

[3] Fact: Tridiagonal hermitian matrix with nonzero diagonals has distinct eigenvalues.

Now, it is done if we show  $A^T A$  is of such a form.

Let  $a_1, \dots, a_m$  be the main diagonals of  $A$  ( $a_i \neq 0, b_i \neq 0$ )  
 $b_1, \dots, b_{m-1}$  be the super ".

Then, the main diagonals of  $A^T A$ :  $(A^T A)_{kk} = \begin{cases} a_1^2, a_m^2 \\ a_k^2 + b_{k-1}^2 (2 \leq k \leq m) \end{cases}$  ■

super "

$$(A^T A)_{k,k+1} = b_k^2 \quad (1 \leq k \leq m-1)$$

sub "

$$(A^T A)_{k+1,k} = a_k^2 \quad (1 \leq k \leq m-1)$$

[4] See smallest singval.mlx.