LECTURE 6

Bond Prices and Yields

EC3333 Financial Economics I

Learning Objectives

- Calculate yields and prices of bonds
- Understand how term structure concepts apply to valuation of securities
- Calculate forward rates from the term structure
- Describe the major theories of term structure, their assumptions and implications

Bond Terminology

- Debt (i.e., *fixed-income*) securities promise either a fixed stream of income or one that is determined according to a specified formula
- Bonds are debt obligations of issuers (borrowers) to bondholders (creditors)
- Face value or par value is the principal repaid at maturity, typically $1000
- Coupon rate determines the interest payment ("coupon payments"), it is expressed as an APR

$$\text{Coupon Payment} = \frac{\text{Coupon Rate} \times \text{Face Value}}{\text{Number of Coupon Payments per Year}}$$

- The indenture is the contract between the issuer and the bondholder that specifies the coupon rate, maturity date, and par value

UNITED STATES OF AMERICA

No. 2406 No. 2406

$20 $20

NATIONAL AMERICANIZATION COMMITTEE

National Americanization Committee

PARTICIPATION BOND

TO BE HELD IN TRUST BY THE ABOVE COMMITTEE FOR THE FULFILLMENT OF ITS OBLIGATIONS TO THE OWNER OR OWNERS

Regular dividends will be distributed through intelligent labor in the factory, in the mine, or in the shop, and through a better understanding of American citizenship, native or foreign-born—a citizenship that must be recognized always by all the nations of the world.

EXTRA DIVIDENDS:

A DIVIDEND in a common language through special facilities for acquiring it and the adoption by aliens of American standards.

A DIVIDEND in a common citizenship through an understanding of American principles and community customs.

A DIVIDEND in an improvement of the labor supply and in the reduction of industrial accidents through systematic training.

A DIVIDEND in the stabilization of the working forces through promotion of the business relations between employer and employee.

A DIVIDEND in general welfare through sanitation and improvement of living conditions in immigrant homes.

A DIVIDEND in the translation of American industrial efficiency to the foreign-born workman.

A DIVIDEND in a home-stake for immigrants in America.

A DIVIDEND in their employment of their savings for their future in this, their country, rather than for their return abroad.

A DIVIDEND in the welding of many peoples into one nation.

A DIVIDEND in undivided loyalty to America.

Signers of coupons shall be inscribed on the NATIONAL SERVICE ROLL displaying their heart interest

IN HUMANITY	IN PATRIOTISM	IN AMERICA UNITED
IN USEFUL EMPLOYEES	IN NEW AMERICANS	IN A NEW NATIONALISM

TRUSTEE FOR BOND

CHAIRMAN

RETURN BOND TO COMMITTEE WHEN LAST COUPON IS DETACHED

AMERICAN BANK NOTE COMPANY, OHIO

Source: https://picryl.com/media/participation-bond-to-be-held-in-trust-by-the-above-committee-for-the-fulfillment Library of Congress, http://www.loc.gov/

Useful sites for government bonds

- https://www.wsj.com/market-data/bonds/treasuries
- https://www.bloomberg.com/markets/rates-bonds/government-bonds/us
- https://home.treasury.gov/
- https://www.treasury.gov/resource-center/data-chart-center/interest-rates/Pages/TextView.aspx?data=yield
- https://www.dmo.gov.uk/
- https://www.dmo.gov.uk/data/gilt-market/index-linked-gilts/
- https://www.mas.gov.sg/bonds-and-bills

U.S. Treasury Bonds

- Sovereign debt is debt issued by national govts
- May be purchased directly from the Treasury
- Note maturity is 1-10 years
- Bond maturity is 10-30 years
- Both make semi-annual coupon payments
- Denomination can be as small as $100, but $1,000 is more common
- The bid and asked prices are quoted as a percentage of par value

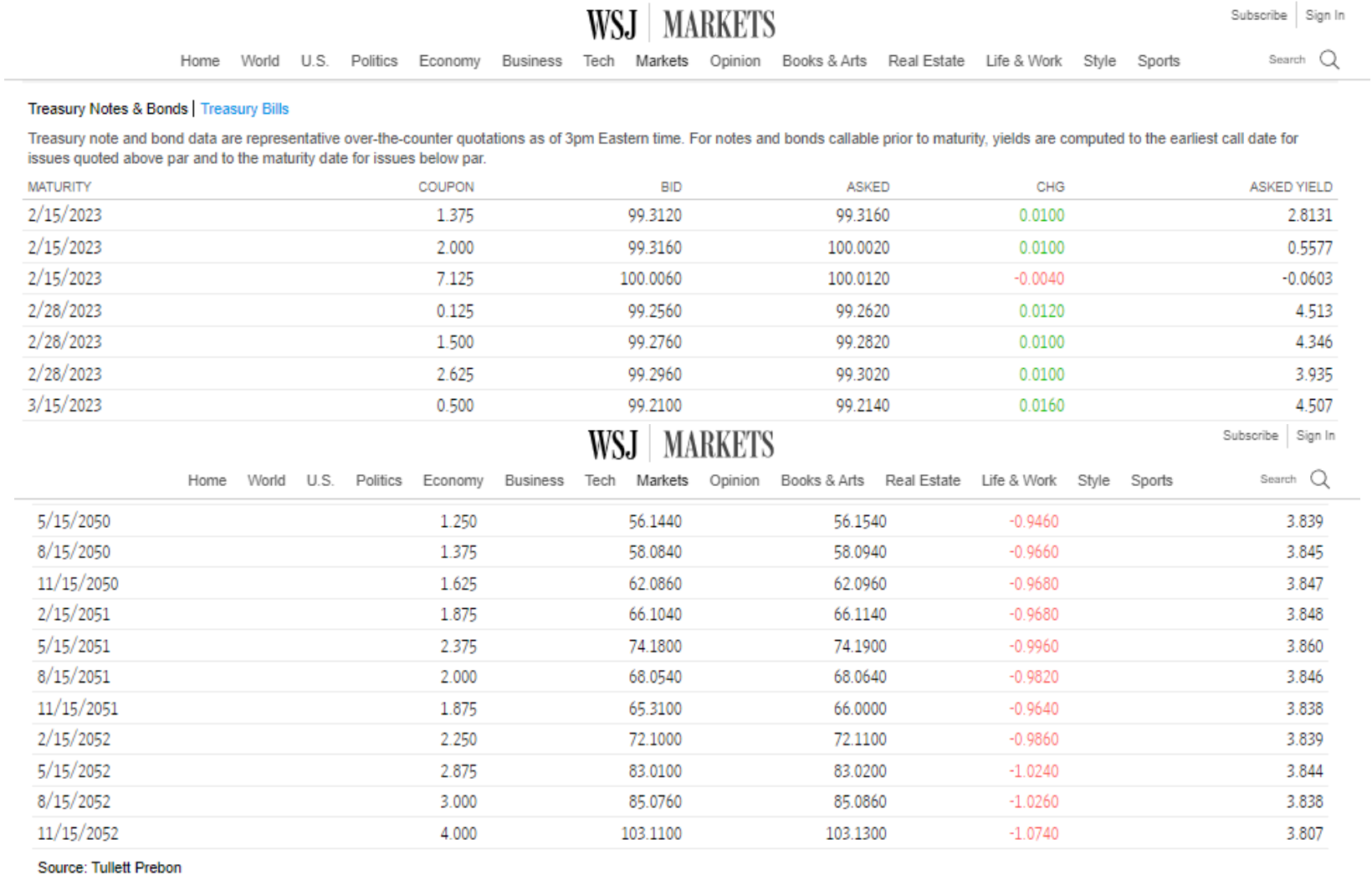
Figure 14.1 Prices and Yields of U.S. Treasure Bonds WSJ May 16, 2016 (from adopted text, Bodie, Kane and Marcus, Investments, McGraw Hill, 12e)

U.S. Treasury Quotes

MATURITY	COUPON	BID	ASKED	CHANGE	ASKED YIELD (%)
May 15 18	1.000	100.3984	100.4141	−0.0859	0.791
May 15 19	0.875	99.8125	99.8281	−0.0859	0.933
Feb 15 21	7.875	130.5781	130.5938	−0.2656	1.225
Aug 15 25	6.875	144.4141	144.4297	−0.5391	1.670
Aug 15 25	2.000	102.2813	102.2969	−0.3438	1.730
May 15 30	6.250	152.3984	152.4609	−0.7969	1.950
Nov 15 41	3.125	111.7891	111.8203	−0.8750	2.496
May 15 46	2.500	97.9922	98.0234	−0.9063	2.595

US Treasury Quotes

Images sourced from: https://www.wsj.com/market-data/bonds/treasuries, Feb 11, 2023.



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Treasury Notes & Bonds | Treasury Bills

Treasury note and bond data are representative over-the-counter quotations as of 3pm Eastern time. For notes and bonds callable prior to maturity, yields are computed to the earliest call date for issues quoted above par and to the maturity date for issues below par.

MATURITY	COUPON	BID	ASKED	CHG	ASKED YIELD
2/15/2023	1.375	99.3120	99.3160	0.0100	2.8131
2/15/2023	2.000	99.3160	100.0020	0.0100	0.5577
2/15/2023	7.125	100.0060	100.0120	-0.0040	-0.0603
2/28/2023	0.125	99.2560	99.2620	0.0120	4.513
2/28/2023	1.500	99.2760	99.2820	0.0100	4.346
2/28/2023	2.625	99.2960	99.3020	0.0100	3.935
3/15/2023	0.500	99.2100	99.2140	0.0160	4.507

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5/15/2050	1.250	56.1440	56.1540	-0.9460	3.839
8/15/2050	1.375	58.0840	58.0940	-0.9660	3.845
11/15/2050	1.625	62.0860	62.0960	-0.9680	3.847
2/15/2051	1.875	66.1040	66.1140	-0.9680	3.848
5/15/2051	2.375	74.1800	74.1900	-0.9960	3.860
8/15/2051	2.000	68.0540	68.0640	-0.9820	3.846
11/15/2051	1.875	65.3100	66.0000	-0.9640	3.838
2/15/2052	2.250	72.1000	72.1100	-0.9860	3.839
5/15/2052	2.875	83.0100	83.0200	-1.0240	3.844
8/15/2052	3.000	85.0760	85.0860	-1.0260	3.838
11/15/2052	4.000	103.1100	103.1300	-1.0740	3.807

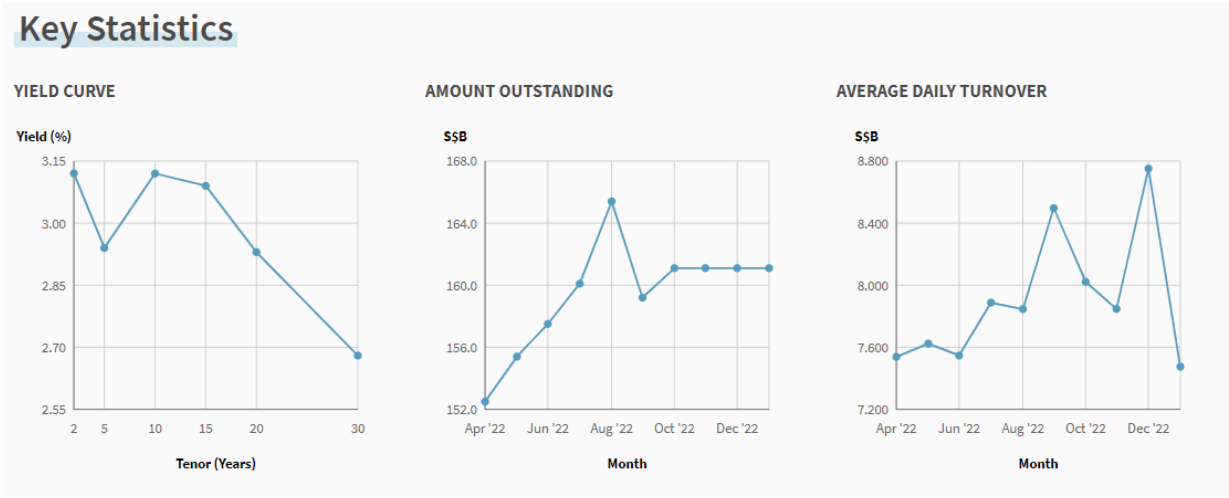
Source: Tullett Prebon

SGS Yield Curve

Source: https://eservices.mas.gov.sg/statistics/fdanet/SgsBenchmarkIssuePrices.aspx & https://www.mas.gov.sg/bonds-and-bills/singapore-government-bonds-information-for-individuals, Downloaded Feb 11, 2023.

	Treasury Bills		Bonds													
	6-Mth	1-Year	2-Year		5-Year		10-Year		15-Year		20-Year		30-Year		50-Year	
Issue Code	BS23102H	BY23100X	NY09100H		N522100N		NX22100W		NZ16100X		NA12100N		NA21200W		NC22300W	
Coupon Rate			3.00%		2.88%		2.63%		2.25%		2.75%		1.88%		3.00%	
Maturity Date	8-Aug-23	30-Jan-24	1-Sep-24		1-Sep-27		1-Aug-32		1-Aug-36		1-Apr-42		1-Oct-51		1-Aug-72	
	Yield	Yield	Yield	Price	Yield	Price	Yield	Price	Yield	Price	Yield	Price	Yield	Price	Yield	Price
3-Feb-23	3.89	3.91	3.02	99.97	2.78	100.39	2.92	97.57	2.95	92.23	2.77	99.64	2.57	86.04	2.59	111.36
6-Feb-23	3.9	3.91	3.08	99.88	2.84	100.15	3.02	96.8	3.03	91.46	2.85	98.49	2.64	84.73	2.65	109.56
7-Feb-23	3.9	3.91	3.09	99.86	2.86	100.07	3.05	96.56	3.05	91.25	2.88	98.14	2.65	84.58	2.66	109.28
8-Feb-23	3.9	3.91	3.06	99.91	2.86	100.06	3.06	96.46	3.05	91.24	2.88	98.14	2.65	84.6	2.65	109.6
9-Feb-23	3.9	3.91	3.07	99.89	2.89	99.93	3.08	96.26	3.07	91.04	2.9	97.78	2.66	84.35	2.66	109.26
10-Feb-23	3.91	3.91	3.12	99.82	2.94	99.74	3.12	95.99	3.09	90.8	2.93	97.33	2.68	83.99	2.68	108.79

* Data reflects bid rates quoted by SGS primary dealers.
* Yield is quoted as % p.a.
* Bond price is quoted in S$ per S$100 of principal amount, excluding any applicable accrued interest (i.e. on clean basis).



Treasury Inflation Protected Securities (TIPS)

- Indexed Bonds
- Payments are tied to the general price index
- Par value of the bond is tied to the general price level
- So both coupon payments and the final repayment of par value increase in direct proportion to CPI

Table 14.1 Principal and Interest Payments for a Treasury Inflation Protected Security (Coupon Rate = 4%)
(from adopted text, Bodie, Kane and Marcus, Investments, McGraw Hill, 12e)

Time	Inflation in Year Just Ended	Par Value	Coupon Payment	+	Principal Repayment	=	Total Payment
0		$1,000.00					
1	2%	1,020.00	$40.80		$ 0		$ 40.80
2	3	1,050.60	42.02		0		42.02
3	1	1,061.11	42.44		1,061.11		1,103.55

Bond Pricing

$$P_B = \sum_{t=1}^{T} \frac{C_t}{(1+r)^t} + \frac{\text{Par Value}}{(1+r)^T}$$

- P_B = Cash price of the bond
- C_t = Interest or coupon payments
- T = Number of periods to maturity
- r = discount rate per period

Bond Pricing

- If you apply the formula for annuity, instead of calculating the sum of (T+1) terms, you only have to calculate the sum of 2 terms

$$P_B = \underbrace{\sum_{t=1}^{T} \frac{C}{(1+r)^t}}_{\text{A T-period Annuity that pays C Per period}} + \frac{\text{Par Value}}{(1+r)^T}$$

$$P_B = \underbrace{\frac{C}{r}\left(1 - \frac{1}{(1+r)^T}\right)}_{\text{The Annuity Formula}} + \frac{\text{Par Value}}{(1+r)^T}$$

Bond Pricing

Example 14.2 Bond Pricing
(from adopted text, Bodie, Kane and Marcus, Investments, McGraw Hill, 12e)

- Suppose the investors demand an annual return of 10%, or 5% per 6-month period
- How much would they pay for:
  - a 30 year, 8% coupon bond with par value of $1000 paying 60 semi-annual coupon payments of $40 each

$$\text{Price} = \sum_{t=1}^{60} \frac{\$40}{(1.05)^t} + \frac{\$1000}{(1.05)^{60}}$$

$$\text{Price} = \$810.71$$

Accrued Interest & Quoted Bond Prices

- Because of accrued interest, the cash price of coupon bond fluctuates around the time of each coupon payment in a sawtooth pattern:
  - It rises as the next coupon payment gets closer and then drops after it has been paid
- If we subtract accrued interest from the bond's cash price and compute the clean price, the saw-tooth pattern of the cash price is eliminated

Yield to Maturity (YTM)

- Interest rate that makes the present value of the bond's payments equal to its price is the yield to maturity (YTM)
- In other words, YTM is the one discount rate that, when applied to the promised cash flows of the bond, recovers the current market price of the bond
- Solve the bond formula for r given the values of P_B and C → The resulting r = YTM

$$P_B = \sum_{t=1}^{T} \frac{C}{(1+r)^t} + \frac{\text{Par Value}}{(1+r)^T}$$

Bond Pricing

Example 14.3 Bond Pricing
(from adopted text, Bodie, Kane and Marcus, Investments, McGraw Hill, 12e)

- Suppose an 8% coupon, 30 year bond is selling for $1,276.76. What is its yield to maturity?

$$1,276.76 = \sum_{t=1}^{60} \frac{40}{(1+r)^t} + \frac{1000}{(1+r)^{60}}$$

  - r = 3% per half year
  - Bond equivalent yield = 6% annually (in Annual Percentage Rate or APR, which does not account for compound interest)
  - EAR = $((1.03)^2) - 1$ = 6.09% annually (Effective Annual Yield or EAR, which accounts for compound interest)

Bond: Prices and Yields

- Prices and yields (required rates of return) have an inverse relationship
- The bond price curve (Figure 14.3) is convex

Figure 14.3 The Inverse Relationship Between Bond Prices and Yields (from adopted text, Bodie, Kane and Marcus, Investments, McGraw Hill, 12e)

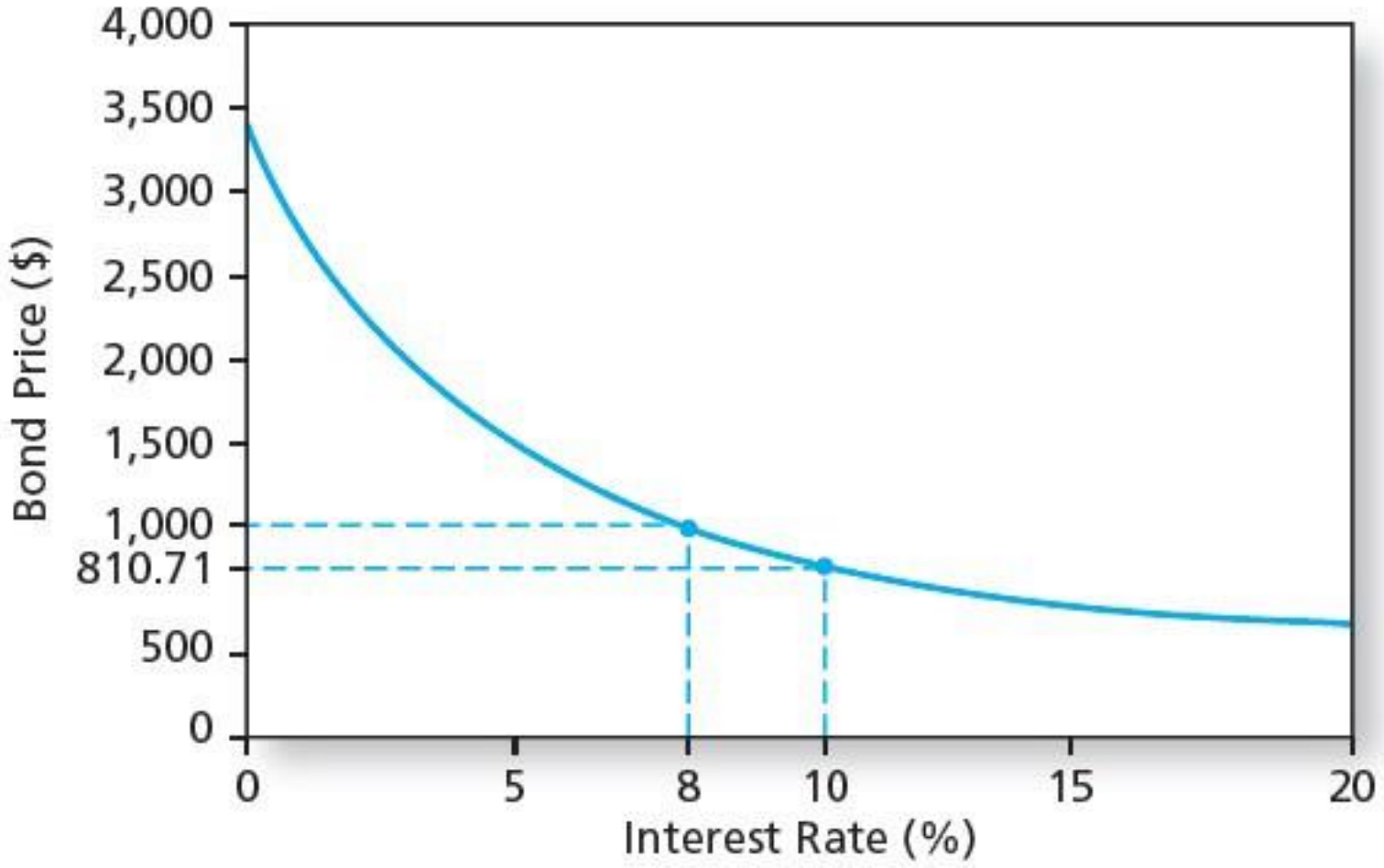
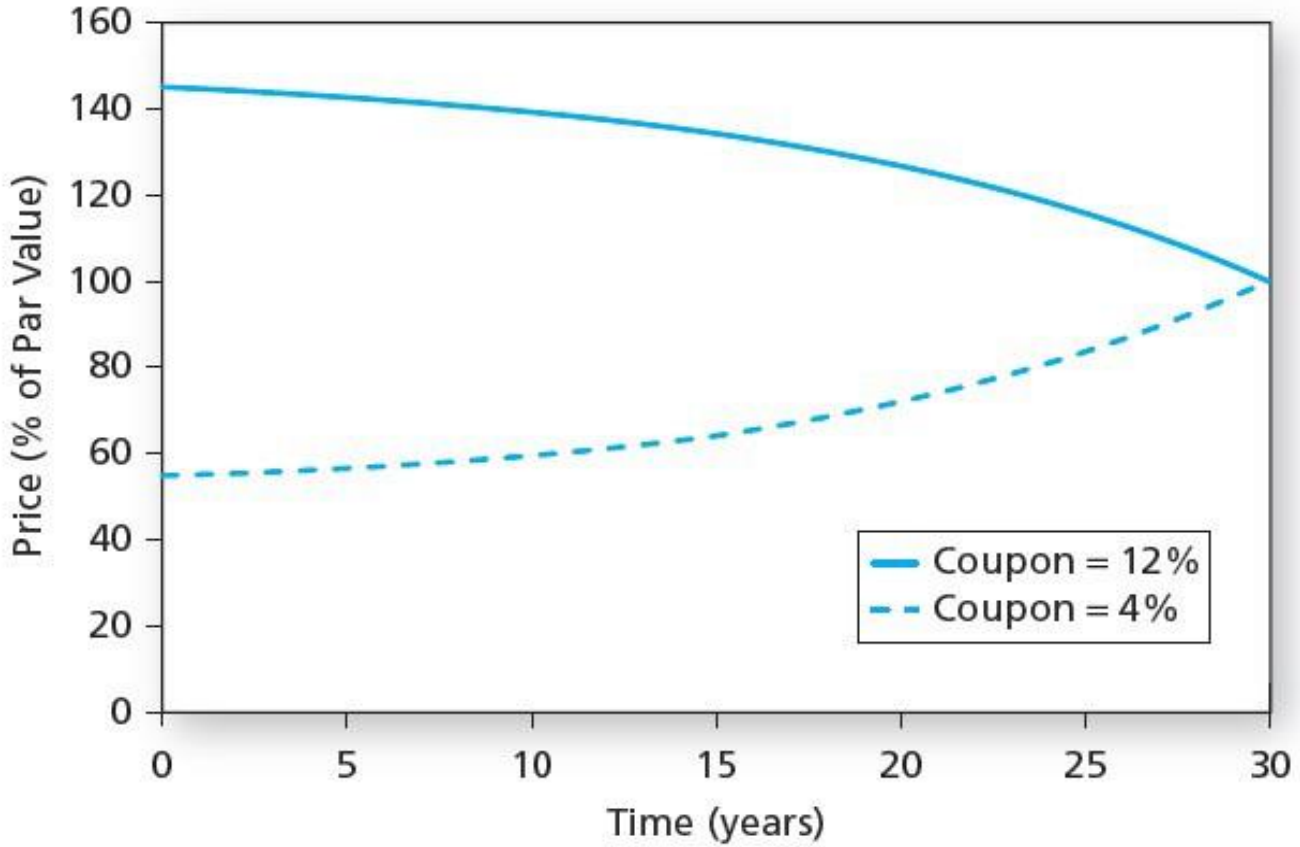


Table 14.2 Bond Prices for an 8% Coupon Bond at Different Interest Rates (from adopted text, Bodie, Kane and Marcus, Investments, McGraw Hill, 12e)

Time to Maturity	Bond Price at Given Market Interest Rate				
	2%	4%	6%	8%	10%
1 year	1,059.11	1,038.83	1,019.13	1,000.00	981.41
10 years	1,541.37	1,327.03	1,148.77	1,000.00	875.35
20 years	1,985.04	1,547.11	1,231.15	1,000.00	828.41
30 years	2,348.65	1,695.22	1,276.76	1,000.00	810.71

- The force of discounting is greatest for the longest-term bond

Figure 14.6 Bond Prices over Time
(from adopted text, Bodie, Kane and Marcus, Investments, McGraw Hill, 12e)



• Two 30-year maturity bonds, each selling at a yield to maturity of 8%. Bond price approaches par value as maturity date approaches

YTM vs. Current Yield

- Yield to Maturity
  - Bond's internal rate of return; accounts for capital gain/loss
  - The interest rate that makes the PV of a bond's payments equal to its price; assumes that all bond coupons can be reinvested at the YTM
- Current Yield
  - Bond's annual coupon payment divided by the bond price
    - For premium bonds (selling above par value)

Coupon rate > Current yield > YTM

    - For discount bonds (selling bellow par value)

YTM > Current yield > Coupon rate

Realized Return from Bond

- Assume an annual coupon bond
- The realized return from holding the bond for one year or the holding period return is

$$R_{t+1} = \frac{C_{t+1} + P_{t+1}}{P_t} - 1 = \frac{C_{t+1}}{P_t} + \frac{P_{t+1} - P_t}{P_t}$$
$$= \text{Current Yield} + \text{Capital Gain/Loss Rate}$$

- P_{t+1} depends on the market interest rate that prevails at t + 1, which is uncertain from today's perspective
- But if the bond is held until maturity, then its value must equal the par value on the maturity date itself.

Realized Yield vs. YTM

- Reinvestment Assumptions
  - When interest rate for reinvestment (of coupon payments) equals YTM, the realized compound return equals YTM
- Holding Period Return
  - Changes in rates affect returns
    - Reinvestment of coupon payments
    - Change in price of the bond
- Horizon Analysis
- Forecasting the realized compound yield over various holding periods or investment horizons

Bond Prices Over Time: YTM vs. HPR

YTM

- Average return if the bond is held to maturity
- Depends on coupon rate, maturity, and par value
- All of these are readily observable

HPR

- Rate of return over a particular investment period
- Depends on the bond's price at the end of the holding period, an unknown future value
- Can only be forecasted

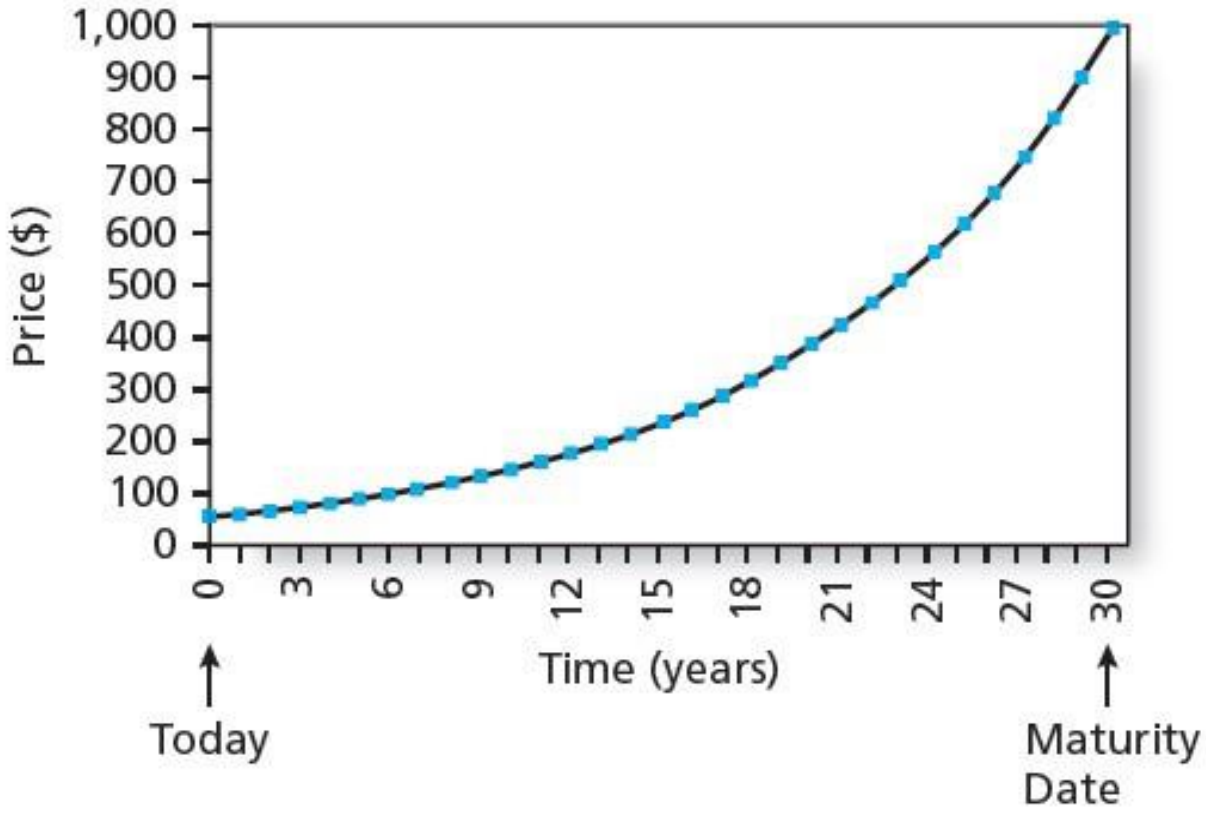
Zero-Coupon Bonds

- Zero-coupon bond does not make coupon payments
- Always sells at a discount (a price lower than face value)
- So they are also called pure discount bonds
- Treasury Bills are U.S. government zero-coupon bonds with a maturity of up to one year.

Treasury Strips

- Longer-term zero-coupon bonds are commonly created from coupon-bearing notes and bonds
- E.g., a 10-year coupon bond would be “stripped” of its 20 semi-annual coupons, and each coupon payment would be treated as a stand-alone zero-coupon bond
- The final payment of principal would be treated as another stand-alone zero-coupon security.
- The maturities of these Treasury strips would thus range from 6 months to 10 years.
- STRIPS: Separate Trading of Registered Interest and Principal of Securities

Figure 14.7 The Price of a 30-Year Zero-Coupon Bond over Time
(from adopted text, Bodie, Kane and Marcus, Investments, McGraw Hill, 12e)



• The bond prices rise exponentially, not linearly, until its maturity Price = $1{,}000/(1+y)^T$, where T = time to maturity, y = 10%

Default Risk

- Rating companies
  - Moody's, Standard & Poor's, Fitch
- Rating Categories
  - Highest rating is AAA or Aaa
  - Investment grade bonds are rated BBB or above (S&P, Fitch) or Baa and above (Moody's)
  - Speculative grade / junk bonds / high-yield bonds have ratings below BBB or Baa
- Higher probability of default for bonds with lower ratings.
  The risk of default rises in a recession

Bond Ratings										
	Very High Quality			High Quality		Speculative		Very Poor	
Standard & Poor's	AAA	AA		A	BBB	BB	B	CCC	D
Moody's	Aaa	Aa		A	Baa	Ba	B	Caa	C

At times both Moody's and Standard & Poor's have used adjustments to these ratings: S&P uses plus and minus signs: A+ is the strongest A rating and A− the weakest. Moody's uses a 1, 2, or 3 designation, with 1 indicating the strongest.

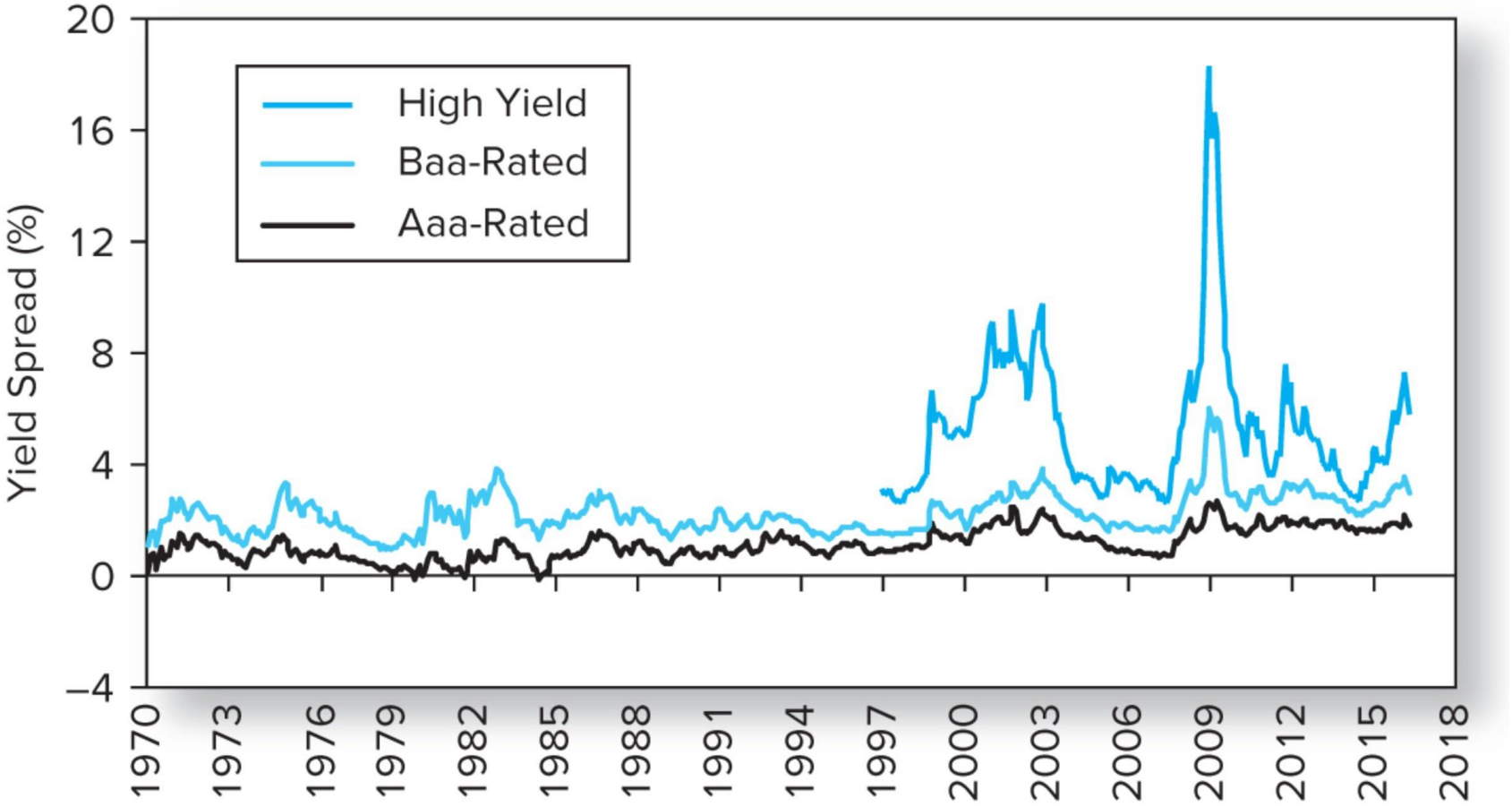
Moody's	S&P	
Aaa	AAA	Debt rated Aaa and AAA has the highest rating. Capacity to pay interest and principal is extremely strong.
Aa	AA	Debt rated Aa and AA has a very strong capacity to pay interest and repay principal. Together with the highest rating, this group comprises the high-grade bond class.
A	A	Debt rated A has a strong capacity to pay interest and repay principal, although it is somewhat more susceptible to the adverse effects of changes in circumstances and economic conditions than debt in higher-rated categories.
Baa	BBB	Debt rated Baa and BBB is regarded as having an adequate capacity to pay interest and repay principal. Whereas it normally exhibits adequate protection parameters, adverse economic conditions or changing circumstances are more likely to lead to a weakened capacity to pay interest and repay principal for debt in this category than in higher-rated categories. These bonds are medium-grade obligations.
Ba B Caa Ca	BB B CCC CC	Debt rated in these categories is regarded, on balance, as predominantly speculative with respect to capacity to pay interest and repay principal in accordance with the terms of the obligation. BB and Ba indicate the lowest degree of speculation, and CC and Ca the highest degree of speculation. Although such debt will likely have some quality and protective characteristics, these are outweighed by large uncertainties or major risk exposures to adverse conditions. Some issues may be in default.
C	C	This rating is reserved for income bonds on which no interest is being paid.
D	D	Debt rated D is in default, and payment of interest and/or repayment of principal is in arrears.

Figure 14.8 Definitions of each bond rating class (from adopted text, Bodie, Kane and Marcus, Investments, McGraw Hill, 12e)

The Default Premium

- Default premium is the spread between the promised YTM and that on otherwise-comparable Treasury bonds that is riskless in terms of default
- When a bond becomes more subject to default risk, its price will fall, thus raising its promised yield to maturity and consequently the default premium
- The promised yield will be realized only if the firm does not default
- In a financial crisis, the default spreads spike, e.g., in 2008

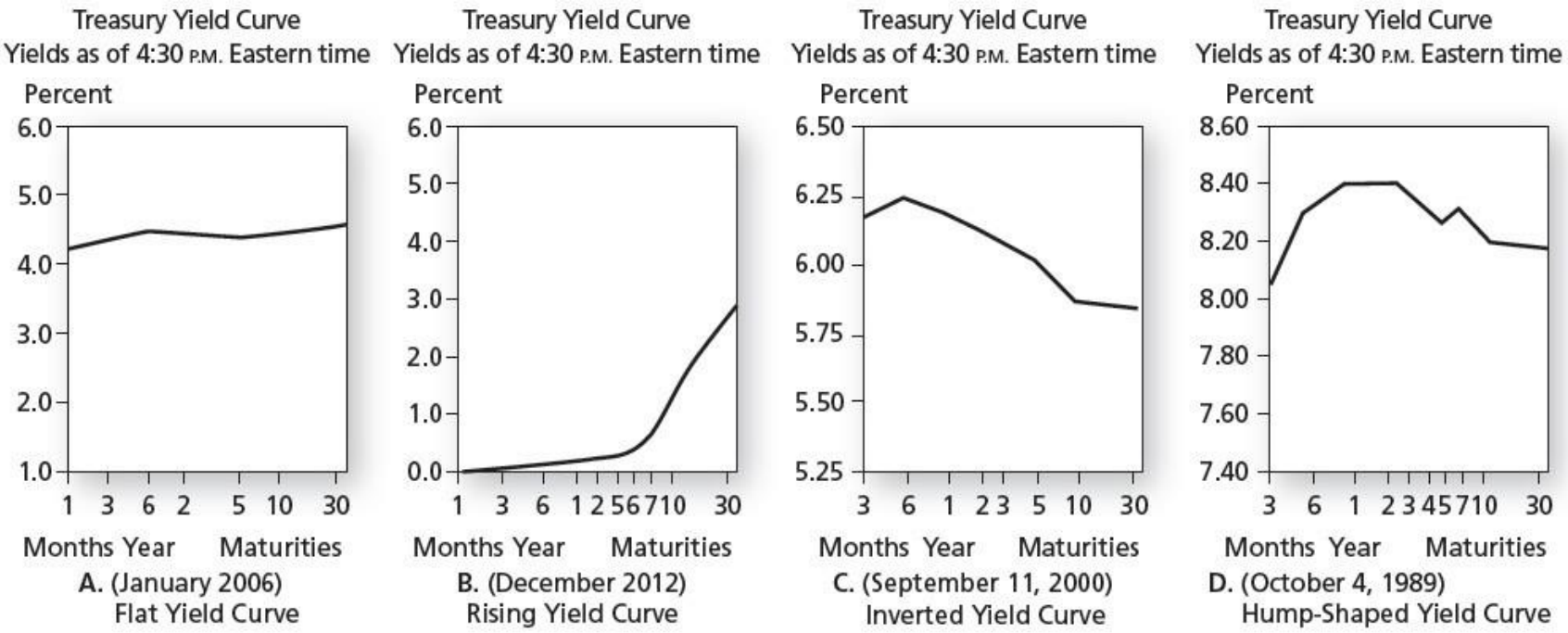
Figure 14.11 Yield Spreads between Corporate & 10-year Treasury Bonds (from adopted text, Bodie, Kane and Marcus, Investments, McGraw Hill, 12e)



The Yield Curve

- The yield curve is a graph that depicts the relationship between YTM and time to maturity
  - A graph of the term structure, where
  - The term structure of interest rates is the structure of interest rates for discounting cash flows of different maturities
- The yield curve is generally upward sloping, but can also be downward sloping or humped-shaped
- Information on expected future short-term rates can be implied from the yield curve
  - E.g., the expectations hypothesis

Figure 15.1 Treasury Yield Curves
(from adopted text, Bodie, Kane and Marcus, Investments, McGraw Hill, 12e)



Yield Curve: Bond Pricing using zero-coupon strips

- Yields on different maturity bonds are not equal
- To price a bond, one can consider cash flow of each bond as a stand-alone zero-coupon bond
- Bond stripping and bond reconstitution offer opportunities for arbitrage
- The value of the bond should be the sum of the values of its parts with arbitrage free pricing

The Yield Curve & Discount Rates

- The term structure can be used to compute the present and future values of a risk-free cash flow over different investment horizons

$$PV = \frac{C_n}{(1 + r_n)^n}$$

- Present Value of a Cash Flow Stream Using a Term Structure of Discount Rates

$$PV = \frac{C_1}{1 + r_1} + \frac{C_2}{(1 + r_2)^2} + \cdots + \frac{C_N}{(1 + r_N)^N} = \sum_{n=1}^{N} \frac{C_N}{(1 + r_n)^n}$$

Pricing Zero-Coupon Bonds

- Prices and Yields to Maturities on Zero-Coupon Bonds ($1,000 Face Value)
- Table 15.1 (from adopted text, Bodie, Kane and Marcus, Investments, McGraw Hill, 12e

Maturity (years)	Yield to Maturity (%)	Price
1	5%	$952.38 = $1,000/1.05
2	6	$890.00 = $1,000/1.06^2
3	7	$816.30 = $1,000/1.07^3
4	8	$735.03 = $1,000/1.08^4

Pricing Coupon Bonds

- Table 15.1 Prices and Yields to Maturities on Zero-Coupon Bonds ($1,000 Face Value) (from adopted text, Bodie, Kane and Marcus, Investments, McGraw Hill, 12e

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4	8	$735.03 = $1,000/1.08^4

- Value a 3-year, 10% coupon bond using discount rates from Table 15.1:

$$\text{Price} = \frac{\$100}{1.05} + \frac{\$100}{1.06^2} + \frac{\$1100}{1.07^3} = \$1{,}082.17$$

  - Price = $1,082.17 and YTM = 6.88%

$$\text{Price} = \frac{\$100}{(1+YTM)^1} + \frac{\$100}{(1+YTM)^2} + \frac{\$1100}{(1+YTM)^3} = \$1{,}082.17$$

  - 6.88% is less than the 3-year rate of 7%

Pricing a bond

- Compute the present value of a risk-free three-year annuity of $500 per year, given the following yield curve:

Zero Coupon Treasury Rates	
Term (Years)	**Rate**
1	0.261%
2	0.723%
3	1.244%

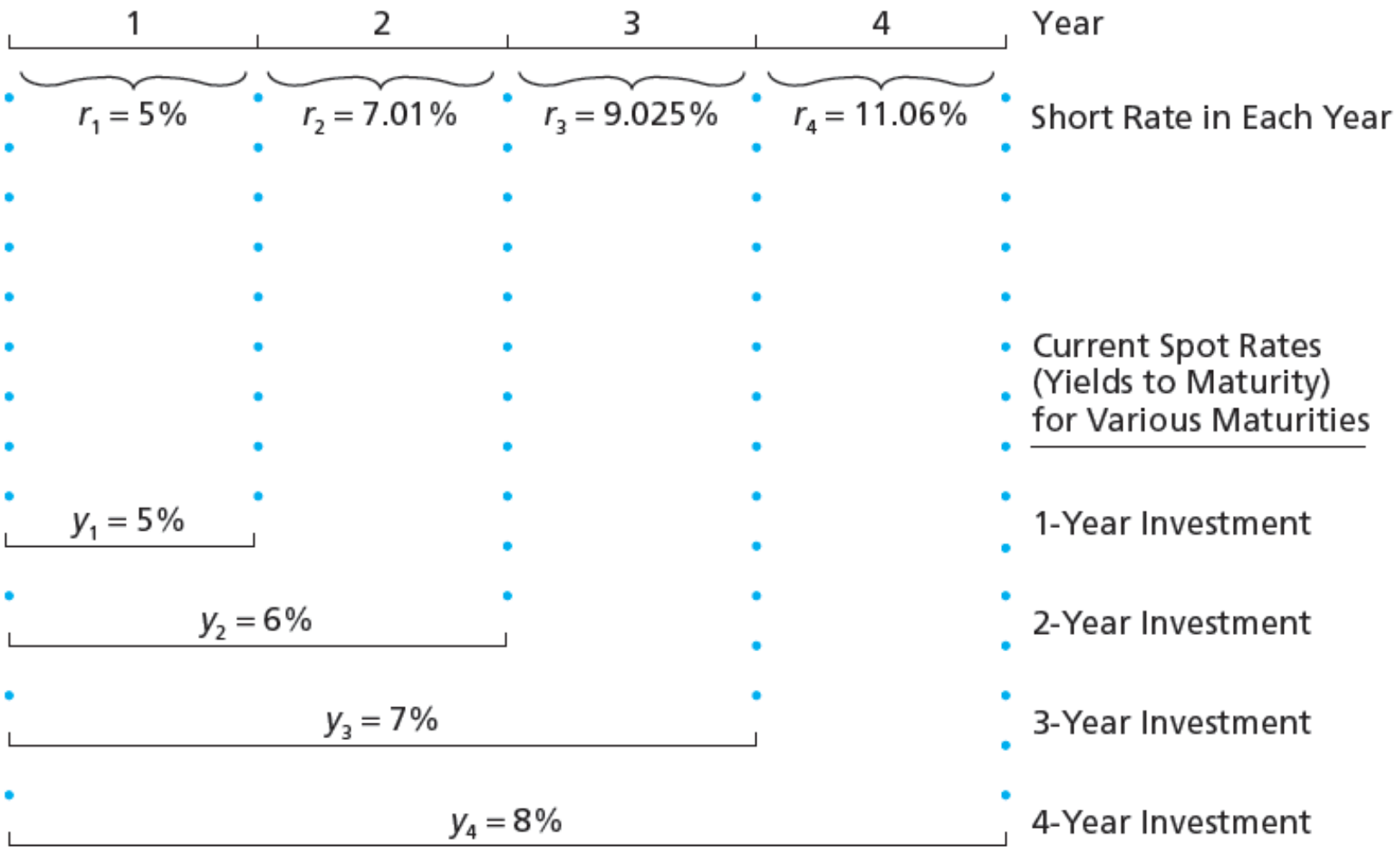
- Each cash flow must be discounted by the corresponding interest rate:

$$PV = \frac{\$500}{1.00261} + \frac{\$500}{1.00723^2} + \frac{\$500}{1.01244^3} = \$1,473.34$$

The Yield Curve

- Information on expected future short-term rates can be implied from the yield curve
- **Spot rate**: YTM on zero-coupon bonds
  - The rate that prevails today for a time period corresponding to the zero's maturity
- **Short rate**: Applies for a given time interval (e.g., one year)
  - The rate for a given time interval or maturity (e.g. one year) at different points in time
  - E.g., short rate for each year

Figure 15.2 Short Rates versus Spot Rates
(from adopted text, Bodie, Kane and Marcus, Investments, McGraw Hill, 12e)



Forward Interest Rates

- An interest rate forward contract is a contract today that fixes the interest rate for a loan or investment in the future.
- A forward rate is an interest rate that we can guarantee today for a loan or investment that will occur in the future.
- Here we **consider interest rate forward contracts for one-year investments**, so the forward rate for year 5 means the rate available today on a one-year investment that begins four years from today.
- By the law of one price, the forward rate for year 1 (f_1) is equal to the yield to maturity of a one-year zero-coupon bond (y_1).

$$f_1 = y_1$$

Forward Interest Rates

- Suppose you want to invest for 2 years
  - Buy and hold a 2-year zero

or

  - Buy a 1-year zero and simultaneously enter into a one-year interest rate forward contract for year 2 at a rate of f_2

$$(1+y_2)^2 = (1+y_1)(1+f_2)$$

$$(1+f_2) = \frac{(1+y_2)^2}{(1+y_1)}$$

  - y_n is the yield to maturity of a zero-coupon bond with an *n*- period maturity

Forward Interest Rate

- In general,

$$(1+f_n) = \frac{(1+y_n)^n}{(1+y_{n-1})^{n-1}}$$

  - f_n = One-year forward rate for period n
  - y_n = Yield for a security with a maturity of n

$$(1+y_n)^n = (1+y_{n-1})^{n-1}(1+f_n)$$

Computing Forward Rates

Example 6A.1 from adopted text, Berk and DeMarzo, Corporate Finance, Pearson, 5e

- **Problem**
  - Calculate the forward rates for years 1 through 5 from the following zero-coupon yields:

Maturity	**1**	**2**	**3**	**4**
YTM	5.00%	6.00%	6.00%	5.75%

Computing Forward Rates

Example 6A.1 from adopted text, Berk and DeMarzo, Corporate Finance, Pearson, 5e

- **Solution**

$$f_1 = YTM_1 = 5.00\%$$

$$f_2 = \frac{(1+YTM_2)^2}{(1+YTM_1)} - 1 = \frac{1.06^2}{1.05} - 1 = 7.01\%$$

$$f_3 = \frac{(1+YTM_3)^3}{(1+YTM_2)^2} - 1 = \frac{1.06^3}{1.06^2} - 1 = 6.00\%$$

$$f_4 = \frac{(1+YTM_4)^4}{(1+YTM_3)^3} - 1 = \frac{1.0575^4}{1.06^3} - 1 = 5.00\%$$

Theories of the Term Structure of Interest Rates – The Expectations Hypothesis

- Risk Neutrality → the Expectations Hypothesis
  - Risk neutral investors → only expected returns matter

    → Bonds of different maturities are perfect substitutes
  - No arbitrage → $f_n = E(r_n)$

    → Forward rate is a forecast of a future short rate, specifically, if expectation hypothesis holds, forward rate equals the expected future spot rate.

Theories of the Term Structure of Interest Rates – The Expectations Hypothesis

- The **Expectations Hypothesis**
  - Observed long-term rate is a function of today's short-term rate and expected future short-term rates

$$f_n = E(r_n)$$

  - The interest rate on a long-term bond will be equal to the geometric average (or, *approximately*, arithmetic average) of the short-term interest rates that people expect to occur over the life of the long-term bond

Theories of the Term Structure of Interest Rates – Liquidity Premium Theory

- Risk averse investors require a risk premium to hold a longer-term bond → the Liquidity Preference Theory
- This **liquidity premium** compensates short-term investors for the uncertainty about future prices

Figure 15.5 Price volatility of long-term Treasury bonds
(from adopted text, Bodie, Kane and Marcus, Investments, McGraw Hill, 12e)

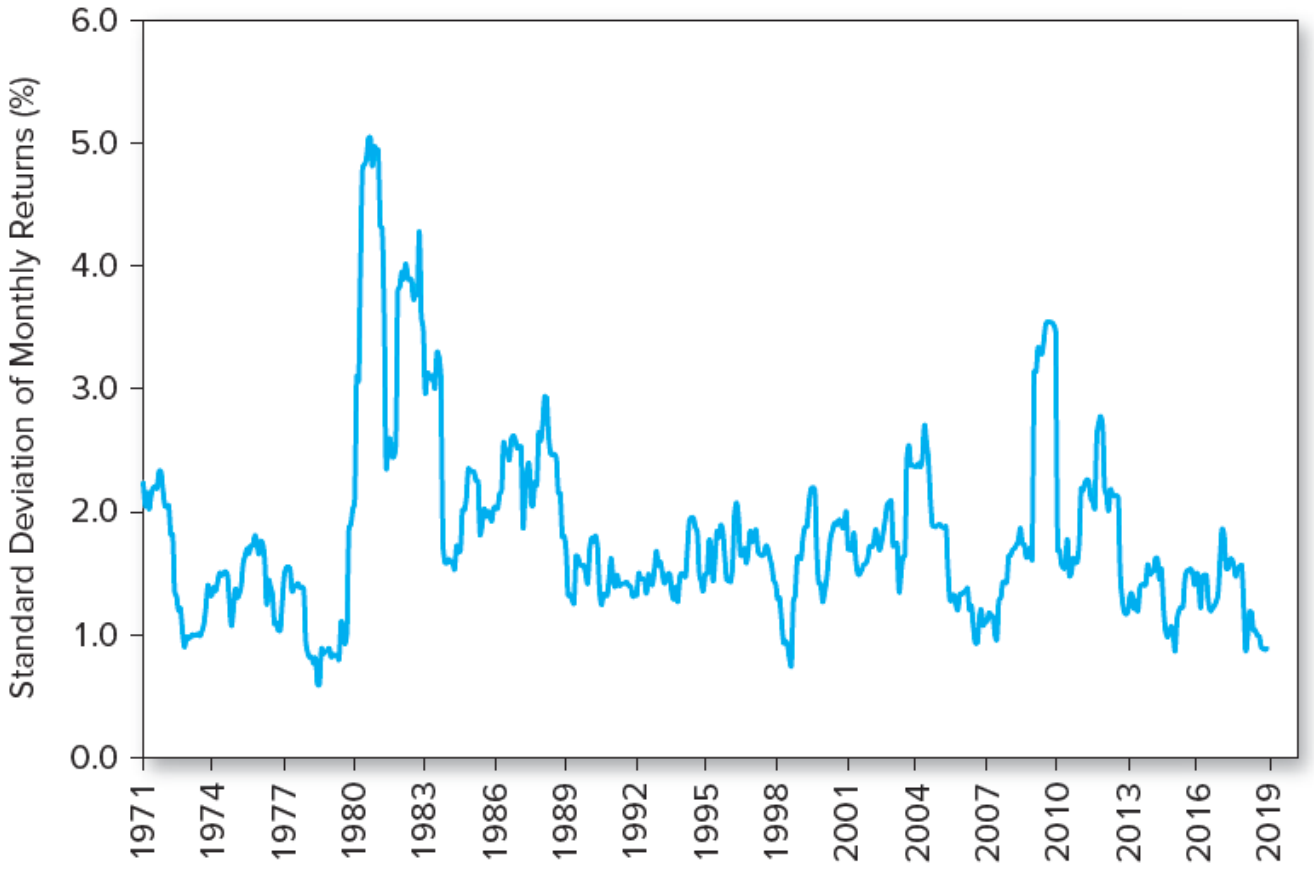


Figure 15.5 Price volatility of long-term Treasury bonds

Theories of the Term Structure of Interest Rates – Liquidity Premium Theory

- The **Liquidity Preference Theory**
- Long-term bonds are more risky

$$f_n > E(r_n)$$

- The excess of f_n over $E(r_n)$ is the *liquidity premium*
  - Predicted to be positive
- Yield curve has an upward bias built into the long-term rates because of the liquidity premium

Interpreting the Term Structure

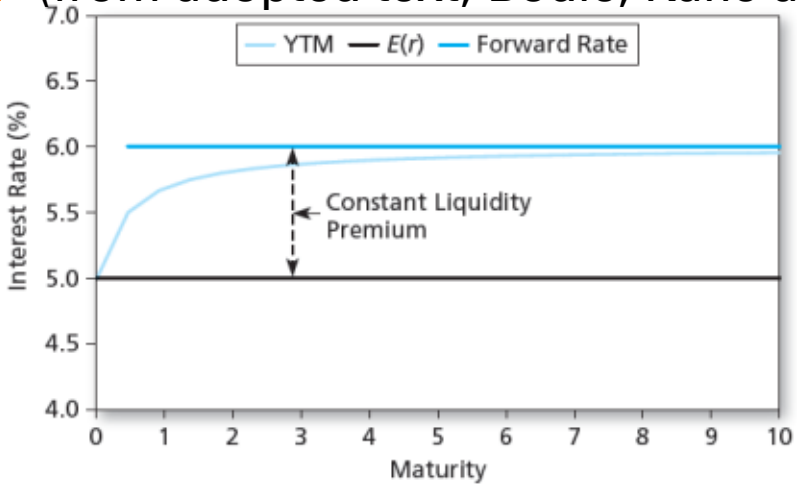
- Yield curve reflects expectations of future short rates, and also reflects other factors such as liquidity premiums
- An upward sloping curve could indicate:
  - Rates are expected to rise

    and/or

  - Investors require large liquidity premiums to hold long term bonds
- The yield curve has been used by the markets as a predictor of the business cycle
  - Long-term rates tend to rise in anticipation of economic expansion
  - Inverted yield curve may indicate that interest rates are expected to fall and signal a recession

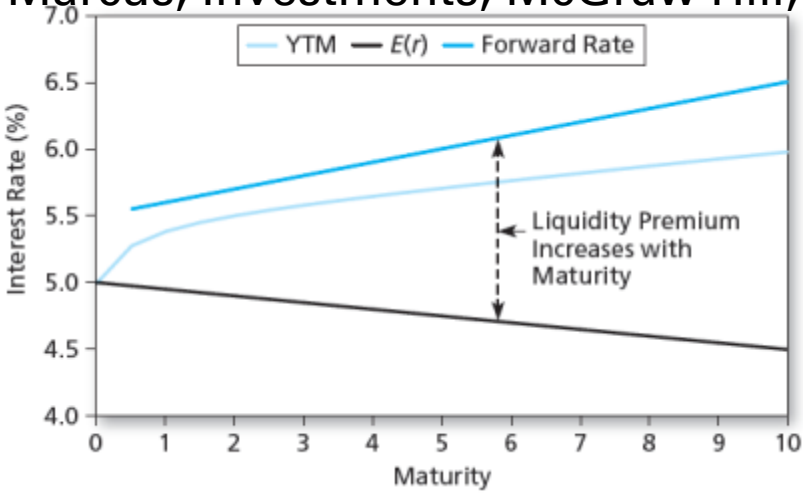
Figure 15.4 Yield curves
(from adopted text, Bodie, Kane and Marcus, Investments, McGraw Hill, 12e)

A



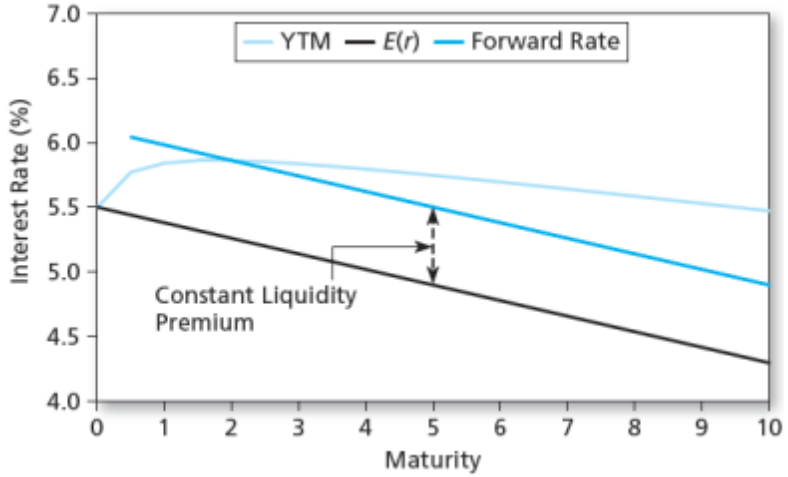
Panel A:
Constant Expected Short Rate.
Liquidity Premium of 1%.

B



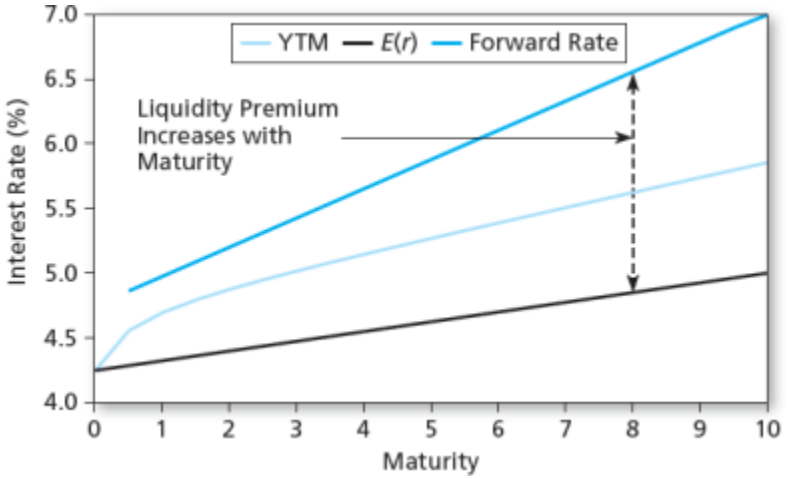
Panel B:
Declining Expected Short Rates.
Increasing Liquidity Premiums.

C



Panel C:
Declining Expected Short Rates.
Constant Liquidity Premiums.

D



Panel D:
Increasing Expected Short Rates.
Increasing Liquidity Premiums.

Figure 15.6 Term Spread: Yields on 10-year vs. 90-day Treasury Securities (from adopted text, Bodie, Kane and Marcus, Investments, McGraw Hill, 12e)

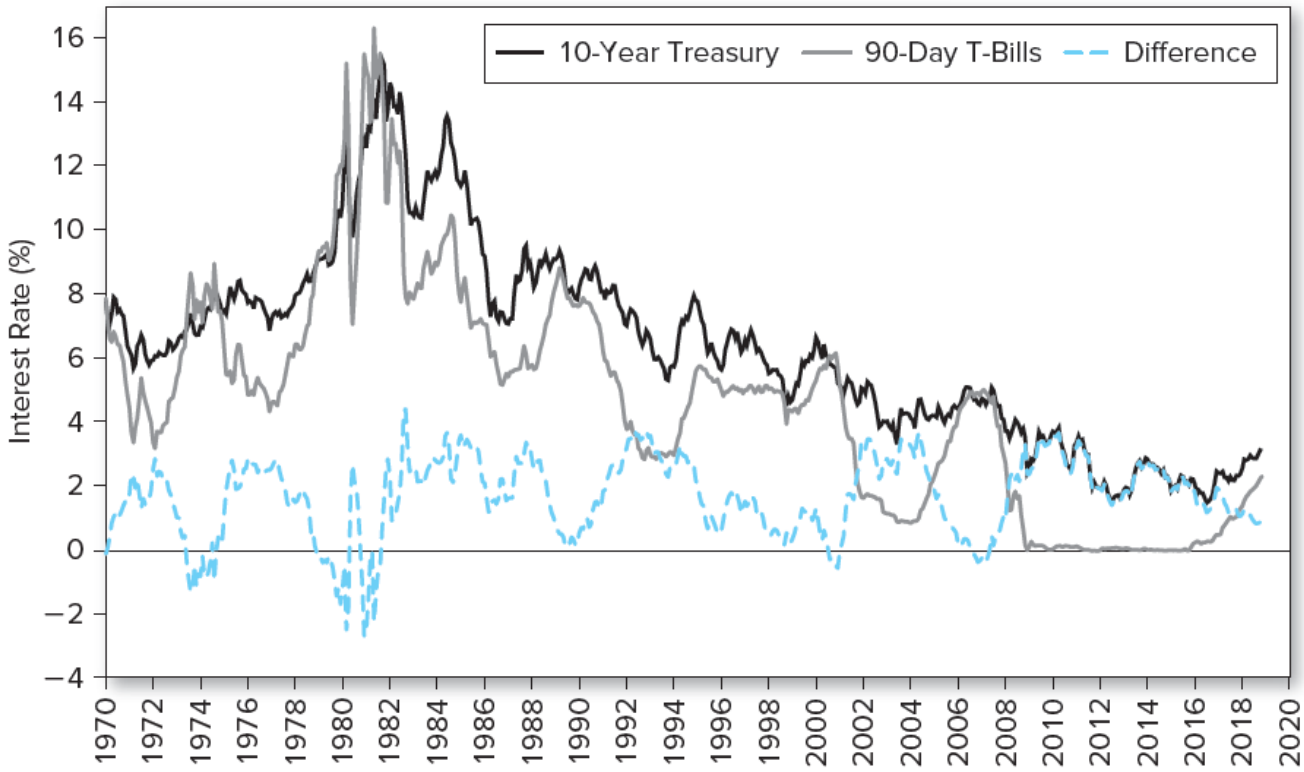
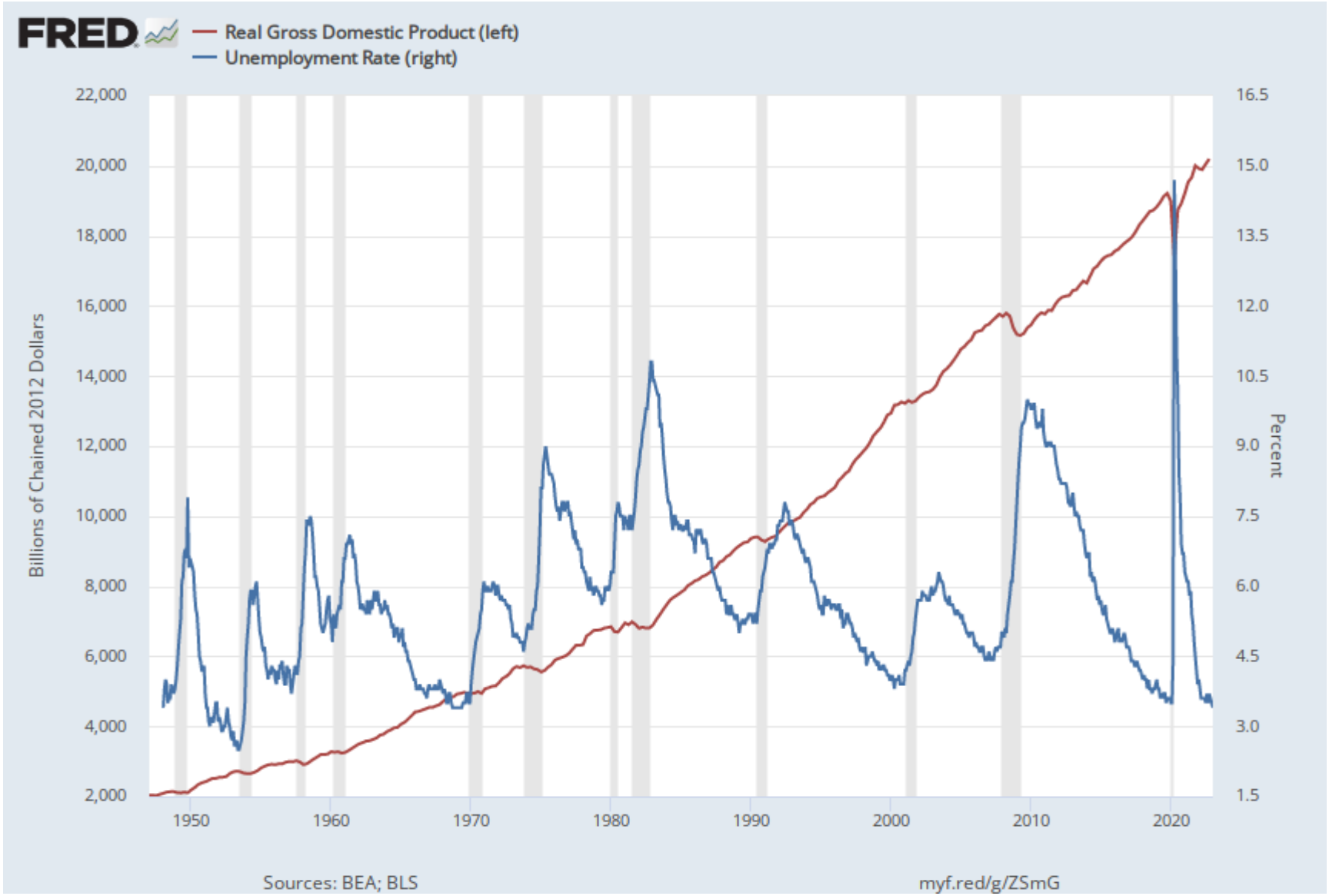


Figure 15.6 Term spread: Yields on 10-year versus 90-day Treasury securities



Source: https://fred.stlouisfed.org/