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Chapter 3. Fluid Flow Parameters

Practice Problems

Use the following values unless told otherwise.

$$\begin{aligned} g &= 32.2\ \text{ft/sec}^2\ \ (9.81\ \text{m/s}^2) \\ \rho_{\text{water}} &= 62.4\ \text{lbm/ft}^3\ \ (1000\ \text{kg/m}^3) \\ p_{\text{atmospheric}} &= 14.7\ \text{psia}\ \ (101.3\ \text{kPa}) \end{aligned}$$

1.

A 10 in (25 cm) composition pipe is compressed by a tree root into an elliptical cross section until its inside height is only 7.2 in (18 cm). What is most nearly the hydraulic radius when flowing half full?

(A)

2.2 in (5.5 cm)

(B)

2.7 in (6.9 cm)

(C)

3.2 in (8.1 cm)

(D)

4.5 in (11.4 cm)

2.

A pipe with an inside diameter of 18.812 in contains water to a depth of 15.7 in. What is most nearly the hydraulic radius? (Work in customary U.S. units only.)

(A)

4.4 in

(B)

5.1 in

(C)

5.7 in

(D)

6.5 in

Solutions

1.

Customary U.S. Solution

The perimeter of the pipe, as in *NCEES Handbook:* Planar Geometry—Area and Perimeter, is

$$p = \pi d = \pi\,(10\text{ in}) = 31.42\text{ in}$$

If the pipe is flowing half full, the wetted perimeter is

$$\text{wetted perimeter} = \tfrac{1}{2}p = \left(\frac{1}{2}\right)(31.42\text{ in}) = 15.71\text{ in}$$

The ellipse will have a minor axis, b, equal to one-half the height of the compressed pipe, or

$$b = \frac{7.2\text{ in}}{2} = 3.6\text{ in}$$

When the pipe is compressed, the perimeter of the pipe remains constant. The perimeter of an ellipse, as in *NCEES Handbook:* Planar Geometry—Area and Perimeter, is

$$p \approx 2\pi\sqrt{\frac{1}{2}\left(a^2+b^2\right)}$$

Solve for the major axis.

$$\begin{aligned} a &= \sqrt{2\left(\frac{p}{2\pi}\right)^2 - b^2} = \sqrt{(2)\left(\frac{31.42\text{ in}}{2\pi}\right)^2 - (3.6\text{ in})^2} \\ &= 6.09\text{ in} \end{aligned}$$

The flow area or area of the ellipse, as in *NCEES Handbook:* Planar Geometry—Area and Perimeter, is

$$\begin{aligned} \text{flow area} &= \tfrac{1}{2}\pi ab \\ &= \left(\frac{1}{2}\pi\right)(6.09\text{ in})\,(3.6\text{ in}) \\ &= 34.4\text{ in}^2 \end{aligned}$$

The hydraulic radius is

$$r_h = \frac{\text{area in flow}}{\text{wetted perimeter}} = \frac{34.4\text{ in}^2}{15.71\text{ in}} = 2.19\text{ in} \quad (2.2\text{ in})$$

The answer is (A).

SI Solution

The perimeter of the pipe, as in *NCEES Handbook:* Planar Geometry—Area and Perimeter, is

$$p = \pi d = \pi\,(25\text{ cm}) = 78.54\text{ cm}$$

If the pipe is flowing half full, the wetted perimeter is

$$\text{wetted perimeter} = \tfrac{1}{2}p = \left(\frac{1}{2}\right)(78.54\text{ cm}) = 39.27\text{ cm}$$

The ellipse will have a minor axis, b, equal to one-half the height of the compressed pipe, or

$$b = \frac{18\text{ cm}}{2} = 9\text{ cm}$$

When the pipe is compressed, the perimeter of the pipe remains constant. The perimeter of an ellipse, as in *NCEES Handbook:* Planar Geometry—Area and Perimeter, is

$$p \approx 2\pi\sqrt{\frac{1}{2}\left(a^2+b^2\right)}$$

Solve for the major axis.

$$
\begin{aligned}
a &= \sqrt{2\left(\frac{p}{2\pi}\right)^2 - b^2} = \sqrt{(2)\left(\frac{78.54 \text{ cm}}{2\pi}\right)^2 - (9 \text{ cm})^2} \\
&= 15.2 \text{ cm}
\end{aligned}
$$

The flow area or area of the ellipse, as in *NCEES Handbook:* Planar Geometry—Area and Perimeter, is

$$
\begin{aligned}
\text{flow area} &= \tfrac{1}{2}\pi ab = \left(\frac{1}{2}\pi\right)(15.2 \text{ cm})(9 \text{ cm}) \\
&= 214.9 \text{ cm}^2
\end{aligned}
$$

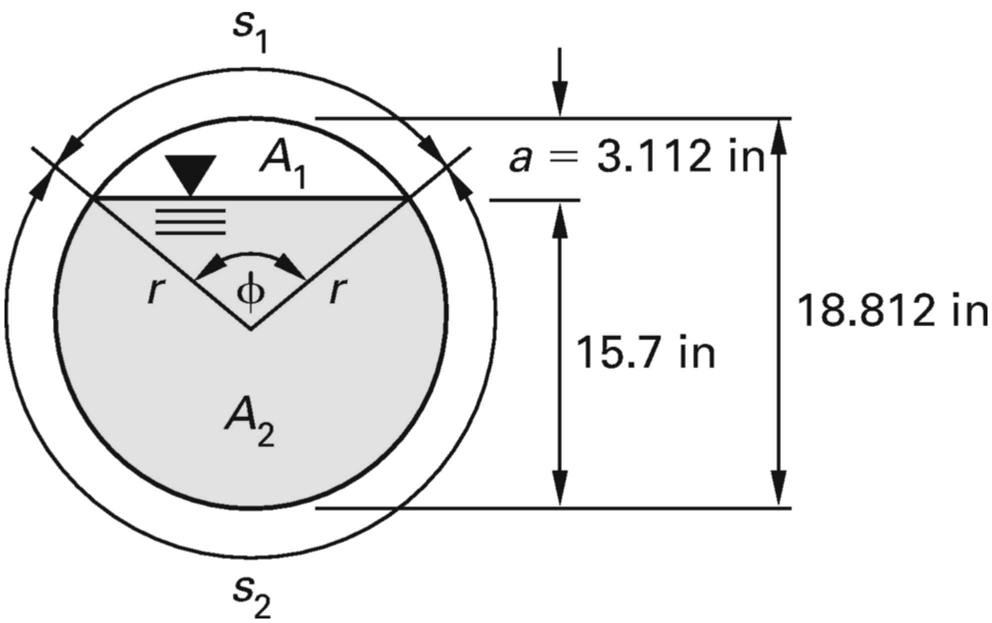
The hydraulic radius is

$$
\begin{aligned}
r_h &= \frac{\text{area in flow}}{\text{wetted perimeter}} = \frac{214.9 \text{ cm}^2}{39.27 \text{ cm}} \\
&= 5.47 \text{ cm} \quad (5.5 \text{ cm})
\end{aligned}
$$

The answer is (A).

2.

As in *NCEES Handbook:* Planar Geometry—Area and Perimeter, for a circular segment,



$$
\begin{aligned}
r &= \frac{d}{2} = \frac{18.812 \text{ in}}{2} = 9.406 \text{ in} \\
\phi &= 2 \arccos \frac{r-d}{r} \\
&= 2 \arccos \frac{9.406 \text{ in} - 3.112 \text{ in}}{9.406 \text{ in}} \\
&= 1.675 \text{ rad}
\end{aligned}
$$

$$
\begin{aligned}
\sin\phi &= 0.9946 \\
A_1 &= \tfrac{1}{2}r^2\left(\phi - \sin\phi\right) \\
&= \left(\frac{1}{2}\right)(9.406\ \text{in})^2\,(1.675 - 0.9946) \\
&= 30.1\ \text{in}^2 \\
A_{\text{total}} &= A_1 + A_2 = \frac{\pi}{4}d^2 = \left(\frac{\pi}{4}\right)(18.812\ \text{in})^2 \\
&= 277.95\ \text{in}^2 \\
A_2 &= A_{\text{total}} - A_1 = 277.95\ \text{in}^2 - 30.1\ \text{in}^2 \\
&= 247.85\ \text{in}^2 \\
s_1 &= r\phi = (9.406\ \text{in})\,(1.675) = 15.76\ \text{in} \\
s_{\text{total}} &= s_1 + s_2 = \pi d = \pi\,(18.812\ \text{in}) \\
&= 59.1\ \text{in} \\
s_2 &= s_{\text{total}} - s_1 = 59.1\ \text{in} - 15.76\ \text{in} = 43.34\ \text{in} \\
r_h &= \frac{A_2}{s_2} = \frac{247.85\ \text{in}^2}{43.34\ \text{in}} = 5.719\ \text{in} \quad (5.7\ \text{in})
\end{aligned}
$$

The answer is (C).