EC3333 Tutorial 6 Suggested Answers Part 1

*For this module, unless otherwise stated, the par value of the bond is assumed to be $1000.

1. A 30-year bond with a face value of $1000 has a coupon rate of 5.5%, with semiannual payments.

   a. What is the coupon payment for this bond?

   b. Draw the cash flows for the bond on a timeline.

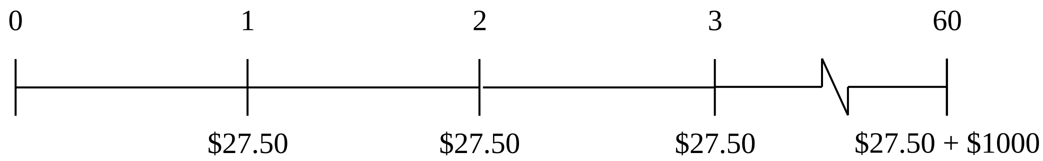
a.

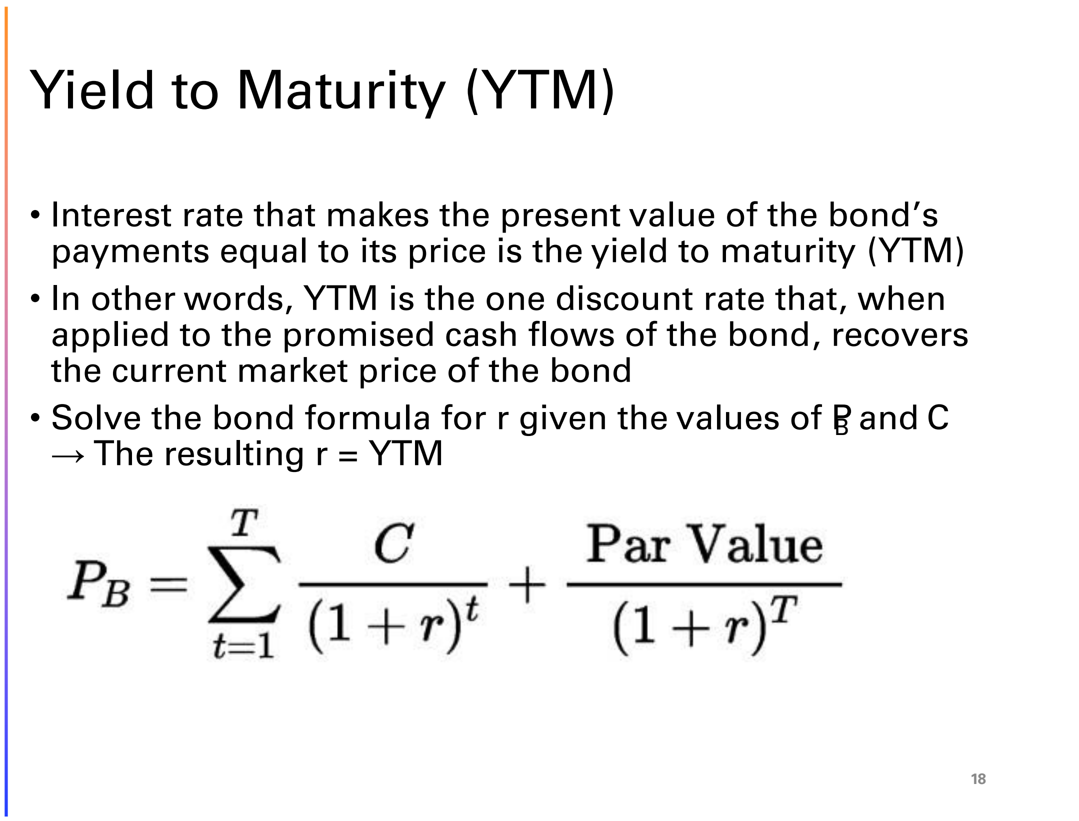
The coupon payment is:

$$CPN = \frac{\text{Coupon Rate} \times \text{Face Value}}{\text{Number of Coupons per Year}} = \frac{0.055 \times \$1000}{2} = \$27.50.$$

b.

The timeline for the cash flows for this bond is (the unit of time on this timeline is six-month periods):





Bond Pricing

Example 14.3 Bond Pricing
(from adopted text, Bodie, Kane and Marcus, Investments, McGraw Hill, 12e)

- Suppose an 8% coupon, 30 year bond is selling for \$1,276.76. What is its yield to maturity?

$$1,276.76 = \sum_{t=1}^{60} \frac{40}{(1+r)^t} + \frac{1000}{(1+r)^{60}}$$

- r = 3% per half year
- Bond equivalent yield = 6% annually (in Annual Percentage Rate or APR, which does not account for compound interest)
- EAR = $((1.03)^2) - 1$ = 6.09% annually (Effective Annual Yield or EAR, which accounts for compound interest)

19

2. Suppose a 10-year, \$1000 bond with an 8% coupon rate and semiannual coupons is trading for a price of \$1034.74.
    a. What is the bond's yield to maturity (expressed as an APR with semiannual compounding)? (You can use the Excel solver function to do this.)
    b. If the bond's yield to maturity changes to 9% APR, what will the bond's price be?

a.

$$P_B = \sum_{t=1}^{T} \frac{C}{(1+r)^t} + \frac{\text{Par Value}}{(1+r)^T}$$

or

$$P_B = \underbrace{\frac{C}{r}\left(1 - \frac{1}{(1+r)^T}\right)}_{\text{The Annuity Formula}} + \frac{\text{Par Value}}{(1+r)^T}$$

Yield to maturity: Using the Excel solver function, enter the formula for the bond price and solve for r, with T=20, Par Value = \$1000, C=40, and set the bond price = \$1034.74.

This results in: YTM per ½ year = r = 3.75%.

YTM (expressed as an APR with semiannual compounding) = 3.75% × 2 = 7.50%.

b. YTM per ½ year = r = 4.5% per 6 months, the new price is \$934.96

3. Suppose the current zero-coupon yield curve for risk-free bonds is as follows:

Maturity (years)	1	2	3	4	5
YTM	5.00%	5.50%	5.75%	5.95%	6.05%

a. What is the price per $1000 face value of a two-year, zero-coupon, risk-free bond?

b. What is the price per $1000 face value of a four-year, zero-coupon, risk-free bond?

c. What is the risk-free interest rate for a five-year maturity?

$$P_B = \sum_{t=1}^{T} \frac{C}{(1+r)^t} + \frac{\text{Par Value}}{(1+r)^T}$$

$$P_{zero-cpn} = \frac{\text{Par Value}}{(1+r)^T}$$

a. $P = 1000/(1.055)^2 = \$898.45$

b. $P = 1000/(1.0595)^4 = \$793.59$

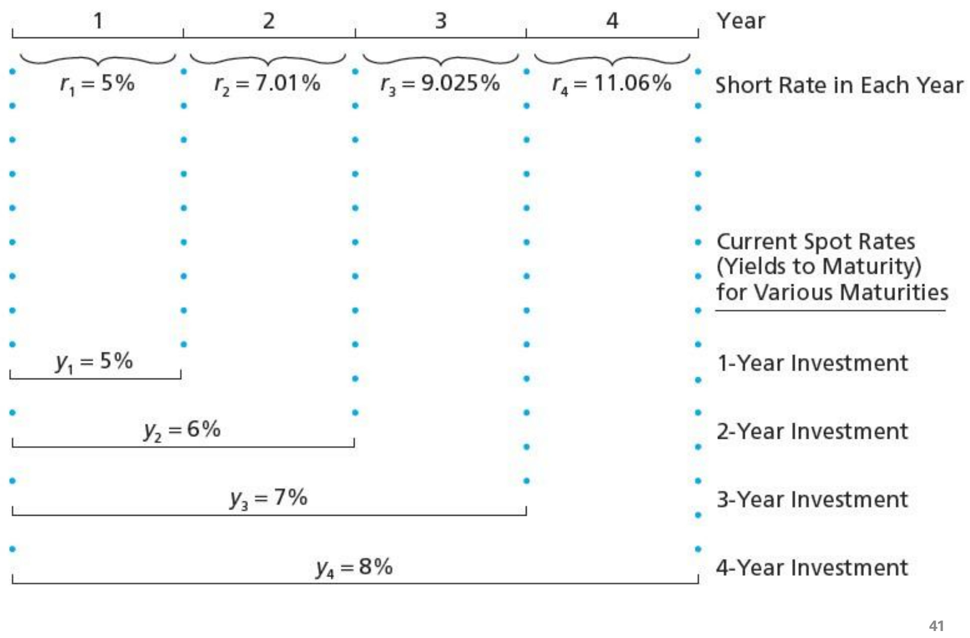
c. 6.05% per year.

The Yield Curve

- The yield curve is a graph that depicts the relationship between YTM and time to maturity
  - A graph of the term structure
- Information on expected future short-term rates can be implied from the yield curve
- Spot rate: YTM on zero-coupon bonds
  - The rate that prevails today for a given maturity
- Short rate: The rate for a given time interval or maturity (e.g. one year) at different points in time

40

Figure 15.2 Short Rates versus Spot Rates
(from adopted text, Bodie, Kane and Marcus, Investments, McGraw Hill, 12e)



4. The following table summarizes prices of various default-free, zero-coupon bonds (expressed as a percentage of face value):

Maturity (years)	**1**	**2**	**3**	**4**	**5**
Price (per $100 face value)	$95.51	$91.05	$86.38	$81.65	$76.51

a. Compute the yield to maturity for each bond.
b. Plot the zero-coupon yield curve (for the first five years).
c. Is the yield curve upward sloping, downward sloping, or flat?

a.

$$P_{zero-cpn} = \frac{\text{Par Value}}{(1+r)^T}$$

Use the following equation.

$$1 + \text{YTM}_T = \left(\frac{\text{FV}_T}{P}\right)^{1/T}$$

$$1 + \text{YTM}_1 = \left(\frac{100}{95.51}\right)^{1/1} \Rightarrow \text{YTM}_1 = 4.70\%$$

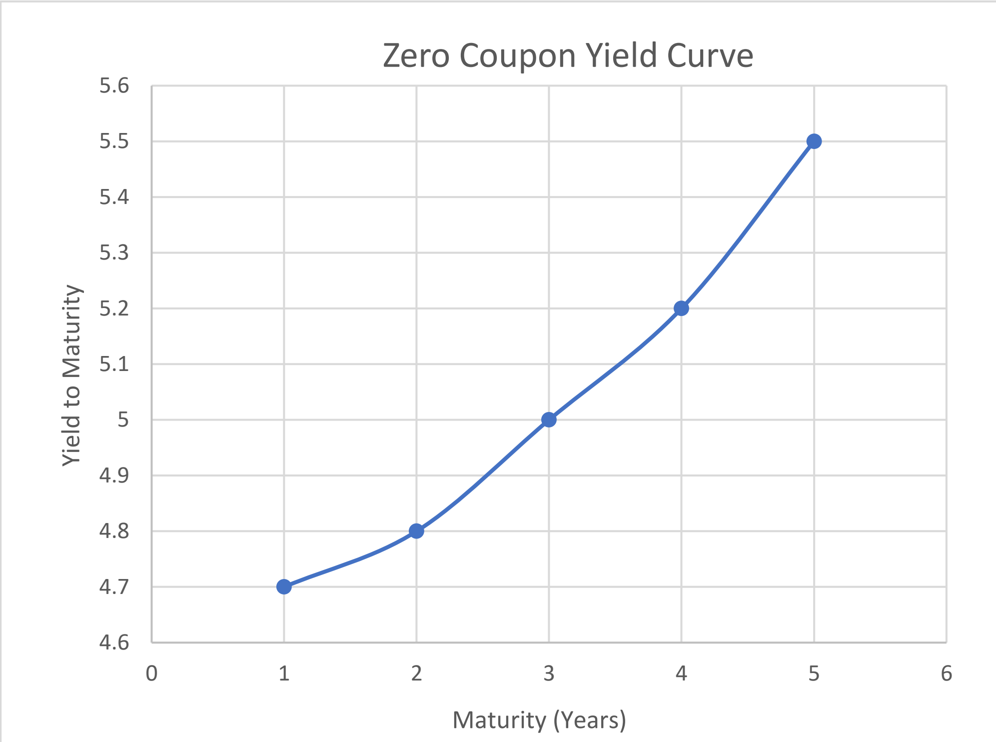
$$1 + \text{YTM}_1 = \left(\frac{100}{91.05}\right)^{1/2} \Rightarrow \text{YTM}_1 = 4.80\%$$

$$1 + \text{YTM}_3 = \left(\frac{100}{86.38}\right)^{1/3} \Rightarrow \text{YTM}_3 = 5.00\%$$

$$1 + \text{YTM}_4 = \left(\frac{100}{81.65}\right)^{1/4} \Rightarrow \text{YTM}_4 = 5.20\%$$

$$1 + \text{YTM}_5 = \left(\frac{100}{76.51}\right)^{1/5} \Rightarrow \text{YTM}_5 = 5.50\%$$

b. The yield curve is as shown below.



c.

The yield curve is upward sloping.

The Yield Curve & Discount Rates

- The term structure canbe used to compute thepresent and future values of a risk-free cash flow over different investment horizons

$$PV = \frac{C_n}{(1+r_n)^n}$$

- Present Value of a Cash Flow Stream Using a Term Structure of Discount Rates

$$PV = \frac{C_1}{1+r_1} + \frac{C_2}{(1+r_2)^2} + \cdots + \frac{C_N}{(1+r_N)^N} = \sum_{n=1}^{N} \frac{C_N}{(1+r_n)^n}$$

38

Pricing a bond

- Compute the present value of a riskfree three-year annuity of $500 per year, given the following yield curve:

Zero Coupon Treasury Rates	
Term (Years)	**Rate**
1	0.261%
2	0.723%
3	1.244%

- Each cash flow must be discounted by the corresponding interest rate:

$$PV = \frac{\$500}{1.00261} + \frac{\$500}{1.00723^2} + \frac{\$500}{1.01244^3} = \$1,473.34$$

39