Tutorial 1 - Submission

A0219739N - Le Van Minh

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1 Rock-paper-scissors

There's no pure strategy Nash Equilibrium. Mixed strategy Nash Equilibrium is: $\sigma = \left\{ \left\{ \frac{1}{3}, \frac{1}{3}, \frac{1}{3} \right\}, \left\{ \frac{1}{3}, \frac{1}{3}, \frac{1}{3} \right\} \right\}$

2 2×2 games

1. Example payoffs: $(a, b, c, d, \alpha, \beta, \gamma, \delta) = (0, 1, 1, 0, 1, 0, 0, 1)$
   required conditions for no pure strategy equilibrium:

$$(a - c)(b - d) < 0 \qquad (\alpha - \gamma)(\beta - \delta) < 0 \qquad (\alpha - \beta)(d - b) < 0$$

2. Player strategies: $\sigma_1 = \{p, 1 - p\}, \sigma_2 = \{q, 1 - q\}$ such that:

$$p = \frac{d - b}{a - b - c + d}$$
$$q = \frac{\delta - \beta}{\alpha - \beta - \gamma + \delta}$$

3. Mixed strategy Nash Equilibrium always exists unless $a + d = c + d$ or $\alpha + \delta = \gamma + \delta$, in which case the condition ($(a - c)(b - d) < 0$ or $(\alpha - \gamma)(\beta - \delta) < 0$) is violated, and the game would have pure strategy Nash Equilibrium.

3 Strategy of the commons

1. Game specification:
   - Set of player $N = \{1, 2, \dots, 16\}$
   - Each player i has a set of strategy $S_i = \{L1, L2\}$ whether to fish in lake 1 or lake 2.
   - Utility function for each player u_i is the number of fish can catch, given a strategy profile s.
2. The Nash Equilibrium is $(L_1, L_2) = (8, 8)$ and $(7, 9)$. Total number of fishes caught is 64 or 67.5.
3. The maximising number of fishermen on lake 1 is $L_1 = 4$, with the total of 72 fishes.

4. Optimal permit cost is one that mame income one makes on lake 2 balances to that of lake 1, when desired strategy profile is played.

$$\begin{aligned} f_1'(4) = f_1(4) - P &= f_2(12)(or + 0.5) \\ 8 - \frac{4}{2} - P &= 4(or 4.5) \\ P &= 2(or 1.5) \\ P &\in [1.5, 2] \end{aligned}$$

4 Beauty contest

This is the older version of the question, the new version has $A = \frac{1}{3n}\sum_{i \in N} s_i$

$$A = \frac{2}{3n}\sum_{i \in N} s_i$$

1. In s^0, if anyone chooses any other number, they will be further than the goal than the rest. So no one wants to divert from this strategy.
2. Let $a < b$ be the minimum and maximum number chosen by the participants. If $A \neq \frac{a+b}{2}$, there's some participants farther from A than the others, and they can improve their earning by choosing the same number as the closest person. Since each person has roughly 1% impact on A, their decision will not affect the result very much. Therefore, $A = \frac{a+b}{2}$ for a strategy profile to be pure-strategy Nash equilibrium, and everyone must either choose a or b.
   But with $A = \frac{a+b}{2}$, either side can switch side and make the result slightly tilt to their side, and make more money as less people are winning.
   Therefore, it is required that $a = b$ to reach Nash Equilibrium. But with $a = b > 1$, choosing a number one unit lower will alway earn you more money.
   $\Rightarrow$ The only pure-strategy Nash Equilibrium are s^0 and s^1.

5 Mixed-strategy equilibrium

Supposed $u_i(s_i) = u_i(s_i')$ with all $s_i, s_i' \in S_i$. Then:

$$u_i(\sigma_i, \sigma_{-i}) = u_i(\sigma_i', \sigma_{-i}) = u_i(s_i)$$

with all $\sigma_i, \sigma_i' \in \Sigma_i$ and $s_i \in S_i$. Therefore:

$$\exists! \sigma_i', u_i(\sigma_i', \sigma_{-i}) > u_i(\sigma)$$

Then:

$$u_i(\sigma_i', \sigma_{-i}) > u_i(\sigma) \Rightarrow \exists s_i, s_i' \in S_i, u_i(s_i) \neq u_i(s_i')$$

Let $s_i = \arg\max_{s_i \in S_i} u_i(s_i)$:

$$u_i(s_i) = \max_{\sigma_i \in \Sigma_i} u_i(\sigma_i, \sigma_{-i}) \geq u_i(\sigma_i', \sigma_{-i}) > u_i(\sigma) \blacksquare$$