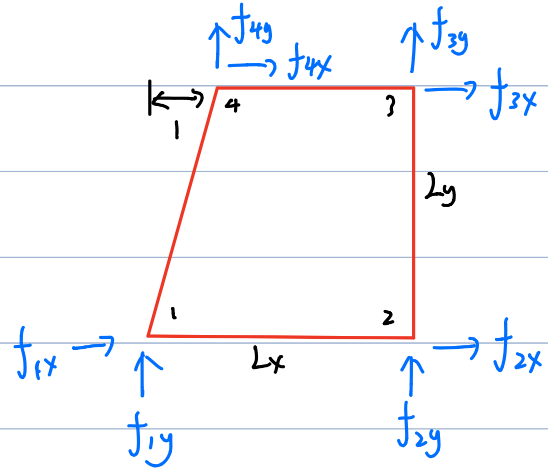


$$
\begin{bmatrix} f_{1x} \\ f_{1y} \\ f_{2x} \\ f_{2y} \\ f_{3x} \\ f_{3y} \\ f_{4x} \\ f_{4y} \end{bmatrix}
=
\overset{8\times 8}{\begin{bmatrix}
\underbrace{\begin{matrix} 0 & \cdots & 0 \\ \vdots & \ddots & \vdots \\ 0 & \cdots & 0 \end{matrix}}_{1\sim 6} &
\begin{matrix} -\frac{C_3}{3r}+\frac{C_1 r}{6} \\ \frac{C_2-C_3}{4} \\ \frac{-(C_3+C_1r^2)}{6r} \\ \frac{C_2+C_3}{4} \\ \frac{C_3-2C_1r^2}{6r} \\ \frac{C_3-C_2}{4} \\ \frac{C_3+C_1r^2}{3r} \\ \frac{-C_3-C_2}{4} \end{matrix} &
\begin{matrix} 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \end{matrix}
\end{bmatrix}}
\begin{bmatrix} d_{1x} & 0 \\ d_{1y} & 0 \\ d_{2x} & 0 \\ d_{2y} & 0 \\ d_{3x} & 0 \\ d_{3y} & 0 \\ d_{4x} & \underline{1} \\ d_{4y} & 0 \end{bmatrix}
$$

Second Last

$$\Sigma F_x = f_{1x}+f_{2x}+f_{3x}+f_{4x} = \cancel{-}\cancel{\frac{C_3}{3r}} + \cancel{\frac{C_1 r}{6}} \cancel{-\frac{(C_3+C_1r^2)}{6r}} + \cancel{\frac{C_3-2C_1r^2}{6r}} + \cancel{\frac{C_3+C_1r^2}{3r}}$$

$= 0$ (equalibrium

$$\Sigma F_y = f_{1y}+f_{2y}+f_{3y}+f_{4y} = \frac{C_2-C_3}{4} + \frac{C_2+C_3}{4} + \frac{C_3-C_2}{4} - \frac{C_3-C_2}{4} = 0$$ (equalibrium)

About Node 1

No deform

$$\circlearrowleft{+}\ \Sigma M = f_{4x}(L_y) + f_{3x}(L_y) - f_{3y}(L_x) - f_{2y}(L_x)$$

$$= \frac{C_3+C_1r^2}{3r}(L_y) + \frac{C_3-2C_1r^2}{6r}(L_y) - \frac{C_3-C_2}{4}(L_x) - \frac{C_2+C_3}{4}(L_x) \neq 0$$

$$= (L_y)\left(\frac{3C_3}{6r}\right) - (L_x)\left(\frac{2C_3}{4}\right)$$

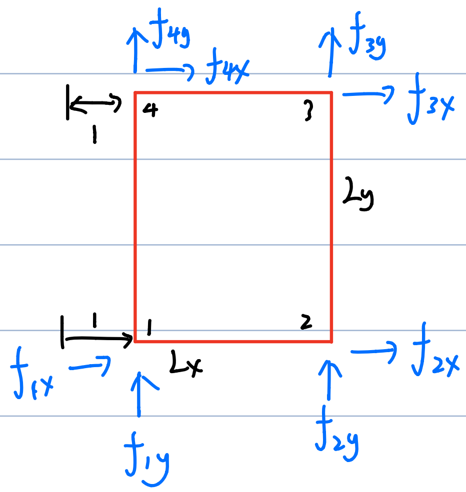
$$= (L_y)\left(\frac{C_3}{2r}\right) - (L_x)\left(\frac{C_3}{2}\right) \checkmark$$

Deform: $= \frac{G_1+C_1r^2}{3r}(L_y) + \frac{G_1-2C_1r^2}{6r}(L_y) - \frac{G_1-C_2}{4}(L_x) - \frac{C_2+G_1}{4}(L_x) + \frac{G_1+C_2}{4}$

$\uparrow f_{4y}(1)$

$= (L_y)\left(\frac{G_1}{2r}\right) - (L_x)\left(\frac{G_1}{2}\right) + \frac{G_1+C_2}{4}$

1b. The deformation is small.



1c.

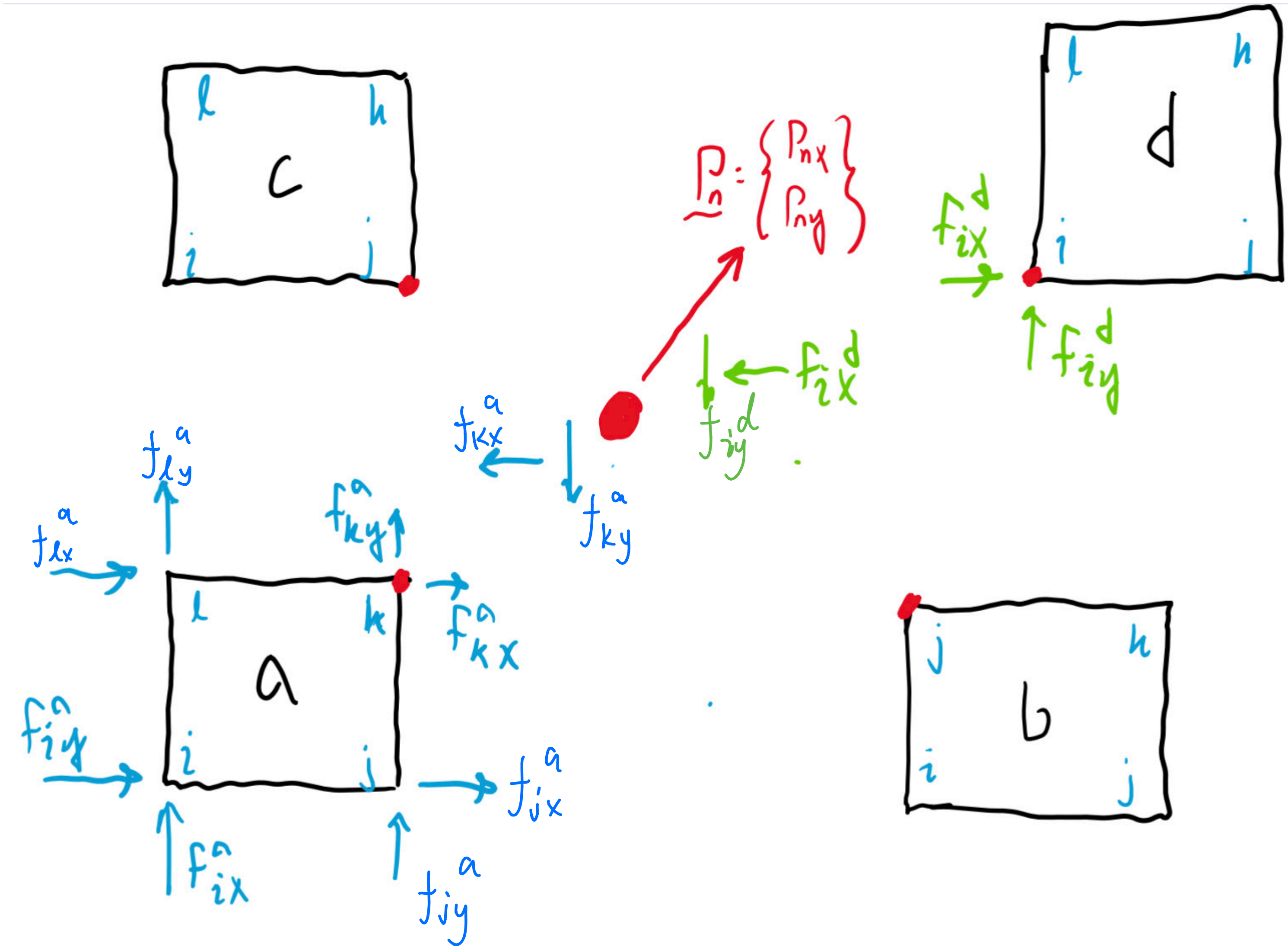
$$\begin{bmatrix} f_{1x} \\ f_{1y} \\ f_{2x} \\ f_{2y} \\ f_{3x} \\ f_{3y} \\ f_{4x} \\ f_{4y} \end{bmatrix} = \begin{bmatrix} \frac{G_1+C_1r^2}{3r} & \cdots 0 \cdots & -\frac{G_1}{3r}+\frac{C_1r}{6} & 0 \\ \frac{C_2+G_1}{4} & \cdots 0 \cdots & \frac{C_2-G_1}{4} & 0 \\ \frac{G_1-2C_1r^2}{6r} & \cdots 0 \cdots & \frac{-(G_1+C_1r^2)}{6r} & 0 \\ \frac{C_2-G_1}{4} & \cdots 0 \cdots & \frac{C_2+G_1}{4} & 0 \\ -\frac{G_1+C_1r^2}{6r} & \cdots 0 \cdots & \frac{G_1-2C_1r^2}{6r} & 0 \\ \frac{-C_2-G_1}{4} & \cdots 0 \cdots & \frac{G_1-C_2}{4} & 0 \\ -\frac{G_1}{3r}+\frac{C_1r}{6} & \cdots 0 \cdots & \frac{G_1+C_1r^2}{3r} & 0 \\ \frac{-C_2+G_1}{4} & \cdots 0 \cdots & \frac{-G_1-C_2}{4} & 0 \end{bmatrix} \begin{bmatrix} d_{1x} & 0 \\ d_{1y} & 0 \\ d_{2x} & 0 \\ d_{2y} & 0 \\ d_{3x} & 0 \\ d_{3y} & 0 \\ d_{4x} & 1 \\ d_{4y} & 0 \end{bmatrix}$$

$\uparrow$ First Row

$\uparrow$ Second Last Row

The vertical forces are not zeros because the element might expand (i.e. poisson Ratio) due to the force from a single direction.

1e



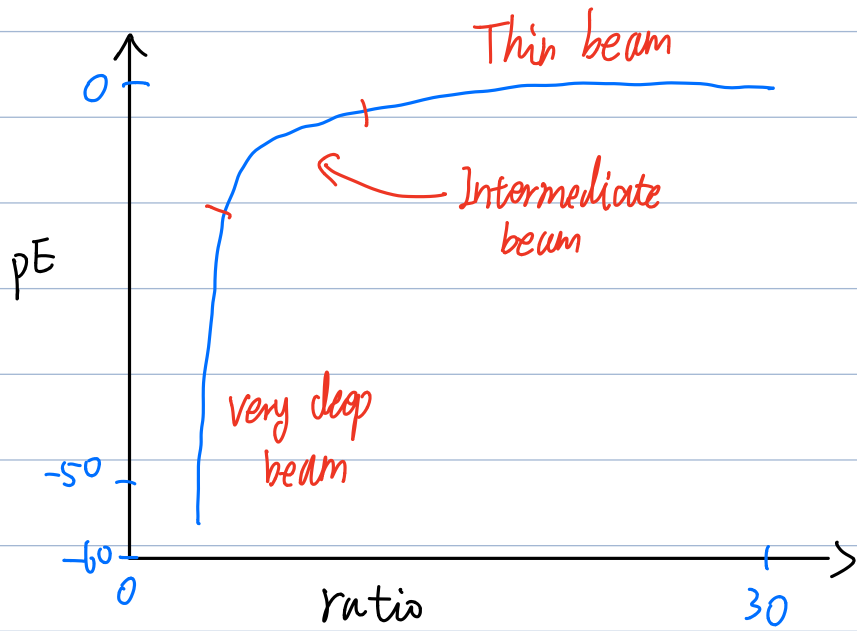
2a. It does not converge because the shape corner acts like a singularities and the mesh cannot be fine enough to make it converge.

2b. When make it rounded, the mesh is able to make it converge because It's not a singularity.

3a. When depth to spin ration approach to 1, the line became rasiddy curved. (Plot)

3b. Plot.

3c.



Plots on Matlab code