P1

ⓐ $y' e^y \cosh x + e^y \sinh x = 0$

$$\frac{dy}{dx} e^y \cosh x + e^y \sinh x = 0$$

$$\underbrace{(e^y \cosh x)}_{N} dy + \underbrace{(e^y \sinh x)}_{M} dx = 0$$

$$\frac{\partial M}{\partial y} = e^y \sinh x \qquad \frac{\partial N}{\partial x} = e^y \sinh x \qquad (\text{exact})$$

$$M = \frac{\partial u}{\partial x}, \quad N = \frac{\partial u}{\partial y} \Rightarrow M dx + N dy = du = 0$$

$$u = \int M dx + k(y) = e^y \int \sinh x \, dx + k(y) = e^y \cosh x + k(y)$$

$$\frac{\partial u}{\partial y} = e^y \cosh x + k'(y) = N \Rightarrow k'(y) = 0$$

$$\boxed{u = e^y \cosh x = C}$$

ⓑ $y' \sec y + (1+2x) \cos y = 0$

$$\frac{dy}{dx} \sec y + (1+2x) \cos y = 0$$

$$\underbrace{(\sec y)}_{Q} dy + \underbrace{(1+2x) \cos y}_{P} \, dx = 0$$

$$\frac{\partial P}{\partial y} = -(1+2x) \sin y, \quad \frac{\partial Q}{\partial x} = 0 \qquad (\text{Not exact})$$

$F = F(x)$ $\quad R = \frac{1}{Q}\left(\frac{\partial P}{\partial y} - \frac{\partial Q}{\partial x}\right) = \frac{1}{\sec y}\left(-(1+2x)\sin y\right)$ No good, (x,y)

$F = F(y)$ $\quad R = \frac{1}{P}\left(\frac{\partial Q}{\partial x} - \frac{\partial P}{\partial y}\right) = \frac{1}{(1+2x)\cos y}\left(0 + (1+2x)\sin y\right) = \underline{\tan y}$ only y good

$$F(y) = \exp\int \tan y\, dy = \exp(-\ln|\cos y|) = e^{-\ln|\cos y|} = \frac{1}{\cos y}$$

$$FP\,dx + FQ\,dy = (1+2x)\,dx + \left(\frac{1}{\cos^2 y}\right) dy$$

$$u = \int (1+2x)\,dx + k(y) = x + x^2 + k(y)$$

$$\frac{\partial u}{\partial y} = k'(y) = FQ = \frac{1}{\cos^2 y}$$

$$k = \int \frac{1}{\cos^2 y}\, dy = \tan y$$

$$\boxed{u = x + x^2 + \tan y = C}$$

ⓒ $x^2 + y^2 - 2xyy' = 0$

$$x^2 + y^2 - 2xy\frac{dy}{dx} = 0$$

Let $u = \frac{y}{x}$, $y = ux$, $y' = \frac{x^2+y^2}{2xy} = \frac{x^2}{2xy} + \frac{y^2}{2xy} = \frac{x}{2y} + \frac{y}{2x}$

$y' = u'x + u$

$$u'x + u = \frac{1}{2u} + \frac{u}{2}$$

$$\frac{du}{dx}x + u = \frac{1}{2u} + \frac{u}{2}$$

$$\frac{1}{dx}x = \frac{1}{du}\left(\frac{1}{2u} - \frac{u^2}{2u}\right)$$

$$\frac{1}{x}dx = du\,\frac{2u}{1-u^2}$$

$$\int \frac{1}{x}dx = \int \frac{2u}{1-u^2}\,du$$

$$\ln x = -\ln(1-u^2) + c$$

$$x = \frac{1}{1-u^2}c$$

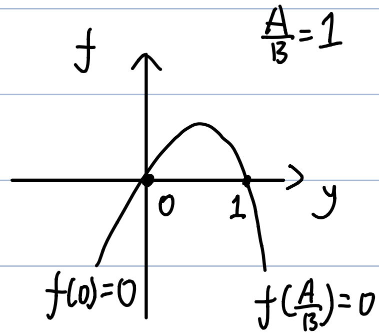
$$x = \frac{c}{1-\frac{y^2}{x^2}} = \frac{cx^2}{x^2-y^2} \Longrightarrow \boxed{x^2 - y^2 = \hat{c}x}$$

P2

$y' = ky(1-y)$. $k > 0$

$y' = \underbrace{ky}_{A} - \underbrace{ky^2}_{B}$

$\boxed{\text{Eqn Solution : } y=0 \text{ or } y=1}$



$\boxed{\begin{array}{ll} y=0 & \text{Unstable} \\ y=1 & \text{Stable.} \end{array}}$