

1. Draw PDAs for the following languages:

(a) $\{1^n 0^{n-3} \mid n \geq 3\}$

(b) $\{1^n 0^{2n} \mid n \geq 1\}$

2. Suppose that you have context-free languages L_1 and L_2 . Show that $L_1 L_2$ is a context-free language. [Hint: Recall the formal definition of a context-free grammar and use proof by construction that show that from grammars G_1 and G_2 , we can get G_3 that recognizes $L_1 L_2$]
3. Suppose that I have a 8×2 board that I want to tile with 1×2 dominoes. I can rotate two dominoes horizontally and place them on the board, or place one domino vertically. A valid tiling is one where we cover the area of the board without going over.

Suppose that I represent vertical placement as V and horizontal placement as H . If I let $\Sigma = \{V, H\}$, I can represent a tiling, read left to right, as a string. For example, $HVVHH$ is a valid tiling (see figure below), while $HHHHHHHH$ is not a valid tiling nor is HV .

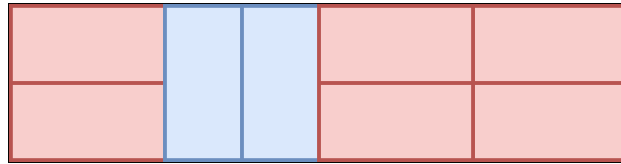


Figure 1: Valid Tiling of 8×2 board

- (a) Draw an NFA for the language $\{w \mid w \in \{V, H\}^*, w \text{ is a valid tiling of an } 8 \times 2 \text{ board}\}$
- (b) Suppose that I add the additional constraint that the tiling must be symmetric. In other words, let $L_2 = \{w \mid w \in \{V, H\}^*, w \text{ is a valid tiling of an } 8 \times 2 \text{ board}, w = w^R\}$. Draw a PDA for L_2