

2 In general, multiple independent variables may be problematic: highly correlated each other
ex) parents level of edu \propto parents income

\leadsto rank of A will be nearly decreased; makes $\sigma_r \approx 0 \leadsto k(A) \uparrow \leadsto K_{A \rightarrow x} \uparrow$

4 A squares & non-singular $\Rightarrow \text{range}(A) = \mathbb{C}^m$.

For sufficient small δA , $A + \delta A$ is also nonsingular i.e., $\text{range}(A + \delta A) = \mathbb{C}^m$.

($A = U \Sigma V^*$, $\delta A = U \delta A' V^* \leadsto A + \delta A = U (\Sigma + \delta A') V^*$. With small $\delta A'$, Σ will have full nonzero σ_i 's.)

So, $b \in \text{range}(A + \delta A)$ even after perturbation of A , so projection is not changed. ■