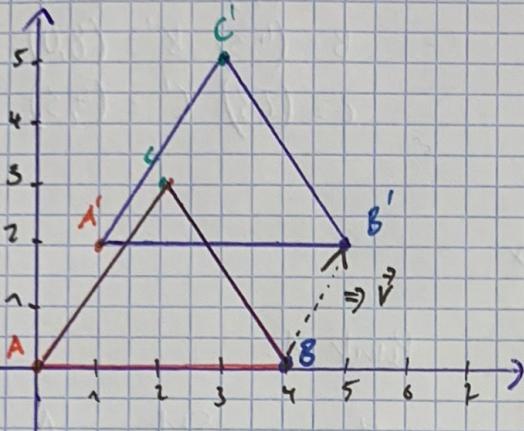


Computer Graphik . / Projective Geometric IÜbung 1Aufgabe 1.

$$A = (0,0) \quad B = (4,0) \quad C = (2,3)$$

$$\vec{v} = (1,2)^T = \vec{v} \cdot \begin{pmatrix} 1 \\ 2 \end{pmatrix}$$



$$A = \text{Ortsvektor} \quad \vec{a} = \begin{pmatrix} 0 \\ 0 \end{pmatrix}$$

$$B = \text{Ortsvektor} \quad \vec{b} = \begin{pmatrix} 4 \\ 0 \end{pmatrix}$$

$$C = \text{Ortsvektor} \quad \vec{c} = \begin{pmatrix} 2 \\ 3 \end{pmatrix}$$

$$\vec{a}' = \vec{a} + \vec{v} = \begin{pmatrix} 0 \\ 0 \end{pmatrix} + \begin{pmatrix} 1 \\ 2 \end{pmatrix} = \begin{pmatrix} 1 \\ 2 \end{pmatrix}$$

$$\vec{b}' = \vec{b} + \vec{v} = \begin{pmatrix} 4 \\ 0 \end{pmatrix} + \begin{pmatrix} 1 \\ 2 \end{pmatrix} = \begin{pmatrix} 5 \\ 2 \end{pmatrix}$$

Komponentenweise Lösung



$$\vec{c}' = \vec{c} + \vec{v} = \begin{pmatrix} 2 \\ 3 \end{pmatrix} + \begin{pmatrix} 1 \\ 2 \end{pmatrix} = \begin{pmatrix} 3 \\ 5 \end{pmatrix}$$

Matrix-Lösung kann nicht als 2×2 Matrix beschrieben werden

\rightarrow homogeneous coordinates

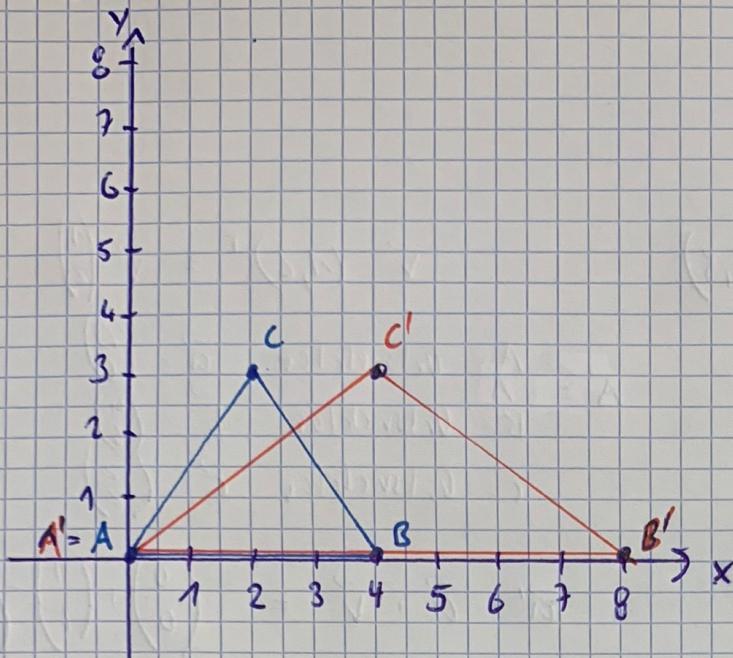
Aufgabe 2

$$\vec{a}' = \vec{a} \cdot \vec{v} = \begin{pmatrix} 0 \\ 0 \\ 0 \end{pmatrix} + \begin{pmatrix} 1 \\ 2 \\ 3 \end{pmatrix} = \begin{pmatrix} 1 \\ 2 \\ 3 \end{pmatrix} = A' = (1,2,3)$$

$$\vec{b}' = \vec{b} \cdot \vec{v} = \begin{pmatrix} 4 \\ 0 \\ 2 \end{pmatrix} + \begin{pmatrix} 1 \\ 2 \\ 3 \end{pmatrix} = \begin{pmatrix} 5 \\ 2 \\ 5 \end{pmatrix} = B' = (5,2,5)$$

$$\vec{c}' = \vec{c} \cdot \vec{v} = \begin{pmatrix} 2 \\ 3 \\ 1 \end{pmatrix} + \begin{pmatrix} 1 \\ 2 \\ 3 \end{pmatrix} = \begin{pmatrix} 3 \\ 5 \\ 4 \end{pmatrix} = C' = (3,5,4)$$

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Aufgabe 3

$$s_x = 2$$

$$s_y = 1$$

$$A = (0,0) \quad A' = (0,0)$$

$$B = (4,0) \quad B' = (8,0)$$

$$C = (2,3) \quad C' = (4,3)$$

Skalierungsmatrix = $\begin{bmatrix} 2 & 0 \\ 0 & 1 \end{bmatrix}$

Punkt A:

$$\begin{pmatrix} x' \\ y' \end{pmatrix} = \begin{pmatrix} 2 & 0 \\ 0 & 1 \end{pmatrix} \begin{pmatrix} 0 \\ 0 \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \end{pmatrix}$$

Punkt B:

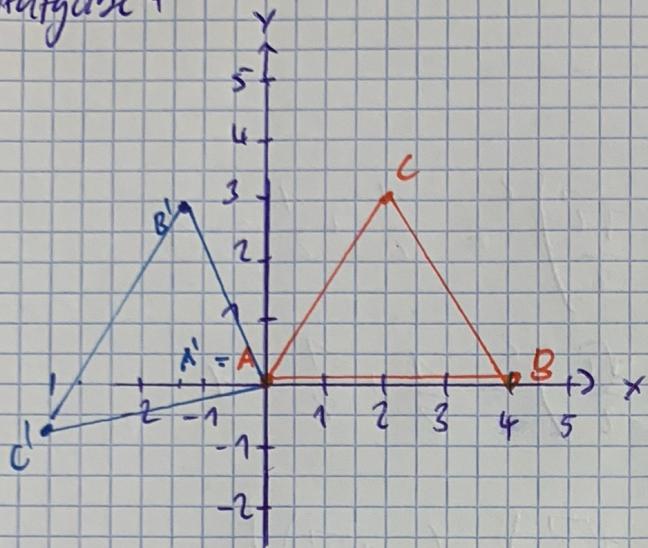
$$\begin{pmatrix} x' \\ y' \end{pmatrix} = \begin{pmatrix} 2 & 0 \\ 0 & 1 \end{pmatrix} \begin{pmatrix} 4 \\ 0 \end{pmatrix} = \begin{pmatrix} 8 \\ 0 \end{pmatrix}$$

Punkt C:

$$\begin{pmatrix} x' \\ y' \end{pmatrix} = \begin{pmatrix} 2 & 0 \\ 0 & 1 \end{pmatrix} \begin{pmatrix} 2 \\ 3 \end{pmatrix} = \begin{pmatrix} 4 \\ 3 \end{pmatrix}$$

Übung 1

Aufgabe 4



$$A = (0,0)$$

$$B = (4,0)$$

$$C = (2,3)$$

$$\sin(135^\circ) = \frac{\sqrt{2}}{2}$$

$$\cos(135^\circ) = -\frac{\sqrt{2}}{2}$$

$$\varphi = 135^\circ \text{ um } (0,0)$$

$$R = \begin{pmatrix} -\frac{\sqrt{2}}{2} & -\frac{\sqrt{2}}{2} \\ \frac{\sqrt{2}}{2} & -\frac{\sqrt{2}}{2} \end{pmatrix}$$

Drehmatrix

$$\begin{bmatrix} 0 & 4 & 2 \\ 0 & 0 & 3 \end{bmatrix}$$

schnell
folgt

$$\begin{bmatrix} -\frac{\sqrt{2}}{2} & -\frac{\sqrt{2}}{2} \\ \frac{\sqrt{2}}{2} & -\frac{\sqrt{2}}{2} \end{bmatrix} \begin{bmatrix} 0 & 4 & 2 \\ 0 & 0 & 3 \end{bmatrix} =$$

$$\begin{bmatrix} 0 & -2\sqrt{2} & -\frac{5\sqrt{2}}{2} \\ 0 & 2\sqrt{2} & -\frac{\sqrt{2}}{2} \end{bmatrix}$$

$$\text{Punkt } C' = x \cdot \left(2 \cdot -\frac{\sqrt{2}}{2} \right) + \left(3 \cdot -\frac{\sqrt{2}}{2} \right) = -\frac{5\sqrt{2}}{2}$$

$$y = 2\sqrt{2} - \frac{3\sqrt{2}}{2} = -\frac{\sqrt{2}}{2}$$

$$A' = (0,0)$$

$$B' = (-2\sqrt{2}, 2\sqrt{2})$$

$$C' = \left(-\frac{5\sqrt{2}}{2}, -\frac{\sqrt{2}}{2} \right)$$

Aufgabe 5

Rechtsmatrix R ist orthogonal

$$\Leftrightarrow \text{Zeilen} = 1$$

$$\text{Skalar} = 0$$

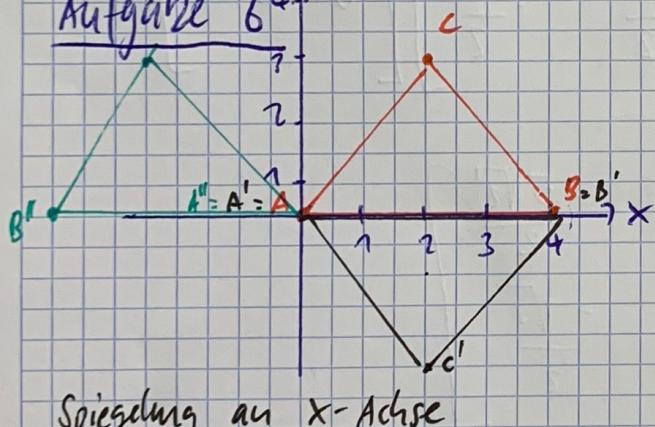
$$1. \text{ Spalte} = \left(-\frac{\sqrt{2}}{2} \right)^2 + \left(\frac{\sqrt{2}}{2} \right)^2 = \frac{1}{2} + \frac{1}{2} = 1$$

$$2. \text{ Spalte} = \left(-\frac{\sqrt{2}}{2} \right)^2 + \left(-\frac{\sqrt{2}}{2} \right)^2 = \frac{1}{2} + \frac{1}{2} = 1$$

$$\text{Skalar} : \left(\frac{-\sqrt{2}}{2} \right)^2 + \left[\left(\frac{-\sqrt{2}}{2} \right) \cdot \left(-\frac{\sqrt{2}}{2} \right) \right]$$

$$0.5 + (-0.5) = 0 \quad \text{nach sumit orthogonal} \quad \square$$

Aufgabe 6



$$\begin{aligned} A &= (0,0) \\ B &= (4,0) \\ C &= (2,3) \end{aligned}$$

an X-Achse

$$\begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix} \begin{bmatrix} 0 & 4 & 2 \\ 0 & 0 & 3 \end{bmatrix} = \begin{bmatrix} 0 & 4 & 2 \\ 0 & 0 & -3 \end{bmatrix}$$

$$\text{Spiegelungsmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix}$$

an Y-Achse

$$\begin{bmatrix} -1 & 0 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} 0 & 4 & 2 \\ 0 & 0 & 3 \end{bmatrix} = \begin{bmatrix} 0 & -4 & -2 \\ 0 & 0 & 3 \end{bmatrix}$$

Spiegelungsmatrix Y-Achse

$$SM = \begin{bmatrix} -1 & 0 \\ 0 & 1 \end{bmatrix}$$