P1

$$y_1' + y_2 = 0 \qquad y_1(0) = 1$$

$$y_1 + y_2' = 2\cos t \qquad y_2(0) = 0$$

$$\mathcal{L} \Downarrow$$

$$\begin{cases} sY_1 - y_1(0) + Y_2 = 0 \\ Y_1 + sY_2 - y_2(0) = \frac{2s}{s^2+1} \end{cases} \Rightarrow \begin{cases} sY_1 - 1 + Y_2 = 0 \\ Y_1 + sY_2 = \frac{2s}{s^2+1} \end{cases}$$

$$Y_2 = 1 - sY_1$$

$$Y_1 + s - s^2 Y_1 = \frac{2s}{s^2+1} \Rightarrow \boxed{Y_1 = \frac{s}{s^2+1}} \xRightarrow{\mathcal{L}^{-1}} \boxed{y_1 = \cos t}$$

$$\frac{s^2}{s^2+1} - 1 + Y_2 = 0$$

$$s^2 - s^2 - 1 + Y_2(s^2+1) = 0$$

$$Y_2(s^2+1) = 1$$

$$\boxed{Y_2 = \frac{1}{s^2+1}} \xRightarrow{\mathcal{L}^{-1}} \boxed{y_2 = \sin t}$$

P2

$$x' = \sigma(y - x) \qquad \sigma = 10$$

$$y' = x(\rho - z) - y \qquad \beta = \frac{8}{3}$$

$$z' = xy - \beta z \qquad \rho = 28$$

(a) critical Points: $f(x) = 0$, $\begin{cases} \sigma(y-x) = 0 \\ x(\rho - z) - y = 0 \\ xy - \beta z = 0 \end{cases} \Rightarrow \begin{cases} x = y \\ x(\rho - z) = y \\ xy = \beta z \end{cases}$

$$x^2 = \beta(27) = \frac{216}{3} \Rightarrow x = y = \pm\sqrt{\frac{216}{3}} = \pm\sqrt{72} = \pm 6\sqrt{2}$$

$$\boxed{\text{Critical Points:} \quad ① \begin{cases} x=0 \\ y=0 \\ z=0 \end{cases} \quad ② \begin{cases} 6\sqrt{2} \\ 6\sqrt{2} \\ 27 \end{cases} \quad ③ \begin{cases} -6\sqrt{2} \\ -6\sqrt{2} \\ 27 \end{cases}}$$

ⓑ for $[0, 0, 0]$

$$\frac{\partial f}{\partial x} = \begin{bmatrix} \frac{\partial f_1}{\partial x} & \frac{\partial f_1}{\partial y} & \frac{\partial f_1}{\partial z} \\ \frac{\partial f_2}{\partial x} & \frac{\partial f_2}{\partial y} & \frac{\partial f_2}{\partial z} \\ \frac{\partial f_3}{\partial x} & \frac{\partial f_3}{\partial y} & \frac{\partial f_3}{\partial z} \end{bmatrix} = \begin{bmatrix} -10 & 10 & 0 \\ 28 & -1 & 0 \\ 0 & 0 & -\frac{8}{3} \end{bmatrix} = A$$

Matlab $\boxed{eig(A) = \begin{bmatrix} -22.83 \\ 11.83 \\ -2.67 \end{bmatrix} \quad \text{Unstable, Saddle point}}$

for $[6\sqrt{2}, 6\sqrt{2}, 27]$

$$\frac{\partial f}{\partial x} = \begin{bmatrix} -10 & 10 & 0 \\ 28 & -1 & -6\sqrt{2} \\ 6\sqrt{2} & 6\sqrt{2} & -\frac{8}{3} \end{bmatrix} = A \quad \boxed{eig(A) = \begin{bmatrix} -22.56 \\ 4.45 + 3.49i \\ 4.45 - 3.49i \end{bmatrix} \text{ unstable.}}$$

for $[-6\sqrt{2}, -6\sqrt{2}, 28]$

$$\frac{\partial f}{\partial x} = \begin{bmatrix} -10 & 10 & 0 \\ 28 & -1 & 6\sqrt{2} \\ -6\sqrt{2} & -6\sqrt{2} & -\frac{8}{3} \end{bmatrix} = A$$

$$\boxed{eig(A) = \begin{bmatrix} -22.56 \\ 4.45+3.49i \\ 4.45-3.49i \end{bmatrix} \text{ unstable.}}$$

P3

$$\theta'' = \sin\theta - \alpha\theta - \beta\theta'$$

(a) $\theta = y_1$, $\theta' = y_2$

$$\boxed{\begin{cases} y_1' = y_2 \\ y_2' = \sin y_1 - \alpha y_1 - \beta y_2 \end{cases}}$$

(b) In Matlab

(c) ① $\alpha = 0$, $\beta = 0$, $y_1 = \theta = \pi$, $y_2 = \theta' = 0$

$$\begin{bmatrix} \frac{\partial f_1}{y_1} & \frac{\partial f_1}{y_2} \\ \frac{\partial f_2}{y_1} & \frac{\partial f_2}{y_2} \end{bmatrix} = \begin{bmatrix} 0 & 1 \\ \cos y_1 - \alpha & -\beta \end{bmatrix} \Rightarrow \begin{bmatrix} 0 & 1 \\ -1 & 0 \end{bmatrix} = A$$

$$eig(A) = \begin{bmatrix} +i \\ -i \end{bmatrix} \quad \boxed{\text{unstable.}}$$

② $\alpha = 3$, $\beta = 1$ $y_1 = 0$ $y_2 = 0$

$$A = \begin{bmatrix} 0 & 1 \\ -3 & -1 \end{bmatrix}, \quad eig(A) = \begin{bmatrix} -0.5+1.66i \\ -0.5-1.66i \end{bmatrix} \quad \boxed{\text{Stable}}$$