

1 As we define $K(A) = \infty$ when A is singular, the prob is meaningless if A is singular.

Spse A is non-singular.

$$\|A\|_2 = \sigma_1 = 100 \quad \text{by Thm 5.3.}$$

$$\|A_F\|_2^2 = \sigma_1^2 + \dots + \sigma_m^2 = 10201$$

$$K(A) = \|A\|_2 \|A^{-1}\|_2 = 100 \|A^{-1}\|_2$$

$$\text{And, } \|A^{-1}\|_2 = \frac{1}{\sigma_m} \text{ as } \sigma_1 \geq \dots \geq \sigma_m > 0.$$

$$\|A_F\|_2^2 = \sum_{i=1}^m \sigma_i^2 = 10201 \geq \sigma_1^2 + 201 \sigma_m^2 \rightsquigarrow 1 \geq \sigma_m^2, 0 < \sigma_m \leq 1 \rightsquigarrow \frac{1}{\sigma_m} \geq 1$$

$$\therefore K(A) = \frac{\sigma_1}{\sigma_m} \geq \sigma_1 = 100. \text{ Note that equality holds when } \Sigma = \begin{bmatrix} 100 & & \\ & 1 & \\ & & \ddots \\ & & & 1 \end{bmatrix}.$$

Let $A = U \Sigma V^*$ (svd of A , exist by thm 4.1)

Then $A^{-1} = V \Sigma^{-1} U^*$ where $\Sigma^{-1} = \begin{bmatrix} 1/\sigma_1 & & \\ & \ddots & \\ & & 1/\sigma_m \end{bmatrix}$

Note that $\sigma_i > 0$ by assumption.

if $\Sigma = \begin{bmatrix} \sigma_1 & & \\ & \ddots & \\ & & \sigma_m \end{bmatrix}$.