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Chapter 20. Radiation and Combined Heat Transfer

Practice Problems

1.

A 6 in (15 cm) thick furnace wall has a 3 in (8 cm) square peephole. The interior of the furnace is at 2200°F (1200°C). The shape factor is 0.38. The surrounding air temperature is 70°F (20°C). The heat loss due to radiation when the peephole is open is most nearly

(A)

450 Btu/hr (150 W)

(B)

1300 Btu/hr (440 W)

(C)

2000 Btu/hr (680 W)

(D)

7900 Btu/hr (2.7 kW)

2.

Dry air at 1 atmospheric pressure flows at 500 ft^3/min (0.25 m^3/s) through 50 ft (15 m) of 12 in (30 cm) diameter uninsulated duct. The emissivity of the duct surface is 0.28. Air enters the duct at 45°F (7°C). The walls, air, and contents of the room through which the duct passes are at 80°F (27°C). An engineer states that the air leaving the duct will be at 50°F (10°C). Considering both convection and radiation, is the engineer's statement that the air leaving the duct will be at 50°F (10°C) correct?

(A)

The temperature is 45°F (7.2°C); this does not agree with the engineer's estimate.

(B)

The temperature is 49°F (9.7°C); this does agree with the engineer's estimate.

(C)

The temperature is 51°F (11°C); this does agree with the engineer's estimate.

(D)

The temperature is 53°F (12°C); this does not agree with the engineer's estimate.

3.

A steel pipe is painted on the outside with dull gray (oil-based) paint. The pipe is 35 ft (10 m) long. The pipe has a 4.00 in (10.2 cm) inside diameter and 4.25 in (10.8 cm) outside diameter. The pipe carries 200 ft^3/min (0.1 m^3/s) of 500°F (260°C), 25 psig (170 kPa) air through a 70°F (20°C) room. The conditions of the air at the end of the pipe are 350°F (180°C) and 15 psig (100 kPa). Using theoretical methods or empirical correlations, the calculated overall coefficient of heat transfer is most nearly

(A)

1.0 Btu/hr-ft^2-°F (6.1 W/m^2·K)

(B)

1.8 Btu/hr-ft^2-°F (11 W/m^2 ·K)

(C)

3.8 Btu/hr-ft^2-°F (22 W/m^2·K)

(D)

54 Btu/hr-ft^2-°F (330 W/m^2 ·K)
4.

The temperature of a gas in a duct with 600°F (315°C) walls is evaluated with a 0.5 in (13 mm) diameter thermocouple probe. The emissivity of the probe is 0.8. The gas flow rate is 3480 lbm/hr-ft^2 (4.7 kg/s·m^2). The gas velocity is 400 ft/min (2 m/s). The film coefficient on the probe is given empirically as

$$h = \frac{0.024G^{0.8}}{D^{0.4}}$$

h in Btu/hr-ft^2-°F
G in lbm/hr-ft^2
D in ft

$$h = \frac{17G^{0.8}}{D^{0.4}}$$

h in W/m^2·K
G in kg/s·m^2
D in m

If the probe reading indicates that the gas temperature is 300°F (150°C), the actual gas temperature is most nearly

(A)

650°R (360K)

(B)

740°R (410K)

(C)

770°R (430K)

(D)

810°R (450K)

5.

A 9 in (23 cm) diameter duct is painted with white enamel. The surface of the duct is at 200°F (95°C). The duct carries hot air through a room whose walls are 70°F (20°C). The air in the room is at 80°F (27°C). The unit heat transfer is most nearly

(A)

685 Btu/hr-ft length (685 W/m length)

(B)

900 Btu/hr-ft length (900 W/m length)

(C)

1100 Btu/hr-ft length (1100 W/m length)

(D)

1500 Btu/hr-ft length (1500 W/m length)

Solutions

1.

Customary U.S. Solution

As in *NCEES Handbook:* Temperature, the absolute temperatures are

$$\begin{aligned} T_{\text{furnace}} &= 2200^\circ\text{F} + 460^\circ = 2660^\circ\text{R} \\ T_\infty &= 70^\circ\text{F} + 460^\circ = 530^\circ\text{R} \end{aligned}$$

The radiation heat loss is

$$\begin{aligned} Q &= AE_{\text{net}} = A\sigma F_{12}\left(T_{\text{furnace}}^4 - T_\infty^4\right) \\ &= \frac{(3\ \text{in})^2\left(0.1713\times10^{-8}\ \dfrac{\text{Btu}}{\text{hr-ft}^2\text{-}^\circ\text{R}^4}\right)\times(0.38)\left((2660^\circ\text{R})^4-(530^\circ\text{R})^4\right)}{\left(12\ \dfrac{\text{in}}{\text{ft}}\right)^2} \\ &= 2033.6\ \text{Btu/hr} \quad (2000\ \text{Btu/hr}) \end{aligned}$$

The answer is (C).

SI Solution

As in *NCEES Handbook:* Temperature, the absolute temperatures are

$$\begin{aligned} T_{\text{furnace}} &= 1200^\circ\text{C} + 273^\circ = 1473\text{K} \\ T_\infty &= 20^\circ\text{C} + 273^\circ = 293\text{K} \end{aligned}$$

The radiation heat loss is

$$\begin{aligned} Q &= AE_{\text{net}} = A\sigma F_{12}\left(T_{\text{furnace}}^4 - T_\infty^4\right) \\ &= \frac{(8\ \text{cm})^2\left(5.67\times10^{-8}\ \dfrac{\text{W}}{\text{m}^2\text{·K}^4}\right)\times(0.38)\left((1473\text{K})^4-(293\text{K})^4\right)}{\left(100\ \dfrac{\text{cm}}{\text{m}}\right)^2} \\ &= 648.2\ \text{W} \quad (680\ \text{W}) \end{aligned}$$

The answer is (C).

2.

Customary U.S. Solution

As in *NCEES Handbook:* Temperature, the absolute temperature of air entering the duct is

$$45^\circ\text{F} + 460^\circ = 505^\circ\text{R}$$

From the ideal gas law, the density of air entering the duct is

$$\rho = \frac{pM}{RT} = \frac{\left(14.7\ \frac{\text{lbf}}{\text{in}^2}\right)\left(12\ \frac{\text{in}}{\text{ft}}\right)^2\left(29\ \frac{\text{lbm}}{\text{lbmol}}\right)}{\left(1545.35\ \frac{\text{ft-lbf}}{\text{lbmol-}^\circ\text{R}}\right)(505^\circ\text{R})}$$
$$= 0.07857\ \text{lbm/ft}^3$$

The mass flow rate of entering air is

$$\dot{m} = \rho\dot{V} = \left(0.07857\ \frac{\text{lbm}}{\text{ft}^3}\right)\left(500\ \frac{\text{ft}^3}{\text{min}}\right)\left(60\ \frac{\text{min}}{\text{hr}}\right)$$
$$= 2357.1\ \text{lbm/hr}$$

The mass velocity entering the duct is

$$G = \frac{\dot{m}}{A_{\text{flow}}} = \frac{\left(2357.1\ \frac{\text{lbm}}{\text{hr}}\right)\left(12\ \frac{\text{in}}{\text{ft}}\right)^2}{\left(\frac{\pi}{4}\right)(12\ \text{in})^2}$$
$$= 3001.1\ \text{lbm/hr-ft}^2$$

To calculate the initial film coefficients, estimate the temperature based on the claim. The film coefficients are not highly sensitive to small temperature differences.

$$T_{\text{bulk,air}} = \tfrac{1}{2}(T_{\text{air,in}} + T_{\text{air,out}}) = \left(\frac{1}{2}\right)(45^\circ\text{F} + 50^\circ\text{F})$$
$$= 47.5^\circ\text{F}$$
$$T_{\text{surface}} = 70^\circ\text{F} \quad \text{[estimate]}$$

As in *NCEES Handbook:* Temperature-Dependent Properties of Air (U.S. Customary Units), the viscosity and Prandtl number are

$$\mu = 1.28 \times 10^{-5}\ \frac{\text{lbm}}{\text{ft-sec}}$$

$$\text{Pr} = 0.708$$

Using the volumetric flow rate and cross-sectional area of the duct, the velocity is

$$\text{v} = \frac{\dot{V}}{A} = \frac{\dot{V}}{\frac{\pi D^2}{4}} = \frac{500\ \frac{\text{ft}^3}{\text{min}}}{\left(\frac{\pi(1\ \text{ft})^2}{4}\right)\left(60\ \frac{\text{s}}{\text{min}}\right)} = 10.6\ \text{ft/sec}$$

From *NCEES Handbook* table "Dimensionless Numbers," the Reynolds number is

$$\text{Re} = \frac{\rho u D}{\mu} = \frac{\left(0.07857\ \frac{\text{lbm}}{\text{ft}^3}\right)\left(10.6\ \frac{\text{ft}}{\text{sec}}\right)(1\ \text{ft})}{\left(1.28 \times 10^{-5}\ \frac{\text{lbm}}{\text{ft-sec}}\right)} = 65{,}000$$

As in *NCEES Handbook* table "Temperature-Dependent Properties of Air at 14.7 psia (U.S. Units)," the thermal conductivity for air is 0.0156. Ignore the viscosity term for such a small temperature change. The correlation for turbulent flow in circular tubes can be found in *NCEES Handbook* table "Forced Convection—Internal Flow." Rearrange the correlation to find the film coefficient for air flowing inside the duct.

$$
\begin{aligned}
\mathrm{Nu}_{\mathrm{D}} &= 0.023\mathrm{Re_D}^{0.8}\mathrm{Pr}^{1/3}\left(\tfrac{\mu_\infty}{\mu_s}\right)^{0.14} = \tfrac{h_i D}{k} \\
h_i &= \left(\tfrac{k}{D}\right)0.023\mathrm{Re_D}^{0.8}\mathrm{Pr}^{1/3}\left(\tfrac{\mu_\infty}{\mu_s}\right)^{0.14} \\
&= \left(\frac{0.0156\ \frac{\text{Btu}}{\text{hr-ft-}^\circ\text{F}}}{1\ \text{ft}}\right)(0.023)\,(65{,}000)^{0.8}(0.708)^{1/3} \\
&= 2.27\ \text{Btu/hr-ft-}^\circ\text{F}
\end{aligned}
$$

For natural convection on the outside of the duct, estimate the film temperature.

$$
\begin{aligned}
T_{\text{film}} &= \tfrac{1}{2}(T_{\text{surface}} + T_\infty) = \left(\frac{1}{2}\right)(70^\circ\text{F} + 80^\circ\text{F}) \\
&= 75^\circ\text{F}
\end{aligned}
$$

From appendixMERM35C (also *NCEES Handbook:* Temperature-Dependent Properties of Air (U.S. Customary Units)), the properties of air at 75°F are

$$
\begin{aligned}
\mathrm{Pr} &\approx 0.72 \\
\frac{g\beta\rho^2}{\mu^2} &= 2.27\times10^6\ \frac{1}{\text{ft}^3\text{-}^\circ\text{F}}
\end{aligned}
$$

The characteristic length is the diameter of the duct.

$$
L = \frac{12\ \text{in}}{12\ \frac{\text{in}}{\text{ft}}} = 1\ \text{ft}
$$

As in *NCEES Handbook* table "Dimensionless Numbers," the Grashof number is

$$
\begin{aligned}
\mathrm{Gr} &= L^3\left(\frac{\rho^2\beta g}{\mu^2}\right)(T_\infty - T_s) \\
&= (1\ \text{ft})^3\left(2.27\times10^6\ \frac{1}{\text{ft}^3\text{-}^\circ\text{F}}\right)(80^\circ\text{F} - 70^\circ\text{F}) \\
&= 2.27\times10^7 \\
\mathrm{PrGr} &= (0.72)\left(2.27\times10^7\right) = 1.63\times10^7
\end{aligned}
$$

As in *NCEES Handbook* table "Forced Convection—External Flow," $C = 0.125$ and $n = 0.333$ for this GrPr value. The correlation equation for free convection over a horizontal cylinder can be found in *NCEES Handbook:* Natural (Free) Convection. Rearrange the correlation to find the film coefficient for a horizontal cylinder.

$$
\begin{aligned}
\mathrm{Nu}_{\mathrm{D}} &= C(\mathrm{Gr\,Pr})^n = \tfrac{h_o D}{k} \\
h_o &= \left(\tfrac{k}{D}\right)C(\mathrm{Gr\,Pr})^{1/3} \\
&= \left(\frac{0.015\ \frac{\text{Btu}}{\text{hr-ft-}^\circ\text{F}}}{1\ \text{ft}}\right)(0.125)\left(1.63\times10^7\right)^{1/3} \\
&= 0.48\ \text{Btu/hr-ft-}^\circ\text{F}
\end{aligned}
$$

Neglecting the wall resistance, the overall film coefficient from equationMERM36071 (also *NCEES Handbook:* Overall Heat-Transfer Coefficient) is

$$
\begin{aligned}
\frac{1}{U} &= \frac{1}{h_o} + \frac{1}{h_i} = \frac{1}{2.27\ \frac{\text{Btu}}{\text{hr-ft}^2\text{-}^\circ\text{F}}} + \frac{1}{0.48\ \frac{\text{Btu}}{\text{hr-ft}^2\text{-}^\circ\text{F}}} \\
&= 2.524\ \text{hr-ft}^2\text{-}^\circ\text{F/Btu} \\
U &= \frac{1}{2.524\ \frac{\text{hr-ft}^2\text{-}^\circ\text{F}}{\text{Btu}}} = 0.393\ \text{Btu/hr-ft}^2\text{-}^\circ\text{F}
\end{aligned}
$$

The heat transfer due to convection is

$$
\begin{aligned}
Q_{\text{convection}} &= UA_{\text{surface}}\left(T_\infty - T_{\text{bulk,air}}\right)\\
&= U(\pi dL)\left(T_\infty - T_{\text{bulk,air}}\right)\\
&= \left(0.393\ \frac{\text{Btu}}{\text{hr-ft}^2\text{-}^\circ\text{F}}\right)\\
&\quad \times\pi(1\ \text{ft})(50\ \text{ft})(80^\circ\text{F} - 47.5^\circ\text{F})\\
&= 2006.3\ \text{Btu/hr}
\end{aligned}
$$

The heat transfer due to radiation is as shown in *NCEES Handbook:* Radiation.

$$
Q_{\text{radiation}} = \sigma F_e F_a A_{\text{surface}}\left(T_\infty^4 - T_{\text{surface}}^4\right)
$$

Assume the room and duct have an unobstructed view of each other. Then $F_a = 1.0$ and $F_e = \epsilon = 0.28$. As in *NCEES Handbook:* Temperature, the absolute temperatures are

$$
\begin{aligned}
T_\infty &= 80^\circ\text{F} + 460^\circ = 540^\circ\text{R}\\
T_{\text{surface}} &= 70^\circ\text{F} + 460^\circ = 530^\circ\text{R}\\
Q_{\text{radiation}} &= \left(0.1713\times10^{-8}\ \frac{\text{Btu}}{\text{hr-ft}^2\text{-}^\circ\text{R}^4}\right)(0.28)(1.0)\\
&\quad \times\pi(1\ \text{ft})(50\ \text{ft})\left((540^\circ\text{R})^4 - (530^\circ\text{R})^4\right)\\
&= 461.5\ \text{Btu/hr}
\end{aligned}
$$

The total heat transfer to the air is

$$
\begin{aligned}
Q_{\text{total}} &= Q_{\text{convection}} + Q_{\text{radiation}}\\
&= 2006.3\ \frac{\text{Btu}}{\text{hr}} + 461.5\ \frac{\text{Btu}}{\text{hr}}\\
&= 2467.8\ \text{Btu/hr}
\end{aligned}
$$

At 47.5°F, the specific heat of air is approximately 0.240 Btu/lbm-°F. Since the heat transfer is known, the temperature of air leaving the duct can be calculated from

$$
\begin{aligned}
Q_{\text{total}} &= \dot{m}c_p\left(T_{\text{air,out}} - T_{\text{air,in}}\right)\\
T_{\text{air,out}} &= T_{\text{air,in}} + \frac{Q_{\text{total}}}{\dot{m}c_p}\\
&= 45^\circ\text{F} + \frac{2467.8\ \frac{\text{Btu}}{\text{hr}}}{\left(2357.1\ \frac{\text{lbm}}{\text{hr}}\right)\left(0.24\ \frac{\text{Btu}}{\text{lbm-}^\circ\text{F}}\right)}\\
&= 49.4^\circ\text{F}\quad(49^\circ\text{F})
\end{aligned}
$$

This agrees with the engineer's estimate.

The answer is (B).

SI Solution

As in *NCEES Handbook:* Temperature, the absolute temperature of air entering the duct is from

$$
7^\circ\text{C} + 273^\circ = 280\text{K}
$$

From the ideal gas law, the density of air entering the duct is

$$
\begin{aligned}
\rho &= \frac{pM}{RT} = \frac{(101.3\ \text{kPa})\left(29\ \frac{\text{kg}}{\text{kmol}}\right)}{\left(8.314\ \frac{\text{kJ}}{\text{kmol}\cdot\text{K}}\right)(280\text{K})}\\
&= 1.2604\ \text{kg/m}^3
\end{aligned}
$$

The mass flow rate of air entering the duct is

$$\dot{m} = \rho\dot{V} = \left(1.2604\ \frac{\text{kg}}{\text{m}^3}\right)\left(0.25\ \frac{\text{m}^3}{\text{s}}\right) = 0.3151\ \text{kg/s}$$

The diameter of the duct is

$$d = \frac{30\ \text{cm}}{100\ \frac{\text{cm}}{\text{m}}} = 0.30\ \text{m}$$

The velocity of air entering the duct is

$$\begin{aligned} \text{v} &= \frac{\dot{V}}{A_{\text{flow}}} = \frac{0.25\ \frac{\text{m}^3}{\text{s}}}{\left(\frac{\pi}{4}\right)(0.30\ \text{m})^2} \\ &= 3.537\ \text{m/s} \end{aligned}$$

To calculate the initial film coefficients, estimate the temperatures based on the claim. The film coefficients are not highly sensitive to small temperature differences.

$$\begin{aligned} T_{\text{bulk,air}} &= \tfrac{1}{2}(T_{\text{air,in}} + T_{\text{air,out}}) = \left(\frac{1}{2}\right)(7^\circ\text{C} + 10^\circ\text{C}) \\ &= 8.5^\circ\text{C} \\ T_{\text{surface}} &= 20^\circ\text{C} \quad \text{[estimate]} \end{aligned}$$

As in *NCEES Handbook:* Temperature-Dependent Properties of Air (SI Units), the viscosity and Prandtl number are

$$\mu = 20.9\ \mu\text{Pa·s}$$

$$\text{Pr} = 0.707$$

Using the volumetric flow rate and cross-sectional area of the duct, the velocity is

$$\begin{aligned} \text{v} &= \frac{\dot{V}}{A} = \frac{\dot{V}}{\frac{\pi D^2}{4}} = \frac{0.25\ \frac{\text{m}^3}{\text{s}}}{\frac{\pi(0.3\ \text{m})^2}{4}} \\ &= 3.54\ \text{m/s} \end{aligned}$$

As in *NCEES Handbook:* Dimensional Analysis, the Reynolds number is

$$\text{Re} = \frac{\rho u D}{\mu} = \frac{\left(1.2604\ \frac{\text{kg}}{\text{m}^3}\right)\left(3.54\ \frac{\text{m}}{\text{s}}\right)(0.3\ \text{m})}{(20.9\ \mu\text{Pa·s})\left(10^{-6}\ \frac{\text{Pa}}{\mu\,\text{Pa}}\right)} = 64{,}045$$

Refer to *NCEES Handbook:* Temperature-Dependent Properties of Air (SI Units) for the thermal conductivity of air. The correlation for forced convection in a pipe can be found in *NCEES Handbook:* Free/Forced Heat-Transfer Coefficients/Correlations. Rearrange the correlation to find the film coefficient for air flowing inside the duct. (Ignore the viscosity term for such a small temperature change.)

$$\begin{aligned} \text{Nu}_{\text{D}} &= 0.023\text{Re}_{\text{D}}{}^{0.8}\text{Pr}^{1/3}\left(\tfrac{\mu_\infty}{\mu_s}\right)^{0.14} = \tfrac{h_i D}{k} \\ h_i &= \left(\tfrac{k}{D}\right)0.023\text{Re}_{\text{D}}{}^{0.8}\text{Pr}^{1/3}\left(\tfrac{\mu_\infty}{\mu_s}\right)^{0.14} \\ &= \left(\frac{0.03\ \frac{\text{W}}{\text{m·K}}}{0.3\ \text{m}}\right)(0.023)\,(64045)^{0.8}(0.707)^{1/3} \\ &= 14.34\ \text{W/m}^2\text{·K} \end{aligned}$$

For natural convection on the outside of the duct, estimate the film coefficient.

$$\begin{aligned} T_{\text{film}} &= \tfrac{1}{2}(T_{\text{surface}} + T_{\infty}) = \left(\frac{1}{2}\right)(20^{\circ}\text{C} + 27^{\circ}\text{C}) \\ &= 23.5^{\circ}\text{C} \end{aligned}$$

From appendixMERM35D (also *NCEES Handbook:* Temperature-Dependent Properties of Air (SI Units)), the properties of air at 23.5°C are

$$\begin{aligned} \text{Pr} &= 0.709 \\ \frac{g\beta\rho^2}{\mu^2} &= 1.43 \times 10^8\ \frac{1}{\text{K}\cdot\text{m}^3} \end{aligned}$$

The characteristic length, L, is the diameter of the duct, which is 0.30 m. The Grashof number is

$$\begin{aligned} \text{Gr} &= L^3\left(\frac{\rho^2\beta g}{\mu^2}\right)(T_{\infty} - T_{\text{surface}}) \\ &= (0.30\ \text{m})^3\left(1.43 \times 10^8\ \frac{1}{\text{K}\cdot\text{m}^3}\right)(27^{\circ}\text{C} - 20^{\circ}\text{C}) \\ &= 2.70 \times 10^7 \\ \text{PrGr} &= (0.709)\left(2.70 \times 10^7\right) = 1.9 \times 10^7 \end{aligned}$$

As in *NCEES Handbook:* Free/Forced Heat-Transfer Coefficients/Correlations, $C = 0.125$ and $n = 0.333$ for this GrPr value. The correlation for free convection over a horizontal cylinder can be found in *NCEES Handbook:* Natural (Free) Convection. Rearrange the correlation to find the film coefficient for a horizontal cylinder.

$$\begin{aligned} \text{Nu}_{\text{D}} &= C(\text{Gr}_{\text{D}}\text{PR})^n = \tfrac{h_o D}{k} \\ h_o &= \left(\tfrac{k}{D}\right) C(\text{Gr}_{\text{D}}\text{PR})^{1/3} \\ &= \left(\frac{0.03\ \frac{\text{W}}{\text{m}\cdot\text{K}}}{0.3\ \text{m}}\right)(0.125)\left(1.9 \times 10^7\right)^{1/3} \\ &= 3.33\ \text{W/m}^2\cdot\text{K} \end{aligned}$$

Neglecting the wall resistance, the overall film coefficient from equationMERM36071 (also *NCEES Handbook:* Overall Heat-Transfer Coefficient) is

$$\begin{aligned} \frac{1}{U} &= \frac{1}{h_o} + \frac{1}{h_i} = \frac{1}{3.3\ \frac{\text{W}}{\text{m}^2\cdot\text{K}}} + \frac{1}{14.34\ \frac{\text{W}}{\text{m}^2\cdot\text{K}}} \\ &= 0.372\ \text{m}^2\cdot\text{K/W} \\ U &= \frac{1}{0.372\ \frac{\text{m}^2\cdot\text{K}}{\text{W}}} = 2.70\ \text{W/m}^2\cdot\text{K} \end{aligned}$$

The heat transfer due to convection is

$$\begin{aligned} Q_{\text{convection}} &= UA_{\text{surface}}\left(T_{\infty} - T_{\text{bulk,air}}\right) \\ &= U\pi dL\left(T_{\infty} - T_{\text{bulk,air}}\right) \\ &= \left(2.70\ \frac{\text{W}}{\text{m}^2\cdot\text{K}}\right)\pi(0.30\ \text{m})(15\ \text{m}) \\ &\quad \times (27^{\circ}\text{C} - 8.5^{\circ}\text{C}) \\ &= 706\ \text{W} \end{aligned}$$

The heat transfer due to radiation is

$$Q_{\text{radiation}} = \sigma F_e F_a A_{\text{surface}}\left(T_{\infty}^4 - T_{\text{surface}}^4\right)$$

Assume the room and duct have an unobstructed view of each other. Then $F_a = 1.0$ and $F_e = \epsilon = 0.28$. The absolute temperatures are as in *NCEES Handbook:* Temperature.

$$
\begin{aligned}
T_{\infty} &= 27^{\circ}\text{C} + 273^{\circ} = 300\text{K} \\
T_{\text{surface}} &= 20^{\circ}\text{C} + 273^{\circ} = 293\text{K} \\
Q_{\text{radiation}} &= \left(5.67 \times 10^{-8}\ \frac{\text{W}}{\text{m}^2{\cdot}\text{K}^4}\right)(0.28)(1)\,\pi \\
&\qquad \times (0.30\ \text{m})(15\ \text{m})\left((300\text{K})^4 - (293\text{K})^4\right) \\
&= 163.8\ \text{W}
\end{aligned}
$$

The total heat transfer to the air is

$$
\begin{aligned}
Q_{\text{total}} &= Q_{\text{convection}} + Q_{\text{radiation}} = 706\ \text{W} + 163.8\ \text{W} \\
&= 869.8\ \text{W}
\end{aligned}
$$

At 8.5°C, the specific heat of air is

$$
\left(1.0048\ \frac{\text{kJ}}{\text{kg}{\cdot}\text{K}}\right)\left(1000\ \frac{\text{J}}{\text{kJ}}\right) = 1004.8\ \text{J/kg}{\cdot}\text{K}
$$

Since the heat transfer is known, the temperature of air leaving the duct can be calculated from

$$
\begin{aligned}
Q_{\text{total}} &= \dot{m}c_p\left(T_{\text{air,out}} - T_{\text{air,in}}\right) \\
T_{\text{air,out}} &= T_{\text{air,in}} + \frac{Q_{\text{total}}}{\dot{m}c_p} \\
&= 7^{\circ}\text{C} + \frac{869.8\ \text{W}}{\left(0.3151\ \frac{\text{kg}}{\text{s}}\right)\left(1004.8\ \frac{\text{J}}{\text{kg}{\cdot}\text{K}}\right)} \\
&= 9.7^{\circ}\text{C}
\end{aligned}
$$

This agrees with the engineer's estimate.

The answer is (B).
3.

Customary U.S. Solution

As in *NCEES Handbook:* Temperature, the absolute temperature of entering air is

$$
500^{\circ}\text{F} + 460^{\circ} = 960^{\circ}\text{R}
$$

The absolute pressure of entering air is

$$
25\ \text{psig} + 14.7\ \frac{\text{lbf}}{\text{in}^2} = 39.7\ \text{psia}
$$

The density of air entering, from the ideal gas law, is

$$
\begin{aligned}
\rho &= \frac{pM}{RT} = \frac{\left(39.7\ \frac{\text{lbf}}{\text{in}^2}\right)\left(12\ \frac{\text{in}}{\text{ft}}\right)^2\left(29\ \frac{\text{lbm}}{\text{lbmol}}\right)}{\left(1545.35\ \frac{\text{ft-lbf}}{\text{lbmol-}^{\circ}\text{R}}\right)(960^{\circ}\text{R})} \\
&= 0.1116\ \text{lbm/ft}^3
\end{aligned}
$$

The mass flow rate of entering air is

$$
\begin{aligned}
\dot{m} &= \rho\dot{V} = \left(0.1116\ \frac{\text{lbm}}{\text{ft}^3}\right)\left(200\ \frac{\text{ft}^3}{\text{min}}\right)\left(60\ \frac{\text{min}}{\text{hr}}\right) \\
&= 1339.2\ \text{lbm/hr}
\end{aligned}
$$

The absolute temperature of leaving air is

$$
350^{\circ}\text{F} + 460^{\circ} = 810^{\circ}\text{R}
$$

The heat loss is

$$\begin{aligned}\dot{Q} &= \dot{m}(h_1 - h_2) = \dot{m}c_p(T_1 - T_2)\\ &= \left(1339.2\ \tfrac{\text{lbm}}{\text{hr}}\right)\left(0.242\ \tfrac{\text{Btu}}{\text{lbm-}^\circ\text{F}}\right)(500^\circ\text{F} - 350^\circ\text{F})\\ &= 48{,}613\ \text{Btu/hr}\end{aligned}$$

Assuming midpoint pipe surface temperature,

$$\tfrac{1}{2}(T_{\text{in}} + T_{\text{out}}) = \left(\frac{1}{2}\right)(500^\circ\text{F} + 350^\circ\text{F}) = 425^\circ\text{F}$$

Since the heat loss is known, the overall heat transfer coefficient can be determined from

$$\begin{aligned}Q &= UA\Delta T = U(\pi dL)\Delta T\\ U &= \frac{Q}{(\pi dL)\Delta T}\\ &= \frac{\left(48{,}613\ \dfrac{\text{Btu}}{\text{hr}}\right)\left(12\ \dfrac{\text{in}}{\text{ft}}\right)}{\pi(4.25\ \text{in})(35\ \text{ft})(425^\circ\text{F} - 70^\circ\text{F})}\\ &= 3.52\ \text{Btu/hr-ft}^2\text{-}^\circ\text{F}\end{aligned}$$

To calculate the overall heat transfer coefficient, disregard the pipe thermal resistance and the inside film coefficient (small compared with outside film and radiation). Work with the midpoint pipe temperature of 425°F.

The absolute temperatures are

$$\begin{aligned}T_1 &= 425^\circ\text{F} + 460^\circ = 885^\circ\text{R}\\ T_2 &= 70^\circ\text{F} + 460^\circ = 530^\circ\text{R}\end{aligned}$$

For radiation heat loss, assume $F_a = 1$ and use *NCEES Handbook:* Emissivity (ε). For 500°F enamel paint of any color,

$$\begin{aligned}F_e &= \epsilon \approx 0.9\\ \frac{Q_{\text{net}}}{A} &= E_{\text{net}} = \sigma F_e F_a\left(T_1^4 - T_2^4\right)\\ &= \left(0.1713 \times 10^{-8}\ \frac{\text{Btu}}{\text{hr-ft}^2\text{-}^\circ\text{R}^4}\right)\\ &\quad \times (0.9)(1.0)\left((885^\circ\text{R})^4 - (530^\circ\text{R})^4\right)\\ &= 824.1\ \text{Btu/hr-ft}^2\end{aligned}$$

From equationMERM37021, the radiant heat transfer coefficient is

$$\begin{aligned}h_{\text{radiation}} &= \frac{E_{\text{net}}}{T_1 - T_2} = \frac{824.1\ \dfrac{\text{Btu}}{\text{hr-ft}^2}}{885^\circ\text{R} - 530^\circ\text{R}}\\ &= 2.32\ \text{Btu/hr-ft}^2\text{-}^\circ\text{F}\end{aligned}$$

For the outside film coefficient, evaluate the film at the pipe midpoint. The film temperature is

$$T_f = \left(\frac{1}{2}\right)(425^\circ\text{F} + 70^\circ\text{F}) = 247.5^\circ\text{F}$$

From appendixMERM35C (also *NCEES Handbook:* Temperature-Dependent Properties of Air (U.S. Customary Units)),

$$\begin{aligned}\text{Pr} &= 0.72\\ \frac{g\beta\rho^2}{\mu^2} &= 0.657 \times 10^6\ \frac{1}{\text{ft}^3\text{-}^\circ\text{F}}\end{aligned}$$

The characteristic length, L, is $d_o = 4.25$ in. The Grashof number is

$$
\begin{aligned}
\mathrm{Gr} &= L^3\left(\frac{\rho^2\beta g}{\mu^2}\right)\Delta T \\
&= \frac{(4.25\ \mathrm{in})^3\left(0.657\times 10^6\ \dfrac{1}{\mathrm{ft}^3\text{-}^\circ\mathrm{F}}\right)(425^\circ\mathrm{F}-70^\circ\mathrm{F})}{\left(12\ \dfrac{\mathrm{in}}{\mathrm{ft}}\right)^3} \\
&= 1.04\times 10^7
\end{aligned}
$$

$$
\mathrm{PrGr} = (0.72)\left(1.04\times 10^7\right) = 7.5\times 10^6
$$

As in *NCEES Handbook* table "Natural (Free) Convection," $n = 1/4$, $C = 0.480$ for this GrPr value. The correlation for free convection over a horizontal cylinder can be found in *NCEES Handbook:* Natural (Free) Convection. Rearrange the correlation to find the film coefficient for a horizontal cylinder.

$$
\begin{aligned}
\mathrm{Nu}_{\mathrm{D}} &= C(\mathrm{GrPr})^n = \tfrac{h_o D}{k} \\
h_o &= \left(\tfrac{k}{D}\right) C(\mathrm{GrPr})^n \\
&= \left(\frac{0.02\ \frac{\mathrm{Btu}}{\mathrm{hr\text{-}ft\text{-}^\circ F}}}{0.354\ \mathrm{ft}}\right)(0.480)\left(7.5\times 10^6\right)^{1/4} \\
&= 1.42\ \mathrm{Btu/hr\text{-}ft^2\text{-}^\circ F}
\end{aligned}
$$

The overall film coefficient is

$$
\begin{aligned}
U &= h_{\mathrm{total}} = h_{\mathrm{radiation}} + h_o \\
&= 2.32\ \frac{\mathrm{Btu}}{\mathrm{hr\text{-}ft^2\text{-}^\circ F}} + 1.42\ \frac{\mathrm{Btu}}{\mathrm{hr\text{-}ft^2\text{-}^\circ F}} \\
&= 3.74\ \mathrm{Btu/hr\text{-}ft^2\text{-}^\circ F} \quad (3.8\ \mathrm{Btu/hr\text{-}ft^2\text{-}^\circ F})
\end{aligned}
$$

The answer is (C).

SI Solution

The absolute temperature of entering air is

$$
260^\circ\mathrm{C} + 273^\circ = 533\mathrm{K}
$$

The absolute pressure of entering air is

$$
170\ \mathrm{kPa} + 101.3\ \mathrm{kPa} = 271.3\ \mathrm{kPa}
$$

The density of air entering, from the ideal gas law, is

$$
\begin{aligned}
\rho &= \frac{pM}{RT} = \frac{(271.3\ \mathrm{kPa})\left(29\ \frac{\mathrm{kg}}{\mathrm{kmol}}\right)}{\left(8.314\ \frac{\mathrm{kJ}}{\mathrm{kmol\cdot K}}\right)(533\mathrm{K})} \\
&= 1.773\ \mathrm{kg/m^3}
\end{aligned}
$$

The mass flow rate of entering air is

$$
\begin{aligned}
\dot{m} &= \rho\dot{V} = \left(1.773\ \frac{\mathrm{kg}}{\mathrm{m}^3}\right)\left(0.1\ \frac{\mathrm{m}^3}{\mathrm{s}}\right) \\
&= 0.1773\ \mathrm{kg/s}
\end{aligned}
$$

The absolute temperature of leaving air is

$$
180^\circ\mathrm{C} + 273^\circ = 453\mathrm{K}
$$

The heat loss is

$$
\begin{aligned}
Q &= \dot{m}\left(h_1 - h_2\right) = \dot{m}c_p\left(T_1 - T_2\right) \\
&= \left(0.1773\ \tfrac{\text{kg}}{\text{s}}\right)\left(1\ \tfrac{\text{kJ}}{\text{kg·K}}\right)\left(533\text{K} - 453\text{K}\right) \\
&= 14.2\ \text{kW}
\end{aligned}
$$

Assuming midpoint pipe surface temperature,

$$
\tfrac{1}{2}\left(T_{\text{in}} - T_{\text{out}}\right) = \left(\frac{1}{2}\right)\left(260^\circ\text{C} + 180^\circ\text{C}\right) = 220^\circ\text{C}
$$

Since the heat loss is known, the overall heat transfer coefficient can be determined from

$$
Q = UA\Delta T = U\left(\pi dL\right)\Delta T
$$

$$
\begin{aligned}
U &= \frac{Q}{\left(\pi dL\right)\Delta T} \\
&= \frac{\left(14\,200\ \text{W}\right)\left(100\ \dfrac{\text{cm}}{\text{m}}\right)}{\pi\left(10.8\ \text{cm}\right)\left(10\ \text{m}\right)\left(220^\circ\text{C} - 20^\circ\text{C}\right)} \\
&= 20.92\ \text{W/m}^2\text{·K}
\end{aligned}
$$

From the customary U.S. solution, work with the midpoint pipe temperature of 220°C.

As in *NCEES Handbook:* Temperature, the absolute temperatures are

$$
\begin{aligned}
T_1 &= 220^\circ\text{C} + 273^\circ = 493\text{K} \\
T_2 &= 20^\circ\text{C} + 273^\circ = 293\text{K}
\end{aligned}
$$

For radiation heat loss, assume $F_a = 1$ and use *NCEES Handbook:* Emissivity (ε). For 260°C enamel paint of any color,

$$
F_e = \epsilon \approx 0.9
$$

From equationMERM37021, the radiant heat transfer coefficient is

$$
\begin{aligned}
h_{\text{radiation}} &= \frac{\sigma F_a F_e\left(T_1^4 - T_2^4\right)}{T_1 - T_2} \\
&= \frac{\begin{gathered}\left(5.67\times10^{-8}\ \dfrac{\text{W}}{\text{m}^2\text{·K}^4}\right)(1)(0.9) \\ \times\left(\left(493\text{K}\right)^4 - \left(293\text{K}\right)^4\right)\end{gathered}}{493\text{K} - 293\text{K}} \\
&= 13.2\ \text{W/m}^2\text{·K}
\end{aligned}
$$

For the outside film coefficient, evaluate the film at the pipe midpoint. The film temperature is

$$
T_f = \left(\frac{1}{2}\right)\left(220^\circ\text{C} + 20^\circ\text{C}\right) = 120^\circ\text{C}
$$

From appendixMERM35D (also *NCEES Handbook* table "Temperature-Dependent Properties of Air (SI Units)"), at 120°C,

$$
\begin{aligned}
\text{Pr} &\approx 0.692 \\
\frac{g\beta\rho^2}{\mu^2} &= 0.528\times10^8\ \frac{1}{\text{K·m}^3}
\end{aligned}
$$

The characteristic length is

$$
L = \text{outside diameter} = \frac{10.8\ \text{cm}}{100\ \dfrac{\text{cm}}{\text{m}}} = 0.108\ \text{m}
$$

As in *NCEES Handbook:* Similitude, the Grashof number is

$$\begin{aligned}\mathrm{Gr} &= L^3\left(\frac{\rho^2 g\beta}{\mu^2}\right)\Delta T\\ &= (0.108\ \mathrm{m})^3\left(0.528\times 10^8\ \frac{1}{\mathrm{K\cdot m^3}}\right)(220^\circ\mathrm{C}-20^\circ\mathrm{C})\\ &= 1.33\times 10^7\end{aligned}$$

$$\mathrm{PrGr} = (0.692)\left(1.33\times 10^7\right) = 9.20\times 10^6$$

As in *NCEES Handbook* table "Natural (Free) Convection," $C = 0.480$ and $n = 0.25$ for this GrPr value. The correlation for free convection over a horizontal cylinder can be found in *NCEES Handbook:* Natural (Free) Convection. Rearrange the correlation to find the film coefficient for a horizontal cylinder.

$$\begin{aligned}\mathrm{Nu}_\mathrm{D} &= C(\mathrm{Gr_D PR})^n = \tfrac{h_o D}{k}\\ h_o &= \left(\tfrac{k}{D}\right)C(\mathrm{Gr_D PR})^n\\ &= \left(\frac{0.034\ \frac{\mathrm{W}}{\mathrm{m\cdot K}}}{0.108\ \mathrm{m}}\right)(0.480)\left(9.2\times 10^6\right)^{1/4}\\ &= 8.32\ \ \mathrm{W/m^2\cdot K}\end{aligned}$$

The overall film coefficient is

$$\begin{aligned}U &= h_\mathrm{total} = h_\mathrm{radiation} + h_o\\ &= 13.2\ \frac{\mathrm{W}}{\mathrm{m^2\cdot K}} + 8.32\ \frac{\mathrm{W}}{\mathrm{m^2\cdot K}}\\ &= 21.52\ \mathrm{W/m^2\cdot K}\quad (22\ \mathrm{W/m^2\cdot K})\end{aligned}$$

The answer is (C).
4.

Customary U.S. Solution

The velocity is relatively low, so incompressible flow can be assumed.

The film coefficient on the probe is

$$\begin{aligned}h &= \frac{0.024 G^{0.8}}{D^{0.4}} = \frac{(0.024)\left(3480\ \frac{\mathrm{lbm}}{\mathrm{hr\text{-}ft^2}}\right)^{0.8}}{\left(\frac{0.5\ \mathrm{in}}{12\ \frac{\mathrm{in}}{\mathrm{ft}}}\right)^{0.4}}\\ &= 58.3\ \mathrm{Btu/hr\text{-}ft^2\text{-}^\circ F}\end{aligned}$$

As in *NCEES Handbook:* Temperature, the absolute temperature of the walls is

$$T_\mathrm{walls} = 600^\circ\mathrm{F} + 460^\circ = 1060^\circ\mathrm{R}$$

Neglect conduction and the insignificant kinetic energy loss. The thermocouple gains heat through radiation from the walls and loses heat through convection to the gas.

$$\begin{aligned}Q_\mathrm{convection} &= AE_\mathrm{radiation}\\ hA\left(T_\mathrm{probe} - T_\mathrm{gas}\right) &= A\sigma\epsilon\left(T^4_\mathrm{walls} - T^4_\mathrm{probe}\right)\\ h\left(T_\mathrm{probe} - T_\mathrm{gas}\right) &= \sigma\epsilon\left(T^4_\mathrm{walls} - T^4_\mathrm{probe}\right)\end{aligned}$$

If $T_\mathrm{probe} = 300°\mathrm{F} + 460° = 760°\mathrm{R}$,

$$
\begin{aligned}
&\left(58.3\ \frac{\text{Btu}}{\text{hr-ft}^2\text{-}^\circ\text{F}}\right)\\
&\quad\times(760^\circ\text{R}-T_{\text{gas}}) = \left(0.1713\times10^{-8}\ \frac{\text{Btu}}{\text{hr-ft}^2\text{-}^\circ\text{R}^4}\right)\\
&\qquad\qquad\times(0.8)\left((1060^\circ\text{R})^4-(760^\circ\text{R})^4\right)\\
&\qquad T_{\text{gas}} = 738.2^\circ\text{R}\quad(740^\circ\text{R})
\end{aligned}
$$

The answer is (B).

SI Solution

The velocity is relatively low, so incompressible flow can be assumed.

The film coefficient on the probe is

$$
\begin{aligned}
h &= \frac{17G^{0.8}}{D^{0.4}} = \frac{(17)\left(4.7\ \frac{\text{kg}}{\text{s}\cdot\text{m}^2}\right)^{0.8}}{\left(\frac{13\ \text{mm}}{1000\ \frac{\text{mm}}{\text{m}}}\right)^{0.4}}\\
&= 333.1\ \text{W/m}^2\cdot\text{K}
\end{aligned}
$$

As in *NCEES Handbook:* Temperature, the absolute temperature of the walls is

$$T_{\text{walls}} = 315^\circ\text{C}+273^\circ = 588\text{K}$$

Neglect conduction and the insignificant kinetic energy loss. The thermocouple gains heat through radiation from the walls and loses heat through convection to the gas.

$$
\begin{aligned}
\frac{Q_{\text{convection}}}{A} &= E_{\text{radiation}}\\
h\left(T_{\text{probe}}-T_{\text{gas}}\right) &= \sigma\epsilon\left(T_{\text{walls}}^4-T_{\text{probe}}^4\right)
\end{aligned}
$$

If $T_{\text{probe}} = 150^\circ\text{C} + 273^\circ = 423\text{K}$,

$$
\begin{aligned}
&\left(333.1\ \frac{\text{W}}{\text{m}^2\cdot\text{K}}\right)\\
&\quad\times(423\text{K}-T_{\text{gas}}) = \left(5.67\times10^{-8}\ \frac{\text{W}}{\text{m}^2\cdot\text{K}^4}\right)(0.8)\\
&\qquad\qquad\times\left((588\text{K})^4-(423\text{K})^4\right)\\
&\qquad T_{\text{gas}} = 411.1\text{K}\quad(410\text{K})
\end{aligned}
$$

The answer is (B).
5.

Customary U.S. Solution

As in *NCEES Handbook:* Temperature, the absolute temperatures are

$$
\begin{aligned}
T_\infty &= 80^\circ\text{F}+460^\circ = 540^\circ\text{R}\\
T_{\text{duct}} &= 200^\circ\text{F}+460^\circ = 660^\circ\text{R}\\
T_{\text{wall}} &= 70^\circ\text{F}+460^\circ = 530^\circ\text{R}
\end{aligned}
$$

As in *NCEES Handbook:* Temperature-Dependent Properties of Air (U.S. Customary Units),

$$\rho = 0.071\ \frac{\text{lbm}}{\text{ft}^3}$$

$$\mu = 1.28\times10^{-5}\ \frac{\text{lbm}}{\text{ft-sec}}$$

$$\mathrm{Pr} = 0.708$$

$$k = 0.0155\ \frac{\mathrm{Btu}}{\mathrm{hr\text{-}ft\text{-}^\circ F}}$$

The average temperature is

$$\begin{aligned} T_{\mathrm{avg}} &= \tfrac{1}{\beta} = \tfrac{T_\infty + T_{\mathrm{duct}}}{2} = \tfrac{540^\circ\mathrm{R} + 660^\circ\mathrm{R}}{2} \\ &= 600^\circ\mathrm{R} \end{aligned}$$

The characteristic length of the cylinder is its diameter, which is 0.75 ft. Substituting the definition of the Grashof number (see *NCEES Handbook* table "Dimensionless Numbers"),

$$\mathrm{GrPr} = \frac{g\beta\rho^2}{\mu^2}(T_s - T_\infty)\, L^3 \mathrm{Pr} = 5.9 \times 10^7$$

As in *NCEES Handbook* table "Natural (Free) Convection," $C = 0.125$ and $n = 0.333$ for this GrPr value. The correlation for free convection over a horizontal cylinder can be found in *NCEES Handbook:* Natural (Free) Convection. Rearrange the correlation to find the convective film coefficient for the outside of the duct.

$$\begin{aligned} \mathrm{Nu}_{\mathrm{D}} &= C(\mathrm{Gr_D PR})^n = \tfrac{hD}{k} \\ h &= \left(\tfrac{k}{D}\right) C(\mathrm{Gr_D PR})^{1/3} \\ &= \left(\frac{0.0155\ \frac{\mathrm{Btu}}{\mathrm{hr\text{-}ft\text{-}^\circ F}}}{0.75\ \mathrm{ft}}\right)(0.125)\left(5.9 \times 10^7\right)^{1/3} \\ &= 1.00\ \mathrm{Btu/hr\text{-}ft^2\text{-}^\circ F} \end{aligned}$$

The duct area per unit length is

$$\frac{A}{L} = \pi D = \frac{\pi\,(9\ \mathrm{in})}{12\ \frac{\mathrm{in}}{\mathrm{ft}}} = 2.356\ \mathrm{ft^2/ft}$$

The convection losses (per unit length) are

$$\begin{aligned} \frac{Q_{\mathrm{convection}}}{L} &= h\left(\frac{A}{L}\right)\Delta T = h\left(\frac{A}{L}\right)(T_{\mathrm{duct}} - T_\infty) \\ &= \left(1.00\ \frac{\mathrm{Btu}}{\mathrm{hr\text{-}ft^2\text{-}^\circ F}}\right)\left(2.356\ \frac{\mathrm{ft^2}}{\mathrm{ft}}\right) \\ &\quad \times (660^\circ\mathrm{R} - 540^\circ\mathrm{R}) \\ &= 282.7\ \mathrm{Btu/hr\text{-}ft} \end{aligned}$$

Assume $\epsilon_{\mathrm{duct}} \approx 0.90$. Then $F_e = \epsilon_{\mathrm{duct}} = 0.90$. $F_a = 1$ since the duct is enclosed. The radiation losses (per unit length) are

$$\begin{aligned} \frac{E_{\mathrm{net}}}{L} &= \left(\frac{A}{L}\right)\sigma F_a F_e\,(T_{\mathrm{duct}}^4 - T_{\mathrm{wall}}^4) \\ &= \left(2.356\ \mathrm{ft^2}\right)\left(0.1713 \times 10^{-8}\ \frac{\mathrm{Btu}}{\mathrm{hr\text{-}ft^2\text{-}^\circ R^4}}\right) \\ &\quad \times (1)(0.90)\left((660^\circ\mathrm{R})^4 - (530^\circ\mathrm{R})^4\right) \\ &= 402.6\ \mathrm{Btu/hr\text{-}ft} \end{aligned}$$

The total heat transfer per unit length is

$$\begin{aligned} \frac{Q_{\mathrm{total}}}{L} &= \frac{Q_{\mathrm{convection}}}{L} + \frac{E_{\mathrm{net}}}{L} = 282.7\ \frac{\mathrm{Btu}}{\mathrm{hr\text{-}ft}} + 402.6\ \frac{\mathrm{Btu}}{\mathrm{hr\text{-}ft}} \\ &= 685\ \mathrm{Btu/hr\text{-}ft\ length} \quad (685\ \mathrm{Btu/hr\text{-}ft\ length}) \end{aligned}$$

The answer is (A).

SI Solution

As in *NCEES Handbook:* Temperature, the absolute temperatures are

$$\begin{aligned} T_{\text{duct}} &= 95^\circ\text{C} + 273^\circ = 368\text{K} \\ T_{\text{wall}} &= 20^\circ\text{C} + 273^\circ = 293\text{K} \\ T_\infty &= 27^\circ\text{C} + 273^\circ = 300\text{K} \end{aligned}$$

The characteristic length of a cylinder is its diameter. From *NCEES Handbook:* Temperature-Dependent Properties of Air (SI Units),

$$\rho = 1.067\ \frac{\text{kg}}{\text{m}^3}$$

$$\mu = 20.2\mu\text{Pa} \cdot \text{s}$$

$$\text{Pr} = 0.714$$

$$k = 0.0285\ \frac{\text{W}}{\text{m} \cdot \text{K}}$$

The average temperature is

$$\begin{aligned} T_{\text{avg}} &= \tfrac{1}{\beta} = \tfrac{T_\infty + T_{\text{duct}}}{2} = \tfrac{300\text{K}+368\text{K}}{2} \\ &= 334\text{K} \end{aligned}$$

Substituting the definition of the Grashof number (see *NCEES Handbook* table "Dimensionless Numbers"),

$$\text{GrPr} = \frac{g\beta\rho^2}{\mu^2}(T_s - T_\infty)\, L^3\text{Pr} = 5 \times 10^7$$

The characteristic length of the cylinder is its diameter, which is 0.23 m. As in *NCEES Handbook* table "Natural (Free) Convection," $C =$ 0.125 and $n =$ 0.333 for this GrPr value. The correlation for free convection over a horizontal cylinder can be found in the same table. Rearrange the correlation to find the convective film coefficient for the outside of the duct.

$$\begin{aligned} \text{Nu}_{\text{D}} &= C(\text{Gr}_{\text{D}}\text{PR})^n = \tfrac{hD}{k} \\ h &= \left(\tfrac{k}{D}\right) C(\text{Gr}_{\text{D}}\text{PR})^{1/3} \\ &= \left(\frac{0.0285\ \frac{\text{W}}{\text{m}\cdot\text{K}}}{0.23\ \text{m}}\right)(0.125)\left(5 \times 10^7\right)^{1/3} \\ &= 5.70\ \text{W/m}^2\cdot\text{K} \end{aligned}$$

The duct area per unit length is

$$\frac{A}{L} = \pi D = \frac{\pi\,(23\ \text{cm})}{100\ \frac{\text{cm}}{\text{m}}} = 0.723\ \text{m}^2/\text{m}$$

The convective losses per unit length are

$$\begin{aligned} \frac{Q_{\text{convective}}}{L} &= h\left(\frac{A}{L}\right)(T_{\text{duct}} - T_\infty) \\ &= \left(5.70\ \frac{\text{W}}{\text{m}^2\cdot\text{K}}\right)\left(0.723\ \frac{\text{m}^2}{\text{m}}\right)(368\text{K} - 300\text{K}) \\ &= 280.2\ \text{W/m} \end{aligned}$$

Assuming $\epsilon_{\text{duct}} \approx 0.90$, $F_e = \epsilon_{\text{duct}} = 0.90$. $F_a = 1$ since the duct is enclosed. The radiation losses per unit length are

$$
\begin{aligned}
\frac{E_{\text{net}}}{L} &= \left(\frac{A}{L}\right)\sigma F_a F_e \left(T_{\text{duct}}^4 - T_{\text{wall}}^4\right) \\
&= \left(0.723\ \frac{\text{m}^2}{\text{m}}\right)\left(5.67\times 10^{-8}\ \frac{\text{W}}{\text{m}^2\cdot\text{K}^4}\right) \\
&\quad \times (1)\,(0.90)\left((368\text{K})^4 - (293\text{K})^4\right) \\
&= 404.7\ \text{W/m}
\end{aligned}
$$

The total heat transfer per unit length is

$$
\begin{aligned}
\frac{Q_{\text{total}}}{L} &= \frac{Q_{\text{convection}}}{L} + \frac{E_{\text{net}}}{L} = 280.2\ \frac{\text{W}}{\text{m}} + 404.7\ \frac{\text{W}}{\text{m}} \\
&= 684.9\ \text{W/m length} \quad (685\ \text{W/m length})
\end{aligned}
$$

The answer is (A).