ME 639: Introduction to Robotics
IIT Gandhinagar

Assignment 2

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1 Solution to Task 1

Rotation matrix R_0^1 is given by:

$$R_0^1 = \begin{bmatrix} cos\theta & -sin\theta & 0 \\ sin\theta & cos\theta & 0 \\ 0 & 0 & 1 \end{bmatrix} \tag{1}$$

Let us denote the three columns of the R_0^1 matrix by C_1, C_2, C_3 respectively. For showing orthogonality, we need to prove that the dot product of any two column vectors of R_0^1 is equal to 0(scalar).

$$C_1.C_2 = \begin{bmatrix} cos\theta \\ s\theta \\ 0 \end{bmatrix} . \begin{bmatrix} -sin\theta \\ cos\theta \\ 0 \end{bmatrix} = -sin\theta cos\theta + sin\theta cos\theta + 0 = 0 \tag{2}$$

$$C_2.C_3 = \begin{bmatrix} -sin\theta \\ cos\theta \\ 0 \end{bmatrix} . \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix} = 0 + 0 + 0 = 0 \tag{3}$$

$$C_1.C_3 = \begin{bmatrix} cos\theta \\ sin\theta \\ 0 \end{bmatrix} . \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix} = 0 + 0 + 0 = 0 \tag{4}$$

2 Solution to Task 2

Determinant of R_0^1 is given by

$$R_0^1 = \begin{vmatrix} cos\theta & -sin\theta & 0 \\ sin\theta & cos\theta & 0 \\ 0 & 0 & 1 \end{vmatrix} = (cos^2\theta + sin^2\theta).1 = 1 \tag{1}$$

3 Task 3

Done

4 Task 4

Done

5 Task 5

About skew symmetric matrices $S(a)$, we know the following interesting properties:

- Linearity: For any pair of vectors $\vec{p}, \vec{q} \in \mathbb{R}^3$ and any pair of scalars $\alpha, \beta \in \mathbb{R}$,

$$S(\alpha\vec{p} + \beta\vec{q}) = \alpha S(\vec{p}) + \beta S(\vec{q}). \tag{1}$$

- Cross product property: For any pair of vectors $\vec{p}, \vec{q} \in \mathbb{R}^3$,

$$S(\vec{p})\vec{q} = \vec{p} \times \vec{q}. \tag{2}$$

For $R \in SO(3), \vec{m}, \vec{n} \in \mathbb{R}^3$,

$$R(\vec{m} \times \vec{n}) = R(\vec{m}) \times R(\vec{n}), \tag{3}$$

only true as R is orthogonal. So, for $R \in SO(3), \vec{a}, \vec{b} \in \mathbb{R}^3$

$$RS(\vec{a})R^T\vec{b} = R(\vec{a} \times R^T\vec{b}) = (R\vec{a}) \times (RR^T\vec{b}) = (R\vec{a} \times \vec{b}) = S(R\vec{a})\vec{b} \tag{4}$$

Thus, we have, $RS(a)R^T = S(Ra)$.

6 Task 6

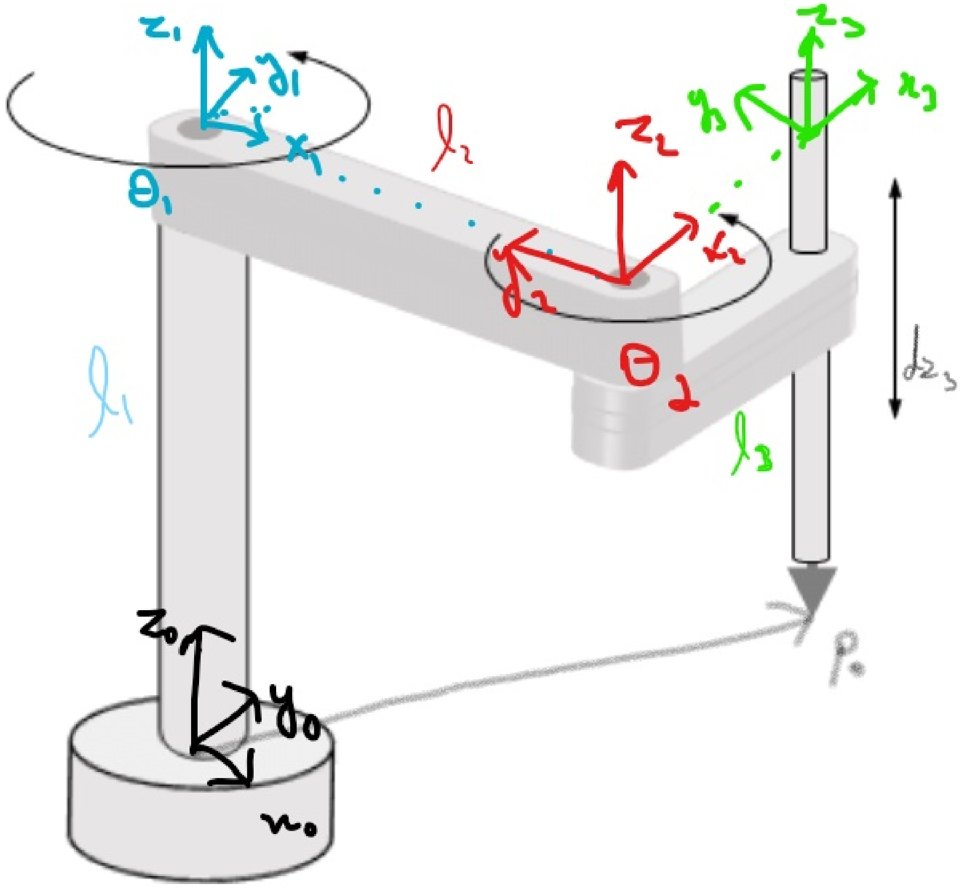

Figure 1: RRP SCARA Configuration

In the RRP SCARA manipulator configuration shown in Figure 1, we have:

$$\begin{bmatrix} P_0 \\ 1 \end{bmatrix} = \begin{bmatrix} R_0^1 & d_0^1 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} R_1^2 & d_1^2 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} R_2^3 & d_2^3 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} P_3 \\ 1 \end{bmatrix} \tag{1}$$

R_0^1 involves positive i.e. anticlockwise rotation by angle θ_1 about the z_0 axis while d_0^1 involves positive translation of l_1 along the z_0 axis.

$$\begin{bmatrix} R_0^1 & d_0^1 \\ 0 & 1 \end{bmatrix} = \begin{bmatrix} cos\theta_1 & -sin\theta_1 & 0 & 0 \\ sin\theta_1 & cos\theta_1 & 0 & 0 \\ 0 & 0 & 1 & l_1 \\ 0 & 0 & 0 & 1 \end{bmatrix} \tag{2}$$

R_1^2 involves positive i.e. anticlockwise rotation by angle $\pi/2$ about the z_1 axis while d_1^2 involves positive translation of l_2 along the x_1 axis.

$$\begin{bmatrix} R_1^2 & d_1^2 \\ 0 & 1 \end{bmatrix} = \begin{bmatrix} cos\pi/2 & -sin\pi/2 & 0 & l_2 \\ sin\pi/2 & cos\pi/2 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} 0 & -1 & 0 & l_2 \\ 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \tag{3}$$

R_2^3 involves positive i.e. anticlockwise rotation by angle θ_2 about the z_2 axis while d_2^3 involves positive translation of l_3 along the x_2 axis.

$$\begin{bmatrix} R_2^3 & d_2^3 \\ 0 & 1 \end{bmatrix} = \begin{bmatrix} cos\theta_2 & -sin\theta_2 & 0 & l_3 \\ sin\theta_2 & cos\theta_2 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \tag{4}$$

P_3 is located at the coordinates of $(0, 0, -dz_3)$ in the (x_3, y_3, z_3) domain.

$$\begin{bmatrix} P_3 \\ 1 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ -dz_3 \\ 1 \end{bmatrix} \tag{5}$$

Now calculating the product, we obtain,

$$\begin{bmatrix} P_0 \\ 1 \end{bmatrix} = \begin{bmatrix} -sin(\theta_1+\theta_2) & -cos(\theta_1+\theta_2) & 0 & l_2cos(\theta_1) - l_3sin(\theta_1) \\ cos(\theta_1+\theta_2) & -sin(\theta_1+\theta_2) & 0 & l_3cos(\theta_1) + l_2sin(\theta_1) \\ 0 & 0 & 1 & l_1 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 0 \\ 0 \\ -dz_3 \\ 1 \end{bmatrix} \tag{6}$$

Hence we have P_0

$$P_0 = (l_2cos(\theta_1) - l_3sin(\theta_1), l_3cos(\theta_1) + l_2sin(\theta_1), l_1 - dz_3) \tag{7}$$

7 Task 7

Please refer to Colab file, RRP SCARA Manipulator Tab for python code implementation.

8 Task 8

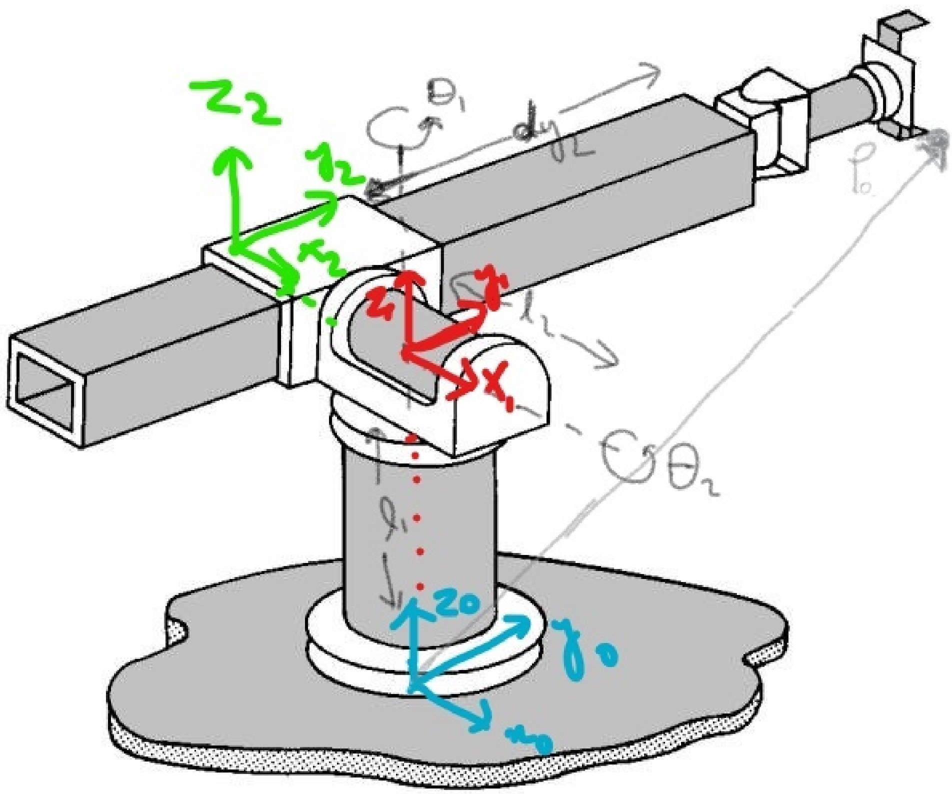

Figure 2: RRP Stanford Configuration

In the RRP Stanford manipulator configuration shown in Figure 2, we have:

$$\begin{bmatrix} P_0 \\ 1 \end{bmatrix} = \begin{bmatrix} R_0^1 & d_0^1 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} R_1^2 & d_1^2 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} P_2 \\ 1 \end{bmatrix} \tag{1}$$

R_0^1 involves positive i.e. anticlockwise rotation by angle θ_1 about the z_0 axis while d_0^1 involves positive translation of l_1 along the z_0 axis.

$$\begin{bmatrix} R_0^1 & d_0^1 \\ 0 & 1 \end{bmatrix} = \begin{bmatrix} cos\theta_1 & -sin\theta_1 & 0 & 0 \\ sin\theta_1 & cos\theta_1 & 0 & 0 \\ 0 & 0 & 1 & l_1 \\ 0 & 0 & 0 & 1 \end{bmatrix} \tag{2}$$

R_1^2 involves positive i.e. anticlockwise rotation by angle θ_2 about the x_1 axis while d_1^2 involves negative translation of l_2 along the x_1 axis.

$$\begin{bmatrix} R_1^2 & d_1^2 \\ 0 & 1 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 & -l_2 \\ 0 & cos\theta_2 & -sin\theta_2 & 0 \\ 0 & sin\theta_2 & cos\theta_2 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \tag{3}$$

P_2 is located at the coordinates of $(0, dy_2, 0)$ in the (x_2, y_2, z_2) domain.

$$\begin{bmatrix} P_2 \\ 1 \end{bmatrix} = \begin{bmatrix} 0 \\ dy_2 \\ 0 \\ 1 \end{bmatrix} \tag{4}$$

Now calculating the product, we obtain,

$$\begin{bmatrix} P_0 \\ 1 \end{bmatrix} = \begin{bmatrix} cos(\theta_1) & -cos(\theta_2)sin(\theta_1) & sin(\theta_1)sin(\theta_2) & -l_2cos(\theta_1) \\ sin(\theta_1) & cos(\theta_1)cos(\theta_2) & -cos(\theta_1)sin(\theta_2) & -l_2sin(\theta_1) \\ 0 & sin(\theta_2) & cos(\theta_2) & l_1 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 0 \\ dy_2 \\ 0 \\ 1 \end{bmatrix} \tag{5}$$

Hence we have P_0

$$P_0 = (-l_2cos(\theta_1) - dy_2cos(\theta_2)sin(\theta_1), dy_2cos(\theta_1)cos(\theta_2) - l_2sin(\theta_1), l_1 + dy_2sin(\theta_2)) \tag{6}$$

Please refer to Colab file, RRP Stanford Manipulator Tab for python code implementation.

9 Task 9

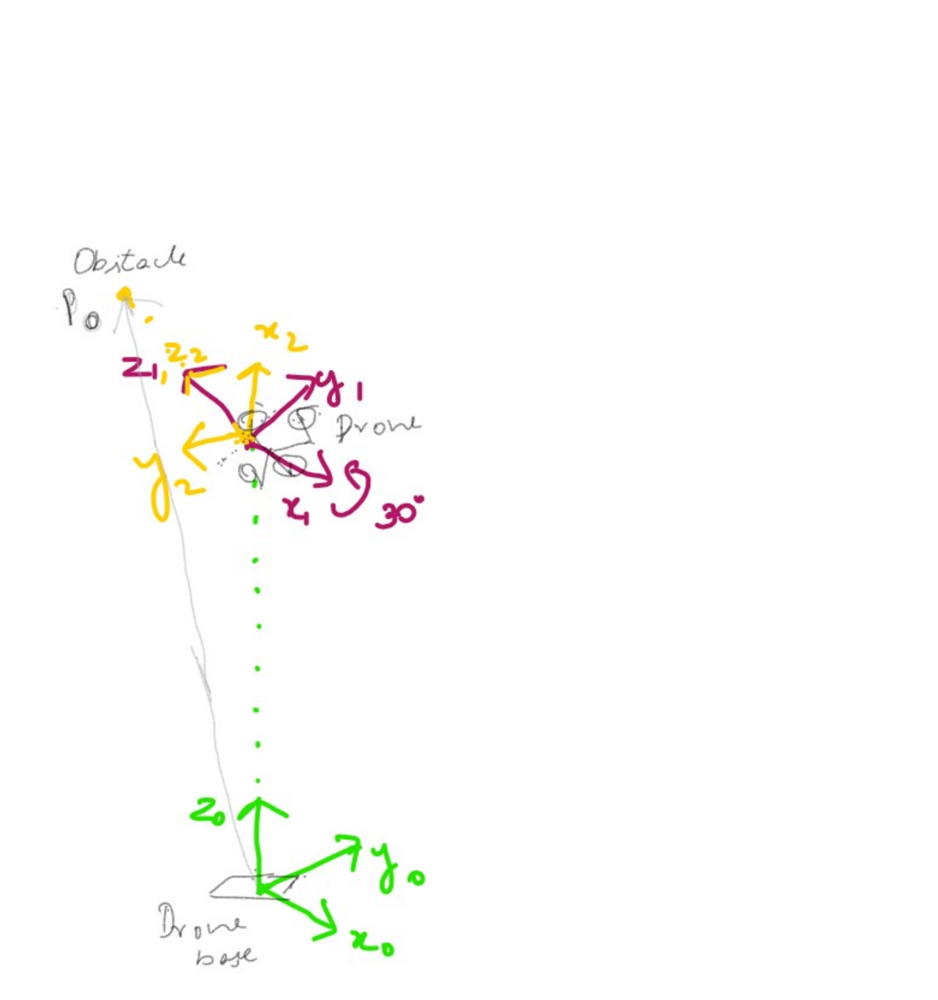

Figure 3: Drone obstacle tracking

In the Drone obstacle tracking diagram shown in Figure 3, we have:

$$\begin{bmatrix} P_0 \\ 1 \end{bmatrix} = \begin{bmatrix} R_0^1 & d_0^1 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} R_1^2 & d_1^2 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} R_2^3 & d_2^3 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} P_3 \\ 1 \end{bmatrix} \tag{1}$$

R_0^1 involves positive translation of $10m$ along the z_0 axis.

$$\begin{bmatrix} R_0^1 & d_0^1 \\ 0 & 1 \end{bmatrix} = \begin{bmatrix} 1 & 1 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 10 \\ 0 & 0 & 0 & 1 \end{bmatrix} \tag{2}$$

R_1^2 involves positive i.e. anticlockwise rotation by angle $30°$ about the x_1 axis..

$$\begin{bmatrix} R_1^2 & d_1^2 \\ 0 & 1 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & cos30° & -sin30° & 0 \\ 0 & sin30° & cos30° & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 0.866 & -0.5 & 0 \\ 0 & 0.5 & 0.866 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \tag{3}$$

R_2^3 involves positive i.e. anticlockwise rotation by angle $60°$ about the z_2 axis.

$$\begin{bmatrix} R_2^3 & d_2^3 \\ 0 & 1 \end{bmatrix} = \begin{bmatrix} cos60° & -sin60° & 0 & 0 \\ sin60° & cos60° & 0 & 0 \\ 0 & 0 & 1 & 3 \\ 0 & 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} 0.5 & -0.866 & 0 & 0 \\ 0.866 & 0.5 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \tag{4}$$

P_3 is located at the coordinates of $(0, 0, 3m)$ in the (x_3, y_3, z_3) domain.

$$\begin{bmatrix} P_3 \\ 1 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 3m \\ 1 \end{bmatrix} \tag{5}$$

Now calculating the product, we obtain,

$$\begin{bmatrix} P_0 \\ 1 \end{bmatrix} = \begin{bmatrix} 1.2500 & -0.4330 & -0.5000 & 0 \\ 0.7500 & 0.4330 & -0.5000 & 0 \\ 0.4330 & 0.2500 & 0.8660 & 10.0000 \\ 0 & 0 & 0 & 1.0000 \end{bmatrix} \begin{bmatrix} 0 \\ 0 \\ 3m \\ 1 \end{bmatrix} \tag{6}$$

Hence we have P_0

$$P_0 = (-1.5m, -1.5m, 12.598m) \tag{7}$$

10 Task 10

10.1 Gearboxes used in robotics applications

10.1.1 Planetary Gear Trains

Planetary gear trains have a high power density and a coaxial configuration.They are either used in a bulky staged-gearhead configuration for greater reliability and adaptibility in heavy industrial lifting applications for effective torque transmission and or used as a single compact gearhead with high ratios for compact transmission for applications involving speed transmission.

10.1.2 Cycloid drive

Used primarily in high torsion and robust applications like CNC machines.An eccentric input motion causes a cycloidal motion of a large planet wheel which is again reconverted to a smaller circular motion using a bearing.

10.1.3 Harmonic drive

Mainly used in zero backlash applications, it is a strain wave gear with a deforming thin cylindrical cup with teeth that serves as output, which engages with fixed solid circular ring with internal gear teeth,while it is deformed by a rotating elliptical plug.

10.2 Gearboxes in drone applications

: It is highly unlikely to see gearboxes used in drone applications as the gearboxes add to the bulk of the drone and the motors can directly rotate the propellers with sufficient speed control for regular flight applications.

11 Task 11

In the RRP SCARA configuration in Figure 1, joints 1,2 are revolute and joint 3 is prismatic.Hence we will have a 6×4 Jacobian matrix given by,

$$J = \begin{bmatrix} z_0 \times (o_3 - o_0) & z_1 \times (o_3 - o_1) & z_2 & 0 \\ z_0 & z_1 & 0 & z_3 \end{bmatrix} \tag{1}$$

On evaluating, we obtain,(J_v is obtained by differentiating the last column of the transformation and J_w is obtained from the rotary column in each transformation).

$$J = \begin{bmatrix} -l_2 sin\theta_1 - l_3 cos(\theta_1) & 0 & 0 & 0 \\ -l_3 sin\theta_1 - l_2 cos(\theta_1) & 0 & 0 & 0 \\ 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 1 & 1 & 0 & -1 \end{bmatrix} \tag{2}$$

12 Task 12

Please refer to Colab file, RRP SCARA Manipulator Jacobian Tab for python code implementation.

13 Task 13

Consider 3-link RRR manipulator, with link angles $\theta_1, \theta_2, \theta_3$, respectively, between base,link 1,link 2 and link 3.Consider link lengths l_1, l_2, l_3, respectively.The three joints are all revolute. Hence we have a 6×4 Jacobian matrix given by,

$$J = \begin{bmatrix} z_0 \times (o_4 - o_0) & z_1 \times (o_4 - o_1) & z_2 \times (o_4 - o_2) & 0 \\ z_0 & z_1 & z_2 & z_3 \end{bmatrix} \tag{1}$$

On evaluating, we obtain,

$$J = \begin{bmatrix} -l_1 s\theta_1 - l_2 s(\theta_1 + \theta_2) - l_3 s(\theta_1 + \theta_2 + \theta_3) & -l_2 s(\theta_1 + \theta_2) - l_3 s(\theta_1 + \theta_2 + \theta_3) & -l_3 s(\theta_1 + \theta_2 + \theta_3) \\ l_1 s\theta_1 + l_2 s(\theta_1 + \theta_2) + l3 s(\theta_1 + \theta_2 + \theta_3) & l_2 s(\theta_1 + \theta_2) + l_3 s(\theta_1 + \theta_2 + \theta_3) & l_3 s(\theta_1 + \theta_2 + \theta_3) \\ 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \\ 1 & 1 & 1 \end{bmatrix} \tag{2}$$

14 Task 14

Please refer to Colab file, RRR Jacobian Tab for python code implementation.