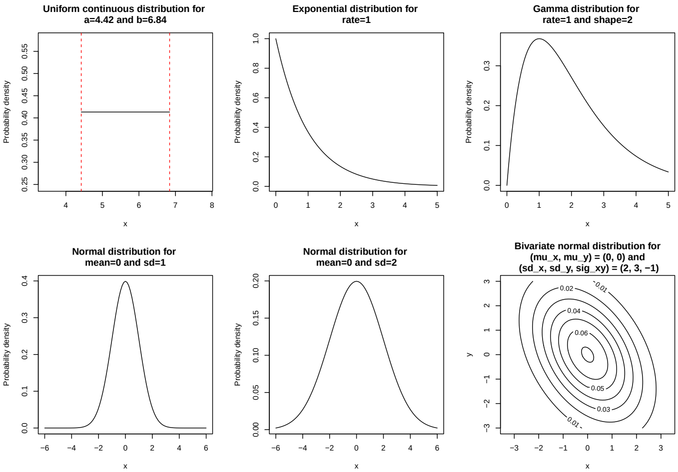
Continuous distributions



Outline

Learning Objectives

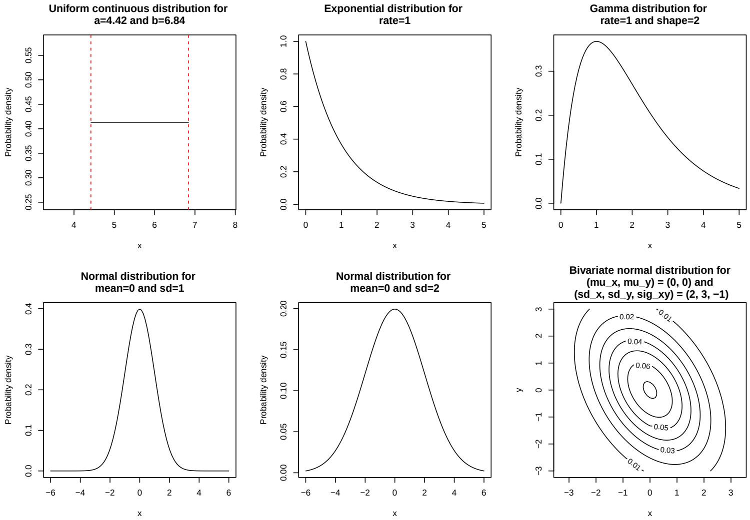
1. Learn what defines the some commonly used continuous distributions.
2. Build a vocabulary of some of the more common continuous distributions

Introduction

A brief review of continuous random variables

- Recall that a continuous random variable can be defined by a probability density function (pdf).
- A continuous random variable can take on an infinite number of values.
- A continuous random variable can belong to some pdf, i.e., $X \sim f(x|\theta)$.
  - x denotes the value that the random variable takes on.
  - θ denotes the parameter(s) of the pdf.

Density plots of continuous distributions

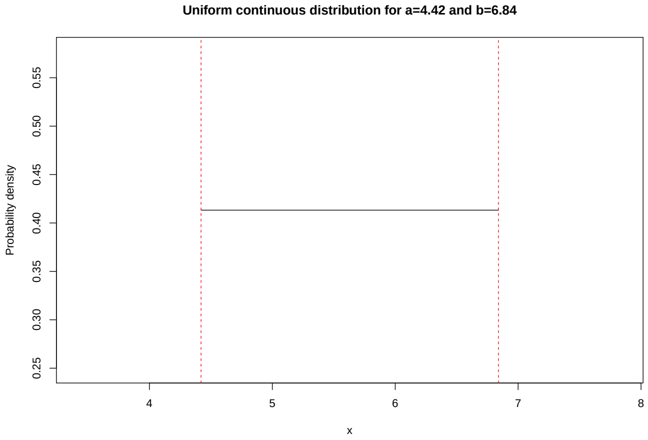


Examples of continuous distributions

Uniform continuous distribution

Uniform continuous distribution

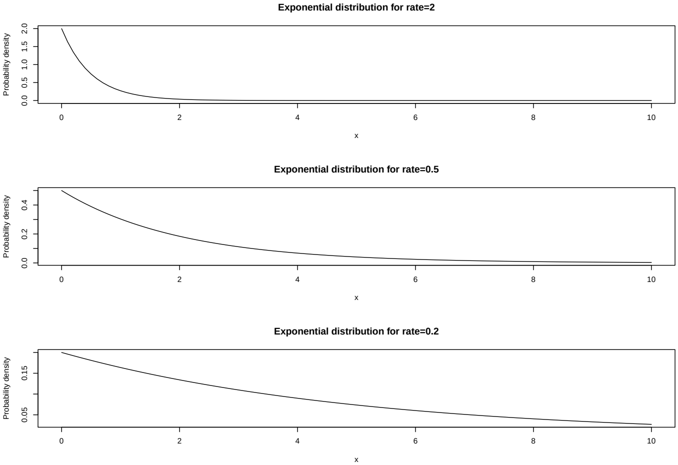
- Simplest continuous probability distribution.
- $X \sim f(x|\theta) = Uniform_{cont}(x|a, b)$.
- The support is $x \in [a, b]$.
- Two parameters a and b.
- E.g., waiting times at a bus stop where $a = 4.42$ minutes and $b = 6.84$ minutes.
  - Infinite number of possible outcomes.
  - Each outcome has an equal probability.



Exponential distribution

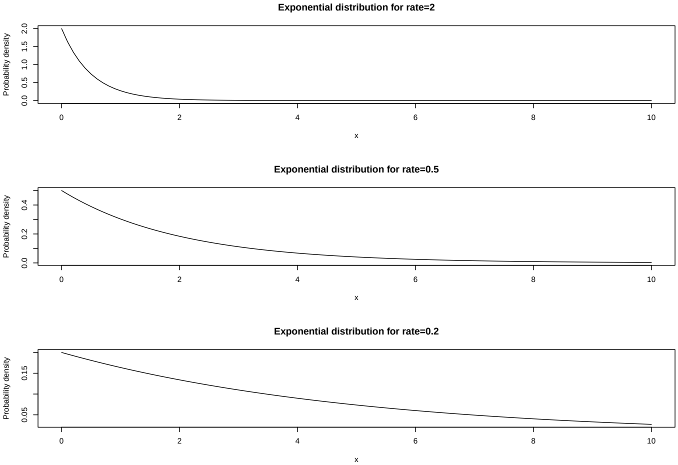
Exponential distribution

- Used to model the amount of time or space that has passed between events, based on a given average quantity in a unit of time or space.
  - Intricately tied to the Poisson distribution.
- $X \sim f(x|\theta) = Exponential(x|\beta)$.
- The support is $x \in [0, \infty)$.
  - The value of x denotes the amount of time or space.
- One parameter β (pronounced beta), called the **rate**, where β is the average number of events in one unit of space or time.



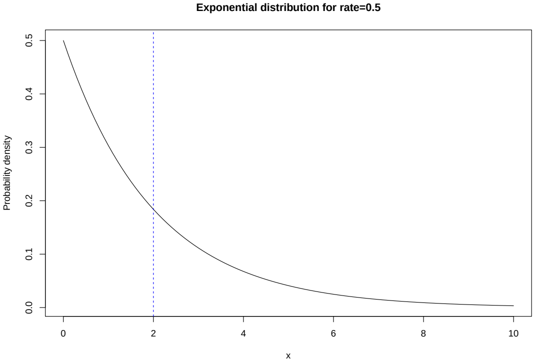
Exponential distribution (cont'd)

- Let's consider stalls A, B, C with *rates* of $\beta = 2, 0.5, 0.2$ customers per minute, respectively.
  - The value of β controls how quickly the probability density goes to zero with increasing x.
- Reminder: since a pdf is used, its values are relative likelihoods, called **probability densities**, rather than actual probabilities.
  - The `d<distribution>()` function now outputs a probability density.



An example: Waiting times at stall B

- Stall B: The rate is $\beta = 0.5$.
- We can use the `dexp()` function to compute the *probability density* at $x = 2$ minutes:

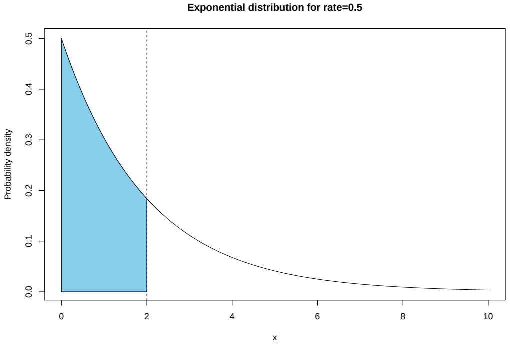



```
dexp(x = 2, # Waiting time x=2 minutes
     rate = 0.5) # Average waiting time was observed to be rate=0.5
```

```
## [1] 0.1839397
```

An example: Waiting times at stall B (cont'd)

- It is the area under the pdf that returns a probability.
- What is the probability that a customer has to wait for between 2 and 4 minutes?
  - First, we can use `pexp()` to find the probability that the waiting time is 2 minutes or less.

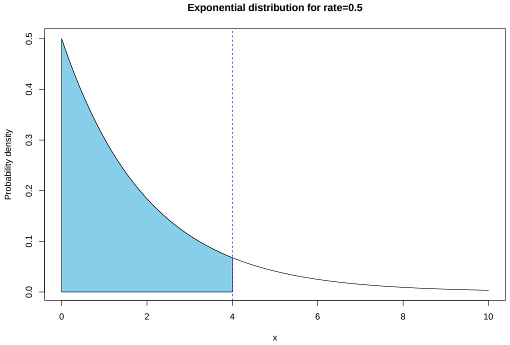



```
(p_2min <- pexp(q=2, # Waiting time of x=2 minutes or less
     rate = 0.5)) # Average waiting time was observed to be rate=0.5
```

```
## [1] 0.6321206
```

An example: Waiting times at stall B (cont'd)

- It is the area under the pdf that returns a probability.
- What is the probability that a customer has to wait for between 2 and 4 minutes?
  - First, we can use `pexp()` to find the probability that the waiting time is 2 minutes or less.
  - Next, we can use `pexp()` to find the probability that the waiting time is 4 minutes or less.

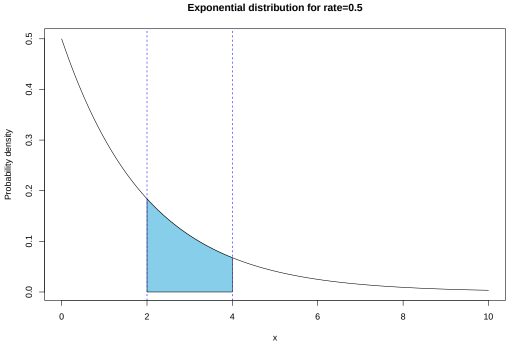



```
(p_4min <- pexp(q=4, # Waiting time of x=4 minutes or less
      rate = 0.5)) # Average waiting time was observed to be rate=0.5
```

```
## [1] 0.8646647
```

An example: Waiting times at stall B (cont'd)

- It is the area under the pdf that returns a probability.
- What is the probability that a customer has to wait for between 2 and 4 minutes?
  - First, we can use `pexp()` to find the probability that the waiting time is 2 minutes or less.
  - Next, we can use `pexp()` to find the probability that the waiting time is 4 minutes or less.
  - Finally, we can take the difference.



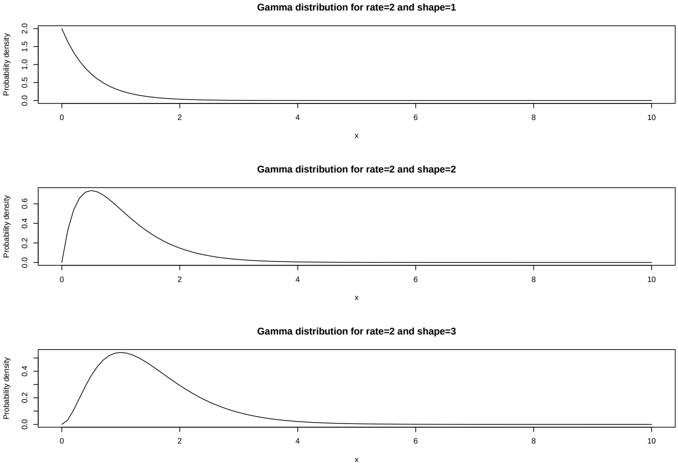

```
# Waiting time between 2 and 4 minutes
p_4min - p_2min
```

```
## [1] 0.2325442
```

Gamma distribution

Gamma distribution

- A modified version of the exponential distribution
- $X \sim f(x|\theta) = Gamma(x|\beta, \alpha)$.
- The support is $x \in [0, \infty)$.
  - The value of x denotes the amount of time or space.
- Two parameters
  - β, called **rate**.
  - α (pronounced alpha), called **shape**.
    - ★ Setting shape $\alpha = 1$ gives us the exponential distribution.



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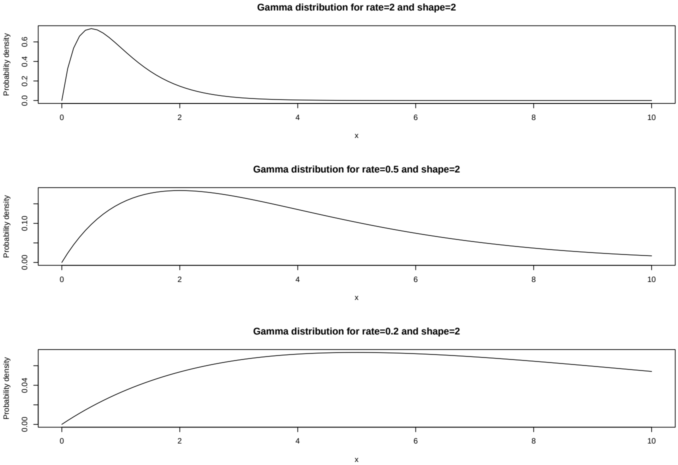
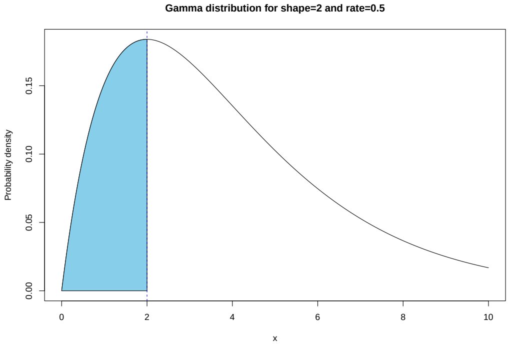
- A modified version of the exponential distribution
- $X \sim f(x|\theta) = Gamma(x|\beta, \alpha)$.
- The support is $x \in [0, \infty)$.
  - The value of x denotes the amount of time or space.
- Two parameters
  1. β, called **rate**.
  2. α (pronounced alpha), called **shape**.
     - ★ Setting shape $\alpha = 1$ gives us the exponential distribution.
- Changing rate has a similar effect as in the exponential model.



An example: Waiting times at stall B based on the gamma model

- It is the area under the pdf that returns a probability.
- What is the probability that a customer has to wait for between 2 and 4 minutes?
  - First, we can use pgamma() to find the probability that the waiting time is 2 minutes or less.

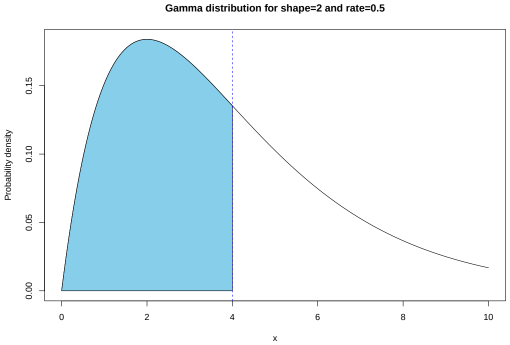



```
(p_2min_g <- pgamma(q=2, # Waiting time of x=2 minutes or less
     shape = 2, # Set shape = 2
     rate = 0.5)) # Average waiting time was observed to be rate=0.5
```

```
## [1] 0.2642411
```

An example: Waiting times at stall B based on the gamma model (cont'd)

- It is the area under the pdf that returns a probability.
- What is the probability that a customer has to wait for between 2 and 4 minutes?
  - First, we can use `pgamma()` to find the probability that the waiting time is 2 minutes or less.
  - Next, we can use `pgamma()` to find the probability that the waiting time is 4 minutes or less.




```
(p_4min_g <- pgamma(q=4, # Waiting time of x=4 minutes or less
     shape = 2, # Set shape = 2
     rate = 0.5)) # Average waiting time was observed to be rate=0.5
```

```
## [1] 0.5939942
```

An example: Waiting times at stall B based on the gamma model (cont'd)

- It is the area under the pdf that returns a probability.
- What is the probability that a customer has to wait for between 2 and 4 minutes?
  - First, we can use `pgamma()` to find the probability that the waiting time is 2 minutes or less.
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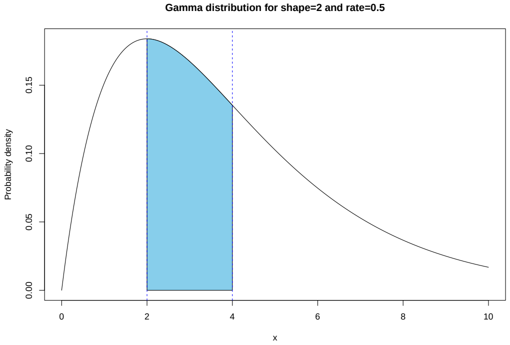
  - Finally, we can take the difference.



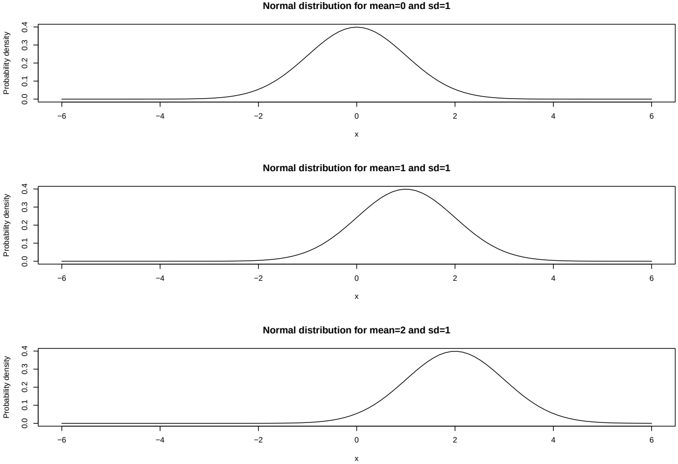

```
# Waiting time between 2 and 4 minutes
p_4min_g - p_2min_g
```

```
## [1] 0.329753
```

Normal distribution

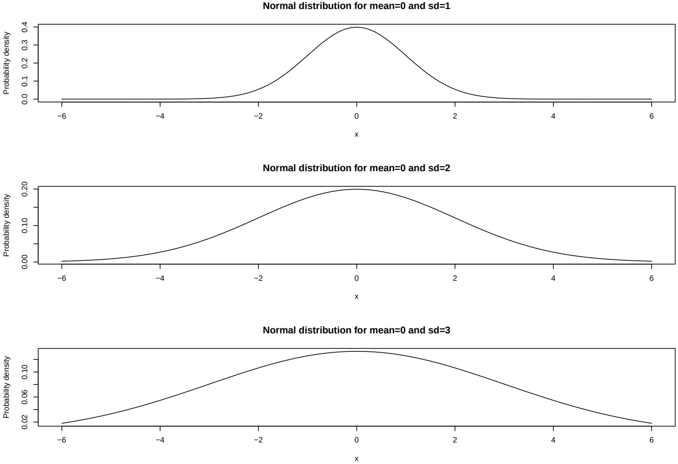
Normal distribution

- Signature symmetric, bell-shape.
- Appears frequently, notably in the natural and social sciences.
- $X \sim f(x|\theta) = Normal(x|\mu, \sigma)$.
- The support is $x \in (-\infty, \infty)$.
- Two parameters
  1. μ (pronounced mu), called **mean**.
  2. σ (pronounced sigma), called **standard deviation**, or just **sd**.
- Changing μ displaces the curve.



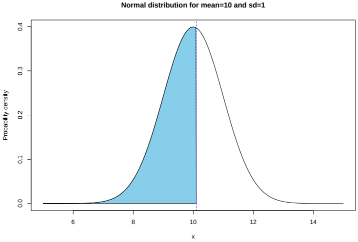
Normal distribution

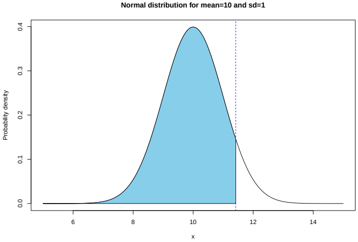
- Signature symmetric, bell-shape.
- Appears frequently, notably in the natural and social sciences.
- $X \sim f(x|\theta) = Normal(x|\mu, \sigma)$.
- The support is $x \in (-\infty, \infty)$.
- Two parameters
  - μ (pronounced mu), called **mean**.
  - σ (pronounced sigma), called **standard deviation**, or just **sd**.
- Changing the mean displaces the plot.
- Changing the sd makes the plot more spread-out.

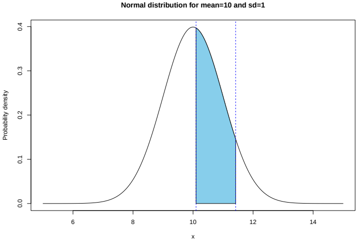


An example: Fluctuating stock prices

- Suppose the price of a certain stock can be described by $X \sim f(x|\theta) = Normal(x|\mu = 10, \sigma = 1)$.
- What is the probability that the stock is between \$10.10 and \$11.42?







An example: Fluctuating stock prices (cont'd)

- The probability can be computed using the `pnorm()` function.

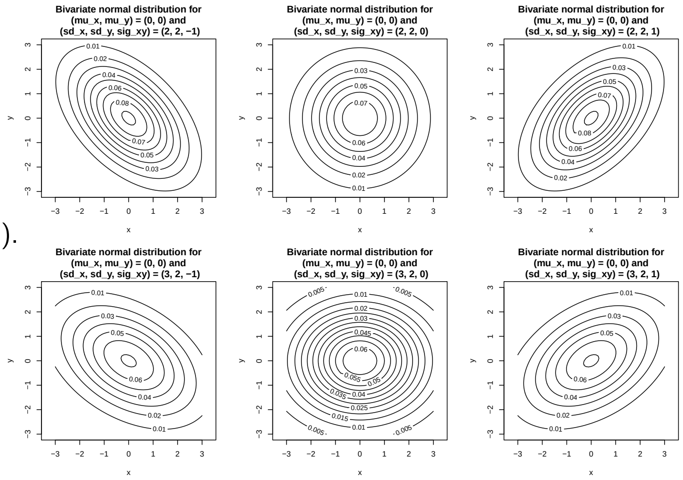
```
p_10.10 <- pnorm(q=10.10, # Stock price is x=10.10 dollars
        mean = 10, # Set mean = 10
     sd = 1) # Set sd = 1
p_11.42 <- pnorm(q=11.42, # Stock price is x=11.42 dollars
        mean = 1, # Set mean = 10
     sd = 1) # Set sd = 1
# Stock price between $10.10 and $11.42
p_11.42 - p_10.10
```

```
## [1] 0.4601722
```

Bivariate normal distribution

Bivariate normal distribution

- Combination of two normal distributions.
- Two random variables X and Y that are correlated in general.
- We shall discuss this in terms of contour plots instead.
- $X, Y \sim f(x, y|\theta) = BivariateNormal(x, y|\mu_x, mu_y, \sigma_x, \sigma_y, \sigma_{xy})$.
- The support is $x, y \in (-\infty, \infty)$.
- Five parameters
  1. μ_x, the **mean** of X.
  2. μ_y, the **mean** of Y.
  3. σ_x, the **sd** of X.
  4. σ_y, the **sd** of Y.
  5. σ_{xy}, the covariance between X and Y.



- Can be used to model pairs of random variables, e.g., height and weight.

More examples

More examples

- There are many more examples of continuous distributions.
- Weibull distribution
- Cauchy distribution
- Chi-squared distribution
- Beta distribution
- Many more
- Different distributions differ in terms of
  - Support
  - Parameters
  - Shape of histogram
- In the case of a continuous distribution, `d<distribution>()` computes probability densities rather than probabilities.
- `p<distribution>()` can be used to compute probabilities for a range of values of x, which corresponds to the area under the curve.
  - e.g., `pexp()`, `pgamma()`, `pnorm()`, etc.

Summary

Summary

In this video, we have:

- Defined some commonly used continuous distributions.
- Built a vocabulary of some of the more common continuous distributions.

References

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