$\boxed{1}$ From the lecture, let $K_n = \langle q_1, \cdots, q_n \rangle$. Form $Q_n = [q_1 | \cdots | q_n]$. $\leadsto$ full rank.

Let θ_j be any Ritz values (the eigenvalue of T_n, where $T_n = Q_n^T A Q_n$)

Since for $\forall x \in K_n$, $\exists y$ s.t $x = Q_n y$, the Rayleigh quotient $r(x)$ when x is restricted to K_n is
$r(x) = \frac{y^T Q_n^T A Q_n y}{y^T y} = \frac{y^T T_n y}{y^T y}$ where $x = Q_n y$.

Now, for convenience, let $r_{T_n}(y) = \frac{y^T T_n y}{y^T y}$

The k^{th} component of $\nabla r_{T_n}(y)$ i.e., $\frac{\partial r_{T_n}(y)}{\partial y_k} = \frac{\partial}{\partial y_k}\left(\frac{y^T T_n y}{y^T y}\right) = \frac{1}{y^T y}\left((T_n y)_k + (T_n^T y)_k - 2 \cdot \frac{y^T T_n y}{y^T y} \cdot y_k\right)$

$$= \frac{1}{y^T y}\left((T_n + T_n^T) y - 2 r_{T_n}(y) \cdot y\right)_k$$

$\therefore \nabla r_{T_n}(y) = \frac{1}{y^T y}\left(\underbrace{(T_n + T_n^T)}_{2T_n} y - 2 r_{T_n}(y) \cdot y\right)$

2T_n as $T_n = T_n^T$ ($\because$ A symmetric $\leadsto$ so does similar T_n)

$$= \frac{2}{y^T y}\left(T_n y - r_{T_n}(y) \cdot y\right).$$

Again, $\nabla r_{T_n}(y) = 0 \leftrightarrow y$ is eigenvector of $T_n \leftrightarrow \overset{r(x)}{r_{T_n}(y)}$ is an eigenvalue of T_n (Ritz value).

$\leadsto$ Ritz values are stationary pts of the Rayleigh quotient $r(x)$ if x is restricted to K_n ($x \in K_n$) ∎