Remo Schwarzentruber

Computer Graphics Übung 2

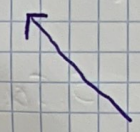
Aufgabe 1

$g = 2x + 3y - 5 = 0$

$h = 5x + 11y - 9 = 0$

Gesucht Schnittpunkt: Q

Skizze:



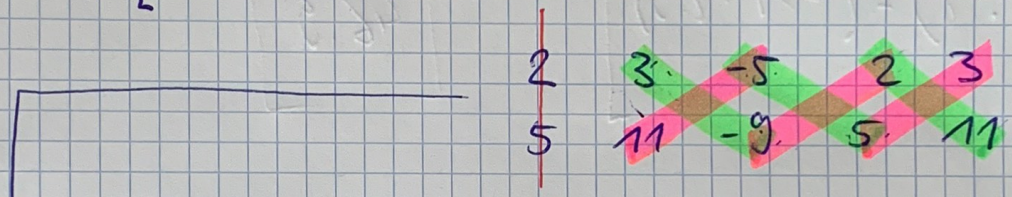
$\vec{g} \cdot \vec{r} = 0$ und $\vec{h} \cdot \vec{r} = 0$

$\Rightarrow$ somit $\vec{r} = \vec{g} \times \vec{h}$

$\vec{g} = [2, 3, -5]^T$

$\vec{h} = [5, 11, -9]^T$

Vektorprodukt: $\vec{g} \times \vec{h}$



$$\begin{bmatrix} (3 \cdot -9) - (-5 \cdot 11) \\ (-5 \cdot 5) - (2 \cdot -9) \\ (2 \cdot 11) - (3 \cdot 5) \end{bmatrix} = \begin{bmatrix} -27 + 55 \\ -25 + 18 \\ 22 - 15 \end{bmatrix} = \begin{bmatrix} 28 \\ -7 \\ 7 \end{bmatrix}$$

$\vec{r} := 28x - 7y + 7 = 0$

Um auf den Punkt zu kommen

$(x, y) = \left(\frac{x_1}{x_3}, \frac{x_2}{x_3}\right)$

$Q = (x, y) = \left(\frac{28}{7}, \frac{-7}{7}\right) \Rightarrow$

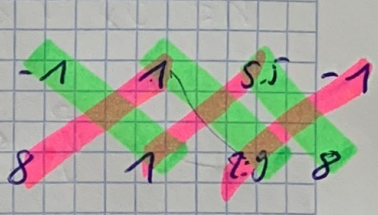
Punkt: $Q(4, -1)$

Aufgabe 2

a) A (5.5 , -1) B(2.9 , 8)

$5.5a - b + c = 0$

$2.9a + 8b + c = 0$



$$\rightarrow \begin{bmatrix} (-1 \cdot 1) - (1 \cdot 8) \\ (1 \cdot 2.9) - (5.5 \cdot 1) \\ (5.5 \cdot 8) - (-1 \cdot 2.9) \end{bmatrix} = \begin{bmatrix} -9 \\ -2.6 \\ 46.9 \end{bmatrix}$$

$$-9x - 2.6y + 46.9 = 0$$

b) weil parallel sein muss $(a_1, b_1) = (a_2; b_2)$

$g \Rightarrow 9x + 2.6y - 46.9 = 0$

Punkt C = (3, -6)

somit die neue Gerade $h = 9x + 2.6y + c = 0$

$(9 \cdot 3) + (2.6 \cdot -6) + c = 0$

$c = 11.4$

d.H. $9x + 2.6y + 11.4 = 0$

Übung 2

Aufgabe 3a) Teil 1

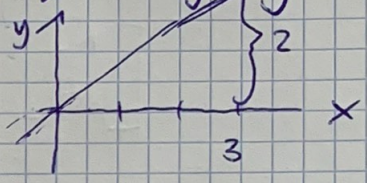
$$\vec{g} = [2, -3, 2]^T \Rightarrow g = 2x - 3y + 2 = 0$$

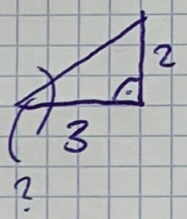
1.)

$$\begin{aligned} 2x - 3y + 2 &= 0 \quad | -2 \\ 2x - 3y &= -2 \quad | -2x \\ -3y &= -2 - 2x \quad | \cdot -1 \\ 3y &= 2 + 2x \quad | :3 \\ y &= \frac{2}{3}x + 2 \end{aligned}$$

$\Rightarrow$ gerade in $y = mx + b$

2.) Steigung





$$\tan(\alpha) = \frac{G}{A}$$

$$\tan(\alpha) = \frac{2}{3}$$

$$\alpha = \tan^{-1}\left(\frac{2}{3}\right)$$

$$\alpha = 0.588$$

$$\alpha = 33.69°$$

3) Schneidung mit x-Achse

$y = 0 \quad 2x + 2 = 0 \quad ; x = -1 \quad \vec{t} = \begin{bmatrix} -1 \\ 0 \end{bmatrix}$

4.) $T = \begin{bmatrix} 1 & 0 & -1 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \quad T^{-1} = \begin{bmatrix} 1 & 0 & -1 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$

5.) Matrix von $\sigma = T^{-1} \cdot S \cdot T$

$$\begin{bmatrix} \cos(2\sigma) & \sin(2\sigma) & 0 \\ \sin(2\sigma) & -\cos(2\sigma) & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 & 1 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} = ST = \begin{bmatrix} \cos(2\sigma) & \sin(2\sigma) & \cos(2\sigma) \\ \sin(2\sigma) & -\cos(2\sigma) & \sin(2\sigma) \\ 0 & 0 & 1 \end{bmatrix}$$

$$T^{-1} \cdot [ST] = \begin{bmatrix} 1 & 0 & -1 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} \cos(2\sigma) & \sin(2\sigma) & \cos(2\sigma) \\ \sin(2\sigma) & -\cos(2\sigma) & \sin(2\sigma) \\ 0 & 0 & 1 \end{bmatrix} =$$

3a) Teil 2

$$[\bar{T}^{-1} S \bar{T}] = \begin{bmatrix} \cos(2\delta) & \sin(2\delta) & \cos(2\delta) - 1 \\ \sin(2\delta) & -\cos(2\delta) & \sin(2\delta) \\ 0 & 0 & 1 \end{bmatrix}$$

$$\delta = \begin{bmatrix} 5/13 & 12/13 & -8/13 \\ 12/13 & -5/13 & 12/13 \\ 0 & 0 & 1 \end{bmatrix}$$

b)

$$\frac{1}{13} \begin{bmatrix} 5 & 12 & -8 \\ 12 & -5 & 12 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 8 & -65.3 \\ 1 & 0.2 \\ 1 & 1 \end{bmatrix} = \frac{1}{13} \begin{bmatrix} 44 & -332.1 \\ 105 & -772.6 \\ 13 & 13 \end{bmatrix}$$

$$= \begin{bmatrix} \frac{44}{13} & \frac{-332.1}{13} \\ \frac{105}{13} & \frac{-772.6}{13} \\ 1 & 1 \end{bmatrix}$$

$$= A' = \left(\frac{44}{13}, \frac{105}{13} \right)$$

$$= B' = \left(\frac{-332.1}{13}, \frac{-772.6}{13} \right)$$

Aufgabe 4

Uebung 2 - Aufgabe 4

In [14]:
```
H = np.array([
    [1.5,0,-1.5],
    [0,1.5,0.5],
    [0,0,1]
])

T = np.array([
    [1,0,1],
    [0,1,(-1/3)],
    [0,0,1]
])

Streckungsfaktor = 1.5

print("Resultat")
print("--------")

result = np.dot(H, T/Streckungsfaktor)
print(result.round(2))
```

```
Resultat
--------
[[1.   0.   0.  ]
 [0.   1.   0.  ]
 [0.   0.   0.67]]
```

Streckungsfaktor = 1.5

Zentrum = (1, (-1/3)

Aufgabe 5

allgemeine Form

$$H = \begin{bmatrix} h_{11} & h_{12} & h_{13} \\ h_{21} & h_{22} & h_{23} \\ h_{31} & h_{32} & h_{33} \end{bmatrix}$$

$h_{33} = 1$

Punkt:

$$(x,y) = \left(\frac{x_1}{x_3}, \frac{x_2}{x_3}\right)$$

$$\vec{r} = \begin{bmatrix} 1 & 4 & 3 & 2 \\ 1 & 1 & 4 & 4 \\ 1 & 1 & 1 & 1 \end{bmatrix}$$

A B C D

$$\vec{r}' = \begin{bmatrix} 1 & 4 & 4 & 1 \\ 1 & 1 & 4 & 4 \\ 1 & 1 & 1 & 1 \end{bmatrix}$$

A' B' C' D'

$$\vec{r}' = H \cdot \vec{r}$$

H r $\vec{r}'$

$$\begin{bmatrix} h_{11} & h_{12} & h_{13} \\ h_{21} & h_{22} & h_{23} \\ h_{31} & h_{32} & 1 \end{bmatrix} \begin{bmatrix} 1 & 4 & 3 & 2 \\ 1 & 1 & 4 & 4 \\ 1 & 1 & 1 & 1 \end{bmatrix} = \begin{bmatrix} 1 & 4 & 4 & 1 \\ 1 & 1 & 4 & 4 \\ 1 & 1 & 1 & 1 \end{bmatrix}$$

$$\begin{bmatrix} h_{11}+h_{12}+h_{13} & 4h_{11}+h_{12}+h_{13} & 3h_{11}+4h_{12}+h_{13} & 2h_{11}+4h_{12}+h_{13} \\ h_{21}+h_{22}+h_{23} & 4h_{21}+h_{22}+h_{23} & 3h_{21}+4h_{22}+h_{23} & 2h_{21}+4h_{22}+h_{23} \\ h_{31}+h_{32}+1 & 4h_{31}+h_{32}+1 & 3h_{31}+4h_{32}+1 & 2h_{31}+4h_{32}+1 \end{bmatrix}$$

Gleichungen

G1) $\frac{h_{11} + h_{12} + h_{13}}{h_{31} + h_{32} + 1} = 1 \Rightarrow h_{11} + h_{12} + h_{13} = h_{31} + h_{32} + 1$

$G1 = h_{11} + h_{12} + h_{13} - h_{31} - h_{32} = 1$

G2) $\frac{h_{21} + h_{22} + h_{23}}{h_{31} + h_{32} + 1} = 1 \Rightarrow h_{21} + h_{22} + h_{23} = h_{31} + h_{32} + 1$

$G2 = h_{21} + h_{22} + h_{23} - h_{31} - h_{32} = 1$

G3) $\frac{4h_{11} + h_{12} + h_{13}}{4h_{31} + h_{32} + 1} = 4 \Rightarrow 4h_{11} + h_{12} + h_{13} = 16h_{31} + 4h_{32} + 4$

$G3 = 4h_{11} + h_{12} + h_{13} - 16h_{31} - 4h_{32} = 4$

G4) $\frac{4h_{21} + h_{22} + h_{23}}{4h_{31} + h_{32} + 1} = 1 \Rightarrow 4h_{21} + h_{22} + h_{23} = 4h_{31} + h_{32} + 1$

$G4 = 4h_{21} + h_{22} + h_{23} - 4h_{31} - h_{32} = 1$

G5) $\frac{3h_{11} + 4h_{12} + h_{13}}{3h_{31} + 4h_{32} + 1} = 4 \Rightarrow 3h_{11} + 4h_{12} + h_{13} = 12h_{31} + 16h_{32} + 4$

$G5 = 3h_{11} + 4h_{12} + h_{13} - 12h_{31} - 16h_{32} = 4$

G6) $\frac{3h_{21} + 4h_{22} + h_{23}}{3h_{31} + 4h_{32} + 1} = 4 \Rightarrow 3h_{21} + 4h_{22} + h_{23} = 12h_{31} + 16h_{32} + 4$

$G6) \; 3h_{21} + 4h_{22} + h_{23} - 12h_{31} - 16h_{32} = 4$

G7) $\frac{2h_{11} + 4h_{12} + h_{13}}{2h_{31} + 4h_{32} + 1} = 1 \Rightarrow 2h_{11} + 4h_{12} + h_{13} = 2h_{31} + 4h_{32} + 1$

G7 $\Rightarrow 2h_{11} + 4h_{12} + h_{13} - 2h_{31} - 4h_{32} = 1$

G8) $\frac{2h_{21} + 4h_{22} + h_{23}}{2h_{31} + 4h_{32} + 1} = 4 \Rightarrow 2h_{21} + 4h_{22} + h_{23} = 8h_{31} + 16h_{32} + 4$

G8 $= 2h_{21} + 4h_{22} + h_{23} - 8h_{31} - 16h_{32} = 4$

G2 / G4 / G6 / G8

G2) $h_{21} + h_{22} + h_{23} - h_{31} - h_{32} = 1$

G4) $4h_{21} + h_{22} + h_{23} - 4h_{31} - h_{32} = 1$

G6) $3h_{21} + 4h_{22} + h_{23} - 12h_{31} - 16h_{32} = 4$

G8) $2h_{21} + 4h_{22} + h_{23} - 8h_{31} - 16h_{32} = 4$

G1) $h_{11} + h_{12} + h_{13} - h_{31} - h_{32} = 1$

G3) $4h_{11} + h_{12} + h_{13} - 16h_{31} - 4h_{32} = 4$

G5) $3h_{11} + 4h_{12} + h_{13} - 12h_{31} - 16h_{32} = 4$

G7) $2h_{11} + 4h_{12} + h_{13} - 2h_{31} - 4h_{32} = 1$

Gleichungssystem via MATRIX gelöst:

$h_{11} = \frac{9}{11}$ $h_{12} = \frac{-5}{11}$ $h_{13} = \frac{5}{11}$

$h_{21} = 0$ $h_{22} = \frac{1}{11}$ $h_{23} = \frac{8}{11}$

$h_{31} = 0$ $h_{32} = \frac{-2}{11}$ $h_{33} = 1$ (gemäss Aufgabe)

$$n = \begin{bmatrix} 9/11 & -5/11 & 5/11 \\ 0 & 1/11 & 8/11 \\ 0 & -2/11 & 1 \end{bmatrix}$$

Kontolle: via jupyter:

In [1]:
```
import numpy as np
PunkteMatrixR = np.array([
    [1,4,3,2],
    [1,1,4,4],
    [1,1,1,1]
])

perspektivischeVerzerrungMatrixN = np.array([
    [(9/11),(-5/11),(5/11)],
    [0,(1/11),(8/11)],
    [0,(-2/11),1],
])

print("Punkte Matrix Source r")
print("--------------------")
print(PunkteMatrixR)
print("\n")

print("perspektivische Verzerrung n")
print("----------------------------")
print(perspektivischeVerzerrungMatrixN.round(2))
print("\n")

print("*************")
print("* Kontrolle *")
print("*************")
homogenenMatrixRStrich = np.dot(perspektivischeVerzerrungMatrixN,PunkteMatrixR)
print("homohene Matrix")
print("---------------")
print(homogenenMatrixRStrich)
print("\n")

print("Koordinaten R'")
print("--------------")
Koordinaten = np.array([homogenenMatrixRStrich[0] / homogenenMatrixRStrich[2], homogenenMatrixRStrich[1]/homogenenMatri
print(Koordinaten)
```

```
Punkte Matrix Source r
--------------------
[[1 4 3 2]
 [1 1 4 4]
 [1 1 1 1]]


perspektivische Verzerrung n
----------------------------
[[ 0.82 -0.45  0.45]
 [ 0.    0.09  0.73]
 [ 0.   -0.18  1.  ]]


*************
* Kontrolle *
*************
homohene Matrix
---------------
[[0.81818182 3.27272727 1.09090909 0.27272727]
 [0.81818182 0.81818182 1.09090909 1.09090909]
 [0.81818182 0.81818182 0.27272727 0.27272727]]


Koordinaten R'
--------------
[[1. 4. 4. 1.]
 [1. 1. 4. 4.]]
```

Source Code jupyter Notebook:

```
import numpy as np
PunkteMatrixR = np.array([
    [1,4,3,2],
    [1,1,4,4],
    [1,1,1,1]
])

perspektivischeVerzerrungMatrixN = np.array([
    [(9/11),(-5/11),(5/11)],
    [0,(1/11),(8/11)],
    [0,(-2/11),1],
])

print("Punkte Matrix Source r")
print("--------------------")
print(PunkteMatrixR)
print("\n")

print("perspektivische Verzerrung n")
print("----------------------------")
print(perspektivischeVerzerrungMatrixN.round(2))
print("\n")

print("*************")
print("* Kontrolle *")
print("*************")
homogenenMatrixRStrich = np.dot(perspektivischeVerzerrungMatrixN,PunkteMatrixR)
print("homohene Matrix")
print("---------------")
print(homogenenMatrixRStrich)
print("\n")

print("Koordinaten R'")
print("---------------")
Koordinaten = np.array([homogenenMatrixRStrich[0] / homogenenMatrixRStrich[2],
homogenenMatrixRStrich[1]/homogenenMatrixRStrich[2]])
print(Koordinaten)
```