

**Practice Problem Set 4**  
**Oligopoly (C.28)**

Question 4.1

In Pulau Kowai, consumers purchase smartphones without a phone plan. There are only two suppliers of smartphones: Apple and Samsung. They compete by setting quantity sequentially. Samsung is the Stackelberg leader and Apple is the follower. The inverse demand function of smartphone in this island is given by  $P = 200 - A - S$ , where  $P$  is price,  $A$  is the amount of iPhones sold, and  $S$  the amount of Galaxy phones sold. The marginal cost of Apple is \$50, and that of Samsung is \$100. Consumers in this island cannot tell the difference between an Apple phone and a Samsung phone. Find the equilibrium  $A$ ,  $S$ , and  $P$ . Do you think that Samsung's position as the Stackelberg leader allows it to sell more than Apple, and earn a bigger profit? Why or why not?

Question 4.2

In Pulau Kawai, consumers purchase smartphones without a phone plan. There are only two suppliers of smartphones: Apple and Samsung. Consumers in this island cannot tell the difference between an Apple phone and a Samsung phone.

Apple and Samsung compete by setting price simultaneously. The inverse demand function of smartphone in this island is given by  $P = 200 - A - S$ , where  $P$  is price,  $A$  is the amount of iPhones sold, and  $S$  the amount of Samsung phones sold. The marginal cost of Apple is \$50, and that of Samsung is \$100. What are the equilibrium prices and quantities of smartphones sold in Pulau Kawai?

Question 4.3

Two companies, Company 1 and Company 2, sell competing products. Their products are substitutes (but not perfect substitutes), so that the number of units that either company sells is a decreasing function of its own price and an increasing function of the other product's price. Let  $p_1$  be the price and  $x_1$  the quantity sold of product 1 and let  $p_2$  and  $x_2$  be the price and quantity sold of product 2. Then  $x_1 = 1000(90 - 0.5p_1 + 0.25p_2)$  and  $x_2 = 1000(90 - 0.5p_2 + 0.25p_1)$ . For each company, the cost of selling an extra unit is zero. Each company chooses the **price** that maximize its profits.

Solve for the equilibrium prices  $p_1$  and  $p_2$ .

Question 4.4

Toyota and Honda are duopolists in the market for cars, a homogenous good. Toyota chooses the quantity  $x_1$  and Honda chooses the quantity  $x_2$ , and the demand for cars is given by  $X = 40 - P$ , where  $P$  is the price of a car and  $X = x_1 + x_2$ . The (total) cost function of each firm is  $c_i = 10x_i$  (where  $i = 1$  if the firm is Toyota and  $i = 2$  if the firm is Honda.)

- i) If Honda does not exist and Toyota monopolizes the market, how much will Toyota produce and at what price? What will its profit be?
- ii) From now onwards, suppose that both firms are in the market. Find the equilibrium outputs and price if Toyota and Honda make their decisions sequentially, with Toyota setting its output before Honda does.
- iii) Suppose instead of quantities, Toyota chooses price  $p_1$  and Honda chooses price  $p_2$  simultaneously. The game is repeated infinitely many times between them and the two firms have a common discount factor  $\delta$ . Suppose the two firms collude as a joint monopoly and divide the profits equally between themselves in the current period as long as no firm has cheated before the current period. Otherwise, they revert to Bertrand competition. Recall that  $0 < \delta < 1$ . There exists a  $\bar{\delta}$  such that for any  $\delta > \bar{\delta}$ , no cheating by both firms in every period can be sustained as an equilibrium. What is the value of  $\bar{\delta}$ ?