LECTURE NINE

Growth and Ideas

Chapter 6 of Charles Jones' text book

Outline for Lecture 6 -- 1

- Motivation & Introduction *(weakness of Solow model demands for another)*
  - The Romer's model *(The main idea behind Romer's model)*
- The Economics of Ideas *(discussion on ideas in the economy)*
  - *In this section, we talk about the increasing return to scales of production function if we consider ideas as an input. Thus, we depart from pure competitive, which was first introduced by Adam Smith, since this theory does not explain growth phenomenon.*
- The Romer Model *(the mathematical model –this is the base Romer model)*
  - Solve Romer Model *(Note: This is a stand alone/closed economy)*
  - Why is There Growth in the Romer Model? *(examine what feature of the model allows us to examine growth)*
  - Balance Growth Path *(Long run behavior of an economy that is growing)*
  - Case Study: A model of World Knowledge *(we consider an open economy where knowledge produced are being shared across economies)*
  - Experiment #1 Changing the Population ($\bar{L}$)
  - Experiment #1 Changing the Research Share ($\bar{l}$)
  - Growth Effects versus Level Effects
  - Case study: globalization and Ideas *(how globalization affect ideas sharing, and thus growth)*

Outline for Lecture 6 -- 2

- Combining Solow and Romer: Overview *(combine ideas from both models)*
- Growth Accounting *(examine sources of growth from production function)*
- Concluding Our Study of Long-Run Growth
- A Postscript on Solow and Romer
- Appendix: Combining Solow and Romer (Algebraically) *(solve the Solow-Romer combined model algebraically)*

Motivation
&
6.1 Introduction

Motivation-1

- Solow started out with extending the production model by adding capital accumulation component
  - Success: providing an explanation for:
    - short-run and medium run growths
    - economic growth differences across countries can be explained by where the countries are relative to their own steady states.
  - Failure: Output per capita and capital per capita converge to constants. Not consistent with data

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Motivation-2

- Looking from bright side, we can learn from Solow:
  - Capital accumulation does not lead to long-run growth. So there is not use increasing the saving rate.
  - With capital accumulation eliminated as the source of growth, economists turn their attention to TFP as possible engine of long-run growth.
  - Motivated economists to endogenize TFP. For example, Paul Romer (1990)

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6.1 Introduction

- In this chapter, we learn:
  - New methods of using existing resources are the key to sustained long-run growth
  - Why "nonrivalry" makes ideas different from other economic goods
  - How the economics of ideas
    - involves increasing returns
    - leads to problems with Adam Smith's invisible hand
  - The Romer model of economic growth
  - How to combine the Romer and Solow models

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The Romer Model—1

- The Romer model divides the world into:
  - Objects
    - capital and labor from the Solow model
    - these are *finite*
  - Ideas
    - items used in making objects
    - these are *virtually infinite*
- This distinction forms the basis for modern theories of economic growth
- Sustained **economic growth** occurs because of **new ideas**.

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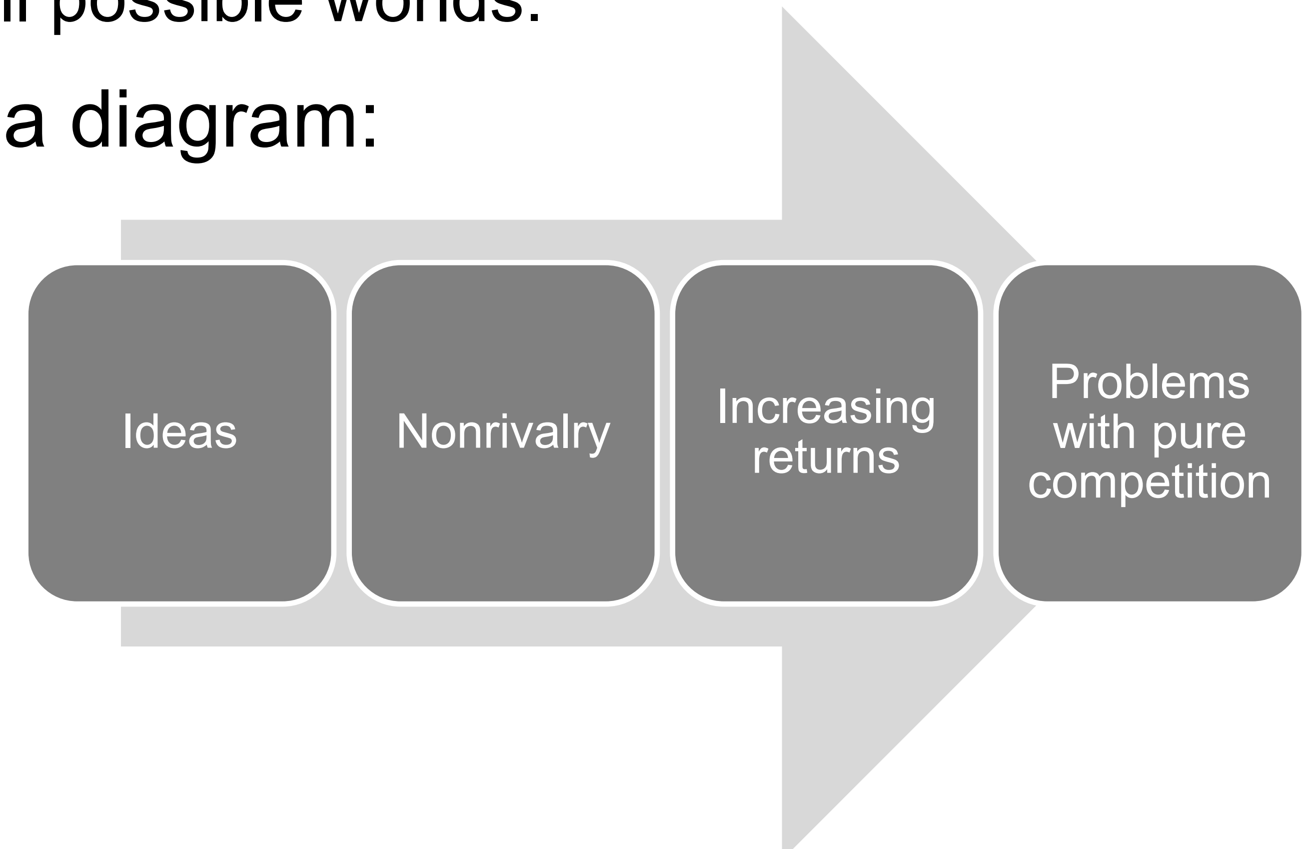
6.2 The Economics of Ideas

6.2 The Economics of Ideas

- Adam Smith's invisible hand theorem:
  - Perfectly competitive markets lead to the best of all possible worlds.
- Idea diagram:



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Nonrivalry

- Nonrivalry:
  - Objects are rivalrous
    - One person's use reduces the usefulness to someone else
  - Ideas are nonrivalrous
    - One person's use does not reduce the usefulness to someone else
- Excludability
  - Legal restrictions on use of a good or idea
    - Ideas are nonrivalrous but may be excludable

	Excludable	Non-excludable
Rival	Private goods (house, car)	Common goods (Fishing ponds, park seats, forests)
Non-rival	Club goods (toll roads, subscripted goods)	Public goods (Street lights, air, news)

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Returns to Scale—1

- Increasing returns to scale:
  - Average production per dollar spent is rising as the scale of production increases
  - Doubling inputs will more than double outputs
  - High fixed initial development costs
- Constant returns to scale:
  - Average production per dollar spent is constant
  - Doubling inputs exactly doubles output
  - The standard replication argument implies constant returns to scale

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Returns to Scale—1 (Example)

- Let's consider an example:
  - Suppose a new antibiotic is created.
  - The cost of developing the new drug is large.
  - The initial start-up costs for the research and development produce no actual good (antibiotic).
  - To create one single dose of the antibiotic, it costs the initial R&D amount plus manufacturing.
    - $2 billion + $10 = 1 vaccine
  - Every dose thereafter costs only the amount it takes to produce the drug.
    - $2 billion = 200 million vaccines
  - At this stage, doubling inputs leads to more than double outputs.

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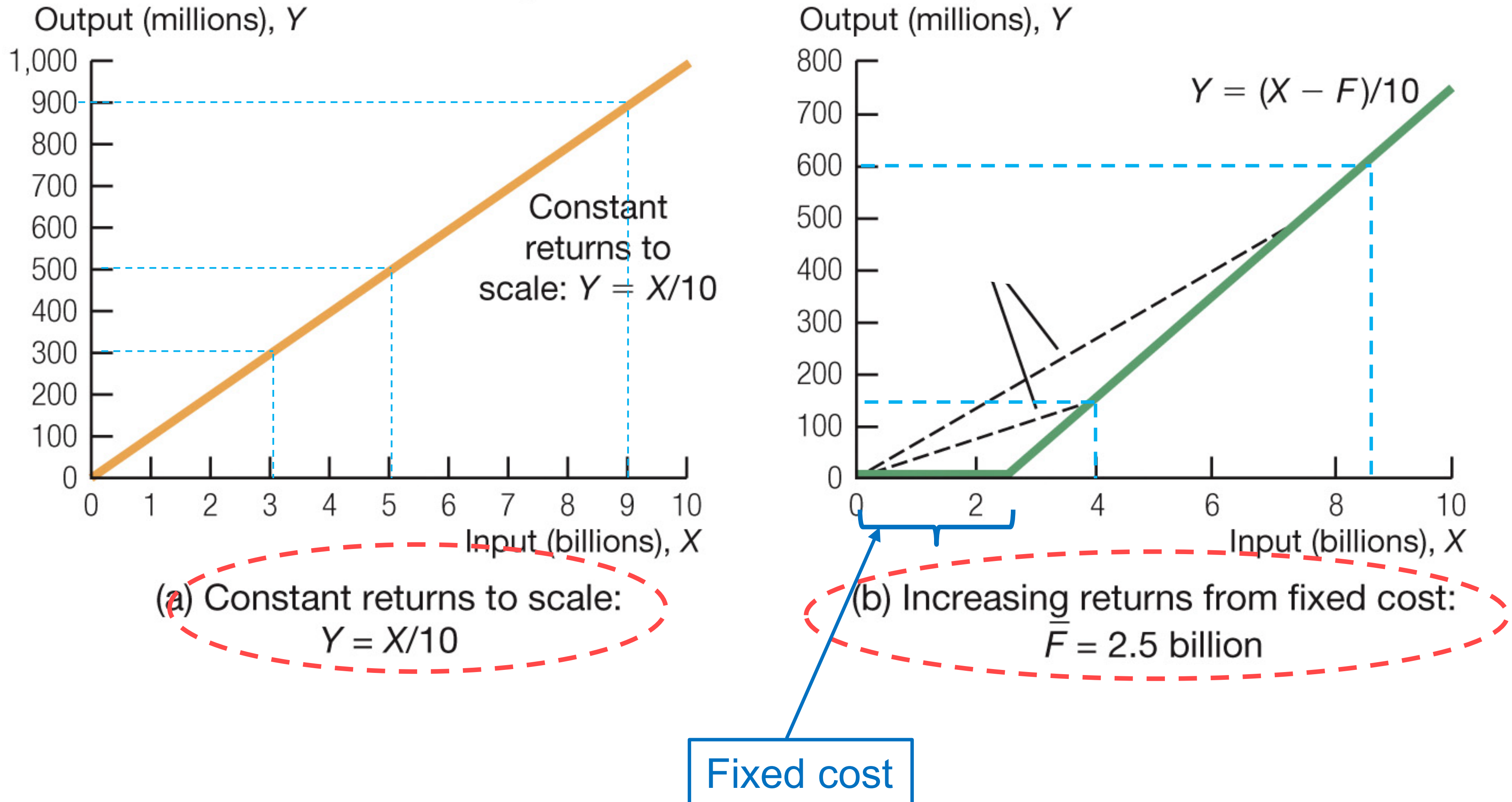
The Antibiotic Example

How a Fixed Cost Leads to Increasing Returns: The Antibiotic Example



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Antibiotic Example (second/right graph)

- The same production function as the first/left graph
- But, with an additional fixed cost $\bar{F}$ of $2.5 million that must be paid before production.

$$Y = \frac{X - \bar{F}}{10} \ once \ X > \bar{F}$$

$$\Rightarrow \frac{Y}{X} = \frac{1 - \frac{\bar{F}}{X}}{10}$$

$$(which \ increases \ as \ X \ gets \ larger)$$

- This leads to increasing returns to scale.
- The average production per dollar spent, Y/X, now increases as the scale of production rises.

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Returns to Scale—2

Last time, A was constant so it is not an input

- Proof of increasing returns
  - Begin with the production function:

$$Y_t = F(K_t, L_t, A_t) = A_t K_t^{1/3} L_t^{2/3}$$

  - Multiply all inputs by a constant (γ):

$$\begin{aligned} Y_t = F(\gamma K_t, \gamma L_t, \gamma A_t) &= \gamma A_t (\gamma K_t^{1/3})(\gamma L_t^{2/3}) \\ &= \gamma \gamma^{1/3} \gamma^{2/3} A_t (K_t^{1/3})(L_t^{2/3}) \\ &= \gamma^2 A_t (K_t^{1/3})(L_t^{2/3}) \end{aligned}$$

$\gamma^2 > \gamma \rightarrow$ increasing returns

  - Output is multiplied by more than γ

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Problems with Pure Competition—1

- Pareto optimal allocation
  - Cannot make someone better off without making someone else worse off
  - Perfect competition results in Pareto optimality because P = MC
- Under increasing returns to scale,
  - A firm faces initial fixed costs and marginal costs.
  - If P = MC, no firm will do research to invent ideas.
  - The fixed research costs will never be recovered.

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Problems with Pure Competition—2

- Consider the antibiotic example:
  - Pharmaceutical firms would not produce a new drug if P=MC.
  - They would have to spend millions of dollars on research before manufacturing the new antibiotic.
  - If P=MC, the initial fixed costs of R&D will never be recovered.
  - Pharmaceuticals would need to sell *at a price greater than marginal cost to recoup the original cost.*

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Problems with Pure Competition—3

- To create incentives for firms to innovate and produce, there must be a wedge between price and marginal cost, implying that there cannot be perfect competition and innovation. So we need:
- Patents
  - Grant monopoly power over a good for a period
  - Generate positive profits
  - Provide incentive for innovation
- However, *P* > MC results in welfare loss
- Other incentives may avoid welfare loss
  - Government funding (such as the National Research Fund)
  - Prizes

There is monopolistic competition

6.3 The Romer Model

6.3 The Romer Model

- The Romer model
  - Distinction between ideas and objects
  - Output requires knowledge and labor
- The production function of the Romer model
  - Constant returns to scale in objects alone
  - Increasing returns to scale in objects and ideas

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The Romer Model—2

- New ideas depend on
  - The existence of ideas in the previous period
  - The number of workers producing ideas
  - Worker productivity
- Unregulated markets underprovide ideas
- The population
  - Workers producing ideas (researchers, innovators)
  - Workers producing output (production/line worker)

$$L_{at} + L_{yt} = \bar{L}$$

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Solving the Romer Model—1

- Express the endogenous variables in terms of the parameters:

$$L_{at} = \bar{\ell}\bar{L}$$

$$L_{yt} = (1 - \bar{l})\bar{L}$$

Resource constraint

  - where $\bar{l}, \bar{L}$ are parameters
  - $\bar{L}$ is the amount of labour (fixed), $\bar{l}$ is the proportion of labour producing ideas (fixed/constant).
  - L_{at} and L_{yt} are endogenous variables

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2

Researchers use "their own labour" and old ideas to generate new flow of ideas:

$$\Delta A_{t+1} = \bar{z} A_t L_{at}$$

Researchers is a fixed proportion of overall labour:

$$L_{at} = \bar{l}\bar{L}$$

4

Ideas: *Generated in "ideas sector"*

1

$$Y_t = \overbrace{A_t}\underbrace{L_{yt}}_{(1-\bar{l})\bar{L}}$$

3

$$L_{yt} = \bar{L} - L_{at}$$

$(market\ clearing)$

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Solving the Romer Model—2

TABLE 6.1

The Romer Model: 4 Equations and 4 Unknowns

Unknowns/endogenous variables:	Y_t, A_t, L_{yt}, L_{at}
Output production function	$Y_t = A_t L_{yt}$
Idea production function	$\Delta A_{t+1} = \bar{z} A_t L_{at}$
Resource constraint	$L_{yt} + L_{at} = \bar{L}$
Allocation of labor	$L_{at} = \bar{\ell}\bar{L}$

Parameters: $\bar{z}, \bar{L}, \bar{\ell}, \bar{A}_0$

Solving the Romer Model—3

- Romer model:
  - Output per person depends on the *stock of knowledge*

$$y_t \equiv \frac{Y_t}{\overline{L}} = A_t(1 - \overline{\ell})$$

  - The growth rate of knowledge is *constant*

$$\frac{\Delta A_{t+1}}{A_t} = \overline{z} L_{at} = \overline{z}\,\overline{\ell}\,\overline{L}$$

- Solow model:
  - Output per person depends on *capital per person*

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Solving the Romer Model—4

- The growth rate of technology:

$$\bar{g} = \bar{z}\bar{l}\bar{L}$$

- The stock of knowledge depends on its initial value and its growth rate

Stock of knowledge

Initial amount of knowledge

Growth rate of knowledge

$$A_t = \bar{A}_0(1 + \bar{g})^t$$

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Solving the Romer Model—5

- Combining:

$$y_t \equiv \frac{Y_t}{\bar{L}} = A_t(1 - \bar{l})$$

and

$$A_t = \overline{A_0}(1 + \bar{g})^{\,t}$$

- Yields:

$$y_t = A_0(1 - \bar{\ell})(1 + \bar{g})^t$$

- Output per person is a function of the parameters of the model.

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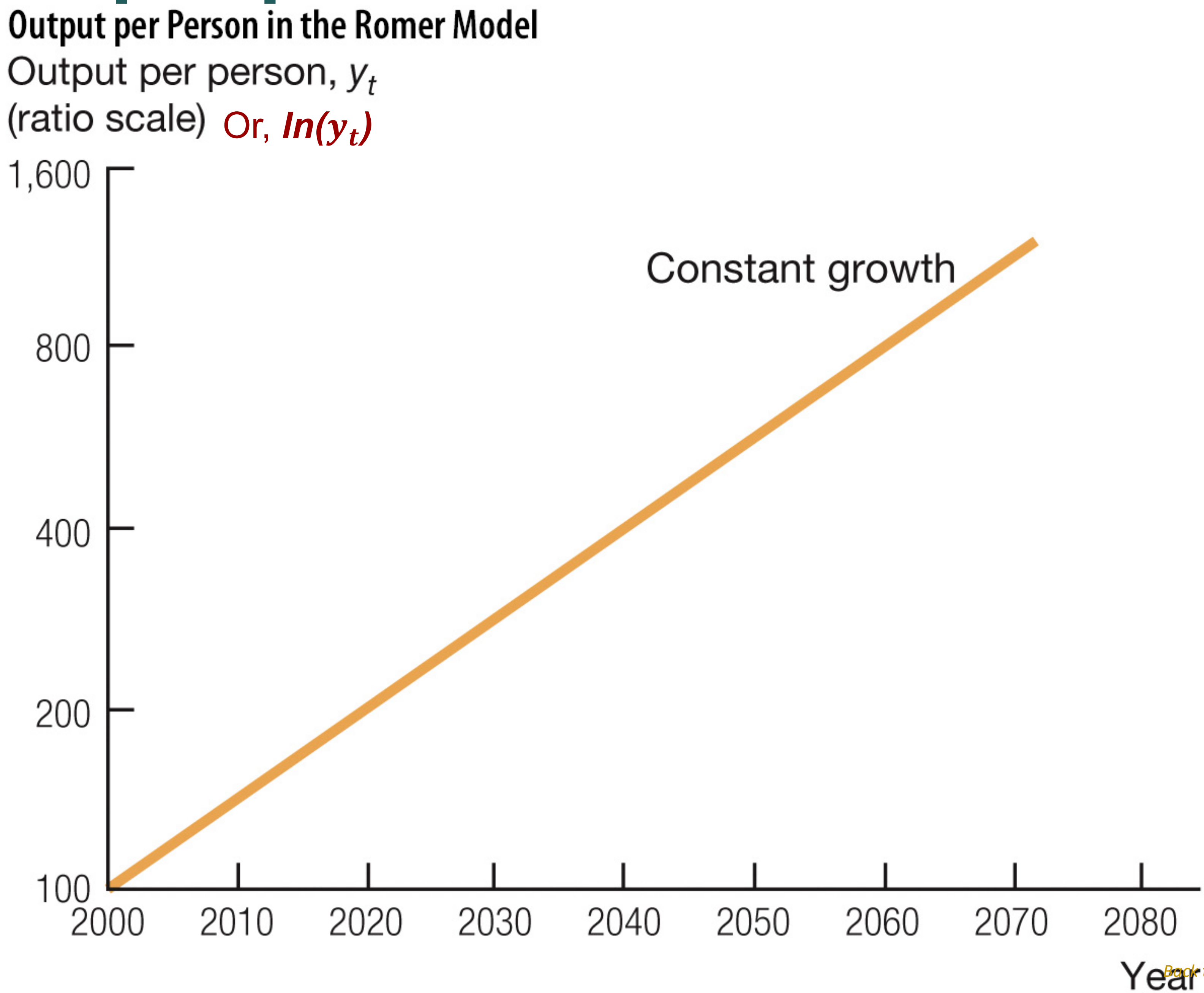
Output per Person—1



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Why log scale gives us straight line?

$$y_t = A_0\left(1 - \bar{l}\right)(1 + \bar{g})^t$$

Log both side we get:

$$\ln(y_t) = \ln\left(A_0\left(1 - \bar{l}\right)\right) + t \times \ln(1 + \bar{g})$$

So as we can see, if we plot $\ln(y_t)$ against t (meaning y-axis is having a log scale), the gradient of the graph would be $\ln(1 + \bar{g}) \approx \bar{g}$ which is a constant. Thus, we have a straight line.

In the previous graph, we assume $\ln\left(A_0\left(1 - \bar{l}\right)\right) = 100$ so when $t = 0$ $(or\ in\ the\ year\ 2000)$, $\ln(y_t) = 100$

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Why is There Growth in the Romer Model?

- **The Romer model produces long-run growth**
  - Does not have diminishing returns to ideas because they are nonrivalrous

$$\Delta A_{t+1} = \bar{z} A_t L_{at}$$

The exponent on A_t is 1 → no diminishing return

  - Labor and ideas together have increasing returns
  - Returns to ideas are unrestricted
- In the Solow model, capital has diminishing returns
  - Eventually, capital and income stop growing

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Balanced Growth

- The Solow model
  - Transition dynamics
- The Romer model
  - Does not exhibit transition dynamics
  - Instead, has balanced growth path
  - All endogenous variables grows at constant growth rates

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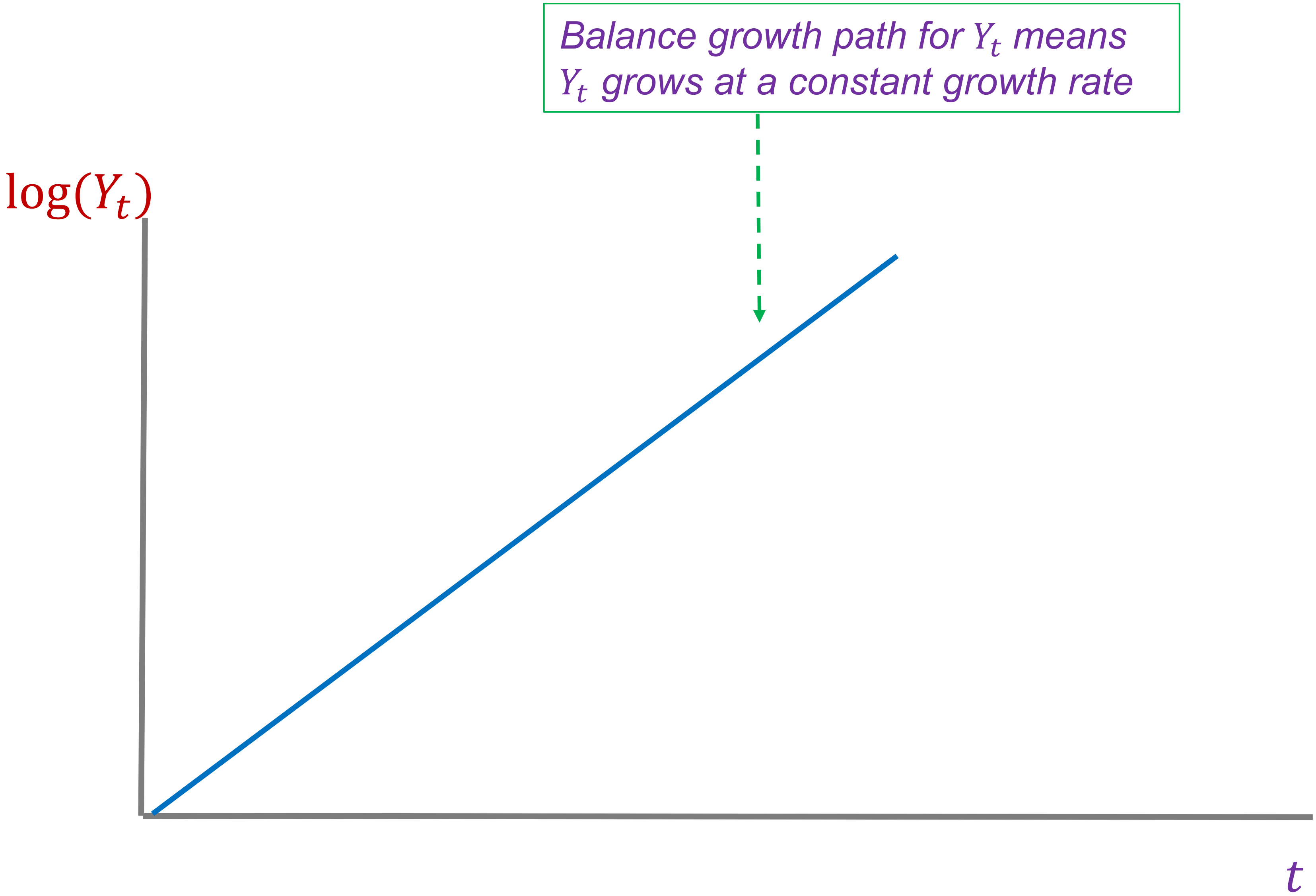

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Case Study: A Model of World Knowledge

- The United States has more several hundred times more researchers than Luxembourg has.
- So, growth rate of US, g_y^{us}, according to Romer's model must grow at a rate several hundred times larger than Luxemburg.

- BUT, growth rates 1960–2014
  - United States: $\boxed{g_y^{us} = 2\%}$
    - 2.0 percent per year increase in per capita GDP
  - Luxembourg: $\boxed{g_y^{Lux} = 2.8\%}$
    - 2.8 percent per year increase in per capita GDP

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WHEN IDEAS ARE PERFECTLY SHARED ACROSS THE GLOBE, THE ORIGINAL ROMER MODEL NEEDS ADJUSTMENT:

$$\Delta A_{t+1}^{US} = \bar{z} A_t^{world} L_{at}^{US}$$

Pool of ideas in the world

$$\Delta A_{t+1}^{Lux} = \bar{z} A_t^{world} L_{at}^{Lux}$$

$$\frac{\Delta A_{t+1}^{Lu}}{A_t^{Lu}} = \bar{z}\frac{A_t^{world}}{A_t^{Lu}} L_{at}^{Lu}$$

This ratio can be very high for Luxemburg. This means that though Luxemburg has smaller number of researchers L_{at}^{Lu} but it enjoys higher $\frac{A_t^{world}}{A_t^{Lu}}$. Thus the growth rate of ideas in Luxemburg can be as high as that of US or even higher.

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- However, it is not accurate to think of ideas remaining within a country.
    - Workers in Luxembourg benefit from ideas invented in the United States.
    - International trade, multinational corporations, licensing agreements, international patent filings, the migration of students and workers, and the open flow of information ensure that an idea created in one place can impact economies worldwide.

- The Romer model *is better applied to the world's stock of ideas,* as opposed to a country-by-country basis.
    - Through the spread of these ideas, growth in the world's store of knowledge drives long-run growth in every country in the world.

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Experiment #1: Changing the Population

$$y_t = \bar{A}_0(1 - \bar{\ell})(1 + \bar{g})^t$$

$$\bar{g} = \bar{z}\bar{\ell}\bar{L}$$

- Changes in the population
  - → changes in the growth rate of knowledge
- An increase in population
  - → immediately and permanently raises the growth rate of per capita output

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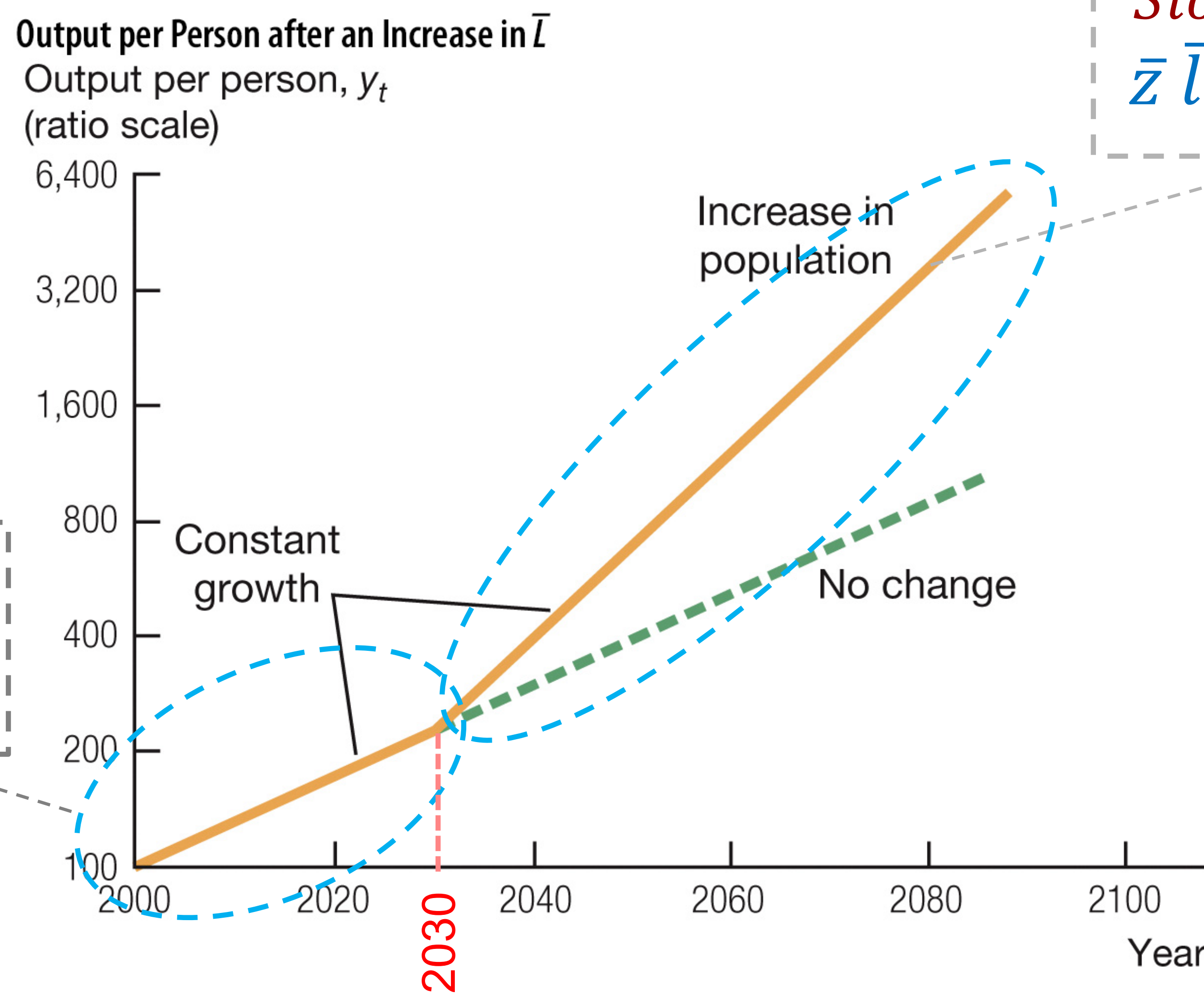
An Increase in $\bar{L}$—1



$Slope = \bar{z}\,\bar{l}L_{new}$

$Slope = \bar{z}\,\bar{l}L_{old}$

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Experiment #2: Changing the Research Share

$$y_t = \bar{A}_0(1 - \bar{\ell})(1 + \bar{g})^t$$
$$\boxed{\bar{g}} = \bar{z}\boxed{\bar{\ell}}\bar{L}$$

$$\boxed{y_t} = \bar{A}_0(1 \boxed{- \bar{\ell}})(1 + \bar{g})^t$$
$$\bar{g} = \bar{z}\bar{\ell}\bar{L}$$

Increase in $\bar{l}$ → Increase $\bar{g}$ Lower $\bar{L}$ → y_t decreases → But, $\bar{g}$ has increased in the future → y_t increases

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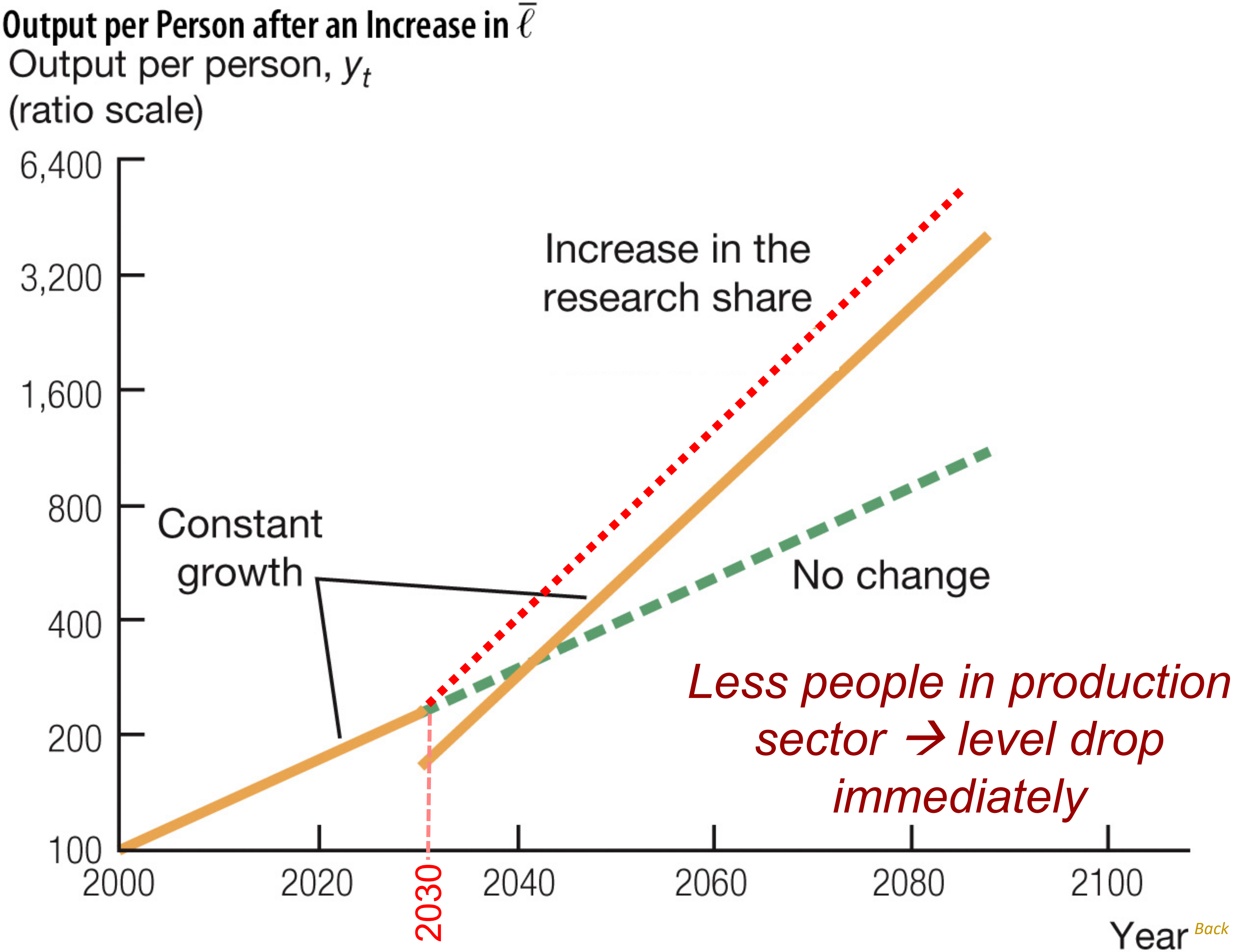
An Increase in $\bar{l}$—2



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An Increase in $\bar{l}$—2

- There are two consequences:
  1. The growth rate is higher: more researchers produce more ideas which leads to faster growth. (Growth effects)
  2. The initial level of output per person declines. (Level effects)
     - There are fewer workers in the consumption goods sector, so production per person must fall initially.

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Growth Effects versus Level Effects

- Growth effects:
  - Changes to the rate of growth of per capita output
- Level effects:
  - Changes in the level of per capita GDP
- The degree of increasing returns matters for growth effects.

We explore more in tutorial

  - If the exponent on ideas is less than 1:
    - there will still be sustained growth
    - growth effects are eliminated due to diminishing returns.

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If there is diminishing return to idea:

TABLE 6.1

The Romer Model: 4 Equations and 4 Unknowns

Unknowns/endogenous variables:	Y_t, A_t, L_{yt}, L_{at}
Output production function	$Y_t = A_t L_{yt}$
Idea production function	$\Delta A_{t+1} = \bar{z} A_t^{\alpha} L_{at}$
Resource constraint	$L_{yt} + L_{at} = \bar{L}$
Allocation of labor	$L_{at} = \bar{\ell}\bar{L}$

Parameters: $\bar{z}, \bar{L}, \bar{\ell}, \bar{A}_0$ $0 < \alpha < 1$

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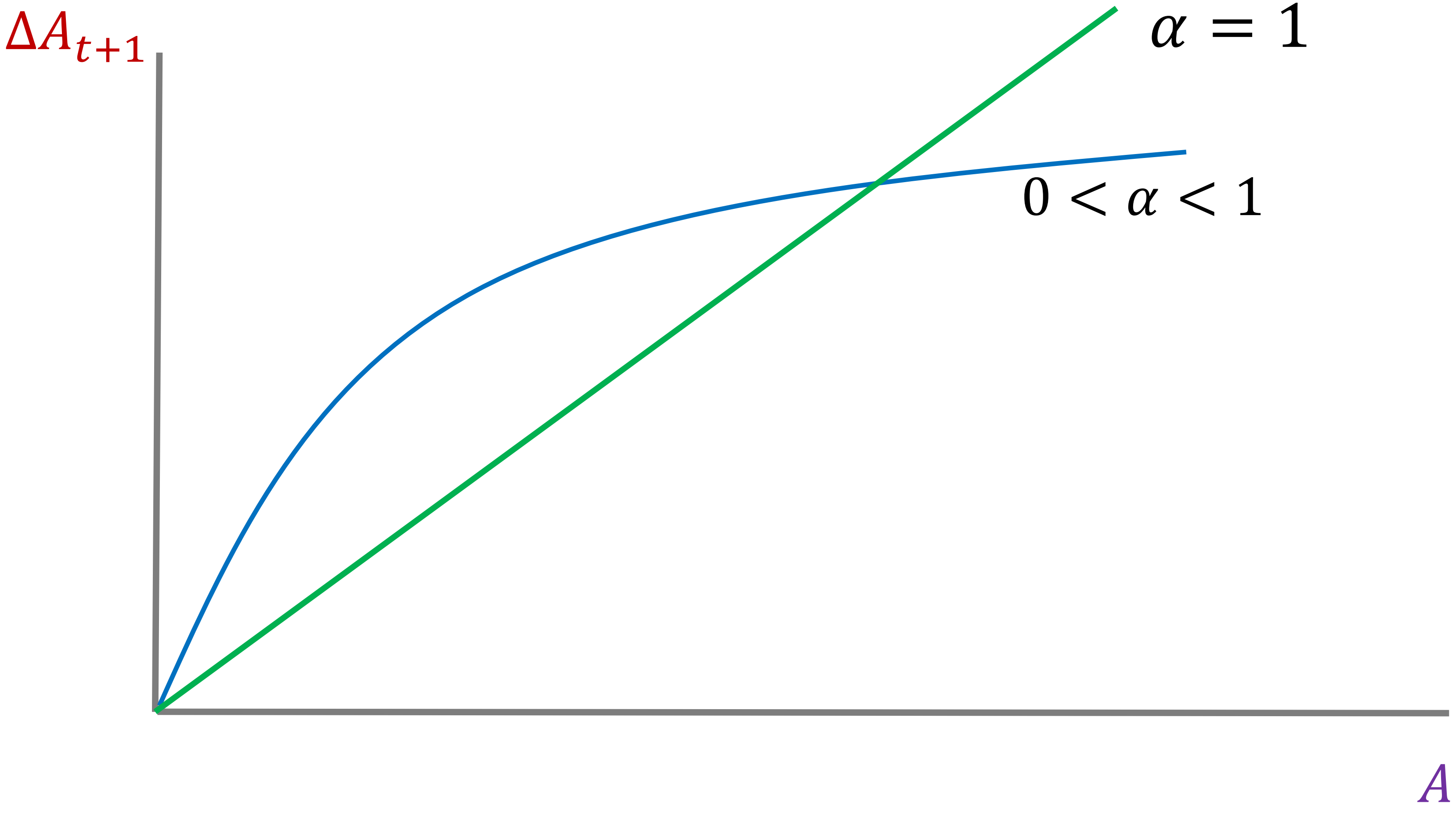
$$\Delta A_{t+1} = \bar{z} A_t^{\alpha} L_{at}$$



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Case Study: Globalization and Ideas

- Consequences of globalization:
  - Ideas can be shared more easily
  - More gains from trade realized
  - More ideas will come from developing economies

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6.4 Combining Solow and Romer: Overview

- The combined Solow–Romer model
  - Nonrivalry of ideas results in long-run growth along a balanced growth path
  - Exhibits transition dynamics if economy is not on its balanced growth path
    - For short periods of time
      - Countries can grow at different rates (Japan and South Korea have grown faster than the United States for the last half century)
    - In the long run
      - Countries grow at the same rate

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6.5 Growth Accounting

- Growth accounting determines
  - The *sources of growth in an economy* and

how they may change over time

- Consider the following production function

$$Y_t = A_t K_t^{1/3} L_{yt}^{2/3}$$

- Total factor productivity (TFP): stock of ideas

Growth Accounting—1

- Apply growth rate rules to the production function
  - The growth rate of each input weighted by its exponent

$$g_{Yt} = g_{At} + \frac{1}{3} g_{Kt} + \frac{2}{3} g_{Lyt}$$

Growth rate of output

Growth rate of knowledge

Growth contribution from capital

Growth contribution from workers

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Growth Accounting—2

- Adjust growth rates by labor hours:

$$\underbrace{g_{Yt} - g_{Lt}}_{\text{growth of } Y/L} = \underbrace{\tfrac{1}{3}(g_{Kt} - g_{Lt})}_{\text{contribution from } K/L} + \underbrace{\tfrac{2}{3}(g_{Lyt} - g_{Lt})}_{\text{labor composition}} + \underbrace{g_{At}}_{\text{TFP growth}}.$$

- TFP growth is often called "the residual"

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Productivity in the United States

- From 1973–1995:
  - Output in the United States grew half as fast as 1948–1973 Post WW2
  - Known as the productivity slowdown
- From 1995–2007:
  - Output grew nearly as rapidly as before 1973–1995
  - Known as the new economy

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Growth Accounting in the United States

TABLE 6.2

Growth Accounting for the United States

Oil shock

	1948–2014	1948–1973	1973–1995	1995–2007	2007–2014
Output per hour, Y/L	2.4	3.3	1.6	2.8	1.4
Contribution of K/L	0.9	1.0	0.8	1.1	0.6
Contribution of labor composition	0.2	0.2	0.2	0.2	0.3
Contribution of TFP, A	1.3	2.1	0.6	1.5	0.5

The table shows the average annual growth rate (in percent) for different variables.
Source: Bureau of Labor Statistics, *Multifactor Productivity Trends*.

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6.6 Concluding Our Study of Long-Run Growth

6.6 Concluding Our Study of Long-Run Growth

- Institutions (property rights, laws) play an important role in economic growth.
- The Solow and Romer models
  - Provide a basis for analyzing differences in growth across countries
  - Do not answer why investment rates and TFP differ across countries

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Case Study: Institutions, Ideas, and Charter Cities

- Institutions
  - Nonrivalrous
  - May help the poorest countries
- Charter Cities
  - Economy agrees to set the rules by which a new city is administered
  - Hong Kong

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6.7 A Postscript on Solow and Romer

6.7 A Postscript on Solow and Romer

- The Solow and Romer models have made many additional valuable contributions:
  - The modern theory of monopolistic competition
  - New understanding of exogenous technological progress

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6.9 Appendix: Combining Solow and Romer (Algebraically)

6.9 Appendix: Combining Solow and Romer (Algebraically)

- The combined model is set up by adding capital into the Romer model production function.
- The combined model features five equations and five unknowns
- The five unknowns:
  - Output Y_t
  - Capital K_t
  - Knowledge A_t
  - Workers L_{yt}
  - Researchers L_{at}

The five equations:

$$Y_t = A_t K_t^{1/3} L_{yt}^{2/3},$$

$$\Delta K_{t+1} = \bar{s} Y_t - \bar{d} K_t,$$

$$\Delta A_{t+1} = \bar{z} A_t L_{at},$$

$$L_{yt} + L_{at} = \bar{L},$$

$$L_{at} = \bar{\ell} \bar{L}.$$

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Setting Up the Combined Model—1

- The production function for output

$$Y_t = A_t K_t^{1/3} L_{yt}^{2/3}$$

- The accumulation of capital over time

$$\Delta K_{t+1} = \bar{s} Y_t - \bar{d} K_t$$

- Ideas

$$\Delta A_{t+1} = \bar{z} A_t L_{at}$$

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Setting Up the Combined Model—2

- The numbers of workers and researchers sum to equal the total population

$$L_{yt} + L_{at} = \bar{L}$$

- Our assumption that a constant fraction of the population works as researchers

$$L_{at} = \bar{\ell}\bar{L}$$

Setting Up the Combined Model—3

- The production function
  - constant returns to scale in objects
  - increasing returns in ideas and objects together

$$g_{Yt} = g_{At} + \frac{1}{3} g_{Kt} + \frac{2}{3} g_{Lyt}$$

- The change in the capital stock is investment minus depreciation

$$g_{At} = \frac{\Delta A_{t+1}}{A_t} = \bar{z} L_{at} = \bar{z} \bar{\ell} \bar{L}$$

- Researchers are used to produce new ideas

$$g_{Kt} = \frac{\Delta K_{t+1}}{K_t} = \bar{s} \frac{Y_t}{K_t} - \bar{d}$$

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Setting Up the Combined Model—4

- The combined model will result in:
  - A balanced growth path
    - Since A_t increases continually over time
  - Transition dynamics
  - Long-run growth:
    - To be on a balanced growth path, output, capital, and stock of ideas all must grow at constant rates

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Setting Up the Combined Model—5

- Start with the production function for output and apply the rules for computing growth rates:

$$g_{Yt} = g_{At} + \frac{1}{3} g_{Kt} + \frac{2}{3} g_{Lyt}$$

Growth rate of output

Growth rate of knowledge

Growth contribution from capital

Growth contribution from workers

$$g_{Yt} \equiv \Delta Y_{t+1}/Y_t$$

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Setting Up the Combined Model—6

- To solve for the growth rate of knowledge
  - Divide the production function for new ideas by A_t

$$g_{At} = \frac{\Delta A_{t+1}}{A_t} = \bar{z}L_{at} = \bar{z}\bar{\ell}\bar{L}$$

- To solve for the growth rate of capital
  - Divide the capital accumulation equation by K_t

$$g_{Kt} = \frac{\Delta K_{t+1}}{K_t} = \bar{s}\frac{Y_t}{K_t} - \bar{d}$$

Setting Up the Combined Model—7

- Therefore:

Constant along a balanced growth path

$$g_{Kt} = \frac{\Delta K_{t+1}}{K_t} = \bar{s}\frac{Y_t}{\underbrace{K_t}_{\text{Must be constant as well}}} - \bar{d}$$

- The asterisk (*) means these variables are evaluated along a balanced growth path

$$g_K^* = g_Y^*$$

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Setting Up the Combined Model—8

- The growth rate in the number of workers is zero
  - The number of workers is a constant fraction of the population
  - The population itself is constant
- Therefore:

$$g_{Lyt} = 0$$

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Setting Up the Combined Model—9

- Plug the results into:

$$g_{Yt} = g_{At} + \frac{1}{3} g_{Kt} + \frac{2}{3} g_{Lyt}$$

$$\boxed{\begin{gathered} g_{At} = \bar{z}\bar{l}\bar{L} \equiv \bar{g} \\ g_K^* = g_Y^* \\ g_{Lyt} = 0 \end{gathered}}$$

$$\downarrow$$

$$g_Y^* = \bar{g} + \frac{1}{3} g_Y^* + \frac{2}{3} \times 0$$

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Setting Up the Combined Model—10

- Solve for the growth rate of output

$$g_Y^* = \bar{g} + \frac{1}{3} g_Y^* + \frac{2}{3} \times 0$$

$$\downarrow$$

$$g_Y^* = \frac{3}{2}\overline{g} = \frac{3}{2}\overline{z}\,\overline{\ell}\,\overline{L}$$

- For the long-run combined model, this equation pins down
  - The growth rate of output
  - The growth rate of output per person

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Setting Up the Combined Model—11

- The growth rate of output is even larger in the combined model than in the Romer model
- Output is higher in this model because
  - Ideas have a direct and an indirect effect
  - Increasing productivity raises output because
    - productivity has increased
    - higher productivity results in a higher capital stock.

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Output per Person—2

- The capital to output ratio is proportional to the investment rate along a balanced growth path

$$\frac{K_t^*}{Y_t^*} = \frac{\bar{s}}{g_y^* + \bar{d}}$$

- This solution can be substituted back into the production function and solved to get:

$$y_t^* \equiv \frac{Y_t^*}{\bar{L}} = \left(\frac{\bar{s}}{g_y^* + \bar{d}}\right)^{1/2} A_t^{*\,3/2}\,(1 - \bar{\ell})$$

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Setting Up the Combined Model—12

$$y_t^* \equiv \frac{Y_t^*}{\bar{L}} = \left(\frac{\bar{s}}{g_y^* + \bar{d}}\right)^{1/2} A_t^{*\,3/2}\,(1 - \bar{\ell})$$

- Growth in A_t
  - Leads to sustained growth in output per person along a balanced growth path
- Output y_t
  - Depends on the square root of the investment rate
- A higher investment rate
  - Raises the level of output per person along the balanced growth path.

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Transition Dynamics

- The Solow model and the combined model both have diminishing returns to capital.
- Transition dynamics applies in both models.
- In the combined model:
  - The further below its balanced growth path an economy is, the faster the economy will grow
  - The further above its balanced growth path an economy is, the slower the economy will grow

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Setting Up the Combined Model—13

- A permanent increase in the investment rate in the combined model implies:
  - The balanced growth path of income is higher (parallel shift).
  - Current income is unchanged.
    - the economy is now below the new balanced growth path
  - The growth rate of income per capita is immediately higher.
    - the slope of the output path is steeper than the balanced growth path

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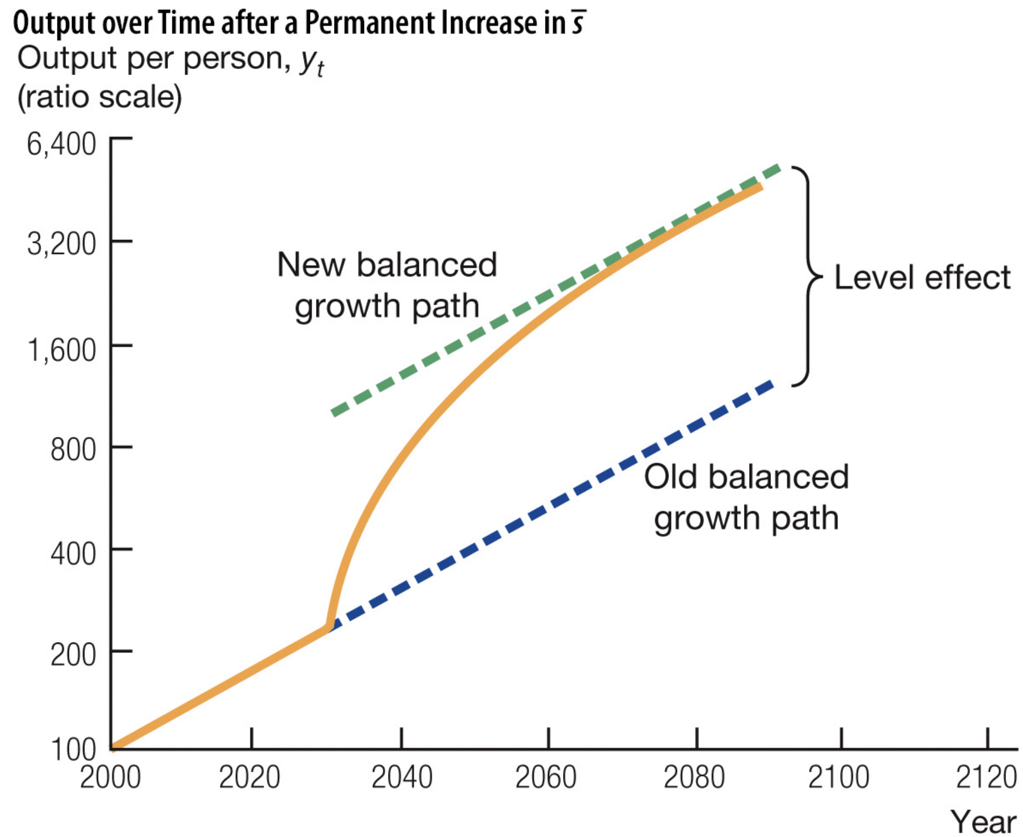
Setting Up the Combined Model—14

- Changes in any parameter result in transition dynamics
  - Long-run growth through ideas
  - Explains differences in growth rates across countries



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