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Chapter 4. Fluid Dynamics

Practice Problems

(Use g = 32.2 ft/sec^2 (9.81 m/s^2) unless told otherwise.)

1.

The capacity of an old municipal pressurized water supply pipe system must be doubled without increasing the average velocity. A new replacement pipe will have 40% less friction than the existing pipe. What is the approximate required increase in pipe diameter?

(A)

20%

(B)

30%

(C)

40%

(D)

100%

2.

A pressurized water pipe system will be modified such that the pipe diameter will be doubled and the average flow velocity halved. Fluid properties are unchanged. When these changes are made, the Reynolds number will

(A)

halve

(B)

double

(C)

quadruple

(D)

remain the same

3.

5 ft^3/sec (130 L/s) of water flows through a schedule-40 steel pipe that changes gradually in diameter from 6 in at point A to 18 in at point B. Point B is 15 ft (4.6 m) higher than point A. The respective pressures at points A and B are 10 psia (70 kPa) and 7 psia (48.3 kPa). All minor losses are insignificant. The velocity and direction of flow at point A are most nearly

(A)

3.2 ft/sec (1 m/s); from A to B

(B)

25 ft/sec (7 m/s); from A to B

(C)

3.2 ft/sec (1 m/s); from B to A

(D)

25 ft/sec (7 m/s); from B to A
4.

Points A and B are separated by 3000 ft of new 6 in schedule-40 steel pipe. 750 gal/min of 60°F water flows from point A to point B. Point B is 60 ft above point A. Approximately what must be the pressure at point A if the pressure at B must be 50 psig?

(A)

90 psig

(B)

100 psig

(C)

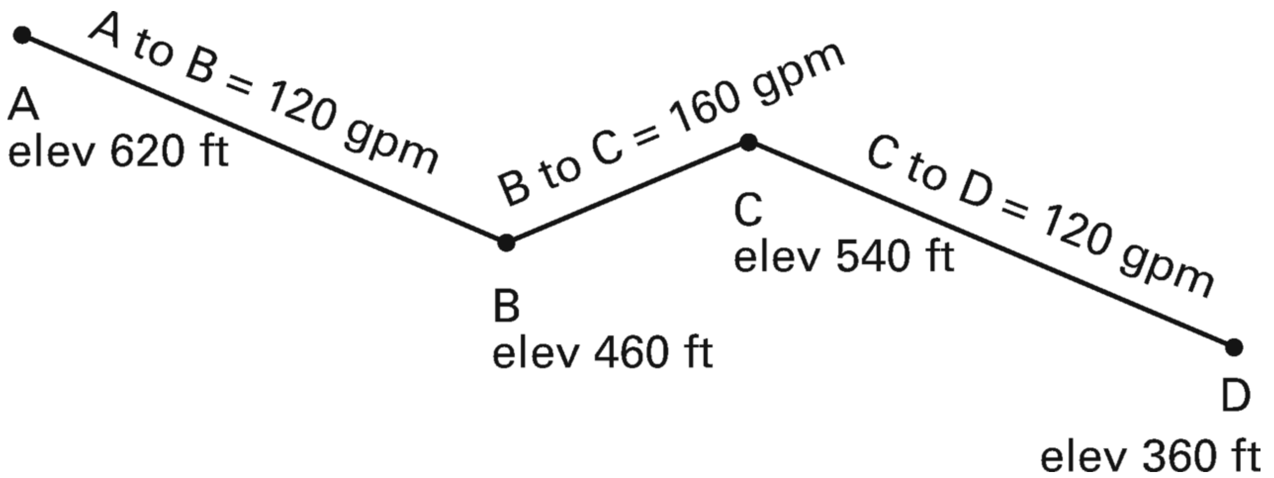
120 psig

(D)

170 psig

5.

A pipe network connects junctions A, B, C, and D as shown. All pipe sections have a Hazen-Williams *C*-value of 150. Water can be added and removed at any of the junctions to achieve the flows listed. Water flows from point A to point D. No flows are backward. All minor losses are insignificant. For simplicity, use the nominal piple diameter listed.



The Hazen-Williams frictional head loss is

$$h_f = \frac{10.44 L_{ft} Q_{gpm}^{1.85}}{C^{1.85} d_{inch}^{4.87}}$$

The lengths and the nominal pipe diameters of the pipe network are as shown.

path	*length*	*nominal diameter*
A to B	20,000 ft	6 in
B to C	10,000 ft	6 in
C to D	30,000 ft	4 in

If the minimum static pressure anywhere in the system is 20 psig, the pressure at point A is most nearly

(A)

14 psig

(B)

17 psig

(C)

24 psig

(D)

30 psig
6.

Based on the formulas commonly used by engineers, is friction head loss ever proportional to velocity, instead of velocity squared?

(A)

yes, in laminar flow

(B)

yes, for non-Newtonian fluids

(C)

yes, in smooth pipes

(D)

no, never

7.

Water flows at 10 ft/s through a pipe network consisting of 50 ft of new 3 in diameter, schedule-40 steel pipe, four 45° standard elbows, and a fully open gate valve. All fittings are flanged. The elevation of the network discharge is 20 ft lower than the elevation of the inlet. The element that contributes the most to specific energy loss is the

(A)

four elbows

(B)

gate valve

(C)

pipe friction (excluding the fittings)
(D)

elevation change
8.

1.5 ft^3/sec (40 L/s) of 70°F (20°C) water flows through 1200 ft (355 m) of 6 in (nominal) diameter new schedule-40 steel pipe. The friction loss is most nearly

(A)

4 ft (1.2 m)

(B)

18 ft (5.2 m)

(C)

36 ft (9.5 m)

(D)

70 ft (21 m)

9.

500 gal/min (30 L/s) of 100°F (40°C) water flows through 300 ft (90 m) of 6 in schedule-40 pipe. The pipe contains two 6 in flanged steel elbows, two gate valves, a 90° angle valve, and a swing check valve. The discharge is located 20 ft (6 m) higher than the entrance. The pressure difference between the two ends of the pipe is most nearly

(A)

12 psi (78 kPa)

(B)

21 psi (140 kPa)

(C)

45 psi (310 kPa)

(D)

87 psi (600 kPa)

10.

70°F (20°C) air is flowing at 60 ft/sec (18 m/s) through 300 ft (90 m) of 6 in schedule-40 pipe. The pipe contains two 6 in (0.15 m) flanged steel elbows, two full-open gate valves, a 90° angle valve, and a swing check valve. The discharge is located 20 ft (6 m) higher than the entrance, and the average air density is 0.075 lbm/ft^3. The pressure difference between the two ends of the pipe is most nearly

(A)

0.26 psi (1.8 kPa)

(B)

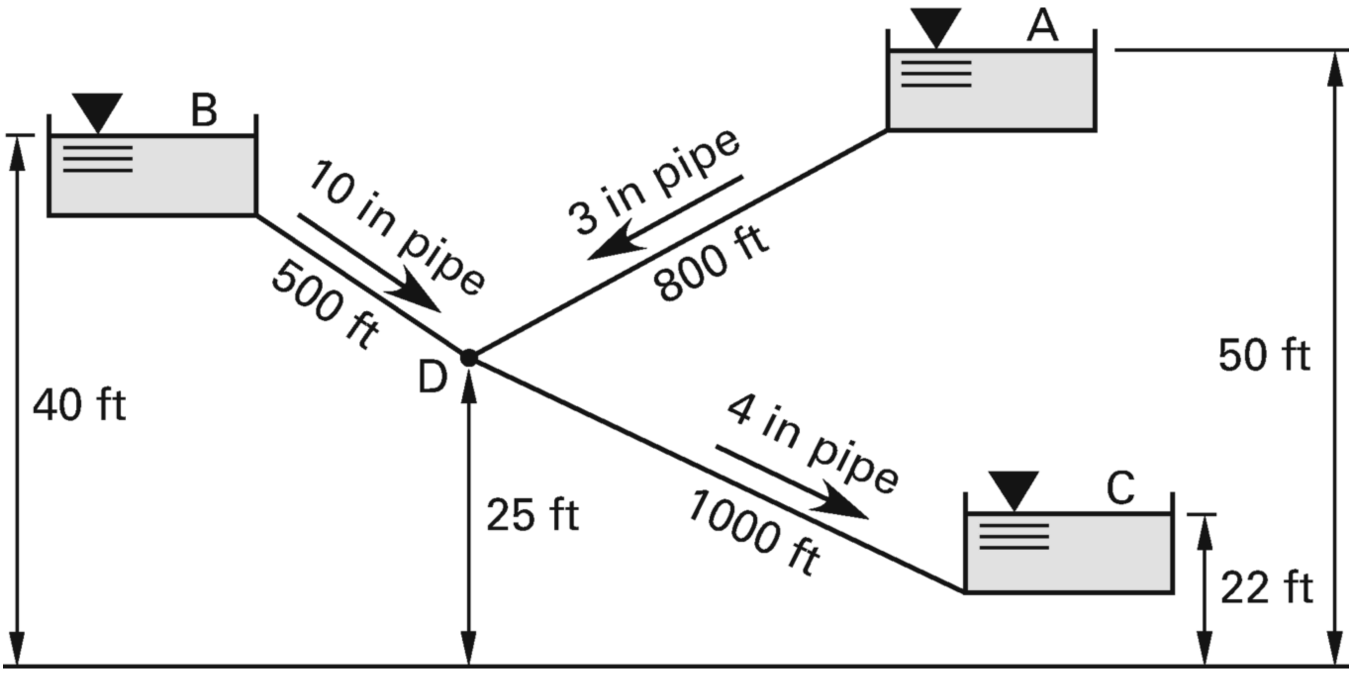
0.52 psi (3.4 kPa)
(C)

1.5 psi (10 kPa)

(D)

13 psi (90 kPa)
11.

Three reservoirs (A, B, and C) are interconnected with a common junction (point D) at elevation 25 ft above an arbitrary reference point. The water levels for reservoirs A, B, and C are at elevations of 50 ft, 40 ft, and 22 ft, respectively. The pipe from reservoir A to the junction is 800 ft of 3 in (nominal) steel pipe. The pipe from reservoir B to the junction is 500 ft of 10 in (nominal) steel pipe. The pipe from reservoir C to the junction is 1000 ft of 4 in (nominal) steel pipe. All pipes are schedule-40 with a friction factor of 0.02. All minor losses and velocity heads can be neglected. The direction of flow and the pressure at point D are most nearly



(A)

out of reservoir B; 500 psf

(B)

out of reservoir B; 930 psf

(C)

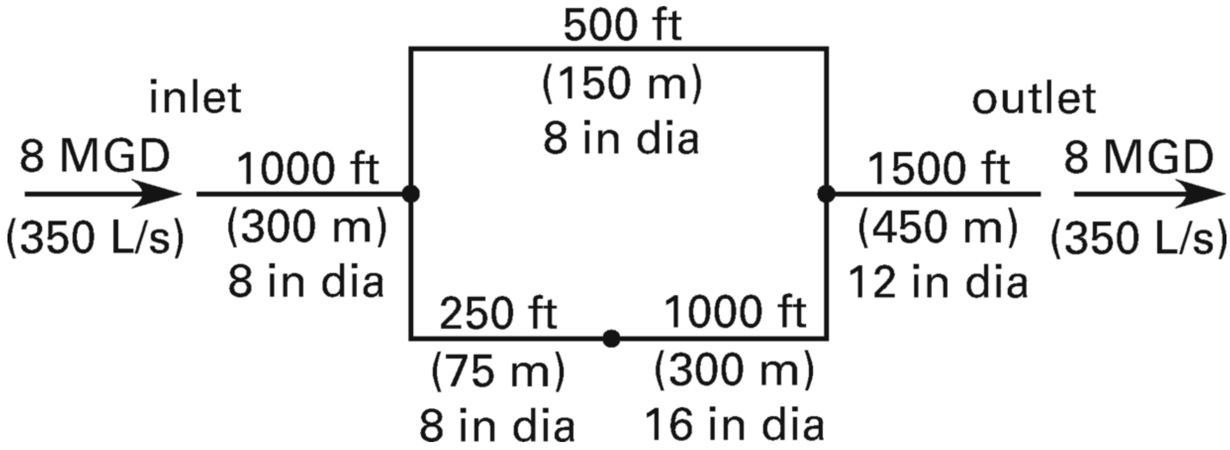
into reservoir B; 1100 psf

(D)

into reservoir B; 1260 psf

12.

8 MGD (millions of gallons per day) (350 L/s) of 70°F (20°C) water flows into the new schedule-40 steel pipe network shown. Minor losses are insignificant.



The quantity of water flowing in the upper branch is most nearly

(A)

1.2 ft^3/sec (0.034 m^3/s)

(B)

2.9 ft^3/sec (0.081 m^3/s)

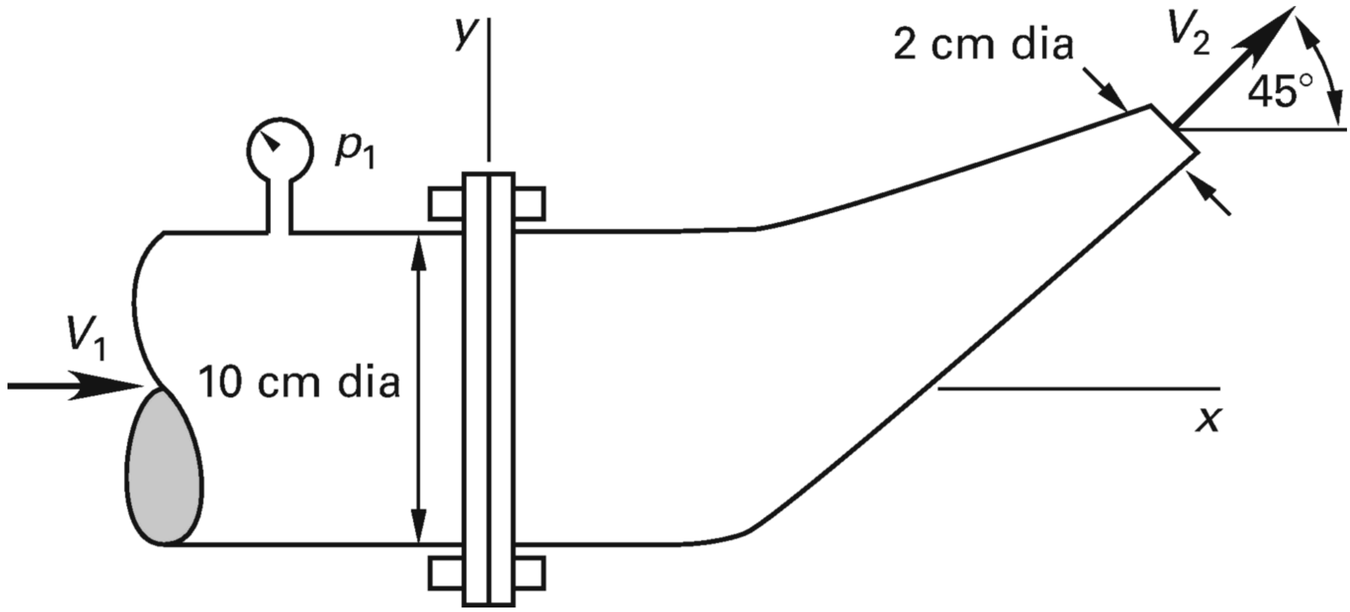
(C)

4.1 ft^3/sec (0.11 m^3/s)

(D)

5.4 ft^3/sec (0.15 m^3/s)
13.

Water flows through the horizontal nozzle shown so that the incoming velocity, v_1, is 4 m/s. Neglect any losses through the nozzle and use specific weight of 9800 N/m^3 and kinematic viscosity equal to 10^{-6} m^2/s for water. The horizontal section of the pipe is cast iron with a specific roughness of 0.00024 m



The difference between the pressure at a section 100 meters upstream of the bolted joint and the inlet pressure, p_1, is most nearly

(A)

190 kPa
(B)

200 kPa

(C)

290 kPa

(D)

420 kPa
14.

A full cylindrical tank that is 40 ft (12 m) high has a constant diameter of 20 ft. The tank has a 4 in (100 mm) diameter hole in its bottom. The coefficient of discharge for the hole is 0.98. Approximately how long will it take for the water level to drop from 40 ft to 20 ft (12 m to 6 m)?

(A)

950 sec

(B)

1200 sec

(C)

1450 sec

(D)

1700 sec
15.

A 24 in diameter siphon is used to transfer irrigation water from a water distribution canal to an irrigation ditch for a field of row crops below. The elevation of the water in the canal is 320 ft MSL (relative to mean sea level), and the elevation of the stilling basin for the row crops is 305 ft MSL. Counting fittings and minor losses, the siphon has a total equivalent length of 42 ft. The siphon is constructed of corrugated metal pipe with a standard galvanized surface. Neglecting entrance and exit losses, what is most nearly the total head loss experienced by the water?

(A)

7.0 ft

(B)

11 ft

(C)

15 ft

(D)

23 ft

16.

A venturi meter with an 8 in diameter throat is installed in a 12 in diameter water line. The venturi is perfectly smooth, so that the discharge coefficient is 1.00. An attached mercury manometer registers a 4 in differential. The volumetric flow rate is most nearly

(A)

1.7 ft^3/sec

(B)

5.2 ft^3/sec

(C)

6.4 ft^3/sec

(D)

18 ft^3/sec
17.

60°F (15°C) benzene (specific gravity at 60°F (15°C) of 0.885) flows through an 8 in/3.5 in (200 mm/90 mm) venturi meter whose coefficient of discharge is 0.99. A mercury manometer indicates a 4 in difference in the heights of the mercury columns. The volumetric flow rate of the benzene is most nearly

(A)

1.2 ft^3/sec (34 L/s)

(B)

9.1 ft^3/sec (250 L/s)

(C)

13 ft^3/sec (360 L/s)

(D)

27 ft^3/sec (760 L/s)
18.

A sharp-edged orifice meter with a 0.2 ft diameter opening is installed in a 1 ft diameter pipe. 70°F water approaches the orifice at 2 ft/sec. The indicated pressure drop across the orifice meter is most nearly

(A)

5.9 psi

(B)

13 psi

(C)

22 psi

(D)

47 psi

19.

A mercury manometer is used to measure a pressure difference across an orifice meter in a water line. The difference in mercury levels is 7 in (17.8 cm). The pressure differential is most nearly

(A)

1.7 psi (12 kPa)

(B)

3.2 psi (22 kPa)

(C)

7.9 psi (55 kPa)

(D)

23 psi (160 kPa)

20.

A sharp-edged ISA orifice is used in a schedule-40 steel 12 in (300 mm inside diameter) water line. The water temperature is 70°F (20°C), and the flow rate is 10 ft^3/sec (250 L/s). The differential pressure change across the orifice (to the vena contracta) should be approximately 25 ft (7.5 m). The smallest orifice that can be used is most nearly

(A)

5.5 in (14 cm)

(B)

7.3 in (19 cm)

(C)

8.1 in (20 cm)

(D)

8.9 in (23 cm)

21.

A pipe necks down from 24 in at point A to 12 in at point B. 8 ft^3/sec of 60°F water flows from point A to point B. The pressure head at point A is 20 ft. Friction is insignificant over the distance between points A and B. The magnitude and direction of the resultant force on the water are most nearly

(A)

2900 lbf, toward A

(B)

3500 lbf, toward A

(C)

2900 lbf, toward B

(D)

3500 lbf, toward B

22.

2000 gal/min (125 L/s) of brine with a specific gravity of 1.2 passes through an 85% efficient pump. The centerlines of the pump's 12 in inlet and 8 in outlet are at the same elevation. The inlet suction gauge indicates 6 in (150 mm) of mercury below atmospheric. The discharge pressure gauge is located 4 ft (1.2 m) above the centerline of the pump's outlet and indicates 20 psig (138 kPa). All pipes are schedule-40. The input power to the pump is most nearly

(A)

12 hp (8.9 kW)

(B)

36 hp (26 kW)

(C)

52 hp (39 kW)

(D)

87 hp (65 kW)

23.

100 ft^3/sec (2.6 m^3/s) of water passes through a horizontal turbine. The water's pressure is reduced from 30 psig (210 kPa) to 5 psig (35 kPa) vacuum. Disregarding friction, velocity, and other factors, the power generated is most nearly

(A)

110 hp (82 kW)

(B)

380 hp (280 kW)

(C)

730 hp (540 kW)

(D)

920 hp (640 kW)

24.

A refrigeration truck is driven at 65 mph into a 15 mph headwind. The frontal area of the truck is 100 ft^2. The coefficient of drag is 0.5. When calculating the drag force on the truck, the velocity that should be used is most nearly

(A)

50 mph

(B)

65 mph
(C)

73 mph

(D)

80 mph
25.

A dish-shaped antenna faces directly into a 60 mph wind. The projected area of the antenna is 0.8 ft^2; the coefficient of drag, C_D, is 1.2; and the density of air is 0.076 lbm/ft^3. The total amount of drag force experienced by the antenna is most nearly

(A)

9.0 lbf

(B)

10 lbf

(C)

14 lbf

(D)

16 lbf

26.

A car traveling through 70°F (20°C) air has the following characteristics.

frontal area	28 ft^2 (2.6 m^2)
mass	3300 lbm (1500 kg)
drag coefficient	0.42
rolling resistance	1% of weight
engine thermal efficiency	28%
fuel heating value	115,000 Btu/gal (32 MJ/L)

Assume the car is traveling at 55 mi/hr (90 km/h). Considering only the drag and rolling resistance, the fuel consumption of the car is most nearly

(A)

0.026 gal/mi (0.062 L/km)

(B)

0.038 gal/mi (0.087 L/km)

(C)

0.051 gal/mi (0.12 L/km)

(D)

0.13 gal/mi (0.30 L/km)

27.

A 1/20th airplane model is tested in a wind tunnel at full velocity and temperature. What is the approximate ratio of the wind tunnel pressure to normal ambient pressure?

(A)

5

(B)

10

(C)

20

(D)

40

28.

68°F (20°C) castor oil (kinematic viscosity at 68°F (20°C) of 1110×10^{-5} ft^2/sec (103×10^{-5} m^2/s)) flows through a pump whose impeller turns at 1000 rpm. A similar pump twice the first pump's size is tested with 68°F (20°C) air. Theoretically, what should be the approximate speed of the second pump's impeller to ensure similarity?

(A)

3.6 rpm

(B)

88 rpm

(C)

250 rpm

(D)

1600 rpm

Solutions

1.

The decrease in friction will affect pumping power required, and increase the volumetric flow, which will increase the velocity. Since there are no details provided about a specific model of pump and system curve, capacity can be estimated as a function of velocity and area. The velocity is unchanged. The volumetric flow rate can be found in *NCEES Handbook:* Conservation of Mass.

$$\begin{aligned} \frac{Q_2}{Q_1} &= \frac{\mathrm{v}A_2}{\mathrm{v}A_1} = \frac{\dfrac{\pi \mathrm{v} d_2^2}{4}}{\dfrac{\pi \mathrm{v} d_1^2}{4}} = \frac{d_2^2}{d_1^2} \\ &= 2 \\ d_2 &= \sqrt{2} d_1 \approx 1.41 d_1 \quad \text{(40\% increase)} \end{aligned}$$

It is worth noting that the pump should be modified by either changing the impeller or reducing the speed, if possible for this type of pump, to prevent the average velocity from exceeding the desired value.

The answer is (C).
2.

As in *NCEES Handbook:* Similitude, the initial Reynolds number is

$$\mathrm{Re}_1 = \frac{D\mathrm{v}}{\nu}$$

After modifications, the Reynolds number is

$$\mathrm{Re}_2 = \frac{(2D)\left(\frac{\mathrm{v}}{2}\right)}{\nu} = \frac{D\mathrm{v}}{\nu} = \mathrm{Re}_1$$

The Reynolds number will remain the same.

The answer is (D).

3.

Customary U.S. Solution

As in *NCEES Handbook* table "Pipe Dimensions and Weights," for schedule-40 pipe,

$$\begin{aligned} D_\mathrm{A} &= 0.5054 \text{ ft} \\ D_\mathrm{B} &= 1.4063 \text{ ft} \end{aligned}$$

Let point A be at zero elevation.

As in *NCEES Handbook:* The Bernoulli Equation, the total energy at point A is

$$E_{t,\mathrm{A}} = E_p + E_\mathrm{v} + E_z = \frac{p_\mathrm{A}}{\rho} + \frac{\mathrm{v}_\mathrm{A}^2}{2g_c} + \frac{z_\mathrm{A} g}{g_c}$$

At point A, from appendixCERM16C, the diameter is 6 in. As in *NCEES Handbook:* Conservation of Mass, the velocity at point A is

$$\begin{aligned} \dot{V} &= \mathrm{v}_\mathrm{A} A_\mathrm{A} = \mathrm{v}_\mathrm{A}\left(\frac{\pi}{4}\right) D_\mathrm{A}^2 \\ \mathrm{v}_\mathrm{A} &= \left(\frac{4}{\pi}\right)\left(\frac{\dot{V}}{D_\mathrm{A}^2}\right) = \left(\frac{4}{\pi}\right)\left(\frac{5\ \frac{\mathrm{ft}^3}{\mathrm{sec}}}{(0.5054 \text{ ft})^2}\right) \\ &= 24.9 \text{ ft/sec} \\ p_\mathrm{A} &= \left(10\ \frac{\mathrm{lbf}}{\mathrm{in}^2}\right)\left(12\ \frac{\mathrm{in}}{\mathrm{ft}}\right)^2 = 1440 \text{ lbf/ft}^2 \\ z_\mathrm{A} &= 0 \end{aligned}$$

As in *NCEES Handbook* table "Physical Properties of Liquid Water (U.S. Units)," for water, $\rho \approx 62.4 \text{ lbm/ft}^3$.

$$\begin{aligned} E_{t,\mathrm{A}} &= \frac{p_\mathrm{A}}{\rho} + \frac{\mathrm{v}_\mathrm{A}^2}{2g_c} + \frac{z_\mathrm{A} g}{g_c} \\ &= \frac{1440\ \frac{\mathrm{lbf}}{\mathrm{ft}^2}}{62.4\ \frac{\mathrm{lbm}}{\mathrm{ft}^3}} + \frac{\left(24.9\ \frac{\mathrm{ft}}{\mathrm{sec}}\right)^2}{(2)\left(32.2\ \frac{\mathrm{lbm\text{-}ft}}{\mathrm{lbf\text{-}sec}^2}\right)} + 0 \\ &= 32.7 \text{ ft-lbf/lbm} \end{aligned}$$

Similarly, the total energy at point B is

$$
\begin{aligned}
\mathrm{v_B} &= \left(\frac{4}{\pi}\right)\left(\frac{\dot{V}}{D_\mathrm{B}^2}\right) = \left(\frac{4}{\pi}\right)\left(\frac{5\ \frac{\mathrm{ft}^3}{\mathrm{sec}}}{(1.4063\ \mathrm{ft})^2}\right) \\
&= 3.22\ \mathrm{ft/sec} \\
p_\mathrm{B} &= \left(7\ \frac{\mathrm{lbf}}{\mathrm{in}^2}\right)\left(12\ \frac{\mathrm{in}}{\mathrm{ft}}\right)^2 = 1008\ \mathrm{lbf/ft}^2 \\
z_\mathrm{B} &= 15\ \mathrm{ft}
\end{aligned}
$$

$$
\begin{aligned}
E_{t,\mathrm{B}} &= \frac{p_\mathrm{B}}{\rho} + \frac{\mathrm{v_B^2}}{2g_c} + \frac{z_\mathrm{B}g}{g_c} \\
&= \frac{1008\ \frac{\mathrm{lbf}}{\mathrm{ft}^2}}{62.4\ \frac{\mathrm{lbm}}{\mathrm{ft}^3}} + \frac{\left(3.22\ \frac{\mathrm{ft}}{\mathrm{sec}}\right)^2}{(2)\left(32.2\ \frac{\mathrm{lbm\text{-}ft}}{\mathrm{lbf\text{-}sec}^2}\right)} \\
&\quad + \frac{(15\ \mathrm{ft})\left(32.2\ \frac{\mathrm{ft}}{\mathrm{sec}^2}\right)}{32.2\ \frac{\mathrm{lbm\text{-}ft}}{\mathrm{lbf\text{-}sec}^2}} \\
&= 31.3\ \mathrm{ft\text{-}lbf/lbm}
\end{aligned}
$$

Since $E_{t,\mathrm{A}} > E_{t,\mathrm{B}}$, the flow is from point A to point B.

The answer is (B).

SI Solution

Let point A be at zero elevation.

As in *NCEES Handbook:* The Bernoulli Equation, the total energy at point A is

$$
E_{t,\mathrm{A}} = E_p + E_\mathrm{v} + E_z = \frac{p_\mathrm{A}}{\rho} + \frac{\mathrm{v_A^2}}{2} + z_\mathrm{A}g
$$

At point A, the diameter is 154 mm (0.154 m). The velocity at point A is

$$
\begin{aligned}
\dot{V} &= \mathrm{v_A}A_\mathrm{A} = \mathrm{v_A}\left(\frac{\pi}{4}\right)D_\mathrm{A}^2 \\
\mathrm{v_A} &= \left(\frac{4}{\pi}\right)\left(\frac{\dot{V}}{D_\mathrm{A}^2}\right) = \left(\frac{4}{\pi}\right)\left(\frac{130\ \frac{\mathrm{L}}{\mathrm{s}}}{(0.154\ \mathrm{m})^2\left(1000\ \frac{\mathrm{L}}{\mathrm{m}^3}\right)}\right) \\
&= 6.98\ \mathrm{m/s} \quad (7\ \mathrm{m/s}) \\
p_\mathrm{A} &= 70\ \mathrm{kPa} \quad (70\,000\ \mathrm{Pa}) \\
z_\mathrm{A} &= 0
\end{aligned}
$$

As in *NCEES Handbook* table "Physical Properties of Liquid Water (SI Units)," for water, $\rho \approx 1000\ \mathrm{kg/m^3}$.

$$
\begin{aligned}
E_{t,\mathrm{A}} &= \frac{p_\mathrm{A}}{\rho} + \frac{\mathrm{v_A^2}}{2} + z_\mathrm{A}g \\
&= \frac{70\,000\ \mathrm{Pa}}{1000\ \frac{\mathrm{kg}}{\mathrm{m}^3}} + \frac{\left(6.98\ \frac{\mathrm{m}}{\mathrm{s}}\right)^2}{2} + 0 \\
&= 94.36\ \mathrm{J/kg}
\end{aligned}
$$

Similarly, at B, the diameter is 429 mm. The total energy at point B is

$$
\begin{aligned}
\mathrm{v_B} &= \left(\frac{4}{\pi}\right)\left(\frac{\dot{V}}{D_\mathrm{B}^2}\right) \\
&= \left(\frac{4}{\pi}\right)\left(\frac{130\ \frac{\mathrm{L}}{\mathrm{s}}}{(0.429\ \mathrm{m})^2\left(1000\ \frac{\mathrm{L}}{\mathrm{m}^3}\right)}\right) \\
&= 0.90\ \mathrm{m/s} \\
p_\mathrm{B} &= 48.3\ \mathrm{kPa} \quad (48\,300\ \mathrm{Pa}) \\
z_\mathrm{B} &= 4.6\ \mathrm{m}
\end{aligned}
$$

$$
\begin{aligned}
E_{t,\mathrm{B}} &= \frac{p_\mathrm{B}}{\rho} + \frac{\mathrm{v_B^2}}{2} + z_\mathrm{B}g \\
&= \frac{48\,300\ \mathrm{Pa}}{1000\ \frac{\mathrm{kg}}{\mathrm{m}^3}} + \frac{\left(0.90\ \frac{\mathrm{m}}{\mathrm{s}}\right)^2}{2} \\
&\quad + (4.6\ \mathrm{m})\left(9.81\ \frac{\mathrm{m}}{\mathrm{s}^2}\right) \\
&= 93.8\ \mathrm{J/kg}
\end{aligned}
$$

Since $E_{t,\mathrm{A}} > E_{t,\mathrm{B}}$, the flow is from point A to point B.

The answer is (B).

4.

$$
\begin{aligned}
\dot{V} &= \frac{750\ \frac{\mathrm{gal}}{\mathrm{min}}}{\left(7.4805\ \frac{\mathrm{gal}}{\mathrm{ft}^3}\right)\left(60\ \frac{\mathrm{sec}}{\mathrm{min}}\right)} \\
&= 1.671\ \mathrm{ft^3/sec}
\end{aligned}
$$

From appendixCERM16B (also *NCEES Handbook* table “Pipe Dimensions and Weights”), $D = 0.5054$ ft, and $A = 0.2006\ \mathrm{ft}^2$.

$$
\mathrm{v} = \frac{\dot{V}}{A} = \frac{1.671\ \frac{\mathrm{ft}^3}{\mathrm{sec}}}{0.2006\ \mathrm{ft}^2} = 8.33\ \mathrm{ft/sec}
$$

For 60°F water, from appendixCERM14A (also *NCEES Handbook* table “Physical Properties of Liquid Water (U.S. Units)”),

$$
\begin{aligned}
\rho &= 62.37\ \mathrm{lbm/ft^3} \\
\nu &= 1.217 \times 10^{-5}\ \mathrm{ft^2/sec}
\end{aligned}
$$

As in *NCEES Handbook:* Similitude, the Reynolds number is

$$
\mathrm{Re} = \frac{\mathrm{v}D}{\nu} = \frac{\left(8.33\ \frac{\mathrm{ft}}{\mathrm{sec}}\right)(0.5054\ \mathrm{ft})}{1.217 \times 10^{-5}\ \frac{\mathrm{ft}^2}{\mathrm{sec}}} = 3.46 \times 10^5
$$

The specific weight is

$$
\gamma = \frac{\rho g}{g_c} = \frac{\left(62.37\ \frac{\mathrm{lbm}}{\mathrm{ft}^3}\right)\left(32.2\ \frac{\mathrm{ft}}{\mathrm{sec}^2}\right)}{32.2\ \frac{\mathrm{lbm\text{-}ft}}{\mathrm{lbf\text{-}sec}^2}} = 62.37\ \mathrm{lbf/ft^3}
$$

As in *NCEES Handbook:* Absolute Roughness and Relative Roughness, for steel,

$$\epsilon = 0.0002$$

$$\frac{\epsilon}{D} = \frac{0.0002\ \text{ft}}{0.5054\ \text{ft}} \approx 0.0004$$

$$f = 0.0175$$

The friction factor is found by following the curved line for relative roughness 0.0004 back to the intersection of the vertical line for Reynolds number 346,000. From this intersection point, travel straight left to the primary y-axis to determine the friction factor.

From equationCERM17022 (also *NCEES Handbook:* Head Loss in Pipe or Conduit),

$$\begin{aligned} h_f &= \frac{fL\text{v}^2}{2Dg} = \frac{(0.0175)(3000\ \text{ft})\left(8.33\ \frac{\text{ft}}{\text{sec}}\right)^2}{(2)(0.5054\ \text{ft})\left(32.2\ \frac{\text{ft}}{\text{sec}^2}\right)} \\ &= 111.9\ \text{ft} \end{aligned}$$

Use the Bernoulli equation (see *NCEES Handbook:* The Bernoulli Equation). Since velocity is the same at points A and B, it may be omitted.

$$\begin{aligned} \frac{p_1}{\gamma_1} &= \frac{p_2}{\gamma_2} + (z_2 - z_1) + h_f \\ \frac{\left(12\ \frac{\text{in}}{\text{ft}}\right)^2 p_1}{62.37\ \frac{\text{lbf}}{\text{ft}^3}} &= \frac{\left(50\ \frac{\text{lbf}}{\text{in}^2}\right)\left(12\ \frac{\text{in}}{\text{ft}}\right)^2}{62.37\ \frac{\text{lbf}}{\text{ft}^3}} + 60\ \text{ft} + 111.9\ \text{ft} \\ p_1 &= 124.5\ \text{lbf/in}^2 \quad (120\ \text{psig}) \end{aligned}$$

The answer is (C).
5.

From equationCERM17030, the friction loss from A to B is

$$\begin{aligned} h_{f,\text{ft,A-B}} &= \frac{10.44 L_{\text{ft}} Q_{\text{gpm}}^{1.85}}{C^{1.85} d_{\text{in}}^{4.87}} \\ &= \frac{(10.44)(20{,}000\ \text{ft})\left(120\ \frac{\text{gal}}{\text{min}}\right)^{1.85}}{(150)^{1.85}(6\ \text{in})^{4.87}} \\ &= 22.4\ \text{ft} \end{aligned}$$

Calculate the velocity head.

$$\begin{aligned} \text{v} &= \frac{\dot{V}}{A} = \frac{120\ \frac{\text{gal}}{\text{min}}}{\left(\frac{\pi}{4}\right)\left(\frac{6\ \text{in}}{12\ \frac{\text{in}}{\text{ft}}}\right)^2\left(7.4805\ \frac{\text{gal}}{\text{ft}^3}\right)\left(60\ \frac{\text{sec}}{\text{min}}\right)} \\ &= 1.36\ \text{ft/sec} \\ h_\text{v} &= \frac{\text{v}^2}{2g} = \frac{\left(1.36\ \frac{\text{ft}}{\text{sec}}\right)^2}{(2)\left(32.2\ \frac{\text{ft}}{\text{sec}^2}\right)} \\ &= 0.029\ \text{ft} \end{aligned}$$

Velocity heads are low and can be disregarded. The friction loss from B to C is

$$\begin{aligned} h_{f,\text{B-C}} &= \frac{(10.44)\,(10{,}000\ \text{ft})\left(160\ \frac{\text{gal}}{\text{min}}\right)^{1.85}}{(150)^{1.85}\,(6\ \text{in})^{4.87}} \\ &= 19.10\ \text{ft} \end{aligned}$$

For C to D,

$$\begin{aligned} h_{f,\text{C-D}} &= \frac{(10.44)\,(30{,}000\ \text{ft})\left(120\ \frac{\text{gal}}{\text{min}}\right)^{1.85}}{(150)^{1.85}\,(4\ \text{in})^{4.87}} \\ &= 242.4\ \text{ft} \end{aligned}$$

Assume a pressure of 20 psig at point A.

$$\begin{aligned} h_{p,\text{A}} &= \frac{\left(20\ \frac{\text{lbf}}{\text{in}^2}\right)\left(12\ \frac{\text{in}}{\text{ft}}\right)^2}{62.4\ \frac{\text{lbf}}{\text{ft}^3}} \\ &= 46.2\ \text{ft} \end{aligned}$$

From the Bernoulli equation, ignoring velocity head,

$$\begin{aligned} h_{p,\text{A}} + z_\text{A} &= h_{p,\text{B}} + z_\text{B} + h_{f,\text{A-B}} \\ 46.2\ \text{ft} + 620\ \text{ft} &= h_{p,\text{B}} + 460\ \text{ft} + 22.4\ \text{ft} \\ h_{p,\text{B}} &= 183.8\ \text{ft} \\ p_\text{B} &= \gamma h_{p,\text{B}} = \frac{\left(62.4\ \frac{\text{lbf}}{\text{ft}^3}\right)(183.8\ \text{ft})}{\left(12\ \frac{\text{in}}{\text{ft}}\right)^2} \\ &= 79.6\ \text{lbf/in}^2 \end{aligned}$$

For B to C,

$$\begin{aligned} h_{p,\text{B}} + z_\text{B} &= h_{p,\text{C}} + z_\text{C} + h_{f,\text{B-C}} \\ 183.8\ \text{ft} + 460\ \text{ft} &= h_{p,\text{C}} + 540\ \text{ft} + 19.10\ \text{ft} \\ h_{p,\text{C}} &= 84.7\ \text{ft} \\ p_\text{C} &= \gamma h_{p,\text{C}} = \frac{\left(62.4\ \frac{\text{lbf}}{\text{ft}^3}\right)(84.7\ \text{ft})}{\left(12\ \frac{\text{in}}{\text{ft}}\right)^2} \\ &= 36.7\ \text{lbf/in}^2 \end{aligned}$$

For C to D,

$$\begin{aligned} h_{p,\text{C}} + z_\text{C} &= h_{p,\text{D}} + z_\text{D} + h_{f,\text{C-D}} \\ 84.7\ \text{ft} + 540\ \text{ft} &= h_{p,\text{D}} + 360\ \text{ft} + 242.4\ \text{ft} \\ h_{p,\text{D}} &= 22.3\ \text{ft} \\ p_\text{D} &= \gamma h_{p,\text{D}} = \frac{\left(62.4\ \frac{\text{lbf}}{\text{ft}^3}\right)(22.3\ \text{ft})}{\left(12\ \frac{\text{in}}{\text{ft}}\right)^2} \\ &= 9.7\ \text{lbf/in}^2 \end{aligned}$$

$p_\text{D} = 9.7\ \text{lbf/in}^2$ is too low; therefore, add $20\ \text{lbf/in}^2 - 9.7\ \text{lbf/in}^2 = 10.3\ \text{lbf/in}^2$ (psig) to each point.

$$p_\text{A} = 20.0\ \frac{\text{lbf}}{\text{in}^2} + 10.3\ \frac{\text{lbf}}{\text{in}^2} = 30.3\ \text{lbf/in}^2 \quad (30\ \text{psig})$$

The answer is (D).

6.

As in *NCEES Handbook:* Head Loss in Pipe or Conduit, the Darcy equation is applicable to fluids in the laminar and turbulent regions.

$$h_f = \frac{fL\text{v}^2}{2Dg}$$

From *NCEES Handbook:* Friction Factors for Laminar Flow, for laminar flow in circular pipes, the friction factor is

$$f = \frac{64}{\text{Re}} = \frac{64\nu}{D\text{v}}$$

Combining these two equations,

$$h_f = \frac{fL\text{v}^2}{2Dg} = \frac{\left(\frac{64\nu}{D\text{v}}\right)L\text{v}^2}{2Dg} = \frac{32\nu L\text{v}}{D^2 g}$$

Friction head loss in laminar flow is proportional to velocity.

The answer is (A).

7.

As in *NCEES Handbook:* Turbulent Flow, the equivalent length of four 45° standard elbows with 3 in diameter is

$$\begin{aligned} \frac{L}{D} &= 16 \\ L &= 16D \\ &= \frac{(16)\left(3\ \frac{\text{in}}{\text{elbow}}\right)(4\ \text{elbows})}{12\ \frac{\text{in}}{\text{ft}}} \\ &= 16\ \text{ft} \end{aligned}$$

The equivalent length of the fully open gate valve is

$$\begin{aligned} \frac{L}{D} &= 10 \\ L &= 10D \\ &= \frac{(10)(3\ \text{in})}{12\ \frac{\text{in}}{\text{ft}}} \\ &= 2.5\ \text{ft} \end{aligned}$$

The elevation change is 20 ft, but this distance is not an equivalent length of pipe. Taking terms from the Bernoulli equation,

$$\begin{aligned} \Delta z &= \frac{fL_e\text{v}^2}{2Dg} \\ L_e &= \frac{2Dg\Delta z}{f\text{v}^2} \end{aligned}$$

Use a value of 0.02, which is appropriate for steel pipe and turbulent flow. Using nominal values for a quick estimate, the equivalent length of pipe equal to the elevation drop is

$$L_e = \frac{2Dg\Delta z}{f\text{v}^2} = \frac{(2)(3\text{ in})\left(32.2\ \frac{\text{ft}}{\text{sec}^2}\right)(20\text{ ft})}{(0.02)\left(10\ \frac{\text{ft}}{\text{sec}}\right)^2\left(12\ \frac{\text{in}}{\text{ft}}\right)}$$
$$= 161\text{ ft}$$

The answer is (D).
8.

Customary U.S. Solution

As in *NCEES Handbook* table "Pipe Dimensions and Weights," for 6 in schedule-40 pipe, the internal diameter, D, is 0.5054 ft. The internal area is 0.2006 ft^2.

As in *NCEES Handbook:* Conservation of Mass, the velocity, v, is calculated from the volumetric flow, $\dot{V}$, and the flow area, A, by

$$\text{v} = \frac{\dot{V}}{A} = \frac{1.5\ \frac{\text{ft}^3}{\text{sec}}}{0.2006\text{ ft}^2}$$
$$= 7.48\text{ ft/sec}$$

Use appendixCERM14A (also *NCEES Handbook* table "Physical Properties of Liquid Water (U.S. Units)"). For water at 70°F, the density is $\rho = 62.299\text{ lbm/ft}^3$, and the viscosity is $\mu = 6.552 \times 10^{-4}$ lbm/ft-sec.

As in *NCEES Handbook* table "Dimensionless Numbers," the Reynolds number is

$$\text{Re} = \frac{Dv\rho}{\mu}$$
$$= \frac{(0.5054\text{ ft})\left(7.48\ \frac{\text{ft}}{\text{sec}}\right)\left(62.299\ \frac{\text{lbm}}{\text{ft}^3}\right)}{6.552 \times 10^{-4}\ \frac{\text{lbm}}{\text{ft-sec}}}$$
$$= 3.59 \times 10^5$$

Since $\text{Re} > 2100$, the flow is turbulent. As in *NCEES Handbook:* Friction Factors for Turbulent Flow, the friction loss coefficient can be determined from the Moody diagram.

As in *NCEES Handbook:* Absolute Roughness and Relative Roughness, for new steel pipe, the specific roughness, ϵ, is 0.0002 ft.

The relative roughness is

$$\frac{\epsilon}{D} = \frac{0.0002\text{ ft}}{0.5054\text{ ft}} = 0.0004$$

From the Moody diagram with Re = 3.57×10^5 and $\epsilon/D = 0.0004$, the friction factor, f, is 0.0174.

As in *NCEES Handbook:* Head Loss in Pipe or Conduit, use Darcy's equation to compute the frictional loss.

$$h_f = \frac{fL\text{v}^2}{2Dg} = \frac{(0.0174)(1200\text{ ft})\left(7.48\ \frac{\text{ft}}{\text{sec}}\right)^2}{(2)(0.5054\text{ ft})\left(32.2\ \frac{\text{ft}}{\text{sec}^2}\right)}$$
$$= 35.9\text{ ft} \quad (36\text{ ft})$$

The answer is (C).

SI Solution

As in *NCEES Handbook* table "Pipe Dimensions and Weights," for 6 in pipe, the internal diameter is 154.1 mm, and the internal area is 186.5×10^{-4} m^2.

As in *NCEES Handbook:* Conservation of Mass, the velocity, v, is calculated from the volumetric flow, $\dot{V}$, and the flow area, *A*, by

$$\begin{aligned}\mathrm{v} &= \frac{\dot{V}}{A} = \frac{40\ \frac{\mathrm{L}}{\mathrm{s}}}{\left(186.5\times 10^{-4}\ \mathrm{m}^2\right)\left(1000\ \frac{\mathrm{L}}{\mathrm{m}^3}\right)} \\ &= 2.145\ \mathrm{m/s}\end{aligned}$$

From appendixCERM14B (also *NCEES Handbook* table "Physical Properties of Liquid Water (SI Units)"), for water at 20°C, the kinematic viscosity is

$$\begin{aligned}\nu &= \frac{\mu}{\rho} = \frac{1.0050\times 10^{-3}\ \mathrm{Pa{\cdot}s}}{998.23\ \frac{\mathrm{kg}}{\mathrm{m}^3}} \\ &= 1.007\times 10^{-6}\ \mathrm{m^2/s}\end{aligned}$$

Calculate the Reynolds number (see *NCEES Handbook* table "Dimensionless Numbers"),

$$\begin{aligned}\mathrm{Re} &= \frac{D\mathrm{v}}{\nu} = \frac{(154.1\ \mathrm{mm})\left(2.145\ \frac{\mathrm{m}}{\mathrm{s}}\right)}{\left(1.007\times 10^{-6}\ \frac{\mathrm{m}^2}{\mathrm{s}}\right)\left(1000\ \frac{\mathrm{mm}}{\mathrm{m}}\right)} \\ &= 3.282\times 10^5\end{aligned}$$

Since $\mathrm{Re} > 2100$, the flow is turbulent. The friction loss coefficient can be determined from the Moody diagram.

From *NCEES Handbook:* Absolute Roughness and Relative Roughness, for new steel pipe, the specific roughness, ϵ, is 6.0×10^{-5} m.

The relative roughness is

$$\frac{\epsilon}{D} = \frac{6.0\times 10^{-5}\ \mathrm{m}}{0.1541\ \mathrm{m}} = 0.0004$$

From the Moody diagram with Re = 3.28×10^5 and ϵ/D = 0.0004, the friction factor, *f*, is 0.0175.

As in *NCEES Handbook:* Head Loss in Pipe or Conduit, use Darcy's equation to compute the frictional loss.

$$\begin{aligned}h_f &= \frac{fL\mathrm{v}^2}{2Dg} = \frac{(0.0175)(355\ \mathrm{m})\left(2.145\ \frac{\mathrm{m}}{\mathrm{s}}\right)^2}{(2)(0.1541\ \mathrm{m})\left(9.81\ \frac{\mathrm{m}}{\mathrm{s}^2}\right)} \\ &= 9.45\ \mathrm{m}\quad (9.5\ \mathrm{m})\end{aligned}$$

The answer is (C).
9.

Customary U.S. Solution

As in *NCEES Handbook* table "Pipe Dimensions and Weights," for 6 in schedule-40 pipe, the internal diameter, *D*, is 0.5054 ft. The internal area is 0.2006 ft^2.

Convert the volumetric flow rate from gal/min to ft^3/sec.

$$\dot{V} = \frac{500\ \frac{\text{gal}}{\text{min}}}{\left(60\ \frac{\text{sec}}{\text{min}}\right)\left(7.4805\ \frac{\text{gal}}{\text{ft}^3}\right)} = 1.114\ \text{ft}^3/\text{sec}$$

As in *NCEES Handbook:* Conservation of Mass, the velocity is

$$\text{v} = \frac{\dot{V}}{A} = \frac{1.114\ \frac{\text{ft}^3}{\text{sec}}}{0.2006\ \text{ft}^2} = 5.55\ \text{ft/sec}$$

Use appendixCERM14A (also *NCEES Handbook* table "Physical Properties of Liquid Water (U.S. Units)"). For water at 100°F, the kinematic viscosity, ν, is 0.739×10^{-5} ft^2/sec, and the density is 62.00 lbm/ft^2.

As in *NCEES Handbook:* Similitude, the Reynolds number is

$$\begin{aligned}\text{Re} &= \frac{D\text{v}}{\nu} = \frac{(0.5054\ \text{ft})\left(5.55\ \frac{\text{ft}}{\text{sec}}\right)}{0.739 \times 10^{-5}\ \frac{\text{ft}^2}{\text{sec}}} \\ &= 3.80 \times 10^5\end{aligned}$$

Since $\text{Re} > 2100$, the flow is turbulent. The friction loss coefficient can be determined from the Moody diagram.

For new steel pipe, the specific roughness, ϵ, is 0.0002 ft.

From *NCEES Handbook:* Absolute Roughness and Relative Roughness, the relative roughness is

$$\frac{\epsilon}{D} = \frac{0.0002\ \text{ft}}{0.5054\ \text{ft}} = 0.0004$$

From the Moody diagram in *NCEES Handbook:* Friction Factors for Turbulent Flow, with Re = 3.80×10^5 and ϵ/D = 0.0004, the friction factor, f, is 0.0173.

Use *NCEES Handbook:* Head Loss in Pipe or Conduit. The equivalent length of two 6 in steel flanged elbows is

$$\begin{aligned}\tfrac{L}{D} &= 20 \\ L &= 20D \\ &= \frac{(20)\left(6\ \frac{\text{in}}{\text{elbow}}\right)(2\ \text{elbows})}{12\ \frac{\text{in}}{\text{ft}}} \\ &= 20\ \text{ft}\end{aligned}$$

The equivalent length of two fully open gate valves is

$$\begin{aligned}\tfrac{L}{D} &= 10 \\ L &= 10D \\ &= \frac{(10)\left(6\ \frac{\text{in}}{\text{valve}}\right)(2\ \text{valves})}{12\ \frac{\text{in}}{\text{ft}}} \\ &= 10\ \text{ft}\end{aligned}$$

The equivalent length of a fully open 90° angle valve is

$$\begin{aligned} \frac{L}{D} &= 165 \\ L &= 165D \\ &= \frac{(165)(6\ \text{in})}{12\ \frac{\text{in}}{\text{ft}}} \\ &= 82.5\ \text{ft} \end{aligned}$$

The equivalent length of a swing check valve is

$$\begin{aligned} \frac{L}{D} &= 125 \\ L &= 125D \\ &= \frac{(125)(6\ \text{in})}{12\ \frac{\text{in}}{\text{ft}}} \\ &= 62.5\ \text{ft} \end{aligned}$$

The total is

$$20\ \text{ft} + 10\ \text{ft} + 82.5\ \text{ft} + 62.5\ \text{ft} = 175\ \text{ft}$$

The equivalent pipe length is the sum of the straight run of pipe and the equivalent length of pipe for the valves and fittings.

$$\begin{aligned} L_e &= L + L_{\text{fittings}} = 300\ \text{ft} + 175\ \text{ft} \\ &= 475\ \text{ft} \end{aligned}$$

Using Darcy's equation (see *NCEES Handbook:* Head Loss in Pipe or Conduit), the frictional loss is

$$\begin{aligned} h_f &= \frac{fL_e\text{v}^2}{2Dg} \\ &= \frac{(0.0173)(475\ \text{ft})\left(5.55\ \frac{\text{ft}}{\text{sec}}\right)^2}{(2)(0.5054\ \text{ft})\left(32.2\ \frac{\text{ft}}{\text{sec}^2}\right)} \\ &= 7.78\ \text{ft} \end{aligned}$$

The head loss is the sum of the head losses through the pipe, valves, and fittings and the change in elevation.

$$\begin{aligned} \Delta h &= h_f + \Delta z = 7.78\ \text{ft} + 20\ \text{ft} \\ &= 27.78\ \text{ft} \end{aligned}$$

The pressure difference between the entrance and discharge is

$$\begin{aligned} \Delta p &= \gamma \Delta h = \rho \Delta h \times \frac{g}{g_c} \\ &= \left(\frac{\left(62.0\ \frac{\text{lbm}}{\text{ft}^3}\right)(27.78\ \text{ft})}{\left(12\ \frac{\text{in}}{\text{ft}}\right)^2}\right)\left(\frac{32.2\ \frac{\text{ft}}{\text{sec}^2}}{32.2\ \frac{\text{lbm-ft}}{\text{lbf-sec}^2}}\right) \\ &= 12\ \text{lbf/in}^2 \quad (12\ \text{psi}) \end{aligned}$$

The answer is (A).

SI Solution

As in *NCEES Handbook* table "Pipe Dimensions and Weights," for 6 in pipe, the internal diameter is 154.1 mm (0.1541 m). The internal area is $186.5 \times 10^{-4}\ \text{m}^2$.

As in *NCEES Handbook:* Conservation of Mass, the velocity, v, is

$$\mathrm{v} = \frac{\dot{V}}{A} = \frac{30\ \frac{\mathrm{L}}{\mathrm{s}}}{(186.5 \times 10^{-4}\ \mathrm{m}^2)\left(1000\ \frac{\mathrm{L}}{\mathrm{m}^3}\right)}$$

$$= 1.61\ \mathrm{m/s}$$

Use appendixCERM14B (also *NCEES Handbook* table "Physical Properties of Liquid Water (SI Units)"). For water at 40°C, the kinematic viscosity, ν, is $6.611 \times 10^{-7}\ \mathrm{m^2/s}$, and the density is $992.25\ \mathrm{kg/m^3}$.

As in *NCEES Handbook:* Similitude, the Reynolds number is

$$\mathrm{Re} = \frac{D\mathrm{v}}{\nu} = \frac{(0.1541\ \mathrm{m})\left(1.61\ \frac{\mathrm{m}}{\mathrm{s}}\right)}{6.611 \times 10^{-7}\ \frac{\mathrm{m}^2}{\mathrm{s}}}$$
$$= 3.75 \times 10^5$$

Since Re > 2100, the flow is turbulent. The friction loss coefficient can be determined from the Moody diagram.

From *NCEES Handbook:* Absolute Roughness and Relative Roughness, for new steel pipe, the specific roughness, ϵ, is 6.0×10^{-5} m.

The relative roughness is

$$\frac{\epsilon}{D} = \frac{6.0 \times 10^{-5}\ \mathrm{m}}{0.1541\ \mathrm{m}} = 0.0004$$

From the Moody diagram in *NCEES Handbook:* Friction Factors for Turbulent Flow, with Re = 3.75×10^5 and ϵ/D = 0.0004, the friction factor, f, is 0.0173.

Use appendixCERM17D (also *NCEES Handbook:* Head Loss in Pipe or Conduit). Since 6 in is 0.1524 m, the equivalent length of two 6 in steel flanged elbows is

$$\begin{aligned} \tfrac{L}{D} &= 20 \\ &= 20D \\ &= (20)\left(0.1524\ \frac{\mathrm{m}}{\mathrm{valve}}\right)(2\ \mathrm{elbows}) \\ &= 6.1\ \mathrm{m} \end{aligned}$$

The equivalent length of two fully open gate valves is

$$\begin{aligned} \tfrac{L}{D} &= 10 \\ L &= 10D \\ &= (10)\left(0.1524\ \frac{\mathrm{m}}{\mathrm{valve}}\right)(2\ \mathrm{valves}) \\ &= 3.0\ \mathrm{m} \end{aligned}$$

The equivalent length of a fully open 90° angle valve is

$$\begin{aligned} \tfrac{L}{D} &= 165 \\ L &= 165D \\ &= (165)(0.1524\ \mathrm{m}) \\ &= 25.1\ \mathrm{m} \end{aligned}$$

The equivalent length of a swing check valve is

$$\begin{aligned}\frac{L}{D} &= 125\\ L &= 125D\\ &= (125)(0.1524 \text{ m})\\ &= 19.1 \text{ m}\end{aligned}$$

The total is

$$6.1 \text{ m} + 3.0 \text{ m} + 25.1 \text{ m} + 19.1 \text{ m} = 53.3 \text{ m}$$

The equivalent pipe length is the sum of the straight run of pipe and the equivalent length of pipe for the valves and fittings.

$$\begin{aligned}L_e &= L + L_{\text{fittings}} = 90 \text{ m} + 53.3 \text{ m}\\ &= 143.3 \text{ m}\end{aligned}$$

Using Darcy's equation (see *NCEES Handbook:* Head Loss in Pipe or Conduit), the frictional loss is

$$\begin{aligned}h_f &= \frac{fL_e\text{v}^2}{2Dg} = \frac{(0.0173)(143.3 \text{ m})\left(1.61 \ \frac{\text{m}}{\text{s}}\right)^2}{(2)(0.1541 \text{ m})\left(9.81 \ \frac{\text{m}}{\text{s}^2}\right)}\\ &= 2.12 \text{ m}\end{aligned}$$

The total head loss is the sum of the head losses through the pipe, valves, and fittings and the change in elevation.

$$\begin{aligned}\Delta h &= h_f + \Delta z = 2.12 \text{ m} + 6 \text{ m}\\ &= 8.12 \text{ m}\end{aligned}$$

The pressure difference between the entrance and discharge is

$$\begin{aligned}\Delta p &= \rho\Delta hg = \left(992.25 \ \frac{\text{kg}}{\text{m}^3}\right)(8.12 \text{ m})\left(9.81 \ \frac{\text{m}}{\text{s}^2}\right)\\ &= 79\,040 \text{ Pa} \quad (78 \text{ kPa})\end{aligned}$$

The answer is (A).
10.

Customary U.S. Solution

As in *NCEES Handbook* table "Pipe Dimensions and Weights," for 6 in schedule-40 pipe, the internal diameter, D, is 0.5054 ft. The internal area is 0.2006 ft^2.

Use appendixCERM14D (also *NCEES Handbook:* Temperature-Dependent Properties of Air (U.S. Customary Units)). For air at 70°F and atmospheric pressure, the kinematic viscosity is 16.39×10^{-5} ft^2/sec.

As in *NCEES Handbook:* Similitude, the Reynolds number is

$$\begin{aligned}\text{Re} &= \frac{D\text{v}}{\nu} = \frac{(0.5054 \text{ ft})\left(60 \ \frac{\text{ft}}{\text{sec}}\right)}{16.39 \times 10^{-5} \ \frac{\text{ft}^2}{\text{sec}}}\\ &= 1.85 \times 10^5\end{aligned}$$

Since $\text{Re} > 2100$, the flow is turbulent. The friction loss coefficient can be determined from the Moody diagram.

For new steel pipe, the specific roughness, ϵ, is 0.0002 ft.

As in *NCEES Handbook:* Absolute Roughness and Relative Roughness, the relative roughness is

$$\frac{\epsilon}{D} = \frac{0.0002 \text{ ft}}{0.5054 \text{ ft}} = 0.0004$$

From the Moody diagram in *NCEES Handbook:* Friction Factors for Turbulent Flow, the friction factor, f, is 0.0184 when $\text{Re} = 1.85 \times 10^5$ and $\epsilon/D = 0.0004$.

Use *NCEES Handbook:* Head Loss in Pipe or Conduit. The equivalent length of two 6 in steel flanged elbows is

$$\begin{aligned} \tfrac{L}{D} &= 20 \\ L &= 20D \\ &= \frac{(20)\left(6\ \frac{\text{in}}{\text{elbow}}\right)(2\ \text{elbows})}{12\ \frac{\text{in}}{\text{ft}}} \\ &= 20\ \text{ft} \end{aligned}$$

Use appendixCERM17D. The equivalent length of two fully open gate valves is

$$\begin{aligned} \tfrac{L}{D} &= 10 \\ L &= 10D \\ &= \frac{(10)\left(6\ \frac{\text{in}}{\text{valve}}\right)(2\ \text{valves})}{12\ \frac{\text{in}}{\text{ft}}} \\ &= 10\ \text{ft} \end{aligned}$$

The equivalent length of a fully open 90° angle valve is

$$\begin{aligned} \tfrac{L}{D} &= 165 \\ L &= 165D \\ &= \frac{(165)(6\ \text{in})}{12\ \frac{\text{in}}{\text{ft}}} \\ &= 82.5\ \text{ft} \end{aligned}$$

The equivalent length of a swing check valve is

$$\begin{aligned} \tfrac{L}{D} &= 125 \\ L &= 125D \\ &= \frac{(125)(6\ \text{in})}{12\ \frac{\text{in}}{\text{ft}}} \\ &= 62.5\ \text{ft} \end{aligned}$$

The total is

$$20\ \text{ft} + 10\ \text{ft} + 82.5\ \text{ft} + 62.5\ \text{ft} = 175\ \text{ft}$$

The equivalent pipe length is the sum of the straight run of pipe and the equivalent lengths of pipe for the valves and fittings.

$$\begin{aligned} L_e &= L + L_{\text{fittings}} = 300\ \text{ft} + 175\ \text{ft} \\ &= 475\ \text{ft} \end{aligned}$$

Using Darcy's equation (see *NCEES Handbook:* Head Loss in Pipe or Conduit), the frictional loss is

$$\begin{aligned} h_f &= \frac{fL_e \text{v}^2}{2Dg} = \frac{(0.0184)(450.2\ \text{ft})\left(60\ \frac{\text{ft}}{\text{sec}}\right)^2}{(2)(0.5054\ \text{ft})\left(32.2\ \frac{\text{ft}}{\text{sec}^2}\right)} \\ &= 966.7\ \text{ft} \end{aligned}$$

The head loss is the sum of the head losses through the pipe, valves, and fittings and the change in elevation.

$$\begin{aligned}\Delta h &= h_f + \Delta z = 966.7\ \text{ft} + 20\ \text{ft}\\ &= 986.7\ \text{ft}\end{aligned}$$

The pressure difference between the entrance and discharge is

$$\begin{aligned}\Delta p &= \gamma\Delta h = \rho\Delta h \times \frac{g}{g_c}\\ &= \left(\frac{\left(0.075\ \frac{\text{lbm}}{\text{ft}^3}\right)(986.7\ \text{ft})}{\left(12\ \frac{\text{in}}{\text{ft}}\right)^2}\right)\left(\frac{32.2\ \frac{\text{ft}}{\text{sec}^2}}{32.2\ \frac{\text{lbm-ft}}{\text{lbf-sec}^2}}\right)\\ &= 0.52\ \text{lbf/in}^2 \quad (0.52\ \text{psi})\end{aligned}$$

The answer is (B).

SI Solution

Use appendixCERM16C (also *NCEES Handbook* table “Pipe Dimensions and Weights”). For 6 in pipe, the internal diameter, D, is 154.1 mm (0.1541 m), and the internal area is $186.5 \times 10^{-4}\ \text{m}^2$.

Use appendixCERM14E (also *NCEES Handbook:* Temperature-Dependent Properties of Air (SI Units)). For air at 20°C, the kinematic viscosity, ν, is $1.512 \times 10^{-5}\ \text{m}^2/\text{s}$.

As in *NCEES Handbook:* Similitude, the Reynolds number is

$$\begin{aligned}\text{Re} &= \frac{D\text{v}}{\nu} = \frac{(0.1541\ \text{m})\left(18\ \frac{\text{m}}{\text{s}}\right)}{1.512 \times 10^{-5}\ \frac{\text{m}^2}{\text{s}}}\\ &= 1.83 \times 10^5\end{aligned}$$

Since $\text{Re} > 2100$, the flow is turbulent. The friction loss coefficient can be determined from the Moody diagram.

From *NCEES Handbook:* Absolute Roughness and Relative Roughness, for new steel pipe, the specific roughness, ϵ, is 6.0×10^{-5} m.

The relative roughness is

$$\frac{\epsilon}{D} = \frac{6.0 \times 10^{-5}\ \text{m}}{0.1541\ \text{m}} = 0.0004$$

From the Moody diagram in *NCEES Handbook:* Friction Factors for Turbulent Flow, with Re = 1.83×10^5 and $\epsilon/D = 0.0004$, the friction factor, f, is 0.0185.

Compute the equivalent lengths of the valves and fittings. (Convert from appendixCERM17D (also *NCEES Handbook:* Head Loss in Pipe or Conduit).)

Since 6 in is 0.1524 m, the equivalent length of two 6 in steel flanged elbows is

$$\begin{aligned}\tfrac{L}{D} &= 20\\ L &= 20D\\ &= (20)\left(0.1524\ \frac{\text{m}}{\text{elbow}}\right)(2\ \text{elbows})\\ &= 6.1\ \text{m}\end{aligned}$$

The equivalent length of two fully open gate valves is

$$\begin{aligned}\frac{L}{D} &= 10\\ L &= 10D\\ &= (10)\left(0.1524\ \frac{\mathrm{m}}{\mathrm{valve}}\right)(2\ \mathrm{valves})\\ &= 3.0\ \mathrm{m}\end{aligned}$$

The equivalent length of a fully open 90° angle valve is

$$\begin{aligned}\frac{L}{D} &= 165\\ L &= 165D\\ &= (165)(0.1524\ \mathrm{m})\\ &= 25.1\ \mathrm{m}\end{aligned}$$

The equivalent length of a swing check valve is

$$\begin{aligned}\frac{L}{D} &= 125\\ L &= 125D\\ &= (125)(0.1524\ \mathrm{m})\\ &= 19.1\ \mathrm{m}\end{aligned}$$

The total is

$$6.1\ \mathrm{m} + 3.0\ \mathrm{m} + 25.1\ \mathrm{m} + 19.1\ \mathrm{m} = 53.3\ \mathrm{m}$$

The equivalent pipe length is the sum of the straight run of pipe and the equivalent lengths of pipe for the valves and fittings.

$$\begin{aligned}L_e &= L + L_{\mathrm{fittings}} = 90\ \mathrm{m} + 53.3\ \mathrm{m}\\ &= 143.3\ \mathrm{m}\end{aligned}$$

As in *NCEES Handbook:* Head Loss in Pipe or Conduit, use Darcy's equation to compute the frictional loss.

$$h_f = \frac{fL_e\mathrm{v}^2}{2Dg} = \frac{(0.0185)(143.3\ \mathrm{m})\left(18\ \frac{\mathrm{m}}{\mathrm{s}}\right)^2}{(2)(0.1541\ \mathrm{m})\left(9.81\ \frac{\mathrm{m}}{\mathrm{s}^2}\right)} = 284.1\ \mathrm{m}$$

The head loss is the sum of the head losses through the pipe, valves, and fittings and the change in elevation.

$$\begin{aligned}\Delta h &= h_f + \Delta z = 284.1\ \mathrm{m} + 6\ \mathrm{m}\\ &= 290.1\ \mathrm{m}\end{aligned}$$

The density of the air, ϱ, is approximately 1.20 kg/m^3.

The pressure difference between the entrance and discharge is

$$\begin{aligned}\Delta p &= \rho\Delta hg = \left(1.20\ \frac{\mathrm{kg}}{\mathrm{m}^3}\right)(290.1\ \mathrm{m})\left(9.81\ \frac{\mathrm{m}}{\mathrm{s}^2}\right)\\ &= 3415\ \mathrm{Pa}\quad (3.4\ \mathrm{kPa})\end{aligned}$$

The answer is (B).

11.

Assume that flows from reservoirs A and B are toward D and then toward C. From continuity,

$$\begin{aligned}\dot{V}_{\text{A-D}} + \dot{V}_{\text{B-D}} &= \dot{V}_{\text{D-C}}\\ A_{\mathrm{A}}\mathrm{v}_{\text{A-D}} + A_{\mathrm{B}}\mathrm{v}_{\text{B-D}} - A_{\mathrm{C}}\mathrm{v}_{\text{D-C}} &= 0\end{aligned}$$

From appendixCERM16B (also *NCEES Handbook* table "Pipe Dimensions and Weights"), for schedule-40 pipe,

$$
\begin{aligned}
A_{\mathrm{A}} &= 0.05134\ \mathrm{ft}^2 \quad D_{\mathrm{A}} = 0.2557\ \mathrm{ft} \\
A_{\mathrm{B}} &= 0.5476\ \mathrm{ft}^2 \quad D_{\mathrm{B}} = 0.8350\ \mathrm{ft} \\
A_{\mathrm{C}} &= 0.08841\ \mathrm{ft}^2 \quad D_{\mathrm{C}} = 0.3355\ \mathrm{ft}
\end{aligned}
$$

$$
0.05134\mathrm{v}_{\text{A-D}} + 0.5476\mathrm{v}_{\text{B-D}} - 0.08841\mathrm{v}_{\text{D-C}} = 0 \quad [\text{Eq. I}]
$$

Ignoring the velocity heads, the conservation of energy equation between A and D is

$$
\begin{aligned}
z_{\mathrm{A}} &= \frac{p_{\mathrm{D}}}{\gamma} + z_{\mathrm{D}} + h_{f,\text{A-D}} \\
50\ \mathrm{ft} &= \frac{p_{\mathrm{D}}}{62.4\ \frac{\mathrm{lbf}}{\mathrm{ft}^3}} + 25\ \mathrm{ft} + \frac{(0.02)\,(800\ \mathrm{ft})\,\mathrm{v}_{\text{A-D}}^2}{(2)\,(0.2557\ \mathrm{ft})\left(32.2\ \frac{\mathrm{ft}}{\mathrm{sec}^2}\right)} \\
\mathrm{v}_{\text{A-D}} &= \sqrt{25.73 - 0.0165p_{\mathrm{D}}} \quad [\text{Eq. II}]
\end{aligned}
$$

Similarly, for B–D,

$$
\begin{aligned}
40\ \mathrm{ft} &= \frac{p_{\mathrm{D}}}{62.4\ \frac{\mathrm{lbf}}{\mathrm{ft}^3}} + 25\ \mathrm{ft} + \frac{(0.02)\,(500\ \mathrm{ft})\,\mathrm{v}_{\text{B-D}}^2}{(2)\,(0.8350\ \mathrm{ft})\left(32.2\ \frac{\mathrm{ft}}{\mathrm{sec}^2}\right)} \\
\mathrm{v}_{\text{B-D}} &= \sqrt{80.66 - 0.0862p_{\mathrm{D}}} \quad [\text{Eq. III}]
\end{aligned}
$$

For D–C,

$$
\begin{aligned}
22\ \mathrm{ft} &= \frac{p_{\mathrm{D}}}{62.4\ \frac{\mathrm{lbf}}{\mathrm{ft}^3}} + 25\ \mathrm{ft} - \frac{(0.02)\,(1000\ \mathrm{ft})\,\mathrm{v}_{\text{D-C}}^2}{(2)\,(0.3355\ \mathrm{ft})\left(32.2\ \frac{\mathrm{ft}}{\mathrm{sec}^2}\right)} \\
\mathrm{v}_{\text{D-C}} &= \sqrt{3.24 + 0.0173p_{\mathrm{D}}} \quad [\text{Eq. IV}]
\end{aligned}
$$

Equations I, II, III, and IV must be solved simultaneously. To do this, assume a value for p_{D}. This value then determines all three velocities in Eqs. II, III, and IV. These velocities are substituted into Eq. I. A trial-and-error solution yields

$$
\begin{aligned}
\mathrm{v}_{\text{A-D}} &= 3.21\ \mathrm{ft/sec} \\
\mathrm{v}_{\text{B-D}} &= 0.408\ \mathrm{ft/sec} \\
\mathrm{v}_{\text{D-C}} &= 4.40\ \mathrm{ft/sec} \\
p_{\mathrm{D}} &= 933.8\ \mathrm{lbf/ft}^2
\end{aligned}
$$

Flow is from B to D.

The answer is (B).
12.

Customary U.S. Solution

First, it is necessary to collect data on schedule-40 pipe and water. The fluid viscosity, pipe dimensions, and other parameters can be found in chapterPECHRM01 and chapterPECHRM03 (also *NCEES Handbook* tables “Pipe Dimensions and Weights” and “Physical Properties of Liquid Water (U.S. Units)”). At 70°F water, $\nu = 1.059 \times 10^{-5}\ \mathrm{ft^2/sec}$.

From tableCERM17002 (also *NCEES Handbook:* Absolute Roughness and Relative Roughness), $\epsilon = 0.0002$ ft. From appendixCERM16B (also *NCEES Handbook* table “Pipe Dimensions and Weights”),

8 in pipe $\quad D = 0.6651\ \mathrm{ft}$
$\quad A = 0.3474\ \mathrm{ft}^2$

12 in pipe $\quad D = 0.9948\ \mathrm{ft}$
$\quad A = 0.7773\ \mathrm{ft}^2$

16 in pipe $\quad D = 1.25 \text{ ft}$

$$A = 1.2272 \text{ ft}^2$$

The flow quantity is converted from gallons per minute to cubic feet per second.

$$\begin{aligned}\dot{V} &= \frac{(8 \text{ MGD})\left(10^6 \ \dfrac{\dfrac{\text{gal}}{\text{day}}}{\text{MGD}}\right)}{\left(24 \ \dfrac{\text{hr}}{\text{day}}\right)\left(60 \ \dfrac{\text{min}}{\text{hr}}\right)\left(7.4805 \ \dfrac{\text{gal}}{\text{ft}^3}\right)\left(60 \ \dfrac{\text{sec}}{\text{min}}\right)} \\ &= 12.378 \text{ ft}^3/\text{sec}\end{aligned}$$

As in *NCEES Handbook:* Conservation of Mass, the velocity for the inlet pipe is

$$\text{v} = \frac{\dot{V}}{A} = \frac{12.378 \ \dfrac{\text{ft}^3}{\text{sec}}}{0.3474 \text{ ft}^2} = 35.63 \text{ ft/sec}$$

As in *NCEES Handbook:* Similitude, the Reynolds number is

$$\begin{aligned}\text{Re} &= \frac{D\text{v}}{\nu} = \frac{(0.6651 \text{ ft})\left(35.63 \ \dfrac{\text{ft}}{\text{sec}}\right)}{1.059 \times 10^{-5} \ \dfrac{\text{ft}^2}{\text{sec}}} \\ &= 2.24 \times 10^6\end{aligned}$$

The relative roughness is

$$\frac{\epsilon}{D} = \frac{0.0002 \text{ ft}}{0.6651 \text{ ft}} = 0.0003$$

From the Moody diagram in *NCEES Handbook:* Friction Factors for Turbulent Flow, $f = 0.015$.

Using equationCERM17023(b) (also *NCEES Handbook:* Head Loss in Pipe or Conduit), the frictional energy loss is

$$\begin{aligned}E_{f,1} &= h_f \times \frac{g}{g_c} = \frac{fL\text{v}^2}{2Dg_c} \\ &= \frac{(0.015)(1000 \text{ ft})\left(35.63 \ \dfrac{\text{ft}}{\text{sec}}\right)^2}{(2)(0.6651 \text{ ft})\left(32.2 \ \dfrac{\text{lbm-ft}}{\text{lbf-sec}^2}\right)} \\ &= 444.6 \text{ ft-lbf/lbm}\end{aligned}$$

As in *NCEES Handbook:* Conservation of Mass, the velocity for the outlet pipe is

$$\text{v} = \frac{\dot{V}}{A} = \frac{12.378 \ \dfrac{\text{ft}^3}{\text{sec}}}{0.7773 \text{ ft}^2} = 15.92 \text{ ft/sec}$$

As in *NCEES Handbook:* Similitude, the Reynolds number is

$$\begin{aligned}\text{Re} &= \frac{D\text{v}}{\nu} = \frac{(0.9948 \text{ ft})\left(15.92 \ \dfrac{\text{ft}}{\text{sec}}\right)}{1.059 \times 10^{-5} \ \dfrac{\text{ft}^2}{\text{sec}}} \\ &= 1.5 \times 10^6\end{aligned}$$

The relative roughness is

$$\frac{\epsilon}{D}=\frac{0.0002\ \text{ft}}{0.9948\ \text{ft}}=0.0002$$

From the Moody diagram in *NCEES Handbook:* Friction Factors for Turbulent Flow, $f = 0.014$.

Using equationCERM17023(b) (also *NCEES Handbook:* Head Loss in Pipe or Conduit), the frictional energy loss is

$$\begin{aligned}E_{f,2}&=h_f\times\frac{g}{g_c}=\frac{fL\text{v}^2}{2Dg_c}\\&=\frac{(0.014)\,(1500\ \text{ft})\left(15.92\ \frac{\text{ft}}{\text{sec}}\right)^2}{(2)\,(0.9948\ \text{ft})\left(32.2\ \frac{\text{lbm-ft}}{\text{lbf-sec}^2}\right)}\\&=83.1\ \text{ft-lbf/lbm}\end{aligned}$$

Assume a 50% split through the two branches. As in *NCEES Handbook:* Conservation of Mass, for the upper branch, the velocity is

$$\text{v}=\frac{\dot{V}}{A}=\frac{\left(\frac{1}{2}\right)\left(12.378\ \frac{\text{ft}^3}{\text{sec}}\right)}{0.3474\ \text{ft}^2}=17.82\ \text{ft/sec}$$

As in *NCEES Handbook:* Similitude, the Reynolds number is

$$\begin{aligned}\text{Re}=\frac{D\text{v}}{\nu}&=\frac{(0.6651\ \text{ft})\left(17.82\ \frac{\text{ft}}{\text{sec}}\right)}{1.059\times10^{-5}\ \frac{\text{ft}^2}{\text{sec}}}\\&=1.1\times10^6\end{aligned}$$

The relative roughness is

$$\frac{\epsilon}{D}=\frac{0.0002\ \text{ft}}{0.6651\ \text{ft}}=0.0003$$

From the Moody diagram in *NCEES Handbook:* Friction Factors for Turbulent Flow, $f = 0.015$.

As in *NCEES Handbook:* Conservation of Mass, for the 16 in pipe in the lower branch, the velocity is

$$\text{v}=\frac{\dot{V}}{A}=\frac{\left(\frac{1}{2}\right)\left(12.378\ \frac{\text{ft}^3}{\text{sec}}\right)}{1.2272\ \text{ft}^2}=5.04\ \text{ft/sec}$$

As in *NCEES Handbook:* Similitude, the Reynolds number is

$$\begin{aligned}\text{Re}=\frac{D\text{v}}{\nu}&=\frac{(1.25\ \text{ft})\left(5.04\ \frac{\text{ft}}{\text{sec}}\right)}{1.059\times10^{-5}\ \frac{\text{ft}^2}{\text{sec}}}\\&=5.95\times10^5\end{aligned}$$

The relative roughness is

$$\frac{\epsilon}{D}=\frac{0.0002\ \text{ft}}{1.25\ \text{ft}}=0.00016$$

From the Moody diagram in *NCEES Handbook:* Friction Factors for Turbulent Flow, $f = 0.015$.

These values of f for the two branches are fairly insensitive to changes in $\dot{V}$, so they will be used for the rest of the problem in both branches.

Use equationCERM17023(b) (also *NCEES Handbook:* Head Loss in Pipe or Conduit) to find the frictional energy loss in the upper branch.

$$\begin{aligned}
E_{f,\text{upper}} &= h_f \times \frac{g}{g_c} = \frac{fL\text{v}^2}{2Dg_c} \\
&= \frac{(0.015)\,(500\text{ ft})\left(17.81\ \frac{\text{ft}}{\text{sec}}\right)^2}{(2)\,(0.6651\text{ ft})\left(32.2\ \frac{\text{lbm-ft}}{\text{lbf-sec}^2}\right)} \\
&= 55.5\text{ ft-lbf/lbm}
\end{aligned}$$

To calculate a loss for any other flow in the upper branch,

$$\begin{aligned}
E_{f,\text{upper 2}} &= E_{f,\text{upper}}\left(\frac{\dot{V}}{\left(\frac{1}{2}\right)\left(12.378\ \frac{\text{ft}^3}{\text{sec}}\right)}\right)^2 \\
&= \left(55.5\ \frac{\text{ft-lbf}}{\text{lbm}}\right)\left(\frac{\dot{V}}{6.189\ \frac{\text{ft}^3}{\text{sec}}}\right)^2 \\
&= 1.45\dot{V}^2
\end{aligned}$$

Similarly, for the lower branch, in the 8 in section,

$$\begin{aligned}
E_{f,\text{lower,8 in}} &= h_f \times \frac{g}{g_c} = \frac{fL\text{v}^2}{2Dg_c} \\
&= \frac{(0.015)\,(250\text{ ft})\left(17.81\ \frac{\text{ft}}{\text{sec}}\right)^2}{(2)\,(0.6651\text{ ft})\left(32.2\ \frac{\text{lbm-ft}}{\text{lbf-sec}^2}\right)} \\
&= 27.8\text{ ft-lbf/lbm}
\end{aligned}$$

For the lower branch, in the 16 in section,

$$\begin{aligned}
E_{f,\text{lower,16 in}} &= h_f \times \frac{g}{g_c} = \frac{fL\text{v}^2}{2Dg_c} \\
&= \frac{(0.015)\,(1000\text{ ft})\left(5.04\ \frac{\text{ft}}{\text{sec}}\right)^2}{(2)\,(1.25\text{ ft})\left(32.2\ \frac{\text{lbm-ft}}{\text{lbf-sec}^2}\right)} \\
&= 4.7\text{ ft-lbf/lbm}
\end{aligned}$$

The total loss in the lower branch is

$$\begin{aligned}
E_{f,\text{lower}} &= E_{f,\text{lower,8 in}} + E_{f,\text{lower,16 in}} \\
&= 27.8\ \frac{\text{ft-lbf}}{\text{lbm}} + 4.7\ \frac{\text{ft-lbf}}{\text{lbm}} \\
&= 32.5\text{ ft-lbf/lbm}
\end{aligned}$$

To calculate a loss for any other flow in the lower branch,

$$
\begin{aligned}
E_{f,\text{lower2}} &= E_{f,\text{lower}}\left(\frac{\dot{V}}{\left(\frac{1}{2}\right)\left(12.378\ \frac{\text{ft}^3}{\text{sec}}\right)}\right)^2 \\
&= \left(32.5\ \frac{\text{ft-lbf}}{\text{lbm}}\right)\left(\frac{\dot{V}}{6.189\ \frac{\text{ft}^3}{\text{sec}}}\right)^2 \\
&= 0.85\dot{V}^2
\end{aligned}
$$

Let x be the fraction flowing in the upper branch. Since the friction losses are equal,

$$
\begin{aligned}
E_{f,\text{upper 2}} &= E_{f,\text{lower 2}} \\
1.45x^2 &= (0.85)\,(1-x)^2 \\
x &= 0.434
\end{aligned}
$$

$$
\begin{aligned}
\dot{V}_{\text{upper}} &= (0.434)\left(12.378\ \frac{\text{ft}^3}{\text{sec}}\right) \\
&= 5.372\ \text{ft}^3/\text{sec} \quad \left(5.4\ \text{ft}^3/\text{sec}\right)
\end{aligned}
$$

The answer is (D).

SI Solution

First, it is necessary to collect data on schedule-40 pipe and water. The fluid viscosity, pipe dimensions, and other parameters can be found in various appendices in chapterPECHRM01 and chapterPECHRM03 (also *NCEES Handbook* tables “Pipe Dimensions and Weights” and “Physical Properties of Liquid Water (SI Units)”). At 20°C water, $\nu = 1.007 \times 10^{-6}\ \text{m}^2/\text{s}$.

From *NCEES Handbook:* Absolute Roughness and Relative Roughness, $\epsilon = 6 \times 10^{-5}$ m. As in *NCEES Handbook* table “Pipe Dimensions and Weights,”

8 in pipe	$D = 202.7$ mm $A = 322.75\ \text{cm}^2$
12 in pipe	$D = 303.2$ mm $A = 721.9 \times 10^{-4}\ \text{m}^2$
16 in pipe	$D = 381$ mm $A = 1140 \times 10^{-4}\ \text{m}^2$

As in *NCEES Handbook:* Conservation of Mass, the velocity for the inlet pipe is

$$
\text{v} = \frac{\dot{V}}{A} = \frac{\left(350\ \frac{\text{L}}{\text{s}}\right)\left(100\ \frac{\text{cm}}{\text{m}}\right)^2}{(322.75\ \text{cm}^2)\left(1000\ \frac{\text{L}}{\text{m}^3}\right)} = 10.85\ \text{m/s}
$$

As in *NCEES Handbook:* Similitude, the Reynolds number is

$$
\begin{aligned}
\text{Re} &= \frac{D\text{v}}{\nu} = \frac{(0.2027\ \text{m})\left(10.85\ \frac{\text{m}}{\text{s}}\right)}{1.007 \times 10^{-6}\ \frac{\text{m}^2}{\text{s}}} \\
&= 2.18 \times 10^6
\end{aligned}
$$

The relative roughness is

$$
\frac{\epsilon}{D} = \frac{6 \times 10^{-5}\ \text{m}}{0.2027\ \text{m}} = 0.0003
$$

From the Moody diagram in *NCEES Handbook:* Friction Factors for Turbulent Flow, $f = 0.015$.

Using equationCERM17023(a) (also *NCEES Handbook:* Head Loss in Pipe or Conduit), the frictional energy loss is

$$\begin{aligned} E_{f,1} &= h_f g = \frac{fL\mathrm{v}^2}{2D} \\ &= \frac{(0.015)\,(300\ \mathrm{m})\left(10.85\ \frac{\mathrm{m}}{\mathrm{s}}\right)^2}{(2)\,(0.2027\ \mathrm{m})} \\ &= 1307\ \mathrm{J/kg} \end{aligned}$$

As in *NCEES Handbook:* Conservation of Mass, the velocity for the outlet pipe is

$$\mathrm{v} = \frac{\dot{V}}{A} = \frac{350\ \frac{\mathrm{L}}{\mathrm{s}}}{\left(721.9 \times 10^{-4}\ \mathrm{m}^2\right)\left(1000\ \frac{\mathrm{L}}{\mathrm{m}^3}\right)} = 4.848\ \mathrm{m/s}$$

As in *NCEES Handbook:* Similitude, the Reynolds number is

$$\begin{aligned} \mathrm{Re} &= \frac{D\mathrm{v}}{\nu} = \frac{(0.3032\ \mathrm{m})\left(4.848\ \frac{\mathrm{m}}{\mathrm{s}}\right)}{1.007 \times 10^{-6}\ \frac{\mathrm{m}^2}{\mathrm{s}}} \\ &= 1.46 \times 10^6 \end{aligned}$$

The relative roughness is

$$\frac{\epsilon}{D} = \frac{6 \times 10^{-5}\ \mathrm{m}}{0.3032\ \mathrm{m}} = 0.0002$$

From the Moody diagram in *NCEES Handbook:* Friction Factors for Turbulent Flow, $f = 0.014$.

Using equationCERM17023(a) (also *NCEES Handbook:* Head Loss in Pipe or Conduit), the frictional energy loss is

$$\begin{aligned} E_{f,2} &= h_f g = \frac{fL\mathrm{v}^2}{2D} \\ &= \frac{(0.014)\,(450\ \mathrm{m})\left(4.848\ \frac{\mathrm{m}}{\mathrm{s}}\right)^2}{(2)\,(0.3032\ \mathrm{m})} \\ &= 244.2\ \mathrm{J/kg} \end{aligned}$$

Assume a 50% split through the two branches. As in *NCEES Handbook:* Conservation of Mass, in the upper branch, the velocity is

$$\mathrm{v} = \frac{\dot{V}}{A} = \frac{\left(\frac{1}{2}\right)\left(350\ \frac{\mathrm{L}}{\mathrm{s}}\right)}{\left(322.7 \times 10^{-4}\ \mathrm{m}^2\right)\left(1000\ \frac{\mathrm{L}}{\mathrm{m}^3}\right)} = 5.423\ \mathrm{m/s}$$

As in *NCEES Handbook:* Similitude, the Reynolds number is

$$\begin{aligned} \mathrm{Re} &= \frac{D\mathrm{v}}{\nu} = \frac{(0.2027\ \mathrm{m})\left(5.423\ \frac{\mathrm{m}}{\mathrm{s}}\right)}{1.007 \times 10^{-6}\ \frac{\mathrm{m}^2}{\mathrm{s}}} \\ &= 1.1 \times 10^6 \end{aligned}$$

The relative roughness is

$$\frac{\epsilon}{D} = \frac{6 \times 10^{-5}\ \text{m}}{0.2027\ \text{m}} = 0.0003$$

From the Moody diagram in *NCEES Handbook:* Friction Factors for Turbulent Flow, $f = 0.015$.

As in *NCEES Handbook:* Conservation of Mass, for the 16 in pipe in the lower branch, the velocity is

$$\text{v} = \frac{\dot{V}}{A} = \frac{\left(\frac{1}{2}\right)\left(350\ \frac{\text{L}}{\text{s}}\right)}{\left(1140 \times 10^{-4}\ \text{m}^2\right)\left(1000\ \frac{\text{L}}{\text{m}^3}\right)} = 1.535\ \text{m/s}$$

As in *NCEES Handbook:* Similitude, the Reynolds number is

$$\begin{aligned}\text{Re} &= \frac{D\text{v}}{\nu} = \frac{(0.381\ \text{m})\left(1.535\ \frac{\text{m}}{\text{s}}\right)}{1.007 \times 10^{-6}\ \frac{\text{m}^2}{\text{s}}} \\ &= 5.81 \times 10^5\end{aligned}$$

The relative roughness is

$$\frac{\epsilon}{D} = \frac{6 \times 10^{-5}\ \text{m}}{0.381\ \text{m}} = 0.00016$$

From the Moody diagram in *NCEES Handbook:* Friction Factors for Turbulent Flow, $f = 0.015$.

These values of f for the two branches are fairly insensitive to changes in $\dot{V}$, so they will be used for the rest of the problem in both branches.

Using equationCERM17023(a) (also *NCEES Handbook:* Head Loss in Pipe or Conduit), the frictional energy loss in the upper branch is

$$\begin{aligned}E_{f,\text{upper}} &= h_f g = \frac{fL\text{v}^2}{2D} \\ &= \frac{(0.015)(150\ \text{m})\left(5.423\ \frac{\text{m}}{\text{s}}\right)^2}{(2)(0.2027\ \text{m})} \\ &= 163.2\ \text{J/kg}\end{aligned}$$

To calculate a loss for any other flow in the upper branch,

$$\begin{aligned}E_{f,\text{upper2}} &= E_{f,\text{upper}}\left(\frac{\dot{V}}{\left(\frac{1}{2}\right)\left(0.350\ \frac{\text{m}^3}{\text{s}}\right)}\right)^2 \\ &= \left(163.2\ \frac{\text{J}}{\text{kg}}\right)\left(\frac{\dot{V}}{0.175\ \frac{\text{m}^3}{\text{s}}}\right)^2 \\ &= 5329\dot{V}^2\end{aligned}$$

Similarly, for the lower branch, in the 8 in section,

$$\begin{aligned}E_{f,\text{lower,8 in}} &= h_f g = \frac{fL\text{v}^2}{2D} = \frac{(0.015)(75\ \text{m})\left(5.423\ \frac{\text{m}}{\text{s}}\right)^2}{(2)(0.2027\ \text{m})} \\ &= 81.61\ \text{J/kg}\end{aligned}$$

For the lower branch, in the 16 in section,

$$\begin{aligned} E_{f,\text{lower},16\text{ in}} &= h_f g = \frac{fL\text{v}^2}{2D} = \frac{(0.015)\,(300\text{ m})\left(1.585\ \frac{\text{m}}{\text{s}}\right)^2}{(2)\,(0.381\text{ m})} \\ &= 14.84\text{ J/kg} \end{aligned}$$

The total loss in the lower branch is

$$\begin{aligned} E_{f,\text{lower}} &= E_{f,\text{lower},8\text{ in}} + E_{f,\text{lower},16\text{ in}} \\ &= 81.61\ \frac{\text{J}}{\text{kg}} + 14.84\ \frac{\text{J}}{\text{kg}} \\ &= 96.45\text{ J/kg} \end{aligned}$$

To calculate a loss for any other flow in the lower branch,

$$\begin{aligned} E_{f,\text{lower2}} &= E_{f,\text{lower}}\left(\frac{\dot{V}}{\left(\frac{1}{2}\right)\left(0.350\ \frac{\text{m}^3}{\text{s}}\right)}\right)^2 \\ &= \left(96.45\ \frac{\text{J}}{\text{kg}}\right)\left(\frac{\dot{V}}{0.175\ \frac{\text{m}^3}{\text{s}}}\right)^2 \\ &= 3149\dot{V}^2 \end{aligned}$$

Let x be the fraction flowing in the upper branch. Since the friction losses are equal,

$$\begin{aligned} E_{f,\text{upper 2}} &= E_{f,\text{lower 2}} \\ 5329x^2 &= (3149)\,(1-x)^2 \\ x &= 0.435 \end{aligned}$$

$$\begin{aligned} \dot{V}_{\text{upper}} &= (0.435)\left(0.350\ \frac{\text{m}^3}{\text{s}}\right) \\ &= 0.15\text{ m}^3/\text{s} \end{aligned}$$

The answer is (D).
13.

First, find the Reynolds number from the velocity (v), flow diameter (D), and kinematic viscosity (ν).

$$\begin{aligned} \text{Re} &= \frac{\text{v}D}{\nu} \\ &= \frac{\left(4\ \frac{\text{m}}{\text{s}}\right)(0.1\text{ m})}{10^{-6}\ \frac{\text{m}^2}{\text{s}}} \\ &= 4\times 10^5 \end{aligned}$$

Next, find the relative roughness. For plain cast iron, the specific roughness is usually taken as 0.00024 m.

$$\frac{\epsilon}{D} = \frac{0.00024\text{ m}}{0.1\text{ m}} = 0.0024$$

With the value of the Reynolds number and the relative roughness, use a Moody diagram to find the friction factor, f.

$$f = 0.025$$

Multiply Darcy's equation by the specific weight of water to find the pressure difference, Δp. L is length and g is gravitational acceleration.

$$h_f = \frac{fL\mathrm{v}^2}{2Dg}$$

$$\begin{aligned}\Delta p &= \gamma h_f \\ &= \gamma \frac{fL\mathrm{v}^2}{2Dg}\end{aligned}$$

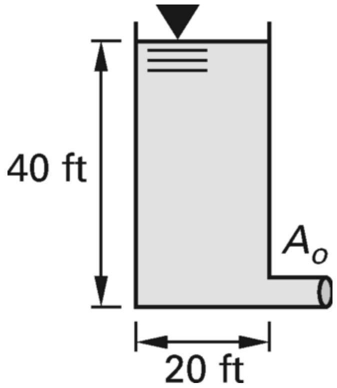
$$\begin{aligned}\Delta p &= \left(9800\ \frac{\mathrm{N}}{\mathrm{m}^3}\right) \frac{(0.025)(100\ \mathrm{m})\left(4\ \frac{\mathrm{m}}{\mathrm{s}}\right)^2}{(2)(0.1\ \mathrm{m})\left(9.8\ \frac{\mathrm{m}}{\mathrm{s}^2}\right)} \\ &= 2.0 \times 10^5\ \mathrm{Pa} \quad (200\ \mathrm{kPa})\end{aligned}$$

The answer is (B).

14.

$$A_o = \left(\frac{\pi}{4}\right)\left(\frac{4\ \mathrm{in}}{12\ \frac{\mathrm{in}}{\mathrm{ft}}}\right)^2 = 0.08727\ \mathrm{ft}^2$$

$$A_t = \left(\frac{\pi}{4}\right)(20\ \mathrm{ft})^2 = 314.16\ \mathrm{ft}^2$$



Using Torricelli's equation in equationCERM17084 (also *NCEES Handbook*: Orifice Discharging Freely into Atmosphere), the time it takes to drop from 40 ft to 20 ft is

$$\begin{aligned}t &= \frac{2A_t\left(\sqrt{z_1} - \sqrt{z_2}\right)}{C_d A_o \sqrt{2g}} \\ &= \frac{(2)\left(314.16\ \mathrm{ft}^2\right)\left(\sqrt{40\ \mathrm{ft}} - \sqrt{20\ \mathrm{ft}}\right)}{(0.98)\left(0.08727\ \mathrm{ft}^2\right)\sqrt{(2)\left(32.2\ \frac{\mathrm{ft}}{\mathrm{sec}^2}\right)}} \\ &= 1696\ \mathrm{sec} \quad (1700\ \mathrm{sec})\end{aligned}$$

The answer is (D).

15.

For low velocities, the total energy head of the water in the canal is 320 ft. Similarly, the total energy head of the water in the irrigation ditch is 305 ft. The head loss is 320 ft – 305 ft = 15 ft.

The answer is (C).

16.

From *NCEES Handbook:* Orifice, Nozzle, and Venturi Meters, the orifice coefficient is $C_{\mathrm{orifice}} = 1.00$.

$$C = \frac{C_{\text{orifice}}}{\sqrt{1-\beta^4}} = \frac{C_{\text{orifice}}}{\sqrt{1-\left(\frac{D_2}{D_1}\right)^4}}$$

$$A_2 = \left(\frac{\pi}{4}\right)\left(\frac{8 \text{ in}}{12 \frac{\text{in}}{\text{ft}}}\right)^2 = 0.3491 \text{ ft}^2$$

The specific weight of mercury is 0.491 lbf/in^3; the specific weight of water is 0.0361 lbf/in^3.

$$\begin{aligned} p_1 - p_2 &= \Delta(\gamma h) \\ &= \left(\begin{array}{l}\left(0.491 \frac{\text{lbf}}{\text{in}^3}\right)(4 \text{ in}) \\ \quad - \left(0.0361 \frac{\text{lbf}}{\text{in}^3}\right)(4 \text{ in})\end{array}\right)\left(12 \frac{\text{in}}{\text{ft}}\right)^2 \\ &= 262.0 \text{ lbf/ft}^2 \end{aligned}$$

From equationCERM17154 (also *NCEES Handbook:* Square-Edge Orifice Meter (Vena Contracta Taps)),

$$\begin{aligned} \dot{V} &= CA_{\text{venturi}}\sqrt{\frac{2g_c(p_1-p_2)}{p}} \\ &= (1.116)(1)\left(0.3491 \text{ ft}^2\right) \\ &\quad \times \sqrt{\frac{(2)\left(32.2 \frac{\text{ft}}{\text{sec}^2}\right)\left(262 \frac{\text{lbf}}{\text{ft}^2}\right)}{62.4 \frac{\text{lbf}}{\text{ft}^3}}} \\ &= 6.406 \text{ ft}^3/\text{sec} \quad (6.4 \text{ ft}^3/\text{sec}) \end{aligned}$$

The answer is (C).
17.

Customary U.S. Solution

As in *NCEES Handbook:* Square-Edge Orifice Meter (Vena Contracta Taps), the volumetric flow rate of benzene through the venturi meter is given by

$$\dot{V} = C_f A_2 \sqrt{\frac{2g(\rho_m - \rho)h}{\rho}}$$

As in *NCEES Handbook* table “Standard Values,” the density of mercury, ϱ_m, at 60°F is approximately 848 lbm/ft^3.

As in *NCEES Handbook:* Density Definitions, the density of the benzene at 60°F is

$$\rho = (\text{SG})\rho_{\text{water}} = (0.885)\left(62.4 \frac{\text{lbm}}{\text{ft}^3}\right) = 55.22 \text{ lbm/ft}^3$$

The throat area is

$$\begin{aligned} A_2 &= \frac{\pi D_2^2}{4} = \frac{\pi\left(\frac{3.5 \text{ in}}{12 \frac{\text{in}}{\text{ft}}}\right)^2}{4} = 0.0668 \text{ ft}^2 \\ \beta &= \frac{3.5 \text{ in}}{8 \text{ in}} = 0.4375 \end{aligned}$$

As in *NCEES Handbook:* Venturi Flow Nozzle Meter,

$$C_f = \frac{C_d}{\sqrt{1-\beta^4}} = \frac{0.99}{\sqrt{1-(0.4375)^4}} = 1.00865$$

Find the volumetric flow of benzene.

$$\begin{aligned}\dot{V} &= C_f A_2 \sqrt{\frac{2g\left(\rho_m - \rho\right)h}{\rho}} \\ &= (1.00865)\left(0.0668\ \text{ft}^2\right) \\ &\quad \times \sqrt{\frac{(2)\left(32.2\ \frac{\text{ft}}{\text{sec}^2}\right)\left(848\ \frac{\text{lbm}}{\text{ft}^3} - 55.22\ \frac{\text{lbm}}{\text{ft}^3}\right) \times (4\ \text{in})}{\left(55.22\ \frac{\text{lbm}}{\text{ft}^3}\right)\left(12\ \frac{\text{in}}{\text{ft}}\right)}} \\ &= 1.181\ \text{ft}^3/\text{sec} \quad (1.2\ \text{ft}^3/\text{sec})\end{aligned}$$

The answer is (A).

SI Solution

As in *NCEES Handbook:* Square-Edge Orifice Meter (Vena Contracta Taps), the volumetric flow rate of benzene through the venturi meter is

$$\dot{V} = C_f A_2 \sqrt{\frac{2g\left(\rho_m - \rho\right)h}{\rho}}$$

As in *NCEES Handbook* table "Standard Values," ϱ_m is the density of mercury at 15°C; ϱ_m is approximately 13 600 kg/m^3.

As in *NCEES Handbook:* Density Definitions, the density of the benzene at 15°C is

$$\begin{aligned}\rho &= (\text{SG})\,\rho_{\text{water}} = (0.885)\left(1000\ \frac{\text{kg}}{\text{m}^3}\right) \\ &= 885\ \text{kg/m}^3\end{aligned}$$

The throat area is

$$\begin{aligned}A_2 &= \frac{\pi D_2^2}{4} = \frac{\pi(0.09\ \text{m})^2}{4} = 0.0064\ \text{m}^2 \\ \beta &= \frac{9\ \text{cm}}{20\ \text{cm}} = 0.45\end{aligned}$$

As in *NCEES Handbook:* Venturi Flow Nozzle Meter,

$$C_f = \frac{C_d}{\sqrt{1-\beta^4}} = \frac{0.99}{\sqrt{1-(0.45)^4}} = 1.01094$$

Find the volumetric flow of benzene.

$$
\begin{aligned}
\dot{V} &= C_f A_2 \sqrt{\frac{2g\left(\rho_m - \rho\right)h}{\rho}} \\
&= (1.01094)\left(0.0064\ \mathrm{m}^2\right) \\
&\quad \times \sqrt{\frac{(2)\left(9.81\ \frac{\mathrm{m}}{\mathrm{s}^2}\right) \times \left(13\,600\ \frac{\mathrm{kg}}{\mathrm{m}^3} - 885\ \frac{\mathrm{kg}}{\mathrm{m}^3}\right)(0.1\ \mathrm{m})}{885\ \frac{\mathrm{kg}}{\mathrm{m}^3}}} \\
&= 0.0344\ \mathrm{m}^3/\mathrm{s} \quad (34\ \mathrm{L/s})
\end{aligned}
$$

The answer is (A).
18.

From appendixCERM14A (also *NCEES Handbook* table "Physical Properties of Liquid Water (U.S. Units)"), for 70°F water,

$$
\begin{aligned}
\nu &= 1.059 \times 10^{-5}\ \mathrm{ft}^2/\mathrm{sec} \\
\gamma &= 62.3\ \mathrm{lbf/ft}^3 \\
D_o &= 0.2\ \mathrm{ft} \\
\mathrm{v}_o &= \mathrm{v}\left(\frac{D}{D_o}\right)^2 = \left(2\ \frac{\mathrm{ft}}{\mathrm{sec}}\right)\left(\frac{1\ \mathrm{ft}}{0.2\ \mathrm{ft}}\right)^2 \\
&= 50\ \mathrm{ft/sec}
\end{aligned}
$$

As in *NCEES Handbook:* Similitude,

$$
\begin{aligned}
\mathrm{Re} &= \frac{D_o \mathrm{v}_o}{\nu} = \frac{(0.2\ \mathrm{ft})\left(50\ \frac{\mathrm{ft}}{\mathrm{sec}}\right)}{1.059 \times 10^{-5}\ \frac{\mathrm{ft}^2}{\mathrm{sec}}} = 9.44 \times 10^5 \\
A_o &= \left(\frac{\pi}{4}\right)(0.2\ \mathrm{ft})^2 = 0.0314\ \mathrm{ft}^2 \\
A &= \left(\frac{\pi}{4}\right)(1\ \mathrm{ft})^2 = 0.7854\ \mathrm{ft}^2 \\
\frac{A_o}{A} &= \frac{0.0314\ \mathrm{ft}^2}{0.7854\ \mathrm{ft}^2} = 0.040
\end{aligned}
$$

From figureCERM17028 (also *NCEES Handbook:* Square-Edge Orifice Meter (Vena Contracta Taps)),

$$
\begin{aligned}
C_f &\approx 0.60 \\
\dot{V} &= A\mathrm{v} = \left(0.7854\ \mathrm{ft}^2\right)\left(2\ \frac{\mathrm{ft}}{\mathrm{sec}}\right) = 1.571\ \mathrm{ft}^3/\mathrm{sec}
\end{aligned}
$$

From equationCERM17160(b) (also *NCEES Handbook:* Square-Edge Orifice Meter (Vena Contracta Taps)), substituting $\gamma = \varrho g/g_c$,

$$
\begin{aligned}
p_p - p_o &= \left(\frac{\gamma}{2g}\right)\left(\frac{\dot{V}}{C_f A_o}\right)^2 \\
&= \frac{\left(\frac{62.3\ \frac{\mathrm{lbf}}{\mathrm{ft}^3}}{(2)\left(32.2\ \frac{\mathrm{ft}}{\mathrm{sec}^2}\right)}\right)\left(\frac{1.571\ \frac{\mathrm{ft}^3}{\mathrm{sec}}}{(0.60)\left(0.0314\ \mathrm{ft}^2\right)}\right)^2}{\left(12\ \frac{\mathrm{in}}{\mathrm{ft}}\right)^2} \\
&= 46.7\ \mathrm{lbf/in}^2 \quad (47\ \mathrm{psi})
\end{aligned}
$$

The answer is (D).

19.

Customary U.S. Solution

As in *NCEES Handbook* table "Standard Values," the densities of mercury and water are

$$\begin{aligned}\rho_{\text{mercury}} &= 848\ \text{lbm/ft}^3\\ \rho_{\text{water}} &= 62.4\ \text{lbm/ft}^3\end{aligned}$$

The manometer tube is filled with water above the mercury column. The pressure differential across the orifice meter is

$$\begin{aligned}\Delta p = p_1 - p_2 &= (\rho_{\text{mercury}} - \rho_{\text{water}})\, h \times \frac{g}{g_c}\\ &= \left(\frac{\left(848\ \frac{\text{lbm}}{\text{ft}^3} - 62.4\ \frac{\text{lbm}}{\text{ft}^3}\right)(7\ \text{in})}{12\ \frac{\text{in}}{\text{ft}}}\right)\left(\frac{32.2\ \frac{\text{ft}}{\text{sec}^2}}{32.2\ \frac{\text{lbm-ft}}{\text{lbf-sec}^2}}\right)\\ &= 458.3\ \text{lbf/ft}^2 \quad (3.2\ \text{psi})\end{aligned}$$

The answer is (B).

SI Solution

As in *NCEES Handbook* table "Standard Values," the densities of mercury and water are

$$\begin{aligned}\rho_{\text{mercury}} &= 13\,600\ \text{kg/m}^3\\ \rho_{\text{water}} &= 1000\ \text{kg/m}^3\end{aligned}$$

The manometer tube is filled with water above the mercury column. The pressure differential across the orifice meter is given by

$$\begin{aligned}\Delta p = p_1 - p_2 &= (\rho_{\text{mercury}} - \rho_{\text{water}})\, hg\\ &= \left(13\,600\ \frac{\text{kg}}{\text{m}^3} - 1000\ \frac{\text{kg}}{\text{m}^3}\right)(0.178\ \text{m})\left(9.81\ \frac{\text{m}}{\text{s}^2}\right)\\ &= 22\,002\ \text{Pa} \quad (22\ \text{kPa})\end{aligned}$$

The answer is (B).

20.

Customary U.S. Solution

Use appendixCERM16B (also *NCEES Handbook* table "Pipe Dimensions and Weights"). For 12 in pipe,

$$\begin{aligned}D &= 0.99483\ \text{ft}\\ A &= 0.7773\ \text{ft}^2\end{aligned}$$

As in *NCEES Handbook:* Conservation of Mass, the velocity is

$$\text{v} = \frac{\dot{V}}{A} = \frac{10\ \frac{\text{ft}^3}{\text{sec}}}{0.7773\ \text{ft}^2} = 12.87\ \text{ft/sec}$$

As in *NCEES Handbook* table "Physical Properties of Liquid Water (U.S. Units)," for water at 70°F, $\nu = 1.059 \times 10^{-5}\ \text{ft}^2/\text{sec}$.

As in *NCEES Handbook:* Similitude, the Reynolds number in the pipe is

$$\begin{aligned}\mathrm{Re} &= \frac{\mathrm{v}D}{\nu} = \frac{\left(12.87\ \frac{\mathrm{ft}}{\mathrm{sec}}\right)(0.99483\ \mathrm{ft})}{1.059\times 10^{-5}\ \frac{\mathrm{ft}^2}{\mathrm{sec}}}\\ &= 1.21\times 10^{6}\quad [\text{fully turbulent}]\end{aligned}$$

Flow through the orifice will have a higher Reynolds number and will also be turbulent.

The volumetric flow rate through a sharp-edged orifice can be found in *NCEES Handbook:* Square-Edge Orifice Meter (Vena Contracta Taps).

$$\dot{V} = CA_o\sqrt{\frac{2g(\rho_m-\rho)h}{\rho}} = CA_o\sqrt{\frac{2g_c(p_1-p_2)}{\rho}}$$

Rearranging,

$$CA_o = \frac{\dot{V}}{\sqrt{\frac{2g_c(p_1-p_2)}{\rho}}}$$

The maximum head loss must not exceed 25 ft.

$$\begin{aligned}\frac{\frac{g_c}{g}\times(p_1-p_2)}{\rho} &= 25\ \mathrm{ft}\\ \frac{g_c(p_1-p_2)}{\rho} &= (25\ \mathrm{ft})\,g\end{aligned}$$

Substituting,

$$CA_o = \frac{10\ \frac{\mathrm{ft}^3}{\mathrm{sec}}}{\sqrt{(2)\left(32.2\ \frac{\mathrm{ft}}{\mathrm{sec}^2}\right)(25\ \mathrm{ft})}} = 0.249\ \mathrm{ft}^2$$

Both C and A_o depend on the orifice diameter. For a 7 in diameter orifice,

$$\begin{aligned}A_o &= \frac{\pi D_o^2}{4} = \frac{\pi\left(\frac{7\ \mathrm{in}}{12\ \frac{\mathrm{in}}{\mathrm{ft}}}\right)^2}{4} = 0.267\ \mathrm{ft}^2\\ \frac{A_o}{A_1} &= \frac{0.267\ \mathrm{ft}^2}{0.7773\ \mathrm{ft}^2} = 0.343\end{aligned}$$

From figureCERM17028 (also *NCEES Handbook* figure "Discharge Coefficient C_{orifice} for Square-Edge Orifice Meters") for $d_2/d_1 = 7/12$ and fully turbulent flow,

$$\begin{aligned}C &= \frac{C_{\mathrm{orifice}}}{\sqrt{1-\beta^4}} = \frac{C_{\mathrm{orifice}}}{\sqrt{1-\left(\frac{D_2}{D_1}\right)^4}}\\ &= \frac{0.605}{\sqrt{1-\left(\frac{7\ \mathrm{in}}{12\ \mathrm{in}}\right)^4}}\\ &= 0.645\\ CA_o &= (0.645)\left(0.267\ \mathrm{ft}^2\right) = 0.172\ \mathrm{ft}^2 < 0.249\ \mathrm{ft}^2\end{aligned}$$

Therefore, a 7 in diameter orifice is too small.

Try a 9 in diameter orifice.

$$\begin{aligned}
A_o &= \frac{\pi D_o^2}{4} = \frac{\pi\left(\dfrac{9\ \text{in}}{12\ \dfrac{\text{in}}{\text{ft}}}\right)^2}{4} = 0.442\ \text{ft}^2 \\
\frac{A_o}{A_1} &= \frac{0.442\ \text{ft}^2}{0.7773\ \text{ft}^2} = 0.569
\end{aligned}$$

From figureCERM17028 (also *NCEES Handbook* figure “Discharge Coefficient C_{orifice} for Square-Edge Orifice Meters”), for $d_2/d_1 = 0.75$ and fully turbulent flow,

$$\begin{aligned}
C &= \frac{C_{\text{orifice}}}{\sqrt{1-\beta^4}} = \frac{C_{\text{orifice}}}{\sqrt{1-\left(\dfrac{D_2}{D_1}\right)^4}} \\
&= \frac{0.615}{\sqrt{1-\left(\dfrac{9\ \text{in}}{12\ \text{in}}\right)^4}} \\
&= 0.73 \\
CA_o &= (0.73)\left(0.442\ \text{ft}^2\right) = 0.323\ \text{ft}^2 > 0.249\ \text{ft}^2
\end{aligned}$$

Therefore, a 9 in orifice is too large.

Interpolating gives

$$\begin{aligned}
D_o &= 7\ \text{in} + \frac{(9\ \text{in} - 7\ \text{in})\left(0.249\ \text{ft}^2 - 0.172\ \text{ft}^2\right)}{0.323\ \text{ft}^2 - 0.172\ \text{ft}^2} \\
&= 8.0\ \text{in}
\end{aligned}$$

Further iterations yield

$$\begin{aligned}
D_o &\approx 8.1\ \text{in} \\
CA_o &= 0.243\ \text{ft}^2
\end{aligned}$$

The answer is (C).

SI Solution

For 300 mm inside diameter pipe, $D = 0.3$ m.

As in *NCEES Handbook:* Conservation of Mass, the velocity is

$$\begin{aligned}
\text{v} &= \frac{\dot{V}}{A} = \frac{\dot{V}}{\dfrac{\pi D^2}{4}} = \frac{250\ \dfrac{\text{L}}{\text{s}}}{\left(\dfrac{\pi(0.3\ \text{m})^2}{4}\right)\left(1000\ \dfrac{\text{L}}{\text{m}^3}\right)} \\
&= 3.54\ \text{m/s}
\end{aligned}$$

From appendixCERM14B (also *NCEES Handbook* table “Physical Properties of Liquid Water (SI Units)”), for water at 20°C, $\nu = 1.007 \times 10^{-6}\ \text{m}^2/\text{s}$.

As in *NCEES Handbook:* Similitude, the Reynolds number in the pipe is

$$\begin{aligned}
\text{Re} &= \frac{\text{v}D}{\nu} = \frac{\left(3.54\ \dfrac{\text{m}}{\text{s}}\right)(0.3\ \text{m})}{1.007\times 10^{-6}\ \dfrac{\text{m}^2}{\text{s}}} \\
&= 1.05\times 10^{6}\quad [\text{fully turbulent}]
\end{aligned}$$

Flow through the orifice will have a higher Reynolds number and also be turbulent.

As in *NCEES Handbook:* Square-Edge Orifice Meter (Vena Contracta Taps), the volumetric flow rate through a sharp-edged orifice is

$$\dot{V} = CA_o\sqrt{\frac{2g(\rho_m - \rho)h}{\rho}} = CA_o\sqrt{\frac{2(p_1 - p_2)}{\rho}}$$

Rearranging,

$$CA_o = \frac{\dot{V}}{\sqrt{\frac{2(p_1 - p_2)}{\rho}}}$$

The maximum head loss must not exceed 7.5 m.

$$\begin{aligned} \frac{p_1 - p_2}{g\rho} &= 7.5\ \text{m} \\ \frac{p_1 - p_2}{\rho} &= (7.5\ \text{m})\,g \end{aligned}$$

Substituting,

$$CA_o = \frac{0.25\ \frac{\text{m}^3}{\text{s}}}{\sqrt{(2)\left(9.81\ \frac{\text{m}}{\text{s}^2}\right)(7.5\ \text{m})}} = 0.021\ \text{m}^2$$

Both C and A_o depend on the orifice diameter. For an 18 cm diameter orifice,

$$\begin{aligned} A_o &= \frac{\pi D_o^2}{4} = \frac{\pi(0.18\ \text{m})^2}{4} = 0.0254\ \text{m}^2 \\ \frac{A_o}{A_1} &= \frac{0.0254\ \text{m}^2}{0.0707\ \text{m}^2} = 0.359 \end{aligned}$$

From figureCERM17028 (also *NCEES Handbook* figure "Discharge Coefficient C_{orifice} for Square-Edge Orifice Meters"), for $d_2/d_1 =$ 18 cm/30 cm and fully turbulent flow,

$$\begin{aligned} C &= \frac{C_{\text{orifice}}}{\sqrt{1-\beta^4}} = \frac{C_{\text{orifice}}}{\sqrt{1-\left(\frac{D_2}{D_1}\right)^4}} \\ &= \frac{0.605}{\sqrt{1-\left(\frac{18\ \text{cm}}{30\ \text{cm}}\right)^4}} \\ &= 0.65 \\ CA_o &= (0.65)\left(0.0254\ \text{m}^2\right) = 0.0165\ \text{m}^2 < 0.021\ \text{m}^2 \end{aligned}$$

Therefore, an 18 cm diameter orifice is too small.

Try a 23 cm diameter orifice.

$$\begin{aligned} A_o &= \frac{\pi D_o^2}{4} = \frac{\pi(0.23\ \text{m})^2}{4} = 0.0415\ \text{m}^2 \\ \frac{A_o}{A_1} &= \frac{0.0415\ \text{m}^2}{0.0707\ \text{m}^2} = 0.587 \end{aligned}$$

From figureCERM17028 (also *NCEES Handbook* figure "Discharge Coefficient C_{orifice} for Square-Edge Orifice Meters"), for $d_2/d_1 =$ 23 cm/30 cm and fully turbulent flow,

$$
\begin{aligned}
C &= \frac{C_{\text{orifice}}}{\sqrt{1-\beta^4}} = \frac{C_{\text{orifice}}}{\sqrt{1-\left(\frac{D_2}{D_1}\right)^4}} \\
&= \frac{0.615}{\sqrt{1-\left(\frac{23\ \text{cm}}{30\ \text{cm}}\right)^4}} \\
&= 0.73 \\
CA_o &= (0.73)\left(0.0415\ \text{m}^2\right) = 0.0303\ \text{m}^2 > 0.021\ \text{m}^2
\end{aligned}
$$

Therefore, a 23 cm orifice is too large.

Interpolating gives

$$
\begin{aligned}
D_o &= 18\ \text{cm} \\
&\quad + (23\ \text{cm} - 18\ \text{cm})\left(\frac{0.021\ \text{m}^2 - 0.0165\ \text{m}^2}{0.0303\ \text{m}^2 - 0.0165\ \text{m}^2}\right) \\
&= 19.6\ \text{cm} \quad (19\ \text{cm})
\end{aligned}
$$

Further iteration yields

$$
\begin{aligned}
D_o &= 20\ \text{cm} \\
C &= 0.675 \\
CA_o &= 0.021\ \text{m}^2
\end{aligned}
$$

The answer is (C).
21.

$$
\begin{aligned}
A_{\text{A}} &= \left(\frac{\pi}{4}\right)\left(\frac{24\ \text{in}}{12\ \frac{\text{in}}{\text{ft}}}\right)^2 = 3.142\ \text{ft}^2 \\
A_{\text{B}} &= \left(\frac{\pi}{4}\right)\left(\frac{12\ \text{in}}{12\ \frac{\text{in}}{\text{ft}}}\right)^2 = 0.7854\ \text{ft}^2
\end{aligned}
$$

As in *NCEES Handbook:* Conservation of Mass,

$$
\begin{aligned}
\text{v}_{\text{A}} &= \frac{\dot{V}}{A} = \frac{8\ \frac{\text{ft}^3}{\text{sec}}}{3.142\ \text{ft}^2} = 2.546\ \text{ft/sec} \\
p_{\text{A}} &= \gamma h = \left(62.4\ \frac{\text{lbf}}{\text{ft}^3}\right)(20\ \text{ft}) = 1248\ \text{lbf/ft}^2 \\
\text{v}_{\text{B}} &= \frac{\dot{V}}{A} = \frac{8\ \frac{\text{ft}^3}{\text{sec}}}{0.7854\ \text{ft}^2} = 10.19\ \text{ft/sec}
\end{aligned}
$$

Use the Bernoulli equation (see *NCEES Handbook:* The Bernoulli Equation) to solve for p_{B}.

$$
\begin{aligned}
p_{\text{B}} &= p_{\text{A}} + \left(\frac{\text{v}_{\text{A}}^2}{2g} - \frac{\text{v}_{\text{B}}^2}{2g}\right)\gamma \\
&= 1248\ \frac{\text{lbf}}{\text{ft}^2} + \left(\frac{\left(2.546\ \frac{\text{ft}}{\text{sec}}\right)^2 - \left(10.19\ \frac{\text{ft}}{\text{sec}}\right)^2}{(2)\left(32.2\ \frac{\text{ft}}{\text{sec}^2}\right)}\right) \\
&\quad \times \left(62.4\ \frac{\text{lbf}}{\text{ft}^3}\right) \\
&= 1153.7\ \text{lbf/ft}^2
\end{aligned}
$$

With $\theta = 0°$, from equationCERM17200(b),

$$\begin{aligned}
F_x &= p_\mathrm{B} A_\mathrm{B} - p_\mathrm{A} A_\mathrm{A} + \frac{\dot{m}\,(\mathrm{v_B} - \mathrm{v_A})}{g_c} \\
&= \left(1153.7\ \frac{\text{lbf}}{\text{ft}^2}\right)(0.7854\ \text{ft}^2) - \left(1248\ \frac{\text{lbf}}{\text{ft}^2}\right)(3.142\ \text{ft}^2) \\
&\quad + \frac{\left(\left(8\ \frac{\text{ft}^3}{\text{sec}}\right)\left(62.4\ \frac{\text{lbm}}{\text{ft}^3}\right)\right)\left(10.19\ \frac{\text{ft}}{\text{sec}} - 2.546\ \frac{\text{ft}}{\text{sec}}\right)}{32.2\ \frac{\text{lbm-ft}}{\text{lbf-sec}^2}} \\
&= -2897\ \text{lbf}\ (2900\ \text{lbf})\ \text{on the fluid}\ \ (\text{toward A}) \\
F_y &= 0
\end{aligned}$$

The answer is (A).
22.

Customary U.S. Solution

As in *NCEES Handbook* table “Pipe Dimensions and Weights,” for schedule-40 pipe,

$$\begin{aligned}
D_i &= 0.9948\ \text{ft} \\
A_i &= 0.7773\ \text{ft}^2
\end{aligned}$$

As in *NCEES Handbook:* Conservation of Mass,

$$\begin{aligned}
\mathrm{v} = \frac{\dot{V}}{A_i} &= \frac{2000\ \frac{\text{gal}}{\text{min}}}{(0.7773\ \text{ft}^2)\left(7.4805\ \frac{\text{gal}}{\text{ft}^3}\right)\left(60\ \frac{\text{sec}}{\text{min}}\right)} \\
&= 5.73\ \text{ft/sec}
\end{aligned}$$

The pressures are in terms of gage pressure. As in *NCEES Handbook:* table “Standard Values,” the density of mercury is 0.491 lbm/in.

$$\begin{aligned}
p_i &= \left(14.7\ \frac{\text{lbf}}{\text{in}^2} - \frac{(6\ \text{in})\left(0.491\ \frac{\text{lbm}}{\text{in}^3}\right)\left(32.2\ \frac{\text{ft}}{\text{sec}^2}\right)}{32.2\ \frac{\text{lbm-ft}}{\text{lbf-sec}^2}}\right) \\
&\quad \times \left(12\ \frac{\text{in}}{\text{ft}}\right)^2 \\
&= 1692.6\ \text{lbf/ft}^2 \\
E_{ti} &= \frac{p_i}{\rho} + \frac{\mathrm{v}_i^2}{2g_c} + \frac{z_i g}{g_c}
\end{aligned}$$

The inlet pump head can be calculated from a variation of the Bernoulli equation (also *NCEES Handbook:* The Bernoulli Equation).

$$E_{ti} = \frac{p_i}{\rho} + \frac{\mathrm{v}_i^2}{2g_c} + \frac{z_i g}{g_c}$$

Since the pump inlet and outlet are at the same elevation, use $\Delta z = 0$. $\varrho = (\mathrm{SG})\varrho_{\mathrm{water}}$.

$$
\begin{aligned}
E_{ti} &= \frac{p_i}{(\mathrm{SG})\,\rho_{\mathrm{water}}} + \frac{\mathrm{v}_i^2}{2g_c} + 0 \\
&= \frac{1692.6\ \frac{\mathrm{lbf}}{\mathrm{ft}^2}}{(1.2)\left(62.4\ \frac{\mathrm{lbm}}{\mathrm{ft}^3}\right)} + \frac{\left(5.73\ \frac{\mathrm{ft}}{\mathrm{sec}}\right)^2}{(2)\left(32.2\ \frac{\mathrm{lbm\text{-}ft}}{\mathrm{lbf\text{-}sec}^2}\right)} \\
&= 23.11\ \mathrm{ft\text{-}lbf/lbm}
\end{aligned}
$$

Calculate the total head at the inlet.

$$
\begin{aligned}
h_{ti} &= E_{ti} \times \frac{g_c}{g} = \left(23.11\ \frac{\mathrm{ft\text{-}lbf}}{\mathrm{lbm}}\right)\left(\frac{32.2\ \frac{\mathrm{lbm\text{-}ft}}{\mathrm{lbf\text{-}sec}^2}}{32.2\ \frac{\mathrm{ft}}{\mathrm{sec}^2}}\right) \\
&= 23.11\ \mathrm{ft}
\end{aligned}
$$

Refer to *NCEES Handbook* table “Pipe Dimensions and Weights.” At the outlet side of the pump,

$$
\begin{aligned}
D_o &= 0.6651\ \mathrm{ft} \\
A_o &= 0.3474\ \mathrm{ft}^2 \\
\mathrm{v}_o &= \frac{Q}{A_o} = \frac{2000\ \frac{\mathrm{gal}}{\mathrm{min}}}{\left(0.3474\ \mathrm{ft}^2\right)\left(7.4805\ \frac{\mathrm{gal}}{\mathrm{ft}^3}\right)\left(60\ \frac{\mathrm{sec}}{\mathrm{min}}\right)} \\
&= 12.83\ \mathrm{ft/sec}
\end{aligned}
$$

The pressures are in terms of gage pressure. The gauge is located 4 ft above the pump outlet, which adds 4 ft of pressure head at the pump outlet.

$$
\begin{aligned}
p_o &= \left(14.7\ \frac{\mathrm{lbf}}{\mathrm{in}^2} + 20\ \frac{\mathrm{lbf}}{\mathrm{in}^2}\right)\left(12\ \frac{\mathrm{in}}{\mathrm{ft}}\right)^2 \\
&\quad + (4\ \mathrm{ft})\left(\frac{(1.2)\left(62.4\ \frac{\mathrm{lbm}}{\mathrm{ft}^3}\right)\left(32.2\ \frac{\mathrm{ft}}{\mathrm{sec}^2}\right)}{32.2\ \frac{\mathrm{lbm\text{-}ft}}{\mathrm{lbf\text{-}sec}^2}}\right) \\
&= 5296\ \mathrm{lbf/ft}^2
\end{aligned}
$$

Calculate the outlet pressure head using the Bernoulli equation.

$$
E_{to} = \frac{p_o}{\rho} + \frac{\mathrm{v}_o^2}{2g_c} + \frac{z_o g}{g_c}
$$

Since the pump inlet and outlet are at the same elevation, $\Delta z = 0$. $\varrho = (\mathrm{SG})\varrho_{\mathrm{water}}$.

$$
\begin{aligned}
E_{to} &= \frac{p_o}{(\mathrm{SG})\,\rho_{\mathrm{water}}} + \frac{\mathrm{v}_o^2}{2g_c} + 0 \\
&= \frac{5296\ \frac{\mathrm{lbf}}{\mathrm{ft}^2}}{(1.2)\left(62.4\ \frac{\mathrm{lbm}}{\mathrm{ft}^3}\right)} + \frac{\left(12.83\ \frac{\mathrm{ft}}{\mathrm{sec}}\right)^2}{(2)\left(32.2\ \frac{\mathrm{lbm\text{-}ft}}{\mathrm{lbf\text{-}sec}^2}\right)} \\
&= 73.28\ \mathrm{ft\text{-}lbf/lbm}
\end{aligned}
$$

Calculate the total head at the outlet.

$$
\begin{aligned}
h_{to} &= E_{to} \times \frac{g_c}{g} = \left(73.28\ \frac{\mathrm{ft\text{-}lbf}}{\mathrm{lbm}}\right)\left(\frac{32.2\ \frac{\mathrm{lbm\text{-}ft}}{\mathrm{lbf\text{-}sec}^2}}{32.2\ \frac{\mathrm{ft}}{\mathrm{sec}^2}}\right) \\
&= 73.28\ \mathrm{ft}
\end{aligned}
$$

Compute the total head across the pump.

$$\Delta h = h_{to} - h_{ti} = 73.28\ \text{ft} - 23.11\ \text{ft} = 50.17\ \text{ft}$$

The mass flow rate is

$$\begin{aligned}\dot{m} &= \rho\dot{V} = (\text{SG})\,\rho_{\text{water}}\,\dot{V}\\ &= \frac{(1.2)\left(62.4\ \frac{\text{lbm}}{\text{ft}^3}\right)\left(2000\ \frac{\text{gal}}{\text{min}}\right)}{\left(7.4805\ \frac{\text{gal}}{\text{ft}^3}\right)\left(60\ \frac{\text{sec}}{\text{min}}\right)}\\ &= 333.7\ \text{lbm/sec}\end{aligned}$$

As in *NCEES Handbook:* Pump Power, the power input to the pump is

$$\begin{aligned}P &= \frac{\Delta h\dot{m}\times\frac{g}{g_c}}{\eta}\\ &= \frac{(50.17\ \text{ft})\left(333.7\ \frac{\text{lbm}}{\text{sec}}\right)\left(\frac{32.2\ \frac{\text{ft}}{\text{sec}^2}}{32.2\ \frac{\text{lbm-ft}}{\text{lbf-sec}^2}}\right)}{(0.85)\left(550\ \frac{\text{ft-lbf}}{\text{hp-sec}}\right)}\\ &= 35.8\ \text{hp}\quad(36\ \text{hp})\end{aligned}$$

(It is not necessary to use absolute pressures as has been done in this solution.)

The answer is (B).

SI Solution

As in *NCEES Handbook* table "Pipe Dimensions and Weights," for schedule-40 pipe,

$$\begin{aligned}D_i &= 303.2\ \text{mm}\\ A_i &= 0.0722\ \text{m}^2\end{aligned}$$

As in *NCEES Handbook:* Conservation of Mass,

$$\text{v} = \frac{\dot{V}}{A_i} = \frac{0.125\ \frac{\text{m}^3}{\text{s}}}{0.0722\ \text{m}^2} = 1.73\ \text{m/s}$$

The pressures are in terms of gage pressure, and the density of mercury is 13 600 kg/m^3.

$$\begin{aligned}p_i &= 1.013\times10^5\ \text{Pa} - (0.15\ \text{m})\left(13\,600\ \frac{\text{kg}}{\text{m}^3}\right)\left(9.81\ \frac{\text{m}}{\text{s}^2}\right)\\ &= 8.13\times10^4\ \text{Pa}\end{aligned}$$

The inlet pump head can be calculated from a variation of the Bernoulli equation (also *NCEES Handbook:* The Bernoulli Equation).

$$E_{ti} = \frac{p}{\rho} + \frac{\text{v}_i^2}{2} + z_i g$$

Since the pump inlet and outlet are at the same elevation, $\Delta z = 0$. $\varrho = (\text{SG})\varrho_{\text{water}}$.

$$
\begin{aligned}
E_{ti} &= \frac{p}{(\mathrm{SG})\,\rho_{\text{water}}} + \frac{\mathrm{v}_i^2}{2} + 0 \\
&= \frac{8.13 \times 10^4\ \mathrm{Pa}}{(1.2)\left(1000\ \dfrac{\mathrm{kg}}{\mathrm{m}^3}\right)} + \frac{\left(1.73\ \dfrac{\mathrm{m}}{\mathrm{s}}\right)^2}{2} \\
&= 69.2\ \mathrm{J/kg}
\end{aligned}
$$

The total head at the inlet is

$$
h_{ti} = \frac{E_{ti}}{g} = \frac{69.2\ \dfrac{\mathrm{J}}{\mathrm{kg}}}{9.81\ \dfrac{\mathrm{m}}{\mathrm{s}^2}} = 7.05\ \mathrm{m}
$$

Assume the pipe nominal diameter is equal to the internal diameter. As in *NCEES Handbook* table "Pipe Dimensions and Weights," for the outlet side of the pump,

$$
\begin{aligned}
D_i &= 202.7\ \mathrm{mm} \\
A_o &= 0.0323\ \mathrm{m}^2
\end{aligned}
$$

As in *NCEES Handbook:* Conservation of Mass,

$$
\mathrm{v}_o = \frac{\dot{V}}{A_o} = \frac{0.125\ \dfrac{\mathrm{m}^3}{\mathrm{s}}}{0.0323\ \mathrm{m}^2} = 3.87\ \mathrm{m/s}
$$

The pressures are in terms of gage pressure. The gauge is located 1.2 m above the pump outlet, which adds 1.2 m of pressure head at the pump outlet.

$$
\begin{aligned}
p_o &= 1.013 \times 10^5\ \mathrm{Pa} + 138 \times 10^3\ \mathrm{Pa} \\
&\quad + (1.2\ \mathrm{m})\left((1.2)\left(1000\ \frac{\mathrm{kg}}{\mathrm{m}^3}\right)\left(9.81\ \frac{\mathrm{m}}{\mathrm{s}^2}\right)\right) \\
&= 2.53 \times 10^5\ \mathrm{Pa}
\end{aligned}
$$

Calculate the outlet pressure head using the Bernoulli equation.

$$
E_{to} = \frac{p_o}{\rho} + \frac{\mathrm{v}_o^2}{2} + z_o g
$$

Since the pump inlet and outlet are at the same elevation, $\Delta z = 0$. $\varrho = (\mathrm{SG})\varrho_{\text{water}}$.

$$
\begin{aligned}
E_{to} &= \frac{p_o}{(\mathrm{SG})\,\rho_{\text{water}}} + \frac{\mathrm{v}_o^2}{2} + 0 \\
&= \frac{2.53 \times 10^5\ \mathrm{Pa}}{(1.2)\left(1000\ \dfrac{\mathrm{kg}}{\mathrm{m}^3}\right)} + \frac{\left(3.87\ \dfrac{\mathrm{m}}{\mathrm{s}}\right)^2}{2} \\
&= 218.3\ \mathrm{J/kg}
\end{aligned}
$$

The total head at the outlet is

$$
h_{to} = \frac{E_{to}}{g} = \frac{218.3\ \dfrac{\mathrm{J}}{\mathrm{kg}}}{9.81\ \dfrac{\mathrm{m}}{\mathrm{s}^2}} = 22.25\ \mathrm{m}
$$

The total head across the pump is

$$
\Delta h = h_{to} - h_{ti} = 22.25\ \mathrm{m} - 7.05\ \mathrm{m} = 15.2\ \mathrm{m}
$$

The mass flow rate is

$$\begin{aligned}\dot{m} &= \rho\dot{V} = (\text{SG})\,\rho_{\text{water}}Q \\ &= (1.2)\left(1000\ \frac{\text{kg}}{\text{m}^3}\right)\left(0.125\ \frac{\text{m}^3}{\text{s}}\right) \\ &= 150\ \text{kg/s}\end{aligned}$$

As in *NCEES Handbook:* Pump Power, the power input to the pump is

$$\begin{aligned}P &= \frac{\Delta h\dot{m}g}{\eta} = \frac{(15.2\ \text{m})\left(150\ \frac{\text{kg}}{\text{s}}\right)\left(9.81\ \frac{\text{m}}{\text{s}^2}\right)}{0.85} \\ &= 26\,314\ \text{W} \quad (26\ \text{kW})\end{aligned}$$

(It is not necessary to use absolute pressures as has been done in this solution.)

The answer is (B).
23.

Customary U.S. Solution

The mass flow rate is

$$\begin{aligned}\dot{m} &= \dot{V}\rho = \left(100\ \frac{\text{ft}^3}{\text{sec}}\right)\left(62.4\ \frac{\text{lbm}}{\text{ft}^3}\right) \\ &= 6240\ \text{lbm/sec}\end{aligned}$$

Refer to *NCEES Handbook:* Similitude. The head loss across the horizontal turbine can be found using a modified version of the Bernoulli equation.

$$\begin{aligned}h_{\text{loss}} &= \frac{\Delta p}{\rho} \times \frac{g_c}{g} \\ &= \left(\frac{\left(30\ \frac{\text{lbf}}{\text{in}^2} - \left(-5\ \frac{\text{lbf}}{\text{in}^2}\right)\right)\left(12\ \frac{\text{in}}{\text{ft}}\right)^2}{62.4\ \frac{\text{lbm}}{\text{ft}^3}}\right)\left(\frac{32.2\ \frac{\text{lbm-ft}}{\text{lbf-sec}^2}}{32.2\ \frac{\text{ft}}{\text{sec}^2}}\right) \\ &= 80.77\ \text{ft}\end{aligned}$$

From tableCERM18005 (also *NCEES Handbook:* Pump Power), the power developed by the turbine is

$$\begin{aligned}P &= \frac{h_{\text{loss}}\dot{m}}{550} \times \frac{g}{g_c} \\ &= \left(\frac{\left(6240\ \frac{\text{lbm}}{\text{sec}}\right)(80.77\ \text{ft})}{550\ \frac{\text{ft-lbf}}{\text{hp-sec}}}\right)\left(\frac{32.2\ \frac{\text{ft}}{\text{sec}^2}}{32.2\ \frac{\text{lbm-ft}}{\text{lbf-sec}^2}}\right) \\ &= 916\ \text{hp} \quad (920\ \text{hp})\end{aligned}$$

The answer is (D).

SI Solution

The mass flow rate is

$$\dot{m} = \dot{V}\rho = \left(2.6\ \frac{\text{m}^3}{\text{s}}\right)\left(1000\ \frac{\text{kg}}{\text{m}^3}\right) = 2600\ \text{kg/s}$$

The head loss across the horizontal turbine is

$$h_{\text{loss}} = \frac{\Delta p}{\rho g} = \frac{\left(210\ \text{kPa} - (-35\ \text{kPa})\right)\left(1000\ \frac{\text{Pa}}{\text{kPa}}\right)}{\left(1000\ \frac{\text{kg}}{\text{m}^3}\right)\left(9.81\ \frac{\text{m}}{\text{s}^2}\right)}$$
$$= 25.0\ \text{m}$$

From tableCERM18006, the power developed by the turbine is

$$P = 9.81 h_{\text{loss}}\dot{m} = \frac{\left(9.81\ \frac{\text{m}}{\text{s}^2}\right)(25.0\ \text{m})\left(2600\ \frac{\text{kg}}{\text{s}}\right)}{1000\ \frac{\text{W}}{\text{kW}}}$$
$$= 638\ \text{W} \quad (640\ \text{kW})$$

The answer is (D).
24.

The speed of the truck relative to the ground is 65 mph. However, power is also required to overcome oncoming wind. The speed of the truck relative to the wind is 65 mph + 15 mph = 80 mph. If the truck were stationary, the wind would exert a force on the truck that would push the truck backward, relative to the ground. The frictional forces between the tires and ground, as well as within the parking brake systems, perform work while preventing this motion.

The answer is (D).

25.

From equationCERM17212(b) (also *NCEES Handbook:* Particle Flow), the drag force on the antenna is

$$F_D = \frac{C_D A \rho \text{v}^2}{2g_c}$$
$$= \frac{(1.2)\left(0.8\ \text{ft}^2\right)\left(0.076\ \frac{\text{lbm}}{\text{ft}^3}\right) \times \left(\frac{\left(60\ \frac{\text{mi}}{\text{hr}}\right)\left(5280\ \frac{\text{ft}}{\text{mi}}\right)}{3600\ \frac{\text{sec}}{\text{hr}}}\right)^2}{(2)\left(32.2\ \frac{\text{lbm-ft}}{\text{lbf-sec}^2}\right)}$$
$$= 8.77\ \text{lbf} \quad (9.0\ \text{lbf})$$

The answer is (A).

26.

Customary U.S. Solution

For air at 70°F,

$$\rho = \frac{p}{RT} = \frac{\left(14.7\ \frac{\text{lbf}}{\text{in}^2}\right)\left(12\ \frac{\text{in}}{\text{ft}}\right)^2}{\left(53.35\ \frac{\text{ft-lbf}}{\text{lbm-}^\circ\text{R}}\right)(70^\circ\text{F} + 460^\circ)}$$
$$= 0.0749\ \text{lbm/ft}^3$$
$$\text{v} = \frac{\left(55\ \frac{\text{mi}}{\text{hr}}\right)\left(5280\ \frac{\text{ft}}{\text{mi}}\right)}{3600\ \frac{\text{sec}}{\text{hr}}}$$
$$= 80.67\ \text{ft/sec}$$

The drag on the car is

$$\begin{aligned}
F_D &= \frac{C_D A \rho \mathrm{v}^2}{2g_c} \\
&= \frac{(0.42)\left(28 \text{ ft}^2\right)\left(0.0749 \ \frac{\text{lbm}}{\text{ft}^3}\right)\left(80.67 \ \frac{\text{ft}}{\text{sec}}\right)^2}{(2)\left(32.2 \ \frac{\text{lbm-ft}}{\text{lbf-sec}^2}\right)} \\
&= 89.0 \text{ lbf}
\end{aligned}$$

The total resisting force is

$$\begin{aligned}
F &= F_D + \text{rolling resistance} \\
&= 89.0 \text{ lbf} + (0.01)(3300 \text{ lbm}) \times \frac{g}{g_c} \\
&= 89.0 \text{ lbf} + (0.01)(3300 \text{ lbm})\left(\frac{32.2 \ \frac{\text{ft}}{\text{sec}^2}}{32.2 \ \frac{\text{lbm-ft}}{\text{lbf-sec}^2}}\right) \\
&= 122.0 \text{ lbf}
\end{aligned}$$

The power manifested by virtue of the car's velocity is

$$\begin{aligned}
P = F\mathrm{v} &= \frac{(122.0 \text{ lbf})\left(80.67 \ \frac{\text{ft}}{\text{sec}}\right)}{778 \ \frac{\text{ft-lbf}}{\text{Btu}}} \\
&= 12.65 \text{ Btu/sec}
\end{aligned}$$

The energy available from the fuel is

$$\begin{aligned}
E_A &= (\text{engine thermal efficiency})(\text{fuel heating value}) \\
&= (0.28)\left(115{,}000 \ \frac{\text{Btu}}{\text{gal}}\right) \\
&= 32{,}2000 \text{ Btu/gal}
\end{aligned}$$

The fuel consumption at 55 mi/hr is

$$\begin{aligned}
\frac{P}{E_A \mathrm{v}} &= \frac{\left(12.65 \ \frac{\text{Btu}}{\text{sec}}\right)\left(3600 \ \frac{\text{sec}}{\text{hr}}\right)}{\left(32{,}200 \ \frac{\text{Btu}}{\text{gal}}\right)\left(55 \ \frac{\text{mi}}{\text{hr}}\right)} \\
&= 0.0257 \text{ gal/mi} \quad (0.026 \text{ gal/mi})
\end{aligned}$$

The answer is (A).

SI Solution

For air at 20°C,

$$\begin{aligned}
\rho = \frac{p}{RT} &= \frac{1.013 \times 10^5 \text{ Pa}}{\left(287.03 \ \frac{\text{J}}{\text{kg}\cdot\text{K}}\right)(20^\circ\text{C} + 273^\circ)} \\
&= 1.205 \text{ kg/m}^3 \\
\mathrm{v} &= \frac{\left(90 \ \frac{\text{km}}{\text{h}}\right)\left(1000 \ \frac{\text{m}}{\text{km}}\right)}{3600 \ \frac{\text{s}}{\text{h}}} \\
&= 25.0 \text{ m/s}
\end{aligned}$$

The drag on the car is

$$\begin{aligned} F_D &= \frac{C_D A \rho \mathrm{v}^2}{2} = \frac{(0.42)\left(2.6\ \mathrm{m}^2\right)\left(1.205\ \frac{\mathrm{kg}}{\mathrm{m}^3}\right)\left(25.0\ \frac{\mathrm{m}}{\mathrm{s}}\right)^2}{2} \\ &= 411.2\ \mathrm{N} \end{aligned}$$

The total resisting force is

$$\begin{aligned} F &= F_D + \text{rolling resistance} \times g \\ &= 411.2\ \mathrm{N} + (0.01)(1500\ \mathrm{kg})\, g \\ &= 411.2\ \mathrm{N} + (0.01)(1500\ \mathrm{kg})\left(9.81\ \frac{\mathrm{m}}{\mathrm{s}^2}\right) \\ &= 558.4\ \mathrm{N} \end{aligned}$$

The power required is

$$\begin{aligned} P &= F\mathrm{v} = (558.4\ \mathrm{N})\left(25\ \frac{\mathrm{m}}{\mathrm{s}}\right) \\ &= 13\,960\ \mathrm{W} \end{aligned}$$

The energy available from the fuel is

$$\begin{aligned} E_A &= (\text{engine thermal efficiency})(\text{fuel heating value}) \\ &= (0.28)\left(32 \times 10^6\ \frac{\mathrm{J}}{\mathrm{L}}\right) \\ &= 8.96 \times 10^6\ \mathrm{J/L} \end{aligned}$$

The fuel consumption at 90 km/h is

$$\begin{aligned} \frac{P}{E_A \mathrm{v}} &= \frac{(13\,960\ \mathrm{W})\left(3600\ \frac{\mathrm{s}}{\mathrm{h}}\right)}{\left(8.96 \times 10^6\ \frac{\mathrm{J}}{\mathrm{L}}\right)\left(90\ \frac{\mathrm{km}}{\mathrm{h}}\right)} \\ &= 0.0623\ \mathrm{L/km} \quad (0.062\ \mathrm{L/km}) \end{aligned}$$

The answer is (A).
27.

To ensure similarity between the model and the true conditions of the full-scale airplane, the Reynolds numbers must be equal.

$$\left(\frac{\mathrm{v}L}{\nu}\right)_{\text{model}} = \left(\frac{\mathrm{v}L}{\nu}\right)_{\text{true}}$$

Use the absolute viscosity.

$$\begin{aligned} \mu &= \frac{\rho\nu}{g_c} \\ \nu &= \frac{\mu g_c}{\rho} \end{aligned}$$

$$\left(\frac{\mathrm{v}L\rho}{\mu g_c}\right)_{\text{model}} = \left(\frac{\mathrm{v}L\rho}{\mu g_c}\right)_{\text{true}}$$

Recall that the absolute viscosity is independent of pressure, so $\mu_{\text{model}} = \mu_{\text{true}}$.

Since g_c is a constant,

$$\left(\frac{\mathrm{v}L\rho}{\mu}\right)_{\text{model}} = \left(\frac{\mathrm{v}L\rho}{\mu}\right)_{\text{true}}$$

Assume the air behaves as an ideal gas.

$$\rho = \frac{p}{RT}$$

$$\left(\frac{\mathrm{v}Lp}{\mu RT}\right)_{\text{model}} = \left(\frac{\mathrm{v}Lp}{\mu RT}\right)_{\text{true}}$$

Since the tunnel operates with air at true velocity and temperature,

$$\begin{aligned} R_{\text{model}} &= R_{\text{true}} \\ \mathrm{v}_{\text{model}} &= \mathrm{v}_{\text{true}} \\ T_{\text{model}} &= T_{\text{true}} \end{aligned}$$

$$\left(\frac{Lp}{\mu}\right)_{\text{model}} = \left(\frac{Lp}{\mu}\right)_{\text{true}}$$

Therefore,

$$(Lp)_{\text{model}} = (Lp)_{\text{true}}$$

Since the scale of the model is 1/20,

$$L_{\text{model}} = \frac{L_{\text{true}}}{20}$$

Substituting gives

$$\begin{aligned} (Lp)_{\text{model}} &= (Lp)_{\text{true}} \\ \left(\frac{L_{\text{true}}}{20}\right) p_{\text{model}} &= L_{\text{true}} p_{\text{true}} \\ p_{\text{model}} &= 20 p_{\text{true}} \end{aligned}$$

The answer is (C).
28.

To ensure similarity between the two impellers, the Reynolds numbers, Re, must be equal.

$$\mathrm{Re}_{\text{oil}} = \mathrm{Re}_{\text{air}}$$

$$\begin{aligned} \frac{\mathrm{v}_{\text{oil}} D_{\text{oil}}}{\nu_{\text{oil}}} &= \frac{\mathrm{v}_{\text{air}} D_{\text{air}}}{\nu_{\text{air}}} \\ \frac{\mathrm{v}_{\text{oil}}}{\mathrm{v}_{\text{air}}} &= \left(\frac{\nu_{\text{oil}}}{\nu_{\text{air}}}\right)\left(\frac{D_{\text{air}}}{D_{\text{oil}}}\right) \end{aligned}$$

v is the tangential velocity, and D is the impeller diameter.

$$\mathrm{v} \propto nD$$

$$\frac{\mathrm{v}_{\text{oil}}}{\mathrm{v}_{\text{air}}} = \left(\frac{n_{\text{oil}}}{n_{\text{air}}}\right)\left(\frac{D_{\text{oil}}}{D_{\text{air}}}\right)$$

Therefore,

$$\begin{aligned} \left(\frac{\nu_{\text{oil}}}{\nu_{\text{air}}}\right)\left(\frac{D_{\text{air}}}{D_{\text{oil}}}\right) &= \left(\frac{n_{\text{oil}}}{n_{\text{air}}}\right)\left(\frac{D_{\text{oil}}}{D_{\text{air}}}\right) \\ n_{\text{air}} &= n_{\text{oil}}\left(\frac{\nu_{\text{air}}}{\nu_{\text{oil}}}\right)\left(\frac{D_{\text{oil}}}{D_{\text{air}}}\right)^2 \end{aligned}$$

Since the air impeller is twice the size of the oil impeller, $D_{\text{air}} = 2D_{\text{oil}}$.

$$
\begin{aligned}
n_{\mathrm{air}} &= n_{\mathrm{oil}}\left(\frac{\nu_{\mathrm{air}}}{\nu_{\mathrm{oil}}}\right)\left(\frac{D_{\mathrm{oil}}}{D_{\mathrm{air}}}\right)^2 \\
&= n_{\mathrm{oil}}\left(\frac{\nu_{\mathrm{air}}}{\nu_{\mathrm{oil}}}\right)\left(\frac{D_{\mathrm{oil}}}{2D_{\mathrm{oil}}}\right)^2 \\
&= \tfrac{1}{4}n_{\mathrm{oil}}\left(\frac{\nu_{\mathrm{air}}}{\nu_{\mathrm{oil}}}\right)
\end{aligned}
$$

Customary U.S. Solution

From appendixCERM14D (also *NCEES Handbook:* Temperature-Dependent Properties of Air (U.S. Customary Units)), for air at 68°F, $\nu = 15.72 \times 10^{-5}$ ft^2/sec.

For castor oil at 68°F, $\nu = 1110 \times 10^{-5}$ ft^2/sec (given).

$$
\begin{aligned}
n_{\mathrm{air}} &= \tfrac{1}{4}n_{\mathrm{oil}}\left(\frac{\nu_{\mathrm{air}}}{\nu_{\mathrm{oil}}}\right) \\
&= \left(\frac{1}{4}\right)\left(1000\ \frac{\mathrm{rev}}{\mathrm{min}}\right)\left(\frac{15.72 \times 10^{-5}\ \frac{\mathrm{ft}^2}{\mathrm{sec}}}{1110 \times 10^{-5}\ \frac{\mathrm{ft}^2}{\mathrm{sec}}}\right) \\
&= 3.54\ \mathrm{rpm} \quad (3.6\ \mathrm{rpm})
\end{aligned}
$$

The answer is (A).

SI Solution

From appendixCERM14E (also *NCEES Handbook:* Temperature-Dependent Properties of Air (SI Units)), for air at 20°C, $\nu = 1.512 \times 10^{-5}$ m^2/s.

For castor oil at 20°C, $\nu = 103 \times 10^{-5}$ m^2/s (given),

$$
\begin{aligned}
n_{\mathrm{air}} &= \tfrac{1}{4}n_{\mathrm{oil}}\left(\frac{\nu_{\mathrm{air}}}{\nu_{\mathrm{oil}}}\right) \\
&= \left(\frac{1}{4}\right)\left(1000\ \frac{\mathrm{rev}}{\mathrm{min}}\right)\left(\frac{1.512 \times 10^{-5}\ \frac{\mathrm{m}^2}{\mathrm{s}}}{103 \times 10^{-5}\ \frac{\mathrm{m}^2}{\mathrm{s}}}\right) \\
&= 3.67\ \mathrm{rpm} \quad (3.6\ \mathrm{rpm})
\end{aligned}
$$

The answer is (A).