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Chapter 2. Fluid Statics

Practice Problems

(Use g = 32.2 ft/sec^2 or 9.81 m/s^2 unless told otherwise.)

1.

A 4000 lbm (1800 kg) blimp contains 10,000 lbm (4500 kg) of hydrogen (specific gas constant = 766.5 ft-lbf/lbm-°R (4124 J/kg·K)) at 56°F (13°C) and 30.2 in Hg (770 mm Hg). If the hydrogen and air are in thermal and pressure equilibrium, what is most nearly the blimp's lift (lifting force)?

(A)

$7.6 \times 10^3 \text{ lbf } \left(3.4 \times 10^4 \text{ N}\right)$

(B)

$1.2 \times 10^4 \text{ lbf } \left(5.3 \times 10^4 \text{ N}\right)$

(C)

$1.3 \times 10^5 \text{ lbf } \left(5.7 \times 10^5 \text{ N}\right)$

(D)

$1.7 \times 10^5 \text{ lbf } \left(7.7 \times 10^5 \text{ N}\right)$

2.

A hollow 6 ft (1.8 m) diameter sphere floats half-submerged in seawater. The mass of concrete (150 lbm/ft^3 or 2400 kg/m^3) that is required as an external anchor to just submerge the sphere completely is most nearly

(A)

2700 lbm (1200 kg)

(B)

4200 lbm (1900 kg)

(C)

5500 lbm (2500 kg)

(D)

6300 lbm (2700 kg)

3.

Water removed from Lake Superior (elevation, 601 ft above mean sea level; water density, 62.4 lbm/ft^3) is transported by tanker ship to the Atlantic Ocean (elevation, 0 ft) through 16 Seaway locks. A tanker's displacement is 32,000 tonnes when loaded, and 5100 tonnes when empty. Each lock is 766 ft long and 80 ft wide. Water pumped from each lock flows to the Atlantic Ocean. Compared to a passage from Lake Superior to the Atlantic Ocean when empty, what is most nearly the change in water loss from Lake Superior when a ship passes through the locks fully loaded?

(A)

27,000 tonnes less loss

(B)

27,000 tonnes additional loss

(C)

54,000 tonnes additional loss

(D)

no change in loss

Solutions

1.

Customary U.S. Solution

The lift (lifting force) of the hydrogen-filled blimp, F_{lift}, is equal to the difference between the buoyant force, F_b, and the weight of the hydrogen contained in the blimp, W_{H}.

$$F_{\text{lift}} = F_b - W_{\text{H}} - W_{\text{blimp}}$$

The weight of the hydrogen is calculated from the mass of hydrogen.

$$W_{\text{H}} = \frac{m_{\text{H}}g}{g_c} = \frac{(10{,}000 \text{ lbm})\left(32.2 \ \frac{\text{ft}}{\text{sec}^2}\right)}{32.2 \ \frac{\text{lbm-ft}}{\text{lbf-sec}^2}}$$
$$= 10{,}000 \text{ lbf}$$

The buoyant force is equal to the weight of the displaced air. The volume of the air displaced is equal to the volume of hydrogen enclosed in the blimp.

The absolute temperature of the hydrogen is

$$T = 56^\circ\text{F} + 460^\circ = 516^\circ\text{R}$$

The pressure of the hydrogen is

$$p = \frac{(30.2 \text{ in Hg})\left(12 \ \frac{\text{in}}{\text{ft}}\right)^2}{2.036 \ \frac{\text{in Hg}}{\frac{\text{lbf}}{\text{in}^2}}} = 2136 \text{ lbf/ft}^2$$

Compute the volume of hydrogen from the ideal gas law.

$$V_{\text{H}} = \frac{m_{\text{H}}RT}{p}$$
$$= \frac{(10{,}000 \text{ lbm})\left(766.5 \ \frac{\text{ft-lbf}}{\text{lbm-}^\circ\text{R}}\right)(516^\circ\text{R})}{2136 \ \frac{\text{lbf}}{\text{ft}^2}}$$
$$= 1.85 \times 10^6 \text{ ft}^3$$

Since the volume of the hydrogen contained in the blimp is equal to the air displaced, the air displaced can be computed from the ideal gas equation. Since the air and hydrogen are in thermal and pressure equilibrium, the temperature and pressure are equal to the values given for the hydrogen.

For air, $R = 53.35$ ft-lbf/lbm-°R.

$$\begin{aligned} m_{\text{air}} &= \frac{pV_{\text{H}}}{RT} = \frac{\left(2136\ \frac{\text{lbf}}{\text{ft}^2}\right)\left(1.85 \times 10^6\ \text{ft}^3\right)}{\left(53.35\ \frac{\text{ft-lbf}}{\text{lbm-}^\circ\text{R}}\right)(516^\circ\text{R})} \\ &= 1.435 \times 10^5\ \text{lbm} \end{aligned}$$

The buoyant force is equal to the weight of the air.

$$\begin{aligned} F_b &= W_{\text{air}} = \frac{m_{\text{air}}g}{g_c} = \frac{\left(1.435 \times 10^5\ \text{lbm}\right)\left(32.2\ \frac{\text{ft}}{\text{sec}^2}\right)}{32.2\ \frac{\text{lbm-ft}}{\text{lbf-sec}^2}} \\ &= 1.435 \times 10^5\ \text{lbf} \end{aligned}$$

The lift (lifting force) is

$$\begin{aligned} F_{\text{lift}} &= F_b - W_{\text{H}} - W_{\text{blimp}} \\ &= 1.435 \times 10^5\ \text{lbf} - 10{,}000\ \text{lbf} - 4000\ \text{lbf} \\ &= 1.295 \times 10^5\ \text{lbf} \quad \left(1.3 \times 10^5\ \text{lbf}\right) \end{aligned}$$

The answer is (C).

SI Solution

The lift (lifting force) of the hydrogen-filled blimp, F_{lift}, is equal to the difference between the buoyant force, F_b, and the weight of the hydrogen contained in the blimp, W_{H}.

$$F_{\text{lift}} = F_b - W_{\text{H}} - W_{\text{blimp}}$$

The weight of the hydrogen is calculated from the mass of hydrogen.

$$\begin{aligned} W_{\text{H}} &= m_{\text{H}}g \\ &= (4500\ \text{kg})\left(9.81\ \frac{\text{m}}{\text{s}^2}\right) \\ &= 44\,145\ \text{N} \end{aligned}$$

The buoyant force is equal to the weight of the displaced air. The volume of the air displaced is equal to the volume of hydrogen enclosed in the blimp.

The absolute temperature of the hydrogen is

$$T = 13^\circ\text{C} + 273^\circ = 286\text{K}$$

The absolute pressure of the hydrogen is

$$\begin{aligned} p &= \frac{(770\ \text{mm Hg})\left(133.4\ \frac{\text{kPa}}{\text{m}}\right)}{1000\ \frac{\text{mm}}{\text{m}}} \\ &= 102.7\ \text{kPa} \end{aligned}$$

Compute the volume of hydrogen from the ideal gas law.

$$
\begin{aligned}
V_{\mathrm{H}} &= \frac{m_{\mathrm{H}} R T}{p} \\
&= \frac{(4500\ \mathrm{kg})\left(4124\ \frac{\mathrm{J}}{\mathrm{kg\cdot K}}\right)(286\mathrm{K})}{(102.7\ \mathrm{kPa})\left(1000\ \frac{\mathrm{Pa}}{\mathrm{kPa}}\right)} \\
&= 5.168 \times 10^4\ \mathrm{m}^3
\end{aligned}
$$

Since the volume of the hydrogen contained in the blimp is equal to the air displaced, the air displaced can be computed from the ideal gas equation. Since the air and hydrogen are assumed to be in thermal and pressure equilibrium, the temperature and pressure are equal to the values given for the hydrogen.

For air, $R = 287.03$ J/kg·K.

$$
\begin{aligned}
m_{\mathrm{air}} &= \frac{p V_{\mathrm{H}}}{R T} \\
&= \frac{(102.7\ \mathrm{kPa})\left(1000\ \frac{\mathrm{Pa}}{\mathrm{kPa}}\right)(5.168 \times 10^4\ \mathrm{m}^3)}{\left(287.03\ \frac{\mathrm{J}}{\mathrm{kg\cdot K}}\right)(286\mathrm{K})} \\
&= 6.465 \times 10^4\ \mathrm{kg}
\end{aligned}
$$

The buoyant force is equal to the weight of the air.

$$
\begin{aligned}
F_b &= W_{\mathrm{air}} = m_{\mathrm{air}} g \\
&= (6.465 \times 10^4\ \mathrm{kg})\left(9.81\ \frac{\mathrm{m}}{\mathrm{s}^2}\right) \\
&= 6.34 \times 10^5\ \mathrm{N}
\end{aligned}
$$

The lift (lifting force) is

$$
\begin{aligned}
F_{\mathrm{lift}} &= F_b - W_{\mathrm{H}} - W_{\mathrm{blimp}} \\
&= 6.34 \times 10^5\ \mathrm{N} - 44\,145\ \mathrm{N} - (1800\ \mathrm{kg})\left(9.81\ \frac{\mathrm{m}}{\mathrm{s}^2}\right) \\
&= 5.7 \times 10^5\ \mathrm{N}
\end{aligned}
$$

The answer is (C).
2.

Customary U.S. Solution

The weight of the sphere is equal to the weight of the displaced volume of water when floating.

The buoyant force is given by

$$
F_b = \frac{\rho g V_{\mathrm{displaced}}}{g_c}
$$

Since the sphere is half submerged,

$$
W_{\mathrm{sphere}} = \frac{1}{2}\left(\frac{\rho g V_{\mathrm{sphere}}}{g_c}\right)
$$

For seawater, $\rho = 64.0$ lbm/ft^3.

The volume of the sphere is

$$
\begin{aligned}
V_{\mathrm{sphere}} &= \tfrac{\pi}{6} d^3 = \left(\frac{\pi}{6}\right)(6\ \mathrm{ft})^3 \\
&= 113.1\ \mathrm{ft}^3
\end{aligned}
$$

The weight of the sphere is

$$\begin{aligned} W_{\text{sphere}} &= \tfrac{1}{2}\left(\frac{\rho g V_{\text{sphere}}}{g_c}\right) \\ &= \left(\frac{1}{2}\right)\left(\frac{\left(64.0\ \frac{\text{lbm}}{\text{ft}^3}\right)\left(32.2\ \frac{\text{ft}}{\text{sec}^2}\right)\left(113.1\ \text{ft}^3\right)}{32.2\ \frac{\text{lbm-ft}}{\text{lbf-sec}^2}}\right) \\ &= 3619\ \text{lbf} \end{aligned}$$

The equilibrium equation for a fully submerged sphere and anchor can be solved for the concrete volume.

$$W_{\text{sphere}} + W_{\text{concrete}} = \left(V_{\text{sphere}} + V_{\text{concrete}}\right)\rho_{\text{water}}$$

$$W_{\text{sphere}} + \rho_{\text{concrete}}V_{\text{concrete}}\left(\frac{g}{g_c}\right) = \left(V_{\text{sphere}} + V_{\text{concrete}}\right)\rho_{\text{water}}\left(\frac{g}{g_c}\right)$$

$$\begin{aligned} &3619\ \text{lbf} + \left(150\ \frac{\text{lbm}}{\text{ft}^3}\right) \\ &\quad\times V_{\text{concrete}}\left(\frac{32.2\ \frac{\text{ft}}{\text{sec}^2}}{32.2\ \frac{\text{lbm-ft}}{\text{lbf-sec}^2}}\right) = \left(113.1\ \text{ft}^3 + V_{\text{concrete}}\right) \\ &\qquad\qquad \times\left(64.0\ \frac{\text{lbm}}{\text{ft}^3}\right) \\ &\qquad\qquad \times\left(\frac{32.2\ \frac{\text{ft}}{\text{sec}^2}}{32.2\ \frac{\text{lbm-ft}}{\text{lbf-sec}^2}}\right) \end{aligned}$$

$$\begin{aligned} V_{\text{concrete}} &= 42.09\ \text{ft}^3 \\ m_{\text{concrete}} &= \rho_{\text{concrete}}V_{\text{concrete}} \\ &= \left(150\ \frac{\text{lbm}}{\text{ft}^3}\right)\left(42.09\ \text{ft}^3\right) \\ &= 6314\ \text{lbm} \quad (6300\ \text{lbm}) \end{aligned}$$

The answer is (D).

SI Solution

The weight of the sphere is equal to the weight of the displaced volume of water when floating.

The buoyant force is given by

$$F_b = \rho g V_{\text{displaced}}$$

Since the sphere is half submerged,

$$W_{\text{sphere}} = \frac{1}{2}\rho g V_{\text{sphere}}$$

For seawater, $\varrho = 1025\ \text{kg/m}^3$.

The volume of the sphere is

$$V_{\text{sphere}} = \tfrac{\pi}{6}d^3 = \left(\frac{\pi}{6}\right)(1.8\ \text{m})^3 = 3.054\ \text{m}^3$$

The weight of the sphere required is

$$
\begin{aligned}
W_{\text{sphere}} &= \tfrac{1}{2}\,\rho g V_{\text{sphere}} \\
&= \left(\frac{1}{2}\right)\left(1025\ \frac{\text{kg}}{\text{m}^3}\right)\left(9.81\ \frac{\text{m}}{\text{s}^2}\right)\left(3.054\ \text{m}^3\right) \\
&= 15\,354\ \text{N}
\end{aligned}
$$

The equilibrium equation for a fully submerged sphere and anchor can be solved for the concrete volume.

$$
\begin{aligned}
W_{\text{sphere}} + W_{\text{concrete}} &= g\left(V_{\text{sphere}} + V_{\text{concrete}}\right)\rho_{\text{water}} \\
W_{\text{sphere}} + \rho_{\text{concrete}}\, g V_{\text{concrete}} &= g\left(V_{\text{sphere}} + V_{\text{concrete}}\right)\rho_{\text{water}} \\
15\,354\ \text{N} + \left(2400\ \frac{\text{kg}}{\text{m}^3}\right) & \\
\times\left(9.81\ \frac{\text{m}}{\text{s}^2}\right) V_{\text{concrete}} &= \left(3.054\ \text{m}^3 + V_{\text{concrete}}\right) \\
&\quad\ \times\left(1025\ \frac{\text{kg}}{\text{m}^3}\right)\left(9.81\ \frac{\text{m}}{\text{s}^2}\right)
\end{aligned}
$$

$$
\begin{aligned}
V_{\text{concrete}} &= 1.138\ \text{m}^3 \\
m_{\text{concrete}} &= \rho_{\text{concrete}} V_{\text{concrete}} \\
&= \left(2400\ \frac{\text{kg}}{\text{m}^3}\right)\left(1.138\ \text{m}^3\right) \\
&= 2731\ \text{kg} \quad (2700\ \text{kg})
\end{aligned}
$$

The answer is (D).
3.

From Archimedes' principle, each tonne of water carried in the tanker displaces a tonne of water in the lock. So, each tonne of water transported out of the lake results in a tonne less of lock loss. Compared to an empty tanker, the net result is zero.

The answer is (D).