ME 564 Homework 5

Due: 11/30/2020

Problem 1 (20 points): Using Laplace transform to solve the system of equations

$$y_1' + y_2 = 0, y_1 + y_2' = 2cost$$

$$y_1(0) = 1, y_2(0) = 0$$

Problem 2: The Lorenz system is a system of ordinary differential equations. It has chaotic solutions for certain parameter values and initial conditions.

$$x' = \sigma(y - x)$$

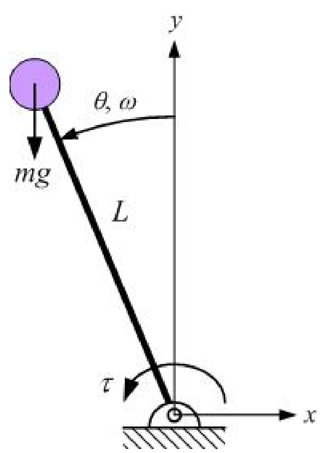
$$y' = x(\rho - z) - y$$

$$z' = xy - \beta z$$

For $\sigma = 10, \beta = 8/3, \rho$=28 (Chaotic parameters)

a) Find the critical points of the system. (15 points)
b) For each critical point, determine the local stability by linearization. You may use Matlab to calculate the eigenvalues. (15 points)

Problem 3: Inverted pendulum



The equation of motion for a free pendulum ($\tau = 0$) is:

$$mL^2\theta'' = mgLsin\theta$$

If $\frac{g}{L} = 1$, the equation of motion of the pendulum becomes:

$$\theta'' = sin\theta$$

If a feedback torque $\tau = -\alpha\theta - \beta\theta'$ is applied at the end of the bar, the equation of motion becomes

$$\theta'' = sin\theta - \alpha\theta - \beta\theta'.$$

a) (10 points) Convert the second order ODE into two first-order ODEs by letting $y_1 = \theta, y_2 = \theta'$

b) (30 points) For the two cases 1) $\alpha = 0, \beta = 0$, and 2) $\alpha = 3, \beta = 1$, solve the equation for t=0:0.1:20 for initial condition $\theta(0) = \pi/2, \theta'(0) = 1$ using fourth-order Runge-Kutta with fixed step size and using ode45. Plot the trajectory for each of the two cases on the phase plane overlaying the solutions of the two methods. You will want to create a Matlab .m file called pendulum.m with the following inputs and outputs:

function dy=pendulum(t, y, α, β)

c) (10 points) For the two cases in part b), by linearization, determine the stability of the critical point at (y_1, y_2)= $(\pi, 0)$ for case 1, and at (y_1, y_2)= (0,0) for case 2. You may use Matlab to calculate the eigenvalues.

Please also turn in your Matlab codes.