EC3333 Tutorial 3

1. A pension fund manager is considering three mutual funds. The first is a stock fund *(S)*, the second is a long-term government and corporate bond fund *(B)*, and the third is a T-bill money market fund that provides a risk-free rate of return of 8%. The characteristics of the risky funds are as follows:

	Expected Return	Standard Deviation
Stock fund (*S*)	20%	30%
Bond fund (*B*)	12%	15%

The correlation between the fund returns is 0.10.

a. What are the investment proportions in the minimum-variance portfolio of the two risky funds, and what is the expected value and standard deviation of its rate of return?

b. Tabulate and draw the investment opportunity set of the two risky funds. Use investment proportions for the stock fund of 0% to 100% in increments of 20%.

c. Draw a tangent from the risk-free rate to the opportunity set. What does your graph show for the expected return and standard deviation of the optimal portfolio?

d. Solve numerically for the proportions of each asset and for the expected return and standard deviation of the optimal risky portfolio.

e. What is the Sharpe ratio of the best feasible CAL?

f. You require that your portfolio yield an expected return of 14%, and that it be efficient, on the best feasible CAL.

   i. What is the standard deviation of your portfolio?
   ii. What is the proportion invested in the T-bill money market fund and each of the two risky funds?

g. If you were to use only the two risky funds, and still require an expected return of 14%, what would be the investment proportions of your portfolio? Compare its standard deviation to that of the optimized portfolio in Part f. What do you conclude?

Hints:

1a.

Analytical solution

Finding the portfolio with minimum portfolio variance is equivalent to solving the following minimization problem, where $\sigma_{SB} = \rho_{BS}\sigma_B\sigma_S$.

$$\min_{x_S, x_B} \sigma_p^2 = x_S^2\sigma_S^2 + x_B^2\sigma_B^2 + 2x_S x_B \sigma_{SB}$$
$$\text{s.t. } x_S + x_B = 1$$

Using substitution method, with $x_B = 1 - x_S$

We get a minimization problem with only one variable.

$$\min_{x_S} \sigma_p^2 = x_S^2\sigma_S^2 + (1 - x_S)^2\sigma_B^2 + 2x_S(1 - x_S)\sigma_{SB}$$

First order condition of the minimization problem is:

$$\begin{aligned} 0 &= \frac{d}{dx_S}\sigma_p^2 = \frac{d}{dx_S}\left(x_S^2\sigma_S^2 + (1 - x_S)^2\sigma_B^2 + 2x_S(1 - x_S)\sigma_{SB}\right) \\ &= 2x_S\sigma_S^2 - 2(1 - x_S)\sigma_B^2 + 2\sigma_{SB}(1 - 2x_S) \\ &\Rightarrow x_S^{min} = \frac{\sigma_B^2 - \sigma_{SB}}{\sigma_S^2 + \sigma_B^2 - 2\sigma_{SB}}, \qquad x_B^{min} = 1 - x_S^{min} \end{aligned}$$

Numerical solution

Using the Excel solver function by minimizing the standard deviation of the risky portfolio, by changing the relative weights of the risky assets, subject to the sum of the weights to1 (see Excel file posted).

1d.

Analytical solution

Using substitution method, with $x_B = 1 - x_S$, and from the first order condition of the maximization problem, the weights of the tangent portfolio are given below. (The proof of the following asset weights in the tangent portfolio for this question is not required in the exam.)

$$x_S^{\tan} = \frac{\left(E(r_S) - r_f\right)\sigma_B^2 - \left(E(r_B) - r_f\right)\sigma_{SB}}{\left(E(r_S) - r_f\right)\sigma_B^2 + \left(E(r_B) - r_f\right)\sigma_S^2 - \left(E(r_S) - r_f + E(r_B) - r_f\right)\sigma_{SB}}$$

$$x_B^{\tan} = 1 - x_S^{\tan}$$

Numerical solution

Using the Excel solver function by maximizing the Sharpe ratio, by changing the relative weights of the risky assets, subject to the sum of the weights to1 (see Excel file posted).