ME 564 Homework 3

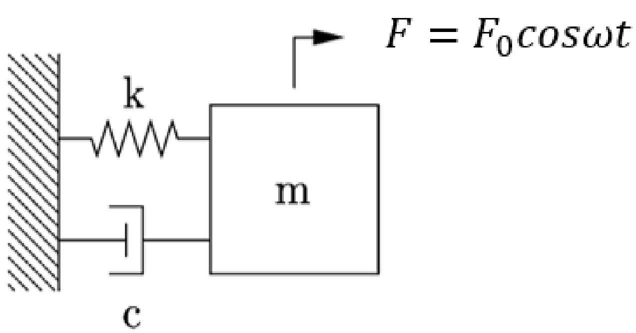
Due date: 10/29/2020

Problem 1: Find a general solution by hand calculation.

1) $x^2y'' - 2xy' + 2y = x^3 sinx$

2) $y'' + 6y' + 8y = 40cos2t$

Problem 2: Forced motion of an ideal damped mass-spring system.



Equation of motion of the mass m by Newton's second law: $F = ma$

$$m\frac{d^2y}{dt^2} = -ky - c\frac{dy}{dt} + F_0 cos\omega t$$

where k is the spring rate, c is the damping constant, y is the displacement of the mass measured from the position at which the spring is unstretched, and F is the driving force.

$m = 10\ kg, k = 90N/m, F_0 = 10N$. The initial displacement of the mass is $y(0) = 0\ m$, and the initial velocity of the mass is $y'(0) = 1\ m/s$. The resonance frequency of the undamped mass-spring system is $\omega_0 = \sqrt{\frac{k}{m}}$.

Write a Matlab code that you can quickly modify the constants and show the displacement and velocity of the mass between $0 < t < 50s$ for the following cases, comparing two sets of solutions using matrix exponential and ode45 respectively.

For this problem, you need to submit 1) a written summary briefly explaining the method you used to solve this problem, 2) your Matlab code and 3) all the output plots for the following 5 cases.

1) $c = 0\ kg/s$, $\omega = 0.9\omega_0$ (Undamped system, beats.)
2) $c = 0\ kg/s$, $\omega = \omega_0$ (Undamped system, resonance.)
3) $c = 10\ kg/s$, $\omega = 0.5\omega_0$ (Underdamping, forced oscillation.)
4) $c = 60\ kg/s$, $\omega = 0.5\omega_0$ (Critical damping, forced oscillation.)
5) $c = 100\ kg/s$, $\omega = 0.5\omega_0$ (Overdamping, forced oscillation.)