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Y=log(1+X$_t{}^2$)= f(x),x=X$_t{}^2$

1. $f^{'}(x) = \frac{2x}{1+x^2}$

$f^{''}(x) = \frac{2(1+x^2)-2x\times 2x}{(1+x^2)^2}$

$$df = f_t dt + f_x dX_t + \tfrac{1}{2} f_{xx} dX_t dX_t$$
$$= 0 + \tfrac{2x}{1+x^2} dX_t + \tfrac{1}{2}\tfrac{2(1+x^2)-2x\times 2x}{(1+x^2)^2} W(t)^4 dt$$
$$= \tfrac{2x}{1+x^2} dX_t + \tfrac{1-x^2}{(1+x^2)^2} W(t)^4 dt$$
$$= \tfrac{2x}{1+x^2} (W(t)dt + W(t)^2 dW(t)) + \tfrac{1-x^2}{(1+x^2)^2} W(t)^4 dt$$
$$= (\tfrac{2x}{1+x^2} W(t) + \tfrac{1-x^2}{(1+x^2)^2} W(t)^4) dt + (\tfrac{2x}{1+x^2} W(t)^2) dW(t)$$

Integrate it:

$$Y = = \int_0^{\mathrm{t}} (\tfrac{2x}{1+x^2} W(t) + \tfrac{1-x^2}{(1+x^2)^2} W(t)^4) dt + \int_0^t (\tfrac{2x}{1+x^2} W(t)^2) dW(t)$$

2.

Y=(1+t)X$_t$=f

$$df = f_t dt + f_x dX_t + \tfrac{1}{2} f_{xx} dX_t dX_t$$
$$= X_{\mathrm{t}} dt + (1+t) dX_t + 0$$
$$= (1+t)(-\tfrac{1}{1+t} X_t dt + \tfrac{1}{1+t} dW(t)) + X_{\mathrm{t}} dt$$
$$= (-X_t dt + dW(t)) + X_{\mathrm{t}} dt$$
$$= dW(t)$$

So dY= dW(t)

Since X(0)=0,→Y(0)=0 -->W(0)=0;

Therefore,W(t) and Y are both Brownian Motion