

### Solution of Problem 3

We can modify Dijkstra's algorithm to solve this problem.

Firstly, create a priority queue in the form of min-heap, named,  $Q$ .

Now, create a vector "dist" which stores minimum bandwidths to reach each switching center from  $a$ . Initialise all entries in "dist" to  $INT\_MAX$  (i.e.,  $\infty$ ).

Enqueue  $a$  into  $Q$  with priority 0.

While  $Q$  is not empty:

- Dequeue a switching center  $v$  from  $Q$  with minimum priority (i.e., <sup>element at</sup> top of min-heap).
- If  $v$  is  $b$ , we can stop the algorithm and return the minimum bandwidth stored in  $dist[v]$  as max bandwidth ~~from~~ of path from  $a$  to  $b$ .
- For each neighbour  $w$  of  $v$ , calculate minimum bandwidth <sup>from  $a$  to  $v$  as min of</sup> of the edge <sup>bandwidth</sup> between  $v$  and  $w$  and min bandwidth stored in  $dist[v]$ .
- If this minimum bandwidth is less than minimum bandwidth stored in  $dist[w]$ , update  $dist[w]$  with this new minimum bandwidth and  $w$  into  $Q$  with priority  $new$ .

If the algo reaches this point, it means there is no path from  $a$  to  $b$ .

## Solution of Problem 4

Initialise an empty set  $R$  to store transitive reduction edges.

For each vertex  $u$  in  $V$ , do the following:

- Perform DFS starting from  $u$ , visiting all reachable vertices.
- During DFS for each ~~reach~~ visited vertex  $v$ , mark  $v$  as visited.
- While perform DFS, if we encounter an edge  $(u, v)$  where  $u$  is current vertex and  $v$  is a visited vertex, remove the edge  $(u, v)$  from  $E$ .
- Add edge  $(u, v)$  to the set  $R$ .

Create a new graph  $G'$  with same vertices as  $G$  and edges in  $R$ .

Time Complexity:

Of DFS  $\rightarrow O(|V| + |E|)$

~~So, since~~ Since, we have performed DFS for  $V$  vertices,

T.C of algorithm =  $O(|V| \times (|V| + |E|))$