# EC3333 Tutorial 4 Suggested Answers

1. The Optima Mutual Fund has an expected return of 20% and a volatility of 20%. Optima claims that no other portfolio offers a higher Sharpe ratio. Suppose this claim is true, and the risk-free interest rate is 5%.

   a. What is Optima's Sharpe Ratio?

   b. If eBay's stock has a volatility of 40% and an expected return of 11%, what must be its correlation with the Optima Fund?

   c. If the SubOptima Fund has a correlation of 80% with the Optima Fund what is the Sharpe ratio of the SubOptima Fund?

Sharpe Ratio measures the ratio of reward-to-volatility provided by a portfolio.

$$\text{Sharpe Ratio} = \frac{\text{Portfolio Excess Return}}{\text{Portfolio Volatility}} = \frac{E[R_P]-r_f}{SD(R_P)}$$

a.

Optima's Sharpe Ratio = (20% – 5%)/20% = 0.75.

The portfolio with the highest Sharpe ratio is the portfolio where the line with the risk-free investment is tangent to the efficient frontier of risky investments. The portfolio is known as the **tangent portfolio**.

If you were to purchase more of investment *i* by borrowing, you would earn the expected return of *i* minus the risk-free return. Adding *i* to the portfolio *P* will improve our Sharpe ratio if

$$\underbrace{E[R_i]-r_f}_{\substack{\text{Additional return}\\ \text{from investment } i}} > \underbrace{SD(R_i)\times \text{Corr}\left(R_i,R_p\right)}_{\substack{\text{Incemental volatility}\\ \text{from investment i}}} \times \underbrace{\frac{E[R_P]-r_f}{SD(R_P)}}_{\substack{\text{Return per unit of volatilty}\\ \text{available from portfolio P}}}$$

If Optima Fund has the highest Sharpe ratio,

$$\underbrace{E[R_i]-r_f}_{\substack{\text{Additional return}\\ \text{from investment } i}} = \underbrace{SD(R_i)\times \text{Corr}\left(R_i,R_p\right)}_{\substack{\text{Incemental volatility}\\ \text{from investment i}}} \times \underbrace{\frac{E[R_P]-r_f}{SD(R_P)}}_{\substack{\text{Return per unit of volatilty}\\ \text{available from portfolio Optima}}}$$

b.

eBay's Sharpe Ratio = (11% – 5%)/40% = 0.15, and its correlation must be 0.15/0.75 = 0.2.

c.

SubOptima's Sharpe Ratio = 0.8 × 0.75 = 0.6.

2. Your investment portfolio consists of $15,000 invested in only one stock — Amazon. Suppose the risk-free rate is 5%, Amazon stock has an expected return of 12% and a volatility of 40%, and the market portfolio has an expected return of 10% and a volatility of 18%. Under the CAPM assumptions,

   a. What alternative investment has the lowest possible volatility while having the same expected return as Amazon? What is the volatility of this investment?

   b. What investment has the highest possible expected return while having the same volatility as Amazon? What is the expected return of this investment?

a.

Under the CAPM assumptions, the market is efficient; that is, a leveraged position in the market has the highest expected return of any portfolio for a given volatility and the lowest volatility for a given expected return. By holding a leveraged position in the market portfolio, you can achieve an expected return of

$$E[R_p] = r_f + x(E[R_m] - r_f) = 5\% + x \times 5\%$$

Setting this equal to 12% gives

$$12 = 5 + 5x \Rightarrow x = 1.4$$

Therefore, the portfolio with the lowest volatility that has the same return as Amazon has

1.4 × ($15,000) = $21,000 in the market portfolio, and borrows $21,000-$15,000 = $6,000, that is, –$6,000 in the risk-free asset.

$$SD(R_P) = xSD[R_m] = 1.4 \times 18 = 25.2\%$$

Note that this is considerably lower than Amazon's volatility.

b.

A leveraged portion in the market has volatility of

$$SD(R_\rho) = xSD(R_m) = x \times 18\%$$

Setting this equal to the volatility of Amazon gives

$$40\% = x \times 18\%$$
$$x = \frac{40}{18} = 2.222$$

Therefore, the portfolio with the highest expected return that has the same volatility as Amazon has $15,000 × 2.2 = $33,000 in the market portfolio, and borrows $33,000 − $15,000 = $18,333.33, that is −$18,333.33 in the risk-free asset.

$$E[R_p] = r_f + x(E[R_m] - r_f) = 5\% + 2.222 \times 5\% = 16.11\%$$

Note that this is considerably higher than Amazon's expected return.

3. Consider a portfolio consisting of the following three stocks:

| | Portfolio Weight | Volatility | Correlation with Market Portfolio |
|---|---|---|---|
| HEC Corp | 0.25 | 12% | 0.4 |
| Green Midget | 0.35 | 25% | 0.6 |
| AliveAndWell | 0.4 | 13% | 0.5 |

The volatility of the market portfolio is 10% and it has an expected return of 8%. The risk-free rate is 3%.

a. Compute the beta and expected return of each stock.

b. Using your answer from part a, calculate the expected return of the portfolio.

c. What is the beta of the portfolio?

d. Using your answer from part c, calculate the expected return of the portfolio and verify that it matches your answer to part b.

$$\beta_i^{Mkt} = \frac{\text{Cov}(R_i, R_{Mkt})}{\text{Var}(R_{Mkt})}$$

$$E[R_i] = r_i = r_f + \underbrace{\beta_i^{\text{Mkt}}\left(E[R_{Mkt}] - r_f\right)}_{\text{Risk premium for security } i}$$

$$E[R_P] = E\left[\sum_i x_i R_i\right] = \sum_i E[x_i R_i] = \sum_i x_i E[R_i]$$

$$\beta_P = \sum_i x_i \beta_i$$

$$E[R_P] = r_P = r_f + \underbrace{\beta_P^{\text{Mkt}}\left(E[R_{Mkt}] - r_f\right)}_{\text{Risk premium for security } P}$$

| | Portfolio Weight | Volatility | Correlation with the Market Portfolio | Beta (Part a answer) | Expected Return (Part a answer) |
|---|---|---|---|---|---|
| HEC Corp | 0.25 | 12% | 0.4 | 0.48 | 5.4 |
| Green Midget | 0.35 | 25% | 0.6 | 1.5 | 10.5 |
| Alive and Well | 0.4 | 13% | 0.5 | 0.65 | 6.25 |
| | | | | Part c answer: | Part b answer: |
| | | | Portfolio | 0.905 | 7.525 |
| | | | | | Part d answer: |
| | | Expected Return calculated from portfolio | | | 7.525 |

4. Suppose the market portfolio is equally likely to increase by 30% or decrease by 10%.

   a. Calculate the beta of a firm that goes up on average by 43% when the market goes up and goes down by 17% when the market goes down.

   b. Calculate the beta of a firm that goes up on average by 18% when the market goes down and goes down by 22% when the market goes up.

   c. Calculate the beta of a firm that is expected to go up by 4% independently of the market.

Beta (β) of a stock is the sensitivity of the stock to the systematic risk. Beta differs from volatility. Volatility measures total risk (systematic plus unsystematic risk), while beta is a measure of only systematic risk.

a. $\text{Beta} = \frac{\Delta \text{ Stock}}{\Delta \text{ Market}} = \frac{43-(-17)}{30-(-10)} = \frac{60}{40} = 1.5$

b. $\text{Beta} = \frac{\Delta \text{ Stock}}{\Delta \text{ Market}} = \frac{-18-22}{30-(-10)} = \frac{-40}{40} = -1$

c. A firm that moves independently has no systemic risk, so beta = 0.

5. Based on the data in Table 10.6 (appended), estimate which of the following investments you expect to lose the most in the event of a severe market down turn:

   a. a $2000 investment in Hershey,

   b. a $1500 investment in Macy's, or

   c. a $1000 investment in Amazon.

For each 1% market decline:

Hershey down 1%*0.33 = 0.33%,

A $2000 investment in Hershey implies a 0.33% × 2,000 = $6.60 loss.

Macy's down 1%*0.75 = 0.75%,

A $1500 investment in Macy's implies a 0.75% × 1,500 = $11.25 loss.

Amazon down 1%*1.62 = 1.62%,

A $1000 investment in Amazon implies a 1.62% × 1,000 = $16.20 loss.

Thus, Amazon investment will lose most.

Table 10.6 Betas with Respect to the S&P 500 for Individual Stocks (Based on Monthly Data for 2013–2018)
(Source: adopted text, Berk and DeMarzo, Corporate Finance, Pearson, 5e)

| Company | Ticker | Industry | Equity Beta |
|---|---|---|---|
| Edison International | EIX | Utilities | 0.15 |
| Tyson Foods | TSN | Packaged Foods | 0.19 |
| Newmont Mining | NEM | Gold | 0.31 |
| The Hershey Company | HSY | Packaged Foods | 0.33 |
| Clorox | CLX | Household Products | 0.34 |
| Walmart | WMT | Superstores | 0.55 |
| Procter & Gamble | PG | Household Products | 0.55 |
| McDonald's | MCD | Restaurants | 0.63 |
| Nike | NKE | Footwear | 0.64 |
| Pepsico | PEP | Soft Drinks | 0.68 |
| Williams-Sonoma | WSM | Home Furnishing Retail | 0.71 |
| Coca-Cola | KO | Soft Drinks | 0.73 |
| Johnson & Johnson | JNJ | Pharmaceuticals | 0.73 |
| Macy's | M | Department Stores | 0.75 |
| Molson Coors Brewing | TAP | Brewers | 0.78 |
| Starbucks | SBUX | Restaurants | 0.8 |
| Foot Locker | FL | Apparel Retail | 0.83 |
| Harley-Davidson | HOG | Motorcycle Manufacturers | 0.88 |
| Pfizer | PFE | Pharmaceuticals | 0.89 |
| Sprouts Farmers Market | SFM | Food Retail | 0.89 |
| Philip Morris | PM | Tobacco | 0.89 |
| Intel | INTC | Semiconductors | 0.93 |
| Netflix | NFLX | Internet Retail | 0.98 |
| Kroger | KR | Food Retail | 1.04 |
| Microsoft | MSFT | Systems Software | 1.04 |
| Alphabet | GOOGL | Internet Software and Services | 1.06 |
| eBay | EBAY | Internet Software and Services | 1.11 |
| Cisco Systems | CSCO | Communications Equipment | 1.14 |
| Southwest Airlines | LUV | Airlines | 1.15 |
| Apple | AAPL | Computer Hardware | 1.24 |
| salesforce.com | CRM | Application Software | 1.25 |
| Walt Disney | DIS | Movies and Entertainment | 1.29 |
| Marriott International | MAR | Hotels and Resorts | 1.32 |
| Amgen | AMGN | Biotechnology | 1.37 |
| Toll Brothers | TOL | Homebuilding | 1.37 |
| Wynn Resorts Ltd. | WYNN | Casinos and Gaming | 1.38 |
| Parker-Hannifin | PH | Industrial Machinery | 1.43 |
| Prudential Financial | PRU | Insurance | 1.51 |
| Nucor | NUE | Steel | 1.57 |
| Amazon.com | AMZN | Internet Retail | 1.62 |
| General Motors | GM | Automobile Manufacturers | 1.64 |
| Autodesk | ADSK | Application Software | 1.72 |
| Hewlett-Packard | HPQ | Computer Hardware | 1.77 |
| Tiffany & Co. | TIF | Apparel and Luxury Goods | 1.77 |
| Brunswick | BC | Leisure Products | 1.84 |
| Chesapeake Energy | CHK | Oil and Gas Exploration | 1.85 |
| Netgear | NTGR | Communications Equipment | 1.94 |
| Ethan Allen Interiors | ETH | Home Furnishings | 2.04 |
| Trimble | TRMB | Electronic Equipment | 2.44 |
| Advanced Micro Devices | AMD | Semiconductors | 2.83 |