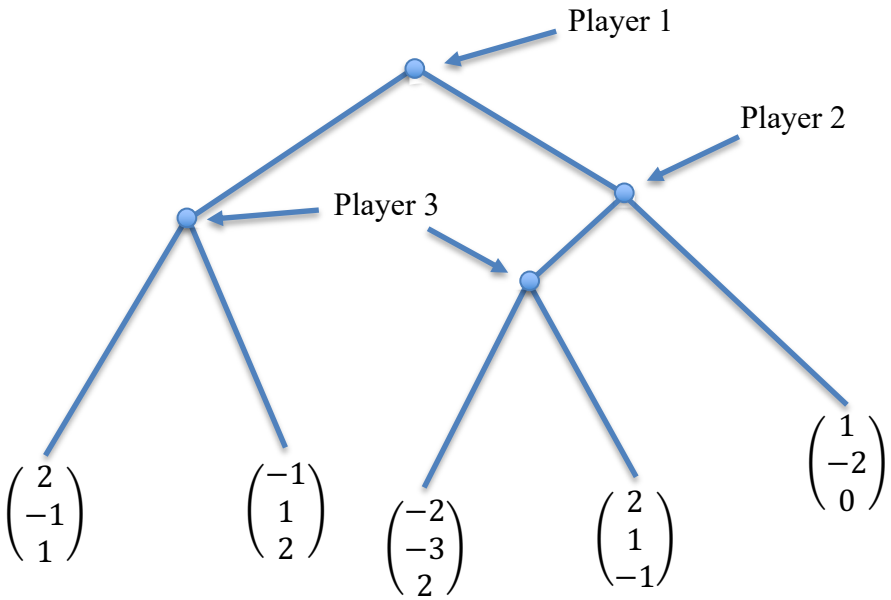
NATIONAL UNIVERSITY OF SINGAPORE
EC3101: MICROECONOMIC ANALYSIS II
Semester 2, AY 2022/2023 MIDTERM EXAMINATION
27 FEBRUARY 2023

Multiple Choice Questions (2 marks each): Choose the BEST answer.

1. If the inflation rate is 5% and the nominal interest rate is 7%, what is the real rate of interest?
   A. -2%.
   B. 2%.
   C. 6%.
   D. 12%.
   E. None of the above.

2. Solve the game represented by the following game tree:



   A. Player 1 receives 2, player 2 receives -1, and player 3 receives 1.
   B. Player 1 receives -1, player 2 receives 1, and player 3 receives 2.
   C. Player 1 receives -2, player 2 receives -3, and player 3 receives 2.
   D. Player 1 receives 2, player 2 receives 1, and player 3 receives -1.
   E. Player 1 receives 1, player 2 receives -2, and player 3 receives 0.

3. Three firms compete in a Bertrand oligopoly. The firms are identical and have a marginal cost of production of 10. If the market demand is given by $Q = 50 - P$, which of the following is a possible equilibrium? Assume that the smallest denomination of currency in this economy is a dollar. Also assume that when n firms set the lowest price, each of these firms get $\frac{1}{n}$ of the sales (i.e., they share the market equally).
   A. $p_A = 9, p_B = 9, p_C = 10$
   B. $p_A = 9, p_B = 10, p_C = 10$
   C. $p_A = 10, p_B = 10, p_C = 11$
   D. $p_A = 10, p_B = 11, p_C = 11$
   E. None of the above

4. Kai has a utility function $U(c_1, c_2) = \min\{c_1, 4c_2\}$, where c_1 and c_2 are Kai's consumptions in period 1 and period 2 respectively (for example, if Kai consumes \$1 in period 1 and \$2 in period 2, Kai's utility will be $U(1,2) = \min\{1, 4 \times 2\} = 1$). Suppose that the interest rate is $r = 100\%$. If Kai does not work in period 1 and obtains an income of \$36 in period 2, how much should Kai consume in period 1?
   A. \$0
   B. \$10
   C. \$12
   D. \$16
   E. \$18

5. Consider a two-period consumption model where Fantine receives an income of \$60 in each period. Suppose that when the interest rate is $r = 20\%$, the optimal consumption for Fantine is (c_1, c_2), while when the interest rate increases to $r' = 25\%$, the optimal consumption for Fantine is (c_1', c_2'). Which of the following statements is/are true?
   A. If $c_1 > 60$, then $U(c_1', c_2') \leq U(c_1, c_2)$.
   B. If $c_1 = 60$, then $U(c_1', c_2') = U(c_1, c_2)$.
   C. If $c_1 < 60$, then $U(c_1', c_2') \geq U(c_1, c_2)$.
   D. All the above.
   E. None of the above.

I. Short Answer Questions

1. In a duopoly, Apple and Samsung compete by setting prices simultaneously. Suppose that the inverse demand function is given by $P = 100 - Q$, and each firm has a constant marginal cost of production equal to 40. There are no fixed costs.

   (a) **(2 marks)** Find the equilibrium prices and profits p_A, p_S, π_A, π_S.

   $$p_A = p_S = 40, \pi_A = \pi_S = 0$$

   (b) **(2 marks)** Suppose instead that Apple and Samsung collude as a monopoly. What is the profit-maximizing price and quantity for the monopoly? What is the total profit?

   $$TR = (100 - Q)Q$$
   $$MR = 100 - 2Q = 40 = MC$$
   $$Q = 30, P = 70, \pi = 900$$

   (c) **(3 marks)** Now suppose that Apple and Samsung compete by setting prices simultaneously for an infinite number of periods. The two firms consider colluding as a monopoly and divide their profits equally. Instead of the Nash reversion strategy, the firms decide to be more forgiving when it comes to the punishment strategy. If one firm cheats, the firms revert to Bertrand competition from the next round onward for only two rounds (instead of forever), and subsequently return to collusion. If the discount factor is $\delta = 0.6$ for both firms, will collusion in all periods be sustainable? Show mathematically.

Without loss of generality, we only need to consider whether Apple has the incentive to cheat,

$$EV_{cheat} = \pi_A(2 + 0\delta + 0\delta^2 + \delta^3 + \delta^4 + \delta^5 + \cdots) = 2\pi_A + \frac{\delta^3}{1-\delta}\pi_A$$

$$EV_{collude} = \pi_A(1 + \delta + \delta^2 + \delta^3 + \delta^4 + \delta^5 + \cdots) = 1.96\pi_A + \frac{\delta^3}{1-\delta}\pi_A$$

$EV_{cheat} > EV_{collude}$, so collusion is not sustainable.
Note that from (i), $\pi_A = \frac{900}{2} = 450$.

2. In a duopoly, Apple and Samsung compete by setting quantities. Samsung observes Apple's level of output before selecting its own. Suppose that the inverse demand function is given by $P = 100 - Q$, and each firm has a constant marginal cost of production given by $MC_S = 40, MC_A = c$ (for simplicity, assume $10 \leq c \leq 70$). There are no fixed costs.

   (a) **(3 marks)** Find the equilibrium price and quantities expressed in c.

   Samsung is the Stackelberg follower:

   $$\pi_s = (100 - q_A - q_S)q_s - 40q_S$$
   $$\frac{\partial \pi_S}{\partial q_S} = 60 - q_A - 2q_S = 0$$

   Now consider the Stackelberg leader, Apple:

   $$\pi_A = \left(100 - q_A - \frac{60 - q_A}{2}\right)q_A - cq_A$$
   $$\frac{\partial \pi_A}{\partial q_A} = 70 - c - q_A = 0$$

   Thus the equilibrium is $q_A = 70 - c, q_S = \frac{c}{2} - 5, P = 35 + \frac{c}{2}$

   (b) **(2 marks)** Suppose $c = 40$. Calculate the Herfindhal index of the market.

   $$HHI = \left(\frac{30}{45}\right)^2 + \left(\frac{15}{45}\right)^2 = \frac{5}{9} \approx 0.56$$

   (c) **(2 marks)** What are the equilibrium price and quantities when $c = 0$? [Hints: no working required; quantities cannot be negative.]
   $q_S = 0$ since q_S cannot be negative. To maximize profit, Apple will set $q_A = 60$. Consequently, $P = 40$.

3. A market consists of 2 firms A and B. Each firm offers shares that are worth \$24 each, but each firm shuts down with probability 0.2. If this occurs, the firm's share will be worth \$0. The probabilities of each firm shutting down are independent. Penny owns 2 shares of Firm A (so the total value is \$48). Suppose Penny's utility function is $U(w) = \sqrt{1 + w}$.

   (a) **(2 marks)** Penny is offered insurance for the shares to insure against the loss when Firm A shuts down. Suppose every dollar of insurance costs $\frac{1}{3}$ dollars. Let K denote

the amount of compensation Penny receives when Firm A shuts down (i.e., Penny's insurance premium is $\frac{1}{3}K$). Assume that K can be any nonnegative real number. What is Penny's utility maximizing amount of K? Is Penny fully insured?

$$EU = 0.8\left(49 - \frac{1}{3}K\right)^{0.5} + 0.2\left(1 + \frac{2}{3}K\right)^{0.5}$$

$$\frac{\partial EU}{\partial K} = -0.8\left(\frac{1}{3}\right)\left(49 - \frac{1}{3}K\right)^{-0.5} + 0.2\left(\frac{2}{3}\right)\left(1 + \frac{2}{3}K\right)^{-0.5} = 0$$

$$4\left(1 + \frac{2}{3}K\right) = 49 - \frac{1}{3}K$$

$$K = \frac{45}{3} = 15$$

Penny is not fully insured (since the insurance is unfair).

(b) **(2 marks)** Instead of the insurance scheme in part (a), suppose only full insurance is offered to Penny. Up to how much would she be willing to pay for full insurance?

Without insurance,

$$EU = 0.8(49)^{0.5} + 0.2(1)^{0.5} = 5.8$$

Thus the insurance cost must satisfy

$$(49 - P)^{0.5} \geq 5.8$$

$$P \leq 49 - 5.8^2 = 15.36$$

(c) **(2 marks)** Now suppose Penny instead owns 1 share of Firm A and 1 share of Firm B. Calculate her expected utility without insurance. Will she be willing to pay more or less for full insurance (covering both Firm A and/or B shutting down) compared to part (b), and why?

$$EU = 0.64(49)^{0.5} + 0.32(25)^{0.5} + 0.04(1)^{0.5} = 6.12$$

Penny will be willing to pay less for full insurance:

i) Comparing $(49 - P^*)^{0.5} = 6.12$ and $(49 - P)^{0.5} = 5.8$, we know that $P^* < P$.

ii) The expected values of the two portfolios are the same, but because Penny's portfolio is diversified in (c), it is less risky than the portfolio in (b). Hence, Penny will be willing to pay less for full insurance for the portfolio in (c).

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