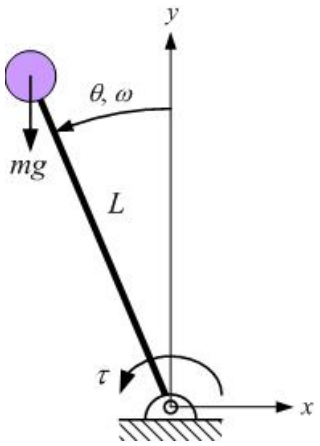
# ME 564 Homework 4

Due date: 11/11/2020

**Problem 1: Using power series to solve the initial value problem. (40 points)**

$$(x^2+1)y'' + 2xy' = 0, y(0) = 0, y'(0) = 1$$

**Problem 2: Inverted pendulum**




The equation of motion for a free pendulum ($\tau = 0$) is:

$$mL^2\theta'' = mgLsin\theta$$

where $mL^2$ is the mass moment of inertial of the system, $\theta''$is the angular acceleration of the pendulum, and $mgLsin\theta$ is the torque that drives the angular acceleration.

If $\frac{g}{L} = 1$, the equation of motion of the pendulum becomes:

$$\theta'' = sin\theta$$

1) For ODE $\theta'' = sin\theta$, is $\theta = 0, \omega = \theta' = 0$ a critical point in the phase plane? If yes, what type of critical point is it? Show all your calculation. (20 points)
2) If a feedback torque $\tau = -2\theta - 1\omega$ is applied at the end of the bar, the equation of motion becomes $\theta'' = sin\theta - 2\theta - 1\omega$. Is $\theta = 0,\ \omega = 0$ a critical point? If yes, what type of critical point is it? Is it stable or unstable? Show all your calculation. (30 points)
3) For the inverted pendulum with feedback torque applied same as on question 2), using ode45 to solve the I.V.P. with initial conditions $\theta(0) = 0, \omega(0) = 1$ and plot the trajectory in the phase plane for $0 < t < 20s$. (Attach the plot with your pdf and upload your code.) (10 points)