1$.

Question 7.3

Today is Sunday. Mona is going to be out of town tomorrow for three days (Monday to Wednesday) and will not need her car during this time. Lisa is interested in renting her car. The value to Lisa of having the car during this time is \$50 per day. The total cost to Mona of letting Lisa use the car is \$20, regardless of how many days Lisa uses it. Lisa can send a message to Mona tonight, offering to rent her car for three days for a specified price. Mona can either accept the offer or reject the offer and make a counteroffer. The only problem is that it takes 24 hours for a counteroffer to be made and accepted. If Mona's counteroffer (made on Monday) is accepted, Lisa could use the car only for two days. If not, Lisa can make a final offer on Tuesday, and if accepted, she could use the car for one day.

Assume that Mona and Lisa would accept an offer when they are indifferent. Furthermore, they do not discount the future (i.e., their discount factors are 1). Find the Rubinstein bargaining solution to this problem (find the value of offer made, the day the offer is made, and the net payoffs of the players).