$\boxed{1}$ (a) True.

Set $C=2, R_0=\frac{\pi}{2}$. Then, $|\sin x| \le 1 < C\cdot 1$ whenever $x > R_0 = \frac{\pi}{2}$ clearly.
It implies $\sin x = O(1)$ as $x \to \infty$

(b) True.

Set $C=2, \varepsilon_0=\frac{\pi}{2}$. Then, $|\sin x| \le 1 < C\cdot 1$ whenever $|x| < \varepsilon_0 = \frac{\pi}{2}$ clearly.
It implies $\sin x = O(1)$ as $x \to 0$

(c) True

Set $C=1$. Then, as $\lim\limits_{x\to\infty} \frac{\log x}{x^{1/100}} \underset{\text{L'Hôpital's Rule}}{=} \lim\limits_{x\to\infty} \frac{x^{99/100}}{100x} = \lim\limits_{x\to\infty} \frac{1}{100 x^{1/100}} = \frac{1}{\infty} = 0,$

$\exists \delta > 0$ s.t $\underbrace{\left|\frac{\log x}{x^{1/100}}\right| \le C}_{\text{equivalent to } |\log x| \le C\cdot|x^{1/100}|}$ whenever $x > \delta$. It implies $\log x = O(x^{1/100})$ as $x \to \infty$ ∎

(d) False

$\lim\limits_{n\to\infty} \frac{n!}{(n/e)^n} = \infty$

(e) True

$A = 4\pi r^2 \quad V = \frac{4}{3}\pi r^3$

(f) True

Choose $C=\pi, \delta=1$.
$|fl(\pi)-\pi| = \varepsilon\pi$ for some $|\varepsilon| \le \varepsilon_{machine}$ by our axiom.
So, $|fl(\pi)-\pi| = \varepsilon\pi \le C\varepsilon_{machine}$ whenever $|\varepsilon_{machine}| < \delta = 1$, which implies $fl(\pi)-\pi = O(\varepsilon_{machine})$ as $\varepsilon_{machine} \to 0$.

(g) False.

Spse the statement is true: $|fl(n\pi)-n\pi| \le C\varepsilon_{machine}$ whenever $|\varepsilon_{machine}| < \delta$. for some $C>0$, $\delta>0$.
Note that we defined precision t as integer with ≥ 1.
So, we can say that $\varepsilon_{machine} = \frac{1}{2}\beta^{1-t} < 1$ always.

By axiom, $|fl(n\pi)-n\pi| = n\pi(1+\varepsilon)$ for some $|\varepsilon| \le \varepsilon_{machine}$.

As $n\pi\underbrace{(1-\varepsilon_{machine})}_{>0\ \&\ \text{constant w.r.t } n} \to \infty$ as $n \to \infty$, $\exists N \in \mathbb{N}$ s.t $n\pi(1-\varepsilon_{machine}) > C\cdot\varepsilon_{machine}$ whenever $n \ge N$.

Note that $|fl(n\pi)-n\pi| = n\pi(1+\varepsilon) \ge n\pi(1-\varepsilon_{machine})$.
So, for any given $C>0$, $\exists n \in \mathbb{N}$ $|fl(n\pi)-n\pi| > C\cdot\varepsilon_{machine}$, which contradicts to the assumption.
$\therefore$ False by proof by contradiction.

$\boxed{2}$ (a) Spse some f satisfy $f(\varepsilon_{machine}) = (1+O(\varepsilon_{machine}))(1+O(\varepsilon_{machine}))$

Then by definition, $\exists C>0\ \&\ \delta>0$ s.t $|f(\varepsilon_{machine})| \le (1+C\cdot\varepsilon_{machine})(1+C\cdot\varepsilon_{machine})$ whenever $|\varepsilon_{machine}| < \delta$.

Now, $(1+C\cdot\varepsilon_{machine})(1+C\cdot\varepsilon_{machine}) = 1+2\cdot C\cdot\varepsilon_{machine} + C^2\varepsilon_{machine}^2$
Note that we defined precision t as integer with ≥ 1.
So, we can say that $\varepsilon_{machine} = \frac{1}{2}\beta^{1-t} < 1$ always. $\rightsquigarrow \varepsilon_{machine}^2 < \varepsilon_{machine}$
$\therefore |f(\varepsilon_{machine})| \le (1+C\cdot\varepsilon_{machine})(1+C\cdot\varepsilon_{machine}) \le 1+\underbrace{(C^2+2C)}_{\text{Let } C'>0}\varepsilon_{machine}$ whenever $|\varepsilon_{machine}| < \delta$
$\rightsquigarrow f(\varepsilon_{machine}) = 1+O(\varepsilon_{machine})$ as $\varepsilon_{machine} \to 0$ ∎

Spse some f satisfy $f(\varepsilon_{machine}) = (1+O(\varepsilon_{machine}))^{-1}$

Then by definition, $\exists C>0$ & $\delta>0$ s.t. $|f(\varepsilon_{machine})| \leq \dfrac{1}{(1+C\cdot\varepsilon_{machine})}$ whenever $|\varepsilon_{machine}|<\delta$.

As $1+\underbrace{C}_{>0}\cdot\underbrace{\varepsilon_{machine}}_{>0} > 1$, $\dfrac{1}{1+C\cdot\varepsilon_{machine}} < 1+C\cdot\varepsilon_{machine}$ since $(1+C\cdot\varepsilon_{machine})^2 > 1$.

It implies $|f(\varepsilon_{machine})| \leq \dfrac{1}{(1+C\cdot\varepsilon_{machine})} \leq 1+C\cdot\varepsilon_{machine}$ whenever $|\varepsilon_{machine}|<\delta$

$\rightsquigarrow f(\varepsilon_{machine}) = 1+O(\varepsilon_{machine})$ as $\varepsilon_{machine}\to 0$ ∎