(a) The Max Von-Mises stress is $312.5\,MPa < 615\,MPa$. No yield

P1 Max displacement is $\boxed{2.1\ mm}$ It's safe.

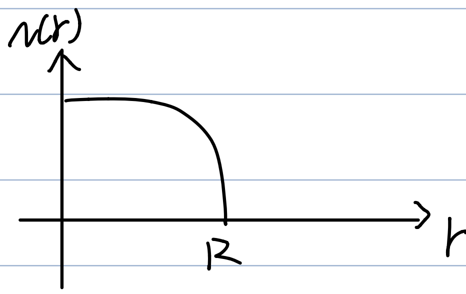
(b) Peak shear stress on the leg is $\boxed{138\ MPa}$

(c) $T = 125\,N\left(0.015 - \frac{0.005}{2} - \frac{0.025}{2}\right) = 7.5\,N\cdot m$

$$T = 2\int_A \phi\, dA \implies 7.5 = 2\int_A \phi_0 N(x,y)\, dA$$

$$7.5 = 2\phi_0 \int_0^R \int_0^{2\pi} N(r)\, r\, d\theta\, dr$$

$$\phi_0 = \frac{3.75}{\int_0^R \int_0^{2\pi} N(r) r\, d\theta\, dr}$$




Guess: $N(r) = 1 - \left(\frac{r}{R}\right)^2 \implies \phi_0 = \frac{3.75}{\int_0^R \int_0^{2\pi} \left(1 - \left(\frac{r}{R}\right)^2\right) r\, d\theta\, dr}$

$\phi_0 = \frac{7.5}{\pi R^2} \leftarrow R = 5.5\,mm$

$\phi_0 = 315679.23$

$\phi(r) = \phi_0 \cdot N(r) \implies \phi(r) = \phi_0\left(1 - \frac{r^2}{R^2}\right)$

In polar: $x = r\cos\theta \quad y = r\sin\theta$

$\phi(x,y) = \phi_0\left(1 - \frac{x^2+y^2}{R^2}\right) = \phi_0 - \frac{\phi_0 x^2}{R^2} + \frac{\phi_0 y^2}{R^2}$

$\frac{d\phi}{dx} = 6yz \quad \frac{d\phi}{dy} = 6xz$

$$\begin{cases} \frac{\partial}{\partial x}\phi(x,y) = \tau_{yz}(x,y) = -\frac{2\phi_0 x}{R^2} \\ \frac{\partial}{\partial y}\phi(x,y) = \tau_{xz}(x,y) = \frac{-2\phi_0 y}{R^2} \end{cases}$$

$R = \frac{55}{2}, \implies \boxed{\tau_{yz} = -10101735232\,0X}$

$\boxed{\tau_{xz} = -10101735 2320\,y}$

(d) The hand-calculation is more precise compare to the FEM, but we are using a circle to approach, it might not be that good.

P2

$$B_e = \begin{bmatrix} \frac{\partial N}{\partial x} & 0 & 0 \\ 0 & \frac{\partial N}{\partial y} & 0 \\ 0 & 0 & \frac{\partial N}{\partial z} \\ \frac{\partial N}{\partial y} & \frac{\partial N}{\partial x} & 0 \\ 0 & \frac{\partial N}{\partial z} & \frac{\partial N}{\partial y} \\ \frac{\partial N}{\partial z} & 0 & \frac{\partial N}{\partial x} \end{bmatrix}$$

$$\frac{\partial N}{\partial x} = -\frac{1}{L_x} \qquad \frac{\partial N}{\partial y} = -\frac{1}{L_y} \qquad \frac{\partial N}{\partial z} = -\frac{1}{L_z}$$

$$N(x,y,z) = \left(1-\frac{x}{L_x}\right)\left(1-\frac{y}{L_y}\right)\left(1-\frac{z}{L_z}\right)$$

$$K_{11} = \frac{E}{(V+1)}\int_0^{L_z}\int_0^{L_y}\int_0^{L_x} \frac{V-1}{2V-1}\left(\frac{\partial N}{\partial x}\right)^2 + \left(\frac{\partial N}{\partial y}\right)^2 + \left(\frac{\partial N}{\partial z}\right)^2 dx\,dy\,dz$$

$$K_{11} = \frac{E}{(V+1)}\int_0^{L_z}\int_0^{L_y}\int_0^{L_x} \frac{V-1}{2V-1}\frac{1}{L_x^2} + \frac{1}{L_y^2} + \frac{1}{L_z^2}\, dx\,dy\,dz$$

$$K_{11} = \frac{E}{V+1}\, xyz\left(\frac{V-1}{2V-1}\frac{1}{L_x^2} + \frac{1}{L_y^2} + \frac{1}{L_z^2}\right)_0^{L_x,\,L_y,\,L_z}$$

$$K_{11} = \frac{E L_x L_y L_z}{V+1}\left[\frac{V-1}{2V-1}\frac{1}{L_x^2} + \frac{1}{L_y^2} + \frac{1}{L_z^2}\right]$$

$$\boxed{K_{11} = \frac{E}{V+1}\left[\frac{V-1}{2V-1}\frac{L_y L_z}{L_x} + \frac{L_x L_z}{L_y} + \frac{L_x L_y}{L_z}\right]}$$

Kirchoff-Love Plate theory

$$K\nabla^4 w = q(x,y) = q_0$$

$$\text{BCs:} \begin{cases} w(x=0)=0 \quad w''(x=0)=0 & ① \\ w(x=L_x)=0 & ② \\ \frac{\partial w}{\partial x}\left(x=\frac{L_x}{2}\right)=0 & ③ \\ \frac{\partial w}{\partial y}(x=0,\ x=L_x)=0 \end{cases}$$

$$w''''=q_0, \quad w'''=q_0x+C_1, \quad w''=\frac{q_0x^2}{2}+C_1x+C_2$$

$$w'=\frac{q_0x^3}{6}+\frac{C_1x^2}{2}+C_2x+C_3$$

$$w=\frac{q_0x^4}{24}+\frac{C_1x^3}{6}+\frac{C_2x^2}{2}+C_3x+C_4$$

From ① ; $C_2=0$ , $C_4=0$

$$\left.\begin{aligned} &\text{From ②,} \quad \frac{q_0L_x^4}{24}+\frac{C_1L_x^3}{6}+\frac{C_2L_x^2}{2}+C_3L_x=0 \\ &\text{From ③,} \quad \frac{q_0L_x^3}{6(2^3)}+\frac{C_1L_x^2}{2(2^2)}+C_2\frac{L_x}{2}+C_3=0 \end{aligned}\right\}$$

$$C_1=-\frac{q_0L_x}{2}$$

$$0=\frac{q_0L_x^4}{24}+\frac{-q_0L_x^4}{12}+C_3L_x, \quad C_3=\frac{q_0L_x^3}{24}$$

$$\text{So,} \quad w=\frac{\left(\frac{q_0x^4}{24}+\frac{-q_0L_xx^3}{12}+\frac{q_0L_x^3}{24}x\right)}{D}$$

Center: $L_x = 2$ center $x = 1$

$$W = \frac{\left(\frac{500}{24} + \frac{-500(2)}{12} + \frac{500(2^3)(1)}{24}\right)}{\frac{2(29000)}{3(1-0.4^2)}} = \boxed{0.0045}$$

Beam Theory.

$$EI\,w'''' = \frac{q(x)}{EI}, \quad w'''' = \frac{q(x)}{EI}, \quad w''' = \frac{q_0 x + C_1}{EI}$$

$$w'' = \frac{q_0 x^2}{2} + C_1 x + C_2, \quad w' = \frac{q_0 x^3}{6} + \frac{C_1 x^2}{2} + C_2 x + C_3$$

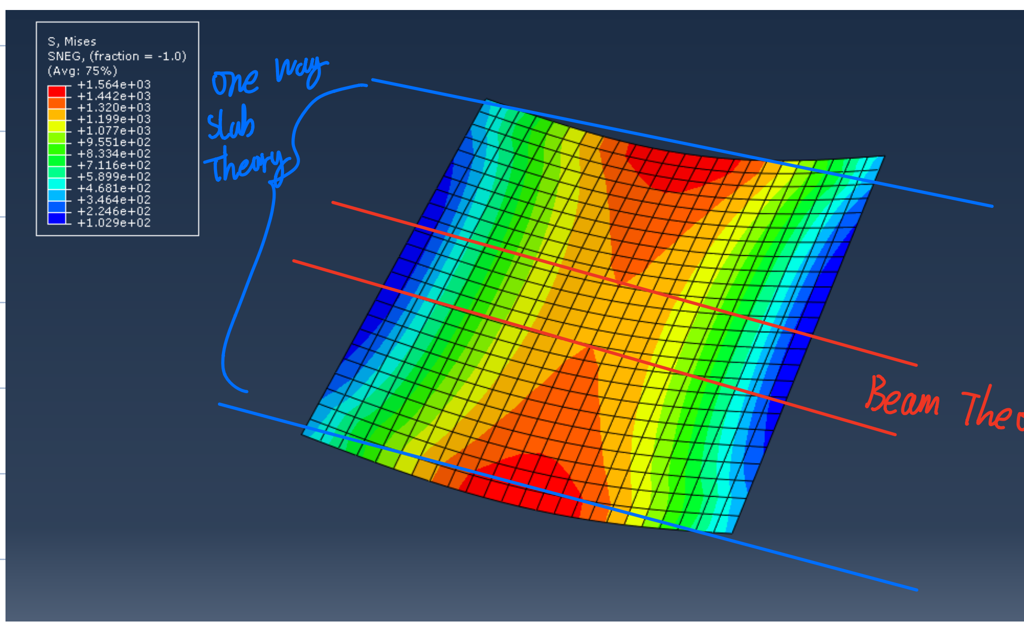
$$w = \frac{q_0 x^4}{24} + \frac{C_1 x^3}{6} + \frac{C_2 x^2}{2} + C_3 x + C_4,$$ BCs are the same

$C_1$, $C_2$ $C_3$ $C_4$ are the same

$$W = \frac{\left(\frac{q_0 x^4}{24} - \frac{q_0 L_x x^3}{12} + \frac{q_0 L_x^3}{24} x\right)}{EI}$$

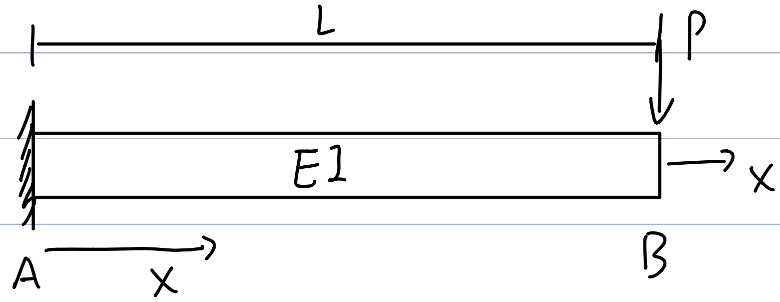
$$W = \frac{\frac{500(1)}{24} - \frac{500(2)}{12} + \frac{500(2)}{24}}{29000\,\frac{24(1)^3}{12}} = \frac{270.833}{58000} = \boxed{0.0046}$$

b



P4

ⓐ




$$M(x) = (L-x)P \quad \cdots\cdots ①$$

$$EIv'' = M(x)$$

$$EIv'' = (L-x)P \quad \cdots\cdots ②$$

$$\int EIv''\,dx = \int (L-x)P\,dx$$

$$\Downarrow$$

$$EIv' = \left(Lx - \frac{x^2}{2}\right)P + C_1 \quad \cdots\cdots ③$$

$$\int \Downarrow \int$$

$$EIv = \left(-\frac{x^3}{6} + \frac{Lx^2}{2}\right)P + C_1 x + C_2 \quad \cdots\cdots ④$$

$$\left.\begin{aligned} v'(0) &= 0 \\ v(0) &= 0 \end{aligned}\right\} BCs$$

From ③ $EIv' = \left(Lx - \frac{x^2}{2}\right)P + C_1 \Rightarrow C_1 = \frac{L^2P}{2}$

From ④ $EIv = \left(-\frac{x^3}{6} + \frac{Lx^2}{2}\right)P + C_1x + C_2 \Rightarrow C_2 = -\frac{L^3P}{6}$

$$EIv = \frac{1}{6}(L-x)^3P + \frac{L^2P}{2}x - \frac{L^3P}{6} \Rightarrow \boxed{v = \frac{P}{6EI}(3L-x)x^2}$$

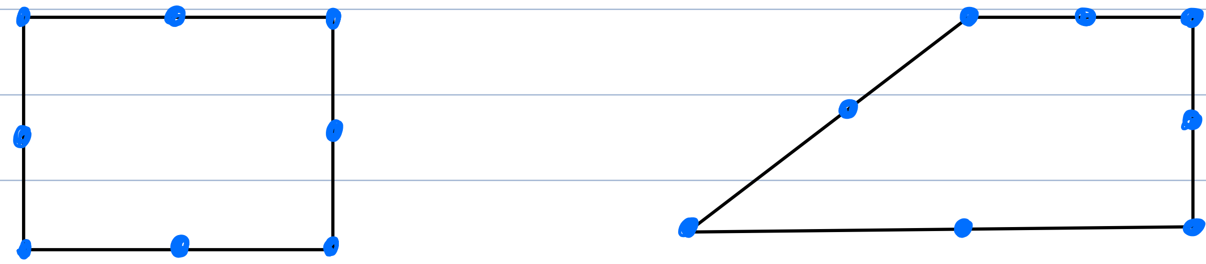
$$EIv' = -\frac{1}{2}(L-x)^2P + \frac{L^2P}{2} \Rightarrow \boxed{v' = \frac{P}{2EI}(2L-x)x}$$

$$v = \frac{100}{6(10^7)83.33}(3(100)-100)100^2 = \boxed{0.04}$$

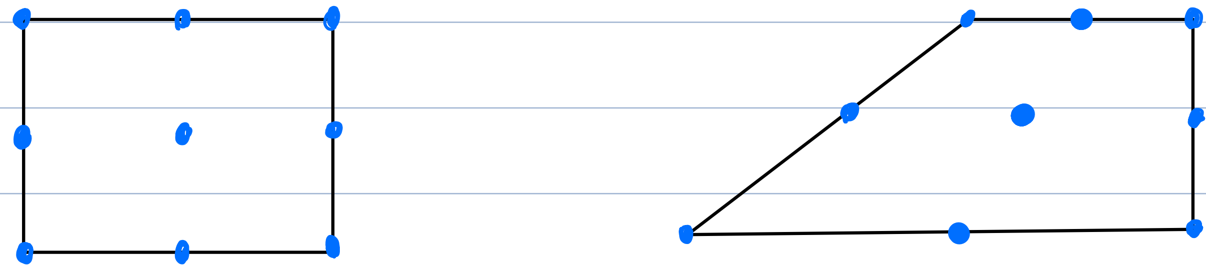
$$I = \frac{1\,(10)^3}{12} = 83.33$$

(b)

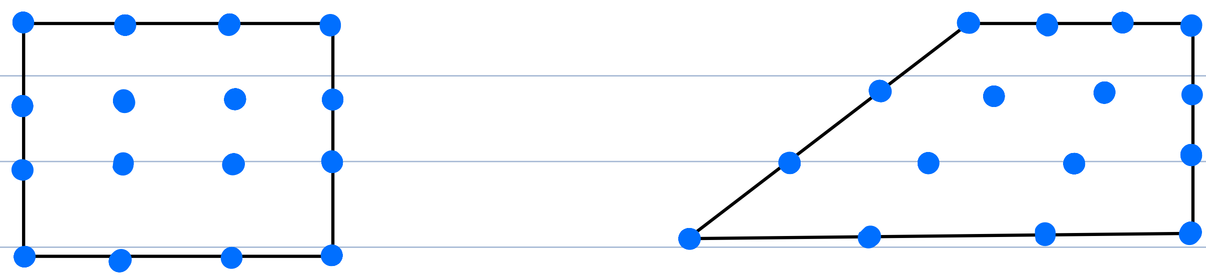
8-nodes



9-nodes



16-nodes



(c) Serendipity elements are rectangular elements with no interior nodes (8-nodes)

Lagrangian elements are those with interior nodes. (9-nodes, 16 nodes)