$\boxed{1}$ (a) $\lambda = \pm 1$

$F = I - 2vv^* \rightsquigarrow F^2 = (I-2vv^*)(I-2vv^*) = I - 4vv^* + 4v\underbrace{v^*v}_{1}v^* = I$

Let λ be eigenvalue of F & x be eigenvector. (So $x \neq 0$)

$Fx = \lambda x$, $F^2x = F(\lambda x) = \lambda Fx = \lambda^2 x \overset{\text{||}}{=} x$

$\therefore \lambda = -1$ or 1

(b) -1

$F = Q\Lambda Q^* \rightsquigarrow \det(F) = \det(\Lambda)$

$\therefore \det(F) = -1$

(c) $\sigma = 1$

By Thm 5.5, singular values of F are the absolute values of its eigenvalue.

($\because F^* = I^* - 2vv^* = I - 2vv^* = F$)

$\boxed{4}$ (a) Use trigonometric identities : $\sin(\alpha \pm \beta) = \sin\alpha\cos\beta \pm \cos\alpha\sin\beta$

$\cos(\alpha \pm \beta) = \cos\alpha\cos\beta \mp \sin\alpha\sin\beta$.

$$F\begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} -\cos\theta x + \sin\theta y \\ \sin\theta x + \cos\theta y \end{bmatrix}, \quad J\begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} \cos\theta x + \sin\theta y \\ -\sin\theta x + \cos\theta y \end{bmatrix}$$

Let $(x, y) = (r\cos\alpha, r\sin\alpha)$.

$$F\begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} r(\sin\theta\sin\alpha - \cos\theta\cos\alpha) \\ r(\sin\theta\cos\alpha + \cos\theta\sin\alpha) \end{bmatrix} = (-r\cos(\theta+\alpha), r\sin(\theta+\alpha))$$

rotating a vector by θ counterclockwisely & reflect about x-axis.

$$J\begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} r(\cos\theta\cos\alpha + \sin\theta\sin\alpha) \\ r(-\cos\alpha\sin\theta + \cos\theta\sin\alpha) \end{bmatrix} = (r\cos(\alpha-\theta), r\sin(\alpha-\theta))$$

rotating a vector by θ clockwisely.

(b) we can do the exactly same thing of QR Factorization by Givens rotation (set $\theta = \alpha$ i.e, $\cos\alpha = \frac{x}{\sqrt{x^2+y^2}}$)

$\sin\alpha = \frac{y}{\sqrt{x^2+y^2}}$

let c

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Modified Algorithm

let $G_{i,j,m-k+1}$ be

$$m-k+1\left\{\overbrace{\begin{bmatrix} 1 & & & & & & & \\ & \ddots & & & & & & \\ & & \overset{i,i}{c} & & & \overset{i,j}{s} & & \\ & & & 1 & & & & \\ & & & & \ddots & & & \\ & & \underset{j,i}{-s} & & & \overset{j,j}{c} & & \\ & & & & & & \ddots & \\ & & & & & & & 1 \end{bmatrix}}^{m-k+1}\right.$$

$$\rightsquigarrow G_{i,j,m-k+1} \cdot \begin{bmatrix} x \\ x \\ \vdots \\ a \\ \vdots \\ b \\ \vdots \\ x \end{bmatrix} = \begin{bmatrix} x \\ x \\ \vdots \\ a\cdot c + b\cdot s \\ \vdots \\ -a\cdot s + b\cdot c \\ \vdots \\ x \end{bmatrix} = \begin{bmatrix} x \\ x \\ \vdots \\ a' \\ \vdots \\ 0 \\ \vdots \\ x \end{bmatrix}$$

$$\text{So, let } Q_{k,i,j} = \left[\begin{array}{c|c} I & 0 \\ \hline 0 & G_{i,j,m-k+1} \end{array}\right]$$

$$\text{Then, desired } Q_k = \prod_{i=k+1}^{m} Q_{k,k,i} \text{ "}$$

To summarize

```
: for $k=1$ to $n$
      $x = A_{k:m,k}$   # $(m-k+1 \times 1)$
      $x' = x_1$   # $(1 \times 1)$
      for $i = 2 : m-k+1$
          $y' = x_i$
          $\cos = x' / sqrt(x'^2 + y'^2)$
          $\sin = y' / sqrt(x'^2 + y'^2)$
          $A_{[k,i],k:n} = G_{1,1,2} \cdot A_{[k,i],k:n}$
          # $2 \times (n-k+1)$   # $\begin{bmatrix} \cos & \sin \\ -\sin & \cos \end{bmatrix}$
```

```
(c) for k=1 to n
        $x = A_{k:m,k}$   #(m-k+1 x 1)
        $x' = x_1$   #(1x1)
        for i = 2 : m-k+1
                                  1     1
            $y' = x_i$
            cos = $x'/\text{sqrt}(x'^2 + y'^2)$
            sin = $y'/\text{sqrt}(x'^2 + y'^2)$
            $A_{[k,i],k:n} = G_{1,1,2} \cdot A_{[k,i],k:n}$
            #2x(n-k+1)   # $\begin{bmatrix} \cos & \sin \\ -\sin & \cos \end{bmatrix}$
```

$\cancel{\begin{matrix} \text{div}: 1 \\ \text{mult}: 2 \\ \text{add}: 1 \end{matrix}}$ dominated

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$(n-k+1) \cdot \overset{6}{2 \times (\underbrace{2}_{\text{mult}}+\underbrace{1}_{\text{add}})}$

total flops

$$\sim 6 \sum_{k=1}^{n} (m-k) \cdot (n-k+1)$$

$$\approx 6 \sum_{k=1}^{n} mn - (m+n)k + k^2$$

$$= 6mn^2 - 3(m+n) \cdot (n^2+n)$$
$$+ n(n+1)(2n+1)$$

$$\approx 6mn^2 - 3(m+n)n^2 + n \cdot n \cdot 2n$$

$$= 3mn^2 - n^3$$

As work for Householder orthogonalization is $2mn^2 - \frac{2}{3}n^3$, it has 50% greater (x1.5) works ■