

1. Prove the following:

- (a) Prove that  $(A - B) \cup (B - A) = (A \cup B) \cap \overline{(A \cap B)}$ . Recall that  $A - B = A \cap \overline{(A \cup B)}$
- (b) Prove that  $A \times (B \cap C) = (A \times B) \cap (A \times C)$
- (c) Suppose that  $a, b, c \in \mathbb{Z}$ . If  $a^2 + b^2 = c^2$ , then  $a$  and  $b$  are even.
- (d) If  $n$  is odd, then  $n^2$  is odd
- (e) Suppose  $a, b \in \mathbb{Z}$ , then  $a^2 - 4b - 2 \neq 0$

2. Suppose that  $\Sigma = \{0, 1\}$ . Describe the following languages in words:

- (a)  $\{00x \mid x \in \Sigma^2\}$
- (b)  $\{0x1 \mid x \in \Sigma^*\}$
- (c)  $\{0^n 1^n \mid n \in \mathbb{N}\}$
- (d)  $\{0^n 1^m \mid n + m = 2^k \text{ where } n, m, k \in \mathbb{N}\}$
- (e)  $\{x \mid x \in \Sigma^* \wedge |x| \text{ is divisible by } 2\}$

3. For the languages in 2., state which of these languages contain the empty string  $\epsilon$