The goal of this library is to model units of measure and manipulation of quantities associated to units of measure.

# Day 1: fundamental units

To understand this one need to know bit about the metric system and dimensional equation. Please read <https://en.wikipedia.org/wiki/Metric_system> and <https://en.wikipedia.org/wiki/Dimensional_analysis>.

We use the 7 base units of the SI (<https://en.wikipedia.org/wiki/International_System_of_Units>) as a starting point: these are the FundamentalMeasureUnit. A fundamental unit is semantically equivalent (for us) to a dimension. We add 3 fundamental units (dimension) and new ones can be dynamically added to the system as required (for instance “$” and/or “€”, “£”, etc.). Note that each currency would be a dimension. Converting a quantity of “$” to “€” is not in the scope of this library.

The 3 fundamental units we add are:

/// <summary>

/// Dimensionles unit. Associated abbreviation is "" (the empty string) and its

/// name is "None".

/// </summary>

public static readonly FundamentalMeasureUnit None;

/// <summary>

/// Dimensionless unit. Used to count items. Associated abbreviation is "#".

/// </summary>

public static readonly FundamentalMeasureUnit Unit;

/// <summary>

/// A bit is defined as the information entropy of a binary random variable

/// that is 0 or 1 with equal probability.

/// Associated abbreviation is "b" (recommended by the IEEE 1541-2002 and

/// IEEE Std 260.1-2004 standards).

/// </summary>

MeasureUnit

NormalizedMeasureUnit

BasicMeasureUnit

FundamentalMeasureUnit

public static readonly FundamentalMeasureUnit Bit;

These fundamental units actually are BasicMeasureUnit with their Exponent set to 1. The BasicMeasureUnit are bound to a FundamentalMeasureUnit with an exponent: they handle the basic items of a dimensional equation m^2, s^-1, etc.

NormalizedMeasureUnit captures a list of one or more BasicMeasureUnit. The list is normalized by decreasing exponents and then by fundamental unit’s name lexicographic order.

Figure 1- Class hierarchy so far.

With this simple model and the help of C# operators overload (\*, / and ^), one can achieve this kind of funny dimensional game (from <https://en.wikipedia.org/wiki/SI_derived_unit>):

var metre = MeasureUnit.Metre;

var second = MeasureUnit.Second;

var kilogram = MeasureUnit.Kilogram;

var ampere = MeasureUnit.Ampere;

var candela = MeasureUnit.Candela;

var squaredMeter = metre^2;

var hertz = MeasureUnit.None / second;

hertz.Abbreviation.Should().Be( "s-1" );

var rad = metre \* (metre ^ -1);

rad.Should().BeSameAs( MeasureUnit.None );

var steradian = squaredMeter / squaredMeter;

steradian.Should().BeSameAs( MeasureUnit.None );

var newton = kilogram \* metre \* (second ^ -2);

newton.Abbreviation.Should().Be( "kg.m.s-2" );

var pascal = newton / squaredMeter;

pascal.Abbreviation.Should().Be( "kg.m-1.s-2" );

var joule = newton \* metre;

var watt = joule / second;

var coulomb = ampere \* second;

var volt = watt / ampere;

var volt2 = joule / coulomb;

volt.Should().BeSameAs( volt2 );

volt.Abbreviation.Should().Be( "m2.kg.A-1.s-3" );

var farad = coulomb / volt;

farad.Abbreviation.Should().Be( "s4.A2.kg-1.m-2" );

var ohm = volt / ampere;

ohm.Abbreviation.Should().Be( "m2.kg.A-2.s-3" );

var farad2 = second / ohm;

farad2.Should().BeSameAs( farad );

var siemens = MeasureUnit.None / ohm;

var siemens2 = ampere / volt;

siemens.Should().BeSameAs( siemens2 );

siemens.Abbreviation.Should().Be( "s3.A2.kg-1.m-2" );

var weber = joule / ampere;

var tesla = volt \* second / squaredMeter;

var tesla2 = weber / squaredMeter;

var tesla3 = newton / (ampere \* metre);

tesla2.Should().BeSameAs( tesla );

tesla3.Should().BeSameAs( tesla );

tesla.Abbreviation.Should().Be( "kg.A-1.s-2" );

var henry = ohm \* second;

var henry2 = volt \* second / ampere;

var henry3 = weber / ampere;

henry2.Should().BeSameAs( henry );

henry3.Should().BeSameAs( henry );

henry.Abbreviation.Should().Be( "m2.kg.A-2.s-2" );

var lumen = candela \* steradian;

var lux = lumen / squaredMeter;

lux.Abbreviation.Should().Be( "cd.m-2" );

Important note: the code above shows that the objects are actually the same object (reference equality) when they define the same unit of measure. This is one of the goal of this library: units must be fully normalized and cached even if new unit of measure can be dynamically created at any time. This is achieved thanks to a ConcurrentDictionary, the “Abbreviation” property of the measures being the key.

Even if it works like a charm and is funny, this is not enough. We must handle:

* Prefixes: standard ones like “d”/”Deci” like in “dm”/”Decimeter” or “m”/”Milli” like in “mm”/”Millimeter”. See <https://en.wikipedia.org/wiki/Metric_prefix> and since we added bit, we should also handle binary prefixes (see <https://en.wikipedia.org/wiki/Binary_prefix>): “Ki”/”Kibbi” like “Kib”/”KibiBit”.

Introducing this is not as easy as it seems. We must be able to transparently handle equivalence like, for instance, the fact that m.s-2 (metre per squared second, ie. an acceleration) is the same as μm.ms-2 (micrometer per squared milliseconds).

* Derived units: a Derived Unit is like an alias, for instance “l”/”Liter” is the same as “dm3”, ie. 10-3 m3. Some of the actually used units introduce a factor (that does not fit into a simple exponentiation like the liter).

For instance (from <https://en.wikipedia.org/wiki/Newton_(unit)>):

* + A newton (N): its definition directly uses fundamental units: 1 N = 1 kg.m.s-2
  + A dyne (dyn) is defined with the newton: 1 dyn = 10-5 N
  + A kilopond (kp) is: 1 kp = 9.80665 N

This definitely requires a more complex model.

# Day 2: Handling prefixes.

Let’s start with standard and binary prefixes. These prefixes (“G”/”Giga”, “k”/”Kilo”, “K”/”Kibi”, “m”/”Milli” etc.) apply only to non-exponentiated units like the fundamental units and can be used wherever a fundamental unit is used.

A deci(squared metre) is NOT Valid, however a squared( decimetre ) is. To properly model this, we refine a little bit our model by introducing a PrefixedMeasureUnit at the same (lowest) level as the FundamentalMeasureUnit.

To unify these two types we introduce a new AtomicMeasureUnit class that is the base class of FundamentalMeasureUnit and PrefixedMeasureUnit.

The NormalizedMeasureUnit now handles one or more (possibly exponentiated) fundamental units as well as PrefixedMeasureUnit that are not “fundamentals”. Its name (“normalized”) is no more accurate: we change it to just be a CombinedMeasureUnit that better reflects what it is.

Since we are dealing with (bad) names, the “Basic measure unit” does not convey a clear idea of what it is. We rename the BasicMeasureUnit to be ExponentMeasureUnit.

The new class diagram is:

MeasureUnit

CombinedMeasureUnit

AtomicMeasureUnit

FundamentalMeasureUnit

PrefixedMeasureUnit

ExponentMeasureUnit

Figure 2- Introducing Prefixes.

The standard S.I. prefixes (all of them) are captured and exposed by the MeasureStandardPrefix class through a set of singletons. This set is not extensible (you can’t define your own prefix).

These prefixes can be applied to any AtomicMeasureUnit to obtain another AtomicMeasureUnit since the result of the prefix application may actually be a prefixed unit or a fundamental unit.

var centimetre = MeasureStandardPrefix.Centi.On( MeasureUnit.Metre );  
centimetre.Abbreviation.Should().Be( "cm" );  
centimetre.Name.Should().Be( "Centimetre" );

If we apply the Hecto prefix to this centimetre, we obtain the (fundamental) metre.

var hectocentimetre = MeasureStandardPrefix.Hecto.On( centimetre );

hectocentimetre.Should().BeSameAs( MeasureUnit.Metre );

Below is the whole test for this new game:

[Test]

public void playing\_with\_decimetre\_and\_centimeter()

{

var decimetre = MeasureStandardPrefix.Deci.On( MeasureUnit.Metre );

decimetre.Abbreviation.Should().Be( "dm" );

decimetre.Name.Should().Be( "Decimetre" );

var decimetreCube = decimetre ^ 3;

decimetreCube.Abbreviation.Should().Be( "dm3" );

decimetreCube.Name.Should().Be( "Decimetre^3" );

// This does'nt compile and this is perfect! :)

//var notPossible = MeasureStandardPrefix.Deci.On( decimetreCube );

var centimetre = MeasureStandardPrefix.Centi.On( MeasureUnit.Metre );

centimetre.Abbreviation.Should().Be( "cm" );

centimetre.Name.Should().Be( "Centimetre" );

var decidecimetre = MeasureStandardPrefix.Deci.On( decimetre );

decidecimetre.Should().BeSameAs( centimetre );

var hectocentimetre = MeasureStandardPrefix.Hecto.On( centimetre );

hectocentimetre.Should().BeSameAs( MeasureUnit.Metre );

var kilocentimeter = MeasureStandardPrefix.Kilo.On( centimetre );

kilocentimeter.Abbreviation.Should().Be( "dam" );

var decametre = MeasureStandardPrefix.Deca.On( MeasureUnit.Metre );

decametre.Should().BeSameAs( decametre );

}

To conclude this second day, one need to consider a not-so-edge-case: standard prefixes are not “complete” enough to be safely used. What happens if you want the “Deci” (10-1) of a “Giga” (109)? There is no prefix available for 108. Similar question: what is a Kilo Yotta (Yotta is the biggest prefix: 1024).

Note that you may not ask it directly, but this may happen indirectly in a complex system. We must be able to cope with these cases, even if we’ll always try to ***eventually*** avoid such “intermediate” prefixes.

The idea is to introduce an “adjustment factor” to our PrefixedMeasureUnit. This adjustment is a ExpFactor that enables us to handle exponents without relying on floating point types (and their intrinsic limitations).

/// <summary>

/// Immutable value type that captures 10^<see cref="Exp10"/>.2^<see cref="Exp2"/>.

/// </summary>

public struct ExpFactor : IComparable<ExpFactor>, IEquatable<ExpFactor>

{

public static readonly ExpFactor Neutral; // The neutral factor (0,0).

public readonly int Exp2; // The base 2 exponent.

public readonly int Exp10; // The base 10 exponent.

public ExpFactor Power( int p )

=> new ExpFactor( Exp2 \* p, Exp10 \* p );

public ExpFactor Multiply( ExpFactor x )

=> new ExpFactor( Exp2 + x.Exp2, Exp10 + x.Exp10 );

public ExpFactor DivideBy( ExpFactor x )

=> new ExpFactor( Exp2 - x.Exp2, Exp10 - x.Exp10 );

}

Note: There is a little bit more code in the struct above, we only show here the most relevant aspects.

There are two exponents: one in base 2 (since we support binary prefixes) and the other in base 10 for matric prefixes.

This allows us to generate “intermediate prefixes” or “out-of-bound” prefixes as required and keep the whole system safe and coherent as long as we find a way to express/show/recognize these beasts.

If an adjustment factor is not the neutral one, it appears in the abbreviation and name of the unit of measure: “(10^-1)Gm”/”(10^-1)Gigameter” will be the “Decigigametre” that do not exist. Of course, you will never meet the “(10^-1)cm” since this is a “mm”/”Millimetre”!

Note the syntax: we have deliberately chosen a syntax that differs from the standard exponent (with the base and the caret ^) so that these beasts can easily been spotted.

Here are the tests that conclude this Day 2:

[Test]

public void playing\_with\_adjustment\_factors()

{

var gigametre = MeasureStandardPrefix.Giga[MeasureUnit.Metre];

gigametre.Abbreviation.Should().Be( "Gm" );

gigametre.Name.Should().Be( "Gigametre" );

var decigigametre = MeasureStandardPrefix.Deci[gigametre];

decigigametre.Abbreviation.Should().Be( "(10^-1)Gm" );

decigigametre.Name.Should().Be( "(10^-1)Gigametre" );

// Instead of "(10^-2)Gigametre", we always try to minimize the absolute value   
// of the adjustement factor: here we generate the "(10^1)Megametre".

var decidecigigametre = MeasureStandardPrefix.Deci[decigigametre];

decidecigigametre.Abbreviation.Should().Be( "(10^1)Mm" );

decidecigigametre.Name.Should().Be( "(10^1)Megametre" );

var decidecidecigigametre = MeasureStandardPrefix.Deci[decidecigigametre];

decidecidecigigametre.Abbreviation.Should().Be( "Mm" );

decidecidecigigametre.Name.Should().Be( "Megametre" );

}

[Test]

public void out\_of\_bounds\_adjustment\_factors()

{

var yottametre = MeasureStandardPrefix.Yotta[MeasureUnit.Metre];

var lotOfMetre = MeasureStandardPrefix.Hecto[yottametre];

lotOfMetre.Abbreviation.Should().Be( "(10^2)Ym" );

var evenMore = MeasureStandardPrefix.Deca[lotOfMetre];

evenMore.Abbreviation.Should().Be( "(10^3)Ym" );

var backToReality = MeasureStandardPrefix.Yocto[evenMore];

backToReality.Abbreviation.Should().Be( "km" );

var belowTheAtom = MeasureStandardPrefix.Yocto[backToReality];

belowTheAtom.Abbreviation.Should().Be( "zm" );

belowTheAtom.Name.Should().Be( "Zeptometre", "The Zeptometre is 10^-21 metre." );

var decizettametre = MeasureStandardPrefix.Deci[belowTheAtom];

decizettametre.Abbreviation.Should().Be( "(10^-1)zm" );

var decidecizettametre = MeasureStandardPrefix.Deci[decizettametre];

decidecizettametre.Abbreviation.Should().Be( "(10^1)ym" );

var yoctometre = MeasureStandardPrefix.Deci[decidecizettametre];

yoctometre.Abbreviation.Should().Be( "ym" );

var below1 = MeasureStandardPrefix.Deci[yoctometre];

below1.Abbreviation.Should().Be( "(10^-1)ym" );

var below2 = MeasureStandardPrefix.Deci[below1];

below2.Abbreviation.Should().Be( "(10^-2)ym" );

}

# Day 3: Handling aliases

It is time now to handle aliases and eventually find a way to normalize units so that we can actually use them to compute quantities.

## The kilogram exception and the byte

Currently, there is no way to manipulate grams since for the moment a gram is a Milli Kilogram: Kilogram is the official standard unit of weight. You don’t want so see the “mkg” unit!

Introducing an alias for the “mkg” (as a “g”/”Gram”) would imply to cope with this exception in too much code location. It is definitely easier to “cheat” and define the Gram as being the fundamental unit instead of the Kilogram. The Kilogram is now defined in the MeasureUnit type initializer as a PrefixedMeasureUnit (and still exposed as a static property):

/// <summary>

/// The kilogram is the unit of mass; it is equal to the mass of the international

/// prototype of the kilogram.

/// This is the only SI base unit that incluedes a prefix. To avoid coping with this

/// exception in the code, we

/// define it as a PrefixedMeasureUnit based on the gram (MeasureStandardPrefix.Kilo

/// of Gram).

/// Associated abbreviation is "kg".

/// </summary>

public static readonly PrefixedMeasureUnit Kilogram;

/// <summary>

/// The gram is our fundamental unit of mass (see Kilogram).

/// Associated abbreviation is "g".

/// </summary>

public static readonly FundamentalMeasureUnit Gram;

Kilogram = RegisterPrefixed( ExpFactor.Neutral, MeasureStandardPrefix.Kilo, Gram );

Byte is also exposed as a static field on the MeasureUnit class. It uses the new AliasMeasureUnit class that now enables us to create new named units from other ones:

/// <summary>

/// A byte is now standardized as eight bits, as documented in ISO/IEC 2382-1:1993.

/// The international standard IEC 80000-13 codified this common meaning.

/// Associated abbreviation is "B" and it is an alias with a ExpFactor 2^3 on Bit.

/// </summary>

public static readonly AliasMeasureUnit Byte;

Byte = new AliasMeasureUnit( "B", "Byte", new FullFactor( new ExpFactor(3,0) ), Bit );

## Defining new units

The new alias and the original fundamental units are the only two ways to explicitly define new unit of measure:

/// <summary>

/// Defines an alias.

/// The same alias can be registered multiple times but it has to exactly match the

/// previously registered one.

/// </summary>

/// <param name="abbreviation">

/// The unit of measure abbreviation.

/// This is the key that is used. It must not be null or empty.

/// </param>

/// <param name="name">The full name. Must not be null or empty.</param>

/// <param name="definitionFactor">

/// The factor that applies to the AliasMeasureUnit.Definition. Must not be

/// FullFactor.Zero.

/// </param>

/// <param name="definition">The definition. Can be any CombinedMeasureUnit.</param>

/// <returns>The alias unit of measure.</returns>

public static AliasMeasureUnit DefineAlias(

string abbreviation,

string name,

FullFactor definitionFactor,

CombinedMeasureUnit definition ) { … }

/// <summary>

/// Define a new fundamental unit of measure (or returns the already defined one).

/// Just like DefineAlias, the same fundamental unit can be redefined multiple times

/// as long as it is actually the same: for fundamental units, the (long) name

/// must be exaclty the same.

/// </summary>

/// <param name="abbreviation">

/// The unit of measure abbreviation.

/// This is the key that is used. It must not be null or empty.

/// </param>

/// <param name="name">The full name. Must not be null or empty.</param>

/// <returns>The fundamental unit of measure.</returns>

public static FundamentalMeasureUnit DefineFundamental(

string abbreviation,

string name ) { … }

The FullFactor that appears in the DefineAlias is a simple extension to the ExpFactor previously described that adds a simple Factor field to it. We use a double for the moment but what we should use here is a rational such as the one provided by <https://www.nuget.org/packages/Rationals/> (that seems a nice project).

A FullFactor describes a simple linear adjustment of the definition units. Thanks to it we can now define the newton, the dyne and the kilopound (from <https://en.wikipedia.org/wiki/Newton_(unit)>):

* A newton (N): its definition directly uses fundamental units: 1 N = 1 kg.m.s-2
* A dyne (dyn) is defined with the newton: 1 dyn = 10-5 N
* A kilopond (kp) is: 1 kp = 9.80665 N

using static CK.Core.MeasureUnit;

…

var kg = Kilogram;

var m = Metre;

var s = Second;

var newton = DefineAlias( "N", "Newton", FullFactor.Neutral, kg \* m \* (s ^ -2) );

var dyne = DefineAlias( "dyn", "Dyne", new ExpFactor( 0, -5 ), newton );

var kilopound = DefineAlias( "kp", "Kilopound", 9.80665, newton );

var newtonPerDyne = newton / dyne;

newtonPerDyne.Abbreviation.Should().Be( "N.dyn-1" );

Note:

* To shorten the code above, we have used the using static CK.Core.MeasureUnit;
* FullFactor defines implicit conversion operators from double and ExpFactor. We could have defined the newton in an even more simple way:

var newton = DefineAlias( "N", "Newton", 1.0, kg \* m \* (s ^ -2) );

The new class diagram is:

MeasureUnit

CombinedMeasureUnit

AtomicMeasureUnit

FundamentalMeasureUnit

PrefixedMeasureUnit

ExponentMeasureUnit

AliasMeasureUnit

Figure 3- Introducing Aliases.

So far so good… but… what is the actual unit of “N.dyn-1”?

You may be surprised but it could be “radian” (or “steradian“) since this has actually no unit: it is Measure.None (just like radian and steradian that are not actual units).

We are now ready to use these units to *calculate* quantities!

# Entering the unit normalization, starting calculation

A Quantity is simply a numerical value (int, float, double, rational, big integer, etc.) associated to its unit od measure.

One of the goal of this library is to help computing such quantities like:

The value of r is 2.105. That is 2.105 MeasureUnit.None for us: the N.dyn-1 must resolve to the dimensionless unit **and** to the factor that link them.

Before manipulating quantities, they must be first promoted/aligned to the same measure of units. You can always multiply/divide quantities (whatever their dimensions are) by simply creating the resulting dimension: 10 m x 40 min = 400 m.min

However, to add or subtract 2 quantities hey must have exactly the same unit: 2 m2 + 3 m2 = 5m2 or 2 m2 + 3 cm2 that is perfectly valid and equals to 20003 cm2 (or 2.0003 m2 as you prefer).

Before tackling this aspect, we need a little bit of refactoring. Current model has the MeasureUnit as a base abstract class of all units. However, the only actual specialization is the CombinedMeasureUnit and it appears that this is all what is needed (this was not obvious on day 1, this indirection seemed useful at this time). We can now simplify the model by acting that a MeasureUnit is always a CombinedMeasureUnit. The latter is gone, merged into the MeasureUnit (that is getting bigger) and to keep code clean it is split in partial files.

The final class diagram is:

MeasureUnit

AtomicMeasureUnit

FundamentalMeasureUnit

PrefixedMeasureUnit

ExponentMeasureUnit

AliasMeasureUnit

Figure 4- Final class diagram.

We could have merged Exponent and Atomic, but we did not and don’t want to. For 2 reasons:

* This (see Day 1) is currently not possible and we want to keep this:

var decimetreCube = decimetre ^ 3;

decimetreCube.Abbreviation.Should().Be( "dm3" );

decimetreCube.Name.Should().Be( "Decimetre^3" );

// This does'nt compile and this is perfect! :)

//var notPossible = MeasureStandardPrefix.Deci.On( decimetreCube );

* This would make the code less “type safe” and hence a little bit more “complex” to write and to understand.

Refactor done. Let’s go back to the task of the day: the normalization.

There is here an obvious choice: the FundamentalMeasureUnit is by design the canonical form of each and every mono-dimensional unit of measure. When multiple dimensions are involved, there is one composite MeasureUnit that contains only atomic FundamentalMeasureUnit (i.e. no more aliases or prefixed units).

From any MeasureUnit to its Canonical form, we can calculate (once for all) the FullFactor that maps the unit to it.

Introducing a new class is useless for this. It is easier to introduce this into our model by adding 2 properties to the MeasureUnit base class:

/// <summary>

/// Gets whether this MeasureUnits only contains normalized units.

/// </summary>

public bool IsNormalized { get; }

/// <summary>

/// Gets the factor that must be applied from this measure to its Normalization.

/// </summary>

public FullFactor NormalizationFactor { get; }

/// <summary>

/// Gets the canonical form of this measure.

/// Its IsNormalized property is necessarily true.

/// </summary>

public MeasureUnit Normalization { get; }

And it works!

[Test]

public void equivalent\_combined\_units()

{

var metre = MeasureUnit.Metre;

var second = MeasureUnit.Second;

var acceleration = metre / (second ^ 2);

acceleration.IsNormalized.Should().BeTrue();

var micrometre = MeasureStandardPrefix.Micro[metre];

var millisecond = MeasureStandardPrefix.Milli[second];

var acceleration2 = micrometre / (millisecond ^ 2);

acceleration2.IsNormalized.Should().BeFalse();

acceleration2.Normalization.Should().BeSameAs( acceleration );

acceleration2.NormalizationFactor.Should().Be( FullFactor.Neutral );

}

[Test]

public void combined\_units\_with\_factor()

{

var metre = MeasureUnit.Metre;

var second = MeasureUnit.Second;

var speed = metre / second;

speed.IsNormalized.Should().BeTrue();

var kilometre = MeasureStandardPrefix.Kilo[metre];

var hour = MeasureUnit.DefineAlias( "h", "Hour", 60\*60, second );

var speed2 = kilometre / hour;

speed2.IsNormalized.Should().BeFalse();

speed2.Normalization.Should().BeSameAs( speed );

speed2.NormalizationFactor.ToDouble()

.Should().BeApproximately( 0.2777777778, 1e-10, "1 m/s = 0.277778 km/h" );

}

We have no more time today (day 4) to detail the (minimalist) code that does this “magic” (what is actually magic is how minimal it is 😊). We’ll talk about this tomorrow.