

1 Yes.

$$A = QR \leadsto A^*A = R^*Q^*QR.$$

Note that Q is unitary matrix in the QR factorization. So, $A^*A = R^*R$.

R is an upper triangular. And, $r_{jj} > 0$ for $1 \leq j \leq m$ (let A is $m \times m$ as we assumed).

So, it is a Cholesky Factorization of A^*A , and we can see that A^*A is hermitian positive definite since A is nonsingular

: Hermitian clearly: $(A^*A)^* = A^*A$

And, for $\forall x \neq 0 \in \mathbb{C}^m$, we know that $Ax \neq 0$ since A is nonsingular.

Then, $x^*(A^*A)x = (Ax)^*(Ax) > 0$. So, it is also positive definite

So, by Thm 23.1, Hermitian positive definite A^*A has a unique Cholesky factorization

$$\therefore R = L^T$$

2 $Ax = b \leadsto A = R^*R$ Factor A into R^*R ①

$R^*y = b$ Forward Substitution ②

$y = Rx$ Backward Substitution ③

① By Thm 23.2, $\tilde{R}^* \tilde{R} = A + \delta A$ with some $\|\delta A\| / \|A\| = O(\epsilon_{\text{machine}})$

② By Thm 17.1, $(\tilde{R}^* + \delta R_1^*) \tilde{y} = b$ with $\|\delta R_1^*\| / \|\tilde{R}^*\| = O(\epsilon_{\text{machine}})$

③ " $(\tilde{R} + \delta R_2) \tilde{x} = \tilde{y}$ with $\|\delta R_2\| / \|\tilde{R}\| = O(\epsilon_{\text{machine}})$

By Thm 14.1, we can say that $\|\tilde{R}\| = \|\tilde{R}^*\| = \|A + \delta A\|^{1/2}$ (results from 2-norm)

$$\frac{\|\tilde{R}\|}{\|A\|} = \frac{\|A + \delta A\|^{1/2}}{\|A\|} = \frac{\|A + \delta A\|}{\|A\| \|\tilde{R}^*\|} = O\left(\frac{1}{\|\tilde{R}^*\|}\right) \text{ as } \epsilon_{\text{machine}} \rightarrow 0 \text{ (Analogous for } \frac{\|\tilde{R}^*\|}{\|A\|})$$

Composing these,

$$b = (\tilde{R}^* + \delta R_1^*) (\tilde{R} + \delta R_2) x = (A + \delta A + \tilde{R}^* \delta R_2 + \delta R_1^* \tilde{R} + \delta R_1^* \delta R_2) x$$

$$\frac{\|\tilde{R}^* \delta R_2\|}{\|A\|} \leq \frac{\|\tilde{R}^*\|}{\|A\|} \|\delta R_2\| = O\left(\frac{\|\delta R_2\|}{\|\tilde{R}\|}\right) = O(\epsilon_{\text{machine}})$$

$$\frac{\|\delta R_1^* \delta R_2\|}{\|A\|} \leq \frac{\|\delta R_1^*\| \|\delta R_2\|}{\|A\|} = \frac{\|\tilde{R}\| \|\tilde{R}^*\| \|\delta R_1^*\| \|\delta R_2\|}{\|A\| \|\tilde{R}\| \|\tilde{R}^*\|} = O(1) \cdot O(\epsilon_{\text{machine}}^2)$$

$$\therefore \frac{\|\Delta A\|}{\|A\|} \leq \frac{\|\delta A + \tilde{R}^* \delta R_2 + \delta R_1^* \tilde{R} + \delta R_1^* \delta R_2\|}{\|A\|} = O(\epsilon_{\text{machine}}).$$