Money and Bonds

Prequel to Chapter 15

OUTLINES FOR PREQUEL

- Discussion on money: *why money can enter utility function*
  - Roles of money: *the needs for holding money*
  - Real money balances: *real money is what matters to household*
- Understanding bonds:
  - Bonds: *What is a bond?*
  - Bond Markets:
    - *How bonds market operates*
    - *Pricing of bonds*; *relationship between price of bonds and nominal interest rate*
- Money – bond relationship
  - Federal Reserve
  - Bank

How Central Bank purchasing/selling of ***bonds*** *lead to* ***monetary contraction or expansion*** *(through multiplier effect of money)*

  - Money markets and bond markets: *How the two are connected (more formal explanation)*

Discussion on money

The Roles of Money

- The roles played by money (more broadly, liquidity)
  - Medium of exchange
    - Eases double-coincidence of wants problem
  - Unit of account
    - A common "language" for all prices to be quoted in
  - Store of value
    - Apples will perish in short amount of time, dollar bills won't

Durable/ lasting purchasing power

Back to sequel's outline

THE ROLES OF MONEY

- How to conceptually "model" money a surprisingly hard problem
  - Much more difficult than, i.e., "consumption-leisure framework" or "consumption-savings framework"
  - How to formally (mathematically) represent these roles of money?

The Roles of Money

- A shortcut: suppose money directly yields utility
  - Period-*t* utility function

$$u\left(c_t, \frac{M^D_t}{P_t}\right)$$

  - Money-in-the-utility-function (MIU) formulation
  - IMPORTANT: It's not M^D_t in the utility function, but rather M^D_t/Pt

Real Money Balances

- M_t/P_t a key variable for macroeconomic analysis
- Unit Analysis (i.e., analyze algebraic units of variables)
  - Units(M_t) = \$
  - Units(P_t) = \$/unit of consumption
  - Units(M_t/P_t) =

$$\frac{\$}{\frac{\$}{\text{unit of consumption}}} = \cancel{\$} \cdot \frac{\text{unit of consumption}}{\cancel{\$}}$$

$$= \text{unit of consumption}$$

Real Money Balances

- Utility (composite of medium of exchange, unit of account, store of value) depends on *real* money (M/P), not nominal money (M)
    - Measures the purchasing power of (nominal) money holdings…
    - …which is presumably what people most care about
- M_t and P_t can potentially grow at different rates between periods
    - In which case real money balances change from one period to the next

Understanding Bonds

Bonds

- A prerequisite for analyzing monetary policy: understanding bonds and bond markets

- Bond markets and money markets tightly linked to each other

- What is a "bond?"
  - Simply put, a debt obligation (i.e., borrow funds today, repay at some future date with interest)

Bonds

- What is a "bond?"
  - …
  - Types of bonds
    - 30-day, 60-day, 90-day Federal government bonds
    - 1-year Federal government bonds
    - 2-year Federal government bonds
    - 5-year Federal government bonds
    - 10-year Federal government bonds
    - 30-year Federal government bonds

Conventional monetary policy operates through short-term bonds

Bonds

- What is a "bond?"
  - …
  - Types of bonds
    - …
    - Foreign sovereign government bonds of various maturities
    - State and local government bonds of various maturities
    - Corporate bonds of various maturities
    - Coupon bonds – pay something back ("coupon payments") every so often until the final date of maturity
    - Zero-coupon bonds – only pay back at final date of maturity

FV=$FaceValue					
$c	$c	$c	$c	…	$c

Coupon ($c) paid out each period

FV=$100	
$1	$1

2-period (e.g. 2 year) bond

Coupon ($1) paid out each period

Bond Markets

- Simplify by supposing that all bonds are one-period zero-coupon government bonds – i.e., short-term bonds
  - Traditional simplification for analysis of monetary policy
  - Understanding how short-term bond is priced
    - Key to understanding how all bonds are priced
    - Key to understanding how all sorts of financial assets are priced

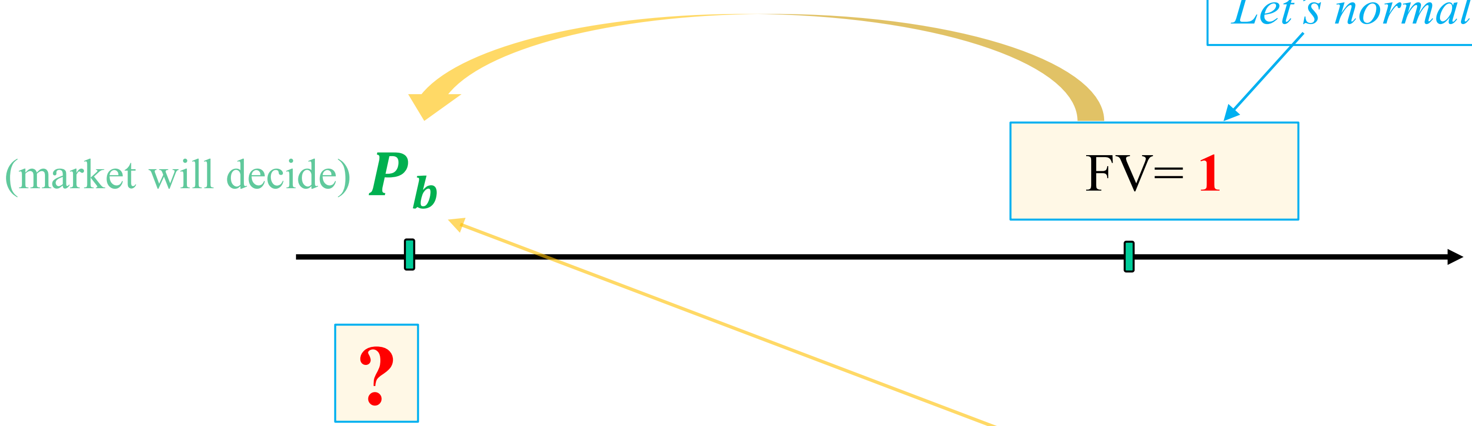
Asset-pricing lurking in the background again…

FV=$100

1-period bond

No coupon

Let's normalize this FV to 1



How much are investors willing to pay for the "promise"/bond?

This means:

Investing P_b

Getting back 1

$$\frac{1}{P_b} = 1 + i$$

Bond Markets

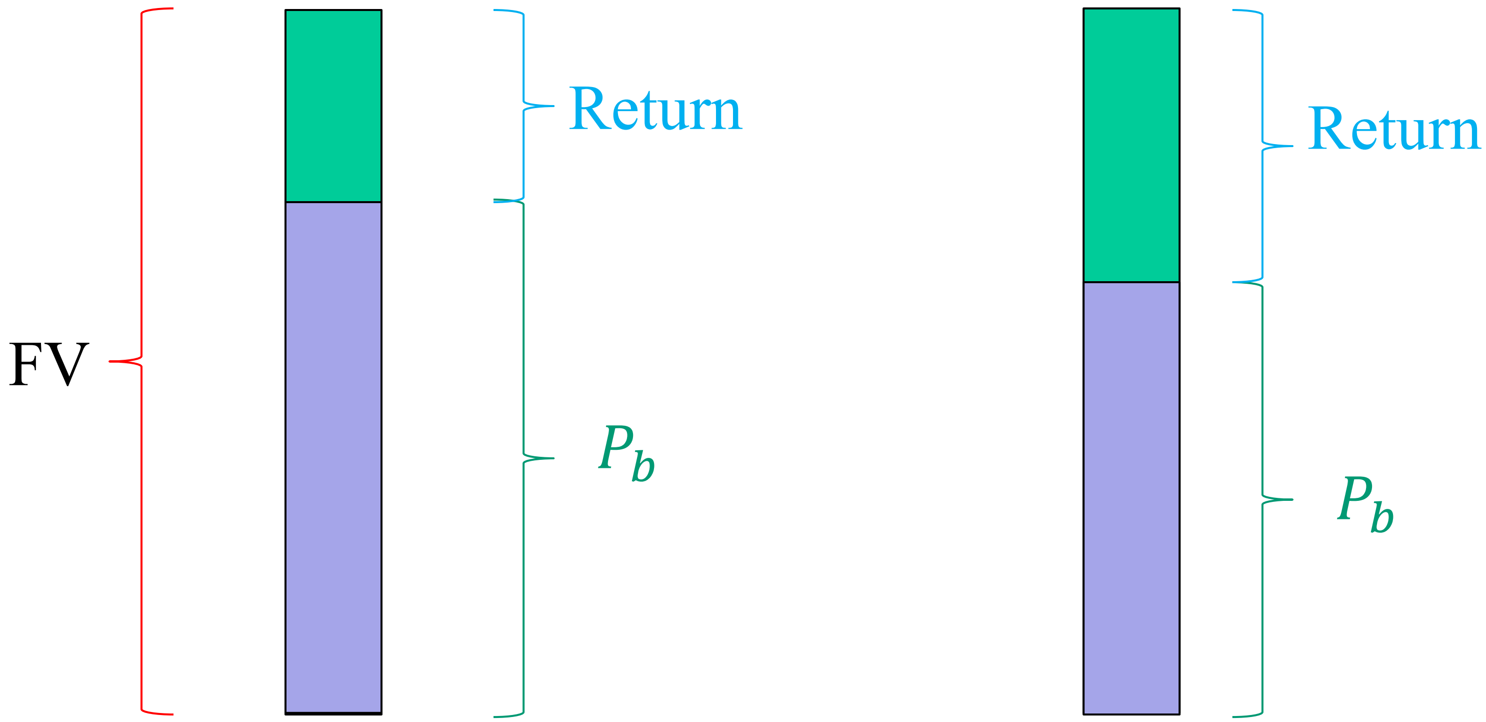
- Key relationship between price of a bond and nominal interest rate

$$P_t^b = \frac{FV_{t+1}}{1+i_t} \quad \dashrightarrow \quad P_t^b = \frac{1}{1+i_t} \quad \dashrightarrow \quad i_t = \frac{1}{P_t^b} - 1$$

Simplify and assume *FV* = 1 (will get main ideas across)

- Notation
  - $P^b{}_t$: nominal price of a one-period bond
  - i_t: nominal interest rate between period t and period $t+1$
  - FV_{t+1}: face-value of bond (i.e., how much will be repaid in $t+1$)

In reality, *many* different values of *FV* ($100, $1000, $10,000,etc…)



Higher P_b → lower return → lower i

Lower P_b → higher return → higher i

Back to sequel's outline

Bond Markets

- Inverse relationship between price of a bond and nominal interest rate – critical

- Short-term bond markets are/have been the conventional channel through which Federal Reserve conducts monetary policy

Money – bond relationship

Federal Reserve

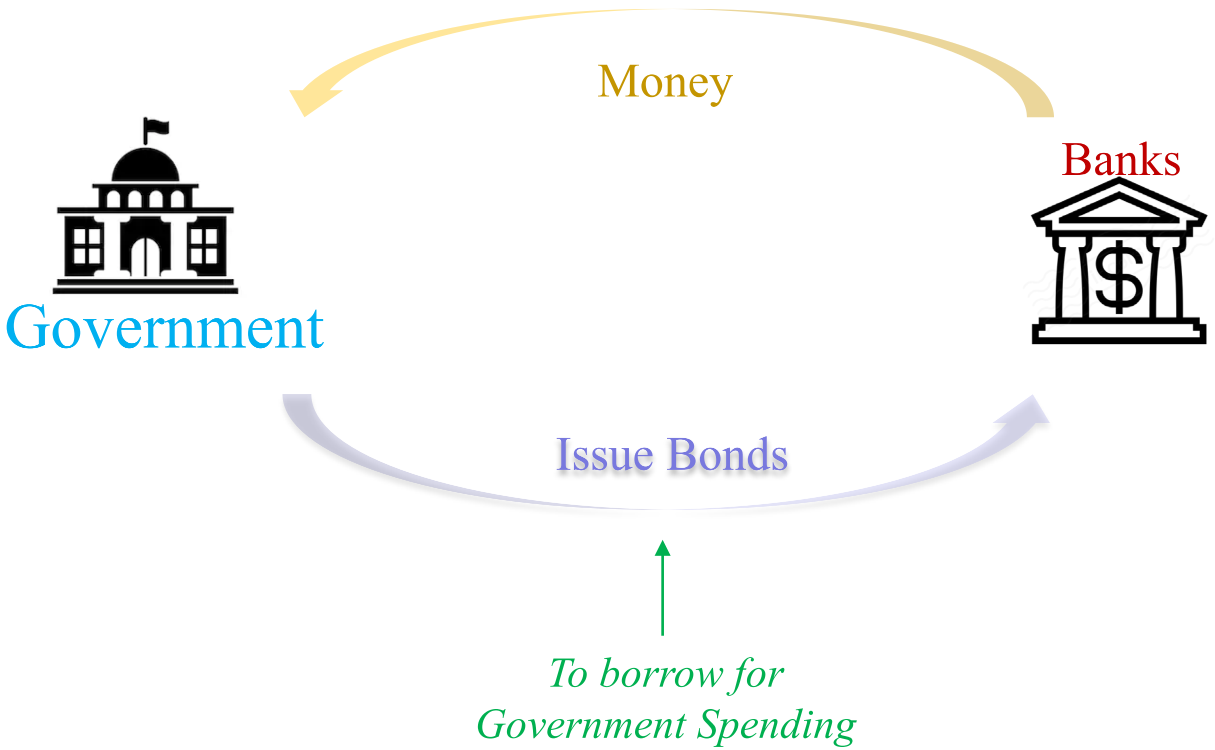
- Manage the money supply through open market operations
- Buy back bonds from the US largest security dealers and banks (primary dealers), not with the general public.
- Upon buying back the bonds, Fed in return gives money to banks.

Extra/Not tested

Banks

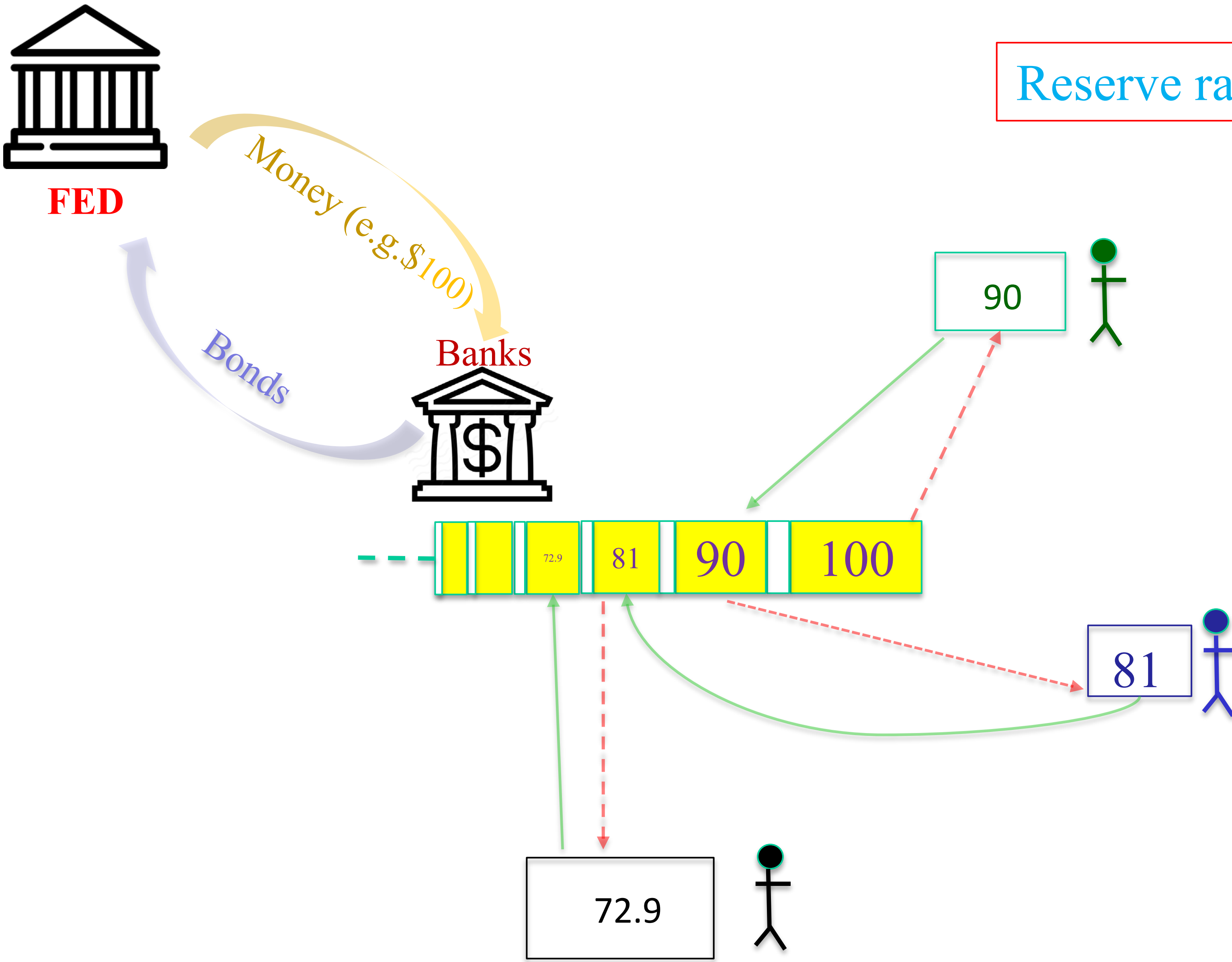
- Required to keep some % of the deposit as reserves. The rest can be lent to the general public

- So upon selling the bonds to Fed, the banks will have extra money (which can be lent to general public). This will start the multiplier effect of money, causing the money supply to increase.


Money
Banks
Government
Issue Bonds
To borrow for
Government Spending

Extra/Not tested

Reserve ratio = 10%



Back to sequel's outline

Extra/Not tested

From $100 released from Fed, the money supply will increase by:

$$\$100 + \$90 + \$81 + \$72.9 + \ldots$$

$$=$$

$$\$100 + \$100 \times 0.9 + \$100 \times 0.9^2 + \$100 \times 0.9^3 + \cdots$$

$$=$$

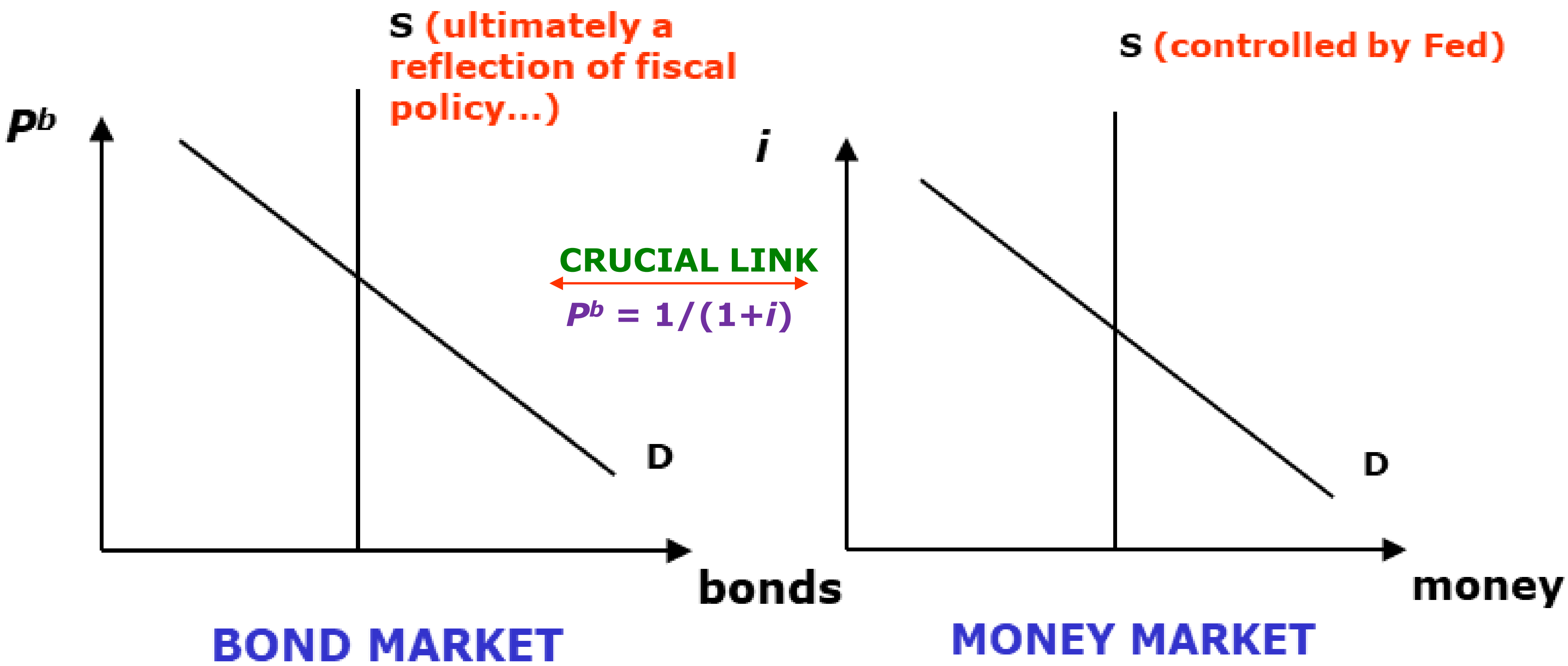
$$\mathbf{\$100}\,\{1 + 0.9 + 0.9^2 + 0.9^3 + \cdots\}$$

$$= \$100\left\{\frac{1}{1-0.9}\right\} = \frac{\$100}{0.1} = \boxed{\$1000}$$

ΔM^S

Money Markets and Bond Markets

- Short-term bond markets and money markets tightly linked to each other



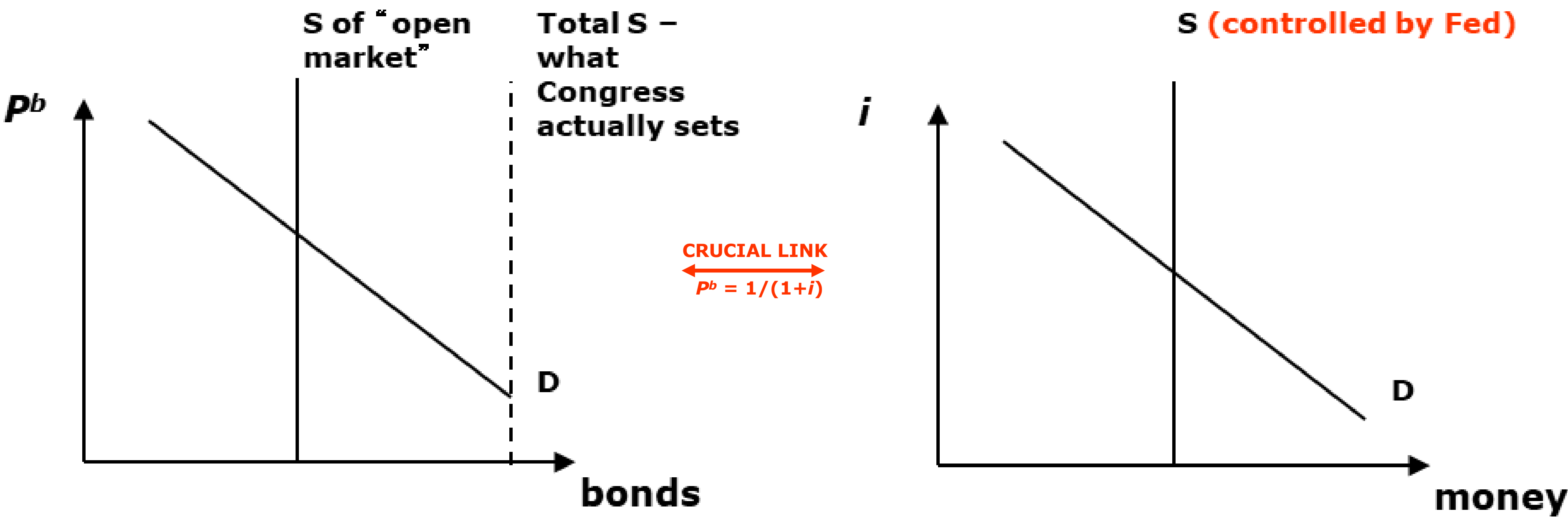
Money Markets and Bond Markets

- *i* can be thought of in two (mirror-image) ways
  - The interest payoff of a bond
  - Opportunity cost of holding money
    - Each unit of wealth held as a dollar is a unit of wealth *not* held as a bond, which entails the loss of chance to earn interest (i.e., opportunity cost)
    - *i* is interpreted as "the price of money"
- Conventional monetary policy
  - Intro macro: Fed open-market operations conducted via short-term bond markets, so Fed operations do affect bond supply

Back to sequel's outline

Money Markets and Bond Markets

- ❑ Intro macro: open-market operations conducted via short-term bond markets

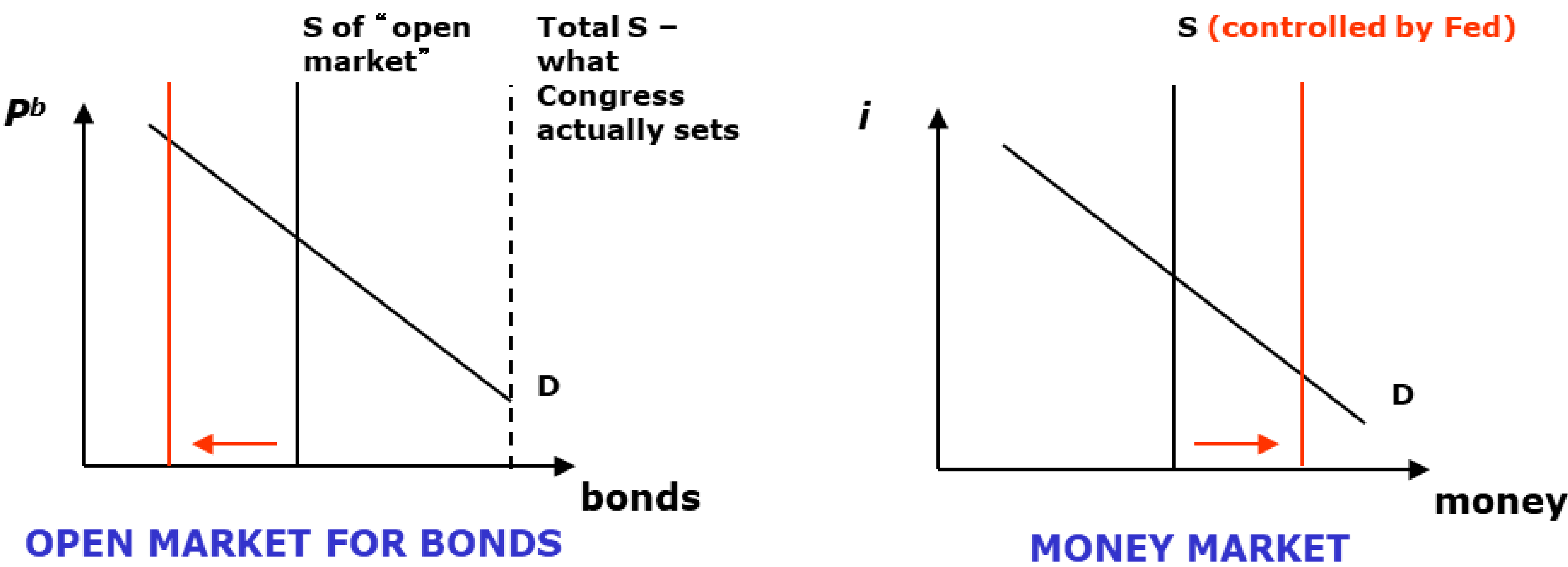


OPEN MARKET FOR BONDS

MONEY MARKET

Money Markets and Bond Markets

- Intro macro: open-market operations conducted via short-term bond markets

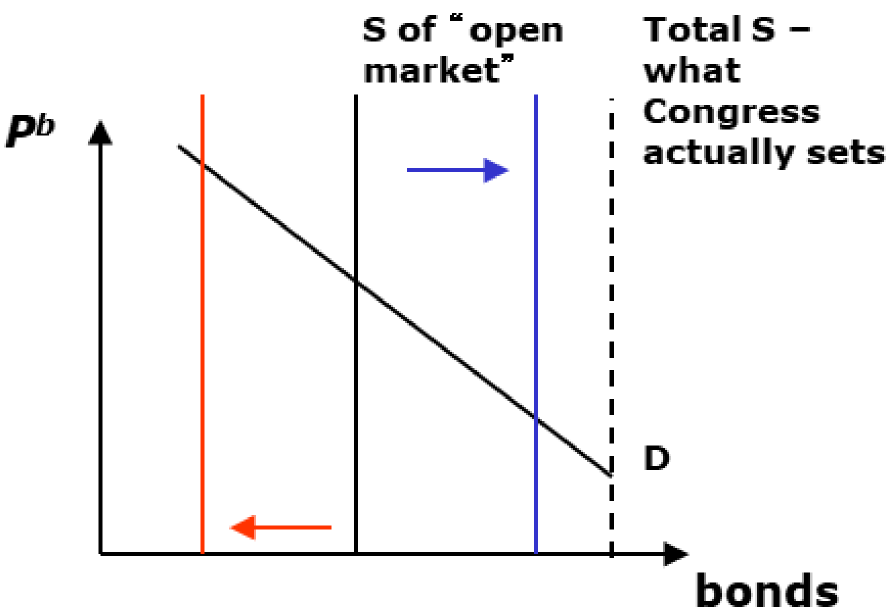


Money Markets and Bond Markets

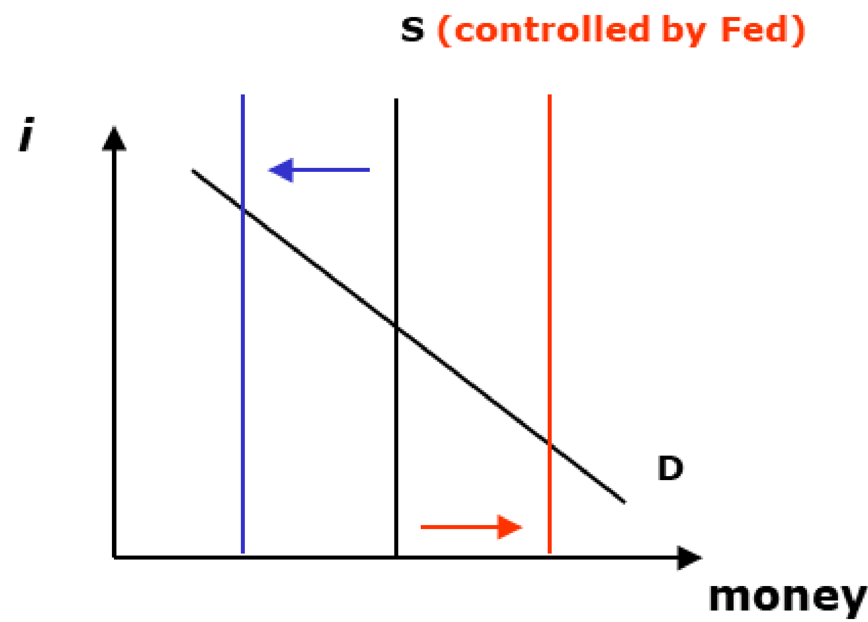
- Expansionary monetary policy by Fed
  - Fed buys short bonds from financial sector, reducing open-market supply…
  - …by printing new money, increasing supply in money market…
  - …which causes short-term *i* to decrease

Money Markets and Bond Markets

- Intro macro: open-market operations conducted via short-term bond markets



OPEN MARKET FOR BONDS



MONEY MARKET

Money Markets and Bond Markets

- Contractionary monetary policy by Fed
  - Fed sells short bonds to financial sector, increasing open-market supply …
  - …in exchange for money, decreasing supply in money market…
  - …which causes short-term *i* to increase

MONETARY POLICY IN THE INFINITE-PERIOD ANALYSIS

CHAPTER 15

OUTLINES FOR CHAPTER 15

- MODEL Specification:
  - Basics – *Explaining terms and concepts*
    - Money in Utility function
  - Budget constraints
- SOLVING MODEL using Lagrange (Sequential approach)
  - Asset pricing revisited: N*ow that bond is incorporated in the model*
  - Consumption-money optimality condition: *Utility tradeoff between consumption and holding money.*
  - Money demand: *derive from consumption-money optimality condition*
- MONETARY POLICY: Short-run effects
  - Is Monetary policy neutral: *Overview of different school of thoughts.*
  - Monetary neutrality debate: *how we can use model results to understand different school of thoughts on money neutrality.*
- MONETARY POLICY: Long-run effects
  - Money and inflation in the long-run: *relationship*
  - Monetarism: *Inflation is monetary phenomenon.*
  - Monetary policy: *do monetary policies work in SR and LR?*

KEYS

- Construct the real money demand curve
- Discuss the impact of monetary policy using the demand curve

MODEL SPECIFICATIONS

Basics

- Extend our infinite-period framework
  - Introduce money and short-term bonds into the Chapter 8 framework
  - So now three types of assets (stocks, short-term bonds, money) for representative consumer to use for savings purposes
- Will allow us to think further about what the "pricing kernel" is
- Will allow us to understand connection between bond prices and stock prices

Back to chp 15's outline

Basics

- Will allow us to consider monetary neutrality (the main issue in the RBC vs. New Keynesian debate)
  - i.e., does money (and thus monetary policy) have important consequences for *real* (i.e., consumption and real GDP) variables?

- Index time periods by arbitrary indexes t, $t+1$, $t+2$, etc.
  - Important: all of our analysis will be conducted from the perspective of the very beginning of period t…

- Sequential Lagrangian analysis (with money in the utility function)

MIU Function

- Individuals obtain utility from purchasing power of money
  - Medium of exchange
  - Unit of account
  - Store of value

- Thus DEMAND holdings of nominal money

$$u\left(c_t, \frac{M_t^D}{P_t}\right)$$

MIU Function

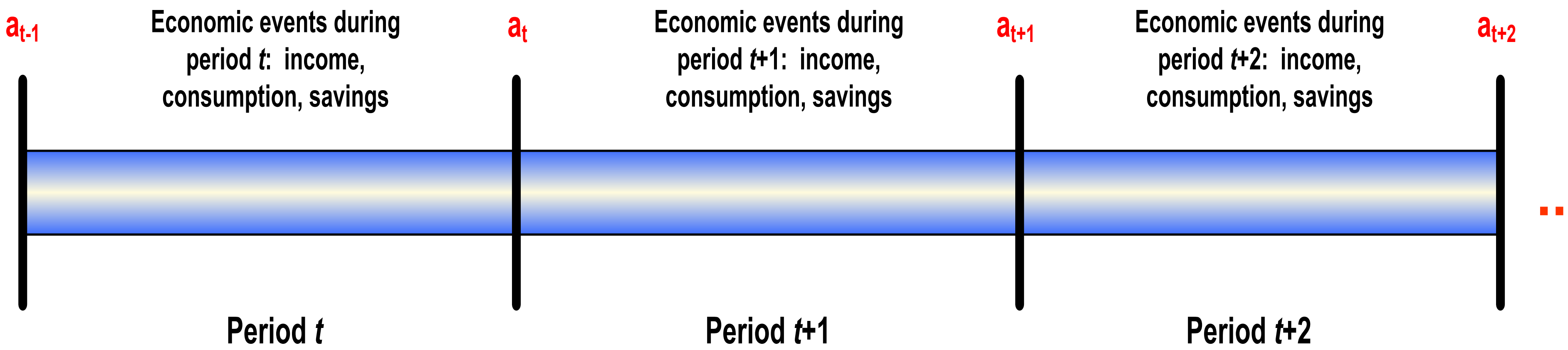
- In consumer optimization problem, it's M^D everywhere
- M^S is determined by the central bank

- In money market equilibrium

$$\frac{M_t}{P_t} = \frac{M_t^D}{P_t} = \frac{M_t^S}{P_t}$$

Basics

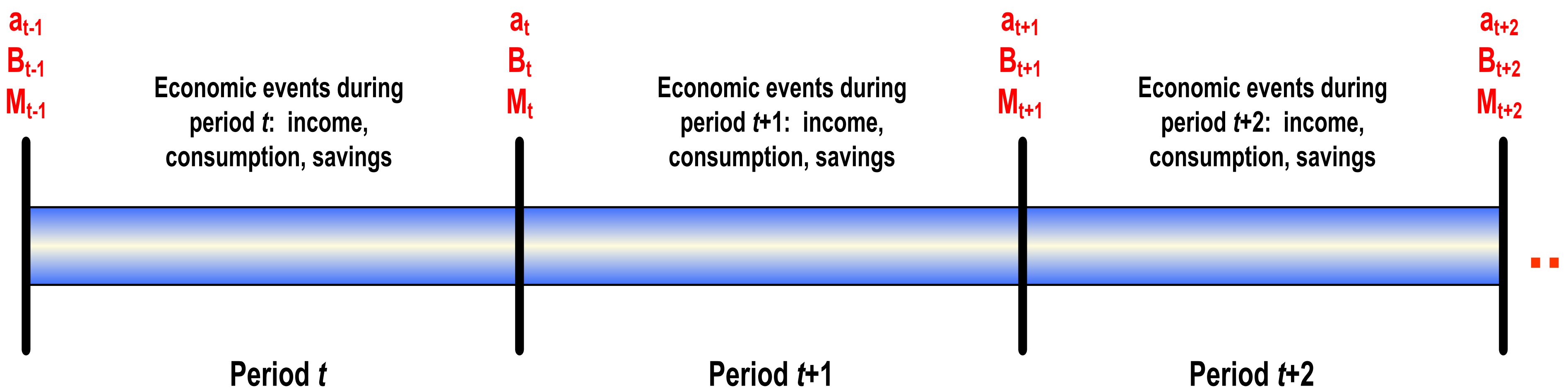
- Timeline of events



- Notation
  - c_t: consumption in period t
  - P_t: nominal price of consumption in period t
  - Y_t: nominal income in period t ("falls from the sky")
  - $a_{t\text{-}1}$: real stock holdings at beginning of period t/end of period t-1

Basics

- Timeline of events



- Notation
  - c_t: consumption in period t
  - P_t: nominal price of consumption in period t
  - Y_t: nominal income in period t ("falls from the sky")
  - a_{t-1}: real stock holdings at beginning of period t/end of period t-1
  - M_{t-1}: nominal money holdings at beginning of period t/end of period t-1
  - B_{t-1}: nominal bond holdings at beginning of period t/end of period t-1

Now *three* types of assets consumers can use for savings purposes

Back to chp 15's outline

Basics

- ...
- ....
  - ....
  - S_t: nominal price of a unit of stock in period *t*
  - D_t: nominal dividend paid in period *t* by each unit of stock held at the start of *t*
  - $P^b{}_t$: nominal price of a bond in period *t*
  - i_t: *nominal interest rate on a bond purchased in t and which pays off in t+1*
  - π_{t+1}: *net inflation rate between period t and period t+1*
  - y_t: *real income in period t (= Yt/Pt)*

Basics

- ....
- Notation
  - c_{t+1}: consumption in period $t+1$
  - P_{t+1}: nominal price of consumption in period $t+1$
  - Y_{t+1}: nominal income in period $t+1$ ("falls from the sky")
  - a_t: real stock holdings at beginning of period $t+1$/end of period t
  - M_t: nominal money holdings at beginning of period $t+1$/end of period t
  - B_t: nominal bond holdings at beginning of period $t+1$/end of period t

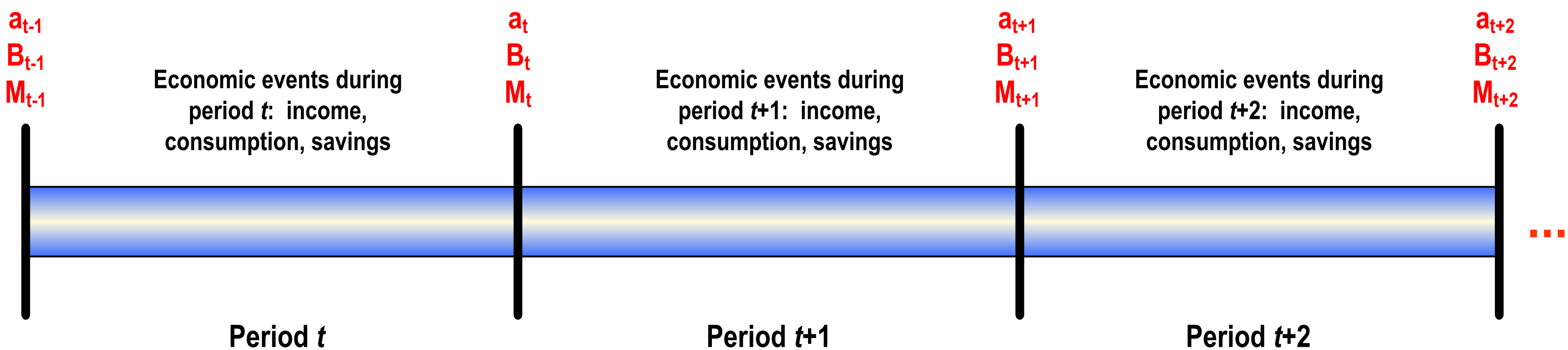
...

BASICS

- ....
- Notation
- ... ...
  - S_{t+1}: nominal price of a unit of stock in period t+1
  - D_{t+1}: nominal dividend paid in t+1 by each unit of stock held at the start of t+1
  - $P^b{}_{t+1}$: nominal price of a bond in period t+1
  - i_{t+1}: nominal interest rate on a bond purchased in t+1 and which pays off in t+2
  - π_{t+2}: net inflation rate between period t+1 and period t+2
  - y_{t+1}: real income in period t (= Y_{t+1}/P_{t+1})

Basics

- Timeline of events



- Notation
  - And so on for period $t+2$, $t+3$, etc…

Budget Constraint(s)

- Extend budget constraints from Chapter 8 stock-pricing framework to now include three distinct types of assets: stocks, money, and short-term bonds

- Need infinite budget constraints to describe economic opportunities and possibilities
  - One for each period

Back to chp 15's outline

$$P_t c_t + P_t^b B_t + M_t + S_t a_t$$

$$= Y_t + M_{t-1} + \mathbf{1} \cdot B_{t-1} + S_t a_{t-1} + D_t a_{t-1}$$

$$P_t c_t + \overbrace{P_t^b B_t}^{\text{New bonds}} + \overbrace{M_t - M_{t-1}}^{\Delta M} + \overbrace{S_t a_t - S_t a_{t-1}}^{S_t \Delta a_t}$$

$$= Y_t + \mathbf{1} \cdot B_{t-1} + D_t a_{t-1}$$

BUDGET CONSTRAINT(S)

- ...
- ...
  - ...
  - In period *t*

$$\underbrace{P_t c_t + P_t^b B_t + M_t + S_t a_t}_{\text{Total outlays in period } t} = \underbrace{Y_t + M_{t-1} + B_{t-1} + S_t a_{t-1} + D_t a_{t-1}}_{\text{Total income in period } t}$$

<u>Total outlays in period *t*:</u> period-*t* consumption + stocks to *carry into period t*+1 + money to *carry into period t*+1 + bond purchases

<u>Total income in period *t*:</u> period-*t* *Y* + income from stock-holdings *carried into period t* (has value S_t and pays dividend D_t) + money-holdings *carried into period t* + bond-holdings *carried into period t* (each unit repays *FV* = 1)

Back to chp 15's outline

Budget Constraint(s)

- In period $t+1$

$$\underbrace{P_{t+1}c_{t+1} + P^b_{t+1}B_{t+1} + M_{t+1} + S_{t+1}a_{t+1}} = \underbrace{Y_{t+1} + M_t + B_t + S_{t+1}a_t + D_{t+1}a_t}$$

Total outlays in period $t+1$: period-$t+1$ consumption + stocks to *carry into period* $t+2$ + money to *carry into period* $t+2$ + bond purchases

Total income in period $t+1$: period-$t+1$ Y + income from stock-holdings *carried into period* $t+1$ (has value S_{t+1} and pays dividend D_{t+1}) + money-holdings *carried into period* $t+1$ + bond-holdings *carried into period* $t+1$ (each unit repays *FV* = 1)

- And analogous budget constraints in period $t+2$, $t+3$, $t+4$, etc.

Back to chp 15's outline

SOLVING MODEL using Lagrange

Lagrange Analysis: Sequential Approach

- Step 1: Construct Lagrange function (starting from t)

$$u\left(c_t, \frac{M_t}{P_t}\right) + \beta u\left(c_{t+1}, \frac{M_{t+1}}{P_{t+1}}\right) + \beta^2 u\left(c_{t+2}, \frac{M_{t+2}}{P_{t+2}}\right) + \cdots$$

$$+\lambda_t[Y_t + (S_t + D_t)a_{t-1} + M_{t-1} + B_{t-1} - P_t c_t - S_t a_t - M_t - P_t^b B_t]$$

$$+\beta\lambda_{t+1}[Y_{t+1} + (S_{t+1} + D_{t+1})a_t + M_t + B_t - P_{t+1}c_{t+1} - S_{t+1}a_{t+1} - M_{t+1} - P_{t+1}^b B_{t+1}]$$

$$+\beta^2\lambda_{t+2}[Y_{t+2} + (S_{t+2} + D_{t+2})a_{t+1} + M_{t+1} + B_{t+1} - P_{t+2}c_{t+2} - S_{t+2}a_{t+2} - M_{t+2} - P_{t+2}^b B_{t+2}]$$

$$+\beta^3\lambda_{t+3}[Y_{t+3} + (S_{t+3} + D_{t+3})a_{t+2} + M_{t+2} + B_{t+2} - P_{t+3}c_{t+3} - S_{t+3}a_{t+3} - M_{t+3} - P_{t+3}^b B_{t+3}]$$

$$+\cdots$$

Lagrange Analysis: Sequential Approach

$$L\left(\begin{array}{c} c_t, c_{t+1}, c_{t+2} \ldots ; \\ a_t, a_{t+1}, a_{t+2}, \ldots ; \\ \boldsymbol{M}_{\boldsymbol{t}}, \boldsymbol{M}_{\boldsymbol{t+1}}, \boldsymbol{M}_{\boldsymbol{t+2}}, \ldots ; \\ B_t, B_{t+1}, B_{t+2}, \ldots ; \\ \lambda_t, \lambda_{t+1}, \ldots \end{array}\right)$$

$$\frac{\partial\left\{u\left(c_t,\frac{M_t}{P_t}\right)+\beta u\left(c_{t+1},\frac{M_{t+1}}{P_{t+1}}\right)+\beta^2 u\left(c_{t+2},\frac{M_{t+2}}{P_{t+2}}\right)+\cdots \\ +\lambda_t\left[Y_t+(S_t+D_t)a_{t-1}+M_{t-1}+B_{t-1}-P_t c_t-S_t a_t-M_t-P_t^b B_t\right] \\ +\beta\lambda_{t+1}\left[Y_{t+1}+(S_{t+1}+D_{t+1})a_t+M_t+B_t-P_{t+1}c_{t+1}-S_{t+1}a_{t+1}-M_{t+1}-P_{t+1}^b B_{t+1}\right] \\ +\beta^2\lambda_{t+2}\left[Y_{t+2}+(S_{t+2}+D_{t+2})a_{t+1}+M_{t+1}+B_{t+1}-P_{t+2}c_{t+2}-S_{t+2}a_{t+2}-M_{t+2}-P_{t+2}^b B_{t+2}\right] \\ +\cdots\right\}}{\partial c_t}$$

$$\boxed{u_1\left(c_t,\frac{M_t}{P_t}\right)-\lambda_t P_t=0}$$

$$\frac{\partial \left\{ u\left(c_t, \frac{M_t}{P_t}\right) + \beta u\left(c_{t+1}, \frac{M_{t+1}}{P_{t+1}}\right) + \beta^2 u\left(c_{t+2}, \frac{M_{t+2}}{P_{t+2}}\right) + \cdots \begin{array}{l} + \lambda_t [Y_t + (S_t + D_t) a_{t-1} + M_{t-1} + B_{t-1} - P_t c_t - \boldsymbol{S_t a_t} - M_t - P_t^b B_t] \\ + \beta \lambda_{t+1} [Y_{t+1} + (\boldsymbol{S_{t+1}} + \boldsymbol{D_{t+1}}) \boldsymbol{a_t} + M_t + B_t - P_{t+1} c_{t+1} - S_{t+1} a_{t+1} - M_{t+1} - P_{t+1}^b B_{t+1}] \\ + \beta^2 \lambda_{t+2} [Y_{t+2} + (S_{t+2} + D_{t+2}) a_{t+1} + M_{t+1} + B_{t+1} - P_{t+2} c_{t+2} - S_{t+2} a_{t+2} - M_{t+2} - P_{t+2}^b B_{t+2}] \\ + \cdots \end{array} \right\}}{\partial \boldsymbol{a_t}}$$

$$\boxed{-\lambda_t S_t + \beta \lambda_{t+1} (S_{t+1} + D_{t+1}) = 0}$$

Back to chp 15's outline

$$\frac{\partial\left\{u\left(c_t,\frac{M_t}{P_t}\right)+\beta u\left(c_{t+1},\frac{M_{t+1}}{P_{t+1}}\right)+\beta^2 u\left(c_{t+2},\frac{M_{t+2}}{P_{t+2}}\right)+\cdots \\ +\lambda_t[Y_t+(S_t+D_t)a_{t-1}+M_{t-1}+B_{t-1}-P_tc_t-S_ta_t-M_t-\boldsymbol{P_t^b B_t}] \\ +\beta\lambda_{t+1}[Y_{t+1}+(S_{t+1}+D_{t+1})a_t+M_t+\boldsymbol{B_t}-P_{t+1}c_{t+1}-S_{t+1}a_{t+1}-M_{t+1}-P_{t+1}^bB_{t+1}] \\ +\beta^2\lambda_{t+2}[Y_{t+2}+(S_{t+2}+D_{t+2})a_{t+1}+M_{t+1}+B_{t+1}-P_{t+2}c_{t+2}-S_{t+2}a_{t+2}-M_{t+2}-P_{t+2}^bB_{t+2}] \\ +\cdots\right\}}{\partial \boldsymbol{B_t}}$$

$$\boxed{-\lambda_t P_t^b+\beta\lambda_{t+1}=0}$$

$$\frac{\partial\left\{u\left(c_t,\frac{M_t}{P_t}\right)+\beta u\left(c_{t+1},\frac{M_{t+1}}{P_{t+1}}\right)+\beta^2 u\left(c_{t+2},\frac{M_{t+2}}{P_{t+2}}\right)+\cdots \begin{array}{l} +\lambda_t[Y_t+(S_t+D_t)a_{t-1}+M_{t-1}+B_{t-1}-P_tc_t-S_ta_t-M_t-P_t^bB_t] \\ +\beta\lambda_{t+1}[Y_{t+1}+(S_{t+1}+D_{t+1})a_t+M_t+B_t-P_{t+1}c_{t+1}-S_{t+1}a_{t+1}-M_{t+1}-P_{t+1}^bB_{t+1}] \\ +\beta^2\lambda_{t+2}[Y_{t+2}+(S_{t+2}+D_{t+2})a_{t+1}+M_{t+1}+B_{t+1}-P_{t+2}c_{t+2}-S_{t+2}a_{t+2}-M_{t+2}-P_{t+2}^bB_{t+2}] \\ +\cdots \end{array}\right\}}{\partial M_t}$$

$$\boxed{\frac{\partial u\left(c_t,\frac{M_t}{P_t}\right)}{\partial M_t}-\lambda_t+\beta\lambda_{t+1}=0}$$

$$\boxed{\frac{\partial u\left(c_t, \frac{M_t}{P_t}\right)}{\partial M_t} - \lambda_t + \beta\lambda_{t+1} = 0}$$

$$\Rightarrow \quad \boxed{\frac{\partial u\left(c_t, \frac{M_t}{P_t}\right)}{\partial\left(\frac{M_t}{P_t}\right)} \cdot \frac{\partial\left(\frac{M_t}{P_t}\right)}{\partial M_t} - \lambda_t + \beta\lambda_{t+1} = 0}$$

$$\Rightarrow \quad \boxed{u_2\left(c_t, \frac{M_t}{P_t}\right) \cdot \frac{1}{P_t} - \lambda_t + \beta\lambda_{t+1} = 0}$$

LAGRANGE ANALYSIS: SEQUENTIAL APPROACH

- Step 2: Compute FOCs with respect to c_t, a_t, B_t, M_t, …

with respect to c_t: $u_1\left(c_t, \frac{M_t}{P_t}\right) - \lambda_t P_t = 0$

with respect to a_t: $-\lambda_t S_t + \beta\lambda_{t+1}(S_{t+1} + D_{t+1}) = 0$

with respect to B_t: $-\lambda_t P_t^b + \beta\lambda_{t+1} = 0$

with respect to M_t: $u_2\left(c_t, \frac{M_t}{P_t}\right) \cdot \frac{1}{P_t} - \lambda_t + \beta\lambda_{t+1} = 0$

Back to chp 15's outline

ASSET PRICING REVISITED

$$-\lambda_t S_t + \beta\lambda_{t+1}(S_{t+1} + D_{t+1}) = 0$$

- Equation 2 →

STOCK-PRICING EQUATION

$$S_t = \left(\frac{\beta\lambda_{t+1}}{\lambda_t}\right)(S_{t+1} + D_{t+1})$$

Period-*t* stock price = **Pricing kernel** x **Future return**

ASSET PRICING REVISITED

- Much of finance theory concerned with pricing kernel
  - Theoretical properties
  - Empirical models of kernels

- Pricing kernel where macro theory and finance theory intersect
  - Lagrange multipliers where macro and finance intersect – an idea that will be important in financial accelerator framework

Asset Pricing Revisited

$$-\lambda_t P_t^b + \beta\lambda_{t+1} = 0$$

Equation 3

- Equation 2 → $$S_t = \left(\frac{\beta\lambda_{t+1}}{\lambda_t}\right)(S_{t+1} + D_{t+1})$$

STOCK-PRICING EQUATION

- Equation 3 → $$P_t^b = \frac{\beta\lambda_{t+1}}{\lambda_t}$$

BOND-PRICING EQUATION

ASSET PRICING REVISITED

- Price of short-term bond *is* the pricing kernel
  - Stock prices and bond prices are connected
  - Most (all?) asset prices fundamentally connected to bond prices
  - Finance: pricing kernel reflects the price/return of the least risky asset in the economy – U.S. Treasury short-term bonds

Terminology: "riskless" asset

Asset Pricing Revisited

- Equation 3 → $$P_t^b = \frac{\beta \lambda_{t+1}}{\lambda_t}$$

BOND-PRICING EQUATION

- Recall $$P_t^b = \frac{1}{1+i_t}$$

- → can express pricing kernel as

$$\frac{\beta \lambda_{t+1}}{\lambda_t} = \frac{1}{1+i_t}$$

Consumption-Money Optimality Condition

Begin with equation 4:

$$\frac{u_2\left(c_t, \frac{M_t}{P_t}\right)}{P_t} - \lambda_t = -\boldsymbol{\beta\lambda_{t+1}}$$

Use $\boldsymbol{\beta\lambda_{t+1}}$ = $\boldsymbol{\lambda_t P^b_t}$ from equation 3

$$\frac{u_2\left(c_t, \frac{M_t}{P_t}\right)}{P_t} - \lambda_t = -\lambda_t P_t^b$$

Divide through by λ_t

$$\frac{u_2\left(c_t, \frac{M_t}{P_t}\right)}{\lambda_t P_t} - 1 = -P_t^b$$

Consumption-Money Optimality Condition

Begin with equation 4:

$$\frac{u_2\left(c_t, \frac{M_t}{P_t}\right)}{\boldsymbol{\lambda_t P_t}} - 1 = -P_t^b$$

Use $\boldsymbol{u_1\left(c_t, \frac{M_t}{P_t}\right)} = \boldsymbol{\lambda_t P_t}$ from equation 1

$$\frac{u_2\left(c_t, \frac{M_t}{P_t}\right)}{\boldsymbol{u_1\left(c_t, \frac{M_t}{P_t}\right)}} - 1 = \boldsymbol{-P_t^b}$$

Use $\boldsymbol{P^b{}_t} = \boldsymbol{1/(1+i_t)}$

$$\frac{u_2(\ldots)}{\boldsymbol{u_1(\ldots)}} - 1 = -\frac{\boldsymbol{1}}{\boldsymbol{1+i_t}}$$

Consumption-Money Optimality Condition

$$\frac{u_2\left(c_t, \frac{M_t}{P_t}\right)}{u_1\left(c_t, \frac{M_t}{P_t}\right)} - 1 = -\frac{1}{1+i_t}$$

$$\frac{u_2\left(c_t, \frac{M_t}{P_t}\right)}{u_1\left(c_t, \frac{M_t}{P_t}\right)} = -\frac{1}{1+i_t} + 1$$

CONSUMPTION-MONEY OPTIMALITY CONDITION

$$\boxed{\frac{u_2\left(c_t, \frac{M_t}{P_t}\right)}{u_1\left(c_t, \frac{M_t}{P_t}\right)} = \frac{i_t}{1+i_t}}$$

Back to chp 15's outline

Money Demand

- Consumption-money optimality condition the foundation of money demand function
- Example: suppose

$$\boxed{u\left(c_t, \frac{M_t}{P_t}\right) = \ln c_t + \ln\left(\frac{M_t}{P_t}\right)}$$

- Thus, $u_1\left(c_t, \frac{M_t}{P_t}\right) = \frac{1}{c_t}$ and $u_2\left(c_t, \frac{M_t}{P_t}\right) = \frac{1}{M_t / P_t}$

$$\boxed{\frac{\partial u\left(c_t, \frac{M_t}{P_t}\right)}{\partial\left(\frac{M_t}{P_t}\right)}}$$

Money Demand

- Consumption-money optimality condition (for this utility function…) is

$$\frac{P_t c_t}{M_t} = \frac{i_t}{1+i_t}$$

Isolate the M_t/P_t term

$$\frac{M_t}{P_t} = c_t \cdot \left(\frac{1+i_t}{i_t} \right)$$

REAL MONEY DEMAND FUNCTION: depends positively on c_t and negatively on i_t (i_t is the opportunity cost of money)

- Use this money demand function to analyze
  - The monetary neutrality debate
  - The long-run (aka steady-state) connection between monetary policy and inflation

Monetary Policy in the Infinite-Period Economy: Short-Run Effects

Back to chp 15's outline

Is Monetary Policy Neutral?

- An enduring question in macroeconomics: does monetary policy have any important effects on the *real* (i.e, *real* GDP, consumption, etc) economy?

- Definition: Money (and hence monetary policy) is neutral if changes in the money supply (i.e., changes in monetary policy) have no effect on the real economy

  - Monetary policy is non-neutral if it does have effects on the real economy

Is Monetary Policy Neutral?

- New Keynesian view: money is non-neutral (because prices are rigid/sticky, often for long periods of time)
- RBC view: money is neutral (because prices are not rigid/sticky in any important way)
- MIU framework allows us to consider how/why monetary policy is or is not neutral

Back to chp 15's outline

Money Demand

CONSUMPTION-MONEY OPTIMALITY CONDITION

$$\underbrace{\frac{u_2(c_t, M_t/P_t)}{u_1(c_t, M_t/P_t)}}_{\text{MRS}} = \underbrace{\frac{i_t}{1+i_t}}_{\text{price ratio}}$$

Using utility function,

$$u\left(c_t, \frac{M_t}{P_t}\right) = \ln c_t + \ln\left(\frac{M_t}{P_t}\right)$$

$$\frac{M_t}{P_t} = c_t \cdot \left(\frac{1+i_t}{i_t}\right)$$

NOTE: consumption-money optimality condition and money demand function are the *same thing*, just viewed from different points of view

Money Demand

- Use money demand function to illustrate effects of money (monetary policy) shocks

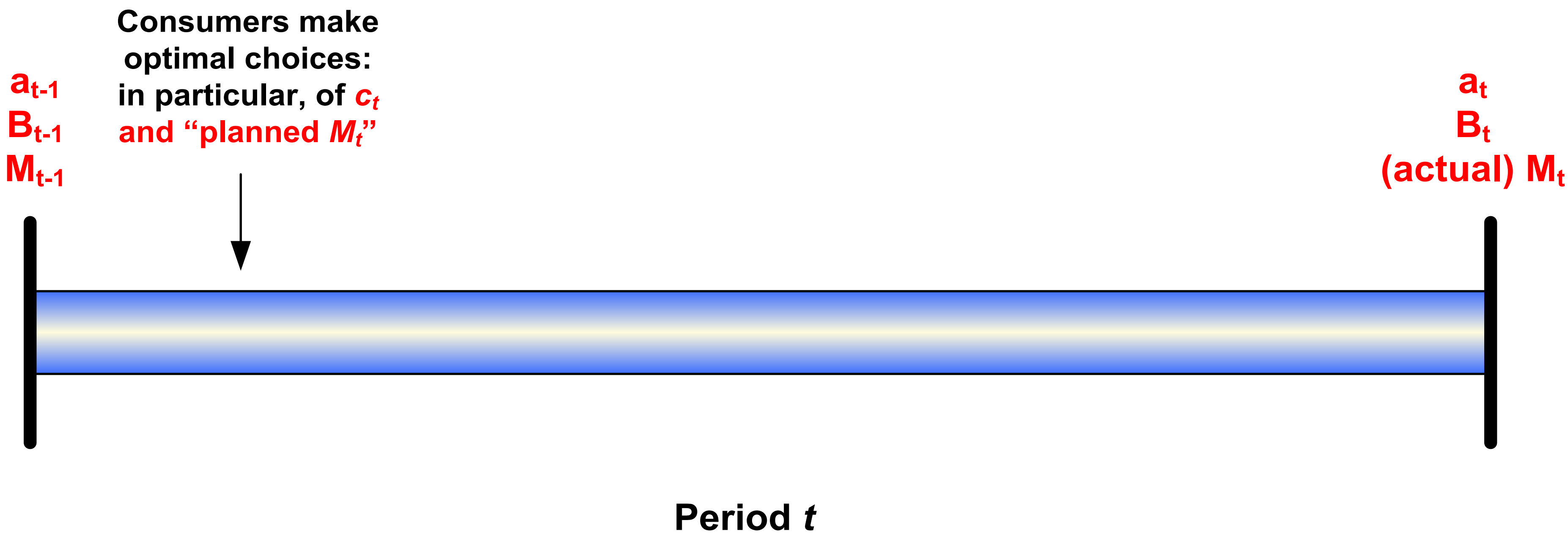
- Gets at core of neutrality debate

- Let's be even more precise about the timing of events…

Back to chp 15's outline

MONETARY NEUTRALITY DEBATE

- Precise timing of events **within period t**



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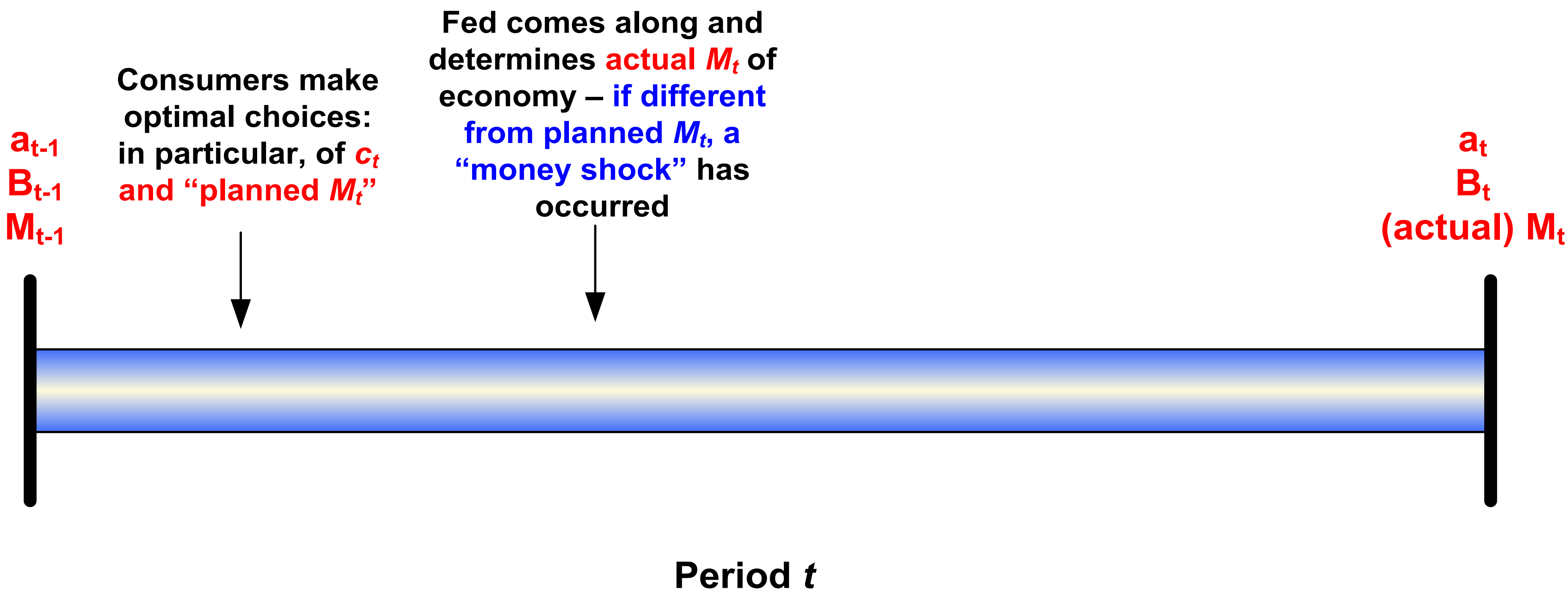
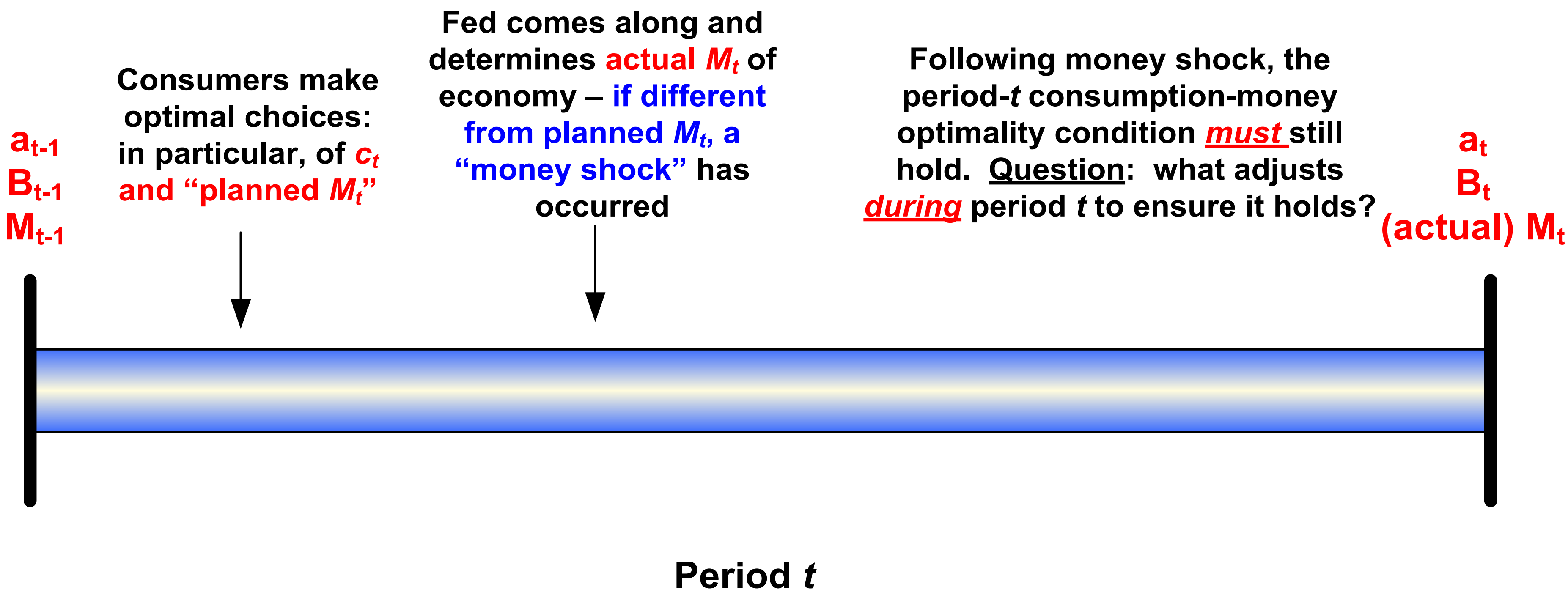
MONETARY NEUTRALITY DEBATE

- **Precise timing of events within period *t***



MONETARY NEUTRALITY DEBATE

- **Precise timing of events within period *t***



Monetary Neutrality Debate

- Fed sets M^S_t after consumers make their choices of c_t and M^D_t (and other choices, too…)
  - If supply M^S_t differs from demanded M^D_t, money shock has occurred
- Question: which adjusts (P_t or c_t) to ensure consumption-money optimality condition holds? (simplify by assuming i_t doesn't adjust)

This is a question about EQUILIBRIUM, with $M^S = M^D$

$$\frac{M_t}{P_t} = c_t \cdot \left(\frac{1+i_t}{i_t} \right)$$

Back to chp 15's outline

MONETARY NEUTRALITY DEBATE

- Keynesian/New Keynesian view

$$\frac{M_t}{P_t} = c_t \cdot \left(\frac{1+i_t}{i_t}\right)$$

  - P_t cannot adjust because prices are sticky
    - (Prices will adjust *later* (i.e, in period t+1 or later), just not in period t)
  - A positive (negative) money shock leads to a rise (fall) in c_t
  - Money (and hence monetary policy) is not neutral

Monetary Neutrality Debate

$$\frac{M_t}{P_t} = c_t \cdot \left(\frac{1+i_t}{i_t} \right)$$

- RBC view
  - P_t can adjust because prices are not sticky
  - No reason for c_t to adjust (they do reflect optimal choices, after all...)
  - A positive (negative) money shock leads to no change (no change) in c_t
  - Money (and hence monetary policy) is neutral

- Empirical evidence for "how sticky" are prices is very mixed…

Back to chp 15's outline

MONETARY NEUTRALITY DEBATE: EXAMPLE

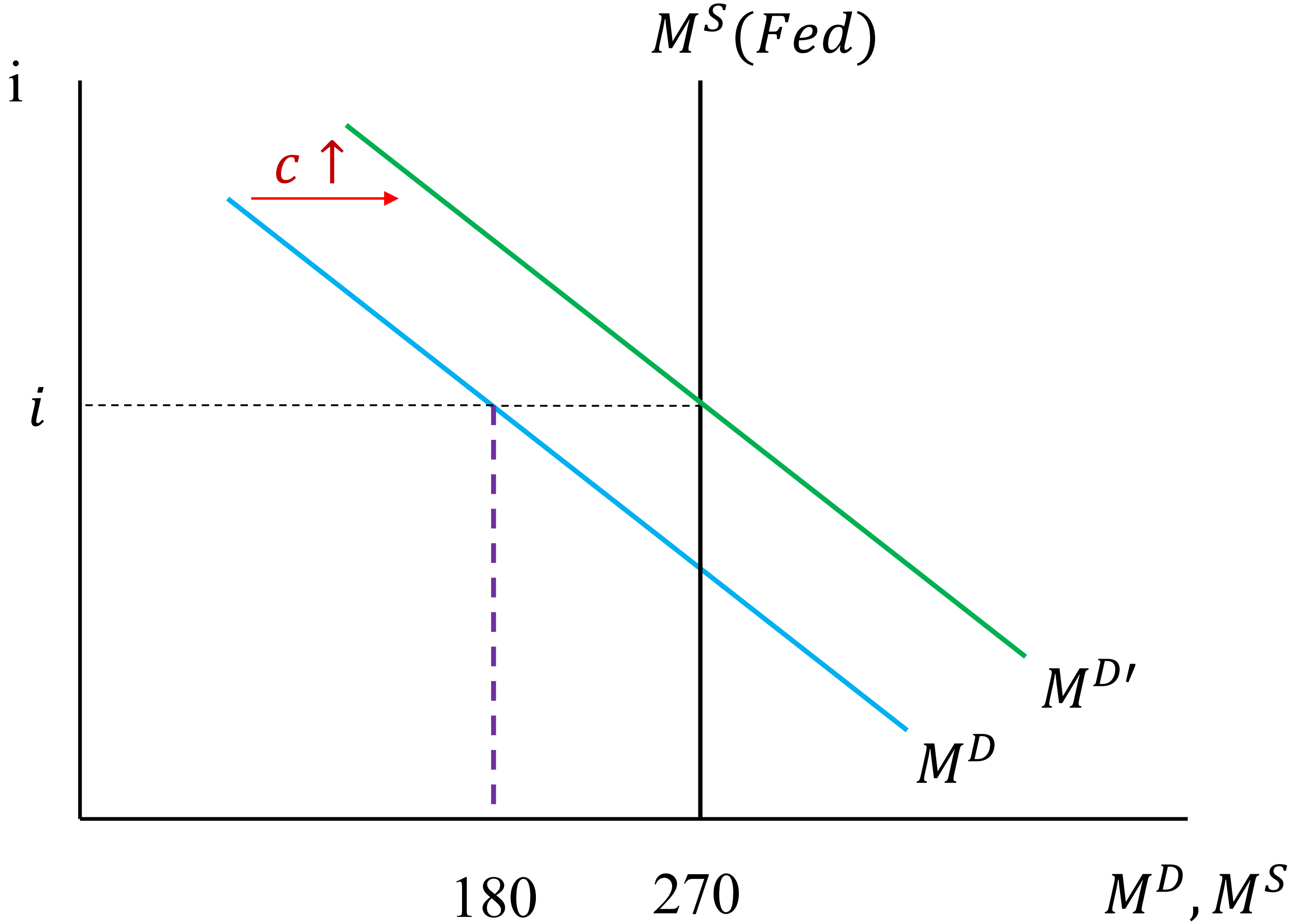
$$\frac{M_t}{P_t} = c_t \cdot \left(\frac{1+i_t}{i_t} \right)$$

- Assume $i_t = 0.125$ is fixed
- Consumers' "demanded" choices are $c_t = 2$ and $M^D{}_t = 180$
- This plan was made with $P_t = 10$ in mind
- Fed sets actual $M^S{}_t = 270$ (a positive money shock because actual $M^S{}_t$ greater than demanded $M^D{}_t$)

Monetary Neutrality Debate: Example

- Keynesian/New Keynesian view
  - $P_t = 10$ won't change (sticky prices)
  - c_t will rise (to $c_t = 3$) to make consumption-money optimality condition hold
  - Monetary policy is non-neutral

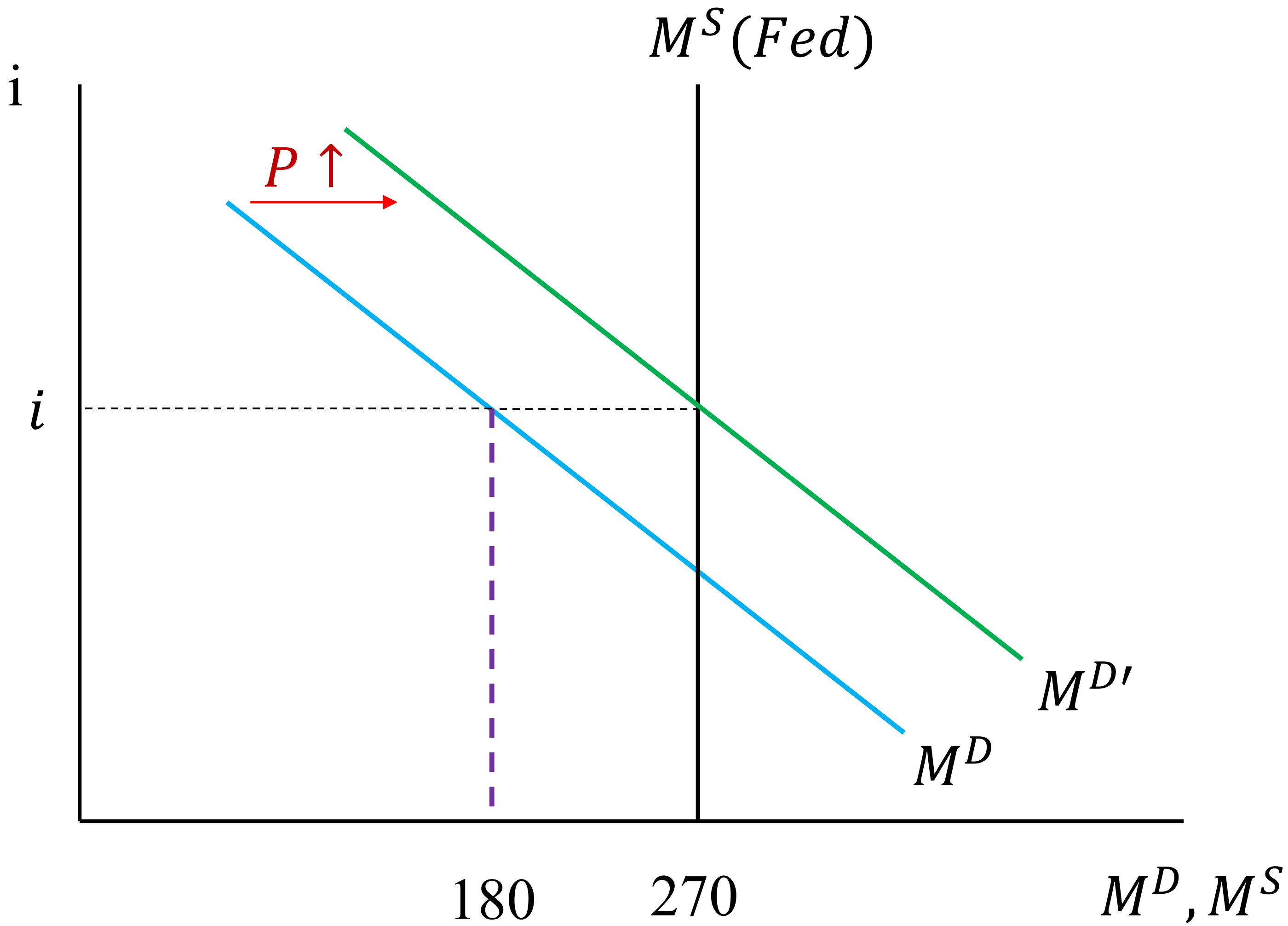
$$\boxed{\frac{M_t}{P_t} = c_t \cdot \left(\frac{1+i_t}{i_t} \right)}$$


$M^S(Fed)$
i
$c\uparrow$
i
$M^{D'}$
M^D
180
270
M^D, M^S

MONETARY NEUTRALITY DEBATE: EXAMPLE

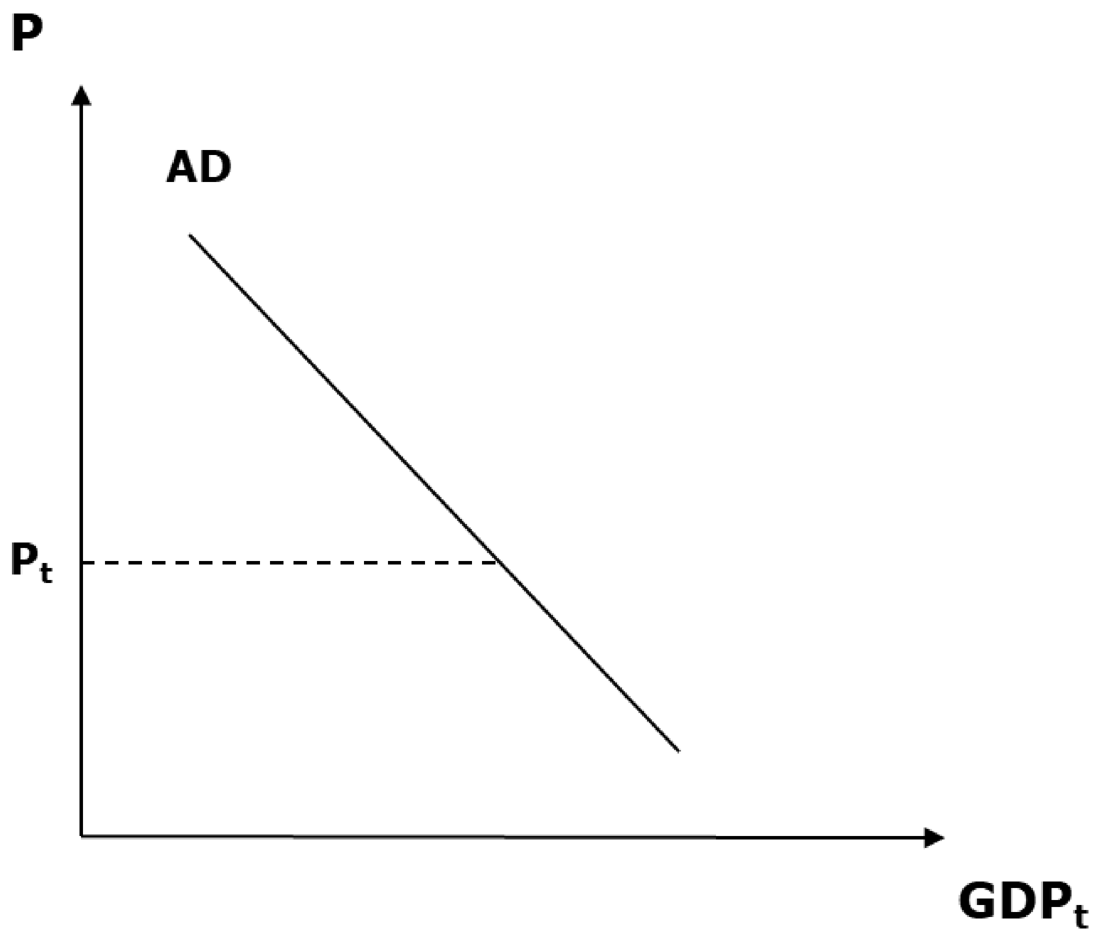
$$\boxed{\frac{M_t}{P_t} = c_t \cdot \left(\frac{1+i_t}{i_t} \right)}$$

- RBC view
  - Consumers' plan of c_t = 2 is what the economy really wants
  - P_t can fully and quickly adjust to accommodate this → P_t = 15
  - Monetary policy is neutral; only effect of monetary policy is on inflation



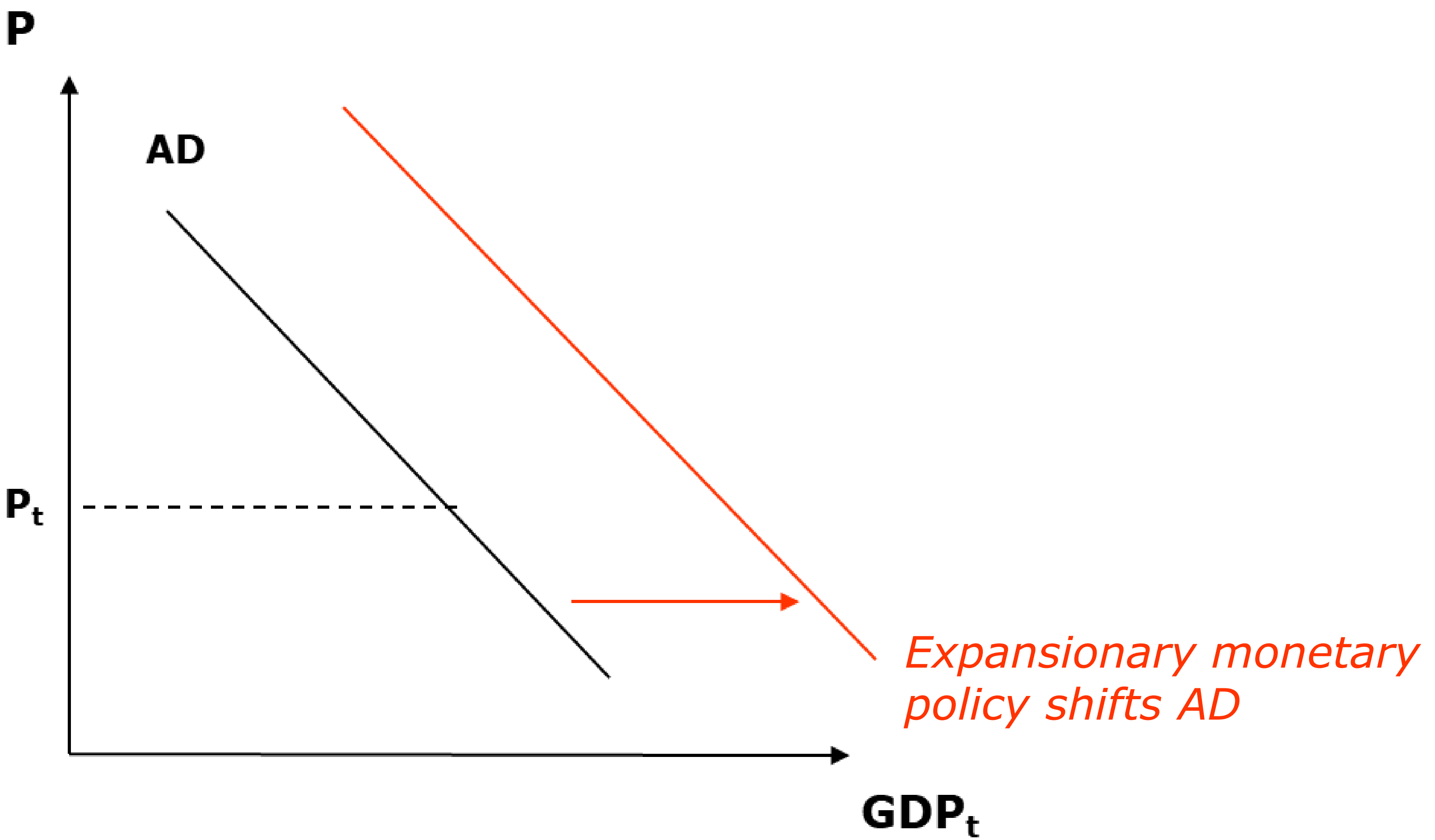
Back to chp 15's outline

Monetary Neutrality Debate



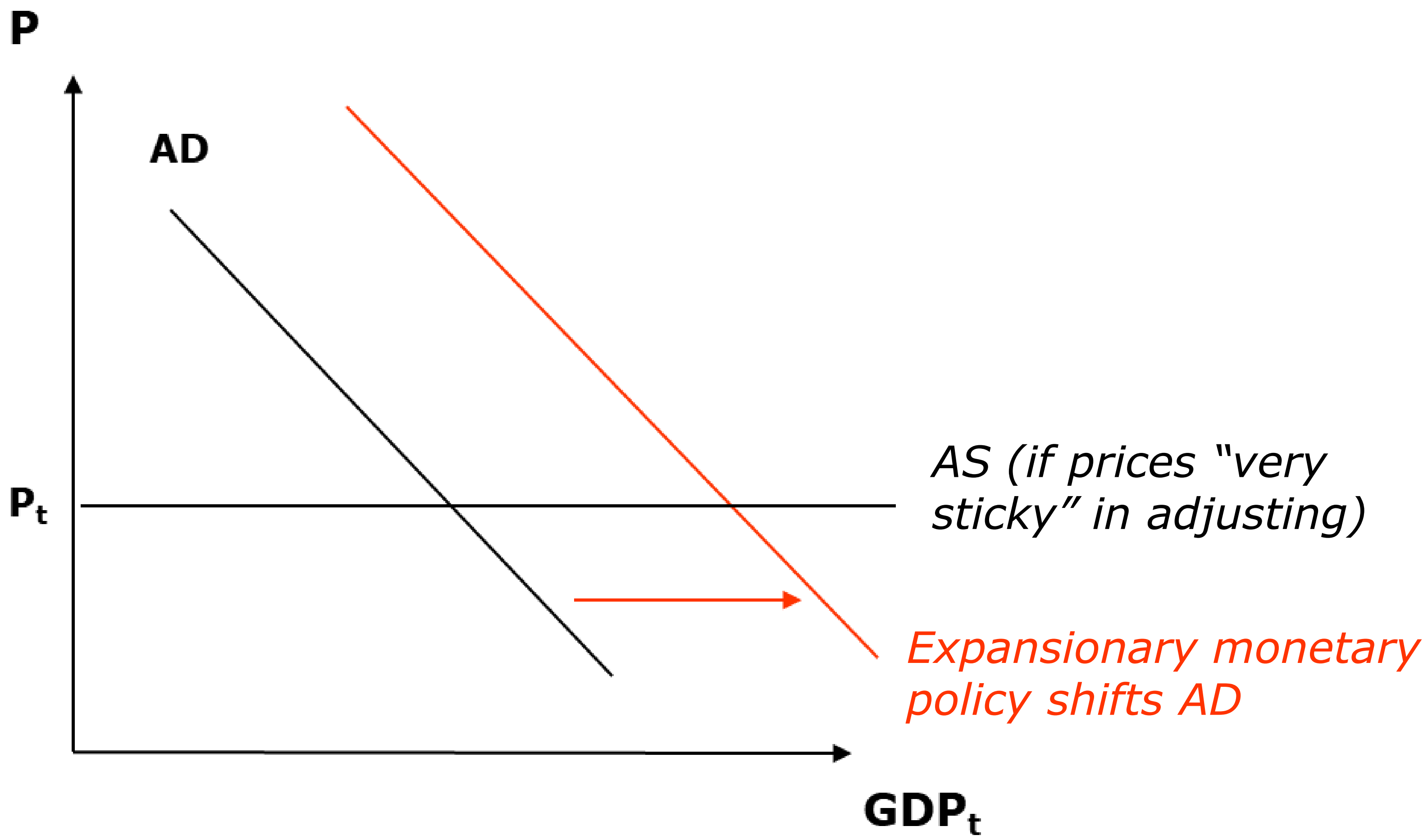
Period-t goods market

Monetary Neutrality Debate



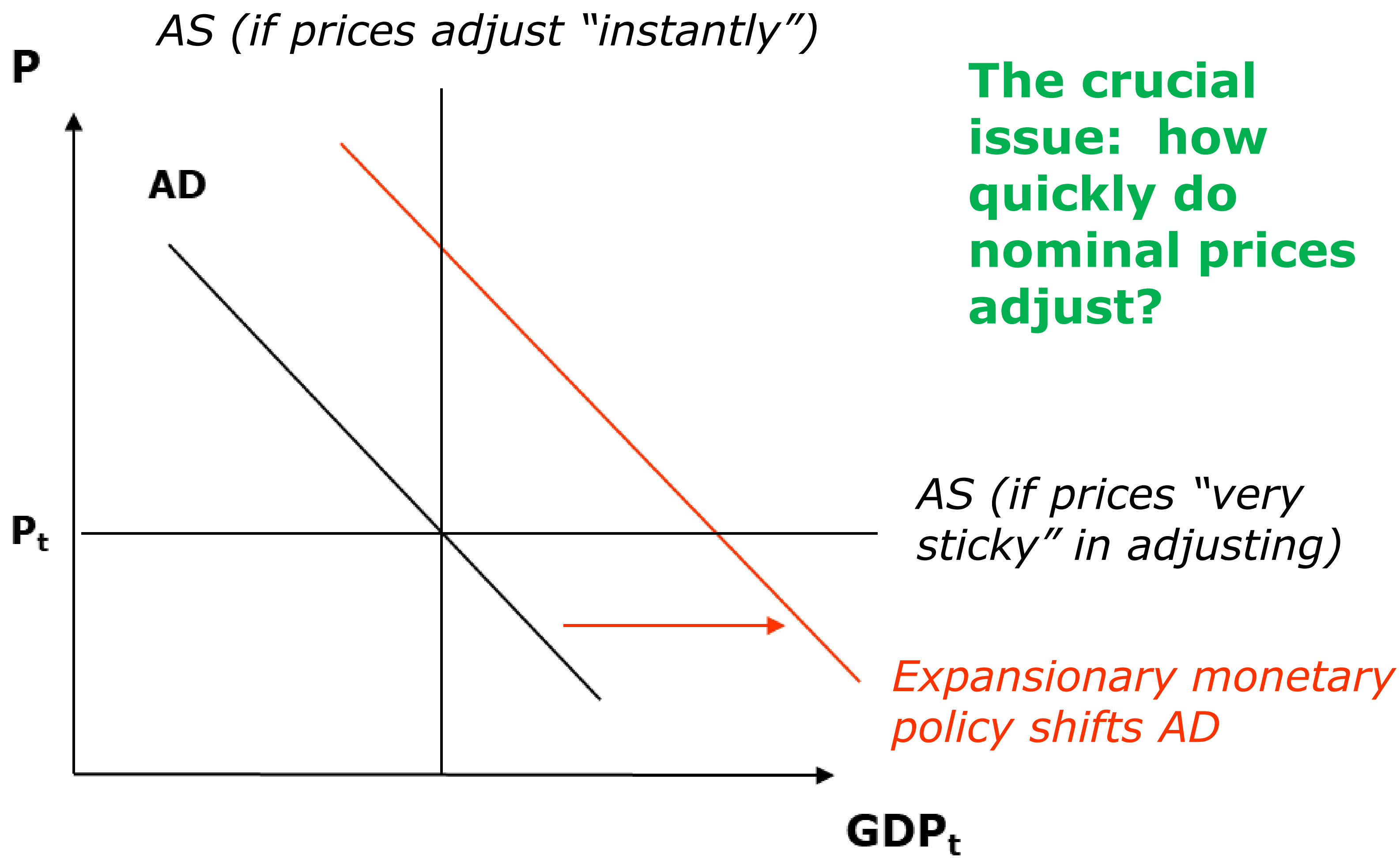
Period-t goods market

MONETARY NEUTRALITY DEBATE



Period-t goods market

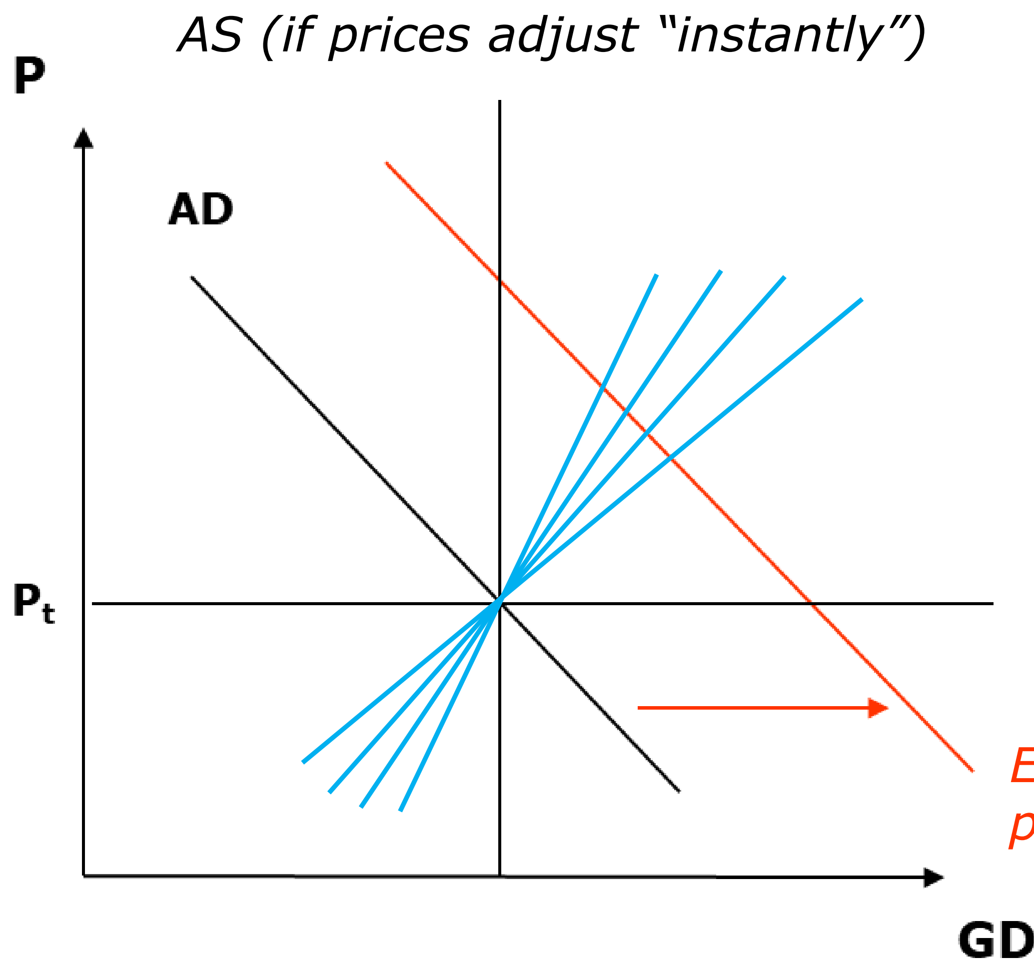
Monetary Neutrality Debate



The crucial issue: how quickly do nominal prices adjust?

Period-t goods market

MONETARY NEUTRALITY DEBATE



The crucial issue: how quickly do nominal prices adjust?

Period-t goods market

MONETARY POLICY IN THE INFINITE-PERIOD ECONOMY: LONG-RUN EFFECTS

Back to chp 15's outline

Money and Inflation in the Long-Run

- Question: what determines inflation in the long run (i.e., in steady-state)?
  - Use both period-(t-1) and period-t money demand functions to analyze

$$u\left(c_t, \frac{M_t}{P_t}\right) = \ln c_t + \ln\left(\frac{M_t}{P_t}\right)$$

Money demand function in t-1

$$\frac{M_{t-1}}{P_{t-1}} = c_{t-1} \cdot \left(\frac{1+i_{t-1}}{i_{t-1}}\right)$$

Money demand function in t

$$\frac{M_t}{P_t} = c_t \cdot \left(\frac{1+i_t}{i_t}\right)$$

Divide period t money demand by period t-1 money demand

$$\frac{M_t / P_t}{M_{t-1} / P_{t-1}} = \frac{c_t}{c_{t-1}} \cdot \left(\frac{1+i_t}{i_t}\right)\left(\frac{i_{t-1}}{1+i_{t-1}}\right)$$

Money and Inflation in the Long-Run

$$\frac{M_t / P_t}{M_{t-1} / P_{t-1}} = \frac{c_t}{c_{t-1}} \cdot \left(\frac{1+i_t}{i_t}\right)\left(\frac{i_{t-1}}{1+i_{t-1}}\right)$$

Recall definition of inflation

$$\pi_t = \frac{P_t}{P_{t-1}} - 1$$

And now define the <u>money growth rate</u> in an analogous way:

$$\mu_t = \frac{M_t}{M_{t-1}} - 1$$

$$\frac{1+\mu_t}{1+\pi_t} = \frac{c_t}{c_{t-1}} \cdot \left(\frac{1+i_t}{i_t}\right)\left(\frac{i_{t-1}}{1+i_{t-1}}\right)$$

Back to chp 15's outline

Money and Inflation in the Long-Run

$$\frac{1+\mu_t}{1+\pi_t} = \frac{c_t}{c_{t-1}} \cdot \left(\frac{1+i_t}{i_t}\right)\left(\frac{i_{t-1}}{1+i_{t-1}}\right)$$

Impose steady state

$i.e., c_{t-1} = c_t = c, i_{t-1} = i_t = i, \pi_t = \pi, and\ \mu_t = \mu$

$$\frac{1+\mu}{1+\pi} = \frac{\cancel{c}}{\cancel{c}} \cdot \cancel{\left(\frac{1+i}{i}\right)\left(\frac{i}{1+i}\right)}$$

$$\boxed{\mu = \pi}$$

This is LR perspective taken by monetarist

IN LONG RUN, RATE OF MONEY GROWTH = RATE OF INFLATION

Back to chp 15's outline

MONETARISM

$$\mu = \pi$$

IN LONG RUN, RATE OF MONEY GROWTH = RATE OF INFLATION

- In steady state, inflation determined solely by how quickly central bank (Fed) expands (or shrinks) the nominal money supply

- This relationship the basis for the monetarist school of thought
  - Milton Friedman's famous dictum: "Inflation is always and everywhere a monetary phenomenon"
    - Policy translation: "A central bank should not worry about/try to control anything other than how quickly the money supply in the economy is growing. Keeping money growth under control will keep inflation under control."

Monetarism

- …
- Rose to prominence in mid- and late 1970's (during macro crises)
- Largest policy influence in U.K., short-lived policy influence in U.S.
- Largely died out as basis for serious policy advice by mid-1980's

- Nevertheless still viewed as fundamental "law" of macroeconomics
  - A concern today: Fed's "easy monetary policy" (read: Fed has increased money supply very rapidly) will lead to a burst of inflation

Monetary Policy

- In short-run, do changes in monetary policy have effects on consumption and real GDP (in conventional conditions)?
  - Keynesian/New Keynesian view: yes because prices are sticky
  - RBC view: no because prices are not sticky
- In long-run, changes in money growth rate
  - Have effects on only inflation
  - Have no effects on consumption and real GDP

Monetary Policy

- Competing principles/theories influence policy-makers' decisions
- Basic models are guideposts for policy debates
- Actual policy-making quite messy
  - Requires lot of judgment
  - Requires hope/belief that basic models are at least *somewhat* useful guides to thinking