1b.

Assumption is that the material will follow the Linear elastic Law: $\sigma_x = E\varepsilon_x$, So area and E should be constant

1c.

$$EA\frac{d\bar{u}(x)}{dx}w(x)\Big|_{x=L} - EA\frac{d\bar{u}(x)}{dx}w(x)\Big|_{x=0} + \int_0^L b(x)w(x)\,dx = \int_0^L EA\frac{d\bar{u}(x)}{dx}\frac{dw(x)}{dx}dx$$

$$\Downarrow$$

$$EA\,v'\bar{v}\Big|_0^L + \int_0^L q\bar{v}\,dx = \int_0^L EA\,v'\bar{v}'dx$$

$$M\bar{v}'\Big|_0^L - V\bar{v}\Big|_0^L + \int_0^L q\bar{v}\,dx = \int_0^L EI\,v''\bar{v}''dx$$

$$EIv''\bar{v}'\Big|_0^L - EIv'''\bar{v}\Big|_0^L + \int_0^L q\bar{v}dx = \int_0^L EIv''\bar{v}''dx$$

$$\cancel{EIv''\bar{v}'\Big|_0^L} - EIv'''\bar{v}\Big|_0^L + \int_0^L q\bar{v}dx = EI\left(\cancel{v''\bar{v}'\Big|_0^L} - \int_0^L v'''\bar{v}'dx\right)$$

$$EIv'''\bar{v}\Big|_0^L - \int_0^L q\bar{v}dx = EI\int_0^L v'''\bar{v}'dx$$

$$\cancel{EIv'''\bar{v}\Big|_0^L} - \int_0^L q\bar{v}dx = \cancel{EI}\left(\cancel{v'''\bar{v}\Big|_0^L} - \int_0^L v''''\bar{v}dx\right)$$

$$\boxed{\int_0^L q\bar{v}dx = \int_0^L v''''\bar{v}dx}$$

1d.

$$M\bar{v}'\Big|_0^L - V\bar{v}\Big|_0^L + \int_0^L q\bar{v}\,dx = \int_0^L EI\,v''\bar{v}''dx$$

$$v = [N]\{d\} \qquad \bar{v} = [N]\{\bar{d}\}$$

$$M([N]'\{\bar{d}\})\Big|_0^L - V[N]\{\bar{d}\}\Big|_0^L + \int_0^L q[N]\{\bar{d}\}dx = \int_0^L EI([N]''\{d\})([N]''\{\bar{d}\})dx$$

$$\underbrace{M[N]^T\Big|_0^L - V[N]\Big|_0^L}\ + \underbrace{\int_0^L q[N]\,dx} = \int_0^L EI[B][d]''[B]\,dx$$

$\uparrow$

$[P]$ $\qquad$ $[P_{FEF}]$ $\qquad$ $[K][d]$

Add transpose for All.

$$\boxed{[\bar{d}]^T\left([K][d]-[P]-[P_{FEF}]\right)=0}$$

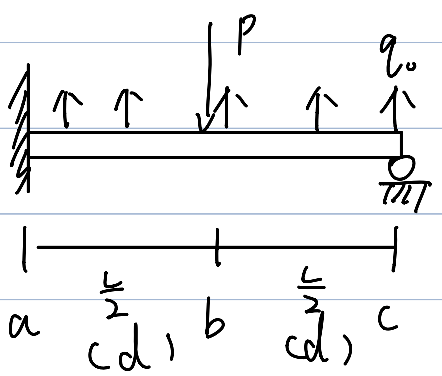
1f.

Matlab code shows the result is the same as the given matrix.

1g.

Matlab code shows the result is the same as the given matrix.

1h



$v(x) = v_i N_1(x) + \theta_i N_2(x) + v_j N_3(x) + \theta_j N_4(x)$

$N_1(x) = 1 - 3\left(\frac{x}{L}\right)^2 + 2\left(\frac{x}{L}\right)^3$

$N_2(x) = L\left[\left(\frac{x}{L}\right) - 2\left(\frac{x}{L}\right)^2 + \left(\frac{x}{L}\right)^3\right]$

$N_3(x) = 3\left(\frac{x}{L}\right)^2 - 2\left(\frac{x}{L}\right)^3$

$N_4(x) = L\left[\left(\frac{x}{L}\right)^3 - \left(\frac{x}{L}\right)^2\right]$

$$[K^e] = \int_{L^e} EI^e\,[B^e]^T[B^e]\,dx = EI\begin{bmatrix} 12/L^3 & 6/L^2 & -12/L^3 & 6/L^2 \\ 6/L^2 & 4/L & -6/L^2 & 2/L \\ -12/L^3 & -6/L^2 & 12/L^3 & -6/L^2 \\ 6/L^2 & 2/L & -6/L^2 & 4/L \end{bmatrix}$$

$d = \frac{L}{2}$

$$u = \begin{bmatrix} v_1 \\ \theta_1 \\ v_2 \\ \theta_2 \\ v_3 \\ \theta_3 \end{bmatrix} \qquad K = \begin{bmatrix} \frac{12}{d^3} & \frac{6}{d^2} & -\frac{12}{d^3} & \frac{6}{d^3} & 0 & 0 \\ \frac{6}{d^2} & \frac{4}{d} & -\frac{6}{d^2} & \frac{2}{d} & 0 & 0 \\ -\frac{12}{d^3} & -\frac{6}{d^2} & \frac{12}{d^3}+\frac{12}{d^3} & -\frac{6}{d^2}+\frac{6}{d^2} & -\frac{12}{d^3} & \frac{6}{d^3} \\ \frac{6}{d^2} & \frac{2}{d} & -\frac{6}{d^2}+\frac{6}{d^2} & \frac{4}{d}+\frac{4}{d} & -\frac{6}{d^2} & \frac{2}{d} \\ 0 & 0 & -\frac{12}{d^3} & -\frac{6}{d^2} & \frac{12}{d^3} & -\frac{6}{d^2} \\ 0 & 0 & \frac{6}{d^2} & \frac{2}{d} & -\frac{6}{d^2} & \frac{4}{d} \end{bmatrix}$$

$$P_{EFE} = \begin{bmatrix} \frac{q_0 L}{2} \\ \frac{q_0 L^2}{12} \\ \frac{q_0 L}{2} + \cancel{\frac{q_0 L}{2}} \\ \cancel{-\frac{q_0 L^2}{12} + \frac{q_0 L^2}{12}} \\ \frac{q_0 L}{2} \\ -\frac{q_0 L^2}{12} \end{bmatrix} \quad = 0$$

$$\boxed{[P] + [P_{EFE}] = [K][u]}$$

1 i

$$v(x) = v_i N_1(x) + \theta_i N_2(x) + v_j N_3(x) + \theta_j N_4(x)$$

We would need this equation to draw shear-moment diagram. App integral to this eqn to get curves of shear and Moment

2b.

$$SF: \quad KT'' + h = 0$$

$$\int_0^L (KT'' + h)\bar{T}\,dx = 0$$

$$\int_0^L KT''\bar{T}\,dx + \int_0^L h\bar{T}\,dx = 0$$

$$\boxed{KT'\bar{T}\Big|_0^L - \int_0^L KT'\bar{T}'\,dx + \int_0^L h\bar{T}\,dx = 0}$$

2C

$$K\hat{T}'\bar{T}\Big|_0^L - \int_0^L K\hat{T}'\bar{T}'\,dx + \int_0^L h\bar{T}\,dx = 0$$

$$\hat{T} = d_1 N_1 + d_2 N_2 + d_3 N_3 = [N][d] \qquad B = [N_1' \;\; N_2' \;\; N_3']$$

$$\bar{T} = \bar{d}_i N_i = [N][\bar{d}] \qquad \bar{T}' = [B][\bar{d}]$$

$$K[d_1 N_1' + d_2 N_2' + d_3 N_3'] N_1\Big|_0^L - \int_0^L K[d_1 N_1' + d_2 N_2' + d_3 N_3'] N_1'\,dx + \int_0^L h N_1\,dx = 0$$

$$\Downarrow$$

$$\boxed{\underbrace{K[B][d][N][\bar{d}]\Big|_0^L}_{[P]} - \underbrace{\int_0^L K[B][d][B][\bar{d}]\,dx}_{[d]} + \underbrace{\int_0^L h[N][\bar{d}]\,dx}_{[P_{EFE}]} = 0}$$

2d

[P] Might be the boundary Conditions(?)

2e (Similar to 1d)

$$[K] = \int_0^L K[B]^T[B]\,dx$$

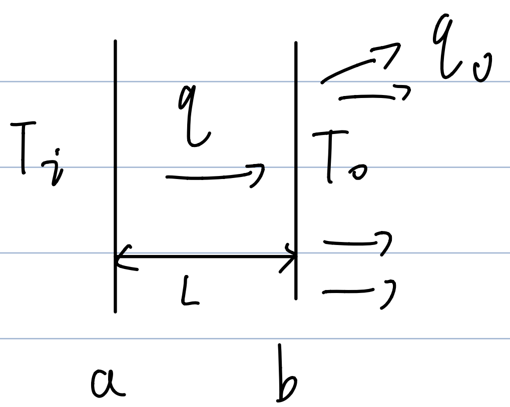
shape function

$$\begin{cases} N_1 = 1 - \frac{x}{L} \\ N_2 = \frac{x}{L} \end{cases}$$

$$[K] = K\int_0^L \begin{bmatrix} (1-\frac{x}{L})' \\ (\frac{x}{L})' \end{bmatrix} \left[(1-\frac{x}{L})' \quad (\frac{x}{L})'\right] dx$$

$$\boxed{K^e = K\begin{bmatrix} \int_0^L [(1-\frac{x}{L})']^2 dx & \int_0^L (1-\frac{x}{L})'(\frac{x}{L})' dx \\ \int_0^L (\frac{x}{L})'(1-\frac{x}{L})' dx & \int_0^L [(\frac{x}{L})']^2 dx \end{bmatrix}}$$

2f.



$$\boxed{\begin{aligned} q &= \frac{K}{L}T_i - \frac{K}{L}T_0 \\ q_0 &= \frac{K}{L}T_0 - \frac{K}{L}T_i \end{aligned}}$$

unknown

Solve for T_0

3b.

$$[\varepsilon] = [F_{3D}][\sigma]$$

$$\begin{bmatrix} \varepsilon_{xx} \\ \varepsilon_{yy} \\ \varepsilon_{zz} \\ \varepsilon_{yz} \\ \varepsilon_{zx} \\ \varepsilon_{xy} \end{bmatrix} = \frac{1}{E} \begin{bmatrix} 1 & -\nu & -\nu & 0 & 0 & 0 \\ -\nu & 1 & -\nu & 0 & 0 & 0 \\ -\nu & -\nu & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1+\nu & 0 & 0 \\ 0 & 0 & 0 & 0 & 1+\nu & 0 \\ 0 & 0 & 0 & 0 & 0 & 1+\nu \end{bmatrix} \begin{bmatrix} \sigma_{xx} \\ \sigma_{yy} \\ \sigma_{zz} \\ \sigma_{yz} \\ \sigma_{zx} \\ \sigma_{xy} \end{bmatrix}$$

$$\begin{bmatrix} \varepsilon_{xx} \\ \varepsilon_{yy} \\ \cancel{\varepsilon_{zz}} \\ \cancel{\varepsilon_{yz}} \\ \cancel{\varepsilon_{zx}} \\ \varepsilon_{xy} \end{bmatrix} = \frac{1}{E} \begin{bmatrix} 1 & -\nu & -\nu & 0 & 0 & 0 \\ -\nu & 1 & -\nu & 0 & 0 & 0 \\ -\nu & -\nu & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1+\nu & 0 & 0 \\ 0 & 0 & 0 & 0 & 1+\nu & 0 \\ 0 & 0 & 0 & 0 & 0 & 1+\nu \end{bmatrix} \begin{bmatrix} \sigma_{xx} \\ \sigma_{yy} \\ \cancel{\sigma_{zz}} \\ \cancel{\sigma_{yz}} \\ \cancel{\sigma_{zx}} \\ \sigma_{xy} \end{bmatrix} \begin{matrix} \\ \\ 0 \\ 0 \\ 0 \\ \\ \end{matrix}$$

$$\boxed{\begin{aligned} \begin{bmatrix} \varepsilon_{xx} \\ \varepsilon_{yy} \\ \varepsilon_{xy} \end{bmatrix} &= \frac{1}{E} \begin{bmatrix} 1 & -\nu & 0 \\ -\nu & 1 & 0 \\ 0 & 0 & 1+\nu \end{bmatrix} \begin{bmatrix} \sigma_{xx} \\ \sigma_{yy} \\ \sigma_{xy} \end{bmatrix} \\ \begin{bmatrix} \sigma_{xx} \\ \sigma_{yy} \\ \sigma_{xy} \end{bmatrix} &= \frac{E}{1-\nu^2} \begin{bmatrix} 1 & \nu & 0 \\ \nu & 1 & 0 \\ 0 & 0 & 1-\nu \end{bmatrix} \begin{bmatrix} \varepsilon_{xx} \\ \varepsilon_{yy} \\ \varepsilon_{xy} \end{bmatrix} \end{aligned}}$$

3c.

First Case: Pure 1D Linear, So $\boxed{\sigma = E\varepsilon}$

Second Case:

$$\begin{bmatrix} \sigma_{xx} \\ \sigma_{yy} \\ \sigma_{xy} \end{bmatrix} = \frac{E}{1-\nu^2} \begin{bmatrix} 1 & \nu & 0 \\ \nu & 1 & 0 \\ 0 & 0 & 1-\nu \end{bmatrix} \begin{bmatrix} \cancel{\varepsilon_{xx}} \\ \varepsilon_{yy} \\ \varepsilon_{xy} \end{bmatrix} \; 0$$

$$\sigma_{yy} = \frac{E}{1-\nu^2} \varepsilon_{yy} > E\varepsilon_{yy} \quad \text{since } 1-\nu^2 < 1$$

$$\sigma_{xx} = \frac{E}{1-\nu^2} \nu \varepsilon_{yy}$$

Third Case

$$\begin{bmatrix} \sigma_{xx} \\ \sigma_{yy} \\ \sigma_{zz} \\ \sigma_{yz} \\ \sigma_{zx} \\ \sigma_{xy} \end{bmatrix} = \frac{E}{(1+\nu)(1-2\nu)} \begin{bmatrix} 1-\nu & \nu & \nu & 0 & 0 & 0 \\ \nu & 1-\nu & \nu & 0 & 0 & 0 \\ \nu & \nu & 1-\nu & 0 & 0 & 0 \\ 0 & 0 & 0 & 1-2\nu & 0 & 0 \\ 0 & 0 & 0 & 0 & 1-2\nu & 0 \\ 0 & 0 & 0 & 0 & 0 & 1-2\nu \end{bmatrix} \begin{bmatrix} \cancel{\varepsilon_{xx}} \; 0 \\ \varepsilon_{yy} \\ \cancel{\varepsilon_{zz}} \; 0 \\ \varepsilon_{yz} \\ \varepsilon_{zx} \\ \varepsilon_{xy} \end{bmatrix}$$

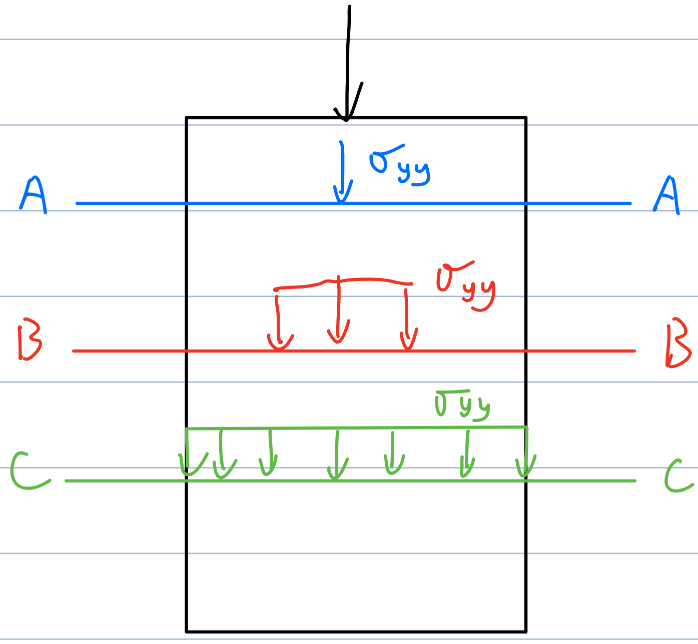
$$\sigma_{yy} = \frac{E}{(1+\nu)(1-2\nu)} (1-\nu) \varepsilon_{yy}$$

$$\sigma_{xx} = \nu \varepsilon_{yy}$$

$\sigma \quad \nu \varepsilon$

$\boxed{\sigma_{zz} = \nu\, \varepsilon_{yy}}$

4.



5.