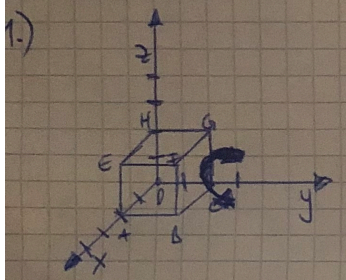
Projektive Geometrie 3

Steve Ineichen

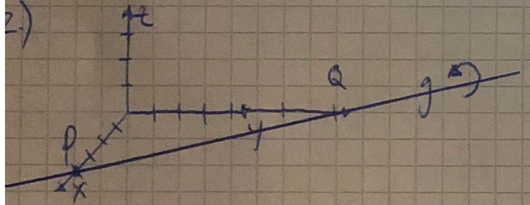
1.)



$$T = \begin{bmatrix} \cos 45° & 0 & \sin 45° & 0 \\ 0 & 1 & 0 & 0 \\ -\sin 45 & 0 & \cos 45° & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$TP = \begin{bmatrix} \frac{1}{\sqrt{2}} & 0 & \frac{1}{\sqrt{2}} & 0 \\ 0 & 1 & 0 & 0 \\ -\frac{1}{\sqrt{2}} & 0 & \frac{1}{\sqrt{2}} & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{array}{c} \begin{array}{cccccccc} A & B & C & D & E & F & G & H \end{array} \\ \begin{bmatrix} 1 & 1 & 0 & 0 & 1 & 1 & 0 & 0 \\ 0 & 1 & 1 & 0 & 0 & 1 & 1 & 0 \\ 0 & 0 & 0 & 0 & 1 & 1 & 1 & 1 \\ 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 \end{bmatrix} \end{array} = \begin{array}{cc} \begin{array}{cccccccc} A' & B' & C' & D' & E' & F' & G' & H' \end{array} & \\ \begin{bmatrix} \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} & 0 & 0 & \frac{1}{\sqrt{2}} & \frac{2}{\sqrt{2}} & \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} \\ 0 & 1 & 1 & 0 & 0 & 1 & 0 & 0 \\ -\frac{1}{\sqrt{2}} & -\frac{1}{\sqrt{2}} & 0 & 0 & 0 & 0 & -\frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} \\ 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 \end{bmatrix} & \begin{array}{l} \leftarrow x \\ \leftarrow y \\ \leftarrow z \\ \end{array} \end{array}$$

2.)



δ = Drehung $\phi = 55°$

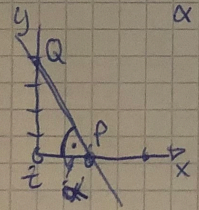
$\delta = \tau^{-1} \circ \rho^{-1} \circ n \circ \rho \circ \tau$

I: Translation null punkt: $\vec{t} \begin{bmatrix} 0 \\ -4 \\ 0 \end{bmatrix}$

$\rightarrow \tau = T \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & -4 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$

$\rightarrow \tau^{-1} = T^{-1} \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 4 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$

II: Rotation



$\alpha = \tan^{-1}\left(\frac{4}{2}\right)$ $\rho = R = \begin{bmatrix} \cos(\alpha) & -\sin(\alpha) & 0 & 0 \\ +\sin(\alpha) & \cos(\alpha) & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$

$R^{-1} = \begin{bmatrix} \cos(\alpha) & \sin(\alpha) & 0 & 0 \\ -\sin(\alpha) & \cos(\alpha) & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$

III: $n = D = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & \cos(\phi) & -\sin(\phi) & 0 \\ 0 & \sin(\phi) & \cos(\phi) & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \Rightarrow \delta = \tau^{-1} \cdot \rho^{-1} \cdot n \cdot \rho \cdot \tau \Longrightarrow \overbrace{\text{Berechnungen}}^{\delta}$ Python

```python
import numpy as np
import math

#Berechnungen Aufgabe 2 Projektive Geometrie 3
t = np.matrix([
    [1, 0, 0, 0],
    [0, 1, 0, -4],
    [0, 0, 1, 0],
    [0, 0, 0, 1]
])
tinv = np.linalg.inv(t)

a = math.atan(2)
r = np.matrix([
    [math.cos(a), -math.sin(a), 0, 0],
    [math.sin(a), math.cos(a), 0, 0],
    [0, 0, 1, 0],
    [0, 0, 0, 1]
])
rinv = np.linalg.inv(r)

phi = math.radians(55)
n = np.matrix([
    [1, 0, 0, 0],
    [0, math.cos(phi), -math.sin(phi), 0],
    [0, math.sin(phi), math.cos(phi), 0],
    [0, 0, 0, 1]
])

o = tinv * rinv * n * r * t
print(o)
```

PROBLEME 2 TERMINAL ••• 1: bash

```
(ml) Steves-MBP:exercises inux$ python exercises_3.py
[[ 0.65886115 -0.17056943 -0.73267186  0.6822777 ]
 [-0.17056943  0.91471529 -0.36633593  0.34113885]
 [ 0.73267186  0.36633593  0.57357644 -1.46534372]
 [ 0.          0.          0.          1.        ]]
```

Projektive Geometrie 3 — Steve [illegible]

3.) a) $M = \frac{1}{9}\begin{bmatrix} 1 & 4 & -8 & -28 \\ 4 & 7 & 4 & 14 \\ -8 & 4 & 1 & -28 \\ 0 & 0 & 0 & 9 \end{bmatrix}$

$A = (2|3|-5) \quad B(6.2|-4,8|-1.7)$

$A^*B^* = M \cdot AB = \frac{1}{9}\begin{bmatrix} 1 & 4 & -8 & -28 \\ 4 & 7 & 4 & 14 \\ -8 & 4 & 1 & -28 \\ 0 & 0 & 0 & 9 \end{bmatrix}\begin{bmatrix} 2 & 6.2 \\ 3 & -4.8 \\ -5 & -1.7 \\ 1 & 1 \end{bmatrix} = \frac{1}{9}\begin{bmatrix} 26 & -27.4 \\ 33 & -1.6 \\ -37 & -98.5 \\ 9 & 9 \end{bmatrix}$

$\overline{AB} = \sqrt{(6.2-2)^2 + (4,8-3)^2 + (-1.7+5)^2} = 9.4536$

$\overline{A^*B^*} = \sqrt{\left(\frac{-27.4}{9} - \frac{26}{9}\right)^2 + \left(\frac{-1.6}{9} - \frac{33}{9}\right)^2 + \left(\frac{-98.5}{9} + \frac{37}{9}\right)} = 9,4536$

$\overline{AB} = \overline{A^*B^*}$

b) $\frac{1}{9}\begin{bmatrix} 1 & 4 & -8 & -28 \\ 4 & 7 & 4 & 14 \\ -8 & 4 & 1 & -28 \\ 0 & 0 & 0 & 9 \end{bmatrix}\begin{bmatrix} x_0 & 0 & 0 \\ 0 & y_0 & 0 \\ 0 & 0 & z_0 \\ 1 & 1 & 1 \end{bmatrix} = \begin{bmatrix} ax_0 & 0 & 0 \\ 0 & by_0 & 0 \\ 0 & 0 & cz_0 \\ a & b & c \end{bmatrix}$

$a = b = c = 9$

$\frac{1}{9}x_0 - \frac{28}{9} = 9x_0$

$\Rightarrow x_0 - 28 = 81x_0$

$\Rightarrow 80x_0 = -28$

$\Rightarrow x_0 = \frac{-28}{80} = \frac{-7}{20}$

$\frac{7}{9}y_0 + \frac{14}{9} = 9y_0$

$7y_0 - 14 = 81y_0$

$74y_0 = +14$

$y_0 = \frac{7}{37}$

$\frac{1}{9}z_0 + \frac{28}{9} - 9z_0 \Rightarrow z_0 = \frac{-7}{20}$

$\varepsilon = \frac{-7}{20}x + \frac{7}{37}y - \frac{7}{20}z + 1 = 0$

4) a) $\vec{v} = (1,1,0)^T$

$|\vec{v}| = \frac{1}{\sqrt{1^2+1^2+0^2}} = \frac{1}{\sqrt{2}}$

x-y Ebene $\Rightarrow$ $a=0$ $b=1$ $c=0$ $d=0$

$\cos(\varphi) = \frac{\vec{u}\cdot\vec{v}}{|\vec{u}|\cdot|\vec{v}|} = \frac{1}{\sqrt{2}} = \frac{\sqrt{2}}{2} \Rightarrow \arccos\left(\frac{\sqrt{2}}{2}\right) = 45°$

$$H = \sqrt{2}\begin{bmatrix} \frac{1}{\sqrt{2}} & -\frac{1}{\sqrt{2}} & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & \frac{1}{\sqrt{2}} & 0 \\ 0 & 0 & 0 & \frac{1}{\sqrt{2}} \end{bmatrix}$$

b) $H \cdot P = P^*$

$$\begin{bmatrix} 1 & -1 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{array}{c} \begin{array}{cccccccc} A & B & C & D & E & F & G & H \end{array} \\ \begin{bmatrix} 0 & 6 & 0 & -6 & 3 & 9 & 3 & -3 \\ 0 & 6 & 9 & 3 & -6 & 0 & 3 & -3 \\ 0 & 3 & 9 & 6 & 6 & 9 & 15 & 12 \\ 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 \end{bmatrix} \end{array}$$

$$= \begin{array}{c} \begin{array}{cccccccc} A^* & B^* & C^* & D^* & E^* & F^* & G^* & H^* \end{array} \\ \begin{bmatrix} 0 & 0 & -9 & -9 & 9 & 9 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 3 & 9 & 6 & 6 & 9 & 15 & 12 \\ 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 \end{bmatrix} \end{array}$$

5.) a.) $a=0$ $b=0$ $c=1$ $d=0$ $\vec{z}\begin{bmatrix} 2 \\ 4 \\ -3 \end{bmatrix}$

1. Translation T um $\vec{t} = -\vec{z}$
2. Perspektive Transformation
3. Translation T^{-1}

$$T = \begin{bmatrix} 1 & 0 & 0 & -2 \\ 0 & 1 & 0 & -4 \\ 0 & 0 & 1 & 3 \\ 0 & 0 & 0 & 1 \end{bmatrix} \quad T^{-1} = \begin{bmatrix} 1 & 0 & 0 & 2 \\ 0 & 1 & 0 & 4 \\ 0 & 0 & 1 & -3 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

Ebene: $0(x+2) + 0(y+4) + 1(z-3) + 0 = 0$

$\Rightarrow z - 3 = 0$

$a' = 0$ $b' = 0$ $c' = 1$ $d' - 3$

$$M = \begin{bmatrix} 3 & 0 & 0 & 0 \\ 0 & 3 & 0 & 0 \\ 0 & 0 & 3 & 0 \\ 0 & 0 & 1 & 0 \end{bmatrix}$$

$H = T^{-1}MT$

$$MT = \begin{bmatrix} 3 & 0 & 0 & 0 \\ 0 & 3 & 0 & 0 \\ 0 & 0 & 3 & 0 \\ 0 & 0 & 1 & 0 \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 & -2 \\ 0 & 1 & 0 & -4 \\ 0 & 0 & 1 & 3 \\ 0 & 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} 3 & 0 & 0 & -6 \\ 0 & 3 & 0 & -12 \\ 0 & 0 & 3 & 9 \\ 0 & 0 & 1 & 3 \end{bmatrix}$$

$$T^{-1}(MT) = \begin{bmatrix} 1 & 0 & 0 & 2 \\ 0 & 1 & 0 & 4 \\ 0 & 0 & 1 & -3 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 3 & 0 & 0 & -6 \\ 0 & 3 & 0 & -12 \\ 0 & 0 & 3 & 9 \\ 0 & 0 & 1 & 3 \end{bmatrix} = \begin{bmatrix} 3 & 0 & 2 & 0 \\ 0 & 3 & 4 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 3 \end{bmatrix}$$

b)

$$P^* = HP = \begin{bmatrix} 3 & 0 & 2 & 0 \\ 0 & 3 & 4 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 3 \end{bmatrix} \begin{bmatrix} 0 & 6 & 0 & -6 & 3 & 9 & 3 & -3 \\ 0 & 6 & 9 & 3 & -6 & 0 & 3 & -3 \\ 0 & 3 & 9 & 6 & 6 & 9 & 15 & 12 \\ 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 \end{bmatrix} = \begin{bmatrix} 0 & 24 & 18 & -6 & 21 & 45 & 39 & 15 \\ 0 & 30 & 63 & 33 & 6 & 36 & 65 & 39 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 3 & 6 & 12 & 9 & 9 & 12 & 18 & 15 \end{bmatrix}$$

Projektive Geometrie 3 Steve Kerchen

6.) a) $a = 5, \quad b = 10$

$x_Q = 3, \quad x_P = -3$

$y_Q = 3, \quad y_P = -2$

$$T = \begin{bmatrix} 10/6 & 0 & 0 & 0 \\ 0 & 2 & 1/5 & 0 \\ 0 & 0 & -3 & -20 \\ 0 & 0 & -1 & 0 \end{bmatrix}$$

b) $$\begin{bmatrix} 10/6 & 0 & 0 & 0 \\ 0 & 2 & 1/5 & 0 \\ 0 & 0 & -3 & -20 \\ 0 & 0 & -1 & 0 \end{bmatrix} \begin{bmatrix} -4.5 & 1.6 \\ 4.2 & -2.4 \\ -8 & -5.5 \\ 1 & 1 \end{bmatrix} = \begin{bmatrix} -7.5 & 8/3 \\ 6.8 & -5.9 \\ 4 & 3.5 \\ 8 & 5.5 \end{bmatrix}$$

$A'(-0.9375, 0.85, 0.5)$ $\quad -1 \leq x_{A'}, y_{A'}, z_{A'} \leq 1$. A' ligt innerha

$B'(0.4848, -1.0027, 0.6364)$ $\quad y_{B'} > 1$. B' ligt ausserhal