GAME APPLICATIONS I

Week 8

(Chapter 30)

Plan for this lecture

1. Reinforcing concepts
   – Why backward induction?
   – Nash equilibrium and strictly dominant strategies

2. 2/3 of the Average

3. Mixed Strategy Nash Equilibrium

Why backward induction?

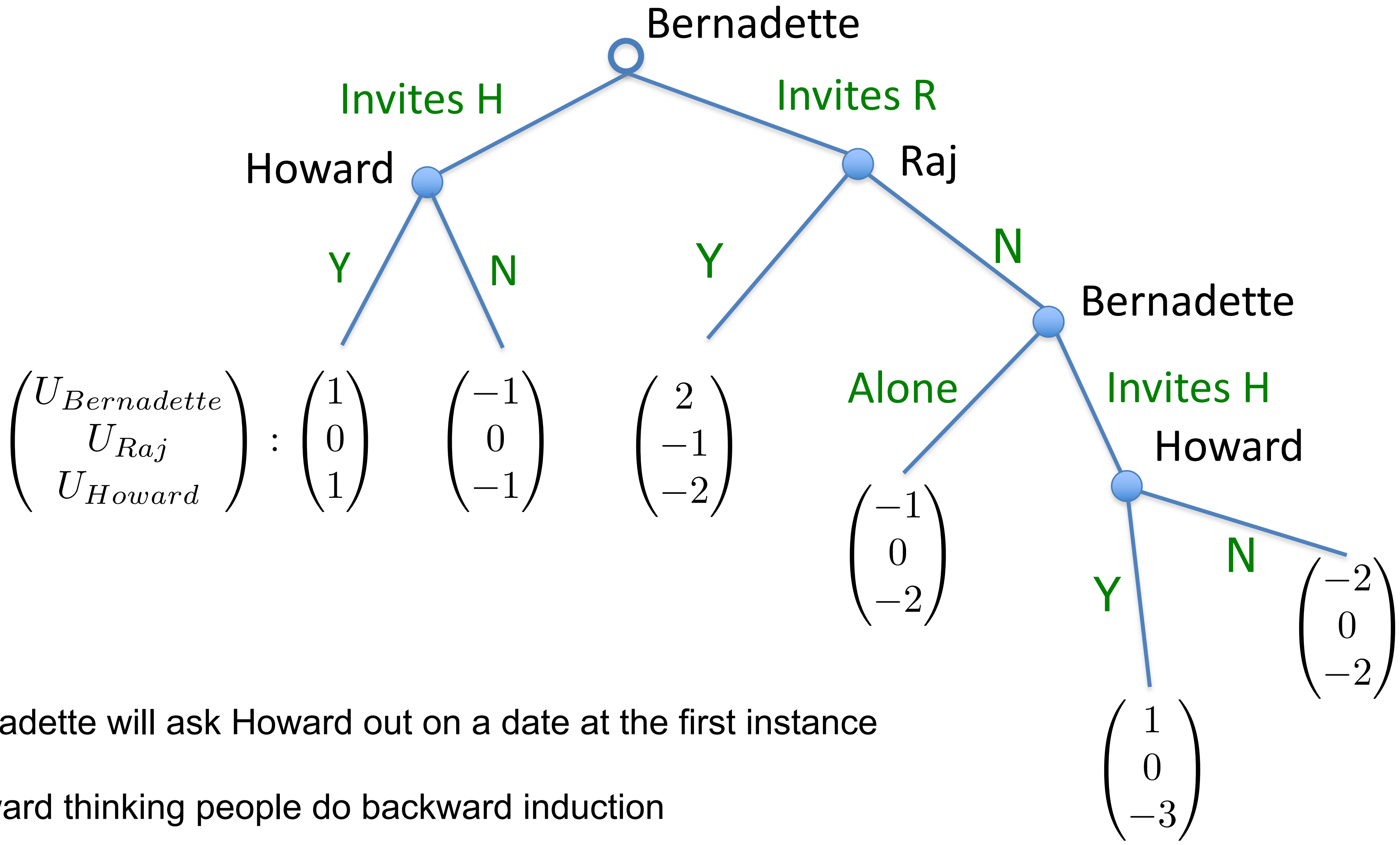
- Backward induction: one considers the consequences of one's possible actions, before deciding which action to take

- You are doing backward induction all the time without realizing it

An Example

- Suppose Bernadette likes both Raj and Howard
- More Raj than Howard
- She can ask either one out on a date
- The person asked could either accept or reject
- Raj does not like Bernadette, and will not accept the invite
- Howard likes Bernadette, would like to accept Bernadette’s invitation. But if Bernadette had asked Raj before asking Howard, Howard would be hurt and would not accept Bernadette’s invite
- You are Bernadette, what would you do?

Suppose Bernadette's situation can be represented as follows:



- Bernadette will ask Howard out on a date at the first instance
- Forward thinking people do backward induction

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Prisoner's Dilemma

		Howard	
		Confess	**Do Not Confess**
Raj	**Confess**	-5, -5	-1, -10
	Do Not Confess	-10, -1	-2, -2

It never makes sense for Raj (and Howard) to choose "DNC"

Dominant and Dominated Strategy

- "Confess" is a strictly *dominant strategy* for each player
  - No matter what strategy the other player chooses, "Confess" is always strictly better than any other strategy

- "Do not confess" is a strictly *dominated strategy* for each player
  - No matter what strategy the other player chooses, there exists another strategy that gives a strictly higher payoff than "Do not confess"

Dominance and Nash Equilibrium

- A strictly dominated strategy is never a Nash equilibrium strategy
- If every player has a strictly dominant strategy, she will play it, and the outcome will be a Nash equilibrium
- Reverse not true. At a Nash equilibrium, players may or may not be playing strictly dominant strategies

Meeting in Singapore (Coordination Game)

(Example of game without strictly dominant strategies)

- *Players*: player 1 and player 2
- *Strategies*: “Jewel” or “Night Safari”
- *Outcomes* and *Payoffs*:
  - If both players go to the same place, they meet and get to enjoy each other’s company at lunch. This is equivalent to getting a monetary payoff of 100 each.
  - If they go to different places, they do not meet and have to eat alone. This is equivalent to a monetary payoff of 0 each.

Nash Equilibriums in “Meeting in Singapore”

		Player 2	
		Jewel	**Safari**
Player 1	**Jewel**	100, 100	0, 0
	Safari	0, 0	100, 100

- Any there strictly dominant strategies?

True or False?

- A game can only have one Pure Strategy NE?

Game with ? PSNE

		Goal Keeper	
		Left	**Right**
Striker	**Left**	5, -5	8, -8
	Right	9, -9	2, -2

- Striker takes penalty against goalkeeper

Game with ? PSNE

		Howard	
		Confess	**Do Not Confess**
Raj	**Confess**	-5, -5	-1, -10
	Do Not Confess	-10, -1	-2, -2

Game with ? PSNE

		Player 2	
		Jewel	**Safari**
Player 1	**Jewel**	100, 100	0, 0
	Safari	0, 0	100, 100

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- Go to PollEv.com
- Enter **ec3101** when prompted for usename
- Key in your response

Find the Nash Equilibrium

Keynesian beauty contest

John Maynard Keynes, Chapter 12, *The General Theory of Employment, Interest and Money* (1936)

- Fictional newspaper beauty contest
- Participants choose the most attractive face from 100 photographs
- Those who picked the most popular faces eligible for a prize
  - Naive strategy: choose the face that you think is most attractive
  - More sophisticated: based on the majority perception of attractiveness
  - Even more sophisticated: based on other participants' opinions of what public perceptions are

It is not a case of choosing those [faces] that, to the best of one's judgment, are really the prettiest, nor even those that average opinion genuinely thinks the prettiest. We have reached the third degree where we devote our intelligences to anticipating what average opinion expects the average opinion to be. And there are some, I believe, who practice the fourth, fifth and higher degrees.

- Stock market: people often price a stock not based on what they think its value is, but based on what they think everyone else thinks

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Mixed Strategy Nash Equilibrium

- So far, when we talk about NE, we are actually talking about Pure Strategy NE
- We now look at Mixed Strategy NE
- What are mixed strategies?

Example: A soccer game

		Goal Keeper	
		Left	**Right**
Striker	**Left**	5, -5	8, -8
	Right	9, -9	2, -2

- No PSNE
- Pure strategy: Striker and Goal Keeper chooses “Left” or “Right”
- Mixed strategy: Striker chooses “Left” with probability p and Goal Keeper chooses “Left” with probability q

Mixed Strategies

- "Left" and "Right" are called *actions*
- Choosing an action with certainty is a *pure strategy*
- A *mixed strategy* is a set of probabilities assigned to each action where the probability of each action is non-negative and the probabilities for all actions sum up to 1
- Pure strategies are also mixed strategies
  - Choosing an action with probability 1
  - (Reverse not true)

Mixed Strategies

		Goal Keeper	
		Left	**Right**
Striker	**Left**	5, -5	8, -8
	Right	9, -9	2, -2

- Striker chooses left with probability *p*; right with probability *1-p*
- Goalkeeper chooses left with probability *q*; right with probability *1-q*

- If Striker chooses left, expected payoff:
  - $EV_{striker}$(left) $= 5q + 8(1-q) = 8 - 3q$

- If Striker chooses right, expected payoff:
  - $EV_{striker}$(right) $= 9q + 2(1-q) = 2 + 7q$

Mixed Strategies

		Goal Keeper Left	Right
Striker	**Left**	5, -5	8, -8
	Right	9, -9	2, -2

- Striker chooses left with probability p; right with probability $1-p$
- Goalkeeper chooses left with probability q; right with probability $1-q$

- If GK chooses left, expected payoff:
  - EV_{gk}(left)= $-5p - 9(1-p)$ = $-9 + 4p$

- If GK chooses right, expected payoff:
  - EV_{gk}(right) = $-8p - 2(1-p)$ = $-2 - 6p$

Mixed Strategy Nash Equilibrium

- In a two-player game, a mixed strategy profile (σ_1^*, σ_2^*) is a *Nash equilibrium* if σ_1^* and σ_2^* are mutual best responses against each other:

$$u_1(\sigma_1^*, \sigma_2^*) \geq u_1(\sigma_1, \sigma_2^*), \quad \forall \sigma_1$$

and

$$u_2(\sigma_1^*, \sigma_2^*) \geq u_2(\sigma_1^*, \sigma_2), \quad \forall \sigma_2$$

Striker's Best Response

- $EV_{striker}$ (left) = $8 - 3q$
- $EV_{striker}$ (right) = $2 + 7q$

- If $EV_{striker}$ (left) > $EV_{striker}$ (right), striker's best response is "left"
- If $EV_{striker}$ (left) < $EV_{striker}$ (right), striker's best response is "right"
- If $EV_{striker}$ (left) = $EV_{striker}$ (right), striker is indifferent

- Striker's best response is "left" (**p=1**) if $8 - 3q > 2 + 7q \rightarrow$ ***q<0.6***
- Striker's best response is "right" (**p=0**) if $8 - 3q < 2 + 7q \rightarrow$ ***q>0.6***
- Striker is indifferent (**0≤p≤1**) if $8 - 3q = 2 + 7q \rightarrow$ ***q=0.6***

Goalkeeper's Best Response

- EV_{gk} (left) = $-9 + 4p$
- EV_{gk} (right) = $-2 - 6p$

- If EV_{gk} (left) > EV_{gk} (right), GK's best response is "left"
- If EV_{gk} (left) < EV_{gk} (right), GK's best response is "right"
- If EV_{gk} (left) = EV_{gk} (right), GK is indifferent

- GK's best response is "left" (**q=1**) if $-9 + 4p > -2 - 6p \rightarrow$ ***p>0.7***
- GK's best response is "right" (**q=0**) if $-9 + 4p < -2 - 6p \rightarrow$ ***p<0.7***
- GK is indifferent (**0≤q≤1**) if $-9 + 4p = -2 - 6p \rightarrow$ ***p=0.7***

Mutual Best Responses

- Striker's best response is "left" (**$p=1$**) if ***$q<0.6$***
- Striker's best response is "right" (**$p=0$**) if ***$q>0.6$***
- Striker is indifferent (**$0 \le p \le 1$**) if ***$q=0.6$***

- GK's best response is "left" (**$q=1$**) if ***$p>0.7$***
- GK's best response is "right" (**$q=0$**) if ***$p<0.7$***
- GK is indifferent (**$0 \le q \le 1$**) if ***$p=0.7$***

- One MSNE: Striker left with probability 0.7; GK left with probability 0.6
- If they play these mixed strategies, no one has incentive to deviate

Penalty Kick in Real Life

Palacios-Huerta (2003) looked at all the penalty kicks in professional soccer games in England, Italy, Spain, etc. during 1995–2000

		Mixture		Scoring rates		Pearson	
Player	#Obs.	L	R	L	R	statistic	p-value
Kicker 1	34	0·32	0·68	0·91	0·91	0·000	0·970
Kicker 2	31	0·35	0·65	0·82	0·80	0·020	0·902
Kicker 3	40	0·48	0·52	0·74	0·76	0·030	0·855
Kicker 4	38	0·42	0·58	0·88	0·91	0·114	0·735
Goalkeeper 1	37	0·38	0·62	0·21	0·22	0·000	0·982
Goalkeeper 2	38	0·39	0·61	0·20	0·22	0·017	0·898
Goalkeeper 3	30	0·60	0·40	0·28	0·25	0·028	0·866
Goalkeeper 4	50	0·46	0·54	0·17	0·15	0·061	0·804

Source: Ignacio Palacios-Huerta, "*Professionals Play Minimax*", Review of Economic Studies, 2003

Example with no Randomizing

Player B

Player A		Up	Down
	Left	6, 4	3, 5
	Right	4, 3	5, 7

Example with no Randomizing

		Player B	
		Up	**Down**
Player A	**Left**	6, 4	3, 5
	Right	4, 3	5, 7

- Now suppose players randomize (i.e. mix) over their actions
- Player A chooses left with probability p; right with probability $1-p$
- Player B chooses up with probability q; down with probability $1-q$

Player A's Best Response

- Player A chooses left with probability p; right with probability $1-p$
- Player B chooses up with probability q; down with probability $1-q$

- For Player A,
  - EV_A (left) $= 6q + 3(1-q) = 3 + 3q$
  - EV_A (right) $= 4q + 5(1-q) = 5 - q$

- Best response "left" (**p=1**) if $3 + 3q > 5 - q \rightarrow$ ***q>0.5***
- Best response "right" (**p=0**) if $3 + 3q < 5 - q \rightarrow$ ***q<0.5***
- Indifferent (**0<p<1**) if ***q=0.5***

Player B's Best Response

- Player A chooses left with probability p; right with probability $1\text{-}p$
- Player B chooses up with probability q; down with probability $1\text{-}q$

- For Player B,
  - EV_B (up) $= 4p + 3\ (1\text{-}p) = 3 + p$
  - EV_B (down) $= 5p + 7\ (1\text{-}p) = 7 – 2p$

- Best response "up" (**q=1**) if $3 + p > 7 – 2p \rightarrow$ ***p>4/3***
- Best response "down" (**q=0**) if $3 + p < 7 – 2p \rightarrow$ ***p<4/3***
- Indifferent (**0<q<1**) if ***p=4/3***

- Player B's best response is "down" (q=0) for $0 \leq p \leq 1$
- Unsurprising since "down" is a dominant strategy for Player B

Example with no Randomizing

- Player B should always choose "down" (q=0) for $0 \leq p \leq 1$
- Player A,
  - Best response "left" (**p=1**) if ***q>0.5***
  - Best response "right" (**p=0**) if ***q<0.5***
  - Indifferent (**0<p<1**) if ***q=0.5***
- If q=0, Player A should choose right (p=0)
  - Randomizing is not a best response for either player
  - This game has a unique PSNE (right, down)