$\boxed{1}$ (a) True.

Set $C=2, R_0=\frac{\pi}{2}$. Then, $|\sin x| \le 1 < C\cdot 1$ whenever $x > R_0 = \frac{\pi}{2}$ clearly.
It implies $\sin x = O(1)$ as $x \to \infty$

(b) True.

Set $C=2, \varepsilon_0=\frac{\pi}{2}$. Then, $|\sin x| \le 1 < C\cdot 1$ whenever $|x| < \varepsilon_0 = \frac{\pi}{2}$ clearly.
It implies $\sin x = O(1)$ as $x \to 0$

(c) True

Set $C=1$. Then, as $\displaystyle\lim_{x\to\infty} \frac{\log x}{x^{1/100}} \underset{\text{L'Hôpital's Rule}}{=} \lim_{x\to\infty} \frac{x^{99/100}}{100\,x} = \lim_{x\to\infty} \frac{1}{100\,x^{1/100}} = \frac{1}{\infty} = 0,$

$\exists \delta > 0$ s.t $\underbrace{\left|\frac{\log x}{x^{1/100}}\right| \le C}_{\text{equivalent to } |\log x| \le C\cdot|x^{1/100}|}$ whenever $x > \delta$. It implies $\log x = O(x^{1/100})$ as $x \to \infty$ ∎

(d) False

$\displaystyle\lim_{n\to\infty} \frac{n!}{(n/e)^n} = \infty$

(e) True

$A = 4\pi r^2 \quad V = \frac{4}{3}\pi r^3$

(f) True

Choose $C=\pi, \delta=1$.
$|fl(\pi)-\pi| = \varepsilon\pi$ for some $|\varepsilon| \le \varepsilon_{machine}$ by our axiom.
So, $|fl(\pi)-\pi| = \varepsilon\pi \le C\varepsilon_{machine}$ whenever $|\varepsilon_{machine}| < \delta = 1$, which implies $fl(\pi)-\pi = O(\varepsilon_{machine})$ as $\varepsilon_{machine} \to 0$.

(g) False.

Spse the statement is true: For $\forall n \in \mathbb{N}$, $\exists$ single constant $C>0$ s.t $|fl(n\pi) - n\pi| \le C\varepsilon_{machine}$ whenever $|\varepsilon_{machine}| < \delta$. for some $\delta > 0$.
For $\forall n \in \mathbb{Z}$, we can say that $fl(n\pi) \ne n\pi$
as integer × irrational = irrational ($n\pi$ = irrational; not in $\mathbb{F}$).
Choose $n \in \mathbb{Z}$ s.t $fl(n\pi) < n\pi$. WLOG, let $n > 0$. (sign doesn't matter).
Existence of such n is clear: spse not exist. Let $fl(n\pi) > n\pi$ for $\forall n \in \mathbb{Z}$.
So, for some floating pt, $|\quad \overset{\bullet}{n\pi}\ \overset{|}{x = fl(n\pi)}$ ($n\pi$ is closer to $fl(n\pi)$)
Then, for $-n$, $\overset{|}{-x}\ \overset{\bullet}{-n\pi} \quad | \rightsquigarrow fl(-n\pi) = -x < -n\pi$.
(By our definition of $\mathbb{F}$, it is symmetrical w.r.t 0)

Now, let $n\pi = 0.f_1f_2f_3\cdots f_t f_{t+1}\cdots \times \beta^{e-t}$. ($f_1 \ne 0$ by our convention of $\mathbb{F}$, $\beta^{t-1} \le$ mantissa $< \beta^{t-1}$)
Then, it is clear that $\beta^x \cdot n\pi = 0.f_1f_2f_3\cdots f_t f_{t+1} \times \beta^{e+x-t}$
So, $fl(n\pi) = 0.f_1f_2f_3\cdots f_t \times \beta^{e-t}$, $fl(\beta^x n\pi) = 0.f_1\cdots f_t \times \beta^{e+x-t}$

$\frac{m}{\beta^t} = 0.f_1\cdots f_t \ge \frac{1}{\beta} = 0.10\cdots0$

$|fl(\beta^x n\pi) - \beta^x n\pi| = 0.\underbrace{00\cdots0}_{t} f_{t+1}\cdots \times \beta^{e+x-t} = \beta^x(0.0\cdots0f_{t+1}\cdots \times \beta^{e-t})$
$= \beta^x \underbrace{|fl(n\pi) - n\pi|}_{\text{Let } k \text{ (constant) as we chose (fixed) some } n.}$

And, as β is $\mathbb{Z} \ge 2$, $\exists x \in \mathbb{N}$ s.t $\beta^x > \frac{C\cdot\varepsilon_{machine}}{k}$. $\rightsquigarrow |fl(\beta^x n\pi) - n\pi| > C\cdot\varepsilon_{machine}$
So, for integer $m = \beta^x \cdot n$, contradiction ∎

$\boxed{2}$ (a) Spse some f satisfy $f(\varepsilon_{machine}) = (1+O(\varepsilon_{machine}))(1+O(\varepsilon_{machine}))$

Then by definition, $\exists C>0$ & $\delta>0$ s.t $|f(\varepsilon_{machine})| \le (1+C\cdot\varepsilon_{machine})(1+C\cdot\varepsilon_{machine})$ whenever $|\varepsilon_{machine}|<\delta$.

Now, $(1+C\cdot\varepsilon_{machine})(1+C\cdot\varepsilon_{machine}) = 1+2\cdot C\cdot\varepsilon_{machine} + C^2\varepsilon_{machine}^2$

Note that we defined precision t as integer with ≥ 1.

So, we can say that $\varepsilon_{machine} = \frac{1}{2}\beta^{1-t} < 1$ always. $\rightsquigarrow$ $\varepsilon_{machine}^2 < \varepsilon_{machine}$

$\therefore |f(\varepsilon_{machine})| \le (1+C\cdot\varepsilon_{machine})(1+C\cdot\varepsilon_{machine}) \le 1+\underbrace{(C^2+2C)}_{\text{Let } C'>0}\varepsilon_{machine}$ whenever $|\varepsilon_{machine}|<\delta$

$\rightsquigarrow$ $f(\varepsilon_{machine}) = 1+O(\varepsilon_{machine})$ as $\varepsilon_{machine}\to 0$ ∎

Spse some f satisfy $f(\varepsilon_{machine}) = (1+O(\varepsilon_{machine}))^{-1}$

Then by definition, $\exists C>0$ & $\delta>0$ s.t $|f(\varepsilon_{machine})| \le \dfrac{1}{(1+C\cdot\varepsilon_{machine})}$ whenever $|\varepsilon_{machine}|<\delta$.

As $1+\underbrace{C}_{>0}\cdot\underbrace{\varepsilon_{machine}}_{>0} > 1$, $\dfrac{1}{1+C\cdot\varepsilon_{machine}} < 1+C\cdot\varepsilon_{machine}$ since $(1+C\cdot\varepsilon_{machine})^2>1$.

It implies $|f(\varepsilon_{machine})| \le \dfrac{1}{(1+C\cdot\varepsilon_{machine})} \le 1+C\cdot\varepsilon_{machine}$ whenever $|\varepsilon_{machine}|<\delta$

$\rightsquigarrow$ $f(\varepsilon_{machine}) = 1+O(\varepsilon_{machine})$ as $\varepsilon_{machine}\to 0$ ∎