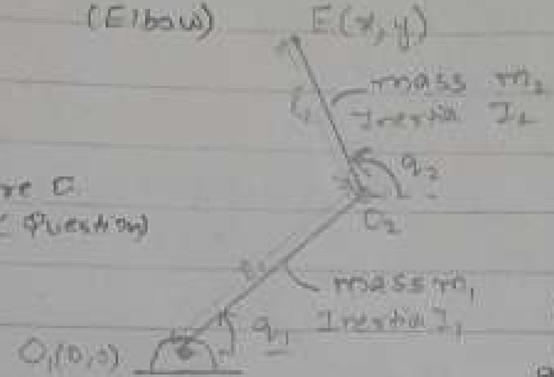
Miniproject-1 ME639

2R Manipulator

Figure 0
(Question)



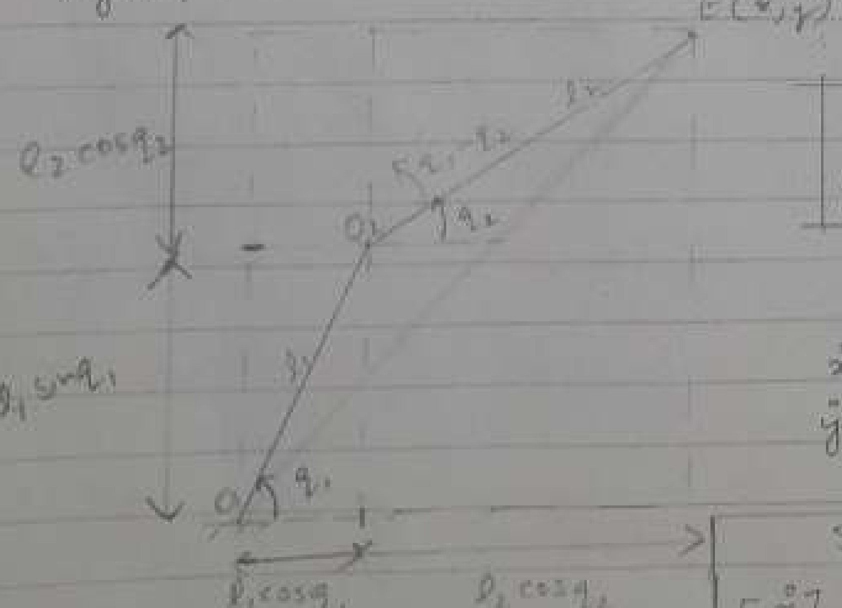
E: end effector
(x, y): end effector position
(q_1, q_2): joint angles of O_1, O_2 from absolute ref. respectively
Assume: O_1: Origin (0, 0)
Assume motors m_{O_1} & m_{O_2} connected at joints O_1 & O_2 respectively.

Presume m_{O_1} & m_{O_2} can have ability to control torques τ_1 & τ_2 applied at O_1 & O_2 or the positions of angles q_1 & q_2.

Task 1: (a) Tracking (x, y) using (q_1, q_2)
(b) Tracking (x, y) using (τ_1, τ_2) i.e. neutralizing the dynamics (or q_1, q_2)

Task 1(a): Solution

Figure 1



From Figure 1,

$$\boxed{\begin{aligned} x &= l_1\cos(q_1) + l_2\cos(q_2) \\ y &= l_1\sin(q_1) + l_2\sin(q_2) \end{aligned}} \quad (1)$$

Taking time derivatives,

$$\dot{x} = -l_1\dot{q}_1\sin(q_1) - l_2\dot{q}_2\sin(q_2)$$
$$\dot{y} = l_1\dot{q}_1\cos(q_1) + l_2\dot{q}_2\cos(q_2)$$

In matrix form, end effector velocity:

$$\boxed{\begin{bmatrix} \dot{x} \\ \dot{y} \end{bmatrix} = \begin{bmatrix} -l_1\sin(q_1) & -l_2\sin(q_2) \\ l_1\cos(q_1) & l_2\cos(q_2) \end{bmatrix} \begin{bmatrix} \dot{q}_1 \\ \dot{q}_2 \end{bmatrix}} \quad (2)$$

To solve task 1(a), we need to find q_1, q_2 in terms of (x, y)

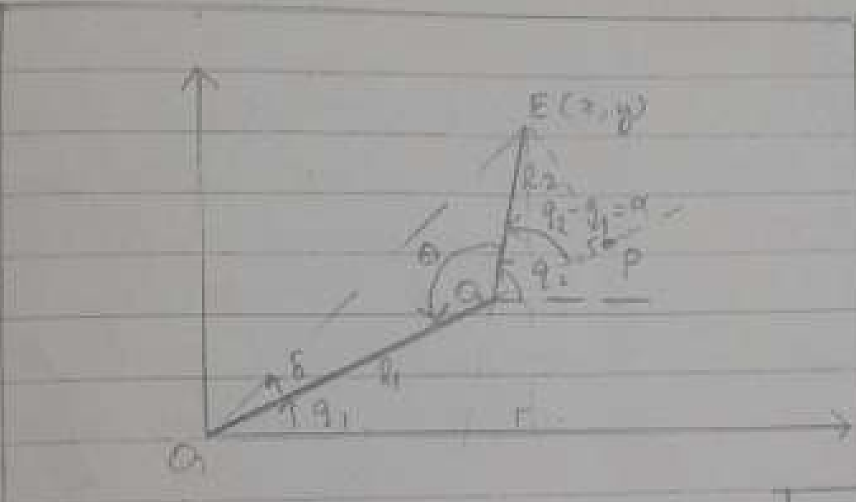


Figure 2

Consider Figure 2

From Pythagoras theorem,

length$(O_1, E) = \sqrt{x^2 + y^2}$

Applying cosine rule on angle θ in triangle $O_1 O_2 E$,

$$\cos(\theta) = \frac{l_1^2 + l_2^2 - (\sqrt{x^2+y^2})^2}{2 l_1 l_2}$$

$$\cos(\pi - \theta) = \cos(\alpha) = \cos(q_2 - q_1) = -\cos(\theta)$$

$$\Rightarrow \boxed{\cos(\alpha) = \frac{x^2 + y^2 - l_1^2 - l_2^2}{2 l_1 l_2}} \quad (3)$$

From $\Delta E O_1 P$, $\tan(\delta) = \frac{l_2 \sin(\alpha)}{l_1 + l_2 \cos(\alpha)}$

$$\tan(q_1 + \delta) = \left(\frac{y}{x}\right)$$

$$\therefore q_1 = \tan^{-1}\left(\frac{y}{x}\right) - \tan^{-1}\left(\frac{l_2 \sin(\alpha)}{l_1 + l_2 \cos(\alpha)}\right)$$

$$\boxed{\begin{aligned} q_1 &= \tan^{-1}\left(\frac{y}{x}\right) - \tan^{-1}\left(\frac{l_2 \sin(q_2 - q_1)}{l_1 + l_2 \cos(q_2 - q_1)}\right) \\ q_2 &= \alpha + q_1 \end{aligned}}$$

To track a given trajectory $E(x(t), y(t))$, the motors at O_1 and O_2 to be controlled in position control mode to achieve q_1 & q_2 angular position at each time step.

Task T2: Applying a constant force F_0 against a wall

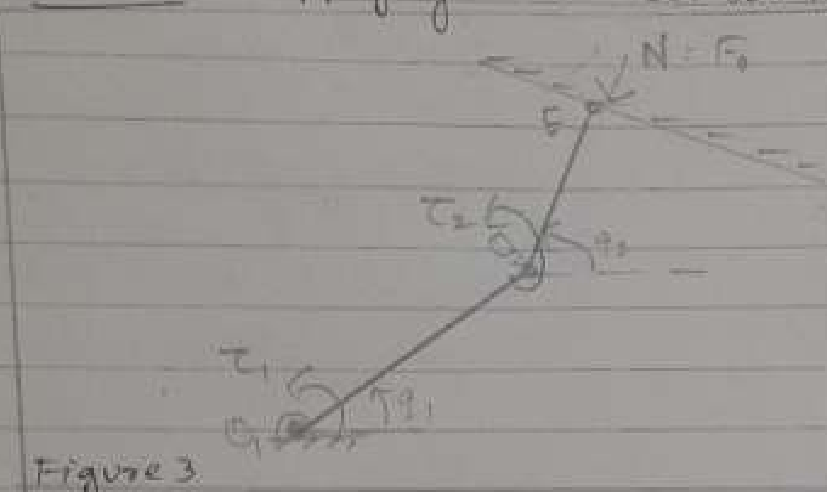


Figure 3

Constant force F_0 is applied through force control (torque control) mode of the motors M_1 at O_1 & M_2 at O_2 respectively.

T2 Solution: (We are assuming a horizontal plane here to ignore gravity & have a general overview of relevant forces on the F.B.D's)

Also, we break F_0 into F_x & F_y

F.B.D of entire manipulator

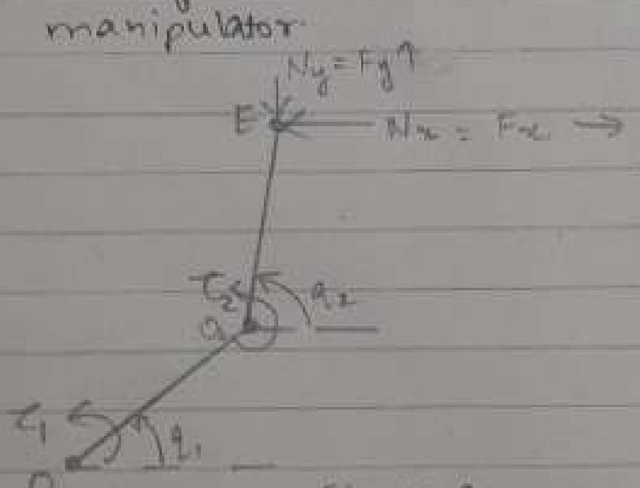


Figure 4.

F.B.D of link O_1O_2

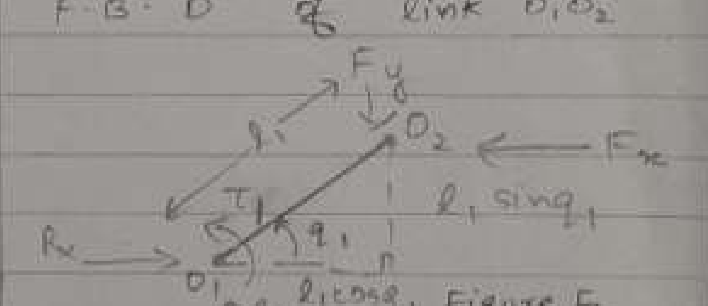


Figure 5

F.B.D of link O_2E

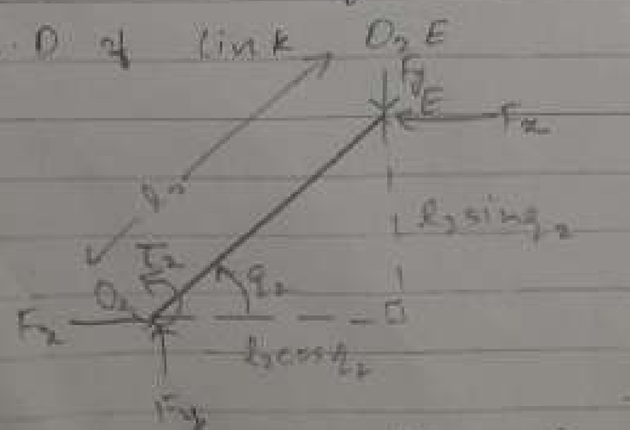


Figure 6

Conserving moments about O_1 & O_2 in figures 4, 5, 6,

$$\sum M_{O_1} = 0 \Rightarrow F_y l_1 \cos q_1 - F_x l_1 \sin q_1 = \tau_1$$

$$\sum M_{O_2} = 0 \Rightarrow F_y l_2 \cos q_2 - F_x l_2 \sin q_2 = \tau_2$$

(4)

To solve T2, we use (3) to position the end effector at the wall and (4) to apply necessary forces (torques)

Task T3 : Make the end effector act as a spring centred at $P(x_0, y_0)$ using a control scheme

Solution to task T3:

Step1: Model the dynamics of the ~~dow~~ 2R manipulator as torques (T1(b))

Step2: Cancel the torque dynamics

Step3: Insert the spring dynamics of $F_{spring_x} = -K(x - x_0)$
(A22)
& $F_{spring_y} = -K(y - y_0)$

at the end effector.

Step1 : We use Lagrange's equations to derive dynamics
(Task T1(b)) Lagrangian $L = K - V$
(Kinetic energy) (potential energy)

$$\boxed{\frac{d}{dt}\left(\frac{\partial L}{\partial \dot{q}_i}\right) - \frac{\partial L}{\partial q_i} = Q_i' \quad \text{— (5)}}$$

Generalized forces derived using principle of virtual work

$$K = \frac{1}{2} I_{O_1} \omega_{O_1}^2 + \frac{1}{2} I_{CG} \omega_{CG}^2 + \frac{1}{2} m_2 v_{C_2}^2$$

$$\Rightarrow K = \underbrace{\frac{1}{2}\left(\frac{1}{3} m_1 l_1^2\right)\dot{q}_1^2}_{\text{pure rotation about } O_1} + \underbrace{\frac{1}{2}\left(\left(\frac{1}{12}\right) m_2 l_2^2\right)\dot{q}_2^2}_{\text{Rotation of } O_2E \text{ about C.G.}} + \underbrace{\frac{1}{2} m_2 v_{C_2}^2}_{\text{translation of } O_2E}$$

$$v_{C_2}^2 = (l_1 \dot{q}_1)^2 + \left(\frac{l_2}{2}\dot{q}_2\right)^2 + 2 l_1 \dot{q}_1 \frac{l_2}{2} \dot{q}_2 \cos(q_2 - q_1)$$

$$V = m_1 g \frac{l_1}{2} \sin(q_1) + m_2 g \left(l_1 \sin(q_1) + \frac{l_2}{2} \sin(q_2)\right)$$

$$L = K - V$$

$$= \frac{1}{6} m_1 l_1^2 \dot{q}_1^2 + \frac{1}{2} m_2 \left(l_1^2 \dot{q}_1^2 + \frac{l_2^2 \dot{q}_2^2}{4} + l_1 l_2 \dot{q}_1 \dot{q}_2 \cos(q_2 - q_1) \right)$$

$$+ \frac{1}{24} m_2 l_2^2 \dot{q}_2^2 - \left(\frac{m g l_1}{2} \sin(q_1) + m_2 g l_1 \sin q_1 + \frac{m_2 g l_2}{2} \sin(q_2) \right)$$

Applying Lagrangian from eqn (5)

$$Q_1 = \tau_1 = \frac{m_1 l_1^2 \ddot{q}_1}{3} + m_2 l_1^2 \ddot{q}_1 + \frac{m_2 l_1 l_2}{2} \ddot{q}_2 \cos(q_2 - q_1)$$

$$- \frac{m_2 l_1 l_2}{2} \dot{q}_1^2 \sin(q_2 - q_1) + \frac{m g l_1 \cos(q_1)}{2} + m_2 g l_1 \cos(q_1)$$

$$Q_2 = \tau_2 = \frac{m_2 l_2^2}{3} \ddot{q}_2 + \frac{m_2 l_1 l_2}{2} \ddot{q}_1 \cos(q_2 - q_1) + \frac{m_2 l_1 l_2}{2} \dot{q}_1^2 \sin(q_2 - q_1)$$

$$+ \frac{m_2 g l_2 \cos(q_2)}{2}$$

(6)

The torque dynamics in (6) solve task 1b

2: Saving τ_1 & τ_2 from (6) as $\tau_{1\,dynamics}$ & $\tau_{2\,dynamics}$

and aim to obtain $\vec{E}(x, y)$ through (3) using (q_1, q_2) to calculate through the dynamics based solution.

Step 3: Calculation of spring dynamics

$$F_{spring_x} = -k_x(x - x_0) = -k_x(x_{displacement}) = -k_x \Delta x$$

$$F_{spring_y} = -k_y(y - y_0) = -k_y(y_{displacement}) = -k_y \Delta y$$

Using $x = \Delta x$, $y = \Delta y$ in (4),

$$F_y l_1 \cos q_1 - F_x l_1 \sin q_1 = \tau_1$$

$$\Rightarrow \boxed{\begin{aligned} k_y \Delta y\, l_1 \cos(q_1) - k_x \Delta x\, l_1 \sin(q_1) &= \tau_{1spring} \\ k_y \Delta y\, l_2 \cos(q_2) - k_x \Delta x\, l_2 \sin(q_2) &= \tau_{2spring} \end{aligned}} \quad (7)$$

To achieve spring-like behaviours from end effector, we need to account for the torque of the 2R manipulator dynamics calculated in (6)

$$\tau_{totalmotor1} = \tau_{1spring} + \tau_{1dynamics}$$

$$\tau_{totalmotor2} = \tau_{2spring} + \tau_{2dynamics}$$