A. Implement Newton-Raphson method to find a root of any differentiable single variable function provided with its first derivative. Function prototype is as follows:

double NewtonRaphson(double (*)(double), double (*)(double), double, double);

Solve for roots of the following functions using your implementation:

a.
$$f(x) = \exp(x) - x^2$$

b.
$$f(x) = -2x - \ln(x)$$

c.
$$f(x) = \cos^2 x + x$$

B. Use finite difference method to calculate the Hessian matrix for any function of 2 double variables For a function f(x, y), its Hessian matrix is

$$\begin{bmatrix} \frac{\partial^2 f}{\partial x^2} & \frac{\partial^2 f}{\partial x \partial y} \\ \frac{\partial^2 f}{\partial x \partial y} & \frac{\partial^2 f}{\partial y^2} \end{bmatrix}$$

Finite difference approximation for derivatives:

•
$$\frac{\partial^2 f}{\partial x^2} = \frac{f(x+\delta,y)+f(x-\delta,y)-2f(x,y)}{\delta^2}$$

•
$$\frac{\partial^2 f}{\partial x^2} = \frac{f(x+\delta,y)+f(x-\delta,y)-2f(x,y)}{\delta^2}$$
•
$$\frac{\partial^2 f}{\partial y^2} = \frac{f(x,y+\delta)+f(x,y-\delta)-2f(x,y)}{\delta^2}$$

•
$$\frac{\partial^2 f}{\partial x \partial y} = \frac{f(x+\delta,y+\delta) + f(x-\delta,y-\delta) - f(x+\delta,y-\delta) - f(x-\delta,y+\delta)}{4\delta^2}$$

Print out Hessian for

•
$$f_1(x,y) = x^2 + y^2 + 6x^2y^2$$

• $f_2(x,y) = xy(x-y)$

•
$$f_2(x, y) = xy(x - y)$$

at points (0,0), (1,1)