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Topic III: Heat Transfer

Chapter 17. Heat Transfer: Conduction

Practice Problems

1.

Experiments have shown that the thermal conductivity, k, of a particular material varies with temperature, T, according to the following relationship.

$$k_T = (0.030)(1 + 0.0015T)$$

What is most nearly the value of k that should be used for a transfer of heat through the material if the hot-side temperature is 350° (°F or °C) and the cold-side temperature is 150° (°F or °C)?

(A)

0.04

(B)

0.06

(C)

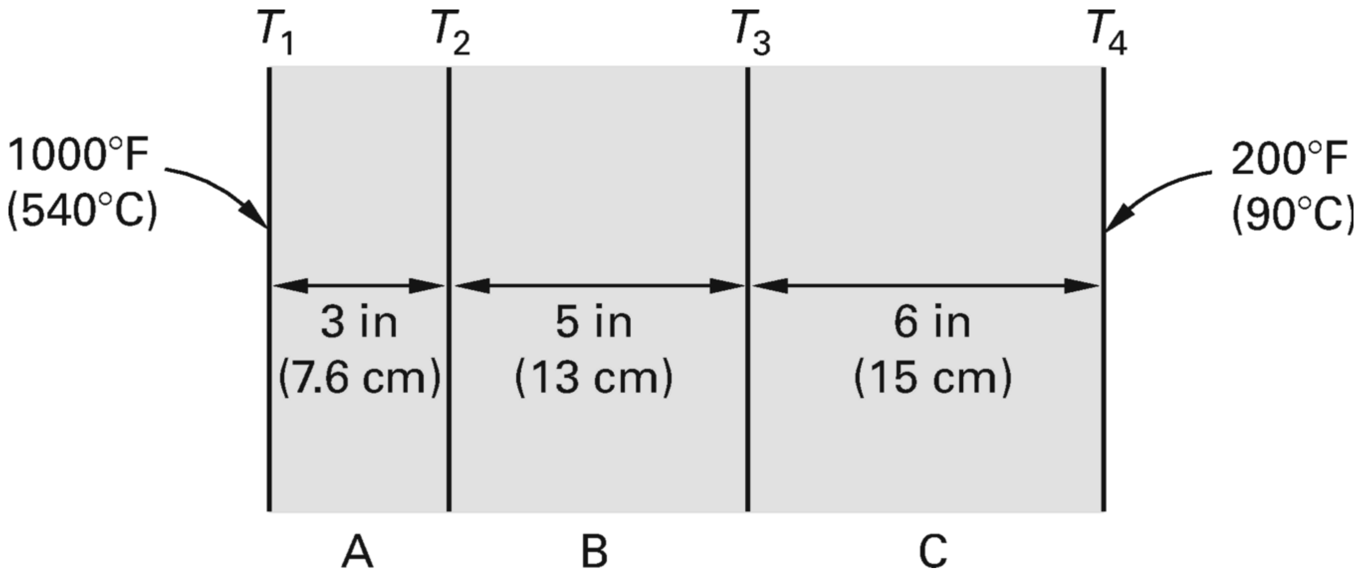
0.10

(D)

0.20

2.

A composite wall is made up of 3.0 in (7.6 cm) of material A exposed to 1000°F (540°C), 5.0 in (13 cm) of material B, and 6.0 in (15 cm) of material C exposed to 200°F (90°C). The mean thermal conductivities for materials A, B, and C are 0.06, 0.5, and 0.8 Btu-ft/hr-ft^2-°F (0.1, 0.9, and 1.4 W/m·K), respectively.



The temperature at the A-B material interface is most nearly

(A)

300°F (100°C)
(B)

350°F (150°C)

(C)

400°F (200°C)

(D)

500°F (250°C)
3.

What is most nearly the heat flow through insulating brick 1.0 ft (30 cm) thick in an oven wall with a thermal conductivity of 0.038 Btu-ft/hr-ft^2-°F (0.066 W/m·K)? The thermal gradient is 350°F (177°C).

(A)

7 Btu/hr-ft^2 (30 W/m^2)

(B)

9 Btu/hr-ft^2 (40 W/m^2)

(C)

11 Btu/hr-ft^2 (60 W/m^2)

(D)

13 Btu/hr-ft^2 (100 W/m^2)

4.

In an old factory, a hot liquid flows through a 30 ft long copper tube that has a 1 in inner diameter and a $^1/_3$ in wall thickness. The copper tube is covered with a $^1/_2$ in thick layer of glass wool insulation, and the glass wool is covered with a $^1/_2$ in thick layer of asbestos. The outside temperatures of the copper tube and asbestos layer are 500°F and 150°F, respectively. The thermal conductivities of copper, glass wool, and asbestos are 220 Btu/hr-ft-°F, 0.0315 Btu/hr-ft-°F, and 0.115 Btu/hr-ft-°F, respectively. The temperature at the interface of the glass wool and asbestos is most nearly

(A)

190°F

(B)

200°F

(C)

260°F

(D)

350°F

5.

In an old factory, a hot liquid flows through a 30 ft long copper tube that has a 1 in inner diameter and a $^1/_3$ in wall thickness. The copper tube is covered with a $^1/_2$ in thick layer of glass wool insulation, and the glass wool is covered with a layer of asbestos of unknown thickness. The outside temperatures of the copper tube and asbestos layer are 500°F and 80°F, respectively. The heat transfer rate per unit length is 110 Btu/hr-ft. The thermal conductivities of copper, glass wool, and asbestos are 220 Btu/hr-ft-°F, 0.0315 Btu/hr-ft-°F, and 0.115 Btu/hr-ft-°F, respectively. The thickness of the asbestos layer is most nearly

(A)

0.2 in

(B)

0.3 in

(C)

1.0 in

(D)

2.5 in

6.

A chromium spherical tank with a 4 ft outer diameter contains hot oil. The tank is covered with a 5 in thick spherical layer of magnesia insulation. The magnesia insulation is covered with a 2 in thick spherical layer of Styrofoam. The outside temperatures of the chromium sphere and the Styrofoam layers are 250°F and 70°F, respectively. The thermal conductivities of magnesia and Styrofoam are 0.04 Btu/hr-ft-°F and 0.02 Btu/hr-ft-°F, respectively. The temperature at the interface of the magnesia and Styrofoam layers is most nearly

(A)

100°F

(B)

110°F

(C)

140°F

(D)

220°F

7.

A metallic, thin-walled spherical tank is covered with a layer of insulation. The thickness of the insulation is equal to the radius of the spherical tank. To increase the volumetric capacity, the original tank will be replaced with a new, insulated, thin-walled spherical tank that has a new tank radius equal to twice the original tank radius. The insulation layer material and thickness are unchanged. Simultaneously, the process is modified so that the outside temperatures for the sphere and its insulated layer are also doubled. The ratio of the heat transfer of the new tank to the heat transfer of the original tank is most nearly

(A)

1:2

(B)

2:1
(C)

3:1

(D)

6:1
8.

The heat supply of a large building is turned off at 5:00 p.m. when the interior temperature is 70°F (21°C). The outdoor temperature is constant at 40°F (4°C). The thermal capacity of the building and its contents is 100,000 Btu/°F (60 MJ/K), and the conductance is 6500 Btu/hr-°F (1.1 kW/K). The interior temperature at 1:00 *a.m.* is most nearly

(A)

45°F (7°C)

(B)

50°F (10°C)

(C)

60°F (15°C)

(D)

65°F (20°C)

9.

Steel ball bearings varying in diameter, d, from $^1/_4$ in to 1 $^1/_2$ in (6.35 mm to 38.1 mm) are quenched from 1800°F (980°C) in an oil bath that remains at 110°F (43°C). The ball bearings are removed when their internal (center) temperature reaches 250°F (120°C). The average film coefficient is 56 Btu/hr-ft^2-°F (320 W/m^2·K).

A linear equation for the time the ball bearings remain in the oil bath as a function of diameter is most nearly

(A)

$0.013d$ ($1800d$)

(B)

$0.023d$ ($3000d$)

(C)

$0.033d$ ($4700d$)

(D)

$0.042d$ ($5900d$)

10.

The walls of a building are made of a brick layer 1 m thick with a conductive coefficient, k, of 0.70 W/m·C, a rock-wool insulation layer 0.080 m thick with $k = 0.50$ W/m·C, and a layer of sheet rock 0.0375 m thick with $k = 0.45$ W/m·C. If the inside and outside temperatures are a constant 27 and −8.0°C, respectively, what is most nearly the heat loss due to conduction through the wall?

(A)

21 W/m^2

(B)

26 W/m^2

(C)

50 W/m^2

(D)

58 W/m^2

11.

A 0.4 in (1.0 cm) diameter uranium dioxide fuel rod with a thermal conductivity of 1.1 Btu-ft/hr-ft^2-°F (1.9 W/m·K) is clad with 0.020 in (0.5 mm) of stainless steel. The fuel rod generates 4×10^7 Btu/hr-ft^3 (4.1×10^8 W/m^3) internally. A coolant at 500°F (260°C) circulates around the clad rod. The outside film coefficient is 10,000 Btu/hr-ft^2-°F (57 kW/m^2·K). The temperature at the longitudinal centerline of the rod is most nearly

(A)

2100°F (1200°C)

(B)

2400°F (1300°C)

(C)

2700°F (1500°C)

(D)

3100°F (1700°C)

12.

Two long pieces of $\frac{1}{16}$ in (1.6 mm) copper wire are connected end-to-end with a hot soldering iron. The minimum melting temperature of the solder is 450°F (230°C). The surrounding air temperature is 80°F (27°C). The unit film coefficient is 3 Btu/hr-ft^2-°F (17 W/m^2·K). The thermal conductivity of the solder is 215 Btu-ft/hr-ft^2-°F (372.1 W/m·K). Disregard radiation losses. The minimum rate of heat application to keep the solder molten is most nearly

(A)

4.3 Btu/hr (1.3 W)

(B)

11 Btu/hr (3.3 W)

(C)

20 Btu/hr (6.0 W)

(D)

47 Btu/hr (14 W)

Solutions

1.

Use the value of k at an average temperature of $\frac{1}{2}(T_1 + T_2)$.

$$\begin{aligned} T &= \left(\frac{1}{2}\right)(150^\circ + 350^\circ) = 250^\circ \\ k &= (0.030)\,(1 + 0.0015T) \\ &= (0.030)\,\big(1 + (0.0015)\,(250^\circ)\big) \\ &= \ 0.04125 \quad (0.04) \end{aligned}$$

The answer is (A).

2.

Customary U.S. Solution

Since the wall temperatures are given, it is not necessary to consider films.

From equationMERM34021 (also *NCEES Handbook:* Conduction), the heat flow through the composite wall is

$$Q = \frac{A\,(T_1 - T_4)}{\sum\limits_{i=1}^{n} \frac{L_i}{k_i}}$$

On a per unit area basis,

$$\begin{aligned} \frac{Q}{A} &= \frac{T_1 - T_4}{\sum\limits_{i=1}^{n} \frac{L_i}{k_i}} \\ &= \frac{T_1 - T_2}{\frac{L_\mathrm{A}}{k_\mathrm{A}}} \\ &= \frac{T_2 - T_3}{\frac{L_\mathrm{B}}{k_\mathrm{B}}} \\ &= \frac{1000^\circ\mathrm{F} - 200^\circ\mathrm{F}}{\begin{array}{c} \dfrac{3\ \mathrm{in}}{\left(\left(0.06\ \frac{\mathrm{Btu\text{-}ft}}{\mathrm{hr\text{-}ft^2\text{-}^\circ F}}\right) \times \left(12\ \frac{\mathrm{in}}{\mathrm{ft}}\right)\right)} + \dfrac{5\ \mathrm{in}}{\left(\left(0.5\ \frac{\mathrm{Btu\text{-}ft}}{\mathrm{hr\text{-}ft^2\text{-}^\circ F}}\right) \times \left(12\ \frac{\mathrm{in}}{\mathrm{ft}}\right)\right)} \\ + \dfrac{6\ \mathrm{in}}{\left(0.8\ \frac{\mathrm{Btu\text{-}ft}}{\mathrm{hr\text{-}ft^2\text{-}^\circ F}}\right)\left(12\ \frac{\mathrm{in}}{\mathrm{ft}}\right)} \end{array}} \\ &= 142.2\ \mathrm{Btu/hr\text{-}ft^2} \end{aligned}$$

To find the temperature at the A-B interface, T_2, use

$$
\begin{aligned}
\frac{Q}{A} &= \frac{T_1 - T_2}{\frac{L_\mathrm{A}}{k_\mathrm{A}}} \\
T_2 &= T_1 - \left(\frac{Q}{A}\right)\left(\frac{L_\mathrm{A}}{k_\mathrm{A}}\right) \\
&= 1000^\circ\mathrm{F} - \left(142.2\ \frac{\mathrm{Btu}}{\mathrm{hr\text{-}ft}^2}\right) \\
&\quad \times \left(\frac{3\ \mathrm{in}}{\left(0.06\ \frac{\mathrm{Btu\text{-}ft}}{\mathrm{hr\text{-}ft}^2\text{-}^\circ\mathrm{F}}\right)\left(12\ \frac{\mathrm{in}}{\mathrm{ft}}\right)}\right) \\
&= 407.5^\circ\mathrm{F} \quad (400^\circ\mathrm{F})
\end{aligned}
$$

The answer is (C).

SI Solution

Since the wall temperatures are given, it is not necessary to consider films.

From equationMERM34021 (also *NCEES Handbook:* Conduction), the heat flow through the composite wall is

$$
Q = \frac{A\left(T_1 - T_4\right)}{\sum\limits_{i=1}^{n} \frac{L_i}{k_i}}
$$

On a per unit area basis,

$$
\begin{aligned}
\frac{Q}{A} &= \frac{T_1 - T_4}{\sum\limits_{i=1}^{n} \frac{L_i}{k_i}} \\
&= \frac{T_1 - T_2}{\frac{L_\mathrm{A}}{k_\mathrm{A}}} \\
&= \frac{T_2 - T_3}{\frac{L_\mathrm{B}}{k_\mathrm{B}}} \\
&= \frac{540^\circ\mathrm{C} - 90^\circ\mathrm{C}}{\begin{array}{c}\frac{7.6\ \mathrm{cm}}{\left(0.1\ \frac{\mathrm{W}}{\mathrm{m\cdot K}}\right)\left(100\ \frac{\mathrm{cm}}{\mathrm{m}}\right)} + \frac{13\ \mathrm{cm}}{\left(0.9\ \frac{\mathrm{W}}{\mathrm{m\cdot K}}\right)\left(100\ \frac{\mathrm{cm}}{\mathrm{m}}\right)} \\ + \frac{15\ \mathrm{cm}}{\left(1.4\ \frac{\mathrm{W}}{\mathrm{m\cdot K}}\right)\left(100\ \frac{\mathrm{cm}}{\mathrm{m}}\right)}\end{array}} \\
&= 444.8\ \mathrm{W/m^2}
\end{aligned}
$$

To find the temperature at the A-B interface, T_2, use

$$
\begin{aligned}
\frac{Q}{A} &= \frac{T_1 - T_2}{\frac{L_\mathrm{A}}{k_\mathrm{A}}} \\
T_2 &= T_1 - \left(\frac{Q}{A}\right)\left(\frac{L_\mathrm{A}}{k_\mathrm{A}}\right) \\
&= 540^\circ\mathrm{C} - \left(444.8\ \frac{\mathrm{W}}{\mathrm{m}^2}\right)\left(\frac{7.6\ \mathrm{cm}}{\left(0.1\ \frac{\mathrm{W}}{\mathrm{m\cdot K}}\right)\left(100\ \frac{\mathrm{cm}}{\mathrm{m}}\right)}\right) \\
&= 202.0^\circ\mathrm{C} \quad (200^\circ\mathrm{C})
\end{aligned}
$$

The answer is (C).

3.

Customary U.S. Solution

From the *NCEES Handbook* section titled “Fundamentals of Heat Transfer Fourier’s Law of Conduction,“ the heat flux per unit area through a plane wall is

$$\begin{aligned}\frac{Q_{1-2}}{A} &= \frac{k\Delta T}{L} = \frac{\left(0.038\ \frac{\text{Btu-ft}}{\text{hr-ft}^2\text{-}^\circ\text{F}}\right)(350^\circ\text{F})}{1.0\ \text{ft}} \\ &= 13.3\ \text{Btu/hr-ft}^2 \quad (13\ \text{Btu/hr-ft}^2)\end{aligned}$$

The answer is (D).

SI Solution

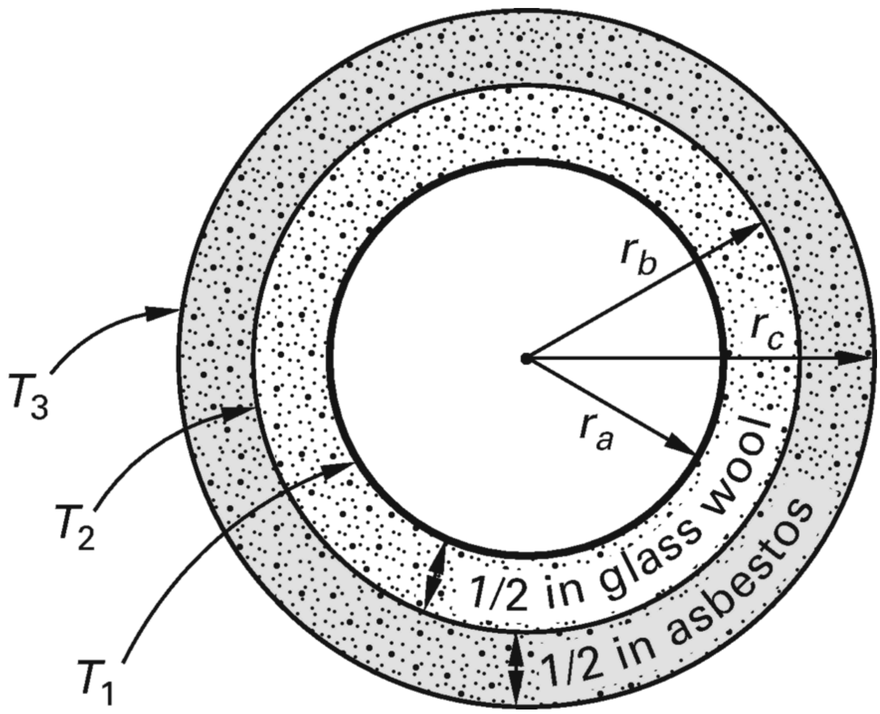
From the *NCEES Handbook* section titled “Fundamentals of Heat Transfer Fourier’s Law of Conduction,“ the heat flux per unit area through a plane wall is

$$\begin{aligned}\frac{Q_{1-2}}{A} &= \frac{k\Delta T}{L} = \frac{\left(0.066\ \frac{\text{W}}{\text{m}\cdot\text{K}}\right)(177^\circ\text{C} + 273.15\ \text{K})\left(100\ \frac{\text{cm}}{\text{m}}\right)}{30\ \text{cm}} \\ &= 99.03\ \text{W/m}^2 \quad (100\ \text{W/m}^2)\end{aligned}$$

The answer is (D).

4.

Because the temperature outside the copper tube is known, the thermal conductivity of the copper and the pipe’s inside radius can be disregarded. r_a is the radius at the interface between the copper and the first layer of insulation. r_b is the radius at the interface between the glass wool insulation and the asbestos layer. r_c is the outer radius of the glass wool layer plus the thickness of the asbestos layer.



$$
\begin{aligned}
r_a &= r_{\text{inside}} + t_{\text{pipe}} = \frac{1}{2}\text{ in} + \frac{1}{3}\text{ in} = 0.833\text{ in} \\
r_b &= r_a + t_{\text{glass wool}} = 0.833\text{ in} + \frac{1}{2}\text{ in} = 1.333\text{ in} \\
r_c &= r_b + t_{\text{asbestos}} = 1.333\text{ in} + \frac{1}{2}\text{ in} = 1.833\text{ in}
\end{aligned}
$$

The copper tube's length is not necessary for the solution. Use equationMERM34033 (also *NCEES Handbook:* Conduction) and the known temperatures to calculate the heat transfer per unit length from the outside of the copper tube through the asbestos.

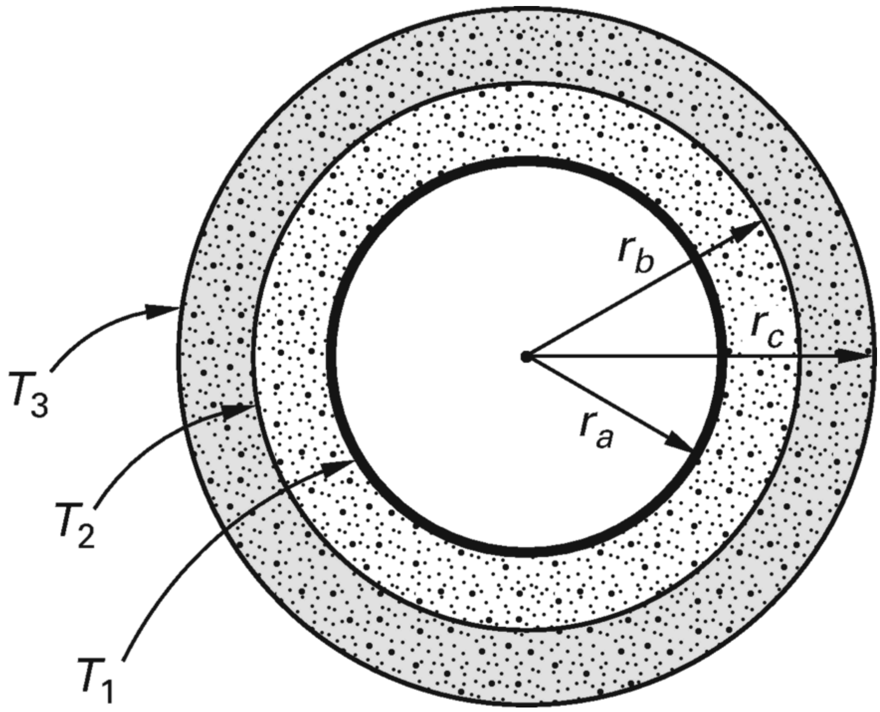
$$
\begin{aligned}
\frac{Q}{L} &= \frac{2\pi\left(T_1 - T_3\right)}{\dfrac{\ln\dfrac{r_b}{r_a}}{k_{\text{glass wool}}} + \dfrac{\ln\dfrac{r_c}{r_b}}{k_{\text{asbestos}}}} \\
&= \frac{2\pi\left(500^\circ\text{F} - 150^\circ\text{F}\right)}{\dfrac{\ln\dfrac{1.333\text{ in}}{0.833\text{ in}}}{0.0315\,\dfrac{\text{Btu}}{\text{hr-ft-}^\circ\text{F}}} + \dfrac{\ln\dfrac{1.833\text{ in}}{1.333\text{ in}}}{0.115\,\dfrac{\text{Btu}}{\text{hr-ft-}^\circ\text{F}}}} \\
&= 124.28\text{ Btu/hr-ft}
\end{aligned}
$$

Use equationMERM34033 (also *NCEES Handbook:* Conduction) again to calculate the temperature at the interface of glass wool and the asbestos from the heat layer through a single layer.

$$
\begin{aligned}
\frac{Q}{L} &= \frac{2\pi\left(T_2 - T_3\right)}{\dfrac{\ln\dfrac{r_c}{r_b}}{k_{\text{asbestos}}}} \\
T_2 &= \frac{\left(\dfrac{Q}{L}\right)\left(\dfrac{\ln\dfrac{r_c}{r_b}}{k_{\text{asbestos}}}\right)}{2\pi} + T_3 \\
&= \frac{\left(124.28\,\dfrac{\text{Btu}}{\text{hr-ft}}\right)\left(\dfrac{\ln\dfrac{1.833\text{ in}}{1.333\text{ in}}}{0.115\,\dfrac{\text{Btu}}{\text{hr-ft-}^\circ\text{F}}}\right)}{2\pi} + 150^\circ\text{F} \\
&= 204.79^\circ\text{F} \quad \left(200^\circ\text{F}\right)
\end{aligned}
$$

The answer is (B).
5.

r_a is the radius at the interface between the copper pipe and the first layer of insulation. r_b is the radius at the interface between the glass wool insulation and the asbestos layer.



$$r_a = r_{\text{inside}} + t_{\text{pipe}} = \frac{1}{2}\text{ in} + \frac{1}{3}\text{ in} = 0.833\text{ in}$$

$$r_b = r_a + t_{\text{glass wool}} = 0.833\text{ in} + \frac{1}{2}\text{ in} = 1.333\text{ in}$$

The copper tube's length is not necessary for the solution. Use equationMERM34033 (also *NCEES Handbook:* Conduction) to calculate the unknown radius, r_c, which is the outer radius of the glass wool layer plus the thickness of the asbestos layer.

$$\frac{Q}{L} = \frac{2\pi\left(T_1 - T_3\right)}{\frac{1}{k_{\text{glass wool}}}\ln\frac{r_b}{r_a} + \frac{1}{k_{\text{asbestos}}}\ln\frac{r_c}{r_b}}$$

$$110\ \frac{\text{Btu}}{\text{hr-ft}} = \frac{2\pi\left(500^\circ\text{F} - 80^\circ\text{F}\right)}{\left(\dfrac{1}{0.0315\ \dfrac{\text{Btu}}{\text{hr-ft-}^\circ\text{F}}}\right)\ln\dfrac{1.333\text{ in}}{0.833\text{ in}} + \left(\dfrac{1}{0.115\ \dfrac{\text{Btu}}{\text{hr-ft-}^\circ\text{F}}}\right)\ln\dfrac{r_c}{1.333\text{ in}}}$$

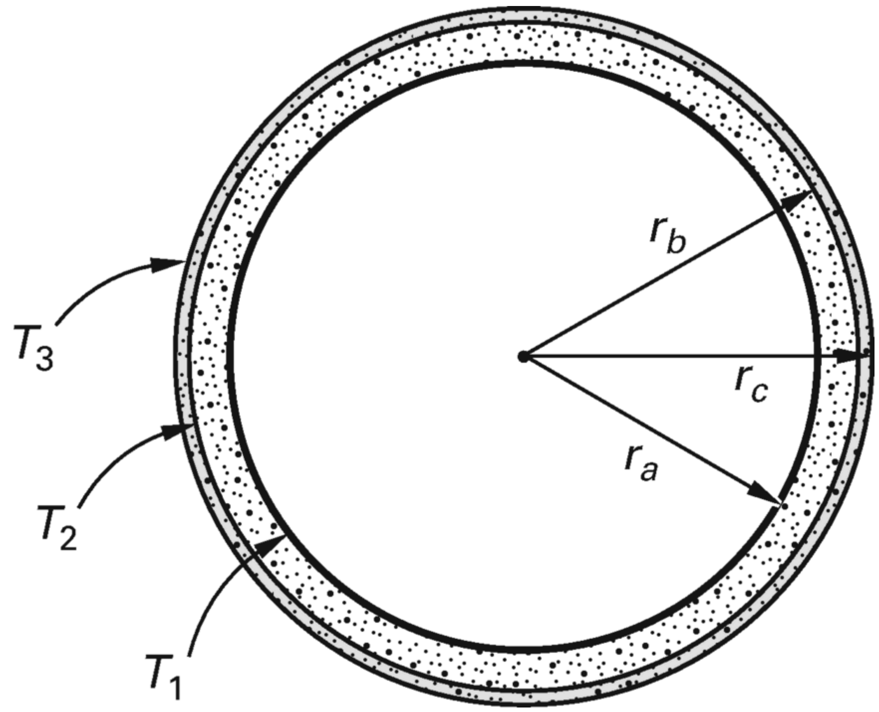
$$\begin{aligned}\ln\frac{r_c}{1.333\text{ in}} &= 1.0425\\ r_c &= (1.333\text{ in})\,e^{1.0425}\\ &= 3.781\text{ in}\end{aligned}$$

The thickness of the asbestos is

$$\begin{aligned}t_{\text{asbestos}} &= r_c - r_b = 3.781\text{ in} - 1.333\text{ in}\\ &= 2.448\text{ in}\quad(2.5\text{ in})\end{aligned}$$

The answer is (D).
6.

r_a is the outer radius of the tank. r_b is the radius at the interface between the magnesia insulation and the Styrofoam layer. r_c is the outer radius of the magnesia insulation layer plus the thickness of the Styrofoam layer.



$$r_a = \frac{r_o}{2} = \frac{4\ \text{ft}}{2} = 2\ \text{ft}$$

$$r_b = r_a + t_{\text{magnesia}} = 2\ \text{ft} + \frac{5\ \text{in}}{12\ \frac{\text{in}}{\text{ft}}} = 2.42\ \text{ft}$$

$$r_c = r_b + t_{\text{Styrofoam}} = 2.42\ \text{ft} + \frac{2\ \text{in}}{12\ \frac{\text{in}}{\text{ft}}} = 2.58\ \text{ft}$$

Use equationMERM34104 (also *NCEES Handbook:* Conduction) to calculate the heat transfer from the chromium sphere through the Styrofoam.

$$\begin{aligned} Q &= \frac{4\pi\left(T_1 - T_3\right)}{\frac{\frac{1}{r_a} - \frac{1}{r_b}}{k_{\text{magnesia}}} + \frac{\frac{1}{r_b} - \frac{1}{r_c}}{k_{\text{Styrofoam}}}} \\ &= \frac{4\pi\left(250^\circ\text{F} - 70^\circ\text{F}\right)}{\frac{\frac{1}{2\ \text{ft}} - \frac{1}{2.42\ \text{ft}}}{0.04\ \frac{\text{Btu}}{\text{hr-ft-}^\circ\text{F}}} + \frac{\frac{1}{2.42\ \text{ft}} - \frac{1}{2.58\ \text{ft}}}{0.02\ \frac{\text{Btu}}{\text{hr-ft-}^\circ\text{F}}}} \\ &= 655.5\ \text{Btu/hr-ft} \end{aligned}$$

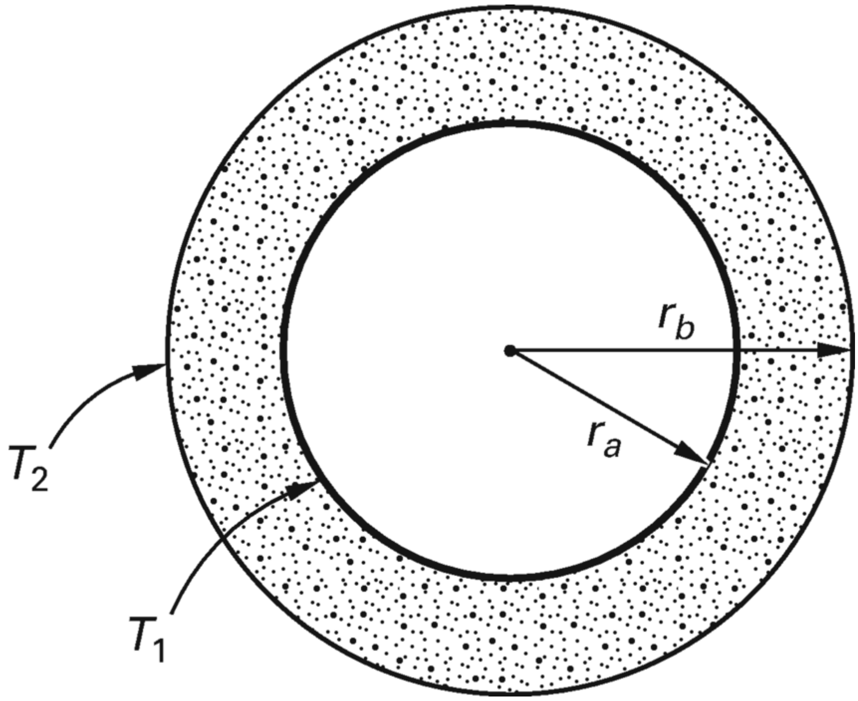
Use equationMERM34104 (also *NCEES Handbook:* Conduction) again to calculate the temperature at the interface of the magnesia and the Styrofoam from the heat transfer through a single layer.

$$T_1 - T_2 = \frac{Q\left(\frac{\frac{1}{r_a} - \frac{1}{r_b}}{k_{\text{magnesia}}}\right)}{4\pi}$$

$$\begin{aligned} T_2 &= T_1 - \frac{Q\left(\frac{\frac{1}{r_a} - \frac{1}{r_b}}{k_{\text{magnesia}}}\right)}{4\pi} \\ &= 250^\circ\text{F} - \frac{\left(655.5\ \frac{\text{Btu}}{\text{hr-ft}}\right)\left(\frac{\frac{1}{2\ \text{ft}} - \frac{1}{2.42\ \text{ft}}}{0.04\ \frac{\text{Btu}}{\text{hr-ft-}^\circ\text{F}}}\right)}{4\pi} \\ &= 136.8^\circ\text{F} \quad (140^\circ\text{F}) \end{aligned}$$

The answer is (C).
7.

$r_{a,1}$ is the outside radius of the original tank.



$$\begin{aligned} r_{a,1} &= r \\ r_{b,1} &= r + r = 2r \end{aligned}$$

The heat transfer rate through an insulated, spherical tank is found from equationMERM34104 (also *NCEES Handbook:* Conduction). For the original tank,

$$\begin{aligned} Q_1 &= \frac{4\pi\left(T_{a,1} - T_{b,1}\right)}{\frac{\frac{1}{r_{a,1}} - \frac{1}{r_{b,1}}}{k}} = \frac{4\pi\left(T_{a,1} - T_{b,1}\right)}{\frac{\frac{1}{r} - \frac{1}{2r}}{k}} \\ &= 8\pi kr\left(T_{a,1} - T_{b,1}\right) \end{aligned}$$

For the new tank,

$$\begin{aligned} r_{a,2} &= 2r \\ r_{b,2} &= 2r + r = 3r \end{aligned}$$

The outside temperatures of the tank and insulation are doubled. From equationMERM34104, the heat transfer rate for the new tank, Q_2, can be simplified to

$$\begin{aligned} Q_2 &= \frac{4\pi\left(T_{a,2}-T_{b,2}\right)}{\frac{\frac{1}{r_{a,2}}-\frac{1}{r_{b,2}}}{k}} \\ &= \frac{4\pi\left(2T_{a,1}-2T_{b,1}\right)}{\frac{\frac{1}{2r}-\frac{1}{3r}}{k}} \\ &= 48\pi kr\left(T_{a,1}-T_{b,1}\right) \end{aligned}$$

The ratio of the heat transfer of the new tank to the heat transfer of the original tank is

$$\frac{Q_2}{Q_1} = \frac{48\pi kr\left(T_{a,1}-T_{b,1}\right)}{8\pi kr\left(T_{a,1}-T_{b,1}\right)} = 6 \quad (6{:}1)$$

The answer is (D).
8.

Customary U.S. Solution

This is a transient problem. The total time is from 5 *p.m.* to 1 *a.m.*, which is eight hours.

The thermal capacitance (capacity), C_e, is given as 100,000 Btu/°F.

Resistance and conductance are reciprocals. The thermal resistance is

$$\begin{aligned} R_e &= \frac{1}{\text{thermal conductance}} = \frac{1}{6500\ \frac{\text{Btu}}{\text{hr-}^\circ\text{F}}} \\ &= 0.0001538\ \text{hr-}^\circ\text{F/Btu} \end{aligned}$$

From equationMERM34050 (also *NCEES Handbook:* Conduction),

$$\begin{aligned} T_t &= T_\infty + \left(T_0 - T_\infty\right) e^{-t/R_eC_e} \\ T_{8\ \text{hr}} &= 40^\circ\text{F} + \left(70^\circ\text{F} - 40^\circ\text{F}\right) \\ &\quad \times \exp\left(\frac{-8\ \text{hr}}{\left(0.0001538\ \frac{\text{hr-}^\circ\text{F}}{\text{Btu}}\right)\left(100{,}000\ \frac{\text{Btu}}{^\circ\text{F}}\right)}\right) \\ &= 57.8^\circ\text{F} \quad (60^\circ\text{F}) \end{aligned}$$

The answer is (C).

SI Solution

The thermal capacitance (capacity) is

$$C_e = \left(60\ \frac{\text{MJ}}{\text{K}}\right)\left(1000\ \frac{\text{kJ}}{\text{MJ}}\right) = 60\,000\ \text{kJ/K} \quad [\text{given}]$$

Resistance and conductance are reciprocals. The thermal resistance is

$$R_e = \frac{1}{\text{thermal conductance}} = \frac{1}{1.1\ \frac{\text{kW}}{\text{K}}} = 0.909\ \text{K/kW}$$

From equationMERM34050 (also *NCEES Handbook:* Conduction),

$$\begin{aligned} T_t &= T_\infty + (T_0 - T_\infty)\, e^{-t/R_e C_e} \\ &= 4^\circ\text{C} + (21^\circ\text{C} - 4^\circ\text{C}) \\ &\quad \times \exp\left(\frac{(-8\ \text{h})\left(3600\ \frac{\text{s}}{\text{h}}\right)}{\left(0.909\ \frac{\text{K}}{\text{kW}}\right)\left(60\,000\ \frac{\text{kJ}}{\text{K}}\right)}\right) \\ &= 14.0^\circ\text{C} \quad (15^\circ\text{C}) \end{aligned}$$

The answer is (C).
9.

Customary U.S. Solution

This is a transient problem. Check the Biot number to see if the lumped parameter method can be used.

The characteristic length, from equationMERM34008, is

$$L_c = \frac{V}{A_s} = \frac{\left(\frac{\pi}{6}\right) d^3}{\pi d^2} = \frac{d}{6}$$

For the largest ball, $d = 1.5$ in.

$$L_c = \frac{d}{6} = \frac{1.5\ \text{in}}{(6)\left(12\ \frac{\text{in}}{\text{ft}}\right)} = 0.0208\ \text{ft}$$

Evaluate the thermal conductivity, k, of steel at

$$\left(\frac{1}{2}\right)(1800^\circ\text{F} + 250^\circ\text{F}) = 1025^\circ\text{F}$$

From appendixMERM34B (also *NCEES Handbook* table “Physical Properties of Metals at 68°F (U.S. Units)”), for steel, $k \approx 22.0\ \text{Btu-ft/hr-ft}^2\text{-}^\circ\text{F}$.

From equationMERM34010 (also *NCEES Handbook:* Conduction), the Biot number is

$$\begin{aligned} \text{Bi} &= \frac{hL_c}{k} \\ &= \frac{\left(56\ \frac{\text{Btu}}{\text{hr-ft}^2\text{-}^\circ\text{F}}\right)(0.0208\ \text{ft})}{22.0\ \frac{\text{Btu-ft}}{\text{hr-ft}^2\text{-}^\circ\text{F}}} \\ &= 0.053 \end{aligned}$$

For small balls, Bi will be even smaller.

Since Bi < 0.10, the lumped parameter method can be used.

The assumptions are

- homogeneous body temperature
- minimal radiation losses
- oil bath temperature remains constant
- h remains constant

From equationMERM34048 and equationMERM34049 (also *NCEES Handbook:* Conduction), the time constant is

$$\begin{aligned} C_e R_e &= c_p \rho V \left(\frac{1}{hA_s}\right) = \left(\frac{c_p \rho}{h}\right)\left(\frac{V}{A_s}\right) = \left(\frac{c_p \rho}{h}\right) L_c \\ &= \left(\frac{c_p \rho}{h}\right)\left(\frac{d}{6}\right) \end{aligned}$$

From appendixMERM34B (also *NCEES Handbook* table “Physical Properties of Metals at 68°F (U.S. Units)”), $\rho = 490\ \mathrm{lbm/ft^3}$ and $c_p = 0.11$ Btu/lbm-°F, even though those values are for 32°F.

The time constant in hours is (measuring *d* in inches)

$$\begin{aligned} C_e R_e &= \frac{\left(\frac{\left(0.11\ \frac{\mathrm{Btu}}{\mathrm{lbm\text{-}^\circ F}}\right)\left(490\ \frac{\mathrm{lbm}}{\mathrm{ft^3}}\right)}{56\ \frac{\mathrm{Btu}}{\mathrm{hr\text{-}ft^2\text{-}^\circ F}}}\right)\left(\frac{d}{6}\right)}{12\ \frac{\mathrm{in}}{\mathrm{ft}}} \\ &= 0.01337d \end{aligned}$$

Take the natural log of the transient equation.

$$\begin{aligned} T_t &= 250^\circ\mathrm{F} \\ T_\infty &= 110^\circ\mathrm{F} \\ \Delta T &= 1800^\circ\mathrm{F} - 110^\circ\mathrm{F} = 1690^\circ\mathrm{F} \\ \ln\left(T_t - T_\infty\right) &= \ln\left(\Delta T e^{\frac{-t}{R_e C_e}}\right) \\ &= \ln\ \Delta T + \ln e^{\frac{-t}{R_e C_e}} \\ &= \ln\ \Delta T - \frac{t}{R_e C_e} \\ \ln\left(250^\circ\mathrm{F} - 110^\circ\mathrm{F}\right) &= \ln\left(1690^\circ\mathrm{F}\right) - \frac{t}{0.01337d} \\ 4.942 &= 7.432 - \frac{t}{0.01337d} \\ t &= 0.0333d \quad (0.033d) \end{aligned}$$

The answer is (C).

SI Solution

For the largest ball, the characteristic length is

$$L_c = \frac{d}{6} = \frac{38.1\ \mathrm{mm}}{(6)\left(1000\ \frac{\mathrm{mm}}{\mathrm{m}}\right)} = 6.35 \times 10^{-3}\ \mathrm{m}$$

Evaluate the thermal conductivity, *k*, of steel at

$$\left(\frac{1}{2}\right)\left(980^\circ\mathrm{C} + 120^\circ\mathrm{C}\right) = 550^\circ\mathrm{C}$$

From appendixMERM34B (also *NCEES Handbook* table “Physical Properties of Metals at 20°C (SI Units)”) and its footnote,

$$\begin{aligned} k &\approx \left(22.0\ \frac{\mathrm{Btu\text{-}ft}}{\mathrm{hr\text{-}ft^2\text{-}^\circ F}}\right)\left(\frac{1.7307\ \mathrm{W{\cdot}hr{\cdot}ft^2{\cdot}^\circ F}}{\mathrm{m{\cdot}K{\cdot}Btu{\cdot}ft}}\right) \\ &= 38.08\ \mathrm{W/m{\cdot}K} \end{aligned}$$

From equationMERM34010 (also *NCEES Handbook:* Conduction), the Biot number is

$$\mathrm{Bi} = \frac{hL_c}{k} = \frac{\left(320\ \frac{\mathrm{W}}{\mathrm{m^2{\cdot}K}}\right)(6.35\times 10^{-3}\ \mathrm{m})}{38.08\ \frac{\mathrm{W}}{\mathrm{m{\cdot}K}}} = 0.053$$

For small balls, Bi will be even smaller.

Since Bi < 0.10, the lumped method can be used. The assumptions are given in the customary U.S. solution. From equationMERM34048 and equationMERM34049 (also *NCEES Handbook:* Conduction), the time constant in hours is

$$\begin{aligned} C_eR_e &= c_p\rho V\left(\frac{1}{hA_s}\right) = \left(\frac{c_p\rho}{h}\right)\left(\frac{V}{A_s}\right) = \left(\frac{c_p\rho}{h}\right)L_c \\ &= \left(\frac{c_p\rho}{h}\right)\left(\frac{d}{6}\right) \end{aligned}$$

From appendixMERM34B (also *NCEES Handbook* table “Physical Properties of Metals at 20°C (SI Units)”) and its footnote,

$$\begin{aligned} \rho &= \left(490\ \frac{\mathrm{lbm}}{\mathrm{ft^3}}\right)\left(16.0185\ \frac{\mathrm{kg{\cdot}ft^3}}{\mathrm{m^3{\cdot}lbm}}\right) \\ &= 7849.1\ \mathrm{kg/m^3} \\ c_p &= \left(0.11\ \frac{\mathrm{Btu}}{\mathrm{lbm\text{-}^\circ F}}\right)\left(4186.8\ \frac{\mathrm{J{\cdot}lbm{\cdot}^\circ F}}{\mathrm{kg{\cdot}K{\cdot}Btu}}\right) \\ &= 460.5\ \mathrm{J/kg{\cdot}K} \\ C_eR_e &= \left(\frac{\left(460.5\ \frac{\mathrm{J}}{\mathrm{kg{\cdot}K}}\right)\left(7849.1\ \frac{\mathrm{kg}}{\mathrm{m^3}}\right)}{320\ \frac{\mathrm{W}}{\mathrm{m^2{\cdot}K}}}\right)\left(\frac{d}{6}\right) \\ &= 1882.6d \end{aligned}$$

Take the natural log of the transient equation.

$$\begin{aligned} T_t &= 120^\circ\mathrm{C} \\ T_\infty &= 43^\circ\mathrm{C} \\ \Delta T &= 980^\circ\mathrm{C} - 43^\circ\mathrm{C} = 937^\circ\mathrm{C} \\ \ln(T_t - T_\infty) &= \ln\,\Delta T - \frac{t}{1882.6d} \\ \ln(120^\circ\mathrm{C} - 43^\circ\mathrm{C}) &= \ln(937^\circ\mathrm{C}) - \frac{t}{1882.6d} \\ 4.344 &= 6.843 - \frac{t}{R_eC_e} \\ t &= 4704.6d \quad (4700d) \end{aligned}$$

The answer is (C).
10.

The problem can be set up as a thermal circuit.

The total thermal resistance is

$$\begin{aligned}R_{\text{thermal}} &= \sum_i \frac{L_i}{k_i} \\ &= \frac{1\ \text{m}}{0.7\ \frac{\text{W}}{\text{m}\cdot\text{C}}} + \frac{0.080\ \text{m}}{0.50\ \frac{\text{W}}{\text{m}\cdot\text{C}}} + \frac{0.0375\ \text{m}}{0.45\ \frac{\text{W}}{\text{m}\cdot\text{C}}} \\ &= 1.67\ \text{m}^2\cdot\text{C/W}\end{aligned}$$

The heat loss per unit area is

$$q = \frac{\Delta T}{R_{\text{thermal}}} = \frac{27^\circ\text{C} - (-8^\circ\text{C})}{1.67\ \frac{\text{m}^2\cdot\text{C}}{\text{W}}} = 21\ \text{W/m}^2$$

The answer is A.
11.

Customary U.S. Solution

The volume of the rod is

$$V = AL = \left(\frac{\pi}{4}\right) d^2 L$$

The volume per unit length is

$$\begin{aligned}\frac{V}{L} &= \frac{\pi}{4} d^2 = \frac{\left(\frac{\pi}{4}\right)(0.4\ \text{in})^2}{\left(12\ \frac{\text{in}}{\text{ft}}\right)^2} \\ &= 8.727\times 10^{-4}\ \text{ft}^3/\text{ft}\end{aligned}$$

The heat output per unit length of rod is

$$\begin{aligned}\frac{Q}{L} &= \left(\frac{V}{L}\right) G = \left(8.727\times 10^{-4}\ \frac{\text{ft}^3}{\text{ft}}\right)\left(4\times 10^7\ \frac{\text{Btu}}{\text{hr-ft}^3}\right) \\ &= 3.491\times 10^4\ \text{Btu/hr-ft}\end{aligned}$$

The diameter of the cladding is

$$d_o = 0.4\ \text{in} + (2)(0.020\ \text{in}) = 0.44\ \text{in}$$

The surface area per unit length of cladding is

$$A = \pi d_o = \frac{\pi (0.44\ \text{in})}{12\ \frac{\text{in}}{\text{ft}}} = 0.1152\ \text{ft}^2/\text{ft}$$

From equationMERM34023 (also *NCEES Handbook:* Conduction), the surface temperature of the cladding is

$$\begin{aligned}T_s &= \frac{Q}{hA} + T_\infty \\ &= \frac{3.491\times 10^4\ \frac{\text{Btu}}{\text{hr-ft}}}{\left(10{,}000\ \frac{\text{Btu}}{\text{hr-ft}^2\text{-}^\circ\text{F}}\right)\left(0.1152\ \frac{\text{ft}^2}{\text{ft}}\right)} + 500^\circ\text{F} \\ &= 530.3^\circ\text{F}\end{aligned}$$

For the cladding,

$$r_o = \frac{d_o}{2} = \frac{0.44 \text{ in}}{(2)\left(12 \ \frac{\text{in}}{\text{ft}}\right)} = 0.01833 \text{ ft}$$

$$r_i = \frac{d}{2} = \frac{0.4 \text{ in}}{(2)\left(12 \ \frac{\text{in}}{\text{ft}}\right)} = 0.01667 \text{ ft}$$

From appendixMERM34B (also *NCEES Handbook* table “Physical Properties of Metals at 68°F (U.S. Units)”), k for stainless steel is 9.4 Btu-ft/hr-ft^2-°F at 68°F.

From equationMERM34005 and equationMERM34019 (also *NCEES Handbook:* Conduction) for a cylinder,

$$\begin{aligned}
T_{\text{inside}} - T_{\text{outside}} &= \frac{Q \ln \frac{r_o}{r_i}}{2\pi k L} = \frac{\frac{Q}{L} \ln \frac{r_o}{r_i}}{2\pi k} \\
&= \frac{\left(3.491 \times 10^4 \ \frac{\text{Btu}}{\text{hr-ft}}\right) \ln \frac{0.01833 \text{ ft}}{0.01667 \text{ ft}}}{2\pi \left(9.4 \ \frac{\text{Btu-ft}}{\text{hr-ft}^2\text{-}^\circ\text{F}}\right)} \\
&= 56.1^\circ\text{F} \\
T_{\text{inside cladding}} &= T_{\text{outside fuel rod}} + 56.1^\circ\text{F} \\
&= 530.3^\circ\text{F} + 56.1^\circ\text{F} \\
&= 586.4^\circ\text{F}
\end{aligned}$$

From equationMERM34056 (also *NCEES Handbook:* Conduction), using $r_{o,\text{fuel rod}} = r_{i,\text{cladding}}$, and using k for the uranium dioxide (not the stainless steel),

$$\begin{aligned}
T_{\text{center}} &= T_o + \frac{G r_o^2}{4k} \\
&= 586.4^\circ\text{F} + \frac{\left(4 \times 10^7 \ \frac{\text{Btu}}{\text{hr-ft}^3}\right)(0.01667 \text{ ft})^2}{(4)\left(1.1 \ \frac{\text{Btu-ft}}{\text{hr-ft}^2\text{-}^\circ\text{F}}\right)} \\
&= 3112^\circ\text{F} \quad (3100^\circ\text{F})
\end{aligned}$$

The answer is (D).

SI Solution

The rod volume per unit length is

$$\begin{aligned}
\frac{V}{L} &= \left(\frac{\pi}{4}\right) d^2 = \frac{\left(\frac{\pi}{4}\right)(1.0 \text{ cm})^2}{\left(100 \ \frac{\text{cm}}{\text{m}}\right)^2} \\
&= 7.854 \times 10^{-5} \text{ m}^3/\text{m}
\end{aligned}$$

The heat output per unit length of rod is

$$\begin{aligned}
\frac{Q}{L} &= \left(\frac{V}{L}\right) G = \left(7.854 \times 10^{-5} \ \frac{\text{m}^3}{\text{m}}\right)\left(4.1 \times 10^8 \ \frac{\text{W}}{\text{m}^3}\right) \\
&= 32\,201.4 \text{ W/m}
\end{aligned}$$

The diameter of the cladding is

$$d_o = \frac{1.0 \text{ cm} + \frac{(2)(0.5 \text{ mm})}{10 \ \frac{\text{mm}}{\text{cm}}}}{100 \ \frac{\text{cm}}{\text{m}}} = 0.011 \text{ m}$$

The surface area per unit length of cladding is

$$A = \pi d_o = \pi\,(0.011\ \mathrm{m}) = 0.0346\ \mathrm{m^2/m}$$

From equationMERM34023 (also *NCEES Handbook:* Conduction), the surface temperature of the cladding is

$$\begin{aligned} T_s &= \frac{Q}{hA} + T_\infty \\ &= \frac{32\,201.4\ \frac{\mathrm{W}}{\mathrm{m}}}{\left(57\ \frac{\mathrm{kW}}{\mathrm{m^2{\cdot}K}}\right)\left(1000\ \frac{\mathrm{W}}{\mathrm{kW}}\right)\left(0.0346\ \frac{\mathrm{m^2}}{\mathrm{m}}\right)} + 260^\circ\mathrm{C} \\ &= 276.3^\circ\mathrm{C} \end{aligned}$$

For the cladding,

$$\begin{aligned} r_o &= \frac{d_o}{2} = \frac{0.011\ \mathrm{m}}{2} = 0.0055\ \mathrm{m} \\ r_i &= \frac{d_i}{2} = \frac{0.01\ \mathrm{m}}{2} = 0.0050\ \mathrm{m} \end{aligned}$$

From appendixMERM34B (also *NCEES Handbook* table “Physical Properties of Metals at 20°C (SI Units)”) and the table’s footnote, the k for stainless steel is 16.3 W/m·K at 30°C.

For a cylinder, from equationMERM34005 and equationMERM34019 (also *NCEES Handbook:* Conduction),

$$\begin{aligned} T_{\text{inside}} - T_{\text{outside}} &= \frac{Q \ln \frac{r_o}{r_i}}{2\pi k L} = \left(\frac{Q}{L}\right)\left(\frac{\ln \frac{r_o}{r_i}}{2\pi k}\right) \\ &= \left(32\,201.4\ \frac{\mathrm{W}}{\mathrm{m}}\right)\left(\frac{\ln \frac{0.0055\ \mathrm{m}}{0.0050\ \mathrm{m}}}{(2\pi)\left(16.3\ \frac{\mathrm{W}}{\mathrm{m{\cdot}K}}\right)}\right) \\ &= 30^\circ\mathrm{C} \\ T_{\text{inside cladding}} &= T_{\text{outside fuel rod}} + 30^\circ\mathrm{C} \\ &= 276.3^\circ\mathrm{C} + 30^\circ\mathrm{C} \\ &= 306.3^\circ\mathrm{C} \end{aligned}$$

From equationMERM34056 (also *NCEES Handbook:* Conduction), using $r_{o,\text{fuel rod}} = r_{i,\text{cladding}}$, and using k for the uranium dioxide (not the stainless steel),

$$\begin{aligned} T_{\text{center}} &= T_o + \frac{G r_o^2}{4k} \\ &= 306.3^\circ\mathrm{C} + \frac{\left(4.1\times 10^8\ \frac{\mathrm{W}}{\mathrm{m^3}}\right)(0.0050\ \mathrm{m})^2}{(4)\left(1.9\ \frac{\mathrm{W}}{\mathrm{m{\cdot}K}}\right)} \\ &= 1655.3^\circ\mathrm{C} \quad (1700^\circ\mathrm{C}) \end{aligned}$$

The answer is (D).
12.

Customary U.S. Solution

Consider this to be two infinite cylindrical fins with

$$\begin{aligned} T_b &= 450^\circ\mathrm{F} \\ T_\infty &= 80^\circ\mathrm{F} \\ h &= 3\ \mathrm{Btu/hr\text{-}ft^2\text{-}^\circ F} \end{aligned}$$

From equationMERM34066 (also *NCEES Handbook:* Planar Geometry—Area and Perimeter), the perimeter length is

$$P = \pi d = \frac{\pi\left(\frac{1}{16}\ \text{in}\right)}{12\ \frac{\text{in}}{\text{ft}}} = 0.01636\ \text{ft}$$

From equationMERM34065 (also *NCEES Handbook:* Planar Geometry—Area and Perimeter), the cross-sectional area of the fin at its base is

$$\begin{aligned} A_b &= \pi r^2 = \pi\left(\frac{d}{2}\right)^2 = \left(\frac{\pi}{4}\right)d^2 \\ &= \frac{\left(\frac{\pi}{4}\right)\left(\frac{1}{16}\ \text{in}\right)^2}{\left(12\ \frac{\text{in}}{\text{ft}}\right)^2} \\ &= 2.131\times10^{-5}\ \text{ft}^2 \end{aligned}$$

From equationMERM34064 (also *NCEES Handbook:* Combination of Heat-Transfer Mechanisms), with two fins joined at the middle,

$$\begin{aligned} Q &= 2\sqrt{hPkA_b}\,(T_b - T_\infty) \\ &= (2)\sqrt{\begin{array}{l}\left(3\ \frac{\text{Btu}}{\text{hr-ft}^2\text{-}^\circ\text{F}}\right)(0.01636\ \text{ft}) \\ \quad\times\left(215\ \frac{\text{Btu-ft}}{\text{hr-ft}^2\text{-}^\circ\text{F}}\right)\left(2.131\times10^{-5}\ \text{ft}^2\right)\end{array}} \\ &\quad\times(450^\circ\text{F} - 80^\circ\text{F}) \\ &= 11.1\ \text{Btu/hr}\quad(11\ \text{Btu/hr}) \end{aligned}$$

This disregards radiation.

The answer is (B).

SI Solution

Consider this an infinite cylindrical fin with

$$\begin{aligned} T_b &= 230^\circ\text{C} \\ T_\infty &= 27^\circ\text{C} \\ h &= 17\ \text{W/m}^2\cdot\text{K} \end{aligned}$$

From equationMERM34066 (also *NCEES Handbook:* Planar Geometry—Area and Perimeter), the perimeter length is

$$P = \pi d = \frac{\pi(1.6\ \text{mm})}{1000\ \frac{\text{mm}}{\text{m}}} = 5.027\times10^{-3}\ \text{m}$$

From equationMERM34065 (also *NCEES Handbook:* Planar Geometry—Area and Perimeter), the cross-sectional area of the fin at its base is

$$\begin{aligned} A_b &= \pi r^2 = \pi\left(\frac{d}{2}\right)^2 = \left(\frac{\pi}{4}\right)d^2 \\ &= \frac{\left(\frac{\pi}{4}\right)(1.6\ \text{mm})^2}{\left(1000\ \frac{\text{mm}}{\text{m}}\right)^2} \\ &= 2.011\times10^{-6}\ \text{m}^2 \end{aligned}$$

From equationMERM34064 (also *NCEES Handbook:* Combination of Heat-Transfer Mechanisms), with two fins joined at the middle,

$$
\begin{aligned}
Q &= 2\sqrt{hPkA_b}\,(T_b - T_\infty) \\
&= (2)\sqrt{\begin{array}{l}\left(17\ \dfrac{\mathrm{W}}{\mathrm{m^2{\cdot}K}}\right)(5.027\times10^{-3}\ \mathrm{m}) \\ \quad\times\left(372.1\ \dfrac{\mathrm{W}}{\mathrm{m{\cdot}K}}\right)(2.011\times10^{-6}\ \mathrm{m^2})\end{array}} \\
&\quad \times (230^\circ\mathrm{C} - 27^\circ\mathrm{C}) \\
&= 3.25\ \mathrm{W} \quad (3.3\ \mathrm{W})
\end{aligned}
$$

This disregards radiation.

The answer is (B).