LECTURE 7

Managing Interest Rate Risk

EC3333 Financial Economics I

Learning Objectives

- Explain the sensitivity of bond prices to interest rates fluctuations.
- Compute various measures of sensitivity in terms of Macauley's duration and modified duration.
- Compute refinements of interest rate sensitivity measures, namely convexity.

Recall in Lecture 6
Figure 14.3 The Inverse Relationship Between Bond Prices and Yields
(from adopted text, Bodie, Kane and Marcus, Investments, McGraw Hill, 12e)

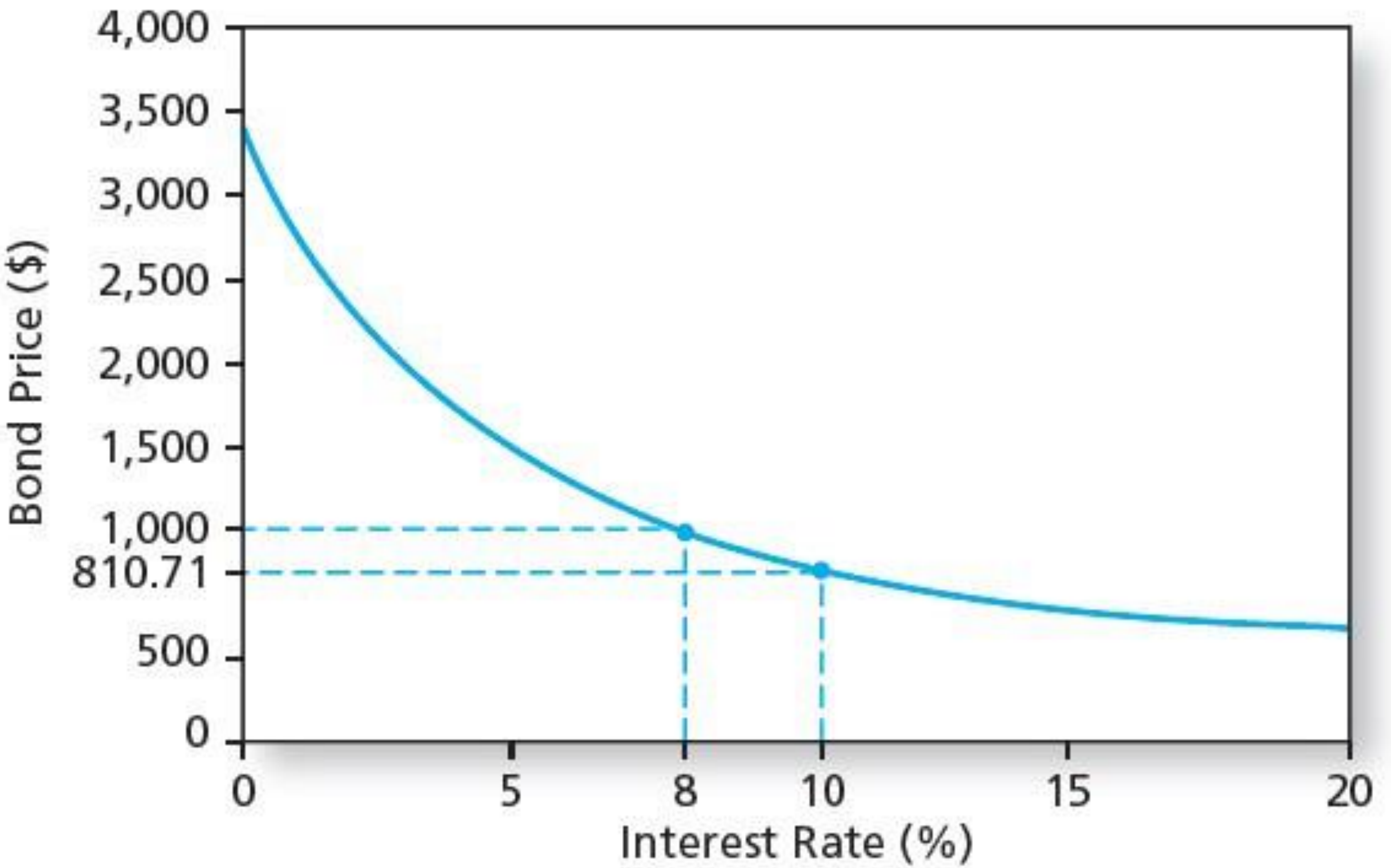
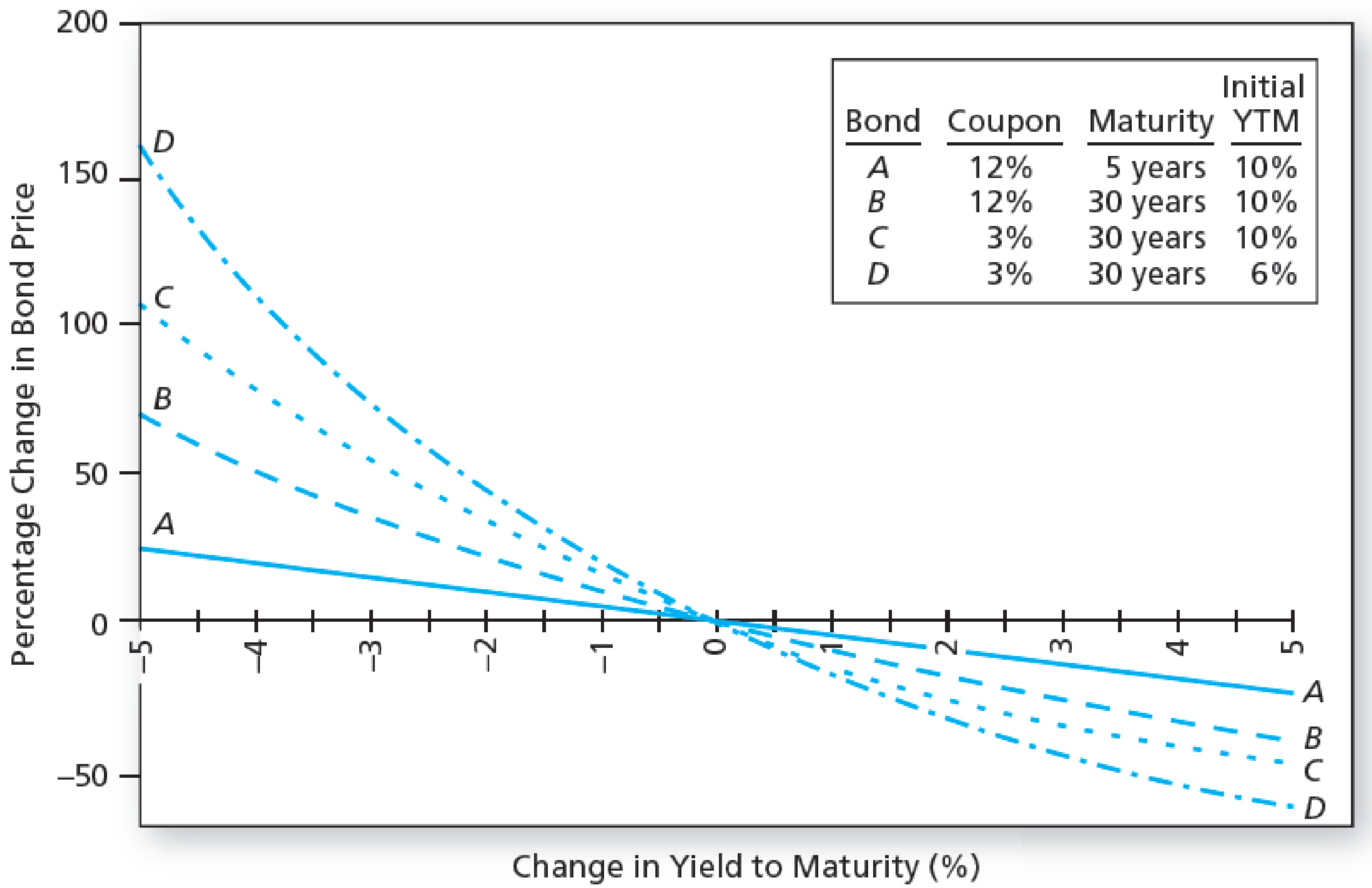


Figure 16.1 Change in Bond Price as a Function of Change in Yield to Maturity
(from adopted text, Bodie, Kane and Marcus, Investments, McGraw Hill, 12e)



General properties of Interest Rate Sensitivity

Properties 1 to 5 are known as Malkiel's bond-pricing relationships (1962)
Property 6 was demonstrated by Homer and Liebowitz (1972)

1. Bond prices and yields are inversely related
2. An increase in a bond's yield to maturity results in a smaller price change than a decrease in yield of equal magnitude
3. Prices of long-term bonds tend to be more sensitive to interest rate changes than prices of short-term bonds
4. Interest rate risk is less than proportional to bond maturity
5. Interest rate risk is inversely related to the bond's coupon rate
6. The sensitivity of a bond's price to a change in its yield is inversely related to the YTM at which the bond is currently selling

Table 16.1 Prices of 8% Coupon Bond (Coupons Paid Semiannually) (from adopted text, Bodie, Kane and Marcus, Investments, McGraw Hill, 12e)

Yield to Maturity (APR)	*T* = 1 Year	*T* = 10 Years	*T* = 20 Years
8%	1,000.00	1,000.00	1,000.00
9%	990.64	934.96	907.99
Fall in price (%)*	0.94%	6.50%	9.20%

*Equals value of bond at a 9% yield to maturity divided by value of bond at (the original) 8% yield, minus 1.

Table 16.2 Prices of Zero-Coupon Bond (Semiannual Compounding) (from adopted text, Bodie, Kane and Marcus, Investments, McGraw Hill, 12e)

Yield to Maturity (APR)	*T* = 1 Year	*T* = 10 Years	*T* = 20 Years
8%	924.56	456.39	208.29
9%	915.73	414.64	171.93
Fall in price (%)*	0.96%	9.15%	17.46%

*Equals value of bond at a 9% yield to maturity divided by value of bond at (the original) 8% yield, minus 1.

Interest Rate Risk

- Higher-coupon-rate bonds have higher fraction of value tied to coupons rather than the final payment of par value.
- Zero-coupon bond = only one cash flow (CF) at maturity
- Coupon bond = a "portfolio" of CFs; compared to Zero-coupon bond, the portfolio of cashflows is more heavily weighted towards the earlier, short-maturity payments.
- Thus, coupon bond has lower "effective maturity" than zero-coupon bond with the same time to maturity.
- Hence, coupon bond is less price sensitive than zero-coupon bond

Duration

- Macaulay's duration equals the weighted average of the times to each coupon or principal payment, using the relative present values of the cash flows as weights.
- The weight = the present value of the cash flow divided by the bond price
- It is a measure of the average maturity of a bond's promised cash flows
- Macaulay's Duration = Maturity for zero coupon bonds
- Macaulay's Duration < Maturity for coupon bonds

Macaulay's Duration

$$D = \sum_{t=1}^{T} \frac{PV(C_t)}{P} \times t$$

$$PV(C_t) = \frac{C_t}{(1+y)^t} \quad \textbf{and} \quad P = \sum_{t=1}^{T} \frac{C_t}{(1+y)^t}$$

- y = YTM
- C_t = Cash flow at time t

Duration as Price Sensitivity

$$P = \sum_{t=1}^{T} \frac{C_t}{(1+y)^t}$$

$$\frac{\partial P}{\partial y} = -\sum_{t=1}^{T} t \times \frac{C_t}{(1+y)^{t+1}}$$

$$-\frac{1}{P}\frac{\partial P}{\partial y} = \frac{1}{P}\frac{1}{(1+y)}\sum_{t=1}^{T} t \times \frac{C_t}{(1+y)^t}$$

$$-\frac{1}{P}\frac{\partial P}{\partial y} = \frac{1}{(1+y)}\sum_{t=1}^{T} t \times \frac{PV(C_t)}{P}$$

$$-\frac{1}{P}\frac{\partial P}{\partial y} = D^*$$

Duration as Price Sensitivity – Modified Duration

- Price change is proportional to duration and not to maturity

$$\frac{\Delta P}{P} = -D^{*} \times \Delta y$$

- D^{*} = D / (1+y) = Modified duration

The Duration of a Coupon Bond

Example 30.11 from adopted text, Berk and DeMarzo, Corporate Finance, Pearson, 5e

- **Problem**
  - What is the duration of 10-year, zero-coupon bond? What is the duration of a 10-year bond with 10% annual coupons trading at par?

The Duration of a Coupon Bond

Example 30.11 from adopted text, Berk and DeMarzo, Corporate Finance, Pearson, 5e

- **Solution**
  - For a zero-coupon bond, there is only single cash flow. Since $PV(C_{10}) = P$ and the duration is equal to the bond's maturity of 10 years.
  - For the coupon bond trades at par, its yield to maturity equals its 10% coupon rate. Table 30.3, shows the calculation of the bond duration.
  - Note that, because the bond pays coupons prior to maturity, its duration is shorter than its 10-year maturity.
  - Moreover, the higher the coupon rate, the more weight is put on these earlier cash flows, shortening the duration of the bond.

Table 30.3 Computing the Duration of a 10-year Coupon Bond with 10% annual coupons trading at par
(from adopted text, Berk and DeMarzo, Corporate Finance, Pearson, 5e)

t (years)	C_t	$PV(C_t)$	$PV(C_t) / P$	$[PV(C_t) / P] \times t$
1	10	9.09	9.09%	0.09
2	10	8.26	8.26%	0.17
3	10	7.51	7.51%	0.23
4	10	6.83	6.83%	0.27
5	10	6.21	6.21%	0.31
6	10	5.64	5.64%	0.34
7	10	5.13	5.13%	0.35
8	10	4.67	4.67%	0.36
9	10	4.24	4.24%	0.37
10	110	42.41	42.41%	4.24
	Bond Price =	100.00	100.00%	Duration = 6.76 yrs

Estimating Interest Rate Sensitivity Using Duration

Example 30.12 from adopted text, Berk and DeMarzo, Corporate Finance, Pearson, 5e

- **Problem**
  - Suppose the yield of a 10-year bond with 10% annual coupons increases from 10% to 10.25%. Use duration to estimate the percentage price change. How does it compare to the actual price change?

Estimating Interest Rate Sensitivity Using Duration

Example 30.12 from adopted text, Berk and DeMarzo, Corporate Finance, Pearson, 5e

- **Solution**
  - In Example 30.11 (Table 30.3), we found that the duration of the bond is 6.76 years. To estimate the percentage price change

$$\%\text{ Price Change } \approx -6.76 \times \frac{0.25\%}{1.10} = -1.54\%$$

  - Indeed, calculating the bond's price with a 10.25% yield to maturity, we get

$$10 \times \frac{1}{0.1025}\left(1 - \frac{1}{(1.1025)^{10}}\right) + \frac{100}{(1.1025)^{10}} = \$98.48$$

  - Which represents a 1.52% price drop.

Duration: Useful Rules of Thumb

1. The duration of a zero-coupon bond equals its time to maturity

2. Holding maturity constant, a bond's duration is lower when the coupon rate is higher

3. Holding the coupon rate constant, a bond's duration generally increases with its time to maturity

4. Holding other factors constant, the duration of a coupon bond is higher when the bond's yield to maturity is lower

5. The duration of a level perpetuity is equal to: $\dfrac{1+y}{y}$

Figure 16.2 Bond Duration versus Bond Maturity
(from adopted text, Bodie, Kane and Marcus, Investments, McGraw Hill, 12e)

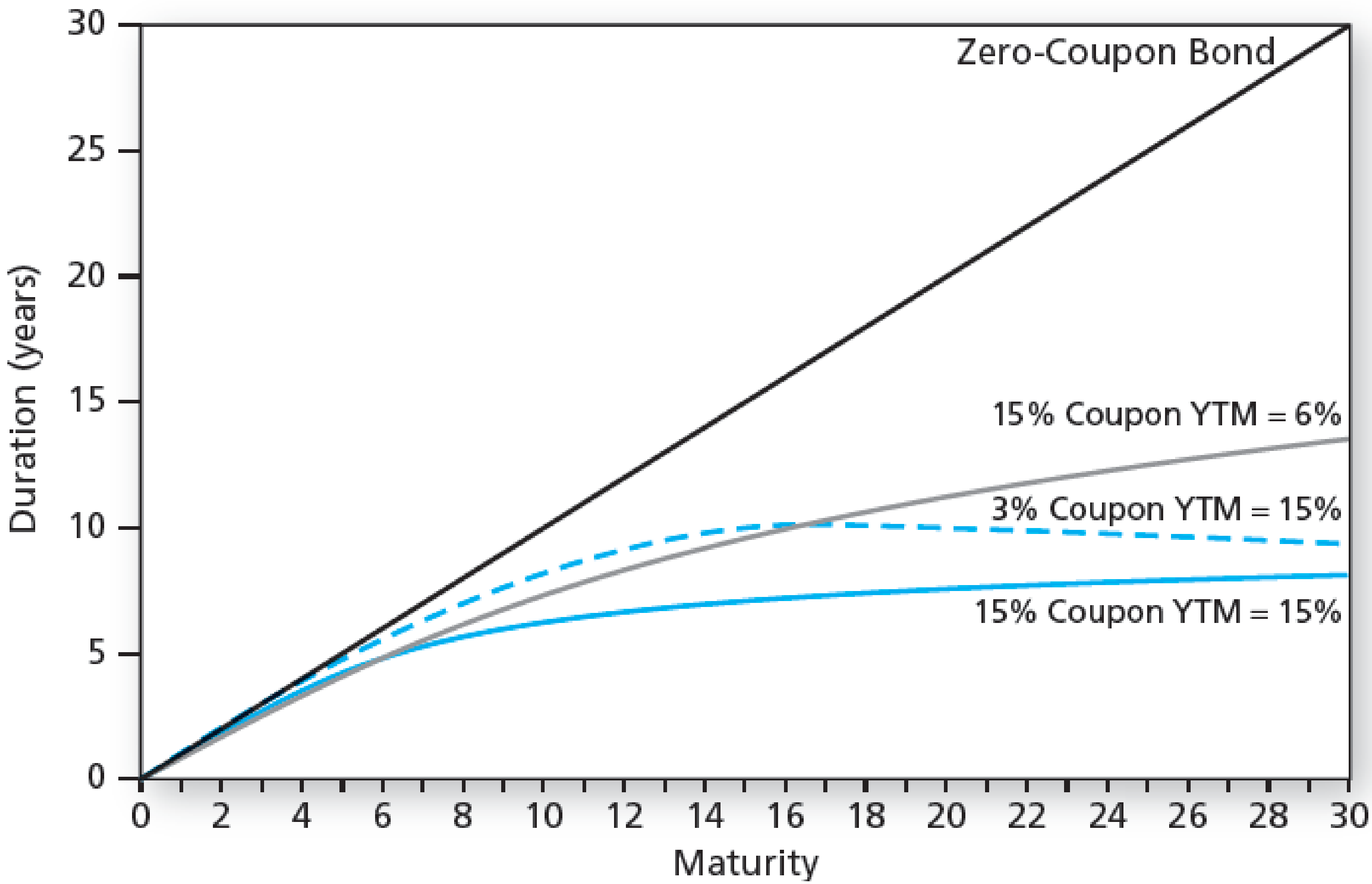


Table 16.3 Bond Durations (Yield to Maturity = 8% APR; Semiannual Coupons) (from adopted text, Bodie, Kane and Marcus, Investments, McGraw Hill, 12e)

	Coupon Rates (per Year)			
Years to Maturity	**6%**	**8%**	**10%**	**12%**
1	0.985	0.981	0.976	0.972
5	4.361	4.218	4.095	3.990
10	7.454	7.067	6.772	6.541
20	10.922	10.292	9.870	9.568
Infinite (perpetuity)	13.000	13.000	13.000	13.000

The Duration of a Portfolio

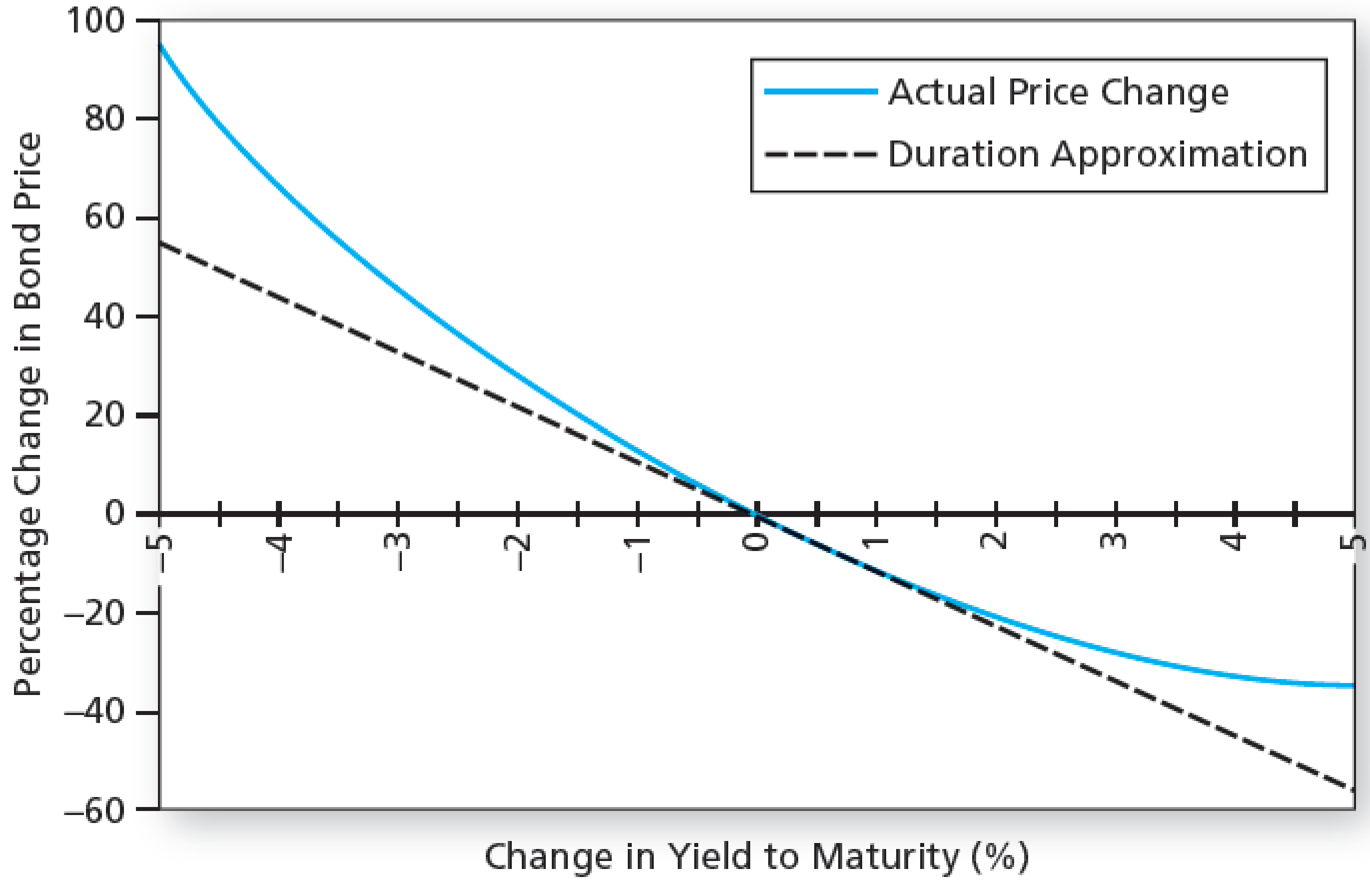
- The duration of a portfolio of investments is the value-weighted average of the durations of each investment in the portfolio.
  - A portfolio of securities with market values A and B and durations D_A and D_B, respectively, has the following duration:

$$D_{A+B} = \frac{A}{A+B} D_A + \frac{B}{A+B} D_B$$

Convexity

- The relationship between bond prices and yields is not linear, but convex
- Duration rule is a good approximation for only small changes in bond yields
- The curvature of the price-yield curve is called the convexity of the bond
- Bonds with greater convexity have more curvature in the price-yield relationship
- Formally, convexity is defined as the rate of change of the slope of the price-yield curve (i.e., second derivative of the price-yield curve), expressed as a fraction of the bond price.

Figure 16.3 Bond Price Convexity
(30-Year Maturity; 8% Coupon; Initial YTM = 8%)
(from adopted text, Bodie, Kane and Marcus, Investments, McGraw Hill, 12e)



Convexity

$$\text{Modified Duration } D^* = -\frac{1}{P} \times \frac{dP}{dy}$$

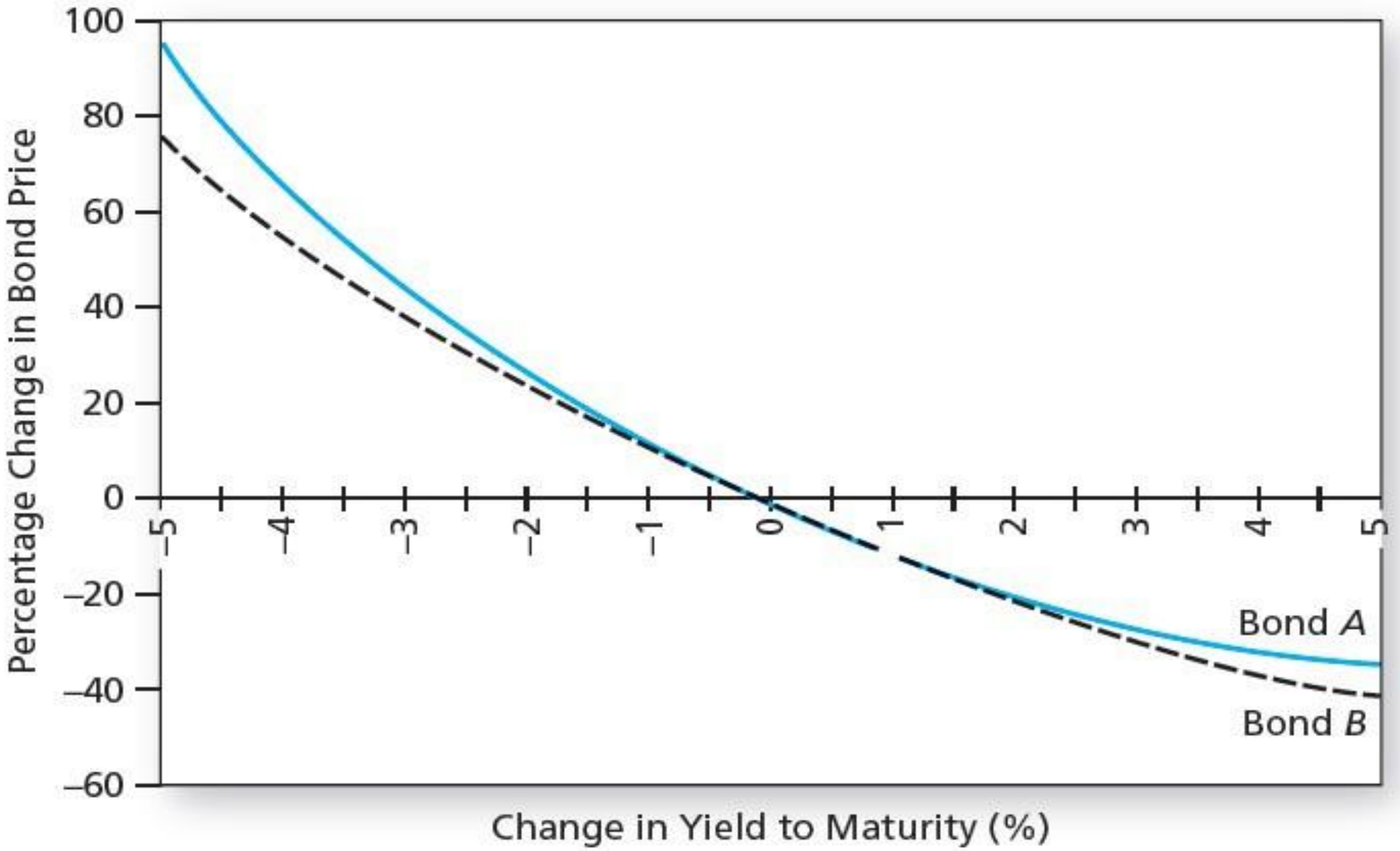
$$\text{Convexity} = \frac{1}{P} \times \frac{d^2P}{dy^2}$$

$$\text{Convexity} = \frac{1}{P \times (1+y)^2} \sum_{t=1}^{n} \left[\frac{C_t}{(1+y)^t} \left(t^2 + t\right) \right]$$

- Correction for Convexity:

$$\frac{\Delta P}{P} = -D^* \times \Delta y + \frac{1}{2} \times \text{ Convexity } \times (\Delta y)^2$$

Figure 16.4 Convexity of Two Bonds
(from adopted text, Bodie, Kane and Marcus, Investments, McGraw Hill, 12e)



- Bond A benefits more from rate decreases and suffer less from rate increases

Why Do Investors Like Convexity?

- Bonds with greater curvature gain more in price when yields fall than they lose when yields rise

- The more volatile interest rates are, the more attractive this asymmetry is

- They also gain more in price when yields fall and lose less when yields rise

- Bonds with greater convexity tend to have higher prices and/or lower yields, all else equal

Immunization

- A way to control interest rate risk that is widely used by pension funds, insurance companies, and banks

- If there is a mismatch between asset and liability maturity structures, the pension fund's or insurance company's ability to meet future obligations will fluctuate with interest rates

- In a portfolio, the interest rate exposure of assets and liabilities are matched by matching the duration of the assets and liabilities

- As a result, value of assets will track the value of liabilities whether rates rise or fall

- A portfolio with a zero duration is called a duration-neutral portfolio or an immunized portfolio

Duration-Based Hedging

- Example: many firms establish pension funds to meet obligations to retirees
- If the assets of a pension fund are invested in bonds and other fixed-income securities, the duration of the assets can be computed
- Similarly, the firm views the obligations to retirees as analogous to interest payments on debt. The duration of these liabilities can be calculated as well
- The manager of a pension fund would commonly choose pension assets so that the duration of the assets is matched with the duration of the liabilities
- In this way, changing interest rates would not affect the net worth of the pension fund

Immunization & Cash-Flow Matching

- A more direct form of immunization is dedication strategy, or cash-flow matching
- In this case, the manager selects either zero-coupon or coupon bonds with total cash flows in each period that match a series of obligations/cash outflows
- If portfolio cash flows are perfectly matched to those of projected liabilities, rebalancing will be unnecessary

Duration-Based Hedging

- Immunization is not a once and for all strategy
- As interest rates change and the durations of assets and/or liabilities change (due merely to the passage of time), the portfolio must be rebalanced continuously to remain duration-neutral

Duration-Based Hedging

- A duration-neutral portfolio is only protected against parallel shifts in the yield curve when interest rate changes affect all yields identically.
- Even if assets have similar maturities, they may have different credit risks.