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Chapter 36. Management Science

Practice Problems

1.

A chemical plant can process 500 tons/day of raw material. The amount of raw material processed in one day can be modeled as having an exponential distribution with a mean of 500 tons. It is desired that the probability of running out of raw material be 0.05. Most nearly, the amount of raw material that should be stocked each day is

(A)

500 tons

(B)

750 tons

(C)

1000 tons

(D)

1500 tons

2.

The physical property of a product within a production line is modeled as time-dependent variable $P(t)$. The time rate of change for the physical property is equal to a constant k multiplied by $P(t)$. The constant k is $0.00001\ \text{s}^{-1}$. The boundary condition is that the physical property initially equals 100 units. Most nearly, what is the physical property's value 24 hours later?

(A)

12

(B)

50

(C)

100

(D)

237

Solutions

1.

Let a be the amount of raw material that should be stocked each day. The probability of running out of raw material in a day is $P(X > a) = 0.05$. By definition, the relationship between the amount of raw material that should be stocked each day and the given probability is

$$\begin{aligned} P(X > a) &= \int_a^\infty f(x)\ dx \\ &= \int_a^\infty \left(\frac{1}{500\ \text{tons}}\right) e^{-x/500\ \text{tons}}\ dx \\ &= -e^{-a/500\ \text{tons}}\Big|_a^\infty \end{aligned}$$

Setting this equal to 0.05 gives

$$\begin{aligned} -e^{-a/500\ \text{ton}} &= 0.05 \\ -\frac{a}{500\ \text{tons}} &= \ln 0.05 \\ a &= -(500\ \text{tons}) \ln 0.05 \\ &= 1497\ \text{tons} \quad (1500\ \text{tons}) \end{aligned}$$

The answer is (D).
2.

This problem can be modeled as a differential equation. The time rate of change of the physical property is

$$\frac{dP(t)}{dt} = kP(t) = \left(\frac{0.00001}{1\ \text{s}}\right) P(t)$$

$$\frac{dP}{dt} - (0.00001)P = 0$$

The above differential equation's characteristic equation is

$$r - 0.00001 = 0$$

The root, r, is 0.00001. Therefore, the solution to the equation is

$$P(t) = Ce^{rt} = Ce^{0.00001t}$$

To solve for C, apply the boundary condition $P(0) = 100$ units.

$$\begin{aligned} P(0) &= 100 = Ce^{(0.00001)(0)} = C(1) \\ C &= 100 \end{aligned}$$

The physical property, P, 24 hours (or 86,400 seconds) later is

$$\begin{aligned} P(86{,}400) &= 100e^{(0.00001)(86{,}400\ \text{s})} \\ &= (100)(2.373) \\ &= 237.3 \quad (237) \end{aligned}$$

The answer is (D).